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Energy Efficiency Maximization for Hybrid TDMA-NOMA System with Opportunistic Time Assignment

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***Abstract*—In this paper, we consider an energy efficient resource allocation technique for a hybrid time division multiple access (TDMA) - non-orthogonal multiple access (NOMA) system. In such a hybrid system, the available time for transmission is divided into several sub-time slots, and a sub-time slot is allocated to serve a group of users (i.e., cluster). Furthermore, signals for the users in each cluster are transmitted based on the NOMA approach. With NOMA, multiple users can be served simultaneously through utilizing power domain multiplexing at transmitter and successive interference cancellation (SIC) at receiver. In this paper, to maximize the energy efficiency (EE), we jointly allocate both the available time slots and the available transmit power in the hybrid TDMA-NOMA system. In particular, we formulate an EE maximization (EE-Max) problem aiming to maximize the overall EE of the system with a per-user minimum rate and transmit power constraints. However, this joint optimization problem is non-convex in nature, and thus, cannot be solved directly. Therefore, we develop an iterative algorithm by approximating the original problem into a convex one with sequential convex approximation (SCA) and a novel second-order cone (SOC) approach. Simulation results demonstrate that the performance of the proposed hybrid TDMA-NOMA system with joint resource allocation outperforms the system with equal time allocation in terms of the overall EE. Simulation results further confirm that the proposed iterative approaches with SCA and SOC techniques converge within a few number of iterations while yielding the solution to the original non-convex problem.**

***Index Terms*—Non-orthogonal multiple access (NOMA), time division multiple access (TDMA), hybrid TDMA-NOMA, energy efficiency, convex optimization.**

I. Introduction

Non-orthogonal multiple access (NOMA) has been envisioned as a promising multiple access technique to significantly improve the spectral efficiency, user fairness and support massive connectivity for the fifth generation (5G) and beyond wireless networks [1]. Different from the conventional orthogonal multiple access (OMA) technologies, such as time division multiple access (TDMA) and orthogonal frequency division multiple access (OFDMA), multiple users in downlink NOMA simultaneously share the same radio resources, namely time and frequency resources [2], [3]. At the transmitter, this simultaneous resource sharing is carried out by exploiting power domain multiplexing, which is referred to as a power-domain superposition coding (SC) technique in the literature [5], [6]. With this multiplexing technique, multiple signals intended for the corresponding users are encoded with different power levels and transmitted simultaneously. The successive interference cancellation (SIC) technique is employed at the receiver end to decode the signals transmitted to multiple users. In the SIC technique, the signals with stronger channel conditions are first decoded and subtracted from the multiplexed received signal prior to decoding their own signals [4], [5], [7]. Furthermore, NOMA can accommodate much more users than OMA by employing non-orthogonal resource allocation, which addresses the dramatically increasing demand for user access required for the Internet-of-Things in future wireless networks [1], [2], [11].

The computational complexity of employing SIC in dense networks grows exponentially with the number of users, while the errors in successive decoding propagate and thus degrade the decoding performance [1]. To address these SIC issues and to exploit additional degrees of freedom, NOMA has been recently combined with a wide range of multiple access techniques. These include multiple-antenna [9], [10], [32], [35] and conventional OMA techniques [18], [20]–[23]. In these combined systems, power domain multiplexing is utilized along with the other existing spatial and orthogonal domains multiplexing to meet the demanding massive connectivity requirements. In particular, these hybrid systems not only exploit different multiplexing domains to enhance the performance, but also facilitate practical implementation of NOMA in dense networks [13], [26]. In hybrid OMA-NOMA systems, the available OMA resources (i.e., time or frequency) are divided into several sub-resource blocks, where each sub-resource block corresponds to a set of multiple users via NOMA technique [14], [26]. Due to the massive connectivity offered by the hybrid OMA-NOMA systems [17], the corresponding high power consumption, however, has become one of the major issues that needs to be carefully addressed.

The power consumption of wireless networks is one of the dominant factors that has a considerable impact in the design

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of future wireless communications systems. On one hand, the excessive power consumption will cause an uncontrolled increase of CO_2 emission levels [24], [27], [28], which raises different environmental issues including global and warming natural disasters [25]. On the other hand, the accelerated growth of the power consumption will be inherently reflected on the overall costs of the wireless communication systems. This, as a result, will impose additional financial pressures on the service providers and consumers. Therefore, several energy efficient approaches have been investigated to limit the unprecedented growth in the power consumption. These solutions include energy harvesting techniques [14], employing green energy resources [25], and efficiently utilizing the available power resources with the energy efficiency (EE) performance metric [28]–[30]. The overall EE of the system, also referred as global EE (GEE), is defined as the ratio between the achieved sum-rate and the corresponding total power consumption [34]. The EE of a communication system with unit of bit-per-joule can be defined as the number of bits transferred per joule of energy consumption in the system [33]. Based on this definition, GEE can be considered as a multi-objective performance metric which simultaneously takes into account the two conflicting performance metrics, namely the sum-rate and the corresponding power consumption [31], [34].

Different EE designs have been considered for NOMA transmissions, which include multiple-input single-output (MISO)-NOMA and multiple-input multiple-output (MIMO)-NOMA EE-based design. For example, in [37], two algorithms are proposed for solving an EE maximization problem with the downlink beamforming design for the MISO-NOMA system. In [35], the EE design is investigated in a multi-cluster multi-user MIMO-NOMA system with pre-defined quality-of-service (QoS) requirements. In addition, some research works for hybrid OMA-NOMA systems investigate different resource allocation techniques [14], [18], [20]. For example, in a hybrid OFDMA-NOMA system, an EE maximization problem is considered in [18], in which the subchannel assignment and the power allocation algorithms were proposed for the system. In [20], the max-min sum of downlink and uplink transmit rates among all users joint resource allocation problem of OFDMA-NOMA system is investigated. Specifically, two scenarios are considered, perfect channel state information (CSI) estimation and imperfect CSI estimation. Then, an asymptotically optimal algorithm and a suboptimal algorithm are proposed. The energy harvesting capabilities of a hybrid TDMA-NOMA system are explored in [14], in which several users are divided into different groups (i.e., clusters) and each cluster is assigned to the equal time slot for transmission, aiming to minimize the transmit power under minimum rate and minimum energy harvesting requirements at each user. In particular, most of the works in the literature assume equal time allocations between the clusters which has several drawbacks. For example, it degrades the overall performance of the system as different clusters have users with diverse channel conditions. In particular, with such equal time allocation, both clusters with stronger users and weaker users will be assigned with the same time slot, which degrades the overall performance.

To the best of our knowledge, previous works in the literature have not considered a joint power and time resource allocation for a hybrid TDMA-NOMA system, especially with the GEE performance metric. Therefore, in this paper, we address this issue by developing a power allocation technique with opportunistic time assignment for a hybrid TDMA-NOMA system. In particular, we formulate a joint GEE maximization (GEE-Max) design for a downlink transmission of hybrid TDMA-NOMA system. In particular, the overall EE of a hybrid TDMA-NOMA system is maximized subject to pre-defined users' minimum rate requirements and the power budget constraint at the base station (BS). We develop an optimization framework that allocates the available transmit power at the BS between the users and opportunistically assigns the available time for transmission between the clusters. However, the formulated GEE-Max optimization problem is non-convex in nature, and cannot be solved directly using available standard optimization software. Hence, we propose an iterative approach for solving the original GEE-Max problem with a novel second-order cone (SOC) approach and approximations. The main contributions of this work are summarized as follows:

- Due to the several drawbacks associated with the equal time allocations, we propose an opportunistic time allocation technique along with power allocations for a hybrid TDMA-NOMA system. In particular, we develop a GEE-Max framework to jointly design both the power levels for users and time slot allocations for the clusters.
- Prior to solving the formulated GEE-Max problem, a feasibility check is carried out as the GEE-Max problem might turn out to be infeasible due to some constraints. Next, we propose two iterative algorithms to solve the feasible GEE-Max problem. In the first algorithm, a novel SOC formulation along with sequential convex approximations (SCA) [36] is utilized to solve the problem. In the second algorithm, we employ the Dinkelbach's algorithm to determine the solution of the original GEE-Max problem.
- We provide simulation results to demonstrate the superior performance of the proposed GEE-Max design with opportunistic time allocations. Furthermore, simulation results confirm that the proposed novel SOC approach with the iterative SIC not only provides the solution to the original GEE-Max optimization problem, but also converges within a few number of iterations.

The remainder of the paper is organized as follows. Section II introduces the model of a hybrid TDMA-NOMA system and formulates the GEE-Max problem. In Section III, two iterative algorithms, namely SCA and Dinkelbach's algorithms are proposed to solve the original GEE-Max problem. Section IV provides simulation results to validate the performance of the developed iterative approaches. Finally, Section V concludes the paper.

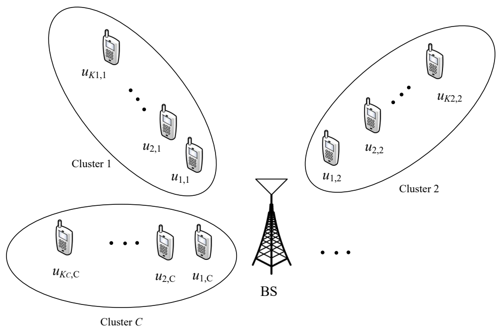

Fig. 1: A hybrid TDMA-NOMA multi-user SISO system.

II. System Model And Problem Formulation

A. System Model

We consider a downlink transmission of a hybrid TDMA-NOMA multi-user single-input single-output (SISO) system. In this hybrid system, a single-antenna BS communicates with K single-antenna users, as shown in Fig. 1. As such, the total number of users is K, which are grouped into C clusters with a time-slot $t_i, \forall i = 1, 2, ..., C$, per cluster. Furthermore, $u_{j,i}$ represents the jth user in the ith cluster. As shown in Fig. 2, we denote the available time for transmission by T. The number of users in the ith cluster, L_i, is denoted by K_i, $\forall i \in \mathcal{C} \triangleq \{1, 2, ..., C\}$, satisfying $K = \sum_{i=1}^{C} K_i$. We assume that signals are transmitted over a quasi-static flat Rayleigh fading channel, where the channel coefficients remain constant over each transmission block but vary independently between different blocks.

The power assigned for $u_{j,i}$ is denoted as $p_{j,i}^2$, and thus, define the total transmit power at the BS by P_t, such that $P_t = \sum_{i=1}^{C} \sum_{j=1}^{K_i} p_{j,i}^2$. The maximum transmit power available at the BS is P^{max}; then, the total transmit power constraint can be mathematically expressed into the following constraint:

$$P_t = \sum_{i=1}^{C} \sum_{j=1}^{K_i} p_{j,i}^2 \leq P^{max}. \quad (1)$$

As the power-domain NOMA technique is applied among the users in the each cluster, the symbol transmitted from the BS during t_i can be written as

$$x_i = \sum_{j=1}^{K_i} p_{j,i} x_{j,i}, \quad (2)$$

where $x_{j,i}$ is the message intended to $u_{j,i}$. Accordingly, at user $u_{j,i}$, the received signal is given by

$$r_{j,i} = h_{j,i} x_i + n_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i \triangleq \{1, 2, ..., K_i\}, \quad (3)$$

where $h_{j,i}$ denotes the Rayleigh fading channel coefficient between the BS and the $u_{j,i}$, and $n_{j,i} \sim \mathcal{CN}(0, \sigma_{j,i}^2)$ denotes the additive white Gaussian noise (AWGN) at receiver. Note that it is assumed that perfect CSI is available at the BS. The corresponding channel gain is defined as $|h_{j,i}|^2 = \frac{\beta}{(d_{j,i}/d_0)^{\kappa}}$

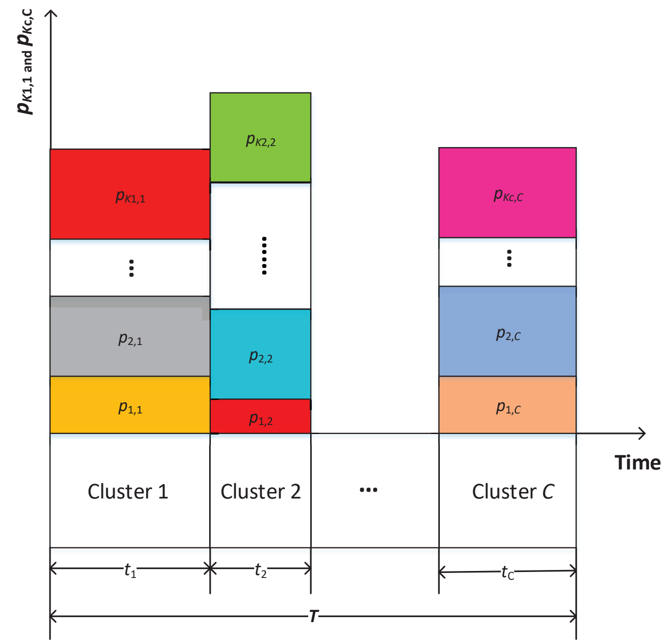

Fig. 2: A time-slot is assigned to serve each cluster, while the users in each cluster communicate with the BS based on the power-domain NOMA.

[16], where $d_{j,i}$ and d_0 are the distances between $u_{j,i}$ and the BS, and a reference distance, respectively. Furthermore, denote the signal attenuation at the reference distance, d_0, by β and the path loss exponent by κ. Without loss of generality, the channel gains for the users at each cluster are assumed to be ordered as

$$|h_{1,i}|^2 \geq |h_{2,i}|^2 \geq ...|h_{K_i,i}|^2, \forall i \in \mathcal{C}. \quad (4)$$

Accordingly, the SIC process is implemented at stronger users, i.e., users with higher channel strengths. In particular, the user $u_{j,i}$ aims to cancel the interference from any other weaker users from $u_{j+1,i}$ to $u_{K_i,i}$ using SIC. It is assumed that SIC is implemented perfectly without any errors. Therefore, the signal-to-interference and noise ratio (SINR) at $u_{j,i}$ to decode the message of weaker users $u_{d,i}, \forall d \in \{j+1, j+2, ..., K_i\}$ can be defined as

$$SINR_{j,i}^{d} = \frac{|h_{j,i}|^2 p_{d,i}^2}{|h_{j,i}|^2 \sum_{s=1}^{d-1} p_{s,i}^2 + \sigma_{j,i}^2},$$
$$\forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}. \quad (5)$$

To successful perform the SIC process, the received SINR levels of the users with weaker channel strengths should be more than the users with stronger channel strengths [7], [8]. This requirement can be only satisfied by imposing the following constraint on the power allocation [38]:

$$p_{K,i}^2 \geq p_{K-1,i}^2 \geq ... \geq p_{1,i}^2, \forall i \in \mathcal{C}. \quad (6)$$

The necessary power constraints for efficient SIC can be given by (6), which is referred to as the SIC constraint throughout this paper; this has been widely adopted in several NOMA downlink transmissions [26], [37]. Thus, the received signal at $u_{j,i}$ after employing SIC can be expressed as

$$r_{j,i}^{SIC} = h_{j,i}p_{j,i}x_{j,i} + h_{j,i}\sum_{s=1}^{j-1} p_{s,i}x_{s,i} + n_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{7}$$

Therefore, the SINR of $u_{j,i}$ can be given by [14], [15]

$$SINR_{j,i} = \min\{SINR_{j,i}^1, SINR_{j,i}^2, ..., SINR_{j,i}^j\}, \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{8}$$

Then, the achieved rate $R_{j,i}$ at $u_{j,i}$ can be given by

$$R_{j,i} = Bt_i \log_2(1 + SINR_{j,i}), \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{9}$$

where B is the available bandwidth of the channel. For notational simplicity, we assume that $B = 1$ throughout this paper. It is worthy to note that grouping (i.e., clustering) strategy plays a crucial role in the performance of the TDMA-NOMA system. Therefore, the proposed user grouping strategy will be discussed later in this paper.

B. Power Consumption Model

The total power consumption at the BS can be defined as [18]

$$P_{total} = \frac{1}{\epsilon}P_t + P_{loss}, \tag{10}$$

where $\epsilon \in [0, 1]$ denotes the efficiency of the power amplifier. Furthermore, P_{loss} represents the total power losses, and it can be expressed as [18], [19]

$$P_{loss} = P_{dyn} + P_{sta}, \tag{11}$$

where P_{dyn} is the dynamic power consumption [34] and P_{sta} is the static power consumption required to maintain the system.

As we have highlighted in the previous section, the GEE can be defined as the ratio between the achieved sum-rate and the corresponding power consumption [18], [35]. Considering the hybrid TDMA-NOMA system, the overall EE, i.e., GEE, can be given by

$$GEE = \frac{\sum_{i=1}^{C}\sum_{j=1}^{K_i} R_{j,i}}{P_{total}}. \tag{12}$$

C. Problem Formulation

By taking into account the importance of energy efficient resource allocation, we develop an GEE-Max optimization design for the hybrid TDMA-NOMA system. In particular, we aim to maximize the overall system EE, i.e., GEE, while satisfying a set of relevant constraints.

Note that the GEE function presented in (12) is fractional in nature; as such, maximizing GEE can be viewed as a joint optimization of the achieved sum-rate (maximization) and the corresponding power consumption (minimization). In this paper, a GEE-Max design is proposed to maximize EE under a set of constraints, including QoS requirements, as well as total time and power constraints at the BS. With these constraints, the GEE-Max optimization problem for the TDMA-NOMA system can be formulated as follows:

$$\textbf{(P1)}: \quad \max_{p_{j,i}, t_i} \quad \frac{\sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i \log_2(1 + SINR_{j,i})}{\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}} \tag{13}$$

$$\text{s.t.} \quad \sum_{i=1}^{C} t_i \leq T, \tag{14}$$

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 \leq P^{max}, \tag{15}$$

$$p_{K,i}^2 \geq p_{K-1,i}^2 \geq ... \geq p_{1,i}^2, \forall i \in \mathcal{C}, \tag{16}$$

$$R_{j,i} \geq \bar{R}_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{17}$$

where $\bar{R}_{j,i}$ is the minimum rate requirement associated with the QoS constraint for user $u_{j,i}$, and the constraint in (14) ensures the maximum time constraint. Furthermore, the constraint in (16) is necessary for the successful application of the SIC technique [37].

In particular, there are several challenges associated with solving P1, which are summarized in the following discussion. Firstly, unlike equal time allocation considered in previous works in the literature including [14], the joint allocations of the time and power resources introduce additional complexity to solve the problem and evaluate the corresponding design parameters. Secondly, as it can be seen in P1, the objective function is not only non-convex, but also fractional. This fractional non-convex nature of the objective function introduces additional complexity in solving the optimization problem. Thirdly, due to the constraints in (16) and (17), the optimization problem P1 might turn out to be infeasible when the minimum rate requirements cannot be achieved with the available power budget at the BS. Hence, determining the solution for P1 should take all these issues into account. Note that this optimization problem is significantly different from those solved in other related work, such as in [26], in terms of both objective function and constraints. Therefore, we provide a comprehensive algorithm to solve this problem in the following section. Before presenting the detailed steps of the proposed algorithm, we first provide a method in the following subsection to validate the feasibility of the problem.

D. Feasibility Analysis of the GEE-Max Problem

Now, we analyse the feasibility issues of the GEE-Max problem P1. Towards this end, we first shed some light on the total time constraint in (14) with the following Lemma:

Lemma 1. *The condition $\sum_{i=1}^{C} t_i = T$ is necessary for achieving the maximum EE for the optimization problem P1.*

Proof: This lemma is proven by contradiction. First, we assume that $\sum_{i=1}^{C} t_i = T$ does not hold for an optimal solution $\mathbf{T}^* = [t_1^*, t_2^*, ..., t_C^*]$, that is, $\sum_{i=1}^{C} t_i < T$. Now, we construct a new solution $\mathbf{T}_{new} = [t_1^*, t_2^* + \left(T^* - \sum_{i=1}^{C} t_i^*\right), t_3^*, ..., t_C^*]$ [12]. Obviously, this new solution still satisfies the time allocation constraint given in (14). Additionally, cluster L_2 has a larger throughput than that of solution $\mathbf{T}^*$ since $R_{j,i}(\mathbf{T}, p_{j,i})$ is a strictly monotonically increasing function with respect to t_i, and clusters $L_1, L_3, ..., L_C$ achieve the same throughputs as that obtained with $\mathbf{T}^*$. Therefore, $\mathbf{T}_{new}$ yields better throughputs than $\mathbf{T}^*$. This contradicts the initial assumption that $\mathbf{T}^*$ is optimal and therefore $\sum_{i=1}^{C} t_i = T$ should be satisfied. This completes the proof of Lemma 1.

Based on Lemma 1, the time allocation constraint is transformed to the following equality constraint to reduce the feasible region of the original problem P1:

$$\sum_{i=1}^{C} t_i = T. \tag{18}$$

Provided a solution set $\{p_{j,i}{}^*, t_i{}^*\}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i$ is feasible, then, the minimum rate constraint in (17) is automatically fulfilled. Obviously, P1 turns out to be infeasible if the corresponding minimum rate constraints cannot be met with the total power constraint in (15). Therefore, we examine this infeasibility issue through evaluating the required minimum power to satisfy the corresponding QoS constraints, as follows:

$$\textbf{(P-Min)}: \quad P^{min} = \min_{p_{j,i}, t_i} \quad \sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 \tag{19}$$

$$\text{s.t.} \quad \sum_{i=1}^{C} t_i = T, \tag{20}$$

$$p_{K,i}^2 \geq p_{K-1,i}^2 \geq ... \geq p_{1,i}^2, \; \forall i \in \mathcal{C}, \tag{21}$$

$$SINR_{j,i} \geq 2^{\frac{\bar{R}_{j,i}}{t_i}} - 1, \; \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{22}$$

where P^{min} is the minimum total transmit power that is required to meet the user data rate requirements. We can claim that the BS has insufficient power budget to achieve the user data rate requirements when $P^{min} > P^{max}$. Under this condition, the optimization problem P1 is classified as an infeasible problem. To handle this infeasibility issue, we consider an alternative resource allocation technique, namely sum-rate maximization (SR-Max) problem. With this technique, we investigate the maximum achievable sum-rate under available power and SIC constraints. This problem can be formulated as

$$\textbf{(SR-Max)}: \quad \max_{p_{j,i}, t_i} \quad \sum_{i=1}^{C}\sum_{j=1}^{K_i} R_{j,i} \tag{23}$$

$$\text{s.t.} \quad \sum_{i=1}^{C} t_i = T, \tag{24}$$

$$p_{K,i}^2 \geq p_{K-1,i}^2 \geq ... \geq p_{1,i}^2, \forall i \in \mathcal{C}, \tag{25}$$

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 \leq P^{max}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{26}$$

The solution of the optimization problem SR-Max can be accessed using the SCA technique. In the following section, we develop two algorithms to solve the original GEE-Max problem P1.

III. Proposed Methodology

In this section, we provide two algorithms to solve the non-convex optimization problem P1, which are based on the SCA technique and Dinkelbach's algorithm (DA). Note that the solution of the original GEE-Max problem P1 depends on the users that are selected for each cluster. Hence, it is necessary to determine the grouping strategy in the considered hybrid TDMA-NOMA system.

Obviously, the optimal user grouping sets can be determined through an exhaustive search or combinatorial search algorithms among all possible user sets. However, this exhaustive search is impractical due to its computational complexity. Furthermore, there are several factors that should be considered when choosing a clustering algorithm, which are summarized in the following discussion. Firstly, grouping users should consider the objective function of the original design problem. In particular, for a given hybrid TDMA-NOMA system, the user clusters for the SR-Max design should be different from those of the GEE-Max design. Secondly, it has been proven in the literature that SIC is successfully implemented with relatively small error when the gap between channel strengths of the users is as high as possible [39]. This imposes that users with diverse channel strengths should be grouped into the same cluster. Considering the above key facts, and similar to the grouping strategy proposed in [26] and [45], we employ a clustering algorithm based on the difference between the channel strengths of the users. In particular, users with higher channel strengths' gaps are grouped into the same cluster. Clusters with only two users have been considered due to practical implementation challenges, including high computational complexity and potential propagations in SIC. With this restriction, the grouping sets of the hybrid TDMA-NOMA can be presented as

$$(\{u_{1,1}, u_{2,1}\}, \{u_{1,2}, u_{2,2}\}, ...\{u_{1,C}, u_{2,C}\}) \equiv \left(\{u_1, u_K\}, \{u_2, u_{K-1}\}, ...\{u_{\frac{K}{2}}, u_{\frac{K}{2}+1}\}\right), \tag{27}$$

where u_1 is the strongest user, while u_K is the weakest user from all users in the considered system. With this grouping strategy, we develop two algorithms in the following subsections to solve the non-convex optimization problem P1.

A. Sequential Convex Approximation (SCA) - based Approach

The SCA technique is a local optimization method for evaluating the solutions of non-convex problems [43]. The key idea behind this iterative approach is to approximate the non-convex functions into convex ones, and then solve iteratively the approximated convex optimization problems. It is worthy to mention that SCA is heuristic; therefore, the solutions generally depend on the initializations [43].

It is obvious that the optimization problem P1 is non-convex due to both non-convex objective functions and constraints. Thus, we deal this non-convexity issue through employing the SCA technique. We start with an approximation of the non-convex objective function. A slack variable γ is introduced to approximate the objective function with a convex one. With this slack variable, the optimization problem P1 can be rewritten as,

$$\textbf{(P2)}: \max_{p_{j,i},t_i,\gamma} \quad \gamma \tag{28}$$

$$\text{s.t.} \quad \frac{\sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i \log_2(1+SINR_{j,i})}{\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}} \geq \sqrt{\gamma}, \tag{29}$$

$$\sum_{i=1}^{C} t_i = T, \tag{30}$$

$$(15),(16),(17). \tag{31}$$

Note that the objective function of the original optimization problem P1 is replaced with the slack variable $\sqrt{\gamma}$ by using epigraph. However, the non-convex constraint in (29) should be approximated by a convex constraints such that problem P2 turns out to be a convex problem. To handle these non-convexity issues, we exploit the SCA technique through introducing additional slack variables. The details of these approximations are provided in the following.

Firstly, by incorporating a positive slack variable z, the constraint in (29) can equivalently be decomposed into the following two constraints:

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i \log_2(1+SINR_{j,i}) \geq \sqrt{\gamma z}, \tag{32}$$

$$\sqrt{z} \geq \frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}. \tag{33}$$

Now, we deal with the non-convexity of (32) by introducing new slack variables $\alpha_{j,i}$ and $\vartheta_{j,i}$ as follows:

$$(1+SINR_{j,i}) \geq \alpha_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1,...,K_i\}, \tag{34a}$$

$$\log_2(1+SINR_{j,i}) \geq \vartheta_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{34b}$$

$$\alpha_{j,i} \geq 2^{\vartheta_{j,i}}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{34c}$$

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i \vartheta_{j,i} \geq \sqrt{\gamma z}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{34d}$$

Note that the constraint in (34c) is convex while the rest of the constraints in (34a), (34b), (34d) are not. To overcome these non-convexity issues, we introduce another slack variable $\eta_{j,i}$, such that the constraint in (34a) can be rewritten as

$$\frac{|h_{j,i}|^2 p_{d,i}^2}{|h_{j,i}|^2 \sum_{s=1}^{d-1} p_{s,i}^2 + \sigma_{j,i}^2} \geq \frac{(\alpha_{j,i}-1)\eta_{j,i}^2}{\eta_{j,i}^2}, \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}. \tag{35}$$

Accordingly, the constraint in (35) can now be decomposed into the following two constraints:

$$|h_{j,i}|^2 p_{d,i}^2 \geq (\alpha_{j,i}-1)\eta_{j,i}^2, \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}, \tag{36a}$$

$$|h_{j,i}|^2 \sum_{s=1}^{d-1} p_{s,i}^2 + \sigma_{j,i}^2 \leq \eta_{j,i}^2, \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}. \tag{36b}$$

Then, based on the approximation of first-order Taylor series expansion method, the constraint in (36a) can be represented as

$$|h_{j,i}|^2 \left(p_{d,i}^{2\,(t)} + 2p_{d,i}^{(t)}(p_{d,i} - p_{d,i}^{(t)})\right) \geq \eta_{j,i}^{2\,(t)} \left(\alpha_{j,i}^{(t)} - 1\right) + 2\left(\alpha_{j,i}^{(t)} - 1\right)\eta_{j,i}^{(t)}\left(\eta_{j,i} - \eta_{j,i}^{(t)}\right) + \eta_{j,i}^{2\,(t)}\left(\alpha_{j,i} - \alpha_{j,i}^{(t)}\right), \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}, \tag{37}$$

where $p_{d,i}^{(t)}$, $\eta_{j,i}^{(t)}$ and $\alpha_{j,i}^{(t)}$ represent the approximations of $p_{d,i}$, $\eta_{j,i}$ and $\alpha_{j,i}$ at the tth iteration, respectively. Note that both sides of (37) are linear in terms of the optimization variables, i.e., $p_{d,i}$, $\eta_{j,i}$, and $\alpha_{j,i}$. Furthermore, the constraint in (36b) can be rewritten as the following SOC constraint:

$$\| |h_{j,i}| p_{1,i}, |h_{j,i}| p_{2,i}, ..., |h_{j,i}| p_{d-1,i}, \sigma_{j,i} \| \leq \eta_{j,i}, \quad \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}, \tag{38}$$

where $||\cdot||$ denotes the Euclidean norm of a vector. With these approximations, the constraint in (34a) can be rewritten as the convex constraints in (37) and (38).

Now, we deal with the non-convexity issue of the constraint in (34d). Similar to the previous approximations, we rewrite the non-convex constraint in (34d) with a new slack variable $\nu_{j,i}$ as

$$t_i \vartheta_{j,i} \geq \nu_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{39a}$$

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} \nu_{j,i} \geq \sqrt{\gamma z}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{39b}$$

To deal with the non-convexity issue of (39a), we add a non-negative term $t_i^2 + \vartheta_{j,i}^2$ to both sides of inequality (39a) without loss of generality. By taking the square root of both sides of the inequality, the following SOC constraint can be defined:

$$t_i + \vartheta_{j,i} \geq \left\| \begin{matrix} 2\sqrt{\nu_{j,i}} \\ t_i - \vartheta_{j,i} \end{matrix} \right\|, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{40a}$$

Furthermore, we approximate the left-side of (39b) with a lower-bounded convex approximation using the first-order Taylor series expansion. As such, (39b) can be reformulated as

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} \nu_{j,i} \geq \sqrt{\gamma^{(t)} z^{(t)}} + \frac{1}{2}\sqrt{\left(\frac{z^{(t)}}{\gamma^{(t)}}\right)}(\gamma - \gamma^{(t)}) + \frac{1}{2}\sqrt{\left(\frac{\gamma^{(t)}}{z^{(t)}}\right)}(z - z^{(t)}). \tag{40b}$$

Similar to the previous approximations, the non-convexity of the constraint in (33) can be dealt with by introducing a new slack variable $\widetilde{z}$,

$$\sqrt{z} \geq \widetilde{z}, \tag{41a}$$

$$\widetilde{z} \geq \frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}. \tag{41b}$$

Now, we exploit a SOC relaxation to cast the non-convex constraints in (41) with convex ones [36] as follows:

$$\frac{z+1}{2} \geq \left\| \begin{matrix} \frac{z-1}{2} \\ \widetilde{z} \end{matrix} \right\|, \tag{42a}$$

$$\frac{\epsilon(\widetilde{z}-P_{loss})+1}{2} \geq \left\| \begin{matrix} \frac{\epsilon(\widetilde{z}-P_{loss})-1}{2} \\ p_{1,i} \\ \dots \\ p_{K,i} \end{matrix} \right\|, \forall i \in \mathcal{C}. \tag{42b}$$

Considering the aforementioned approximations, the constraint in (29) can be equivalently rewritten as a set of the following convex constraints: (34c), (37), (38), (40a), (40b), (42a) and (42b). Accordingly, the minimum rate constraints can be redefined as the following convex constraints:

$$\nu_{j,i} \geq \bar{R}_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{43}$$

Next, the non-convexity issue of (16) can be handled by approximating each non-convex term in the inequality by a lower-bounded convex term using the first-order Taylor series expansion. With this approximation, each term in (16) can be equivalently written as

$$p_{K,i}^2 \geq {p_{K,i}^2}^{(t)} + 2{p_{K,i}}^{(t)}(p_{K,i} - {p_{K,i}}^{(t)}), \forall i \in \mathcal{C}. \tag{44}$$

With the above relaxations, the original non-convex optimization problem P1 can be equivalently written as the following approximated convex one:

$$\textbf{(P3)}: \quad \max_{\Gamma} \quad \gamma \tag{45}$$

$$\text{s.t.} \quad (34c), (37), (38), (40a), (40b), (42a), (42b), \tag{46}$$

$$(30), (15), (44), (43), \tag{47}$$

where Γ comprises all the optimization variables, such that $\Gamma = \{p_{j,i}, t_i, \gamma, \alpha_{j,i}, \vartheta_{j,i}, \eta_{j,i}, z, \widetilde{z}, \nu_{j,i}\}$, $\forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i$. More specifically, the solution of P1 is obtained through iteratively solving the approximated convex optimization problem P3 and updating initialized variables. In particular, the solution of each iteration is fed into the optimization problem P3 to update the corresponding initial parameter for the convergence iteration. However, the initial parameters of the first iteration have to be carefully selected to guarantee the success of the iterative algorithm. In particular, for this iterative SCA algorithm, selecting initial optimization parameters, $\Gamma^{(0)}$, has a considerable impact on both efficiency of the solution and convergence of the algorithm itself. Hence, the initialization of variables should be carefully defined. These initial values can be chosen by defining random power allocations $p_{j,i}^{(0)}$ that fulfill the constraints in the problem P-min. Then, the corresponding slack variables are determined by substituting these power allocations in (34), (37), (38), (40) and (42). Consequently, the solutions obtained in each iteration are used as initialized variables to solve the optimization problem P3 in the subsequent iteration. Note that the objective function of the optimization problem P3, i.e., γ, is a non-decreasing function [41]. Therefore, a solution of the SCA algorithm can be selected as a reasonable solution for the original optimization problem if the difference between two consecutive solutions is less than a pre-defined threshold, τ. This stopping criteria of the proposed iterative algorithm can be mathematically given by $|\gamma^{*(t+1)} - \gamma^{*(t)}| \leq \tau$. We summarize the proposed iterative SCA technique based algorithm to solve P1 in Algorithm 1.

TABLE I: GEE-Max Joint Resource Allocation Algorithm.

Algorithm 1: SCA method to solve GEE-Max Problem.

1: Group the users into clusters based on the grouping strategy defined in (27)
2: Initialize: Set the parameters $\Gamma^{(0)}$
3: Repeat
4: Solve the problem P3 in (45) - (47)
5: Update all parameters $\Gamma^{(t)}$
6: Until $|\gamma^{*(t+1)} - \gamma^{*(t)}| \leq \tau$.

It is worth pointing out that the convergence of the proposed SCA-based approach to solve the GEE-Max problem has been carefully investigated in [44], where it has been stated that the SCA guarantees at least a local optimal solution, and in most cases, a global optimal solution. On the other hand, we discuss now the required aspects to guarantee the convergence of the SCA technique in Algorithm 1. Firstly, we initialize the iterative algorithm with appropriate initial parameters $\Gamma^{(0)}$, which ensures the feasibility of the problem at each iteration. It can be realized that the solutions returned at the iteration t are also feasible solutions for the problem at the successive iteration $t+1$. This implies that Algorithm 1 yields a non-decreasing sequence of the objective values, i.e., $\gamma^{(t+1)} > \gamma^{(t)}$. In addition, the total transmit power at BS is limited by an upper bound of P^{max}, which confirms that γ will converge to the solution with a finite number of iterations.

B. Dinkelbach's algorithm (DA) - based Approach

In this subsection, we develop an alternative approach based on DA to solve the original GEE-Max problem. This approach not only validates the solution obtained through the SCA algorithm, but also provides an alternative technique to deal with the fractional nature of the objective function in the original optimization problem P1. With DA, a new non-negative variable λ is introduced to parametrize the fractional objective function into a non-fractional one [40]. Based on the variable λ, the problem P1 can be defined as follows:

$$\textbf{(P4)}: \quad \max_{p_{j,i},t_i} \quad \sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i \log_2(1+SINR_{j,i}) - \lambda\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}\right) \tag{48}$$

$$\text{s.t.} \quad \sum_{i=1}^{C} t_i^{(t)} = T, \tag{49}$$

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} {p_{j,i}^2}^{(t)} \leq P^{max}, \tag{50}$$

$${p_{K,i}^2}^{(t)} \geq {p_{K-1,i}^2}^{(t)} \geq ... \geq {p_{1,i}^2}^{(t)}, \forall i \in \mathcal{C}, \tag{51}$$

$$R_{j,i}^{(t)} \geq \bar{R}_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{52}$$

Obviously, the objective function is convex with respect to λ. Then, the following theorem presents the solution to the problem P4.

Theorem 1. *The optimal objective value of* ***P4*** *equals to zero, i.e.,*

$$\sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i^* \log_2(1+SINR_{j,i}^*) - \lambda^*\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} {p_{j,i}^*}^2 + P_{loss}\right) = 0, \tag{53}$$

where $\{t_i^*, p_{j,i}^*, \lambda^*\}, \forall i, \forall j$ *denote the corresponding optimal solutions for* ***P4****.*

With Theorem 1, the solution of the original fractional problem **P1** (i.e., $\{t_i^*, p_{j,i}^*\}, \forall i, \forall j$) can be determined by solving the non-fractional optimization problem **P4**, where the optimal objective value of **P4** is zero [40]. The proof of Theorem 1 can be found in [40].

According to Theorem 1, the fractional objective function can now be transformed into a subtractive form, and thus, obtaining the variables $p_{j,i}, t_i$ that maximize the GEE in the original problem P1 is equivalent to solving the parameterized optimization problem P4. Therefore, we first initialize the parameter λ with zero, then use the convex approximation techniques to solve the parameterized optimization problem P4 [40]. Then, we update λ in the tth iteration as follows:

$$\lambda^{(t)} = \frac{\sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i^{(t-1)} \log_2(1+SINR_{j,i}^{(t-1)})}{\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} {p_{j,i}^2}^{(t-1)} + P_{loss}}. \tag{54}$$

In particular, the variables $t^{(t)}, p_{j,i}^{(t)}$ in the tth iteration can be found by solving the following optimization problem:

$$\textbf{(P5)}: \quad \max_{p_{j,i}^{(t)},t_i^{(t)}} \quad \sum_{i=1}^{C}\sum_{j=1}^{K_i} t_i^{(t)} \log_2(1+SINR_{j,i}^{(t)}) - \lambda^{(t-1)}\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} {p_{j,i}^2}^{(t)} + P_{loss}\right) \tag{55}$$

$$\text{s.t.} \quad (49), (50), (51), (52). \tag{56}$$

Now, we highlight the following observations by comparing the optimization problems P1 and P5. Firstly, note that the parametrization carried out using DA deals with the fractional nature of the objective function, as seen in P5. However, it can be shown that the first part of the objective function in P5 still remains non-convex because the optimization variables are coupled. Secondly, we can use the convex approximations implemented in P3 to deal with the non-convex constraints in P5.

Considering the above, we deal with the non-convexity issue of the objective function of P5 by using the same approach that has been developed to approximate the constraints in problem P3, such as the SCA technique, through introducing a set of slack variables as follows:

$$t_i \log_2(1+SINR_{j,i}) \geq y_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{57a}$$

$$(1+SINR_{j,i}^d) \geq \beta_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, ..., K_i\}, \tag{57b}$$

$$\log_2(1+SINR_{j,i}) \geq \chi_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{57c}$$

$$\beta_{j,i} \geq 2^{\chi_{j,i}}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{57d}$$

$$t_i \chi_{j,i} \geq y_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{57e}$$

The constraint in (57b) can be equivalently rewritten as the following set of convex constraints:

$$|h_{j,i}|^2\left({p_{d,i}^2}^{(t)} + 2p_{d,i}^{(t)}(p_{d,i} - p_{d,i}^{(t)})\right) \geq {\theta_{j,i}^2}^{(t)}\left(\beta_{j,i}^{(t)} - 1\right) + 2\left(\beta_{j,i}^{(t)} - 1\right)\theta_{j,i}^{(t)}\left(\theta_{j,i} - {\theta_{j,i}}^{(t)}\right) + {\theta_{j,i}^2}^{(t)}\left(\beta_{j,i} - {\beta_{j,i}}^{(t)}\right), \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}, \tag{58a}$$

$$\| |h_{j,i}|p_{1,i}, |h_{j,i}|p_{2,i}, ..., |h_{j,i}|p_{d-1,i}, \sigma_{j,i} \| \leq \theta_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \forall d \in \{j+1, j+2, ..., K_i\}, \tag{58b}$$

where $\theta_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i$, are newly introduced variables. As can be seen, the constraints in (57e) are jointly convex with respect to the involved variables where the right side is an affine function and the left side is a quadratic-over-affine function [41]. The inequality (57e) can be formulated into a SOC constraint as follows:

$$t_i + \chi_{j,i} \geq \left\| \begin{matrix} 2\sqrt{y_{j,i}} \\ t_i - \chi_{j,i} \end{matrix} \right\|, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i. \tag{59}$$

TABLE II: Dinkelbach's Method to Solve GEE-Max Problem.

Algorithm 2: Dinkelbach's method to solve GEE-Max Problem.

1: Initialize: $\lambda^{(0)}$ to satisfy $G(\lambda^{(0)}) \geq 0$, iteration number $t = 0$ and set the parameters $\Phi^{(0)}$
2: Repeat
3: Solve the problem P6 with $\Phi^{(t-1)}$, then obtain the optimal Φ^*
4: Update $\Phi^{(t)} = \Phi^*$
5: Until required accuracy is achieved
6: Update $\lambda^{(t+1)}$ according to (54), and set $t \leftarrow t + 1$
7: Until convergence.

It is obvious that $\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}\right)$ is a convex function in terms of $p_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i$. Thus, the function $\lambda\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}\right)$ is also a convex function of $p_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i$ as λ is a constant and consequently $\sum_{i=1}^{C}\sum_{j=1}^{K_i} y_{j,i} - \lambda\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} p_{j,i}^2 + P_{loss}\right)$ is a convex function. From these observations, the original non-convex optimization problem P1 can be approximated using the DA as the following optimization problem:

$$\textbf{(P6)}: \quad \max_{\Phi^{(t)}} \quad \sum_{i=1}^{C}\sum_{j=1}^{K_i} y_{j,i}^{(t)} - \lambda^{(t-1)}\left(\frac{1}{\epsilon}\sum_{i=1}^{C}\sum_{j=1}^{K_i} {p_{j,i}^2}^{(t)} + P_{loss}\right) \tag{60}$$

$$\text{s.t.} \quad (49), (50), (51), (58a), (58b), (57d), (59), \tag{61}$$

$$y_{j,i}^{(t)} \geq \bar{R}_{j,i}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i, \tag{62}$$

where $\Phi^{(t)} = \{p_{j,i}^{(t)}, t_i^{(t)}, \theta_{j,i}^{(t)}, \chi_{j,i}^{(t)}, y_{j,i}^{(t)}\}, \forall i \in \mathcal{C}, \forall j \in \mathcal{K}_i$. With the solution of P6, we update the involved variables at the successive iteration. This iterative process is carried out until the algorithm converges. Algorithm 2 summarizes the proposed iterative algorithm for solving the original problem P1. In this DA-based iterative algorithm, we first initialize the variables with $\lambda^{(0)}$ and $\Phi^{(0)}$. Then, for current λ, the optimal variables Φ^* are determined by solving the problem P6 until the required accuracy is achieved. The proof for the convergence of the proposed algorithm is provided in Appendix.

C. Complexity Analysis of the Proposed Schemes

In this subsection, we provide the analysis for the computational complexity of solving the original GEE-Max optimization problem **P1**.

1) The complexity of the SCA based approach: With the SCA-based approach, the solution of the original GEE-Max optimization problem **P1** is obtained through solving the approximated optimization problem **P3**, iteratively. Therefore, the complexity of solving **P1** can be defined by quantifying the complexity of solving the approximated **P3** and the average number of required iterations, where the interior-point method is utilized to solve the SOCP with SOC and linear constraints [46], [47]. In particular, the complexity of solving the SOCP constraints at each iteration is given by $\mathcal{O} = (\mathcal{A}^2\mathcal{B})$ [46], where $\mathcal{A}$ and $\mathcal{B}$ represent the number of optimization variables and the dimensions of the SOC constraints, respectively. In fact, as **P3** is solved in a number of iterations, the overall complexity of solving the original problem is upper bounded by $\mathcal{O}\big(\mathcal{A}^2\mathcal{B}\log(\frac{1}{\tau})\big)$, where τ is the required solution accuracy.

2) The complexity of the Dinkelbach's algorithm (DA) based approach: In the developed DA-based algorithm, the original optimization problem is transformed into a standard SOCP problem for the given non-negative variable λ. However, two iterative algorithms are deployed to determine the solution in the DA based approach. Thus, the upper-bound complexity of DA based approach can be defined as $\mathcal{O}\big(\mathcal{A}^2\mathcal{B}\log(\frac{1}{\tau})\log(\frac{1}{\varpi})\big)$, where ϖ represents the required solution accuracy.

IV. Simulation Results

In this section, we provide simulation results to demonstrate the effectiveness of the proposed joint GEE-Max design for the considered hybrid TDMA-NOMA system. Additionally, the performance of the proposed schemes have been compared with that of the other baseline designs, namely, SR-Max and P-Min designs. In particular, we evaluate the performance of the proposed GEE-Max design with opportunistic time allocations against schemes with equal time allocations.

In the simulations, a hybrid TDMA-NOMA system is considered with 10 users. The users are assumed to be uniformly distributed over a circle area with a radius of 10 meters around the BS, where the minimum distance d_0 is selected 1 meter (d_0 = 1 m), where d_0 is the reference distance. The corresponding channel gain is $|h_{j,i}|^2 = \frac{\beta}{(d_{j,i}/d_0)^\kappa}$, where $\beta = -30$ dB and $\kappa = 2$. The noise variance at each user $\sigma_{j,i}^2$ depends on the noise power spectral density N_0 and the channel bandwidth B, which is expressed as $\sigma_{j,i}^2 = N_0 B$. In these simulations, N_0 is assumed to be -70 dBm/Hz and the bandwidth B is set to 1 MHz. The power amplifier efficiency ϵ for both algorithms is 0.35 [33]. In addition, the stopping-criteria threshold for both algorithms is set to 0.01 [37]. Furthermore, the CVX software is used to solve the convex problems in these simulations [42].

Fig. 3 compares the EE of our proposed design with the existing conventional designs in the literature, namely the resource allocation techniques with P-Min and SR-Max in hybrid TDMA-NOMA system. As seen in Fig. 3, the proposed GEE-Max based design outperforms the conventional design criteria of P-Min and SR-Max in terms of achieved EE. In addition, the EE of the SR-Max based design is not monotonically increasing with the available power and decreases when the transmit power exceeds a certain available power budget. This is due to the fact that this design fully uses all the available power for maximizing the achievable sum rate instead of maximizing the EE. In other words, maximizing the achievable sum-rate does not always maximize the EE. We can also observe that the P-Min based design achieves lower EE than the proposed scheme. This is due to the fact that the P-Min design seeks for the minimum power that is required to achieve the minimum rate requirements.

Fig. 4 depicts the average EE versus transmit power for different QoS requirements using the algorithms developed through the SCA and DA techniques. We can observe that the EE of both algorithms first increases until reaches a certain

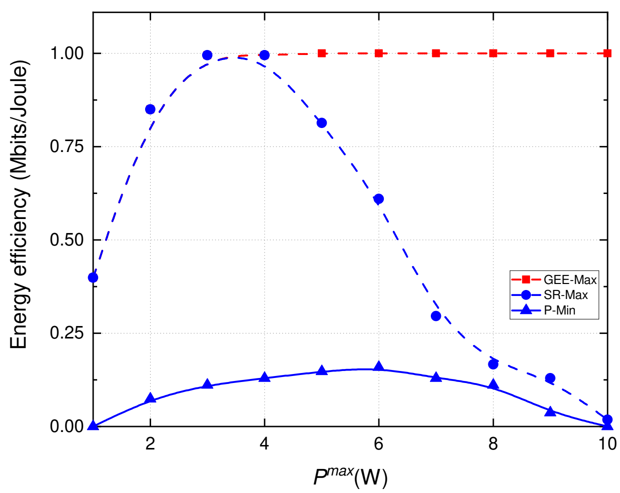

Fig. 3: Energy efficiency of the hybrid TDMA-NOMA system with different design criteria, $\bar{R}_{j,i} = 5bits/s/Hz$.

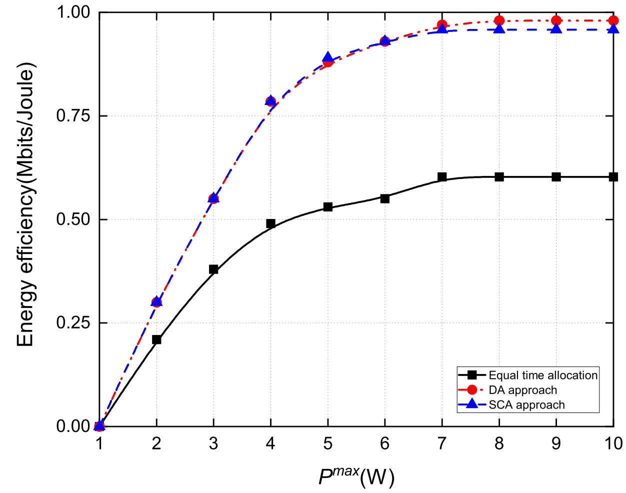

Fig. 5: Energy efficiency of the proposed algorithm and equal time allocation scheme with different transmit power, $\bar{R}_{j,i} = 3bits/s/Hz$.

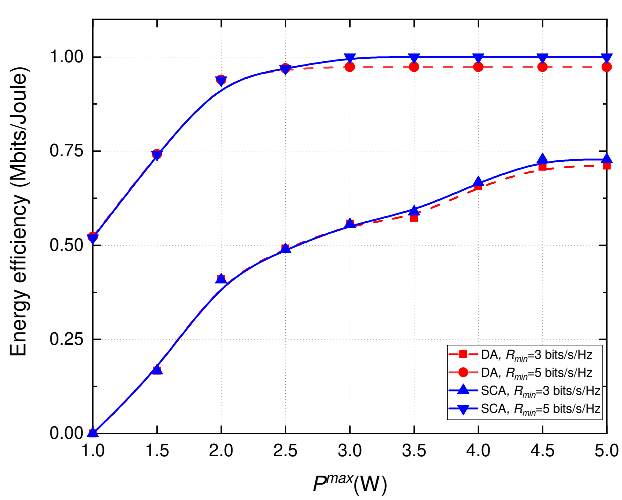

Fig. 4: Energy efficiency of the proposed algorithm with different QoS requirements.

value, and then it remains constant after a certain maximum power P^{max}. The EE performance with $R_{min} = 5$ bits/s/Hz [1] is better than that with $R_{min} = 3$ bits/s/Hz. This performance difference can be justified through the following argument. With higher R_{min}, the increasing rate in the sum rate is higher than the increasing rate in the total power consumption, which results in an improvement of EE. Note that the performance gap between these two algorithms is not significant in terms of the achieved EE. By setting the stopping-criteria to zero (i.e., $\lambda^{(t+1)} - \lambda^{(t)} = 0$), then both approaches should achieve the same performance.

Next, Fig. 5 illustrates the achieved EE against different transmit power levels for the proposed scheme with opportunistic time allocation and the conventional schemes with equal time allocation. As seen in Fig. 5, the achieved EE of the GEE-Max design with opportunistic time allocation outperforms that of the conventional equal time allocation. This can be achieved by solving the GEE-Max problem using either the SCA or the DA algorithm, as shown in Fig. 5. In these algorithms, both time and power resources are utilized efficiently to achieve the best EE for a given system.

Next, we compare the achieved per-user power allocations and per-cluster time allocations in the hybrid TDMA-NOMA schemes with the opportunistic time allocations versus that of the conventional schemes with equal time allocation in Table III and IV, respectively. For the sake of comparison, we assume that both schemes use the same minimum data rate requirements ($\bar{R}_{j,i} = 2$ bits/s/Hz). The achieved rate of each user and the time allocations using the proposed opportunistic time allocation are given for five different sets of random channels. As seen in Table III, most users achieve better rates in our proposed opportunistic time allocations based hybrid TDMA-NOMA schemes when compared with the conventional scheme with equal time allocations.

The convergence of SCA and DA-based EE maximization algorithms is studied in Fig. 6 and Fig. 7, respectively. In particular, five different sets of channels are considered to evaluate the convergence. In these simulations, the maximum transmit power P^{max} is set to 10 W. As seen in Fig. 6, the SCA-based algorithm converges to the optimal EE faster by using the relaxation of constraints. The convergence of DA follows the same procedure as for the convergence of the SCA technique for each λ. In addition, the simulation results confirm that both algorithms converge within a few number of iterations.

[1] Note that R_{min} and $\bar{R}_{j,i}$ carry the same meaning

TABLE III: Power Allocations For Each User In The Hybrid TDMA-NOMA Through The Proposed Opportunistic Time Allocations And The Conventional Equal Time One.

	Channels	$p_{1,1}$	$p_{2,1}$	$p_{1,2}$	$p_{2,2}$	$p_{1,3}$	$p_{2,3}$	$p_{1,4}$	$p_{2,4}$	$p_{1,5}$	$p_{2,5}$
Scheme with opportunistic time allocations	Channel 1	9.407	2.000	8.678	2.000	7.443	2.000	2.461	2.000	2.000	2.000
	Channel 2	1.2509	2.6832	1.2509	2.6832	1.2509	2.5327	1.2898	2.2687	1.3310	2.1908
	Channel 3	0.8979	2.1111	0.8979	2.1111	0.8979	1.9323	0.8979	1.6852	0.8979	1.5376
	Channel 4	1.1093	2.6235	1.1093	2.6235	1.1093	2.5285	1.1093	2.4644	1.1093	2.4644
	Channel 5	0.8393	1.8862	0.8393	1.8433	0.8393	1.7023	0.8393	1.6952	1.0169	1.5410
Scheme with equal time allocations	Channel 1	7.517	2.000	7.517	2.000	7.091	2.000	3.293	2.000	2.687	2.000
	Channel 2	1.1546	3.2741	1.1546	2.8266	1.1546	2.1453	1.1546	1.9478	1.1855	1.8550
	Channel 3	0.9145	2.6055	0.9145	2.3931	0.9145	1.8665	0.9145	1.3254	0.9145	1.2887
	Channel 4	1.1637	3.1452	1.1637	2.9782	1.1637	2.3582	1.1637	2.1550	1.1637	2.1295
	Channel 5	0.8568	2.2729	0.8568	1.9871	0.8568	1.5232	0.8568	1.4743	0.9514	1.3963

TABLE IV: Time Allocation And Achieved Minimum Throughout In The Hybrid TDMA-NOMA And The Conventional Schemes.

	Scheme with opportunistic time allocations						Scheme with equal time allocations					
Channels	$t_1(s)$	$t_2(s)$	$t_3(s)$	$t_4(s)$	$t_5(s)$	EE (Mbits/Joule)	$t_1(s)$	$t_2(s)$	$t_3(s)$	$t_4(s)$	$t_5(s)$	EE (Mbits/Joule)
Channel 1	2.254	2.329	2.118	1.980	1.516	0.356	2	2	2	2	2	0.327
Channel 2	1.751	1.819	2.097	2.114	2.219	0.275	2	2	2	2	2	0.251
Channel 3	2.737	2.399	1.892	1.425	1.547	0.490	2	2	2	2	2	0.438
Channel 4	2.599	2.388	1.783	1.628	1.601	0.304	2	2	2	2	2	0.284
Channel 5	2.634	2.220	1.695	1.630	1.819	0.563	2	2	2	2	2	0.533

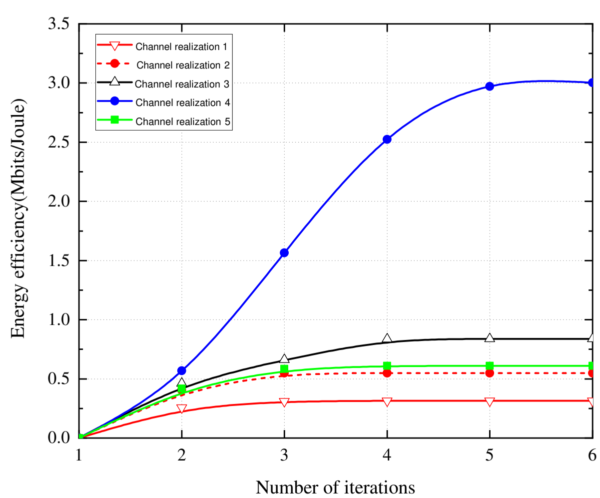

Fig. 6: The convergence of the SCA algorithm for five different sets of channels.

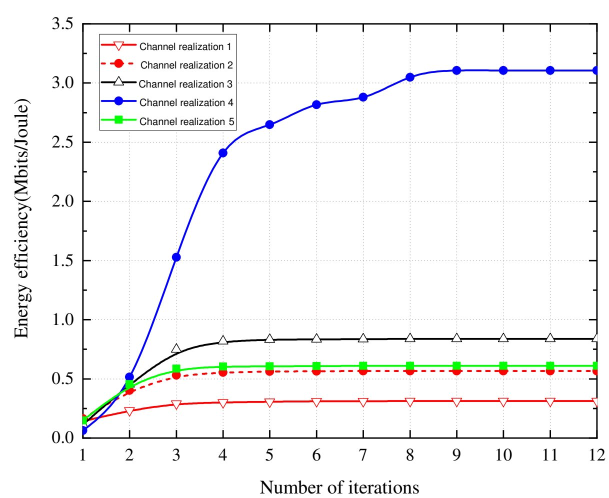

Fig. 7: The convergence of the DA for five different sets of channels.

V. Conclusions

In this paper, we have studied the GEE-Max problem with joint power-time resource allocation for a hybrid TDMA-NOMA system. In particular, the users are grouped into a number of clusters, the available transmission time is divided into several time-slots, and the power-domain NOMA is exploited to serve multiple users within each cluster. However, due to the non-convexity of the formulated GEE-Max optimization problem, we have proposed two different algorithms based on the SCA and DA techniques, respectively. Simulation results have demonstrated the effectiveness of the proposed schemes. In particular, the proposed hybrid TDMA-NOMA system with opportunistic time allocation outperforms the conventional resource allocations with equal time assignment in terms of the required minimum transmit power and achieved sum rate. In other words, the proposed schemes achieve better EE than the conventional schemes with equal time allocations. As evidenced by a large number of work in the literature, reinforcement learning techniques might be exploited to improve the computational complexity while meeting the delay requirements. This is one of the promising research directions that we would explore in the future in resource allocation techniques for NOMA-based systems.

APPENDIX

In order to prove the convergence of the DA-based iterative approach to the optimal solution, the following conditions can be equivalently proven [40]:

- Firstly, we prove that $\lambda^{(t+1)} > \lambda^{(t)}$ for all t.

Lemma 2. *Let* $G(\lambda^{(t)}) = \max_{\{t_i^*, p_{j,i}^*\}} \left\{ \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i, p_{j,i}) - \lambda^{(t)} P_{total}(p_{j,i}) \right\}$, *then* $G(\lambda)$ *is a strictly monotonic decreasing function of* λ*, i.e., if* $\lambda^{(t)} < \lambda^{(t+1)}$*, then* $G(\lambda^{(t)}) > G(\lambda^{(t+1)})$.

Proof: Let $\{t_i^*, p_{j,i}^*\}$ be the optimal power allocation and time slot assignment for the proposed schemes for $G(\lambda^{(t+1)})$. Then

$$\begin{aligned} G(\lambda^{(t+1)}) &= \max_{\{t_i^*, p_{j,i}^*\}} \left\{ \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i, p_{j,i}) - \lambda^{(t+1)} P_{total}(p_{j,i}) \right\} \\ &= \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i^*, p_{j,i}^*) - \lambda^{(t+1)} P_{total}(p_{j,i}^*) \\ &< \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i^*, p_{j,i}^*) - \lambda^{(t)} P_{total}(p_{j,i}^*) \\ &\leq \max_{t_i, p_{j,i}} \left\{ \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i, p_{j,i}) - \lambda^{(t)} P_{total}(p_{j,i}) \right\} \\ &= G(\lambda^{(t)}). \end{aligned} \quad (63)$$

This completes the proof of Lemma 2.

Lemma 3. *Let* $\{t_i, p_{j,i}\}$ *be an arbitrary power allocation and time slot assignment and* $\lambda^{(t+1)} = \frac{\sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(p_{j,i}^{(t+1)}, t_i^{(t+1)})}{P_{total}(p_{j,i}^{(t+1)})}$*, then* $G(\lambda^{(t+1)}) \geq 0$.

Proof: $G(\lambda^{(t+1)}) = \max_{\{t_i, p_{j,i}\}} \left\{ \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i, p_{j,i}) - \lambda^{(t+1)} P_{total}(p_{j,i}) \right\} \geq \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}(t_i^{(t+1)}, p_{j,i}^{(t+1)}) - \lambda^{(t+1)} P_{total}(p_{j,i}^{(t+1)}) = 0$. Hence, $G(\lambda^{(t+1)}) \geq 0$. This completes the proof of Lemma 3 and this lemma implies that $G(\lambda^{(t)}) \geq 0$. By definition, we have

$$\begin{aligned} G(\lambda^{(t)}) &= \sum_{i=1}^{C} \sum_{j=1}^{K_i} R_{j,i}({t_i}^{(t)}, p_{j,i}^{(t)}) - \lambda^{(t)} P_{total}(p_{j,i}^{(t)}) \\ &= \lambda^{(t+1)} P_{total}(p_{j,i}^{(t)}) - \lambda^{(t)} P_{total}(p_{j,i}^{(t)}) > 0. \end{aligned} \quad (64)$$

Since $P_{total}(p_{j,i}^{(t)}) > 0$, then $\lambda^{(t+1)} > \lambda^{(t)}$.

- Secondly, we prove that $\lim_{t\to\infty} \lambda^{(t)} = \lambda^*$, where λ^* is the maximum EE and we shall prove that the $\lambda^{(t)}$ equals to λ^* when iteration number t approaches to infinity. We prove it by contradiction. Assume that $\lim_{t\to\infty} \lambda^{(t)} = \lambda^*$ does not hold, that is, $\lim_{t\to\infty} \lambda^{(t)} = \widetilde{\lambda} < \lambda^*$. Based on this argument, $G(\widetilde{\lambda}) = 0$. However, $G(\lambda)$ is a strictly monotonic decreasing function based on Lemma 2, and therefore we obtain

$$0 = G(\widetilde{\lambda}) > G(\lambda^*) = 0, \quad (65)$$

which contradicts the initial assumption. Therefore, this confirms that the Dinkelbach's algorithm based method converges to the optimal solution. ■

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