A dynamical systems approach to optimising irrigation strategy under the influence of land-atmosphere feedbacks

Bettina Meyer[1], Douglas J. Parker[2], and Jan O. Haerter[1]

bettina.meyer@nbi.ku.dk

[1]Copenhagen University, Niels Bohr Institute, Physics, København Ø, Denmark

[2]School of Earth and Environment, University of Leeds

UNIVERSITY OF COPENHAGEN

VILLUM FONDEN

Abstract

A dynamical systems approach to optimising irrigation strategy under the influence of land-atmosphere feedbacks.

Bettina Meyer , Douglas J. Parker , and Jan O. Haerter

[1]Copenhagen University, Niels Bohr Institute, Physics, København Ø, Denmark (bettina.meyer@nbi.ku.dk) [2]School of Earth and Environment, University of Leeds

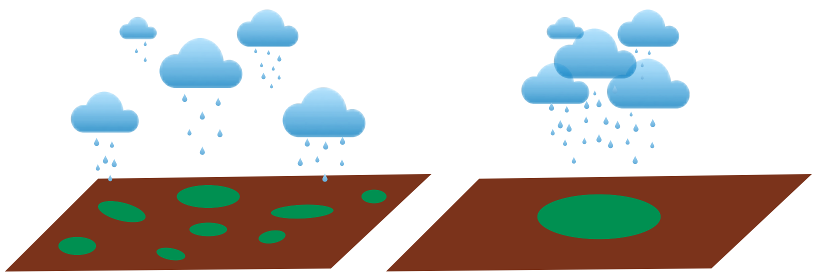
The soil-moisture feedback describes how precipitation amount, timing and intensity react to spatial anomalies in surface moisture. For heterogeneous moisture distributions with moist/dry patches on the scale of 10– 50km, numerical studies supported by observations indicate a negative soil-moisture feedback, where it rains more over dry patches (Imamovic, 2018; Rieck et al., 2014). The circulation established by the heterogeneous soil-moisture patches not only modifies the spatial rain distribution but allows for more water to be extracted from the atmosphere, thereby increasing the domain mean precipitation.

We here suggest that the negative soil-moisture feedback can be exploited when irrigating agricultural land: if farmers cooperate by following a spatially heterogeneous irrigation pattern, they can increase both their collective time-mean precipitation and thus the total water available for growing crops. However, the spatially non-local nature of the feedback allows individual farmers to exploit this strategy, thereby saving their own resources; a typical ‘tragedy of commons’ situation.

We formulate this setup in terms of an optimisation problem and study its parameter phase space, both analytically and numerically, in order to understand optimal rules and the consequences of the players’ choice to cooperate vs. compete. Different constraints in terms of water availability (reservoir) and average soil moisture as defined by the evaporation timescale are explored.

Reducing the details of the land-atmosphere interaction into simple feedback parameters helps to elucidate the complex interactions between the precipitation, soil moisture and the human intervention by irrigation. Taking into account the negative soil-moisture feedback in irrigation models opens up new strategies to optimise water management and thereby increase crop yield.

Soil moisture-precipitation feedback



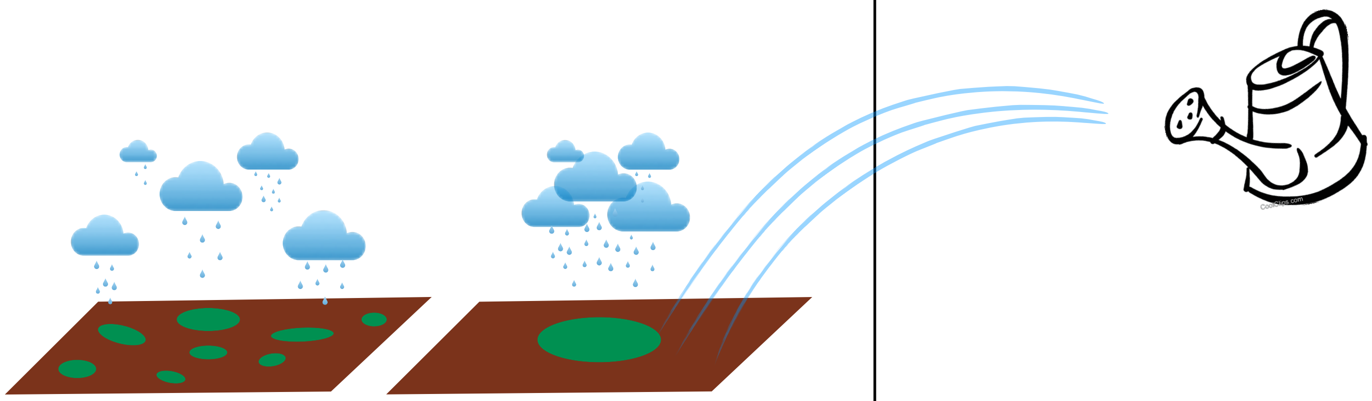
The **soil-moisture feedback** describes how the amount, timing and intensity of precipitation reacts to moist (or dry) surface anomalies. This involves both a local and non-local feedback, where spatial correlations seem to be case dependent.

The sign of the feedback is not fully resolved to this point as it depends on spatial scales, the circulation regime, surface properties and other environmental factors (e.g., Seneviratne 2010, Guillod 2015).

A positive feedback means that wet soil increases precipitation intensity, while a negative feedback means that it rains more over dry patches. On a scale of 10-50 km, various numerical studies indicate a negative soil-moisture feedback. Imamovic 2017, Rieck 2014).

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

Soil moisture-precipitation feedback



The **soil-moisture feedback** describes how the amount, timing and intensity of precipitation reacts to moist (or dry) surface anomalies. This involves both a local and non-local feedback, where spatial correlations seem to be case dependent.

The sign of the feedback is not fully resolved to this point as it depends on spatial scales, the circulation regime, surface properties and other environmental factors (e.g., Seneviratne 2010, Guillod 2015).

A positive feedback means that wet soil increases precipitation intensity, while a negative feedback means that it rains more over dry patches. On a scale of 10-50 km, various numerical studies indicate a negative soil-moisture feedback. Imamovic 2017, Rieck 2014).

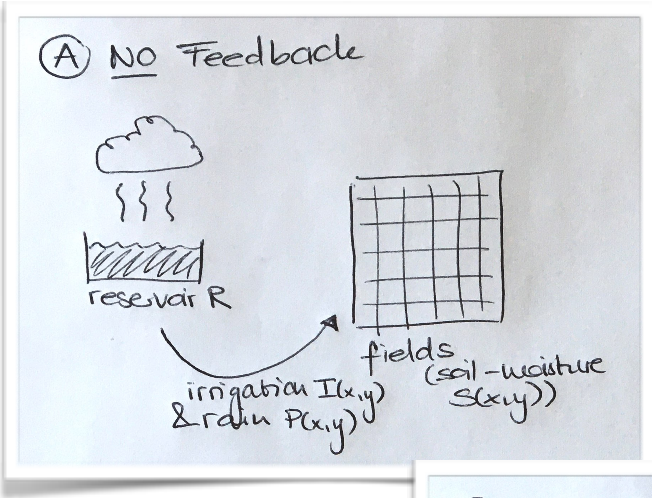
Irrigation & Water Management

Water and irrigation management is an important field of research, specifically in semi-arid regions under limited water resources. Crop models intend on finding optimal strategies for the irrigation of agricultural land in a given setup (climate regime, soil type, crop type etc.).

This study explores, **to what extent the (negative) soil-moisture feedback can be exploited to optimise irrigation strategies for agricultural land**: if farmers cooperate by following a spatially heterogeneous irrigation pattern, they can increase both their collective time-mean precipitation and thus the total water available for growing crops. However, the spatially non-local nature of the feedback allows individual farmers to exploit this strategy, thereby saving their own resources; a typical **‘tragedy of commons’** situation.

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

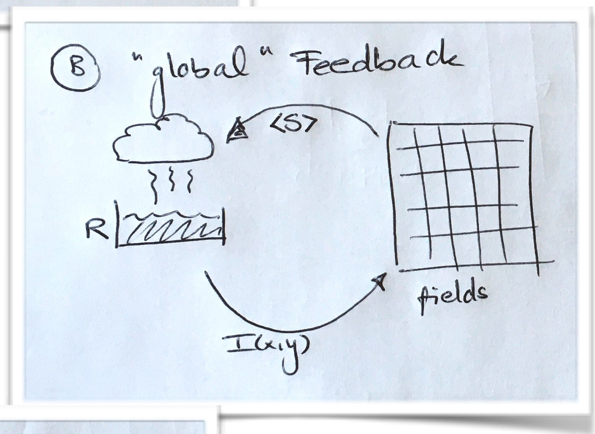
Exploiting the soil-moisture feedback in water management



(A) *No* soil moisture-precipitation feedback

In a moste simple setup, irrigation (water distribution) is managed by taking into only the effect of precipitation on soil moisture.

Management: take into account (seasonal and short-term) precipitation forecasts.

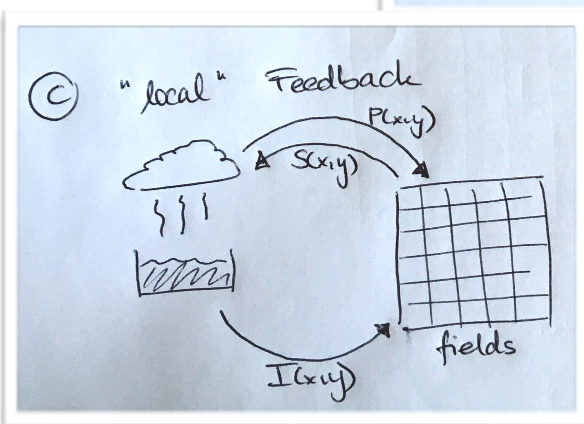


(B) "global" soil moisture-precipitation feedback

>> coupling to global (mean) soil moisture levels

"wet gets wetter"
(e.g., Guillod 2014, Allan et al. 2020)

"moisture recycling:" part of the precipitated water stems from regional evapotranspiration; over large-enough domains, moisture thus gets recycled multiple times and thus an increased soil-moisture input leads to increased hydrological cycle (e.g., Seneviratne 2010)



(C) "local" soil-moisture feedback

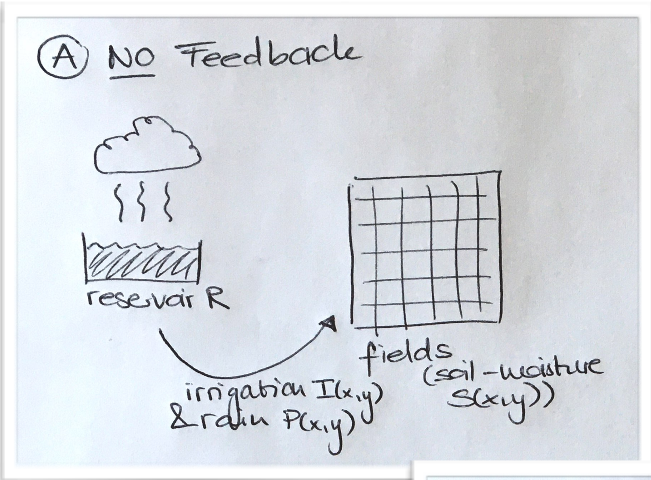
>> coupling to local soil moisture anomalies (heterogeneities)

both positive and negative soil moisture-precipitation feedbacks have been reported (e.g., Seneviratne 2010)

negative feedback: enhanced cloud formation and precipitation over dry land patches
observed in high-resolution models (cloud-resolving models, regional climate models), while global climate models do not represent negative feedbacks

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

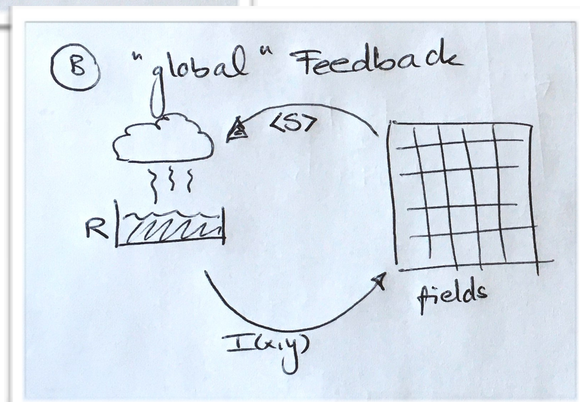
Exploiting the soil-moisture feedback in water management



(A) *No* soil moisture-precipitation feedback

In a moste simple setup, irrigation (water distribution) is managed by taking into only the effect of precipitation on soil moisture.

Management: take into account (seasonal and short-term) precipitation forecasts.

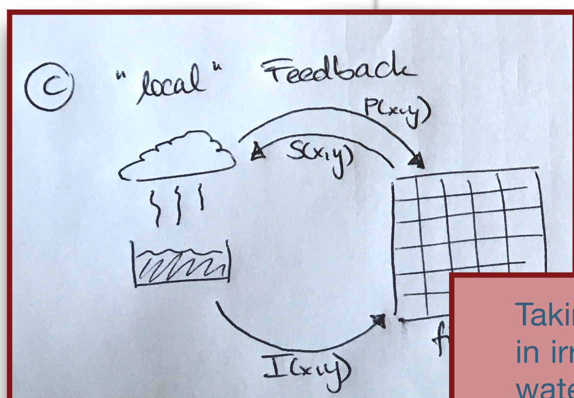


(B) "global" soil moisture-precipitation feedback

>> coupling to global (mean) soil moisture levels

"wet gets wetter"
(e.g., Guillod 2014, Allan et al. 2020)

"moisture recycling:" part of the precipitated water stems from regional evapotranspiration; over large-enough domains, moisture thus gets recycled multiple times and thus an increased soil-moisture input leads to increased hydrological cycle (e.g., Seneviratne 2010)



(C) "local" soil-moisture feedback

>> coupling to local soil moisture anomalies (heterogeneities)

both positive and negative soil moisture-precipitation feedbacks have been reported (e.g., Seneviratne 2010)

negative feedback: enhanced cloud formation and precipitation over dry land

regional climate models),

cks

Taking into account the negative soil-moisture feedback in irrigation models opens up new strategies to optimise water management and thereby increase crop yield.

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

The model

Reservoir & soil moisture

$$\dot{R}_{ij}(t) = a\, P_{ij}(t) - I_{ij}(t)$$
$$\dot{S}_{ij}(t) = (1-a)\, P_{ij}(t) + I_{ij}(t) - \epsilon S_{ij}(t)$$

$R:$ water reservoir (local or global)

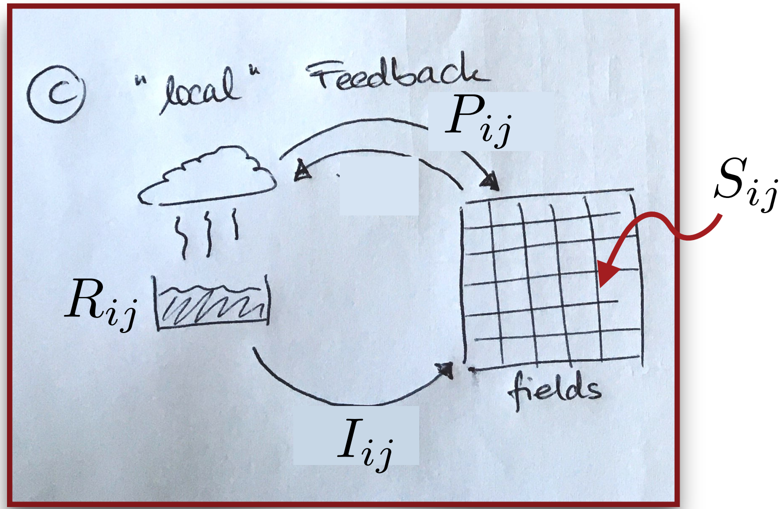
$S:$ soil-moisture

$\epsilon \geq 0:$ evaporation coefficient

$0 \leq a \leq 1:$ partial storage of rain water into the reservoir

$P:$ precipitation field

$I:$ irrigation



Soil moisture-precipitation feedback

$$P_{ij}(t) = P_0 + A_{ij,kl} S_{kl}(t)$$
$$I_{ij}(t) = f_{\text{decision}}(R_{ij}(t), S_{ij}(t), P_{ij}(t), P_{pred}(t + \Delta t), \epsilon)$$

$A_{ij,kl}:$ coupling matrix (soil moisture-precipitation feedback)

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

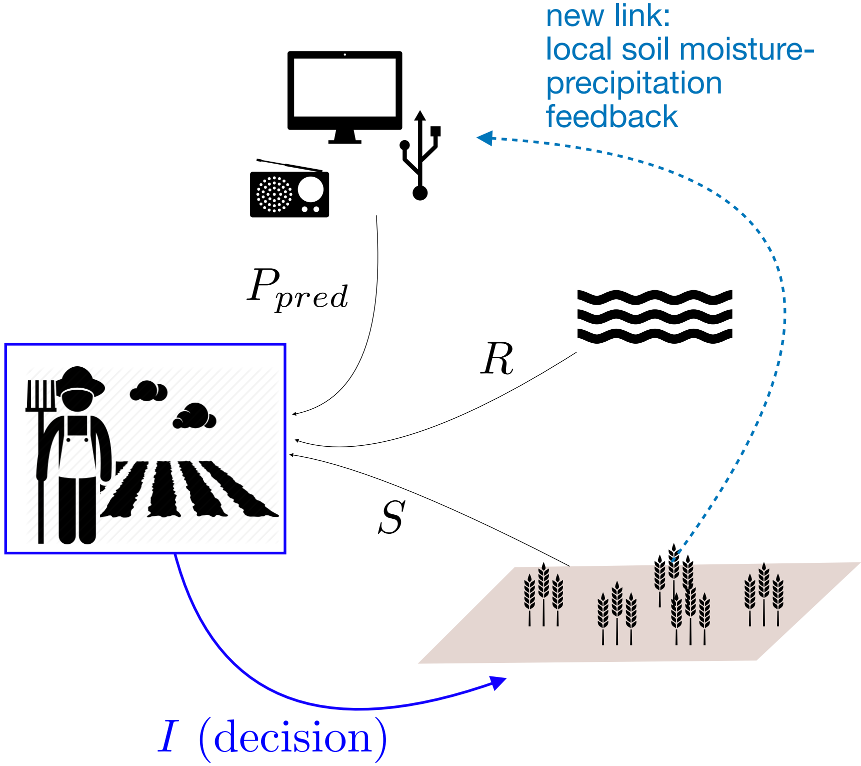
The Farmer's Decision

Our hypothetical farmer has to decide how much to irrigate the field.

The decision is based on:

- *What is the weather forecast saying?* (near-term and seasonal precipitation forecast)
- *How much water do I have available?* (reservoir water level)
- *Is the soil moist enough for the crop to grow?* (soil moisture measurements)

We would like to optimise this decision by including the local soil moisture-precipitation feedback.



Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

Analytical Analysis

In the most simple case, the system is formulated in terms of two agents that represent for example two farmers that own two neighbouring fields. This setup can be extended to *n* fields that are aligned in a one-dimensional chain. If all feedbacks and couplings are symmetric, the matrix representation can be used for *(n x n)* agents that are distributed on a 2d-grid.

Steady-state solution

The steady state is defined by: $\dot{R} = \dot{S} = 0$

Irrigating only as much as is collected in terms of rain water. $I = aP$

Precipitation needs to balance the soil-moisture loss. $P = \epsilon S$

Introducing the soil moisture-precipitation feedback set constraints on

- the physical model parameters {**α**, β, γ, ε}.
- the soil moisture pattern

>> A steady-state does not exist for all combinations of physical parameters.

Perturbation from steady state

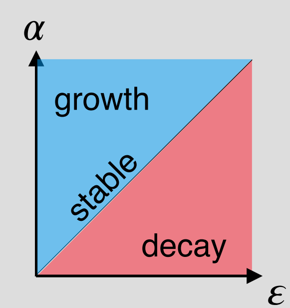
Steady state solution: $\{I_b,\, S_b,\, P_b = P(S_b)\}$

Perturbations:

$$I = I_b + I'$$
$$S = S_b + S'$$
$$P = P(S) = P_b + P',\ \text{with } P' = AS'$$

>> Perturbation decays / grows depending on parameters.

$$P = P_0 + A\,S$$
$$S = (s_1, s_2)$$
$$A = \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix}, \qquad \alpha, \beta, \gamma \in \mathbb{R}$$
$$\text{symmetric case: } \beta = \gamma$$



$$S'(t) = \Delta\, \exp\left(-\tilde{\epsilon} t\right),$$
$$\tilde{\epsilon} = (1-a)(\alpha - \epsilon),$$
$$R'(t) = a\alpha\, S'(t)$$

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

Analytical Analysis

Soil moisture-precipitation feedbacks

symmetric case (no mean advection): To make the coupling symmetric ($A = A^T$), it is assumed that no background advection wind field, as for example a propagating squall-line or sea breeze, is present.

From eigenvalue analysis of the coupling matrix A, its elements can be related to the spatially ***local* coupling of soil moisture S onto co-located precipitation P** and ***non-local* coupling,** where **precipitation depends on the soil moisture gradient between field 1 and 2** .

$$P = P_0 + A\,S$$

$$S = (s_1, s_2)$$

$$A = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}, \qquad \alpha, \beta \in \mathbb{R}$$

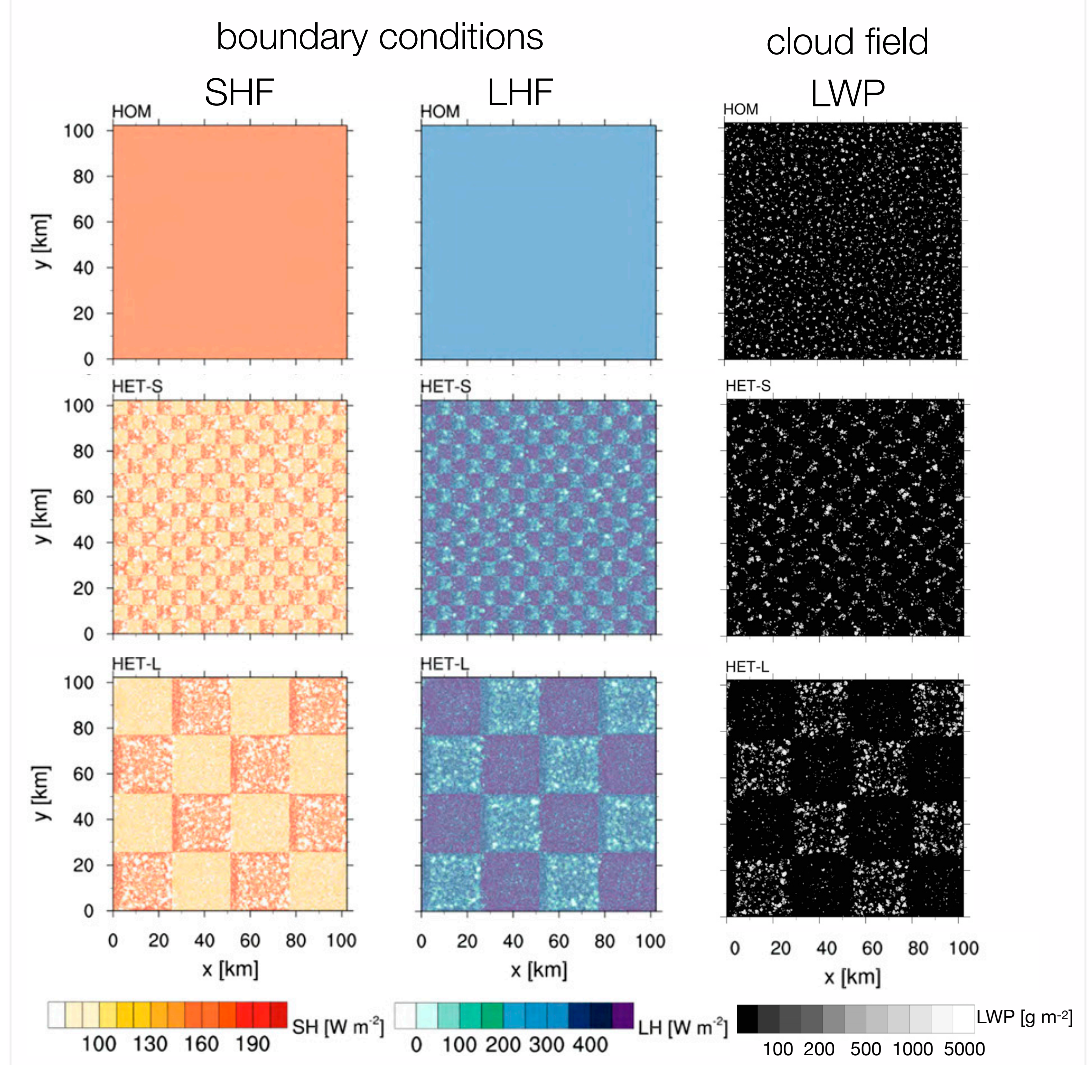
$$\text{symmetric case: } \beta = \gamma$$

$$\begin{cases} \text{local coupling: } \lambda \\ \text{non-local coupling: } \eta \end{cases} \Rightarrow \begin{cases} \alpha = \lambda + \eta \\ \boxed{\beta = -\eta} \end{cases}$$

the off-diagonal elements couple the neighbouring farmers and their irrigation strategies to each other

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020

Determine soil moisture-precipitation coupling



(Idealised) numerical simulations:

The elements of coupling matrix A can be extracted from (idealised) numerical simulations:

$$P_{ij} = P_0 + A_{ij,kl}\, S_{kl}$$

$$A = (A_{ij,kl} - P_0)\, S_{kl}^{-1}$$

$$A = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}, \quad \alpha, \beta \in \mathbb{R}$$

P_0 : stochastic variable

Figure:
Malte Rieck, Cathy Hohenegger and Chiel C. van Heerwaarden, *The Influence of Land Surface Heterogeneities on Cloud Size Development*, Monthly Weather Review (2014)

Future Steps

Question 1: **How much can the crop yield be increased by choosing informed irrigation strategies that take into account positive and negative soil moisture-precipitation feedbacks?**

To understand by how much the irrigation strategy can be optimised by including the (negative) soil moisture-precipitation feedbacks, we compare the dynamics of a system for two different irrigation strategies:

(i) an `informed' choice based on knowing about the soil moisture-precipitation feedback, and

(ii) a `trivial' choice, where the farmer may still have access to a weather forecast, predicting typical precipitation timeseries, but is not aware of the non-local feedback, that couples his decision to the behaviour of his neighbours.

The system includes soil moisture-precipitation feedbacks.

Question 2: **How relevant is a correct representation of the (negative) soil moisture-precipitation feedback in weather forecast models, that are then used by farmers to make their irrigation decisions?**

Compare two systems that underlie different physical rules while being controlled by the same irrigation strategy:

(i) a reference model that misses the (negative) soil moisture-precipitation feedback, and

(ii) a feedback-model where precipitation is coupled to the soil-moisture.

In the reference model the precipitation is modelled as a purely stochastic process that is not coupled to the soil-moisture.

Bettina Meyer, bettina.meyer@nbi.ku.dk, EGU 2020