# Sphere semantics for aspect

Ahti-Veikko Pietarinen
Department of Philosophy
University of Helsinki
P. O. Box 9, FIN-00140 University of Helsinki
ahti-veikko.pietarinen@helsinki.fi*

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**Abstract**

Sphere semantics is based on Tarski's geometry of solids. It is evoked here to provide semantics for natural-language aspect, in particular the English progressive. Three virtues are argued for: (i) The approach extends interval semantics and omits its pitfalls, (ii) the semantics solves the imperfective puzzle, and (iii) the solution does not appeal to the strategy of eventual outcomes.

## 1 Introduction

To get a grip of aspectual distinctions of verbs has been elusive in logical approaches to natural-language semantics. Point-based structures of time and the classical Priorian tense logic (Prior, 1967) have turned out to be too weak. The interval-based semantics also lacks expressive resources and has drawbacks (Cresswell, 1985; Halpern & Shoham, 1991).

I will propose a new model and semantics of time based the mereological notion of spheres. Sphere semantics is based on Tarski's geometry of solids from 1927. Many aspectual distinctions may be characterised in a unifying logical manner in sphere semantics. I will confine the discussion to the English progressive.

Given a universe of closed spheres, an alternativeness relation is defined in terms of tangentiality. The fundamental notions of external and internal tangents, external and internal diameters and concentricity may then be defined.

Unlike in interval semantics, in sphere semantics events, states, processes and episodes (Bach, 1981) are not evaluated in segments of times but in closed spheres, which are primitives of the universe. Consequently, the problem of taking intervals as the primitive notion dispensing with points of time at the extremities of an interval disappears.

The English progressive is characterised as continuous action on spheres. Action is non-terminating in so far as a sphere is not exited via external tangents. Accordingly, tangential exit characterises completion. The mereological 'part of' relation differentiates between events holding in homogeneous and event holding in heterogeneous spheres. Non-duratives are null-diametric spheres.

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Further, we get a qualitative notion of possible worlds. External diametricity defines a passage from one event to another being maximally protracted, while internal diametricity captures that two subevents are maximally distant. The latter means that two spheres are produced: one for atelic and the other for telic events. That some subevent both begins and ends involves centric tangentiality.

## 2 The semantics of the progressive

**Tenses as sentential operators** Basic natural-language tenses are routinely considered in logical semantics as sentential operators:

$p$ = *John builds a house*.
P(ast) $p$ = *John built a house*.
F(uture) $p$ = *John will build a house*.

The semantics is routinely given in terms of possible-worlds semantics. This is fine, as far as it goes. However, this approach does not differentiate between simple tenses and PROGRESSIVES:

**?** $p$ = *John is / was / will be / would be building a house*.

A number of semantic methods to cope with progressive have been proposed, with little consensus so far (Galton, 1984, 1987; Halpern & Shoham, 1991; Landman, 1992; Lascarides & Asher, 1993; Verkuyl, 1993).[1]

**The Imperfective Puzzle** At the heart of the semantics of the progressive is the Imperfective Puzzle (Dowty, 1979; Lascarides, 1991).[2] Suppose we have

> **Scenario 1:** John stepped onto the street and walked to the other side.

Both (1) and (2) are true of this scenario:

*John crossed the street.* (1)

*John was crossing the street.* (2)

The most clear-cut possibility for the definition of truth of the progressive would be the following.

**P1:** PROG($p$) is true at $t$ if and only if the event $e$ of PROG($p$) is part of an event $e'$ of $p$.

For instance, the event $e$ is the progressive *crossing the street* in (2), and the event $e'$ is the imperfective *crossed the street* in (1).

However, consider now

> **Scenario 2:** John stepped onto the street, but twisted his ankle and quickly turned back.

[1] Halpern & Shoham (1991) propose an interval semantics for computational tasks, not for natural-language applications. Pietarinen (2005) suggests how to apply and extend their proposal to various aspectual constructions in natural language. See also Janasik et al. (2002); Pietarinen (2001, 2003); Pietarinen & Sandu (2004).

[2] I see no ‘paradox’ here, and thus call what Dowty and Lascarides term the ‘imperfective paradox’ the imperfective puzzle.

Now rival practical consequences are possible. For, on the one hand, there are the PERFORMATIVES (RESULTATIVES):

$$\textit{John was crossing the street} \not\Longrightarrow \textit{John crossed the street}. \quad (3)$$

On the other hand, there are the ACTIVITIES (NON-RESULTATIVES):

$$\textit{Mary was whistling the tune} \Longrightarrow \textit{Mary whistled the tune}. \quad (4)$$

Accordingly, the progressive has been attempted to be captured in terms of INERTIA WORLDS (Dowty, 1979). They are those possible worlds in which the 'natural courses of events' will hold.

**P2:** PROG($p$) is true at $t$ iff in all INERTIA WORLDS the event $e$ of PROG($p$) is part of the event $e'$ of $p$.

But now, suppose

> **Scenario 3:** John stepped onto the street inattentively while a bus was approaching him in high speed.

Mere inertia worlds are thus not sufficient, since progressives are defeasible by whatever surprising facts there may be. Thus, one may attempt the following amendment:

**P3:** PROG($p$) is true at $t$ iff in all 'normally-continuing worlds' the event $e$ of PROG($p$) continues after the time $t$ and is part of $e'$.

However, also this definition leads to well-known problems (Landman, 1992).

**Problems with the semantics of the progressive** Summarising, it is clear that continuing by way of the strategy of refining the truth-conditional definitions for progressives by attempting to take all possible exceptions that might occur into account, and otherwise imitating the strategies of tense logical semantics, is bound to remain unsatisfactory.

# 3 Interval semantics

**What is it?** Traditional modal temporal logic interprets formulas over TIME POINTS. However, for real life events, we need durations, intervals, continuants, differential equations, and other dynamic resources (Hamblin, 1972). The durations may furthermore be infinitesimally small.

For example, in qualitative physics, the event of *Water level increased by 5 cm* needs to be captured. As regards computational tasks, the events such as *The robot carried out the manufacturing task* or *The processor is reasoning about the information* are vital. In biology, evolution is, overall, a continuous process: *Species are evolving*.

As a first approximation, one might use the distinction between OPEN vs. CLOSED INTERVALS to capture the essence of performatives (Bennett, 1981):

**P4:** PROG(*cross the street*) is true at $I$ iff there exists some larger interval $I'$ such that $I'$ is OPEN, $I \subseteq I'$, and *John* $\in ||$*cross the street*$||$ at $I'$.

*John* $\in$ $||$*cross the street*$||$ means that the proper name *John* is in the extension of *cross the street*.

Now, it indeed appears that

$$\textit{John is crossing the street} \not\Longrightarrow \textit{John crossed / has crossed / had crossed the street.} \tag{5}$$

On the other hand,

$$\textit{John built a house in one month } (I^*) \textit{ last year} \Longrightarrow \textit{John is building a house at any } I \subseteq I^*. \tag{6}$$

Heterogeneous structures of intervals (i.e., no subinterval property) are those in which we do not take $\bigcup_{i=i}^{n} I$ over $I'$.

Intervals may also be represented by a pair of moments $\langle t_1, t_2 \rangle$ in which a proposition is true (Halpern & Shoham, 1991).

**Problems with Interval Semantics** Interval semantics presents us with a gamut of problems of various sorts:

- There is no notion of time at the extremities of an open interval.
- How to proceed from one open interval to another?
- Open intervals fail to capture TELICITY, such as *finish*, *complete*, *end*, *arrive*.
- There are no CULMINATIONS (*John's life be ending*).

We are thus unhappy with the dichotomy between open and closed intervals.

But how to devise a semantics for processes and other continuous expressions that nevertheless would account for at least some sense of termination or culmination?

# 4 Sphere semantics

**Basic concepts** My proposal is along the following lines. We replace the traditional truth-conditional notion of the interval semantics 'satisfaction by an interval $I$ and interpretation $M$' (in symbols, $\langle M, I \rangle \models p$), by 'satisfaction by a CLOSED SPHERE $s$' (in symbols, $\langle M, s \rangle \models p$).

Spheres are primitives of the system, and they give rise to a mereological system similar to Alfred Tarski's geometry of solids (Tarski, 1956/1927).

Unlike in interval logic, there are no unique, definite starting and ending points, while there are the notions of INITIATION and EXIT via INTERNAL and EXTERNAL TANGENTIALITY. Thus, in sphere semantics there is a well-defined notion of time at tangents.

The following comprise the basic concepts of sphere semantics:

INTERPRETATION $M = \langle \mathfrak{T}, V \rangle$;

TEMPORAL STRUCTURE $\mathfrak{T} = \langle \mathcal{S}, \sqsubseteq \rangle$, in which $\mathcal{S}$ is a set of spheres, and $\sqsubseteq$ is a 'part of' relation;

VALUATION $V$: $\Phi \rightarrow 2^S, S \subseteq \mathcal{S}$;

PROPER PART: $s_1, s_2 \in \mathcal{S}, s_1 \sqsubset s_2$ if $s_1 \sqsubseteq s_2$ and $s_1 \neq s_2$;

DISJOINTNESS: $s_1 <> s_2$ if there is no $s_3$ such that $s_3 \not\sqsubseteq s_1$ and $s_3 \not\sqsubseteq s_2$.

For convenience, we also define the ‘earlier-than’ ordering relation:

ORDER RELATION: $s_1 << s_2$.

This could as well be defined in terms of external tangentiality.

Continuity, or the fact that something is in progress, now comes as a primitive in spheres and their interconnection.

Based on this basic structure, we can define the notions of external and internal tangentiality, external and internal diametricity and concentricity. Further aspects may be defined following these procedures.

We work with the following

**Language $\mathcal{L}$:** $\phi \in \Phi ::= \mid p \mid \neg p \mid \varphi \wedge \psi \mid P\varphi \mid F\varphi \mid \langle \mathrm{ProgPerf} \rangle \varphi \mid \langle \mathrm{ProgAct} \rangle \varphi \mid \langle \mathrm{ProgExtD} \rangle \varphi \mid \langle \mathrm{ProgIntD} \rangle \varphi \mid \langle \mathrm{Punct} \rangle \varphi \mid$.

($\mathrm{Prog}$ = progressive, $\mathrm{Perf}$ = performative, $\mathrm{Act}$ = activity.)

The basic truth clauses for the atomic propositions, negation, conjunction, and the tense-logical past and future operators are as follows.

**Truth values:**

For all $p \in \Phi$, we have $\langle M, s \rangle \models p$ iff $s \in V(p)$.

$\langle M, s \rangle \models \neg\varphi$ iff $\langle M, s \rangle \not\models \varphi$.

$\langle M, s \rangle \models \varphi \wedge \psi$ iff $\langle M, s \rangle \models \varphi$ and $\langle M, s \rangle \models \psi$.

$\langle M, s \rangle \models P\varphi$ iff $\exists s' \in \mathcal{S}, s' << s$, and $\langle M, s' \rangle \models \varphi$.

$\langle M, s \rangle \models F\varphi$ iff $\exists s' \in \mathcal{S}, s << s'$, and $\langle M, s' \rangle \models \varphi$.

We next describe the definitions for different aspects of the progressive.

**Absence of External Tangents:** $\langle M, s \rangle \models \langle \mathrm{ProgPerf} \rangle \varphi$ iff there exists $s^* \in \mathcal{S}$ and

- given $s \sqsubseteq s^*$, $\langle M, s^* \rangle \models \varphi$ and
- for all $s' \in \mathcal{S}$, $s^* <> s'$, there are no $s_1, s_2 \in \mathcal{S}, s^* \sqsubseteq s_1, s^* \sqsubseteq s_2, s_1 <> s', s_2 <> s'$ such that $s_1 \sqsubset s_2$ or $s_2 \sqsubset s_1$, for which $\langle M, s' \rangle \models \varphi$.

In words, the assertion $\varphi$ is not true in any $s \sqsubseteq s^*$ that has an external tangent $s'$ in which $\varphi$ is true.

For example, it is seen that:

*John is crossing the street* $\not\Rightarrow$ *John crossed the street.* (7)

*John is crossing the street* $\not\Rightarrow$ *John will have crossed the street.* (8)

**Internal Tangentiality:** $\langle M, s\rangle \models \langle \mathtt{ProgAct}\rangle\varphi$ iff there exists $s^* \in \mathcal{S}$ and

- given $s^* \sqsubseteq s$, $\langle M, s^*\rangle \models \varphi$ and
- for all $s' \in \mathcal{S}$, $s^* \sqsubset s'$, there exist $s_1, s_2 \in \mathcal{S}, s^* \sqsubseteq s_1, s^* \sqsubseteq s_2, s_1 \sqsubseteq s', s_2 \sqsubseteq s'$ such that $s_1 \sqsubset s_2$ or $s_2 \sqsubset s_1$, for which $\langle M, s'\rangle \models \varphi$.

This means that $\varphi$ is true in all spheres $s^* \sqsubseteq s$ that are internally tangential to $s'$.

For example:

*John is walking* $\implies$ *John (has/had) walked*. (9)

**External Diametricity:** $\langle M, s\rangle \models \langle \mathtt{ProgExtD}\rangle\varphi$ iff there exist $s', s^* \in \mathcal{S}$ and

- $s', s^*$ are EXTERNALLY TANGENTIAL to $s$ and
- given $s_1, s_2 \in \mathcal{S}, s_1 <> s, s_2 <> s, s' \sqsubseteq s_1, s^* \sqsubseteq s_2$, then $s_1 <> s_2$.

As an example, the antecedent sentence implies the two sentences in the consequent:

*John is running a marathon* (at $s$) $\implies$ *John is preparing for the gun* (at $s'$), *John is exhausted* (at $s^*$). (10)

**Internal Diametricity:** $\langle M, s\rangle \models \langle \mathtt{ProgIntD}\rangle\varphi$ iff there exist $s', s^* \in \mathcal{S}$ and

- $s, s^*$ are INTERNALLY TANGENTIAL to $s$ and
- given $s_1, s_2 \in \mathcal{S}, s_1 <> s, s_2 <> s, s'$ EXTERNALLY TANGENTIAL to $s_1$ and $s^*$ to $s_2$, then $s_1 <> s_2$.

For example, the antecedent sentence implies the two sentences in the consequent:

*John is running a marathon* (at $s$) $\implies$ *John started to run a marathon* (at $s'$), *John ran the home stretch* (at $s^*$). (11)

**Concentricity:** $\langle M, s\rangle \models \langle \mathtt{Punct}\rangle\varphi$ iff there exists $s' \in \mathcal{S}$ for which $s'$ consists of all spheres CONCENTRIC with $s$, such that $\langle M, s'\rangle \models \varphi$. Concentricity is defined by the following two items:

- $s = s'$
- $s' \sqsubset s$, and given $s_1, s_2 \in \mathcal{S}$ EXTERNALLY DIAMETRICAL to $s'$ and INTERNALLY TANGENTIAL to $s$, then $s_1$ and $s_2$ are INTERNALLY DIAMETRICAL to $s$
- $s \sqsubset s'$, and given $s_1, s_2 \in \mathcal{S}$ EXTERNALLY DIAMETRICAL to $s$ and INTERNALLY TANGENTIAL to $s'$, then $s_1$ and $s_2$ are INTERNALLY DIAMETRICAL to $s'$.

Consider the following two examples:

*John broke the vase* $\not\Longrightarrow$ *John is breaking the vase*. (12)

*John owns the house* $\not\Longrightarrow$ *?John is owning the house*. (13)

Concentricity captures PUNCTUALS and STATIVES in terms of being defined at null-diametric spheres (‘truth-at-a-point’).

# 5 Solving the Imperfective Puzzle

The analogue to 'truth in homogeneous vs. heterogeneous intervals' is now 'truth in all vs. some subparts of a sphere'.

The approach solves the Imperfective Puzzle. To see this, observe that implications with resultatives (such as *win*) are blocked:

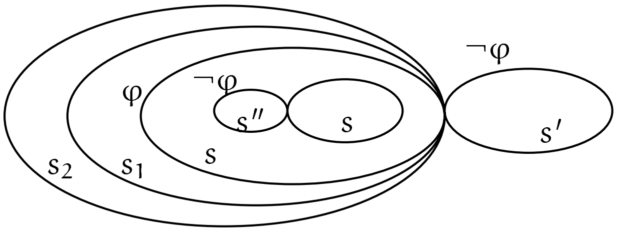
$$\textit{John was winning the race} \quad \not\Longrightarrow \quad \textit{John won the race}. \tag{14}$$

However, implications with non-resultatives (such as *run*) do hold:

$$\textit{John was running} \quad \Longrightarrow \quad \textit{John ran}. \tag{15}$$

The arrow in (15) may of course also be reversed.

In order to see (14), construct the model $M$ with the following interpretations ($\varphi$ = *win the race*):



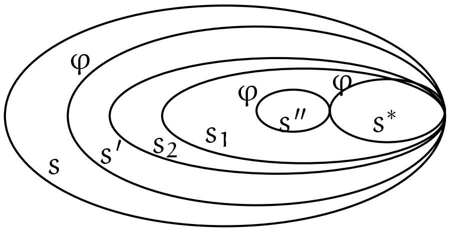

Here $s'' <> s$. Now, $\langle M, s\rangle \models \langle \mathrm{ProgPerf}\rangle\varphi$ while $\langle M, s\rangle \not\models P\varphi$, given the fact that the left hand side is consistent with $s'' << s$, $\langle M, s''\rangle \not\models \varphi$.

In other words, it is consistent and natural to assert the following two sets of sentences in the same model each:

$$\{\textit{John was winning the race, John did not win the race}\}. \tag{16}$$

$$\{\textit{John was losing the race, John won the race}\}. \tag{17}$$

In order to see (15), construct $M'$:




Here $s'' \sqsubset s$. Now, $\langle M', s\rangle \models \langle \mathrm{ProgAct}\rangle\varphi$ and $\langle M', s\rangle \models P\varphi$, given that the left hand side is consistent with $s'' << s^*$, $\langle M', s''\rangle \models \varphi$.

The following two sentences should be and indeed are compatible in the same model:

$$\{\textit{John was running, John ran}\}. \tag{18}$$

Sphere semantics does not solve the puzzle in terms of one of the versions of the subjunctively formulated EVENTUAL OUTCOME STRATEGY ('Whatever the process is, if it were to continue uninterrupted, then it would lead to the culmination').[3] The eventual outcome strategy would involve COLLATERAL KNOWLEDGE and falters on infinite processes. We solve the problem in the semantic terms of tangential exit or initiation of an activity at any point of the surface of the spheres.

[3]As proposed in Dowty (1979).

## 6 Outlook

One of the characteristic features of sphere semantics is that it defines a notion of time at the locations of moments in which an action or performance completes, while dispensing with the notion of time at an extremity.

The proposal of sphere semantics for the progressive aspect should be extended to a number of directions.

1. It needs to be studied in which ways sphere semantics is richer and expressively stronger that that of the interval semantics. Among other things, it is likely to be computationally intractable.

2. We may incorporate and study various structures of time, including branching time. For instance, we may organise the external tangencies into a backwards-linear structure. An interesting possibility that would arise is to have branching-time models with some overlap between the spheres.

3. Since distances between subevents need not be maximal, for internal tangencies non-maximality defines an event-internal earlier-than relation, useful in tackling the internal constituency of events referred to by uninflected verbs (the primary aspect).

By suitable restrictions, external tangencies thus define the sphere-theoretic analogue to a branching-time logic but with a richer structure: as different branches may also overlap, situations that refer to spheres with a common ancestor may share common parts. Overlaps may likewise concern internal properties of the primary aspect.

Furthermore, INFINITESIMAL notion of time according to which spheres do not consist of any metric of real numbers needs to be incorporated into the framework. It appears that having infinitesimals dispenses with the instantaneous time in the structures and, together with the topology of spheres, is thus more realistic for a comprehensive semantics for bringing out a plethora of aspectual distinctions in natural language.

Cross-linguistic applications are also on the agenda. In particular, empirical cases exist that support the view that progressive is a defeasible form of imperfective aspect (Alcazar, 2004).

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