þÿStudents In-class and Out of Class Mathematica

Rasmussen, Chris

Routledge
2021-04-16

þÿRasmussen, C, Fredriksen, H, Howard, E, Pepin, B & Rämö, J 20
In-class and Out of Class Mathematical Activities . in V Durand-Guerrier , R Hochmuth , E
Nardi & C Winsløw (eds) , Research and Development in University Mathematics Education
: Overview Produced by the International Network for Didactic Research in University
Mathematics . 1st edn , European Research in Mathematics Education , Routledge . https://doi.org/10.4324/978042

http://hdl.handle.net/10138/342832
https://doi.org/10.4324/9780429346859

Students' In-class and Out-of-Class Mathematical Practices

Chris Rasmussen[1], Helge Fredriksen[2], Emma Howard[3], Birgit Pepin[4], and Johanna Rämö[5]

[1]San Diego State University, United States, crasmussen@sdsu.edu, [2]University of Agder, Norway, [3]University College Dublin, Ireland, [4]Eindhoven University of Technology, The Netherlands, [5]University of Helsinki, Finland

Keywords: resources, assessment, flipped classroom, practices.

INTRODUCTION

In this chapter, we review the literature on university students' mathematical practices, both in-class and out-of-class. We use the term *practice* to refer to the various ways in which students engage in mathematics. In order to narrow the scope of this broad charge, we focus the literature review on various theoretical perspectives that are relevant for studying students' in-class and out-of-class practices, students' use of resources out-of-class, students' role in assessment practices, students' responses to active learning initiatives, and students' in-class mathematical practices. These themes emerged from our review of the 2016 and 2018 INDRUM proceedings, as well as several recent reviews of the broader terrain of university mathematics education research (e.g., Biza, Giraldo, Hochmuth, Khakbaz, & Rasmussen, 2016; Rasmussen & Wawro, 2017; Winsløw, Gueudet, Hochmuth, & Nardi, 2018; Winsløw & Rasmussen, 2018). Given these multiple sources, the papers highlighted here include contributions to the two INDRUM proceedings as well as publications more broadly. Not all themes were strongly represented in the INDRUM proceedings and hence in these cases we relied more heavily on the broader literature.

While the focus of this chapter is on what students do, we recognize that the ways in which students engage in mathematics does not happen in a vacuum. Instead, it is necessary to situate students' mathematical practices in relation to interactions with other students, the teacher, the mathematics, and resources. These elements constitute the vertices of what Rezat and Sträßer (2012) refer to as the socio-didactic tetrahedron. The first of six sections of this chapter develop in greater detail various theoretical perspectives that further help frame students' mathematical practices, both in-class and out-of-class, as well with respect to the social and institutional milieu. The focus of each of the four sections that follow can be framed in terms of the vertices of the socio-didactic tetrahedron and their respective links, in particular those along the students-mathematics-resources triangular face. Students' use of resources focuses more directly on the interaction between artifacts and the mathematics; students' role in assessment practices highlights the students-to-teacher and students-to-mathematics edges; students' responses to active learning initiatives consider more broadly the social and institutional milieu in which various forms of active learning occur; and, finally, students' in-class mathematical practices highlights student-student interaction and the student-mathematics edge of the tetrahedron. For the purpose of this chapter, the socio-didactic tetrahedron provides an orienting frame for seeing one way in which the various sections work together. The final section of this chapter lays our future research directions related to the preceding sections.

THEORETICAL DEVELOPMENTS RELATED TO STUDENT PRACTICES

In this section we review the theoretical developments related to research on student practices over the past five years, in particular, those theories that underpin the research reported in the subsequent sections, and the main theoretical frames that were evident in the proceedings of INDRUM 2016 and 2018 and in selected review chapters and articles.

A relatively new, albeit often used theoretical frame, is that of RAME (Resource Approach in Mathematics Education: Trouche, Gueudet, & Pepin, 2019), as evidenced in the subsequent section on students' use of resources out-of-class. RAME is associated with the Documentational Approach to Didactics (DAD: Trouche, Gueudet, & Pepin, 2018). DAD (and RAME) is concerned with students' interactions with resources (Gueudet & Pepin, 2016; Kock & Pepin, 2018). Gueudet and Pepin (2016) define student resources as anything likely to re-source (that is, to source again or differently) students' mathematical practice, leaning on Adler's (2000) definition of mathematics re-sources (in Adler's case used by teachers). Typically, a distinction is made between material resources, human/social resources, and cognitive resources (Pepin & Gueudet, 2018). For material resources, a further distinction has been made between curriculum resources (those resources proposed to students and aligned with the course curriculum), and general resources (which students might find/access randomly online). Curriculum resources are developed, proposed and used by teachers and students for the learning (and teaching) of the course mathematics, inside and outside the classroom. They can include text resources, such as textbooks, readers, websites and computer software, but also feedback on written work. General resources are the non-curricular material resources utilized by students, such as general websites (e.g., Wikipedia, YouTube). In terms of human/social resources, we refer to formal or casual human interactions, such as conversations with friends, peers or tutors. Cognitive resources relate to mathematical concepts that students may use to support their work with mathematics. The aim is to investigate students' practices, activities and learning through the lens of their interactions with resources in/for learning or studying.

Over the past decades, other theories have gained importance in research studies on student practices at university level in higher education mathematics. For example, Anastasakis, Robinson, and Lerman (2017) used Activity Theory to examine engineering students' use of resources. They found that students predominantly used resources provided by the university, and examination goals were the main consideration, with understanding mathematics and acquiring mathematical skills as lesser albeit important goals.

Other theories include the Theory of Didactical Situations (TDS) and the Anthropological Theory of Didactics (ATD). TDS (Brousseau, 1998) investigates relationships that develop in the interactions between students and content (within the teacher-student-content socio-didactic triangle) – the *milieu*. This typically includes student peers, the concepts at stake, and students' previous knowledge and understandings, all of which sets the tone for the *milieu*. Strømskag (2018) developed a model for instructional design based on TDS, which she used in teacher education. She showed that the model was instrumental in creating student teachers' awareness of the impact of the *milieu* on the nature of the knowledge developed by the pupils.

Chevallard's (1985) ATD is a model of mathematical knowledge that has been applied to investigate student activity. Simply put, it claims that there are two aspects to student (or human) activity: the practical, and the theoretical - these are the main components of so-called praxeologies. Importantly, ATD draws attention to institutional conditions, the ecology in which the activities take place. Barquero (2018) used this concept to explore the ecology of mathematical modelling practices through the systematic variation of teaching institutions. That is, she explored the variety of constraints in different university classrooms when modelling, where particular proposals of study and research paths were suggested to overcome some of the constraints.

Another widely used theoretical frame for studying student assessment practices is that of Formative Assessment (FA), which underpins the section on students' role in assessment practices (Black & Wiliam, 1998; Hattie & Timperley, 2007; Sadler & Good, 2006). One of the main principles in FA is feedback to support the development of student self-regulation (Butler & Winne, 1995). Another is that it encourages peer and self-assessment. At INDRUM, these theories have been frequently used to investigate meaningful assessment and student involvement in assessment practices (e.g., Copeland, Howard, Meehan, & Parnell, 2018; Gaspar Martins, 2018) as well as self-regulation and self-efficacy (e.g., Lahdenperä, 2018).

Another widely used widely framework for the design and analysis/evaluation of mathematical tasks is Realistic Mathematics Education (RME, Van den Heuvel-Panhuizen & Drijvers, 2014; Fredriksen, 2018). RME is a domain-specific instruction theory, based on six principles: the activity principle (the importance of doing mathematics); the reality principle (the importance of starting with 'real-life' problems that are meaningful for students); the level principle (student levels of understanding); the intertwinement principle (integrated curriculum); the interactivity principle (learning mathematics is a social activity favouring group work and whole class discussions); the guidance principle (or guided intervention). RME has provided one of the foundations for the development of active learning approaches in mathematics education at both school and university level. For example, Fredriksen (2018) took the perspective of analysing how RME task design could influence the learning experience of a group of first-year students in computer engineering in a flipped classroom setting.

Related to the notion of active learning, the construct of Inquiry-Based Mathematics Education (IBME) (e.g., Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019) has been prominent. In North America, researchers have successfully used this framing at university level to advance understanding of how students develop knowledge through active learning approaches. This has been explored in the section on students' in-class mathematical practices. At INDRUM, theoretical lenses that are related to inquiry approaches have been prominent (e.g., Gjesteland, Vos, & Wold, 2018; Monreal Galán, Ruiz-Munzón, & Barquero, 2018). For example, Jaworski and colleagues (2018) have used the notion of community of inquiry (consisting of teacher researchers, students, and analytic assistants) to develop tasks in a computer medium to promote students' learning of mathematics – students were seen here as "partners in task design".

Finally, an increasingly used theoretical frame is the theory of commognition (Sfard, 2008). Sfard's commognitive framework conceptualizes mathematics as a discourse - a form of communication consisting mainly of its word use, visual mediators, routines, and narratives. It offers researchers the opportunity to employ discourse analysis of the mathematics classroom, in particular how students engage in discourse (with peers, or with the teacher). For example, Schüler-Meyer (2018) explored students' discursive practices of defining of convergent sequences in a transition course (between high school and university). Using a commognitive framework, he identified central development stages of the definition from experiential to abstract.

We now turn to the sections on students' use of resources, students' role in assessment practices, students' responses to active learning initiatives, and students' in-class mathematical practices.

STUDENTS' USE OF RESOURCES OUT-OF-CLASS

Considering the recent surge of technology being incorporated into mathematics education, we consider the relationship between students and mediating artifacts; where, "mediating artifacts might be mathematics textbooks, digital technologies, as well as tasks and problems, language" (Rezat & Sträßer, 2012, p. 643). Digital resources include online videos (both lecture-capture and video cast), online instructor notes, apps (for example GeoGebra), websites, interactive textbooks, and online quizzes. Much debated is the use of recorded lectures, mainly in terms of students' potential learning benefits and the possible effect of decreased lecture attendance (Morris, Swinnerton, & Coop, 2019).

An area of INDRUM research has been identifying students' choices of artifacts or resources to support their student-mathematics relationship and understanding the reasons for these choices. This research reflects a growing area of interest in general for the mathematics education community.

Recorded lectures

A controversial education topic is the provision of recorded lectures. There is a view by some instructors that the provision of recorded lectures would lead to a decrease in lecture attendance (Witthaus & Robinson, 2015). Danielson, Preast, Bender, and Hassall (2014) examined both instructors' and students' views of recorded lectures in a veterinary science course. They investigated the relationship between learning, students' use of recorded lectures, and course characteristics. Course characteristics are considered as the teaching approach (straight lecture, interactive lecture and mixed lecture with group work), curricular coordination (tight versus loose), and course content (clinical or problem solving, basic science – applied, basic science – research, non-traditional). Danielson et al. (2014) concluded that when lectures are in the traditional format as opposed to an interactive format, students achieve better learning gains from viewing recorded lectures.

Wold (2018) examined students' viewing of recorded lectures in a first-year mathematics course. Her findings include that students watch nearly the entirety of the video rather than selecting parts of the lecture to view, perhaps suggesting that lecture-capture recordings are used as a replacement for lectures. Wold (2018) found that 21% of the class watched the lecture capture recordings regularly with relatively few viewings occurring prior to examinations. Although Howard, Meehan, and Parnell (2019) showed in a business mathematics module that the timing of online resources is related to the timing of the assessment, possibly indicating a difference between how students actively engage with lecture-capture recordings and short videos. While Wold (2018) examined students' use of a single resource, recorded lectures, it should be acknowledged that resources are predominantly not used in isolation.

Resource systems

The basis of resource system studies is the identification of which resources students use, and the understanding of why the patterns of usage occur or how students are using the resources. In INDRUM papers, the focus has been on first-year undergraduate students' use of resources, including online resources in a system (Gueudet & Pepin, 2016; Hillesund, 2018; Kanwal, 2018; Kock & Pepin, 2018; Quéré, 2016). These studies consider students as independent self-regulated learners. This interest in first-year undergraduate students may be owing to the increased proportion of mathematics time available for out-of-class practices, or to the effect of transitioning from a teacher-led approach at post-primary level to a self-directed and inquiry approach at university level.

For example, Quéré (2016) analyzed open-ended surveys of engineering students for evidence that students had reflected upon the autonomy of the university learner in comparison to the post-primary learner. This self-directed learning was associated with organisation skills, lesson preparation, management of peer-study groups and the responsibility to correct assignments. However, it can be observed that students try to transfer their post-primary level approach of using resources to university education (Gueudet & Pepin, 2016), including a reliance on worked examples.

Kock and Pepin (2018) examined students' use of all available resources in two modules, calculus and linear algebra, with the aim of understanding how students coordinate their learning. They found that a segment of the student population felt lost owing to the variety and volume of resources available to them at university. To explain this, Kock and Pepin (2018, p.1) argue that "the course organization and the alignment of curriculum materials with the learning goals had an impact on the students' choice and use of resources." They argue that when there is an alignment between these factors, students are able to direct their learning to appropriate resources for understanding (i.e., optimize the student-artifact-mathematics relationship).

In this vein, Kanwal (2018) examined a resource system in an online calculus module which featured a dedicated online interactive system "MyMathLab", tutorial videos and a textbook. MyMathLab contained formative assessments and students' assignments. Kanwal (2018) contends that examination goals drive students' resource choices and techniques, referring to this as students being pragmatic. An example of this pragmatic approach is that students identified videos as being useful for understanding concepts, however, they were also considered as time-consuming in comparison to the support of MyMathLab. Kanwal (2018) also found that websites were being used for explanation of concepts and to check answers. Whether it is the use of university provided resources through a Learning Management System or external resources such as websites, students are embracing online resources and research shows that students' attitudes towards these are positive even when the academic results are mixed (Trenholm, Alcock, & Robinson, 2012).

INDRUM studies have examined students' use of textbooks in the context of a resource system. For example, students use textbooks to find worked examples which are similar to their assignment questions and are seen as less useful than instructors' notes (Gueudet & Pepin, 2016), or/and textbooks are used as a repository of questions/examples with answers and are a source of important or relevant formulae (Kanwal, 2018). Quéré (2016) highlighted the role of exercises as central to mathematics, unlike in other disciplines. González-Martín and Hernandes-Gomes (2018) adopted the premise that university instructors draw on textbooks as a main source to guide their teaching. Their work follows from a study by Mesa and Griffiths (2012) which investigated mathematics faculty members use of textbooks . Combining these findings from INDRUM, it could be claimed that students are using online resources provided by instructors for explanation of concepts with textbooks being used primarily for assessment purposes. This aligns with studies which acknowledge a difference in how undergraduate students read mathematics, including textbooks, to the approach taken by mathematicians (Alcock, Hodds, Roy, & Inglis, 2015; Biza et al., 2016).

Qualitative and quantitative approaches to examining student use of resource systems

To analyse resource systems, INDRUM studies have taken a qualitative approach with interviews being a main source of data. For example, Hillesund (2018) examined students' choice of resources through combining interviews with students' Schematic Representation of Resource System (Pepin, Xu, Trouche, & Wang, 2017). This consists of a student drawing a representation of the resources they used for their module. He surmised that this method allows students to organize their thoughts in addition to imposing minimum interference by the researcher. Analyzing the representations, he noted that students organized their resource representation through considering either categories, situations, the purpose of the resources or the features of the resources.

However, this qualitative approach does not capture the data associated with online resources including the date, time, and length of use of a specific resource, and in some settings the data can be per page of a textbook or pdf. An alternative approach to examine students' choice of resources, which can use this data, is cluster analysis. This method can be used to find groups of students who use specific resources or use resources in a similar manner. For example, Inglis et al. (2011) investigated students' use of lecture-capture, live lectures in their university's mathematics learning support centre. They found through clustering that students tended to use predominantly only one of these resources and students who use face-to-face resources achieved higher module marks. Comparatively, Howard, Meehan and Parnell (2018) in their cluster analysis of online videos versus live lectures found four patterns of resource use. These are described as Lecture-Users, Video-Users, Dual-Users and Switchers. Cluster analysis can also be used to investigate the timing of resources. In contrast to this literature, we found far fewer INDRUM papers that relate the types of resources accessed by students to their academic achievement. Rather, INDRUM papers have focused on understanding students' choice of resources in light of the transition from post-primary learners to university learners.

STUDENTS' ROLE IN ASSESSMENT PRACTICES

Assessment has a key role in shaping learning behavior. In recent decades, higher education literature has highlighted the relation between assessment and learning by moving from assessment of learning to assessment for learning. At the same time, university mathematics education practices may still emphasize summative assessment and closed-book examinations (e.g. Iannone & Simpson, 2011). Also, mathematics students seem to prefer traditional assessment methods (Iannone & Simpson, 2015).

The influence of assessment on learning is not always positive. Assessment can, for example, lead to grade hunting or poor learning strategies. Negative effects of assessment can be seen in Fuller, Deshler, Darrah, Trujillo, and Wu (2016), who investigated the interaction of students' personal traits and anxiety in a developmental mathematics course. One of their findings was that exam anxiety seemed to be a contributing factor in explaining students not meeting the course requirements.

In this section, we provide selected examples of how assessment can be used to promote learning. The focus is on assessment processes in which students have an active role. We start by presenting studies from the INDRUM proceedings that touch on formative assessment, and then move into themes that embrace the students' active role: engagement with feedback, peer assessment and self-assessment. In terms of the chapter's theoretical framing, the socio-didactic triangle, these themes are located mainly on the edges "student-teacher" and "student–mathematics".

Formative assessment

When emphasizing assessment for learning, attention has been paid to formative assessment. Voigt (2016) studied a flipped classroom setting that contained regular quizzes. In his study, students' confidence in their mathematical abilities declined less in a flipped learning format than traditional lecture format. One explanation to this was that the quizzes flipped learning students completed made it possible for the students to make sure that they understand the core topics.

Gaspar Martins (2018) described a model of offering engineering students volunteer weekly quizzes that contributed to their final grade. The students could retry the quizzes until achieving the right answer. The students found the quizzes beneficial to their learning and felt that they made them study more.

Engagement with feedback

A core component of formative assessment is the feedback students receive. However, students might not use or be able to make sense of the teachers' feedback (e.g. Byrne, Hanusch, Moore, & Fukawa-Connelly, 2018). In order to understand students' ability to uptake feedback, Carless and Boud (2018) proposed a framework for feedback literacy. The elements of the framework are students' capability to appreciate the role of feedback, make judgements of the quality of their work, manage affects, and take action in response to feedback information.

Many of the elements of Carless and Boud's framework can be seen in the INDRUM 2018 study of Copeland et al. concerning business mathematics students' engagement with feedback. Students were given the opportunity to resubmit their quizzes, and were asked to identify, explain, and correct their errors. The results indicate that task type may affect students' remediations: conceptual tasks resulted more often in remediations that stated only a solution to the task, and lacked identification and explanation of errors. Students whose grades were lower tended to give solutions as remediations. The hypothesis is that the remediation process is not as beneficial for the lower achieving students as it is for the higher achieving students.

Landers and Reinholz (2015) studied students' participation in a feedback reflection activity in a mathematics course at a community college. Students were asked to read the feedback given to them by teachers and keep a written reflection log. According to Landers and Reinholz, students who took part regularly in the reflection activity saw it as an opportunity for self-assessment and grew as effective learners.

Peer assessment

In order to improve feedback processes, Nicol (2010) calls out for iterative dialogues, not only between students and teachers but also between students and their peers. Reinholz (2015) has introduced a peer assessment approach called Peer-Assisted Reflection (PAR). In PAR, students attempt a problem, reflect on their work, confer with a peer, and revise and submit a final solution. In Reinholz's study, PAR resulted in improvement of student performance in introductory college calculus.

Alqassab, Strijbos and Ufer (2018) studied the impact of peer-feedback training on pre-service mathematics teachers' ability to provide feedback as well as their beliefs concerning peer-feedback. The effect of peer-feedback training was linked to the level of domain knowledge of the students. They suggest that domain knowledge of students should be taken into account in peer-feedback activities.

Jones and Alcock (2013) used Comparative Judgement (CJ; Pollitt, 2012) as a peer assessment method that requires no assessment criteria. Students were presented pairs of student solutions, and in each case they had to assess which one was better. Based on this, solutions were ranked from lowest to highest. Jones and Alcock argue that CJ can suit assessing tasks that require conceptual understanding.

The benefits of peer assessment for students do not only come from receiving feedback. When producing feedback, students reflect and evaluate their own work (Nicol, Thomson, & Breslin, 2014). To elaborate how peer assessment and self-assessment are connected, Reinholz (2016) presented a theoretical framework, the Assessment Cycle. It builds on Kollar and Fischer's (2010) framework and identifies six possible phases in the peer assessment process: task engagement, peer analysis, feedback provision, feedback reception, peer conferencing, and revision.

Self-assessment

Making qualitative judgements about one's own performance is an essential component in lifelong learning and developing self-assessment skills and can be argued to be core to university studies. Self-assessment has been shown to have a positive influence on learning, for example, by improving self-efficacy and self-regulation (Panadero, Jonsson, & Botella, 2017).

There have been several studies on the accuracy of self-assessment. In the context of mathematics, Hosein and Harle (2018) studied students' ability to predict their grades and found that accuracy of predictions was associated with academic performance and mathematics confidence. However, Panadero, Brown and Strijbos (2016) argue that the accuracy in score prediction might not be as essential as the students' ability to describe the quality of their work and detect their strengths and weaknesses.

In the 2018 INDRUM conference, Häsä, Nieminen, Rämö, Pesonen, and Pauna described a digital self-assessment model (DISA) in which students regularly evaluated the quality of their learning outcomes, received feedback on their performance, and finally reflected on their learning and decided their own grades. The intended learning outcomes were made transparent through a detailed rubric. According to Häsä et al., students' perceptions of self-assessment indicated increased ownership of learning, but also revealed that self-assessment was a new and strange method to them. A study of the DISA model suggests that students associate self-assessment and the lack of exam with a deep approach to learning (Nieminen, Häsä, Rämö, & Tuohilampi, 2018).

STUDENTS' RESPONSES TO ACTIVE LEARNING INITIATIVES

Stanberry (2018, p. 960) defined active learning as a "process of education whereby students engage in activities like reading, writing, discussion, or problem solving that encourage analysis, synthesis, reflection and evaluation of class content." The term active learning encompasses many pedagogical strategies, such as collaborative learning, think-pair-share (Prince, 2004), and, lately, flipped classrooms (Bergmann & Sams, 2012). As such, the concept of active learning is quite flexible, as it includes various in-class initiatives engaging students in various forms of reasoning. However, this section will focus mainly on the above-mentioned frameworks of flipped classroom and IBME. There seems to be a certain momentum gaining towards the study of student-active and inquiry teaching approaches between the two conferences of 2016 and 2018. Considering students' practices, INDRUM 2016 included the presentation of two papers relevant to active learning initiatives, while the INDRUM 2018 proceedings presented four. The broader community of ERME also observes this trend. Skott, Mosvold, and Sakonidis (2018, p. 163-4) wrote about "the reform in mathematics education" which "promotes a vision of school mathematics that focuses on students' creative engagement in exploratory and problem-solving activities…".

Considering our chapter's instructional tetrahedron theoretical framing, flipped classroom and IBME will be centered mainly along the student-student and student-mathematics axis. However, the teacher-student interaction may be considered to have a vital role in these frameworks, due to the necessity of guidance and facilitation of the teacher.

Flipped classroom

The main advantage of the flipped classroom in connection to active learning is the additional space for in-class learning activities afforded by this pedagogical approach. Since much of the direct instruction can be reserved for out-of-class videos, the teacher may facilitate activities in-class that focus on key concepts with groups of students (Strayer, 2012). Several INDRUM authors have studied the effect of the flipped classroom approach in university mathematics education settings. Voigt (2016) did a statistical comparative analysis of N=427 undergraduate pre-calculus students on their experiences, attitudes and mathematical knowledge in a flipped classroom format compared to students in a traditional lecture format. He found that students in the flipped format were more positive about their in-class experience, felt more confident and cooperative in mathematical problem solving, and attained mathematical knowledge slightly better than students in traditional courses.

Fredriksen (2018) examined the learning experience of a group of first-year students in computer engineering in a flipped classroom setting. Following a single session with close attention to two groups of students collaborating, he found that students were able to utilize previous knowledge from the videos, in addition to other mediating means, successfully creating a mathematical model of a simulated double Ferris wheel movement.

Lim, Kim, and Lee (2016) examined the transition from a traditional to a flipped design in calculus and non-linear systems theory. The class size was relatively small, less than 20 learners in each. They found that the intensive in-class activities aided novice learners to gain insight from peer approaches. Furthermore, they claimed to be able to engage more in-depth discussions in class due to the preparatory videos, as they could repeatedly review difficult topics. Jungića, Kaurb, Mulhollanda, and Xinc (2014) described an intervention using flipped classroom in large introductory calculus classes with up to 342 enrolled students. Their in-class strategy was peer-to-peer instruction, utilizing clickers to get initial responses on multiple-choice questions highlighted on the auditorium screen. After the initial display of results on-screen, they were told to discuss and attempt to convince their neighbour about their choice, thus spurring discussion. After this discussion phase, students would vote again, and plenary discussions orchestrated by the teacher would eventually converge the answer towards the one aimed at by the teacher. Data collected from a Likert-scale questionnaire showed that students perceived the intervention as beneficiary to their learning experience.

Critical voices were raised among students in these studies towards the increased workload and the lack of interaction possibilities during out-of-class viewing of lecture content. However, the overall impression from students participating in these studies seem to be a positive attitude towards the effect of being better prepared, leading to increased interactions with peers. Furthermore, these results seem to be consistent across a range of class sizes, although ways to implement the flipped format would vary greatly.

Inquiry-based learning approaches

Inquiry-based mathematics education considers ways in which students are actively engaged in working with mathematics in a process similar to how mathematicians and scientists do it. This may involve looking for relationships, making conjectures and generalizations, interpreting solutions and communicating these by various means (Dorier & Maass, 2014; Laursen & Rasmussen, 2019).

Rasmussen and Wawro (2017) performed a comprehensive review of research in undergraduate mathematics education towards such learning approaches. It seems that a large body of research indicates an improved success rate among students exposed to such initiatives (we refer readers to the meta-analysis of Freeman et al., 2014). They also reported on studies indicating that inquiry approaches resulted in significant better achievements among women, a result consistent with findings in other active-learning environments such as Extreme Apprenticeship (XA) (Lahdenperä, 2018). This INDRUM paper presented a quantitative study comparing the XA framework with a more traditional lecture-based course layout on the same cohort of students. Besides the effects of diminishing gender differences, the results indicated enhanced self-efficacy and self-regulation among students in the XA course. Although IBME seems to have an advantageous effect on student practices, there are obstacles particularly related to pedagogical content knowledge among university teachers. Considering how and when to shift between students' discovery and teacher telling in inquiry teaching environments can be a challenge for instructors (Marrongelle & Rasmussen, 2008). Also, inability to facilitate unpacking students' reasoning in whole-class discussions may severely limit the learning potential of these initiatives. As such, there is a need to convey research in inquiry-based teaching during professional development for teaching mathematicians.

STUDENTS' IN-CLASS MATHEMATICAL PRACTICES

While still not the norm at the university level, there is increasing interest in IBME (Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019) where students learn mathematics by actively engaging in the authentic practices of mathematics. Our review of the INDRUM proceedings and the recent university mathematics education literature revealed the following two overarching themes related to this emerging trend: (1) Ways in which students engage in doing mathematics in class; and (2) Factors that contribute to student in-class engagement in mathematics. The first theme characterizes the nature or students' mathematical activity while the second theme examines factors that enable or constrain students' participation in the authentic practice of mathematics. This second theme is well-represented in INDRUM papers, while the first theme comes primarily from the broader literature. Thus, future INDRUM papers are needed to represent and further develop this first theme. In terms of the chapter's broader theoretical framing of the socio-didactic tetrahedron, these two themes load primarily on the student-mathematics interaction and the student-student interaction.

We use the term mathematical practice to refer to particular types of joint enterprises (Wenger, 1999) in which mathematicians customarily engage. Moschkovich (2002) describes customary activities of mathematicians such as conjecturing, arguing, abstracting, and generalizing. Rasmussen, Zandieh, King, and Teppo (2005) describe customary practices of mathematicians such as symbolizing, defining, modeling, theoremizing, and algorithmatizing. Common across these characterizations of practice is the focus on authentic mathematical activity and the reinvention of mathematics.

Much of this work is informed by the instructional design theory of Realistic Mathematics Education (RME) (Freudenthal, 1973; Gravemeijer, 1999).

Ways in which students engage in doing mathematics in class

This theme includes studies that investigate the prospects and possibilities for student reinvention of mathematics and their participation in the authentic practices of mathematics. Central to this line of research is the identification of productive, often informal, ways of reasoning that can be leveraged by students and instructional designers to reinvent significant and deep mathematical ideas and the creation of instructional sequences that foster student reinvention. For example, Larsen (2009; 2013) conducted a series of teaching experiments guided by the instructional design theory of RME in which he developed a sequence of tasks in which students generated a minimal set of rules for the set of symmetries of an equilateral triangle to reinvent the group axioms and the concept of isomorphism, as well as an approach to fostering student reinvention of isomorphism by having students determine how many groups could be formed with four elements. Similarly informed by RME, Rasmussen, Dunmyre, Fortune, and Keene (2019) identified key ways of reasoning and an instructional sequence in a first course in differential equations in which students reinvent a bifurcation diagram. Wawro, Rasmussen, Zandieh, Larson, & Sweeney (2012) discovered means by which students can reinvent the ideas core to span and linear (in)dependence based on their informal and intuitive reasoning with vectors. These studies are good examples of the processes and activities that can promote student reinvention of significant mathematical ideas and shows the power of RME as a guiding framework for the design and implementation of instructional sequences that support student reinvention.

The practice of creating and using algorithms (or algorithmatizing) is exemplified in the study by Lockwood, Swinyard, and Caughman (2015) in which a pair of undergraduate students successfully reinvented four basic counting formulas. Similarly to the previous studies on reinvention, this study takes seriously Freudenthal's recommendation (1973) to avoid an anti-didactic inversion where symbolic formalism precedes informal reasoning.

The practice of theoremizing, which refers to when students are engaged in genuine argumentation where conjectures are made and then justifications are created to support or refute the conjectures, is another area in which researchers are studying and documenting student reinvention (Rasmussen, Wawro, & Zandieh, 2015). Finally, we consider studies that take up students' participation in the practice of defining and the reinvention of definitions. Here we briefly review three of the many studies that have investigated student reinvention of definitions. Zandieh and Rasmussen (2010) for their part detailed the productive ways that students leveraged their concept image in service of creating and using a definition of spherical triangle. Swinyard and Larsen (2012) in their investigation of student reinvention of the limit definition proposed two key developments that were necessary for students to reinvent the formal definition. The first of these is a shift from an "x-first" perspective to a "y-first" perspective. The second key idea is the development of "an arbitrary closeness perspective to operationalize what it means to be infinitely close to a point" (p. 476). Lastly, Chorlay (2019) reported on a teaching experiment carried out within the framework of didactic engineering with year 12 students in France that led to student reinvention of a correct definition of the infinite limit for sequences. This empirical study suggested that some nonstandard expressions for the definition may be more consonant with student informal and intuitive reasoning that the standard definition.

Factors that contribute to students' in-class engagement in mathematics

The second theme related to student mathematical practices focuses on factors that either inhibit or positively contribute to students' productive and successful participation in mathematical activity. The range of factors investigated include peer-to-peer discussions, interaction patterns, and affective characteristics.

For example, Hoppenbrock (2016) analysed the role of clicker questions in triggering peer-to-peer discussion and the development of students' conceptual understanding. Informed by Steinberg's (2005) framing of the epistemological triangle in relation to acts of communication, he found that students' understanding of "for all … there exists" (AE) and "there exists… for all" (EA) statements were enriched by a clicker question that invoked high quality discussion of different interpretations of AE and EA expressions. The clicker question was designed to meet four task characteristics that previous research identified as supporting peer-to-peer interaction: meaningful, complex, need for different ability to be solved, and aim of level raising. The notion of level raising fits well with the previously discussed RME perspective on mathematizing. Analysis of peer-to-peer discussion of different interpretations of the AE and EA clicker question revealed two main level raising outcomes. The first was students' ability to find connections between their concept image and the symbolic expressions and the positive role of misunderstanding and mistakes in advancing their understanding of definitions.

Discussing and working with peers was also identified by Rämö, Oinonen, and Vihavainen (2016) to be positively associated with higher level of course performance in an introductory mathematics course focused on sets, functions, and proof. Grounded in the theoretical perspective of cognitive apprenticeship (Brown, Collins, & Duguid, 1989), the course was taught using a form of IBME, the Extreme Apprenticeship approach (Rämö, Oinonen, & Vikberg, 2015). In this approach, class time is structured by having students participate in practices that resemble those carried out by mathematicians. These practices align well with the earlier review in this section that focused on the nature of students' engaging in mathematical practices such as conjecturing, defining, modelling, etc. Consistent with the work of Fuller et al. (2016) that examined affective factors, Rämö et al. (2016) found students' motivation and learning strategies to be related to sufficient time and opportunities for peer-to-peer collaboration. As suggested by the previous studies as well as other larger scale studies (e.g., Freeman et al., 2014), peer-to-peer interaction can lead to higher levels of course performance. Rasmussen, Apkarian, Dreyfus, and Voigt (2016) examined ways in which engaging with someone else's reasoning relates to advances in reasoning for the person that decenters from their own way of reasoning. Based on fine-grained, qualitative analysis of individual interviews that approximated classroom peer-to-peer discourse, they found that higher levels of engaging with someone else's reasoning support growth in one's own reasoning. The significance of this work lies in explicating *why* peer-to-peer interaction is repeatedly found to lead to better performance on course outcomes. In a related analysis that used classroom video-recordings from this same course, Dreyfus, Rasmussen, Tabach, and Apkarian (2018) develop a theoretical and methodological approach (networking two different theoretical perspectives, abstraction in context and the emergent perspective) for investigating individual and collective mathematical progress.

DISCUSSING THE PAST AND LOOKING FORWARD

We conclude this chapter by looking forward at promising directions for future research and how future research might engage in cross-pollination of the themes of students' use of resources, students' role in assessment practices, students' responses to active learning initiatives, and students' in-class mathematical practices.

The cross-pollination of the four themes that capture students' in-class and out-of-class practices would, in our view, benefit from theoretical networking. While the frameworks discussed in the first section of this chapter have been used in their pure form, combining two or more theories has been shown to be beneficial for developing deeper understandings of particular phenomena (Bikner-Ahsbahs & Prediger, 2014). Moreover, some frameworks, such as Commognition, TDS, and ATD, which are used more broadly in RUME, have not been widely leveraged in studies that examine students' in-class and out-of-class practices, but these theories are well-positioned to examine relationships that develop in the interactions within the teacher-student-content socio-didactic tetrahedron. Hence we see these frameworks as good candidates for networking theories more commonly used in the domain of students' in-class and out-of-class practices. Doing so would add analytic power and provide greater continuity and coherence to the broader RUME field.

For example, the analyses of students' use of online resources outside of the classroom have mostly been based on a single module or a single resource. Research has rarely considered the ways and possibilities that students' out-of-class practices might complement assessment practices and/or in-class inquiry practices. Through examining the student-artifacts-content relationship across a programme, the effect of prior experience of resources or/and module design on this relationship can be established. Studies show that students' out-of-class practices are influenced by assessment goals. Kanwal (2018) suggested that a variety of tasks is needed to encourage students into a knowledge-based approach rather than an assessment-focused approach. Similarly, Gueudet and Pepin (2016) found that a knowledge-based approach was used when a significant alteration to a mathematical method was required. Future research might examine how students' out-of-class practices are impacted by the alteration of module tasks, which intersects well with what we know about students in-class practices.

Another area for cross-pollination is the use of online resources and interactive textbooks, which nowadays occurs on or off campus and in and out of the classroom. For example, in mobile learning students use their devices wherever they are, and they use them alone and/or with their peers to study the mathematics. These kinds of different study practices, in combination with digital resources, new forms of assessment, and new forms of participating in inquiry learning environments, need to be taken into consideration when investigating student learning of university mathematics. Such investigations seem ripe for cross-pollinating different in-class and out-of-class practices.

In terms of assessment, we regard this a still under-researched area in university mathematics education; this is reflected in the rather small number of INDRUM papers on the topic. More research is needed, as well as implications on how to transfer the research into practice. Attention should be given to preparing students for life-long learning by teaching them to make judgements of their own work and giving them an active role in the assessment process (Boud & Falchikov, 2006). However, we should not assume that assessment is viewed and experienced in the same way in all cultural and educational contexts. Panadero et al. (2016) point out that the effects of self-assessment on students may vary from culture to culture and call for cross-cultural research and awareness of socio-cultural values when studying self-assessment. We suggest the same for assessment in general. Also, from the viewpoint of the socio-didactical tetrahedron, the axis containing the nodes "conventions and norms about being a student and about learning" and "institutions" requires attention. Students and society expect that summative assessment is used for the purpose of certifying achievement. On the other hand, as Black and McCormick (2010) claim, since summative assessment has a great impact on what and how students' study, we should also focus on the learning benefits of summative assessment. This might require radical re-thinking of learning and teaching in higher education that brings together recent advances on IBME, which gives students more power over their own learning. A related assessment question is: how is learning affected if we give students more power over their assessment?

The body of research on flipped classroom and IBME has gained substantial magnitude. Most investigations consider only student satisfaction and performance, leaving little insight into the fundamental aspects that make these initiatives work or not (Fredriksen & Hadjerrouit, 2019). Thus, there is a definite lack of qualitative research considering how the participation in classroom activities is influenced by factors such as task design, out-of-class videos, and institutional settings. The networking of theories may feature a broader understanding of students' learning practices upon being subject to these approaches.

The research to date on students' in-class mathematical practices has shown the power in student reasoning to reinvent mathematics as they participate in mathematical practices. A next step for future research is the theoretical elaboration of learning trajectories that support student reinvention. We conjecture that a research agenda focused on learning trajectories and instruction based on these is likely to result in more generalizable and useful outputs for practitioners and researchers. Moreover, such an agenda has considerable potential to bring together not only the different themes presented here on students' in-class and out-of-class practices, but also the findings and advances on teacher practices (see Chapter 12 in this book).

Finally, we point to an area of research glaringly absent from the INDRUM proceedings – equity and inclusion. A research agenda on equity and inclusion can cut across the four themes of students' use of resources, students' role in assessment practices, students' responses to active learning initiatives, and students' in-class mathematical practices. For example, in what ways do innovative assessment practices and inquiry approaches privilege (or not) particular student demographic groups? Does the use of online platforms appeal more to certain student groups than others and if so, how does that impact learning and persistence in mathematics? While some research suggests that women benefit from IBME (Laursen, Hassi, Kogan, & Westin, 2014), other research (e.g., Johnson et al., in press) points to the possibility that student-centred classrooms may, depending on the instructor, reinforce gendered and racialized implicit biases. Making equity and inclusion a focus of research on students' mathematical practices, and of university mathematics education research at large, is an ethical and moral imperative.

REFERENCES

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(3), 205 - 224 .

Alcock, L., Hodds, M., Roy, S., & Inglis, M. (2015). Investigating and improving undergraduate proof comprehension. *Notices of the American Mathematical Society, 62*(7), 742-752.

Alqassab, M., Strijbos, J. W., & Ufer, S. (2018). Training peer-feedback skills on geometric construction tasks: role of domain knowledge and peer-feedback levels. *European Journal of Psychology of Education*, *33*(1), 11-30.

Anastasakis, M., Robinson., C. L., & Lerman, S. (2017). Links between students' goals and their choice of educational resources in undergraduate mathematics. *Teaching Mathematics and its Applications: An International Journal of the IMA, 36*(2), 67-80.

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education*, *45*(6), 797-810.

Barquero, B. (2018). The ecological relativity of modelling practices: adaptations of a study and research path to different university settings. INDRUM 2018 Proceedings, pp. 85-94.

Bergmann, J., & Sams, A. (2012). *Flip your classroom, reach every student in every class every day*. Washington DC: International Society for Technology in Education.

Bikner-Ahsbahs, A., & Prediger, S. (2014). *Networking of theories as a research practice in mathematics education*. Cham, Switzerland: Springer International Publishing.

Biza, I., Giraldo, V., Hochmuth, R., Khakbaz, A., & Rasmussen, C. (2016). *Research on teaching and learning mathematics at the tertiary level: State-of-the-art and looking ahead*. ICME-13 Topical Surveys. Springer.

Black, P., & McCormick, R. (2010). Reflections and new directions. *Assessment & Evaluation in Higher Education*, *35*(5), 493–499.

Black, P., & Wiliam, D. (1998). Inside the Black Box: Raising Standards through Classroom Assessment, *Phi Delta Kappan international*, *80*(2), 139-148.

Boud, D., & Falchikov, N. (2006). Aligning assessment with long-term learning. *Assessment and Evaluation in Higher Education, 31*(4), 399–413.

Brousseau, G. (1998). *La théorie des situations didactiques*, Grenoble: La Pensée Sauvage.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42.

Butler, D. L., & Winne, P. H. (1995). Feedback and self-regulated learning: A theoretical synthesis. *Review of Educational Research, 65*(3), 245-281.

Byrne, M., Hanusch, S., Moore, R. C., & Fukawa-Connelly, T. (2018). Student interpretations of written comments on graded proofs. *International Journal of Research in Undergraduate Mathematics Education, 4*(2), 228–253.

Carless, D., & Boud, D. (2018). The development of student feedback literacy: Enabling uptake of feedback. *Assessment & Evaluation in Higher Education*, *43*(8), 1315–1325.

Chevallard, Y. (1985) *La Transposition Didactique. Du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage (2nd ed., 1991).

Chorlay, R. (2019 – online first). Scaffolding without lecturing: Analyzing a pathway to a student-worded definition of limits at the secondary-tertiary transition. *International Journal of Research in Undergraduate Mathematics Education*.

Copeland, C., Howard, E., Meehan, M., & Parnell, A. (2018). Engaging with feedback: How do students remediate errors on their weekly quiz. INDRUM 2018 Proceedings, pp. 306–315.

Danielson, J., Preast, V., Bender, H., & Hassal, L. (2014). Is the effectiveness of lecture capture related to teaching approach or content type? *Computers & Education, 72*, 121-131.

Dorier, J.-L., & Maass, K. (2014). Inquiry-based mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 300-304). Dordrecht: Springer Netherlands.

Dreyfus, T., Rasmussen, C., Apkarian, N., & Tabach, M. (2018). The complexity of knowledge construction in a classroom setting. INDRUM 2018 Proceedings, pp. 286-295.

Fredriksen, H. (2018). Analysing tasks for the Flipped Classroom from the perspective of Realistic Mathematical Education. INDRUM 2018 Proceedings, pp. 296–305.

Fredriksen, H., & Hadjerrouit, S. (2019 – online first). An activity theory perspective on contradictions in flipped mathematics classrooms at the university level. *International Journal of Mathematical Education in Science and Technology*, 1-22.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences of the United States of America, 111*(23), 8410-8415.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht: Reidel.

Fuller, E., Deshler, J., Darrah, M., Trujillo, M., & Wu, X. (2016). Anxiety and Personality Factors Influencing the Completion Rates of Developmental Mathematics Students. INDRUM 2016 Proceedings, pp. 274–283.

Gaspar Martins, S. (2018). Weekly homework quizzes as formative assessment for Engineering students are a fair and effective strategy to increase learning? INDRUM 2018 Proceedings, pp. 105–114.

Gjesteland, T., Vos, P., & Wold, M. (2018). Mathematical modelling and activation – a study on a large class, a project-based task and students’ flow. INDRUM 2018 Proceedings, pp. 175-184.

González, A. S., & Hernandes-Gomes, G. (2018). The use of integrals in Mechanics of Materials textbooks for engineering students: the case of the first moment of an area. INDRUM 2018 Proceedings, pp. 115-124.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 2*, 155-177.

Gueudet, G., & Pepin, B. (2016). Students’ work in mathematics and resources mediation at entry to university. INDRUM 2016 Proceedings, pp. 444-453.

Hattie, J., & Timperley, H. (2007). The power of feedback, *Review of Educational Research*, *77*(1), 81-112.

Hillesund, E. (2018). Using schematic representation of resource systems to examine how first year engineering students use resources in their studies of mathematics. INDRUM 2018 Proceedings, pp. 185-186.

Hoppenbrock, A. (2016). Multiple choice questions and peer instruction as pedagogical tools to learn the mathematical language. INDRUM 2016 Proceedings, pp. 254-463.

Hosein, A. & Harle, J. (2018). The relationship between students' prior mathematical attainment, knowledge and confidence on their self-assessment accuracy. *Studies in Educational Evaluation, 56,* 32–41.

Howard, E., Meehan, M., & Parnell, A. (2018). Live lectures or online videos: Students' resource choices in a first-year university mathematics module. *International Journal of Mathematical Education in Science and Technology, 49*(4), 530-553.

Howard, E., Meehan, M., & Parnell, A. (2019). An exploration of the relationship between continuous assessment and resource use in a service mathematics module. *Eleventh Congress of European Research in Mathematics Education, Utrecht*, Netherlands: CERME.

Häsä, J., Nieminen, J. H., Rämö, J., Pesonen, H., & Pauna, M. (2018). Determining your own grade – Student perceptions on self-assessment in a large mathematics course. INDRUM 2018 Proceedings, pp. 366–367.

Iannone, P. and Simpson, A. (2015). Students' preferences in undergraduate mathematics assessment. *Studies in Higher Education, 40*(6), 1046–1067.

Iannone, P., & Simpson, A. (2011). The summative assessment diet: how we assess in mathematics degrees. *Teaching Mathematics and its Applications: An International Journal of the IMA, 30*(4), 186–196.

Inglis, M., Palipana, A., Trenholm, S., & Ward, J. (2011). Individual differences in students' use of optional learning resources. *Journal of Computer Assisted Learning, 27*(6), 490-502.

Jaworski, B., Treffert-Thomas, S., Hewitt, D., Feeney, M., Shrish-Thapa, D., Conniffe, D., Dar, A., Vlaseros, N., & Anastasakis, M. (2018). Student Partners in Task Design in a computer medium to promote Foundation students' learning of mathematics. INDRUM 2018 Proceedings, pp. 316-325.

Johnson, E., Andrews-Larson, C., Keene, K., Keller, R., Fortune, N., & Melhuish, K. (in press). Inquiry and gender inequity in the undergraduate mathematics classroom. *Journal for Research in Mathematics Education.*

Jones, I. & Alcock, L. (2013). Peer assessment without assessment criteria. *Studies in Higher Education, 39*(10), 1774–1787.

Jungića, V., Kaurb, H., Mulhollanda, J., & Xinc, C. (2014). On flipping the classroom in large first year calculus courses. *International Journal of Mathematical Education in Science and Technology, 46*(4), 508-520.

Kanwal, S. (2018). Engineering students' engagement with resources in an online learning environment. INDRUM 2018 Proceedings, pp. 145-154.

Kock, Z.-J. & Pepin, B. (2018). Student use of resources in calculus and linear algebra. INDRUM 2018 Proceedings, pp. 336-345.

Kollar, I., & Fischer, F. (2010). Peer assessment as collaborative learning: A cognitive perspective. *Learning and Instruction, 20*(4), 344–48.
Lahdenperä, J. (2018). Comparing male and female students' self-efficacy and self-regulation skills in two undergraduate mathematics course contexts. INDRUM 2018 Proceedings, pp. 346-355.
Landers, M., & Reinholz, D. (2015). Students' reflections on mathematics homework feedback. *Journal of Developmental Education, 38*(3), 22–36.
Larsen, S. (2009). Reinventing the concepts of groups and isomorphisms: The case of Jessica and Sandra. *Journal of Mathematical Behavior, 28*(2), 119–137.
Larsen, S. (2013). A local instructional theory for the guided reinvention of the group and isomorphism concepts. *Journal of Mathematical Behavior, 32*(4), 712–725.
Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education, 45*, 406-418.
Laursen, S., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics education. *International Journal of Research in Undergraduate Mathematics Education, 5*(1), 129-146.
Lim, C., Kim, S., & Lee, J. (2016). Designing the Flipped Classroom in Higher Education. In J. M. Spector, D. Ifenthaler, D. G. Sampson, & P. Isaias (Eds.), *Competencies in Teaching, Learning and Educational Leadership in the Digital Age: Papers from CELDA 2014* (pp. 245-258). Cham: Springer International Publishing.
Lockwood, E., Swinyard, C. A., & Caughman, J. S. (2015). Patterns, sets of outcomes, and combinatorial justification: Two students' reinvention of counting formulas. *International Journal of Research in Undergraduate Mathematics Education*, *1*(1), 27-62.
Marrongelle, K., & Rasmussen, C. (2008). Meeting new teaching challenges: Teaching strategies that mediate between all lecture and all student discovery. In M. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 167–178). Washington DC: Mathematical Association of America.
Mesa, V., & Griffiths, B. (2012). Textbook mediation of teaching: An example from tertiary mathematics instructors. *Educational Studies in Mathematics, 79*(1), 85-107.
Monreal Galán, N., Ruiz-Munzón, N., & Barquero, B. (2018). Central dialectics for mathematical modelling in the experience of a study and research path at university level. INDRUM 2018 Proceedings, pp. 155-164.
Morris, N. P., Swinnerton, B., & Coop, T. (2019). Lecture recordings to support learning: A contested space between students and teachers. *Computers & Education, 140*.
Moschkovich, J., (2002). An introduction to examining everyday and academic mathematical practices. *Journal for Research in Mathematics Education.*

Monograph, Vol. 11, Everyday and academic mathematics in the classroom, p. 1-11.
Nicol, D. (2010). From monologue to dialogue: improving written feedback processes in mass higher education. *Assessment & Evaluation in Higher Education*, *35*(5), 501–517.
Nicol, D., Thomson, A., & Breslin, C. (2014). Rethinking feedback practices in higher education: a peer review perspective. *Assessment & Evaluation in Higher Education*, *39*(1), 102–122.
Nieminen, J. H., Häsä, J., Rämö, J., & Tuohilampi, L. (2018). *Replacing exam with self-assessment: Reflection-centred learning environment as a tool to promote deep learning*. Proceedings of the 20th meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education. San Diego: RUME.
Panadero, E., Brown, G. T., & Strijbos, J. W. (2016). The future of student self-assessment: A review of known unknowns and potential directions. *Educational Psychology Review, 28*(4), 803–830.
Panadero, E., Jonsson, A., & Botella, J. (2017). Effects of self-assessment on self-regulated learning and self-efficacy: Four meta-analyses. *Educational Research Review, 22*, 74–98.
Pepin, B., & Gueudet, G. (2018). Curricular resources and textbooks. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Berlin, Heidelberg: Springer.
Pepin, B., Xu, B., Trouche, L., & Wang, C. (2017). Developing a deeper understanding of mathematics teaching expertise: an examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics, 94*(3), 257-274.
Pollitt, A. (2012). The method of Adaptive Comparative Judgement. *Assessment in Education: Principles, Policy & Practice, 19*(3), 281–300.
Prince, M. (2004). Does Active Learning Work? A Review of the Research. *Journal of Engineering Education*, *93*(3), 223-231.
Quéré, P.-V. (2016). Une étude de l'autonomie en mathématiques dans la transition secondaire-supérieur. INDRUM 2016 Proceedings, pp. 484-493.
Rasmussen, C., Apkarian, N., Dreyfus, T., & Voigt, M. (2016). Ways in which engaging in someone else's reasoning is productive. INDRUM 2016 Proceedings, pp. 504-513.
Rasmussen, C., Dunmyre, J., Fortune, N., & Keene, K. (2019 –online first). Modeling as a means to develop new ideas: The case of reinventing a bifurcation diagram. *PRIMUS*.
Rasmussen, C., & Wawro, M. (2017). Post-calculus research in undergraduate mathematics education. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 551- 581). Reston, VA: National Council of Teachers of Mathematics.

Rasmussen, C., Wawro, M., & Zandieh, M. (2015). Examining individual and collective level mathematical progress. *Educational Studies in Mathematics, 88*(2), 259-281.

Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing mathematical activity: A view of advanced mathematical thinking. *Mathematical Thinking and Learning, 7,* 51-73.

Rämö, J., Oinonen, L., & Vihavainen, A. (2016). Activity and performance on a student-centred undergraduate mathematics course. INDRUM 2016 Proceedings, pp. 494-503.

Rämö, J., Oinonen, L., & Vikberg, T. (2015). Extreme apprenticeship –Emphasising conceptual understanding in undergraduate mathematics. In K. Krainer & N. Vondrová (Eds.), Proceedings of the ninth congress of the European society for research in mathematics education (pp. 2242–2248). Prague: Charles University in Prague, Faculty of Education and ERME.

Reinholz, D. (2015). Peer-Assisted Reflection: A design-based intervention for improving success in calculus. *International Journal of Research in Undergraduate Mathematics Education*, *1*(2), 234–267.

Reinholz, D. (2016). The assessment cycle: a model for learning through peer assessment. *Assessment & Evaluation in Higher Education*, *41*(2), 301–315.

Rezat, S., & Sträßer, R. (2012). From the didactical triangle to the socio-didactical tetrahedron: artifacts as fundamental constituents of the didactical situation. *ZDM Mathematics Education, 44*, 641–651.

Sadler, P. M., & Good, E. (2006). The impact of self-and peer-grading on student learning. *Educational Assessment, 11*(1), 1–31.

Schüler-Meyer, A. (2018). Defining as discursive practice in transition – Upper secondary students reinvent the formal definition of convergent sequences. INDRUM 2018 Proceedings, pp. 537-546.

Sfard, A. (2008) *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. New York, NY: Cambridge University Press.

Skott, J., Mosvold, R., & Sakonidis, C. (2018). Classroom practice and teachers' knowledge, beliefs and identity. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty y ears of communication, cooperation and collaboration in Europe* (pp. 162-180). London: Routledge (ERME series inaugural volume).

Stanberry, M. L. (2018). Active learning: a case study of student engagement in college Calculus. *International Journal of Mathematical Education in Science and Technology, 49*(6), 959-969.

Strayer, J. F. (2012). How learning in an inverted classroom influences cooperation, innovation and task orientation. *Learning Environments Research, 15*(2), 171-193.

Steinbring, H. (2005). *The construction of new mathematical knowledge in classroom interaction* (Mathematics Education Library, Vol. 38). Berlin, New York: Springer.

Strømskag, H. (2018) A model for instructional design in mathematics implemented in teacher education. INDRUM 2018 Proceedings, pp. 422-431.
Swinyard, C., & Larsen, S. (2012). Coming to understand the formal definition of limit: Insights gained from engaging students in reinvention. *Journal for Research in Mathematics Education, 43*(4), 465–493.
Trenholm, S., Alcock, L., & Robinson, C. L. (2012). Mathematics lecturing in the digital age. *International Journal of Mathematical Education in Science and Technology, 43*(6), 703-716.
Trouche, L., Gueudet, B., & Pepin, B. (Eds.) (2019). *The 'resource' approach in mathematics education* (RAME). Cham: Springer International Publishing.
Trouche L., Gueudet G., Pepin B. (2018 - online first). Documentational Approach to Didactics. In: Lerman S. (Eds.) *Encyclopedia of mathematics education*. Cham: Springer International Publishing.
Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education. In S. Lerman (Ed.). *Encyclopedia of mathematics education* (pp. 521-525). Dordrecht, Heidelberg, New York, London: Springer.
Voigt, M. (2016). Comparative analysis of learning gains and students' attitudes in a flipped precalculus classroom. INDRUM 2016 Proceedings, pp. 511–519.
Wawro, M., Rasmussen, C., Zandieh, M., Larson, C., & Sweeney, G. (2012). An inquiry-oriented approach to span and linear independence: The case of the magic carpet ride sequence. *PRIMUS, 22*(8), 577-599.
Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.
Winsløw, C., Gueudet, G., Hochmuth, R., & Nardi, E. (2018). Research on university mathematics education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp.82-96). London: Routledge (ERME series inaugural volume).
Winsløw, C., & Rasmussen, C. (2018). University mathematics education. In S. Lerman (Ed.) *Encyclopaedia of mathematics education*. Cham: Springer Science+Business Media.
Witthaus, G. R., & Robinson, C. L. (2015). Lecture capture literature review: A review of the literature from 2012-2015. Centre for Academic Practice, Loughborough University Loughborough. Retrieved from https://dspace:lboro:ac:uk/dspace-jspui/handle/2134/25712.
Wold, M. (2018). Students' usage patterns of video recorded lectures in an undergraduate mathematics course. INDRUM 2018 Proceedings, pp. 370-371.
Zandieh, M., & Rasmussen, C. (2010). Defining as a mathematical activity: A framework for characterizing progress from informal to more formal ways of reasoning. *Journal of Mathematical Behavior, 29*, 57-75.