Controlling pattern formation and spatio-temporal disorder in nonlinear optics

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Abstract: We present a feedback control method for the stabilization of unstable patterns and for the control of spatio-temporal disorder. The control takes the form of a spatial modulation to the input pump, which is obtained via filtering in Fourier space of the output electric field. The control is powerful, flexible and non–invasive: the feedback vanishes once control is achieved. We demonstrate by means of computer simulation, the effect of the control in two different optical systems.

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1.- Introduction

Controlling spatio-temporal systems can be thought at first to be a task of by far greater magnitude than just controlling chaos in nonlinear dynamics. A large number of periodic unstable orbits in the neighborhood of the chaotic attractor is at the base of successful and flexible control of chaos techniques devised since the pioneering work of Ott, Grebogi and Yorke[1]. We show here that the spontaneous breaking of the translational symmetry in spatially extended systems implies the existence of a large number of unstable solutions and that flexible and powerful control techniques can be implemented in presence of both spatio-temporal order and disorder.

Our method relies upon the fact that a spatially extended system can have a simplified representation in Fourier space. This formed the basis of the control technique used in the paper by Lourenço[2] *et. al.* to suppress 1–D spatio-temporal chaotic motion. The features which we will discuss and use in our control are found in the

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spatial Fourier transform of the electric field and are common to many pattern forming optical systems. Similar properties can also be found in other types of driven dissipative nonlinear systems with rotational invariance[3]. This technique is therefore of general relevance.

It is also important to note that the Fourier transform (or far–field) in an optical system is routinely obtainable in experiments using a single lens. This provides the prospect of the technique being applied in a fully optical manner. It can thus take full advantage of the speed that such systems offer, because all–optical (analog) control is limited only by the response speed of the system.

2.- Stabilization and Tracking of Unstable Patterns via Fourier Space Techniques.

The behaviour of spatially extended nonlinear dynamical systems, including those found in optics, often has a more convenient description in Fourier space than in real space. Because of the spontaneous breaking of the translational symmetry, pattern forming systems possess a large number (in principle an infinity) of stationary states, some stable and the largest majority unstable, for any given set of parameter values. These features suggest the possibility of using control techniques which operate in the spatial Fourier domain to stabilize unstable states or to choose between alternative stable states. Optics has an advantage in that the Fourier transform of an optical field is simply its far–field image (in the paraxial approximation). This means that the operations of Fourier transformation and inverse transformation can each be performed with a simple lens system.

To investigate this idea we have chosen the model of a passive two–level medium in an optical cavity given by Equation (1)[4]

$$\partial_t E\psi = \quad -E\left[(1+i\theta)+\frac{2C(1-i\Delta)}{|E|^2+1+\Delta^2}\right]+E_I+i(\partial_{xx}+\partial_{yy})E\psi \qquad (1)$$

where E is the electric field, θ is the cavity detuning, Δ the atomic detuning, $2C$ the medium density expressed as an optical absorptivity and E_I is the spatially dependent input pump field. Also, the time t has been scaled by the cavity decay time. For simplicity we concentrate on the purely absorptive ($\Delta = 0$) case. Time–independent, spatially homogeneous solutions of Equation (1) become unstable for suitable values of C[5]. Above this "modulational instability" (MI) threshold perturbations of the form $e^{i\mathbf{K}.\mathbf{r}}$ experience growth if $|\mathbf{K}| \simeq K_c \pm \varepsilon$ where $K_c = \sqrt{-\theta}$[5], and ε is small close to threshold.

The condition on $|\mathbf{K}|$ corresponds to an annulus in Fourier space. Competition between modes in this annulus leads eventually to a steady state consisting of either two ("rolls") or six (hexagons) peaks. Depending on the parameter values either one, or both, of the roll or hexagon solutions may be stable. The model therefore allows investigations into both the stabilization of unstable patterns and the selection of a given stable pattern from a set of alternatives.

The characteristics of the states we wish to stabilize provide the physical basis for our control technique, which has two basic components[6]. To achieve full control over pattern formation, we must be able to suppress the growth of unwanted Fourier modes, and encourage the growth of modes necessary for the formation of the desired pattern. The stabilization of homogeneous and roll solution requires only the first of these, however, the control of patterns such as squares and hexagons, requires both.

To suppress undesired modes, we first of all take the Fourier transform of the output electric field. We then filter the field in Fourier space so that we are left with the modes which we wish to suppress. We then take the inverse Fourier transform, and add it to the pump beam as *negative* feedback, with corresponding strength s_1. Thus, the pump field acquires a spatial modulation which is determined by the filtering in Fourier space. The pump field can then be written as

$$E_I(x,y) = E_{I0}(1 - s_1 f_1(x,y)) \quad (2)$$
$$f_1(x,y) = \mathcal{F}^{-1}\mathcal{U}\mathcal{F}E \quad (3)$$

where E_{I0} is the magnitude of the plane–wave pump, s_1 is the feedback strength, $\mathcal{F}$ denotes the operation of Fourier transform of the electric field $E, \mathcal{U}$ describes the filtering operations in Fourier space and $\mathcal{F}^{-1}$ is the inverse Fourier transform.

Therefore, to stabilize the homogeneous solution, we suppress the growth of the modes on the annulus K_c by removing all the Fourier components but the annulus and adding the resulting field f_1 to the input pump, in accordance with (2). This, therefore, suppresses all pattern formation. To stabilize rolls, instead, we remove two diametrically opposite modes from the Fourier circle of magnitude K_c to obtain f_1. This suppresses the growth of the homogeneous solution as well as all patterns but the rolls. Note that the feedback vanishes as the rolls stabilize, ensuring that these rolls are indeed a solution of equation (1). Due to the rotational degeneracy, we are free to choose the orientation of the stabilized rolls.

If we now try to stabilize squares (four wave–vectors) by removing the corresponding wave–vectors from the feedback, we end with the stabilization of rolls in one of the two possible orientations, the roll pattern being more stable than squares. We must ensure the presence of all four wave–vectors and this is done by filtering the Fourier field to obtain the amplitudes a_i of the four modes necessary for squares and passing the field through an interferometer with a field rotating element in one arm. This ensures the presence of four modes of amplitude b_i given by

$$b_1 \propto -a_1 + a_4 \qquad b_3 \propto -a_3 + a_2$$
$$b_2 \propto -a_2 + a_1 \qquad b_4 \propto -a_4 + a_3 \quad (4)$$

We then take the inverse Fourier transform to construct $f_2(x,y)$ which is fed back as *positive* feedback to the pump. The feedback field now becomes

$$E_I(x,y) = E_{I0}(1 - s_1 f_1(x,y) + s_2 f_2(x,y)) \quad (5)$$

Such rotation guarantees the simultaneous growth of both pairs of roll wavevectors, thus stabilizing the square pattern. Similar ideas can be applied to the stabilization and tracking of hexagons[6]. The following animation generated from a computer simulation, shows a dynamical sequence of control where we obtain a hexagonal pattern from an initially stable set of rolls. The control is then switched off and the system returns eventually to a roll solution. Figure 1 displays the magnitude of the feedback used in the animation. The figure shows that when stabilization is achieved, the feedback vanishes.

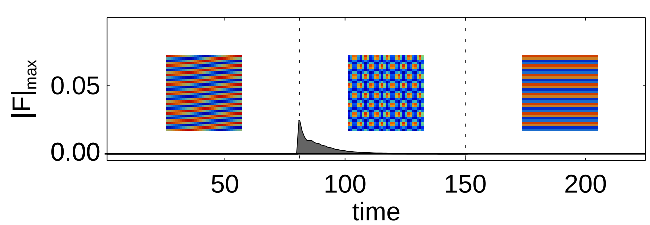

Fig. 1. The figure displays the patterns shown in the animation, along with the feedback (shaded curve) which vanishes when stabilization is achieved. The feedback is normalised to the maximum wave–vector of the pattern.

3.- Elimination of Spatio-Temporal Disorder in via Fourier Space Techniques.

The coupling of spatial and temporal degrees of freedom in nonlinear dynamical systems often leads to loss of spatio-temporal coherence. A simple–minded suppression of such behaviour can be obtained by reducing either the nonlinearity or the number of spatial degrees of freedom (as in the use of apertures in optical cavities). Such approaches always result in serious limitations for practical applications. A less restrictive restoration of spatial and temporal order is highly desirable in fields as diverse as laser and plasma physics as well as hydrodynamics.

We have applied our Fourier space method devised originally to stabilize unstable patterns, in order to eliminate spatio-temporal disorder. Here we apply the control to a liquid-crystal light valve (LCLV)[7], used in a configuration modelling a Kerr slice with feedback mirror[8]. The LCLV is a hybrid electrical/optical device which allows the phase shift of a 'read' beam to be controlled by the intensity of a 'write' beam. A feedback loop is provided by allowing the phase shifted input field to fall on the 'write' side of the LCLV after some distance of free space propagation. For input intensities I above some threshold, the plane wave output beam loses stability and forms a roll pattern in one dimension, and a hexagonal pattern in two dimensions. For larger values of I, these patterns become unstable and turbulent dynamics occur[8].

We can inhibit this instability by including an additional feedback loop into the LCLV setup with a fraction of the field propagating to the 'write' side of the LCLV extracted, filtered in Fourier space and then re-combined with the backward field. A schematic diagram of the system is shown in Figure 2. Here we focus on the 1–D LCLV, and the Fourier filter is chosen so as to allow all wave–vectors relevant to a roll pattern to pass through it unaffected. All other undesired wavevectors will see an additional loss due to the feedback, which is proportional to the feedback strength s[6]. The following animation sequence displays the stabilization of a roll solution in the 1–D LCLV from an initially turbulent state. An asterisk appears in the top right hand corner of the frame when the control operates. After some time, the control is removed and we return to a turbulent state.

In this way, we have been able to stabilize rolls (1–D) and hexagonal patterns (2–D)[10] well beyond their normal regime of stability. As in reference[6], by suitable modification of the filter we have also successfully stabilized rolls and squares in the (2–D) case[10], both unstable solutions in this system. It is important to note that this technique is non-invasive; the patterns it stabilizes are exact solutions of the original system without control and the control signal vanishes when control is established. As a quantitative measure of this fact, we compared the output field integrated over the transverse plane in the cases of 'turbulent' and controlled dynamics. The hexagonal pattern contains around 97% of the 'energy' of the 'turbulent' one. The technique has also been used to successfully control a broad area laser with finite size gain profile[10]. For certain ranges of parameter values, an erratic emission of 'optical vortices' occur, which travel across the beam. By applying control in exactly the same way as above, the vortex stream can be eliminated and a steady laser output obtained. The power output of the controlled state is the same, within the bounds of numerical accuracy, as for the uncontrolled state. This emphasizes the advantage of our technique over more conventionally used 'pinhole' methods and indicates the generality of the technique.

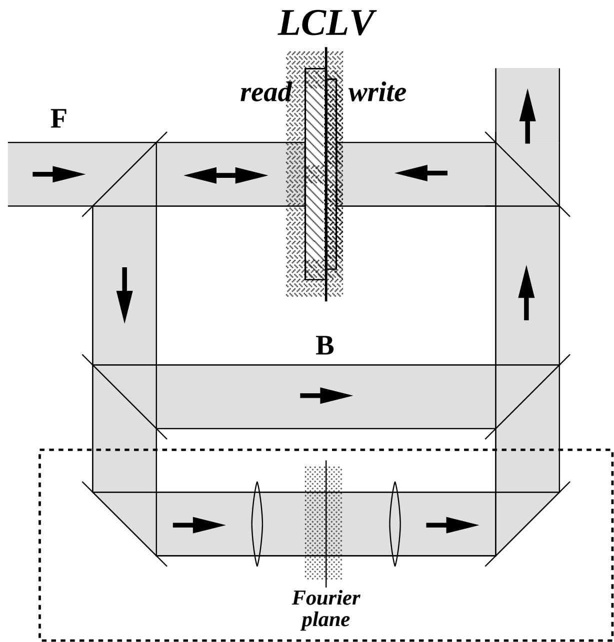

Fig. 2. Schematic diagram of the LCLV in a configuration modelling a Kerr–slice feedback with mirror, with an added control loop (dashed box). F denotes the plane–wave input field, and B the backward field. The Filtering is performed in the Fourier plane in the feedback loop.

4.- Conclusion

In this article we have shown that by applying suitable feedback, we can control pattern formation and suppress turbulent behaviour in nonlinear optical systems. The feedback takes the form of a spatial modulation of the input pump beam, which is constructed from the output field, after suitable filtering in Fourier space. We, have shown by means of simulation, the control of unstable hexagons in a two–level medium in an optical cavity, when rolls are stable, and the control of rolls in the 1–D LCLV in an otherwise turbulent state.