Capacity Optimization of EV Charging Networks: A Greedy Algorithmic Approach

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***Abstract*—In the recent years, there has been a steady increase in the use of electrical vehicles (EV). Their further adoption is becoming more dependent on the quality of service provided by the charging infrastructure. In this paper, the focus is on optimizing the charging infrastructure from the point of minimizing the service drop modelled using the standard M/M/c/c loss queue. To be exact, a mathematical model is proposed for the problem of optimizing capacities at individual stations in an EV charging network. The novelty is in considering the relation of capacity of a charging station to its arrival rate. Due to the non-linearity of the problem, a greedy algorithm combined with a local search is developed for finding near optimal configurations of the system. The new model is evaluated using real-world data for population density and existing charging infrastructure for metropolitan areas. The conducted computational experiments, show that charging networks optimized using the proposed model, significantly better reflect the state-on-the-ground than standardly used models, while maintaining a low service drop rate.**

***Index Terms*—Electrical vehicles, Charging Infrastructure, Greedy algorithm, Optimization**

I. Introduction

In the recent years, there has been an astonishing increase in the use of electrical vehicles (EV). EVs are essential for the progressive decarbonisation of road transport and have a positive impact on air quality in urban areas. Due to these facts, many countries have provided a wide range of incentives to ensure the expansion of EV use. Some examples are purchase subsidies, free parking, use of carpooling lanes, support for the provision of public charging networks, etc. Further adoption of EVs is highly dependent on the development of the public charging network [1], [2].

Although there is a similarity between the use of fuelling stations for internal combustion engine (ICE) vehicles and charging stations for EVs, there are also significant differences. In case of ICE vehicles, refuelling is only possible by visiting a gas station, while EVs have several options to recharge due to availability of different charger types [3]. Firstly, it is possible to use slow Level 1 chargers for home charging. Medium speed Level 2 chargers are frequently used for out-of-home charging at office and public parking lots, having a wide range of positive impacts [4]–[6]. It should be noted that smart scheduling of charging at such facilities can be used as a demand management system [7]–[9].

Finally, the use of fast Level 3 or fast DC chargers, which can fully charge a typical EV in around 30 minutes, has a high level of similarity to petrol stations. The cost of such chargers is typically up to ten times higher than Level 2 ones. Consequently, they are mostly installed by specialized providers, who try to attract the maximal number of EVs (costumers). Due to relatively longer charging times of EVs, the quality of service at charging stations becomes of the highest importance and is highly dependent on the available charging infrastructure [10]. Because of the high cost of fast public chargers, significant effort has been dedicated to optimizing the capacity of such systems. One part of the related research uses models based on location-allocation problems [11]. The developed models frequently correspond to NP-Hard problems that have been solved using different heuristic and metaheuristics methods them [12]–[14].

Methods for finding optimal locations of charging stations can focus on long distance and regional travel [15], [16] or on infrastructure within a city [17], [18]. Except for finding the optimal locations of electric vehicle charging stations (eCS), it is also necessary to specify their capacities. Although these two problems can be addressed jointly as in [19], [20], in real- world applications the locations of the eCS are often predetermined, due to some fixed outside factors, and there is only a need to optimize their capacities.

This paper addresses the problem of optimizing the capacities in a charging network with known locations of the eCS with-in a metropolitan area. In commonly used models for this problem, the arrival rate to an eCS is predetermined based on the locations and properties related to costumers like population density, points of interest, origin/destination pairs of trips, etc. The optimized networks acquired by such models frequently have a large discrepancy to real-world charging networks, some examples can be seen in [19], [20]. In this paper, a new model is proposed that considers the relation between the capacities of the eCSs in the network to the arrival rates. Due to the non-linearity of the problem, a greedy heuristic approach combined with a local search is proposed for finding near optimal solutions for the modelled system. The proposed model is used to analyse real-world charging networks in metropolitan areas based on population density and locations of the eCS.

The paper is organized as follows. In Section II, the details of the model are given. In the next section, the optimization method is presented. Section IV, on the other hand, is dedicated to the analysis of the conducted computational experiments. The paper is finalized with concluding remarks.

II. Model

In this section, the model for optimizing the capacity of eCSs in a charging network within a metropolitan area is presented. The optimization is done by minimizing the blocking probability, based on the standard M/M/c/c queue, using the Erlang B formula. Generally, in models of this type, it is considered that the arrival rate to an eCS depends on the location and properties related to costumers like population density, points of interest, origin/destination pairs of trips, etc.

In the proposed model, the influence of the capacity of an eCS on the arrival rate is also considered. The reason for this is that an eCS with high capacity, has a large number of customers and becomes attractive for other business like convenience stores, fast food restaurants and similar. This is frequently observed in case of gas stations. Due to the significantly longer time needed to charge an EV compared to ICE vehicles, it is expected that this becomes even more relevant for the former. On the other hand, the existence of such business attracts more costumers to the eCS. The mathematical rationale behind can be explained by statistical gains. For instance, as outlined in [21], larger serving capacity leads to better performance when compared to the case where serving capacity is distributed to multiple facilities.

Based on these assumptions, the corresponding model is defined as follows. Firstly, the details of the model in which the arrival rates are independent of the capacity of the charging station is presented. Later, this model is extended to include the previously mentioned dependence, in which larger facilities are assumed to attract more customers. Let us assume there are $|C|$ charging stations, in relation let us define $C = \{1, \ldots, |C|\}$ as the set of all charging stations. Let N be the total number of chargers that can be distributed among them. It is also assumed that each eCS has at least one charger and that there is a maximal capacity M that an eCS can not exceed. In relation, let us define variables n_c for each charging station $c \in C$ which indicate the capacity, or in other words, the number of chargers at the eCS. Let us use the notation n for the vector $n = (n_1, \ldots n_{|C|})$ containing this information for all the eCSs. The vector n satisfies the following constraint.

$$\sum_{c \in C} n_c = N \tag{1}$$

Next, let there be a set of P of population centers (PC). Each population center $p \in P$ has a weight (population size) w_p. It is assumed that the distances between each population center $p \in P$ and charging station $c \in C$ is known in advance and is equal to d_{pc}. The first step in specifying the model is to define which part of the population of PC p uses a charging station $c \in C$. It is natural to assume that this distribution is close related to the distance d_{pc} between this charging station and the population center. In the proposed model, it is assumed that this relation is inversely proportional to the quadratic distance between the PC and the charging station. Let us now define w_{pc} as the part of the population of PC p that uses eCS c as follows.

$$t_p = \sum_{x \in C} \frac{1}{d_{px}^2} \tag{2}$$

$$w_{pc} = w_p \frac{1}{d_{pc}^2} \frac{1}{t} \tag{3}$$

In (2), t_p is used to simplify the notation for the normalization of the weight distribution, it is simply equal to sum of all inverse square distances of population center p to all the charging stations $x \in C$. The part of the PC p that used CS c, w_{pc}, is proportional to the weight w_p and inversely proportional to square distance d_{pc}. This value is normalized using t. Now, the population that uses charging station $c \in C$ can simple be defined as

$$\lambda_c = \sum_{p \in P} w_{pc} \tag{4}$$

Eq. (4) states that the total size of the population that uses the eCS c, λ_c, is equal to the sum of all the users from each population center that use the charging station. The following step is specifing the blocking probability in the system based on the values of λ_c. The standard way of calculating blocking probability (or loss of service) is by minimizing the following sum:

$$F(n) = \sum_{c \in C} \lambda_c \mathcal{E}(n_c, \frac{\lambda_c}{\mu}). \tag{5}$$

In (5), λ_c is weight related to the amount of users visiting the charging station $c \in C$. Decision variables n_c show how many chargers should be placed at the charging station c. $\mathcal{E}$ is the Erlang B formula where the first parameter is the number of used resources (or chargers) and $\frac{\lambda_c}{\mu}$ is the load intensity. Note that since λ_c represents the arrival rate of EVs, μ denotes the average service (charges per hour) rate. The objective is to minimize the sum of blocking probabilities scaled by the population that is effected by the blocking for all charging stations. The notation $F(n)$ is used for the value of the blocking probability in the system that takes values between 0 and 1.

In the extended model, it is assumed that the population visiting station c is dependent on the number of chargers assigned to the stations in the system. Let use the simplified notation $\hat{\lambda}_c$ for the population that the eCS c attracts in the extended model. Note that $\hat{\lambda}_c$ is a function $\tilde{\lambda}_c(n)$ dependent on the number of chargers at each of eCSs. In the proposed model, it is assumed that the attractiveness of an eCS c is proportional to its capacity n_c. Based on this assumption let us specify $\hat{\lambda}_c(n)$ in the following text

$$\hat{t}_p = \sum_{x \in C} \frac{n_c}{d_{px}^2} \tag{6}$$

$$\hat{w}_{pc} = w_p \frac{n_c}{d_{pc}^2} \frac{1}{\hat{t}} \tag{7}$$

In (6), $\hat{t}_p$ is used to simplify the notation for the normalization of the weight distribution, it is equal to sum of all inverse square distances of population center p to the charging station $x \in C$ multiplied with the number of chargers at that eCS n_c. The part of the PC p that uses eCS c, w_{pc}, is proportional to the weight w_p and inversely proportional to square distance d_{pc} and proportional to the number of chargers at that eCS n_c. This value is normalized using $\hat{t}_p$. In the extended model, the population $\hat{\lambda}_c$ that uses the charging station $c \in C$ is acquired by substituting w_{pc} with $\hat{w}_{pc}$ in Eq. 4 as follows.

$$\hat{\lambda}_c = \sum_{p \in P} \hat{w}_{pc}. \tag{8}$$

The blocking probability of the system in the extended model $F(\hat{n})$ is acquired by substituting λ_c with $\hat{\lambda}_c$,

$$\hat{F}(n) = \sum_{c \in C} \hat{\lambda}_c \mathcal{E}(n_c, \frac{\hat{\lambda}_c}{\mu_c}). \tag{9}$$

III. Optimization of charging capacities

In this section, the method for optimizing the capacity of eCSs in the system is presented. The goal is to find the capacity of each of the $|C|$ stations, given n, based on the objective functions $F(n)$ and $\hat{F}(n)$, dependent on the used model. Due to the non-linearity of the problem, a heuristic approach is used. To be exact, a greedy algorithm is developed that iteratively expands the capacity of the eCSs. To further improve the quality of a solution generated in this way, a local search is also developed. In the following subsections, details of these two steps are provided.

A. Greedy algorithm

The idea of the greedy algorithm is to start with an initial distribution of chargers in the system. In the proposed approach, the initial solution is $n' = (1, \ldots, 1)$, which represents a system that has a singe charger at each station. The algorithm iterative expands this solution by adding one charger to one of the stations $c \in C$, or in other words, the value n_c is incremented by one. Let us use the notation n^{+c}, defined for each $c \in C$, as the configuration of the system acquired after adding a single charger to eCS c. Now the set of all potential stations for expansion for a partial solution n is,

$$\hat{C} = \{c \mid (c \in C) \wedge (n_c < M)\}. \tag{10}$$

The goal of the heuristic function is to make it possible to generate a solution that has a small value of the objective function. Because of this a natural heuristic function for selecting the best candidate for expansion of all $c \in C$ is the value of $F(n^{+c})$ or $F(\hat{n}^{+c})$. The proposed greedy algorithm is best understood by observing its pseudocode given in Alg 1.

Algorithm 1 Greedy Algorithm

```
Generate initial solution n = (1, ;1)
N' = N - |C|
while N' > 0 do
    s = argmin_{c∈Ĉ} F(n^{+c})
    n = n^{+s}
    N' = N'
end while
```

B. Local search

In this subsection, a local search for improving the quality of solutions acquired by the greedy algorithm is presented. It implements the simple idea of relocating a charger from an origin eCS o to a destination eCS d and testing if an improvement has been achieved. Let us define the function $Move(n, o, d)$ for a vector (system configuration) n and station indices o and d. The function $Move(n, o, d)$ returns a vector which is equal to, n except for the o-th element which is decreased by one and the d-th element which is increased by one. To be able to specify the local search, it is also necessary to define the set of all possible relocations. To be more precise, the set P of origin/destination pairs (o, d) for which $M(o, d, n)$ does not result in an invalid configuration. Formally, the set P is defined as

$$P = \{(o, d) \mid (o, d \in C) \wedge (n_o > 1) \wedge n_d < M\}. \tag{11}$$

Eq. (11), states that the number of chargers at the origin station n_o is larger than one. In addition, the number of chargers at the destination station n_d must be less than the maximal capacity. Using these definitions, the local search can be specified (see Algorithm 2).

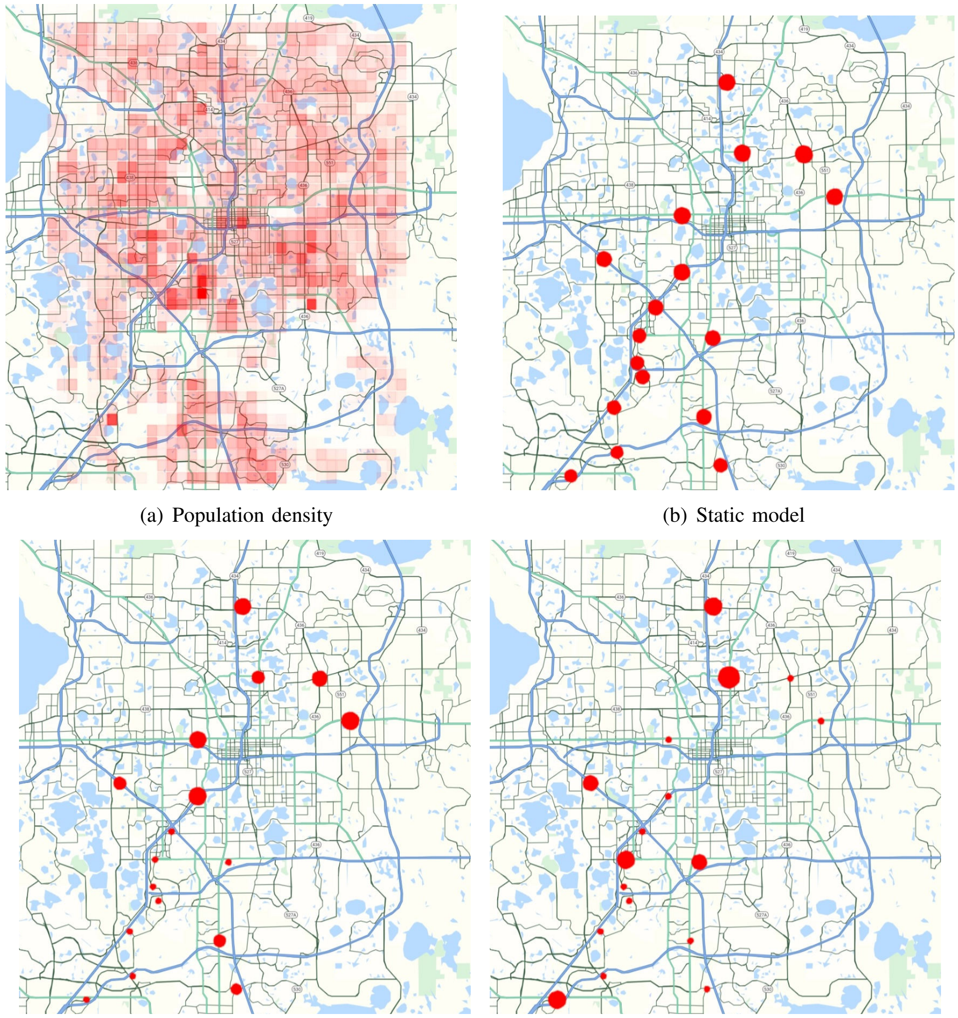
Algorithm 2 Local Search

```
repeat
    Calculate P using Eq. (11)
    for (o, d) ∈ P do
        n' = Move(n, o, d)
        if F(n') < F(n) then
            n = n'
            break;
        end if
    end for
until No Improve
```

In the Algorithm 2, it is assumed that n is the initial solution acquired from the greedy algorithm. Each iteration of the main loop attempts to find a relocation of a charger that produces an improvement. Firstly, the set of valid relocations P is calculated based on Eq. (11). Next, the inner loop iteratively tests if any relocations $(o, d) \in P$ produces an improvement. In case this is true for relocation (o, d), it is applied to the current best solution n and the inner loop is exited. This procedure is repeated until no further improvement is possible.

IV. Results

In this section, the results of the conducted computational experiments are presented. Their goal is to evaluate the effectiveness of the extended dynamic model that considers the relation between the charging capacities of eCS with-in the system on the costumer arrival rates. The proposed model



(a) Population density
(b) Static model
(c) Dynamic model
(d) Real-world

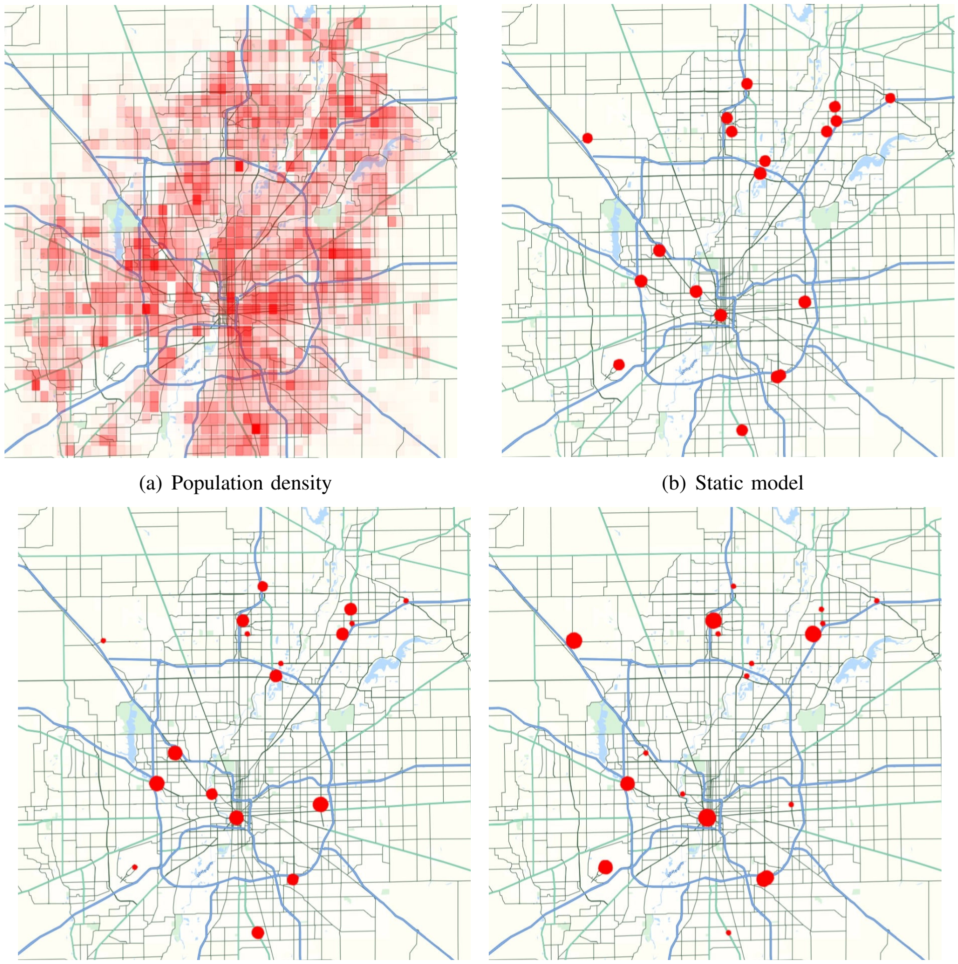
Fig. 1. Illustration of eCS capacities for Indianapolis, USA. The size of the red circles represents the capacity.

and optimization methods have been implemented in C# in Visual Studio 2019. The computational experiments have been performed on a personal computer running Windows 10 having Intel(R)Xeon(R) Gold 6244 CPU @3.60 GHz processor with 128 GB memory.

The evaluation is conducted through case studies for the cities of Indianapolis, IN and Orlando, FL both in the USA. The case studies are based on real-world data for the eCS [22] and population density [23]. The used data for the eCS is the following: the number of chargers, and latitude and longitude. Only eCS having fast (Level 3) chargers that are within a rectangular area containing the entire city of interest are used. The total number of chargers that is deployed in the systems $|C|$ is equal to the number of fast chargers that exists in the real-world data. Next, the maximal number of chargers at a station M is equal to the maximal number of chargers found at a station in the real-world data.

The real-world population density data, from [23], has been used for specifying the PC. This data provides the population density over 30 meter cells. In the model, each population center corresponds to a cell in a 40×40 rectangular grid covering the entire metropolitan area. The location of the PC p is equal to the center of the corresponding grid cell. The population w_p of PC p is equal to the sum of the population of all the cells from the density data are inside it. The distance between a PC p and eCS c is calculated directly based on the corresponding latitude and longitude. The last step in defining the model is specifying the normalization factor μ. It is assumed that the drop of service rate is less than $\%5$. The value of μ is selected so that the drop rate, calculated using objective functions $F(n)$ and $\hat{F}(n)$, acquired using the proposed optimization methods, is less than the $\%5$.

The results of the conducted computational experiments can be seen in Figs. 1 and 2 for Indianapolis and Orlando, USA, respectively. The time needed to optimize the charging capacities was less than 15 seconds for both case studies. The first thing that was observed that drop rate for the dynamic model was around 10% lower than in case of the static one.



(a) Population density

(b) Static model

(c) Dynamic model

(d) Real-world

Fig. 2. Illustration of eCS capacities for Orlando, USA. The size of the red circles represents the capacity.

It should be noted that this is not a direct comparison, since different objective functions are used, but an indication that the dynamic model better reflects the behavior of the EV users in the sense of drivers avoiding eCS with a high level of congestion. In case of both cities in the static model, the number of chargers at each eCS is evenly distributed and reflects the population density over the metropolitan area. This is in high contrast to the state-on-the-ground, where there are several eCS with large capacities and the rest of them have a small number of chargers. The distribution of chargers, in the dynamic model, has a structure that also has a small number of eCS with a large capacity. The effect of the dynamic model is most noticeable, in areas where there are several eCS with a small area where most of the chargers are centralized at one or two eCS. One of the noticeable difference between the optimized networks using this model to the state on the ground is the lack of high capacities eCS related to points of interest. One example is the airport in Indianapolis. This indicates that inclusion of this type of information in the model is needed for real-world applications.

V. Conclusion

In this paper, a new model for optimizing the capacities of eCS in a charging network within a metropolitan area is proposed. The main novelty of the approach is that it considers the impact of the capacity of an eCS as to its attractiveness to EVs drivers, or in other words, to its arrival rate. The model is used to optimize the charging network by minimizing the drop rate calculated using the M/M/c/c queue model. Near optimal network configurations are found using a greedy heuristic approach combined with a local search. The proposed optimization method is highly computationally effective and solves problem instances for real-world systems within a few seconds. The network configurations acquired in this way much better reflect the state-on-the-ground compared to the standard used models, while maintaining a low service drop rate.

In the future, we plan to expand this model to include more information about EV driver behavior like points of interest, origin/destination pairs of trips, and similar. Another direction of research is using more advanced tools for analysing the service drop probability, for instance the hypercube model

[24], [25].

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