Coherent Terahertz Smith-Purcell Radiation Assisted by Quasi-BIC

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***Abstract*-A free-electron-driven terahertz (THz) source based on coherent Smith-Purcell radiation (SPR) assisted by bound state in the continuum (BIC) is proposed in this paper. A reflection-type quasi-BIC with ultrahigh quality factor is formed by continuously tuning the structural parameter of the compound grating, which results from the wavevectors mismatch between the incident plane wave and the guided mode. When a sheet electron beam flies over the surface of the grating, the SPR will be enhanced at the resonant frequency of the quasi-BIC and become coherent accordingly. The forming process of the BIC through parameter tuning is analyzed, and the frequency spectrum of the coherent SPR wave demonstrates the enhancement effect of the quasi-BIC. The proposed quasi-BIC-assisted coherent THz SPR scheme may bring possibilities to the design and applications based on compact THz sources.**

I. INTRODUCTION

Since bound states in the continuum (BICs) are theoretically proposed in quantum mechanics by von Neumann and Wigner in 1929 [1], they have become a general wave phenomenon and exist in many wave-based systems, such as elastic waves, acoustic waves, and electromagnetic waves [2-4]. A BIC mode can be interpreted as a resonance mode whose frequency lies in the continuous radiation frequency spectrum. However, different from the normal resonance mode, the energy of the BIC cannot leak away because there is no radiative channel for it to couple with, which results in an ideal infinite quality factor Q at the BIC resonance frequency [5]. Symmetry-protected BICs are the most common ones to be found in the system where the coupling channels of specific leaky resonance to the radiation modes are prohibited by the symmetry of the system. For instance, any even mode at the Γ point of a photonic crystal slab is symmetry-protected BIC when the C_2 symmetry is preserved [6]. Besides, two kinds of BICs, specifically, Fabry-Pérot BICs and Friedrich-Wintgen BICs, can also be formed by continuously tuning the structural parameters until two or more radiation modes cancel each other due to the interference phenomena [7, 8]. The BIC structures with natural ultrahigh-Q properties have great potential for numerous electromagnetic applications, such as lasering, sensing, and filtering [9-11]. Moreover, optical BICs also enable an increasing light-matter interaction time, which can be used in a free-electron-driven system to enhance the beam-wave interaction strength including Cherenkov and Smith-Purcell radiations.

In 1953, Smith and Purcell demonstrated that diffraction-type radiation could be generated when the electrons move parallel to a 1D periodic system, which was later known as Smith-Purcell radiation (SPR) [12]. Based on the experimental results, the dependence of the wavelength on particle velocity, grating period, and radiation angle could be quantified as:

$$\lambda = \frac{L}{|n|}\left(\frac{1}{\beta} - \cos\theta\right), \tag{1}$$

where λ denotes the wavelength of the radiation produced at angle θ with respect to the orientation of the electrons, L is the period of the periodic system, and n is the diffraction order. $\beta = c/v$, where c is the speed of light and v is the velocity of the electron. The traditional SPR is incoherent, whose spectrum is broadband, and the radiation angle covers a wide range. However, when the radiation is bound to a narrow frequency band in a specific direction, the SPR becomes coherent [13]. The energy of coherent SPR can be significantly increased by employing compound grating, periodic bunched electron beams, and resonance structure [14]. Since BICs are unique leakless resonance modes with infinite Q-factor, they can be a perfect choice to generate coherent SPR. In 2018, Song et al. proposed a periodic grating structure which successfully achieved unusual Cherenkov radiation with enhanced efficiency based on a quasi-BIC [15]. Yang designed a 1D silicon-on-insulator grating to realize a coherent SPR generated from the electron beam coupling with BICs and quasi-BICs [16].

In this work, a terahertz (THz) coherent SPR source based on quasi-BIC was designed and studied. The dielectric compound grating was designed precisely to support the quasi-BIC through continuously tuning the structural parameters, and the forming process of the BIC is analyzed in Section II. When a sheet electron beam moves parallel to the grating, due to the ultrahigh-Q of the quasi-BIC mode, the SPR is strongly enhanced at the frequency of the resonance and becomes coherent, which is discussed in Section III. Finally, the paper is concluded in Section IV. This BIC-based scheme can also operate at the microwave and mid-infrared regions and has potential applications in compact THz radiation sources.

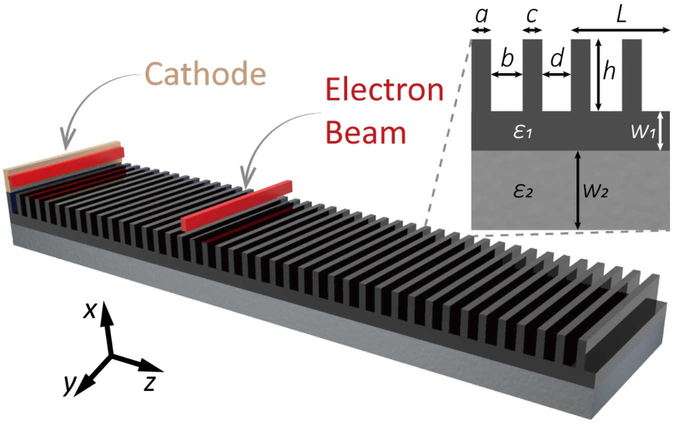

Figure 1. Model of the compound grating. The sheet electron beam, which is marked as a red sheet, is emitted from the left end, and travels parallel to the grating in the *z*-direction.

TABLE I
STRUCTURAL PARAMETERS OF THE COMPOUND GRATING

Parameter	L	h	a	b	c	d	w_1	w_2
Size(mm)	0.8	0.58	0.16	0.252	0.16	0.228	0.32	0.64

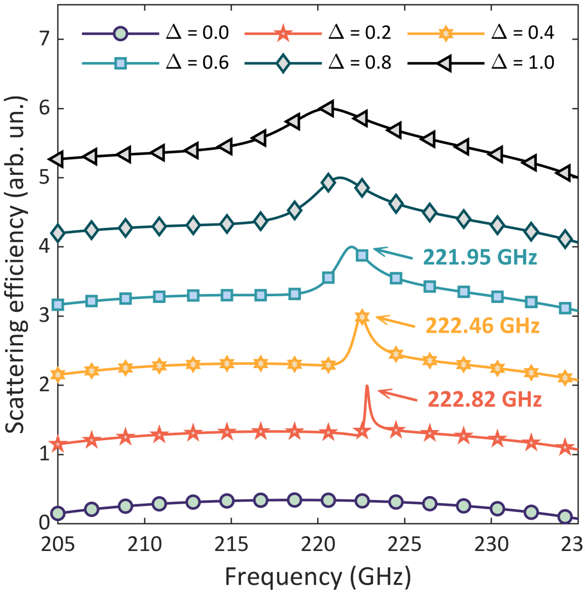

Figure 2. Reflection coefficient of the ideal compound grating for TM mode excitation at different structural parameters Δ = (*b*−*d*)/(*L*−*a*−*c*).

II. MODEL AND QUASI-BIC

The model of the compound grating is shown in Fig. 1, and the structural parameters are listed in Table I. The relative permittivities of the top and bottom regions are $\varepsilon_1 = 3.9$ and $\varepsilon_2 = 2.1$, respectively. Each part of the compound grating consists of four parts, whose widths are *a*, *b*, *c*, and *d*. The period is *L*. The depth of the grating is *h*, and the widths of two dielectric waveguide regions are w_1 and w_2. By continuously tuning the structural parameters of the grating, a reflection-type BIC can be achieved due to the mismatch of the wavevector between the incident plane wave and the guided mode of the dielectric waveguide [17] When the TM-polarized electromagnetic plane wave impinges the grating with incident angle $\theta_i = 0°$, the reflectance spectra of the infinite long compound grating at different Δ values are shown in Fig. 2, where Δ is a ratio of the structural parameters:

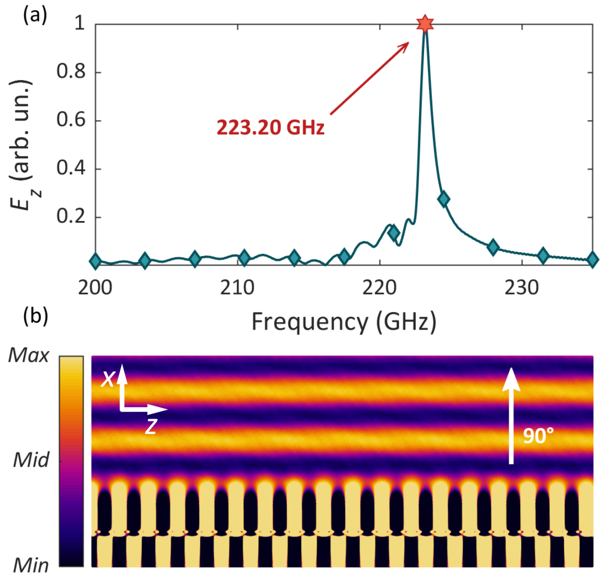

Figure 3. (a) Spectrum of E_z probed above the compound grating. (b) The field distribution at 223.2 GHz on *x*-*z* plane. The radiation wave propagates along the *x*-direction, which is perpendicular to the grating direction (*z*).

$$\Delta = (b-d)/(L-a-c). \tag{2}$$

As shown in Fig. 2, when Δ = 1, the reflection coefficient does not have obvious peaks which indicates weak resonance in the grating structure. However, as the decrease of Δ, the resonance becomes stronger around 222.82 GHz, and the resonant peak is getting steeper compared with previous scenarios. The Q factors can be estimated by:

$$Q = f_{peak} / \left| f_{peak} - f_{dip} \right|. \tag{3}$$

Hence, the value of the Q-factor will undoubtedly grow rapidly with the decrease of Δ according to (3). When Δ= 0, the resonance disappears completely and a BIC with infinite Q-factor is formed. Since such kinds of BICs are completely decoupled to the radiation channel, it cannot be excited [5]. However, if Δ is close to zero, the quasi-BICs, such as Δ = 0.2, 0.4, and 0.6 as shown in Fig. 2, will appear with ultrahigh Q factors and strong resonance which can still couple with the radiation and be excited by the incident wave [17].

III. COHERENT SMITH-PURCELL RADIATION

To utilize the compound grating and quasi-BIC for THz coherent SPR generation, the Δ was tuned to 0.05 with a Q factor of 5.1×10^3. A 122-keV sheet electron beam was launched in the left end of the grating and a magnetic field of 0.5 T was used to focus the electrons. The period number of the compound grating used in this paper was 50. As shown in Fig. 3(b), due to the diffraction of the grating, the evanescent wave carried by the electron beam was scattered into a plane wave which propagates along the −*x*-direction at 223.2 GHz. According to the analysis mentioned in Section II, the incident plane wave would be both enhanced due to the quasi-BIC with the ultrahigh Q factor and scattered into SPR that propagates along the *x*-direction. Therefore, the generated SPR would be significantly enhanced at the frequency of the quasi-BIC and thus become coherent. The frequency spectrum probed above the grating validated this phenomenon as shown in Fig. 3(a), where the magnitude of E_z at 223.2 GHz was much stronger than other frequencies.

IV. CONCLUSION

In this paper, a dielectric compound grating was used to form the quasi-BIC for THz coherent SPR. By continuously tuning the structural parameters of the grating, the forming process of the BIC and quasi-BIC of the grating was analyzed, and the quasi-BICs were formed around 222.82 GHz for the normal incident waves. A sheet electron beam with the appropriate energy was employed to excite the quasi-BIC and to generate the coherent SPR at the frequency of the quasi-BIC. The proposed quasi-BIC-assisted THz coherent SPR scheme can not only be scaled into the microwave and mid-infrared regions but also foster applications based on compact THz radiation sources.

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