Design of an Ultra-Low Spread Magnetic Cusp Gun Based on the Compensation Principle

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***Abstract*-In this paper, a design of a magnetic cusp gun with ultra-low velocity spread is proposed. Based on the Lagrange mechanics, the spread in the generalized angular momentum and the spread in guiding center radius can compensate each other for low velocity spread.**

I. Introduction

Gyrotrons based on the electron cyclotron maser have demonstrated their capability of generating high-power and high-frequency electromagnetic radiation with high efficiency [1]-[3]. Based on the synchronization between waveguide modes and particles, the radiation frequency can be approximately determined by as [3]

$$\omega \approx \frac{seB_0}{\gamma m_0} \tag{1}$$

where ω is the radiation angular frequency, e is the electron charge, m_0 is the rest mass of electrons, B_0 is the strength of the external magnetic field, s is the harmonic number, γ is the relativistic factor. Based on (1), B_0 increases linearly when ω elevates. To effectively lower the magnetic field strength, harmonic interaction is employed in gyrotron design. However, high-harmonic interaction still encounters mode competition. To partly suppress the mode competition, using the large orbit beams (LOB) become an option in gyrotron designs and experiments. In the past decades, the way to generate LOBs was widely investigated. There remains many theoretical and experimental research work about generating and the properties of LOBs. Compared with small orbit beams, the compression ratio is relatively large, and the spread is large as well. In this paper, the cause of velocity spread is under exploration based on the Lagrange mechanics in Section II. Simulation results and related analysis is demonstrated in Section III. The conclusion is finally drawn in Section IV.

II. Theoretical Analysis Based on the Lagrange Mechanics.

Normally, the motion and trajectories of particles in an electron gun are theoretically analyzed based on the Lagrange mechanics. The Lagrangian for the dynamics of relativistic electrons could be written as [4]

$$L = -m_0c^2\left(1-\frac{\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2}{c^2}\right)+e\phi-er\dot{\theta}A_\theta \tag{2}$$

where ϕ is the scalar electronic potential, r, θ, z are the coordinates in the cylindrical system, and A_θ is the magnetic vectorial potential. According to the conservation of generalized angular momentum (GAM), there remains a relation as follows

$$P_{\theta c} = -eB_{zc}r_c^2 = -eB_0\left(r_g^2 - r_L^2\right) \tag{3}$$

where $P_{\theta c}$ is the GAM in the emission surface of the cathode, r_c is the radius of the emission surface, B_{zc} is the magnetic field strength in the emission surface, r_g is the guiding center radius, r_L is the Larmor radius of gyrating electrons under the magnetic field of B_0. Based on (3), the relationship between the spread of GAMs, guiding center radiuses and velocities can be expressed as

$$\frac{\Delta v_t}{v_t} = \frac{r_L^2 - r_g^2}{r_L^2}\cdot\frac{\Delta P_{\phi c}}{2P_{\phi c}} + \frac{r_g^2}{r_L^2}\cdot\frac{\Delta r_g}{r_g} \tag{4}$$

where v_t is the transverse velocity of electrons. When the LOB is ideal enough, i.e., rL ≫ rg, equation (4) can be simplified as

$$\frac{\Delta v_t}{v_t} = \frac{\Delta P_{\phi c}}{2P_{\phi c}} + \frac{r_g^2}{r_L^2}\cdot\frac{\Delta r_g}{r_g} \tag{5}$$

, which is just like the expression in [5]. Equation (4) quantitatively demonstrated the relationship between the spread of GAMs, guiding center radiuses and velocities, and provide a possibility to lower the velocity spread. The spread in initial GAM is determined by the profile of the magnetic field in the emission surface and the geometry shape of the emission surface. The spread of the guiding center radius is more complicated, there remain many factors. According to the previous study, the guiding center radius can be determined as [4], [6]

$$\dot{r} = \frac{\omega_c\zeta}{2}\sin^{-1}\frac{1}{\eta} + v_{r0} \tag{6}$$

where ζ is a scale factor, ω_c is the electron cyclotron frequency, $\eta = v_0/r_0\omega_c$, v_0 is the injection beam velocity and r_0 is the injection beam radius.

III. Simulation Results and Related Analysis

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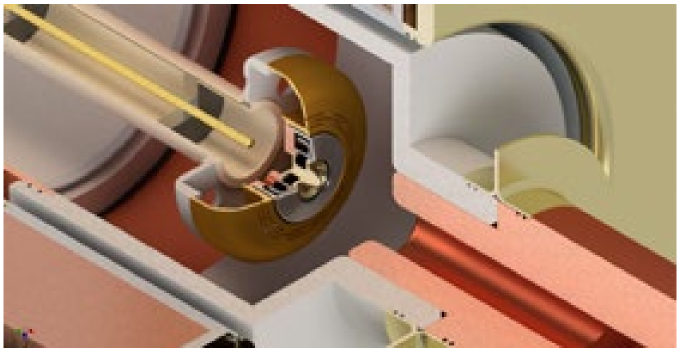

Fig. 1 Design scheme of the electron gun.

The design scheme of the magnetic cusp gun is shown in Fig. 1. The electron gun is operating under 80 kV, 1 A, and 9 T. From the simulations, the guiding center radius is 7.36% of the operating wavelength. The pitch factor is ~1.5. The velocity spread is <2%. To analyze the compensation, the distribution of the initial GAM and the guiding center radius is shown in Fig. 2. Fig. 2 demonstrated various distributions in different oblique angles of the inner focusing electrode. There remain two parts on the right-hand side of (4). The first part is related to the spread of GAM, which is represented as δ_{GAM}. The second part is related to r_g, which is represented as δ_{rg}. The two parts are compressed during the sweep of those parameters. The distribution diagram is divided into two parts. The first parts are the pure-color regions which are filled with red or blue only. The second are overlapping regions. In pure-color regions, the electrons are not critically compensated. In pure red regions, no electrons have δ_{GAM} closely equal to δ_{rg} of their own, which means that such electrons are under-compensated. In pure blue regions, the number of electrons having specific δ_{GAM} is not in agreement with that of an electron having equal δ_{rg}. Consequently, the electron in this region is not likely to be critically compensated. Only in overlapping regions, there keep possibilities that the δ_{GAM} and δ_{rg} of the electrons compensate each other.

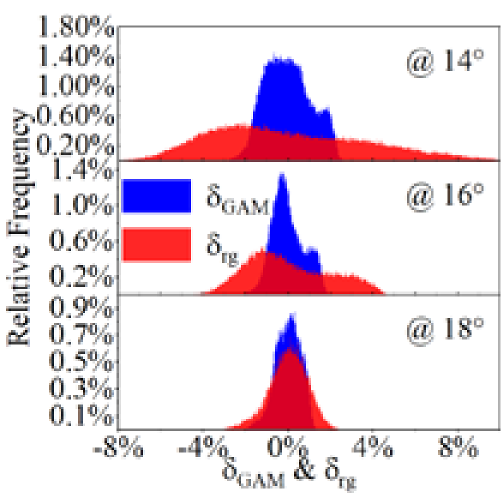

Fig. 2 Distribution of initial GAM and the guiding center radius.

IV. Conclusion

In this paper, a design of a magnetic cusp gun with ultra-low velocity spread is proposed. Based on the Lagrange mechanics, the spread in GAM and guiding center radius can compensate each other to generate LOBs with low velocity spread. The distribution of GAM is influenced by the profile of the magnetic field in emission surface and the geometry shape of the emission surface. The distribution of guiding radius can be change by many factors. The oblique angle of the inner focusing electrode is an option. When the spread of GAM and guiding center radius matches well, the generated beams normally demonstrate low velocity spread.

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