A hybrid success history-based adaptive differential evolution and manta ray foraging optimization for multi-objective truss optimization problems

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Abstract

In this paper, a hybrid multi-objective metaheuristic algorithm based on the manta ray foraging optimization (MRFO) and the success history-based parameter adaptive differential evolution (SHADE) is developed to solve multi-objective truss optimization problems, called MO-SHADE-MRFO. SHADE is a variant of differential evolution with high performance in solving optimization problems, and MRFO is a novel metaheuristic algorithm inspired from the behavior of manta rays. In the proposed algorithm, the updating mechanism of MRFO is embedded into the SHADE, to enhance global convergence of SHADE for multi-objective truss optimization problems. The design problem is to minimize both structural mass and compliance subjected to stress constraints. Six benchmark truss optimization problems, including 10-bar, 25-bar, 37-bar, 120-bar, 200-bar and 942-bar trusses, are utilized to test the effectiveness of the proposed algorithm. The performance of the proposed algorithm is compared with nine state-of-the-art algorithms, in terms of metrics including hypervolume, inverted generational distance, and spacing-to-extent. The experiment results demonstrate that the proposed algorithm can obtain the best statistical values of metrics and the lowest standard deviation values in most test problems, which is more accurate than the compared algorithms. The Pareto solutions obtained by the proposed algorithm are well-distributed and smooth in each problem.

Key words: *Truss design; Multi-objective problem; Meta-heuristics; Success history-based adaptive differential evolution; Manta ray foraging optimization*

1. Introduction

The optimal truss design is a very challenging task usually solved by the cumbersome trial-and-error in the practical engineering design [1]. Multi-objective truss optimization is quite advantageous than single-objective optimization because more design objectives can be embedded into the design. Designers are usually interested in optimizing multiple design criteria in order to obtain a set of Pareto optimal solution for decision-making. Some well-known multi-objective optimization algorithms can be selected in solving multi-objective truss optimization problems, including non-dominated sorting genetic algorithm II (NSGA-II) [2], multiobjective particle swarm optimization (MOPSO) [3], multi-objective evolutionary algorithm based on decomposition (MOEA/D) [4]. Coello and Christiansen [5] presented the genetic algorithm for multi-objective optimization of truss structures. Mokarram and Banan [6] developed the FC-MOPSO based on the selection and preservation of diversity for solving multi-objective truss optimization problems, which provided good performance on searching for the acceptable approximations of Pareto fronts under limited function evaluations. Tejani et al. [7] proposed a multi-objective adaptive symbiotic organisms search based on different archive techniques for solving multi-objective truss optimization problems. Panagant et al. [8] compared 14 multi-objective metaheuristic algorithms for solving benchmark truss optimization problems, and the results showed that SHAMODE and SHAMODE-WO outperforms other compared algorithms in most cases. According to the No Free Lunch (NFL) theory [9], a metaheuristic optimizer can tackle an optimization problem efficiently while fail to converge for another optimization problem. The NFL theory motivates us to develop more novel and high-quality multi-objective metaheuristic for multi-objective truss optimization problems.

In multi-objective truss optimization, the measurement indicators are various, such as hypervolume, inverted generational distance, spacing-to-extent, and so on. Therefore, some metaheuristic algorithms may result in the premature or oscillation of indicators during optimization [10]. A variant of DE called success history-based adaptive differential evolution (SHADE) [11] was proposed for optimization, which was a high-quality metaheuristic algorithm and ranked 3rd in the CEC 2014 competition. However, similar to other metaheuristics, SHADE still suffers from the oscillation of some indicators during the optimization process, because the convergence is more concerned for single-objective optimization in SHADE, but the diversification is also important in multi-objective optimization.

This paper developed a novel hybrid optimizer combining the merits of SHADE and MRFO to solve multi-objective optimization problems, called MO-SHADE-MRFO. In the proposed MO-SHADE-MRFO, the MRFO is embedded into the SHADE to balance the exploration and exploitation, which can enhance the global convergence. The efficiency of MO-SHADE-MRFO is illustrated by six benchmark design examples: planar 10-bar, spatial 25-bar, planar 37-bar, spatial 120-bar, planar 200-bar, and spatial 942-bar truss design. The main objectives of the truss design are minimizing the weight and compliance of the structure. The proposed algorithm is also compared with 9 state-of-the-art multi-objective algorithms: MOPSO, NSGA-II, MOEA/D, MOGOA, MOMVO, MOWCA, MOSSA, UPSEMOA, and SHAMODE, in terms of metrics including hyper-volume, inverted generational distance, spacing, and Pareto front.

2. Problem description

The multi-objective truss optimization design is a challenging task due to the conflicting objectives, complicated constraints, and discrete design variables of cross-sectional areas [1]. In general, the multi-objective truss optimization problem is formulated as:

$$\begin{aligned} &\text{Find} \quad && A=\{A_1, A_2, \ldots, A_m\} \\ &\text{Min} \quad && f_1(A)=\sum_{i=1}^{m} A_i\rho_i L_i, \quad f_2(A)=compliance=\boldsymbol{u}^T\boldsymbol{F} \\ &\text{s.t.} \quad && |\sigma_i|-\sigma_i^{\max}\leq 0, \quad A_i^{\min}\leq A_i\leq A_i^{\max} \end{aligned} \tag{1}$$

where A_i is the design variable of the cross-sectional area for i-th element, m is the number of design variables, f_1 and f_2 denote the structural mass and compliance, respectively, ρ_i and L_i are the mass density and length of the i-th element, respectively, σ_i and σ_i^{max} are the tress and the allowable value of the i-th element, and A_i and A_i^{max} are the lower and upper bounds of cross-sectional areas of design variables. The compliance is computed by the vector product of displacement $\boldsymbol{u}$ and force $\boldsymbol{F}$.

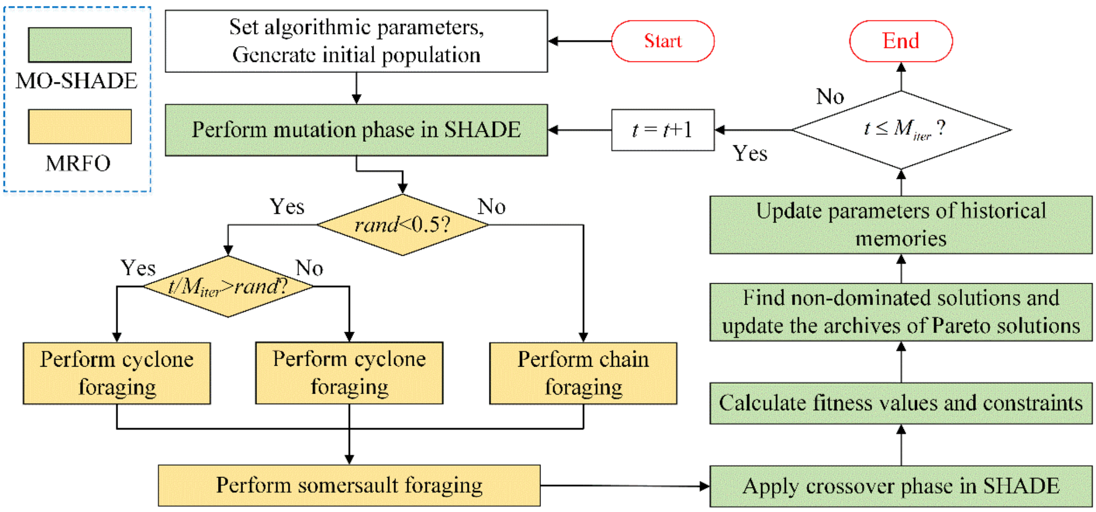


Fig. 1 The flowchart of MO-SHADE-MRFO algorithm

SHADE is a very effective and successful metaheuristic algorithm, especially for solving single-objective optimization problems. However, there has been little research on SHADE for solving multi-objective truss optimization problems. In this work, a variant of SHADE is presented for multi-objective truss optimization problems, called multi-objective success history-based adaptive differential evolution with manta ray foraging optimization (MO-SHADE-MRFO), which stands for the integration of the cyclone and chain foraging of MRFO into the SHADE. The modification is made at the reproduction process, where each mutant vector has a chance to be further updated with the cyclone or chain movement of MRFO. Besides, all the non-dominated solutions are saved and updated in the selection process. The flowchart of MO-SHADE-MRFO is illustrated in Fig. 1.

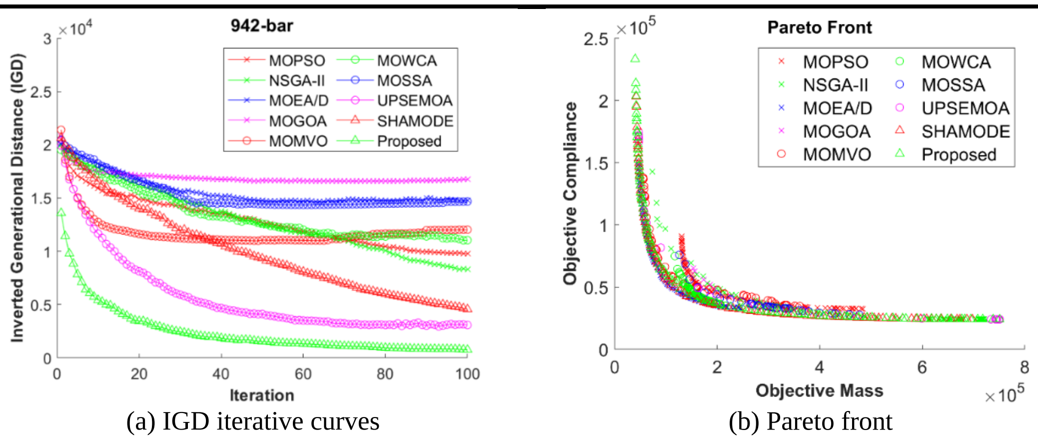
3. Numerical results

In this work, six benchmark truss optimization problems, including 10-bar, 25-bar, 37-bar, 120-bar, 200-bar and 942-bar trusses, are utilized to test the effectiveness of the proposed algorithm. The performance of MO-SHADE-MRFO is also compared with nine competitive multi-objective algorithms, including MOPSO, NSGA-II, MOEA/D, MOGOA, MOMVO, MOWCA, MSSA, UPSEMOA, and SHAMODE. For fair comparison, the population size and maximum iterations are set as 100 and 100, respectively. All multi-objective metaheuristic algorithms execute 30 times independently. Three different metrics are used to evaluate the performance of metaheuristic algorithms, including hyper volume (HV) to measure spread of Pareto front, spacing-to-extent (STE) to measure spacing and extent of a front, and inverted generational distance (IGD) to measure the distances between the Pareto front and the reference front.

For simplicity, the results of 942-bar truss problems are discussed. According to the statistical results in Table 1, MO-SHADE-MRFO achieves good performance in the HV, IGD and STE metrics, which ranks first among the compared algorithms. Moreover, the convergence capacities are illustrated in Fig. 2, demonstrating that MO-SHADE-MRFO has smooth and well-distributed Pareto solutions, which is better than MOPSO, NSGA-II, MOWCA, MOGOA and MOMVO. In all, MO-SHADE-MRFO is a competitive algorithm for multi-objective truss optimization problems.

Table 1 Statistical metrics for 942-truss problems

Algorithm		MOPSO	NSGA-II	MOEA/D	MOGOA	MOMVO	MOWCA	MOSSA	UPSEMOA	SHAMODE	Proposed
HV	Best	1.07E+11	1.17E+11	1.21E+11	8.73E+10	1.15E+11	1.07E+11	9.51E+10	1.14E+11	1.27E+11	**1.29E+11**
	Mean	1.18E+11	1.20E+11	1.23E+11	9.94E+10	1.20E+11	1.15E+11	1.03E+11	1.26E+11	1.29E+11	**1.30E+11**
	Worst	1.23E+11	1.25E+11	1.26E+11	1.07E+11	1.22E+11	1.21E+11	1.12E+11	1.30E+11	1.31E+11	**1.31E+11**
	SD	4.43E+09	2.28E+09	1.20E+09	3.74E+09	1.71E+09	3.08E+09	3.23E+09	3.14E+09	9.20E+08	**4.07E+08**
	FR	6.23	5.60	4.17	9.83	6.13	7.57	9.13	3.27	1.93	**1.13**
IGD	Best	5.57E+03	3.75E+03	9.73E+03	1.36E+04	7.87E+03	6.97E+03	1.14E+04	1.57E+03	2.90E+03	**5.04E+02**
	Mean	9.79E+03	8.28E+03	1.48E+04	1.67E+04	1.20E+04	1.10E+04	1.47E+04	3.08E+03	4.57E+03	**7.69E+02**
	Worst	1.32E+04	1.27E+04	2.04E+04	2.02E+04	1.78E+04	1.51E+04	1.97E+04	5.95E+03	6.74E+03	**1.39E+03**
	SD	1.97E+03	2.10E+03	2.79E+03	1.58E+03	2.17E+03	2.14E+03	1.88E+03	7.97E+02	9.85E+02	**1.84E+02**
	FR	5.50	4.57	8.27	9.47	6.93	5.97	8.30	2.07	2.93	**1.00**
STE	Best	4.00E-03	4.21E-03	4.24E-03	**1.65E-03**	4.16E-03	6.29E-03	2.92E-03	2.67E-03	3.62E-03	2.86E-03
	Mean	6.34E-03	9.89E-03	9.82E-03	6.96E-03	9.85E-03	1.16E-02	9.21E-03	5.74E-03	4.82E-03	**3.87E-03**
	Worst	9.34E-03	3.94E-02	3.56E-02	2.69E-02	4.17E-02	2.58E-02	2.92E-02	3.08E-02	6.81E-03	**5.49E-03**
	SD	1.36E-03	7.32E-03	5.99E-03	5.29E-03	7.38E-03	4.09E-03	5.16E-03	4.92E-03	8.86E-04	**5.08E-04**
	FR	5.33	6.93	7.00	4.83	6.87	8.63	6.63	3.47	3.50	**1.80**



(a) IGD iterative curves (b) Pareto front

Fig. 2. Iterative curves and Pareto front of 942-bar truss problem

4. Conclusions

In this work, a novel MO-SHADE-MRFO optimizer is presented for multi-objective optimization of truss structures, combining two powerful single-objective metaheuristic algorithms: success-history based adaptive differential evolution (SHADE) and manta ray foraging optimization (MRFO). In MO-SHADE-MRFO, the Pareto archives are used to save the non-dominated solutions. The operators of SHADE are used in the proposed algorithm to ensure the convergence for optimization problems. The operators of MRFO including cyclone, chain, and somersault foraging are embedded into the proposed algorithm to enlarge the population's diversity and enhance convergence ability. The combination of MRFO and SHADE can provide good convergence and coverage, which enhances the diversification and intensification in solving multi-objective truss optimization problems.

The performance of MO-SHADE-MRFO is investigated using six multi-objective truss optimization problems (10-bar truss, 25-bar truss, 37-bar truss, 120-bar truss, 200-bar truss, and 942-bar truss). The objective is to minimize the structural mass and compliance subjected to elemental stress, with discrete design variables of cross-sectional areas. This study compares the proposed algorithm with other nine state-of-the-art multi-objective metaheuristic algorithms, including MOPSO, NGSA-II, MOEA/D, MOGOA, MOMVO, MOWCA, MOSSA, UPSEMOA and SHAMODE, by evaluating the performance of algorithms with three metrics, HV, IGD and STE. Based on the statistical results, the proposed MO-SHADE-MRFO is overall the best algorithm, which is effective in solving multi-objective truss optimization problems, with well spread, consistent and smooth Pareto solutions in each benchmark problem. Evidently, the proposed algorithm provides competitive performance in solve truss multi-objective optimization problems. In future works, we can extend the proposed algorithm for higher dimension and practical challenging optimization problems.

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