

Eduardo Castro e Costa,  
José P. Duarte, Mário Krüger

# COMPUTATIONAL MODELLING AND DIGITAL MATERIALIZATION OF ALBERTI'S COLUMN SYSTEM: A REFLECTION ON RESULTS OBTAINED

S

## INTRODUCTION

In this paper we present some of the results of the Digital Alberti research project, whose main objective is to assess the influence of the Renaissance on Portuguese architecture in the Counter-reform period. In this project, Renaissance architect Leon Batista Alberti's treatise on architecture, *De re aedificatoria* – actually its Portuguese translation, “Da arte edificatória” (Alberti, 2011) – is analyzed, decoded and compared to Portuguese buildings through the use of new technologies.

R

In that sense, parts of the treatise were translated into computational models, namely shape grammars, which help to quantify the similarity between the treatise and the built examples (Coutinho, Castro e Costa, Duarte, & Krüger, 2011). Among the many issues addressed in the treatise by Alberti, the description of the column system is particularly thorough, making it a suitable candidate for translation into computational models.

E

Both deriving from and complementing the development of the column system shape grammars, several types of models were developed – digital, physical, computational and virtual. This led to a deeper understanding of the treatise, and thus aiding to the development of the grammars.

Elaborated close to the project's completion, this paper focuses on the development of both the computational and the physical models of the column system, posing as a reflection on the results obtained from these tasks.

## Methodology

The modelling of the column system encompasses three main tasks: a) the development of the computational model, b) its implementation, and c) the production of the physical models.

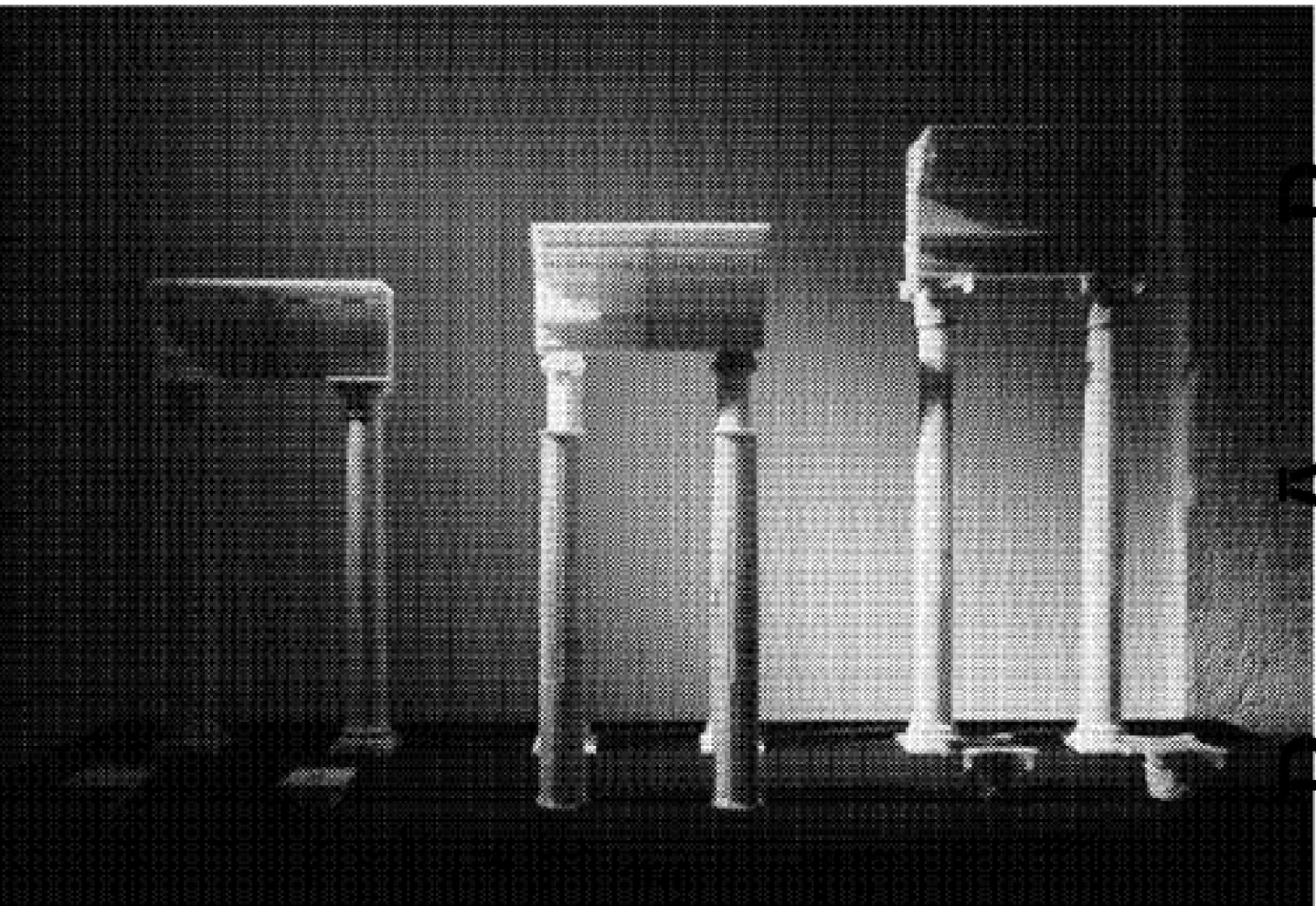
P

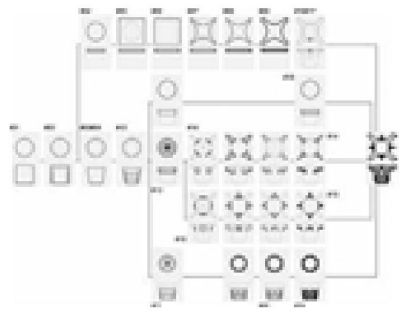
The computational model incorporates Alberti's instructions regarding the art of building, translating them into a parametric generative system that carries them out. This system is implemented into computer programs that automatically generate three-dimensional digital models of the Albertian column system. Although developed in close proximity to the shape grammar, the computational model was built directly from the treatise and it is not by itself an implementation of the grammar.

A

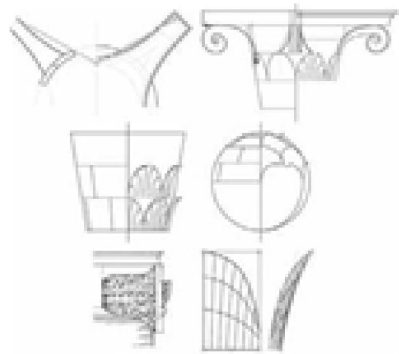
The digital models resulting from the manipulation of parameters of the computational model enable the generation of several other types of models, including two-dimensional drawings or virtual models. They can also be materialized using digital fabrication technology (Duarte, Celani, & Pupo, 2012). Along this research project, several physical models were produced, from plastic miniatures of the column system elements to stone capitals close to natural size.

P





1. Hierarchical modelling scheme of the Corinthian capital (Castro e Costa, 2012).



2. filling in the gaps for the Corinthian capital: sprouting stalks and acanthus leaves

## deVelopINg the coMputAtIoNAI Model

### rule decoding

For the development of the computational model it was first necessary to read Alberti's treatise, namely the chapters where the author describes the column system. Each element of this description is addressed in an algorithmical fashion, and characterized by a specific rule, numerically describing the element and relating with other elements. Analysis of these rules generated tables, which allowed for the rules to be seen as a whole. Table 1 is an example of such approach, containing some of the rules that generate the Corinthian capital, according to the English edition of the treatise (Alberti, 1988). The table features one rule per row, its text and location in the treatise, and a mathematical representation (Castro e Costa, 2012, p. 47).

### rule relationships

Analyzing the rules as a whole allowed to better identify relationships among them. Some relationships pointed to the existence of subdivision rules, suggesting that the elements of the column system could be organized into hierarchical tree structures, which are represented in the modelling schemes (Figure 1) (Castro e Costa, 2012, p. 50).

Besides subdivision rules, two other types were identified: proportion and detailing rules.

In subdivision rules (for example rule #02), Alberti describes elements of the column system as function of previously defined elements, by subdividing them and determining their order, assigning them positional relationships of "above" and "below". In proportion rules (for example rule #01), the author assigns dimensions to the elements in function of previously defined elements. In detailing rules (for example in rules #14 and #15), Alberti describes the geometry of some of the elements.

### completing the model

Alberti is very diligent prescribing proportion and subdivision rules. However, in the detailing rules, the information given by the author is sometimes insufficient to determine the exact shapes of every element of the column system. To solve this puzzle, it was necessary to look for solutions outside of the treatise, namely illustrations of later editions of Alberti's treatise, and observation of buildings. Illustrations of treatises by other authors were also referenced, but only when not conflicting with Alberti's rules. Analyzing these sources, it was possible to fill in the gaps, gaining some insight on how to detail some complex geometry.

A good example for this approach is the Corinthian capital computational model. The geometrical complexity of this capital, which features the acanthus leaves or the sprouting stalks (Figure 2), requires both modelling skills and knowledge of geometry. Observation of elements both drawn and sculpted was essential for the understanding of such geometries, whose modelling is still being improved.

Table 1  
Interpretation table of Alberti's rules  
for the Corinthian capital

S

R

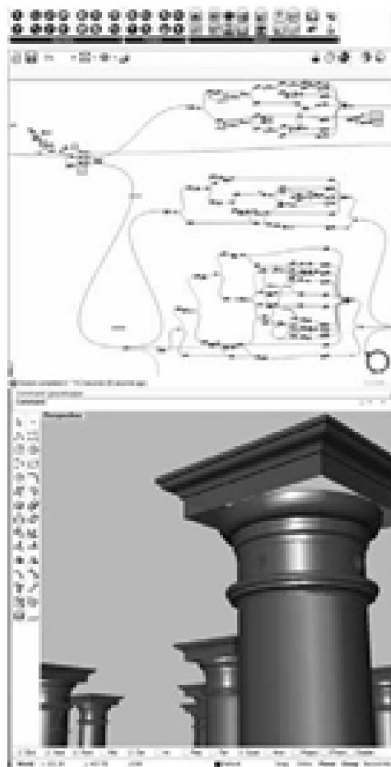
F

P

A

P

Page nr.	Line nr.	rule transcribed from the treatise's page and line <-> Mathematical translation of the transcribed rule (using the following variables: D = diameter; H = height; W = width; L = length; M = module, auxiliary variable; d_ = differential)	Internal rule numbering
208	06	The height of the Corinthian capital is equal to the diameter at the base of the column <-> $H_{capital} = D_{moscope}$	01
	07	and is divided into seven modules. The abacus takes up one module and the remainder is occupied by the vase, <-> $M = 1/7 \square H_{capital}$ ; $d_{Habacus} = 1 \square M$ ; $d_{Hvase} = 6 \square M$	02
	08	whose base has the same width as the top of the column, without its projections, <-> $W_{vase} = D_{moscope}$	03
	09	and whose upper rim has the same width as the bottom of the column. <-> $W_{vase} = D_{moscope}$	04
		[..]	
	17	The vase is girt with a fillet and an astragal, <-> (rule dependent on shaft and rule #18)	10
	17	which cover it with two interlapping rows of leaves standing out in relief, each row contains eight leaves. <-> $Leaf = 1/8 \square Leafrow$	11
	19	The first row is two modules high, as is the second. The remaining space is taken up by the stalks sprouting out. <-> $d_{Hleafrow} = 2 \square M$ ; $d_{Hstalkrow} = 2 \square M$	12
	21	These stalks are sixteen in number, four of them unfold on each face of the capital, two from the same knot on the right, and two from the same knot on the left; <-> $Lstalk = 1/16 \square Lstalkrow$ ; (shape rule: see scheme)	13
209	01	the two end ones hang below the corners of the abacus in a form of spiral, <-> (shape rule: see scheme)	14
	02	while the middle ones also curl, so that their ends meet in the center. <-> (shape rule: see scheme)	15
		[..]	
	05	Each leaf should be articulated into five or, possibly, seven lobes. <-> $Llobe \in \{1/5 \square Leaf, 1/7 \square Leaf\}$	19
	06	The tip of the leaves hang forward half a module. <-> $d_{Wleaf} = 1/2 \square M$	20
	07	As with all carving, deeply incised lineaments will add great charm to the leaves of the capital. <-> (shape rule: see scheme)	21



3. Computational and digital models (Grasshopper and rhino) of the Albertian column system

## IMpLeMEntING the coMputAtIoNAI Model

### Selecting the technology

The parametric model was translated into a computer program in Grasshopper (GH), a visual programming language that builds upon NURBS modelling software, Rhinoceros (Rhino). In GH, programs can be visually developed as algorithms that automatically generate digital 3D models in Rhino according to parameters input by a user. Alberti's instructions were therefore translated into GH components, and feeding the prescribed parameters into these components, the computer program generates any column according to Alberti's canon (Figure 3).

### debugging and optimizing the program

As more rules were implemented, the more complex the computer program became, making it increasingly difficult even to navigate around. In order to simplify the program, an effort was frequently made into debugging the program, eliminating unnecessary or redundant code towards its optimization.

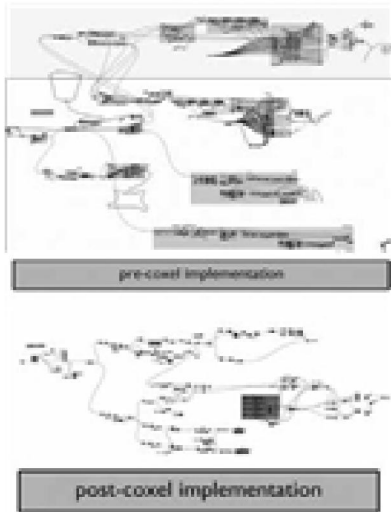
While this effort was useful to keep most of the operations under control, some of Alberti's instructions, namely those concerning more complex geometry (i.e., the acanthus leaves) implied the development of complex – and inherently computationally heavy – parts of code, and thus compromising the performance of the models themselves.

### Implementing classes

In the several attempts to debug the computer program, it became evident that most elements of the column system share common properties, allowing interpreting them as topologically similar entities. On the other hand, it often happened that the same groups of operations were used repeatedly, pointing to the use of subroutines (Scott, 2009). These two factors suggested the implementation of computational classes.

The selected language for implementing these classes was VB.NET, an object-oriented language for which GH has built-in scripting tools, and so a custom class was created, and named *coxel* (Column Element). Implementation of this class allowed both defining relationships more intuitively, as well as reducing the amount of code for the computational models (Figure 4), thus further simplifying the system (Castro e Costa, 2012).

Because a new type of object was defined, it was then necessary to gain a deeper understanding of how shape and geometry is represented both conceptually and computationally, since rendering of the elements had to be programmed directly, using RhinoCommon.



4. Class implementation as means of simplifying the system

S  
R  
E  
P  
A  
P

## producing the physIc AI Models

The computational model previously described generates digital models of the column system elements. These three-dimensional digital models were used for the production of corresponding physical models through different digital fabrication technologies.

Fabricating the column elements had two main goals:

a) to determine the qualities of the previously modelled geometry, and b) to assess the suitability of each technology for producing the different elements of the column system, according to parameters such as shape or size. Three digital fabrication techniques were tested: two additive techniques – Fused Deposition Modeling (FDM) and Three-Dimensional Printing (3DP) –, and one subtractive technique – CNC milling.

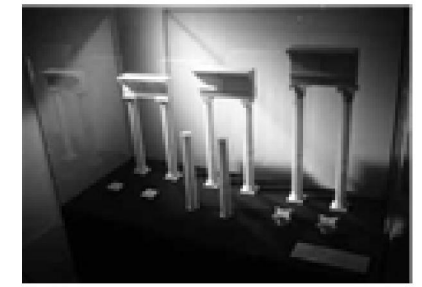
### Additive technologies

Several smaller models were produced using FDM, namely some test models, as well as a collection of small scale models of all the elements of the column system (Figure 6). This miniature collection is featured as an interactive installation, demonstrating the combinatorial nature of Alberti's column system (Coutinho et al., 2011), so that elements of different types can be combined – for example, a column with a Ionic capital may feature a Doric base, and be topped by a Corinthian entablature.

Three-Dimensional Printing (3DP) technology features a higher modelling resolution when compared with FDM, thus allowing for the fabrication of more detailed objects, such as the Corinthian capital. Figure 7 shows instances of the Corinthian capital produced through each of the two technologies. The FDM model is less developed in terms of shape, and was printed at half the scale of the 3DP model. Nevertheless, the problems caused by the anisotropy inherent to the FDM printing process (Ahn, Montero, Odell, Roundy, & Wright, 2002) were detected in the first case, and the computational model was edited in order to correct the problem. All in all, FDM and 3DP proved to be more suitable for smaller models that require more detail.

### cNC milling

While searching for Portuguese companies equipped with digital fabrication technologies, contact was established with a stone transformation company working with a 6-axis CNC milling machine especially fit for stone work. So work was developed towards the production of a stone Corinthian capital. The success of this experiment resulted from the collaboration with the company, which also provided a good insight on the problems that arise within an industrial context. The stone capital was fabricated along 5 different phases and 15 hours (Figure 8). Each phase featured an increased level of detail from the previous one, derived from the tool used in each phase.



5. Collection of miniature fDM produced models of the column system



6. Corinthian capital models produced through additive prototyping technologies



7. five production phases of the Corinthian capital

The Corinthian capital that resulted from the stone milling does not correspond to the final form of the digital model, due to limitations of the technology itself. In fact, it is common practice that milled pieces undergo through post-production work, comprising some corrections and final polishing. However, this post-production phase was skipped so that it is possible to witness the level of detail allowed by the milling technology (Figure 9).

The production of the Corinthian capital in stone poses as an example of possible collaborations between academic research and industrial activities. Following the production of the Corinthian capital, further production is planned, as well as further development of the computational model according to the needs of the stone-transforming company. The ultimate goal is to add to the company's competitiveness, increasing productivity through innovation.

# S R E P A P

## conclusion

The development and implementation of the computational models implied a thorough analysis and an effort to have a deeper understanding of Alberti's treatise. The result was a tool that complements the development of the shape grammars necessary for accomplishing the project's objectives. The findings in this research can be of value for other projects that are based on rule-based form finding.

Also the production of the corresponding physical elements aided in fine-tuning the shape of the column elements, and the results are a valuable addition to the Digital Alberti exhibition. Also, fabrication of the Corinthian capital poses as a case study of a successful collaboration between an academic research project and an industrial company, which is to be continued beyond the scope of the project.

## Acknowledgements

"Digital Alberti" project (PTDC/AUR-AQI/108274/2008 – FCOMP-01-0124-FEDER-008842) funded by FEDER Grants through COMPETE – Programa Operacional Factores de Competitividade and by National Grants through FCT – Fundação para a Ciência e a Tecnologia, hosted by CES at the University of Coimbra and coordinated by Mário Krüger.

## reference

Ahn, S.-H., Montero, M., Odell, D., Roundy, S., & Wright, P. K. (2002). Anisotropic material properties of fused deposition modeling ABS. *Rapid Prototyping Journal*, 8(4), 248-257. doi:10.1108/13552540210441166

—

Alberti, L. B. (1988). *On the Art of Building in Ten Books*. (J. Rykwert, N. Leach, & R. Tavernor, Trans.). MIT Press.

—

Alberti, L. B. (2011). *Da Arte Edificatória*. (A. E. Santo & M. Krüger, Trans.). Fundação Calouste Gulbenkian.

Castro e Costa, E. (2012). *Modelação computacional e materialização digital de elementos clássicos de arquitectura: Sistematizando a coluna de Alberti* (MSc). Faculdade de Arquitectura da Universidade Técnica de Lisboa, Lisboa.

—

Coutinho, F., Castro e Costa, E., Duarte, J. P., & Krüger, M. (2011). A computational interpretation of *De re aedificatoria*: Translating Alberti's column system into a shape grammar. In *RESPECTING FRAGILE PLACES – 29th eCAADe Conference Proceedings* (pp. 788-798). University of Ljubljana, Faculty of Architecture (Slovenia). Retrieved from [http://cumincad.scix.net/cgi-bin/works/Show?ecaade2011\\_083](http://cumincad.scix.net/cgi-bin/works/Show?ecaade2011_083)



8. final capital stone model

Duarte, J. P., Celani, G., & Pupo, R. T. (2012). Inserting Computational Technologies in Architectural Curricula. In N. Gu & X. Wang (Eds.), *Computational Design Methods and Technologies: Applications in CAD, CAM and CAE Education*. IGI Global. Retrieved from <http://www.igi-global.com/chapter/inserting-computational-technologies-architectural-curricula/62959>

—

Scott, M. L. (2009). *Programming Language Pragmatics*. Morgan Kaufmann.