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### Article

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5 **Active vibration control of a small-scale flexible structure**  
6 **subject to moving-loads and experimental validation**

7

8 Lukas Sievert

9

10 *Department of Maritime and Mechanical Engineering, Liverpool John Moores University, 3 Byrom Street,*  
11 *L3 3AF, Liverpool, United Kingdom*  
12 *e-mail: l.sievert@2017.ljmu.ac.uk*

13

14 Dan Stancioiu

15

16 *Department of Maritime and Mechanical Engineering, Liverpool John Moores University, 3 Byrom Street,*  
17 *L3 3AF, Liverpool, United Kingdom*  
18 *e-mail: d.stancioiu@ljmu.ac.uk*

19

20 Christian Matthews

21

22 *Department of Maritime and Mechanical Engineering, Liverpool John Moores University, 3 Byrom Street,*  
23 *L3 3AF, Liverpool, United Kingdom*  
24 *Email: c.matthews@ljmu.ac.uk*

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26 **Abstract**

27 This study directly addresses the problem of optimal control of a structure under the action of moving  
28 masses. The main objective is to experimentally implement and validate an active control solution for a  
29 small-scale test stand. The supporting structure is modeled as an Euler-Bernoulli simply supported beam,  
30 acted upon by moving masses of different weights and velocities. The experimental implementation of the  
31 active controller poses a particular set of challenges as compared to the numerical solutions.

32 It is shown both numerically and experimentally that using electromagnetic actuation, a reduced order  
33 controller designed using a time-varying algorithm provides a reduction of the maximum deflection of up  
34 to 18% as compared to the uncontrolled structure. The controller performance and robustness were tested  
35 against a representative set of possible moving load parameters.

36 In consequence of the variations in moving mass weight and speed the controller gain requires a  
37 supplementary adaptation. A simple algorithm that schedules the gain as a function of the weight and speed  
38 of the moving mass can achieve both a good performance and an adjustment of the control effort to the  
39 specific design requirements.

40

41 **Keywords:**

42 Time-varying optimal control, Active vibration control, Moving mass, State estimation

43

44 **1. Introduction**

45 The dynamics of a structure under the action of a moving load is relevant to many engineering applications  
46 such as linear guideways, robotics and overhead cranes. However, this subject is particularly studied and  
47 applied to vehicle/pedestrian-bridge interaction and train-track interaction [1–5]. If the inertia effect of the  
48 moving structure needs to be taken into account [6], in modal space it leads to a time-varying system of  
49 equations. The problem of moving loads in relation to bridge-structure interaction has been studied  
50 extensively both analytically and experimentally [7,8]. Of special concern for structural engineers is not  
51 only the modelling but also for the improvement of the dynamic response of the supporting structure to  
52 specific moving-load actions. One example could be the effect of different traffic loads in the case of a  
53 bridge structure. The research literature shows a series of studies that put forward passive methods designed  
54 to address this specific problem. The passive approach is attractive as it provides a low cost solution [9–  
55 12], but it is less efficient when the structure is subjected to loads with a random variation in parameters  
56 like moving speed and weight.

57 Active vibration control methods offer higher efficiency by reducing broadband frequencies and by  
58 providing a higher and flexible actuation [13] which in the context of a moving mass structure means that  
59 the control could adapt actively to different weights and speeds. The active control of a structure subjected

60 to a moving mass, compared to the general structural modal control [14–16], is of special interest and  
61 difficulty. The dynamic matrices of the structure, mass, damping and stiffness change over time, depending  
62 on the speed and weight of the moving mass, therefore an active control solution must take into account the  
63 time-varying nature of the system [17–19].

64 Several studies have investigated the active control of a moving mass system numerically. Sung [20]  
65 presented the dynamic modelling and the time-invariant optimal control of a simply supported beam under  
66 a moving mass. He used two piezoelectric actuators and their locations were determined by an optimal  
67 quadratic cost functional. Deng et al. [21] used a linear-quadratic Gaussian modal controller for a time-  
68 varying structure including identification and control update in real-time. The numerical model, which  
69 alters due to structural changes, is updated by an observer. The method was validated numerically.

70 The time-varying nature of the system was taken into account in [18] where Nikkhoo proposes a method  
71 based on solving the Riccati equation at every time-step. In [17,18] it is shown that for a high traveling  
72 speed, and for certain locations and number of actuators, the time-varying control shows a significant  
73 improvement compared to the time-invariant control. In [19] the classical optimal control approach is  
74 applied to single and multi-span beams under the influence of a moving load and a moving mass. The  
75 proposed solutions were based on displacement-velocity and velocity-acceleration feedback using  
76 piezoelectric actuators. Stancioiu et al. [17] cast the problem into a terminal-time optimal control  
77 framework [22] and further presented a numerical study for synthesis of time-varying control solution. The  
78 study also introduced an augmented system, which took into account the effect of the moving mass in the  
79 control synthesis problem. A drawback of the study was that it assumed full knowledge of the state-  
80 variables. A combination of sliding mode control and positive position feedback for a beam subjected to a  
81 moving mass was presented in [23]. The sliding mode control, used when the mass moves along the beam  
82 is robust to parameter uncertainties and the positive position feedback control is efficient to suppress the  
83 free vibration after the mass leaves the beam. Liu et. al. [24] devised a finite-time optimal regulator for an  
84 uncertain beam-mass system. The distributed material parameters were discretized for representative points  
85 and the regulator calculated with the probability density equation method.

86 Despite a large number of studies dedicated to numerical solutions, only few studies approached the  
87 problem of experimental implementation and validation of the moving mass vibration control. One of the  
88 main difficulties for the experimental implementation of the controller is that if the dynamic equations are

89 cast in modal space, the states are not directly accessible. Therefore an observer or state-estimator needs to  
90 be considered. This in turn leads to high computational time which counteracts the real-time ability of the  
91 controller. Frischgesell et. al. [25] studied a time-varying discrete observer for a moving mass system  
92 equipped with a force actuator. The aim was to minimize the maximum traverse deflection. The time-  
93 varying system and input matrices were calculated offline at specific times due to the high computation  
94 time required. Reckmann and Popp [26] extend this work with an adaptation method and a discrete time  
95 optimal controller designed to achieve a lower deflection of the flexible structure.

96 Pisarski [27] studied numerically and experimentally the semi-active control of a structure subjected to a  
97 moving load. In this study, an open-loop optimal bang-bang controller was used. The study considered the  
98 moving speed and weight of the mass and it was shown that the controlled system outperforms the passive  
99 case by 40% in terms of the proposed evaluation metric. This work was extended by [28] where a closed-  
100 loop adaptive control was proposed. The control gains were calculated offline for a constant speed and mass  
101 of the load with the ability to adapt online to the actual mass parameters.

102 This paper presents an experimental approach to the problem of active control of a structure under moving  
103 loads. The proposed solution is based on an optimal time-varying control algorithm and relies on a state-  
104 feedback controller. A new method to estimate the states of the system (modal coordinates and modal  
105 velocities) based on the inverse of the matrix of modal shape vectors and measured displacements is  
106 proposed. This simpler algorithm allows fast sampling times and proves to be robust against structural  
107 changes. This method of state estimation was first presented by the authors in [29], where a suboptimal  
108 controller was implemented to reduce the deflection of the beam at given locations.

109 In spite of the fact that the time-varying nature of the system is taken into account in the control approach,  
110 an objective function based on deflection responses requires an adaptation of the control effort to the mass  
111 and velocities of the load acting on the supporting structure. The feasibility of a simple gain scheduling  
112 procedure is investigated and shows a good performance for a control effort adjusted to the dynamic  
113 parameters of the problem.

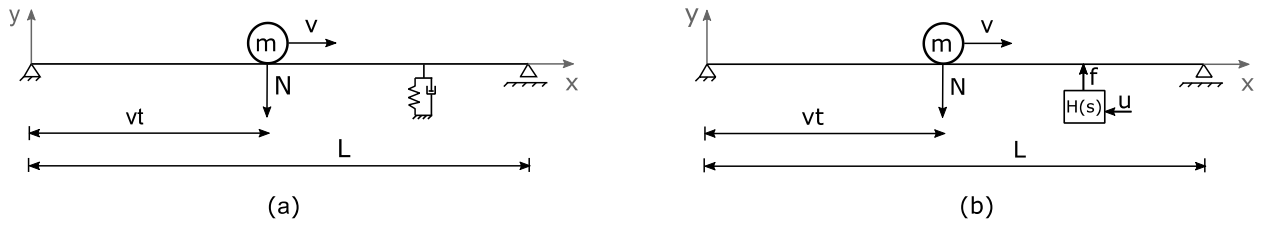
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115

## 116 2. The moving-mass structure interaction model

117 The investigated structure is modelled as an Euler-Bernoulli simply supported beam structure of mass per  
 118 unit length  $\rho A$  and flexural rigidity  $EI$ . The structure of length  $L$  is subjected to the action of a mass  $m$   
 119 moving with constant speed  $v$ , as illustrated in Fig. 1. The structure is also supported by an inertial shaker  
 120 which in passive state will be modelled as a spring-damper support. In the active state the actuator dynamics  
 121 is represented by a transfer function  $H(s)$ , specified in state space form in Eq. (5).

122



123

124 **Fig. 1** Model of the beam structure subjected to a moving mass, with an inactive actuator (a) and an active  
 125 actuator (b).

126

127 Under the assumption of permanent contact between the mass and the beam, the general system of equations  
 128 in modal coordinates governing the dynamics of a beam subjected to a mass  $m$  travelling at constant speed  
 129  $v$  at any time  $t$  within the interval  $[0, t_f]$  with  $t_f = L/v$  is [2,29,30]:

130

$$(\mathbf{M} + \Delta\mathbf{M}(t))\ddot{\mathbf{q}} + (\mathbf{D} + \Delta\mathbf{D}(t) + \mathbf{D}_a)\dot{\mathbf{q}} + (\mathbf{K} + \Delta\mathbf{K}(t) + \mathbf{K}_a)\mathbf{q} = -mg\boldsymbol{\psi}(vt) + \boldsymbol{\psi}(x_a)f \quad (1)$$

131

132 In this case the vectors  $\mathbf{q}$  and  $\dot{\mathbf{q}}$  represent modal displacements and modal velocities of the structure which  
 133 are not directly accessible from the measurements and are estimated using mode shape functions. The  
 134 structure's response is approximated at sensor locations  $x_{si}$  using the mode shape functions  $\boldsymbol{\psi}(x)$ , as:  
 135  $w(x_{si}, t) = \boldsymbol{\psi}^T(x_{si})\mathbf{q}(t)$ . The constant matrices  $\mathbf{M}$ ,  $\mathbf{D}$  and  $\mathbf{K}$  can be expressed as functions of the modal  
 136 shape vectors  $\boldsymbol{\psi}(x)$ , mass per unit length  $\rho A$ , damping  $c\rho A$  and stiffness  $EI$ :

137



$$\begin{aligned}
\mathbf{M} &= \rho A \int_0^L \boldsymbol{\psi}(x) \boldsymbol{\psi}^T(x) dx, \\
\mathbf{D} &= \rho A c \int_0^L \boldsymbol{\psi}(x) \boldsymbol{\psi}^T(x) dx, \\
\mathbf{K} &= EI \int_0^L \boldsymbol{\psi}(x) \boldsymbol{\psi}''''^T(x) dx
\end{aligned} \tag{2}$$

138

139 The time-dependent matrices  $\Delta\mathbf{M}(t)$ ,  $\Delta\mathbf{D}(t)$  and  $\Delta\mathbf{K}(t)$  are defined as [17,30]:

140

$$\begin{aligned}
\Delta\mathbf{M}(t) &= m \boldsymbol{\psi}(vt) \boldsymbol{\psi}^T(vt), \\
\Delta\mathbf{D}(t) &= 2mv \boldsymbol{\psi}(vt) \boldsymbol{\psi}'^T(vt), \\
\Delta\mathbf{K}(t) &= mv^2 \boldsymbol{\psi}(vt) \boldsymbol{\psi}''^T(vt)
\end{aligned} \tag{3}$$

141

142 The added damping and stiffness matrices due to the electrodynamic actuator located at  $x_a$  are [29]:

143

$$\begin{aligned}
\mathbf{D}_a &= c_a \boldsymbol{\psi}(x_a) \boldsymbol{\psi}^T(x_a), \\
\mathbf{K}_a &= k_a \boldsymbol{\psi}(x_a) \boldsymbol{\psi}^T(x_a)
\end{aligned} \tag{4}$$

144

145 An accurate model of the modal shaker could be very complex [27]. For this investigation a simpler first  
146 order model valid at low frequencies is used. The dynamics of the actuator acting on the beam structure is  
147 modelled as a state-space system from input voltage  $u$  to output force  $f$ :

148

$$\begin{aligned}
\dot{z} &= -\alpha z + \beta u; \\
f &= \gamma z
\end{aligned} \tag{5}$$

149

150 In the state-space representation, considering  $n$  vibrational modes, with inclusion of the electrodynamic  
151 shaker's dynamics, the system matrices are:

152

$$\mathbf{A}(t) = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} \\ -(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}(\mathbf{K} + \Delta\mathbf{K}(t) + \mathbf{K}_a) & -(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}(\mathbf{D} + \Delta\mathbf{D}(t) + \mathbf{D}_a) & \gamma(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}\boldsymbol{\psi}(x_a) \\ \mathbf{0}_{l \times n} & \mathbf{0}_{l \times n} & -\alpha \end{bmatrix}; \quad (6)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_{n \times 1} \\ \mathbf{0}_{n \times 1} \\ \beta \end{bmatrix}; \quad \mathbf{B}_f(t) = \begin{bmatrix} \mathbf{0}_{n \times 1} \\ -(\mathbf{M} + \Delta\mathbf{M}(t))^{-1}\boldsymbol{\psi}(vt) \\ 0 \end{bmatrix};$$

153

154

The state vector becomes  $\mathbf{x}^T(t) = [\mathbf{q}(t) \ \dot{\mathbf{q}}(t) \ z(t)]$ . The time  $t_f$  represents the time the mass leaves the beam.

155

From this instant of time, the beam vibrates freely and the system governing the motion is a linear-time

156

invariant system. The system equations for  $t > t_f$  changes from (1) to:

157

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \boldsymbol{\psi}(x_a)f \quad (7)$$

158

159

with initial conditions the values of the states at the instant of time  $t_f$ .

160

### 161 3. The finite time control algorithm

162

When only one actuator is used, the time-varying plant with the states and control matrices presented in

163

(6), can be written in state-space form as:

164

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)u(t) \quad (8)$$

165

166

The aim of the controller is to minimize the deflection response at different locations along the beam. In

167

order to achieve this the performance objective can be formulated like a quadratic objective in deflection

168

at sensors locations

169

$$J = \frac{1}{2} \int_{t_0}^{t_f} \mathbf{w}^T(x_{si}, t) \mathbf{Q} \mathbf{w}(x_{si}, t) dt = \frac{1}{2} \int_{t_0}^{t_f} \mathbf{x}^T(t) \mathbf{C}^T \mathbf{Q} \mathbf{C} \mathbf{x}(t) dt \quad (9)$$

170

171 subject to equation (8) and the control's saturation limits  $|u(t)| \leq u_0$ . In equation (9) matrix  $\mathbf{C}$  is the output  
 172 matrix of the system described by (8) and consists of modal shape vectors  $\boldsymbol{\psi}(x_{si})$ .

173 This type of objective function was studied in [17] and it was shown that it leads to a two-boundary value  
 174 problem which makes the control design problem mathematically challenging. Also, the synthesized control  
 175 function is discontinuous. Such a control solution, even if it correctly describes the required control action,  
 176 may be difficult to implement as the electromagnetic type of actuation chosen here cannot accurately  
 177 describe a control function with discontinuities. For this reason, a quadratic objective function that also  
 178 includes the control has been chosen. The quadratic performance index is defined as:

179

$$J = \frac{1}{2} \mathbf{x}^T(t_f) \mathbf{F} \mathbf{x}(t_f) + \frac{1}{2} \int_{t_0}^{t_f} [\mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + u^T(t) \mathbf{R} u(t)] dt \quad (10)$$

180

181 The emphasis on the deflection will be addressed by choosing a state weighting matrix  $\mathbf{Q}$  with higher values  
 182 corresponding to the first states corresponding to the displacements and a significantly lower value for the  
 183 terms corresponding to the velocities. The control limitation is assured by the selection of the control  
 184 weighting parameter  $\mathbf{R}$ . In equation (10)  $t_f$  is specified and the final state  $\mathbf{x}(t_f)$  is constrained by the weighting  
 185 matrix  $\mathbf{F}$  in order to reduce the free vibration of the structure when one mass leaves the beam [17]. For a  
 186 system with  $p$  states and  $r$  actuators, the matrices  $\mathbf{F}$  and  $\mathbf{Q}$  are  $p \times p$  symmetric, positive semidefinite matrices  
 187 and matrix  $\mathbf{R}$  is  $r \times r$  positive definite. For the case when only one actuator is used,  $\mathbf{R}$  becomes a scalar.

188 When the value of the control function  $u(t)$  is unconstrained, the optimal control  $u^*(t)$  is  
 189 defined as [22] :

190

$$u^*(t) = -\mathbf{R}^{-1} \mathbf{B}'(t) \mathbf{P}(t) \mathbf{x}^*(t) = -\mathbf{k}(t) \mathbf{x}^*(t) \quad (11)$$

191

192  $\mathbf{k}(t) = -\mathbf{R}^{-1} \mathbf{B}'(t) \mathbf{P}(t)$  is called the Kalman gain and  $\mathbf{P}(t)$ , is a  $p \times p$  symmetric, positive definite matrix (for all  $t$   
 193  $\in [t_0, t_f]$ ), and is the solution of the matrix differential Riccati equation

194

$$\dot{\mathbf{P}}(t) = \mathbf{P}(t)\mathbf{A}(t) - \mathbf{A}^T(t)\mathbf{P}(t) - \mathbf{Q} + \mathbf{P}(t)\mathbf{B}(t)\mathbf{R}^{-1}\mathbf{B}^T(t)\mathbf{P}(t) \quad (12)$$

195

196 The optimal state is the solution of

197

$$\dot{\mathbf{x}}^*(t) = [\mathbf{A}(t) - \mathbf{B}(t)\mathbf{R}^{-1}\mathbf{B}^T(t)\mathbf{P}(t)]\mathbf{x}^*(t) \quad (13)$$

198

199 The matrix differential Eq. (12) can be solved backwards with  $t_{start}=t_f$  and the initial condition  $\mathbf{P}(t=t_f)=\mathbf{F}$ .

200 Then the optimal time-varying gain  $\mathbf{k}(t)$  is calculated forward using the values of  $\mathbf{P}(t)$ . Although  $p$  optimal

201 states  $x^*(t)$  are calculated, in theory the structure consists of an infinite number states which can cause

202 instability. The performance of the control system still needs to be tested for a representative set of values

203 of the masses and traveling speeds.

204

205

#### 206 4. The state estimation

207 The particular type of problem studied here where the effect of the loads on the structure cannot be used as

208 an input, makes the use of an estimator difficult. The solution presented here assumes that the number of

209 sensors equals the number of modes used for the numerical model.

210 The state vector is estimated from the experimentally measured deflection vector  $\mathbf{w}(t)_{n \times 1} = [w_1(x_{s1}, t) \dots$

211  $w_n(x_{sn}, t)]^T$  and the velocity vector  $\dot{\mathbf{w}}(t)_{n \times 1} = [\dot{w}_1(x_{s1}, t) \dots \dot{w}_n(x_{sn}, t)]^T$  at locations  $x_{sn}$  :

212

$$\begin{aligned} \mathbf{q}(t) &= \mathbf{\Psi}(x_{sn})^{-1}\mathbf{w}(t) \\ \dot{\mathbf{q}}(t) &= \mathbf{\Psi}(x_{sn})^{-1}\dot{\mathbf{w}}(t) \end{aligned} \quad (14)$$

213

214 In this equation  $\mathbf{\Psi}(x_{sn})$  is the  $n \times n$  matrix that contains the mode shapes calculated at sensor locations

215  $x_{sn}$ :

216

$$\Psi(x_{sn})_{n \times n} = \begin{bmatrix} \psi_1(x_{s1}) & \psi_2(x_{s1}) & \dots & \psi_n(x_{s1}) \\ \psi_1(x_{s2}) & \psi_2(x_{s2}) & \dots & \psi_n(x_{s2}) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_1(x_{sn}) & \psi_2(x_{sn}) & \dots & \psi_n(x_{sn}) \end{bmatrix} \quad (15)$$

217

218 When  $n$  sensors are used and  $n$  modes are estimated, the state-space vector can be determined as a unique  
 219 solution of equations. From Eq. (14) and Eq. (15) it can be seen that only the mode shapes of the structure  
 220 and the measured deflections are needed to calculate the modal coordinates and modal velocities. The  
 221 advantage of this method is that it avoids the implementation of an observer and can be applied to time-  
 222 varying systems with fast sampling times.

223

## 224 5. Experimental validation

225 In order to validate the beam-mass system modelled by Eq. (6), the method of state estimation of Eq. (14)  
 226 and the finite time controller, numerical simulations are compared with experimental measurements.

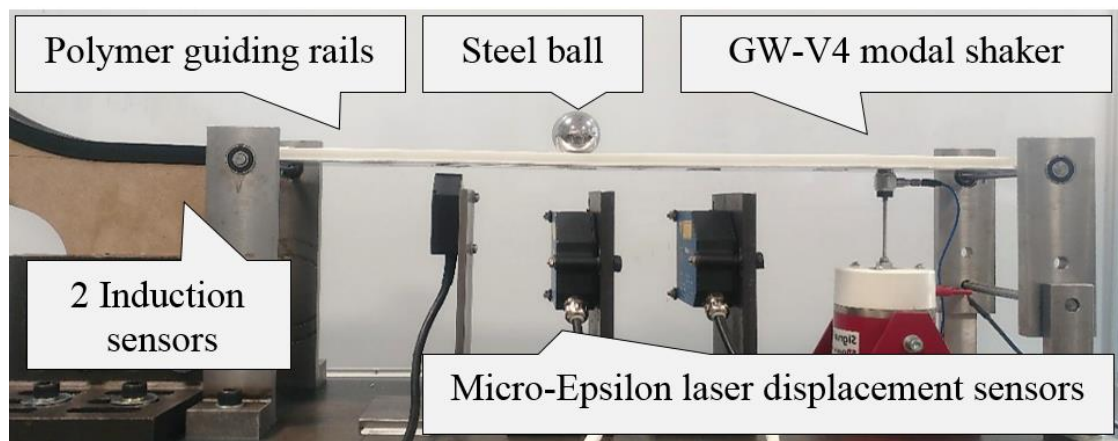
227

### 228 5.1 The experimental test stand

229 Fig. 2. shows the experimental set-up. Different steel balls with known mass  $m$  are accelerated by a ramp  
 230 and move over the simply supported beam structure at nearly constant speed. The geometrical  
 231 characteristics of the aluminium beam are: span length  $L = 0.6$  m and cross section  $A = 0.06$  m  $\times$  0.002 m.  
 232 By adding polymer guiding rails, the flexural rigidity and the damping coefficient are increased. Three  
 233 displacement sensors measure the deflection at  $x_{s1} = 0.15$  m,  $x_{s2} = 0.25$  m and  $x_{s3} = 0.35$  m.

234 The optimal gains of the finite time-varying and time-invariant control are calculated numerically in  
 235 MATLAB and stored on a CompactRIO embedded controller. With input from the laser displacement  
 236 sensors (optoNCDT 1700 and optoNCDT 1610), the states are estimated in real time every 15 ms and the  
 237 output voltage is calculated and sent to the power amplifier (Data Physics PA30E) for the actuation of the  
 238 electrodynamic shaker (Data Physics V4).

239



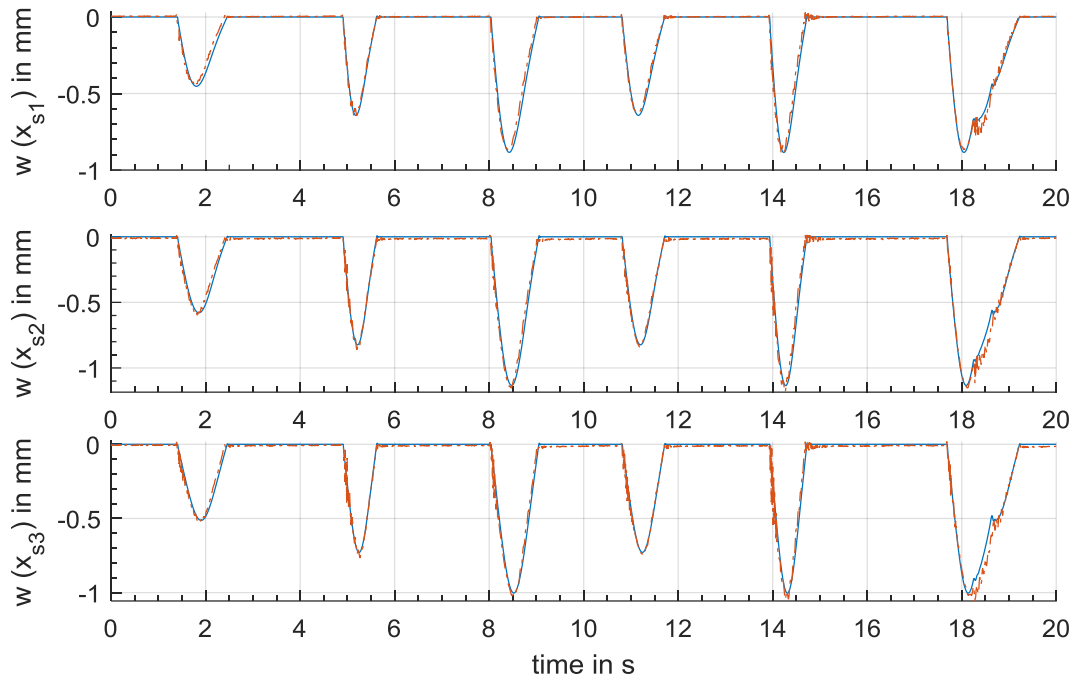
240

241 **Fig. 2** Experimental set-up, aluminium polymer beam subjected to a moving mass.  
 242

243 Fig. 3 shows the deflection response  $w(x_{si}, t)$ , numerically estimated (blue line) at three sensor locations  
 244 ( $i = 1, 2, 3$ ) when 7 balls are launched along the beam, against the experimentally measured deflections (red  
 245 line). For the last run two balls are moving on the structure. The parameters of the numerical beam model  
 246 are defined as mass per length unit  $\rho A = 0.535 \text{ kgm}^{-1}$  and flexural rigidity  $EI = 11.68 \text{ Nm}^{-2}$ . Due to the  
 247 polymer guiding rail the height is changed to 3.3 mm and a constant modal damping ratio  $\zeta = 0.03$  is  
 248 assumed throughout. No control action is involved. The influence of the electrodynamic actuator is modelled  
 249 as a spring-damper system with a damping coefficient of  $c_a = 80 \text{ Nsm}^{-1}$  and a stiffness of  $k_a = 12000 \text{ Nm}^{-1}$ .  
 250 With these adjustments, the deflections of the experimental data are in good agreement with the numerical  
 251 model.

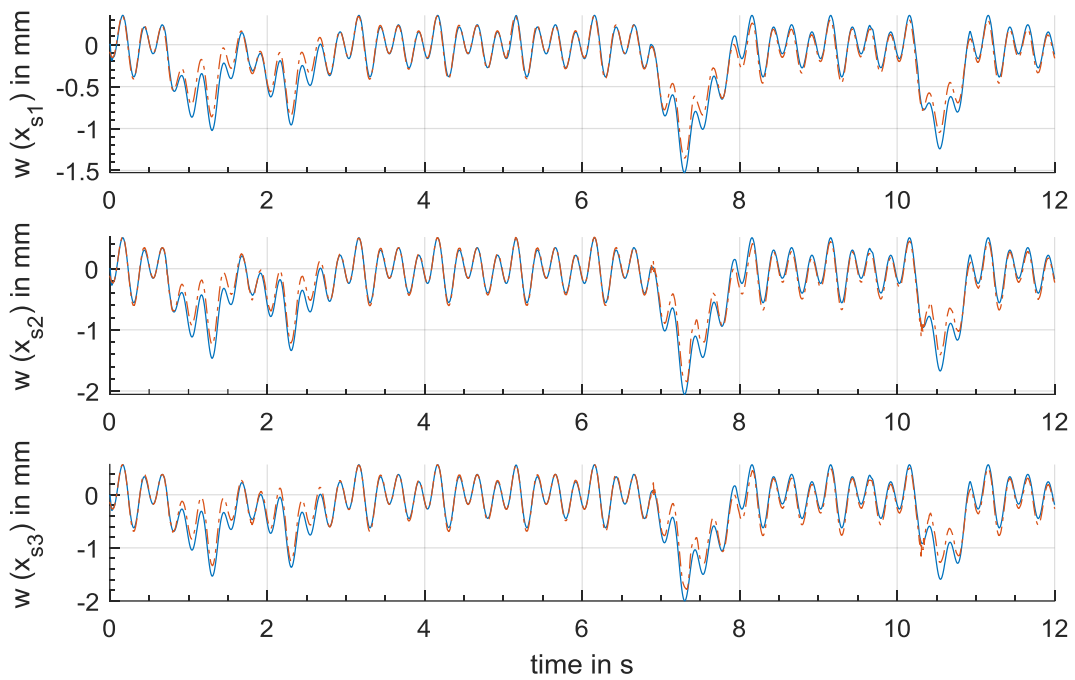
252 The beam-shaker system was validated using an active shaker with and without the action of the moving  
 253 mass. Therefore, the shaker's stiffness changes to  $k_a = 3500 \text{ Nm}^{-1}$  and  $\gamma = 4.6$  in (5). Numerical  
 254 investigations have shown that the dynamics of the beam can be accurately approximated using only the  
 255 first three modes.

256

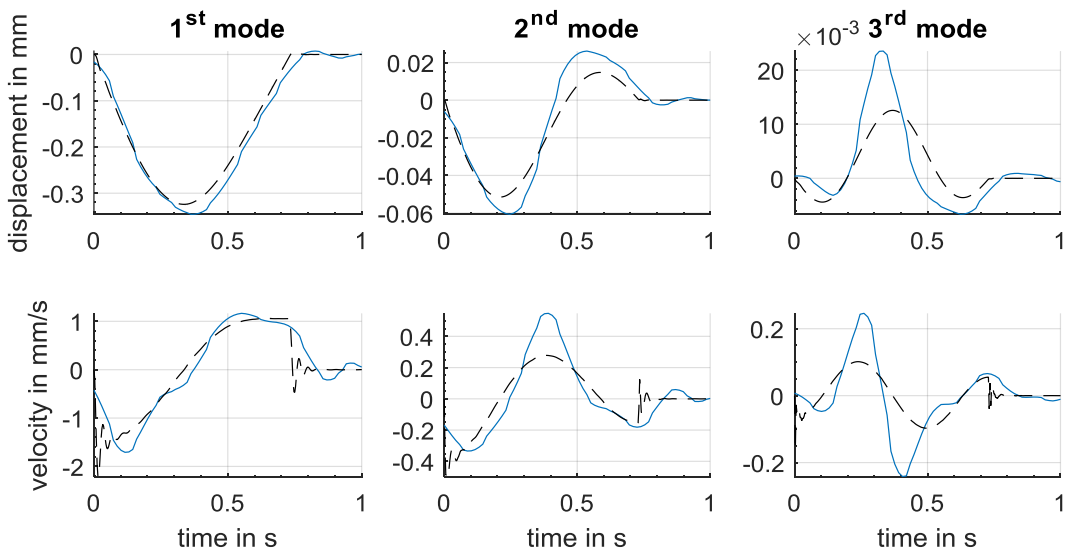


257  
 258 **Fig. 3** Experimental validation between the displacements of masses traveling at different speeds obtained  
 259 by the numerical model (blue continuous) and the experimental measurements (red dashed).  
 260

261 Fig. 4 shows a comparison between the experimental data and the numerical model for time deflection  
 262 response at sensors locations when four masses are launched at different speeds along the beam and the  
 263 shaker's input is fed with a prescribed voltage. In this case the voltage supplied was a combination of  
 264 sinusoidal functions. From Eq. (14) and Eq. (15) it follows that, since three sensors are installed, three  
 265 modal coordinates,  $q_i, (i = 1,2,3)$  can be calculated directly and by using the derivative three modal  
 266 velocities,  $\dot{q}_i, (i = 1,2,3)$ . For one mass moving along the beam these are represented in Fig. 5.  
 267



268  
 269 **Fig. 4** Validation of the beam mass system with an active electromagnetic shaker, numerical model (blue  
 270 continuous), and the experimental measurements (red dashed).  
 271



272  
 273 **Fig. 5** Comparison modal coordinates and modal velocities, numerical model (black dashed) and the  
 274 measured signal (blue continuous).  
 275

276 The first mode is dominant and shows the best accordance with the modal displacement estimated using  
 277 experimental data. For the modal velocity, the first mode also shows the best match. A 10<sup>th</sup> order digital  
 278 low-pass filter with a cut-off frequency  $f_{3dB} = 10$  Hz, reduces the noise but it causes a slight delay.



280 5.2 Experimental results for optimal control implementation

281 For the time-invariant control, the constant gain is calculated, without taking into account the time-varying  
282 parts in system equation (6):

283

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{I}_{n \times n} & \mathbf{0}_{n \times i} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} & \gamma\mathbf{M}^{-1}\boldsymbol{\Psi}(x_a) \\ \mathbf{0}_{1 \times n} & \mathbf{0}_{1 \times n} & -\alpha \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{n \times 1} \\ \mathbf{0}_{n \times 1} \\ \beta \end{bmatrix}; \quad (16)$$

284

285 The actuator is located at  $x_a = 0.5$  m. The error and performance index are defined as  
286  $\mathbf{Q} = \text{diag}(1000, 100, 10, 0.1, 0.01, 0.01, 0)$  and  $R = 0.00009$  for the time-invariant control as well as for the  
287 time-varying control. The terminal cost matrix is defined as  $\mathbf{F} = \mathbf{Q}$ .

288 The displacement response of the supporting structure is mainly induced by the first mode. This knowledge  
289 was utilized by defining the error performance matrix  $\mathbf{Q}$ , setting higher weight toward the first modes.

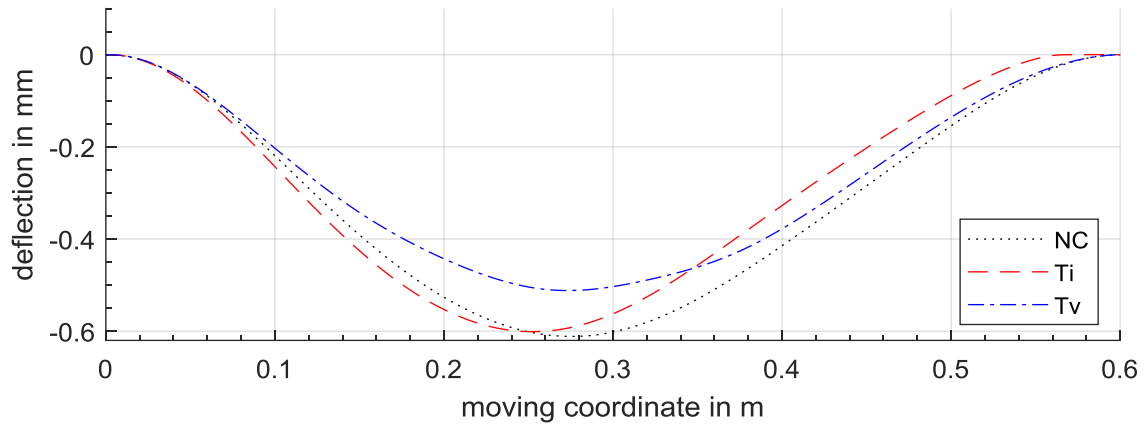
290 The weight of the moving masses used in the experiments ranges from 0.261 kg to 0.509 kg. The masses  
291 are accelerated by a ramp and move over the simply-supported beam structure with approximately constant  
292 speed. The values of the speeds used is between  $0.3 \text{ ms}^{-1}$  and  $0.55 \text{ ms}^{-1}$ . The actuator is located at  $x_a =$   
293  $0.5$  m, which is not the optimal position in terms of maximum deflection reduction making it even more  
294 necessary to employ the time-varying control solution [17]. The performance of the control methods is  
295 assessed by using the maximum absolute value of the displacement at the sensor locations  $x_{si}$ .

296 Of the three available sensor locations  $x_{s2} = 0.25$  m is chosen for further evaluation of the control methods.

297 It displays the maximum deflection of the beam, as seen in Fig. 3, as well as the maximum deflection at the  
298 moving coordinate  $vt$  (Fig. 6).

299

300



301

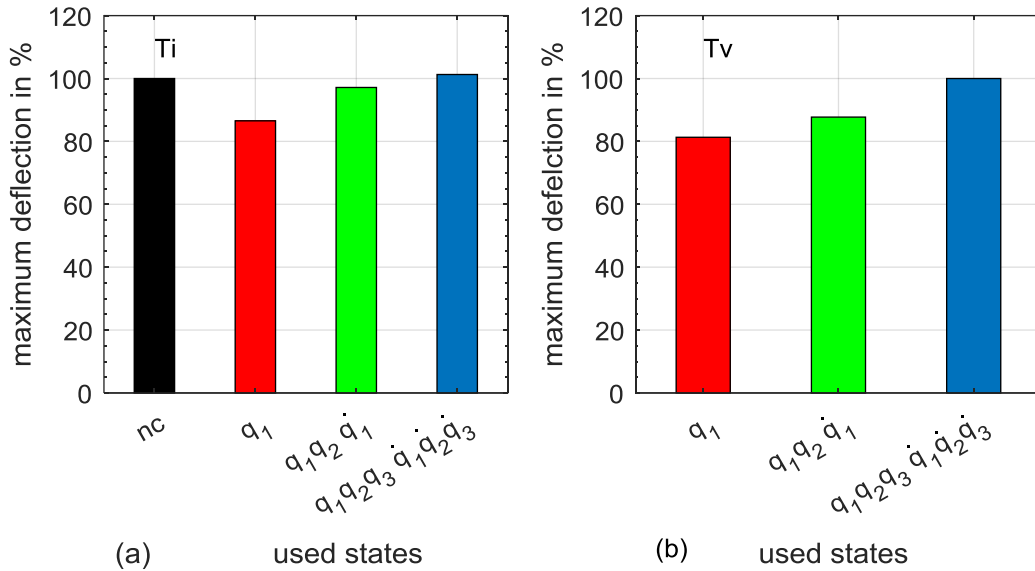
302

303 **Fig. 6** Numerical deflection of the moving coordinate  $vt$  of the mass  $m = 0.5$  kg traveling with velocity  $v =$   
304  $0.3 \text{ ms}^{-1}$ , no control (NC), time-invariant control (Ti) and time-varying system control (Tv).  
305

306 Following Fig. 5 it is clear that a full state feedback controller cannot be used given the lack of accuracy of  
307 the state estimation. Also, the controllability matrix of the system (16) is not full rank which indicates that  
308 not all of the states might be controllable as well. The best matches of the modal coordinates towards the  
309 numerical model are achieved for the estimated states  $[q_1 \ q_2 \ \dot{q}_1]$  as defined in section 5.1.

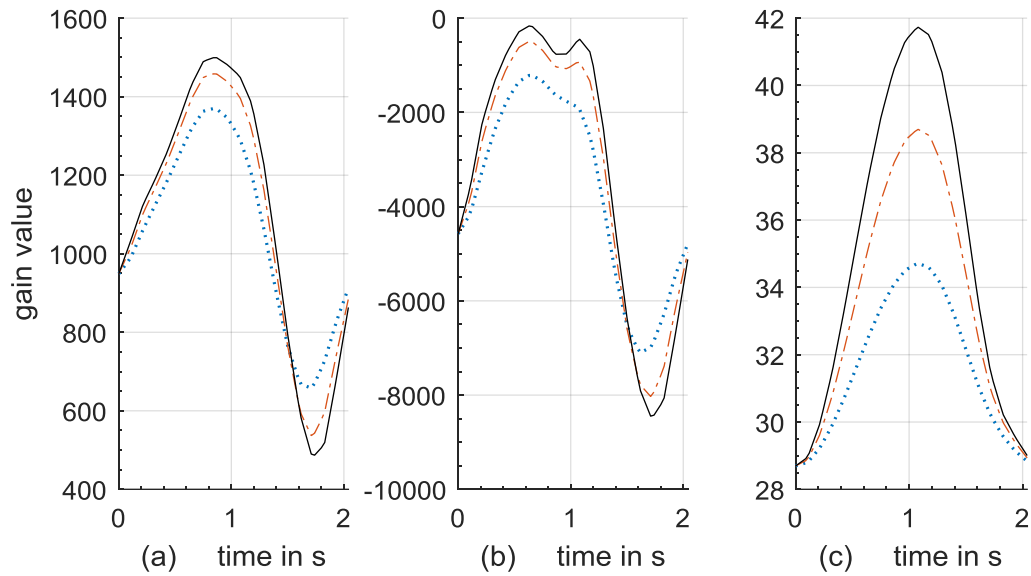
310 The influence on the deflection reduction, using a reduced order controller, is considered for a mass  $m =$   
311  $0.261$  kg moving at a speed  $v = 0.55 \text{ ms}^{-1}$ . Three runs were taken per method. The value for the maximum  
312 displacement was averaged over the three runs. Fig. 7 displays the experimental relative maximum  
313 deflection at sensor  $x_{s2}$  for the time-invariant control method (left) in comparison with the time-varying  
314 control method (right) using different combinations of controlled states. It can be observed that a time-  
315 invariant controller only using one state  $q_1$  provides a reduction of the maximum deflection of about 15%.  
316 The deflection reduction decreases even more when using more states leading to even a slight increase  
317 when using all states, which might be due to inaccuracies of the mode estimation. In contrast, the time-  
318 varying control method is applicable for the states  $[q_1 \ q_2 \ \dot{q}_1]$  as well, with a reduction of about 15%. Using  
319 only the first state results in the best deflection reduction at  $x_{s2}$  of about 20%. Although using further states  
320 results in a complete solution of the problem, due to the lack of accuracy of the estimated states, the beam  
321 deflection is not improved. A different value of  $\mathbf{Q}$  with an even higher weight towards the first modal

322 displacement and modal velocity might lead to a higher reduction of the deflection if more of the states are  
 323 used.  
 324



325 (a) used states (b) used states  
 326 **Fig. 7** Relative maximum deflection measured at sensor location  $x_{s2}$  normalized to the uncontrolled  
 327 structure (nc) of the time-invariant control (Ti) (a) and the time-varying control (Tv) (b) from using one  
 328 state to using all states.  
 329

330 Fig. 8 shows the time histories of the varying gains  $k_1$ ,  $k_2$  and  $k_4$  corresponding to the states  $[q_1 \ q_2 \ \dot{q}_1]$ . The  
 331 tests were run for the masses  $m = 0.261$  kg,  $m = 0.371$  kg and  $m = 0.509$  kg traveling at the speed  $v = 0.3$   
 332  $\text{ms}^{-1}$ . Towards the time of  $t = 0.8$  s the traveling mass reaches the moving coordinate  $vt = 0.24$  m where  
 333 the beam has the highest deflection (see Fig. 6). Consequently, the gains  $k_1$  and  $k_2$  increase up to this time.  
 334 With that, a higher actuation is achieved when the action of the mass is high. Subsequently the gains  
 335 decrease. When the mass passes by  $x_d = 0.5$  m the gains  $k_1$  and  $k_2$  reach their minimum. The least amount  
 336 of force is required to counteract the influence of the moving mass. In this way, an effective and stable  
 337 control is achieved.  
 338



339

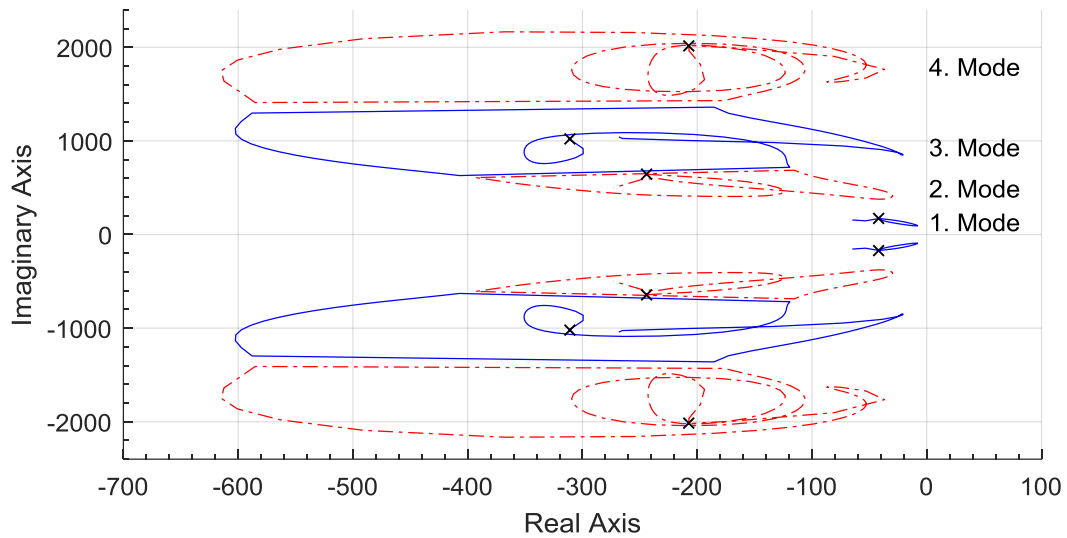
340

341 **Fig. 8** Development of the time-varying gains  $k_1(t)$  (a),  $k_2(t)$  (b) and  $k_4(t)$  (c) for the four different masses  
 342  $m = 0.261$  kg (blue dotted),  $m = 0.371$  kg (red dashed) to  $m = 0.509$  kg (black continuous) at velocity  $v =$   
 343  $0.3 \text{ ms}^{-1}$ .

344

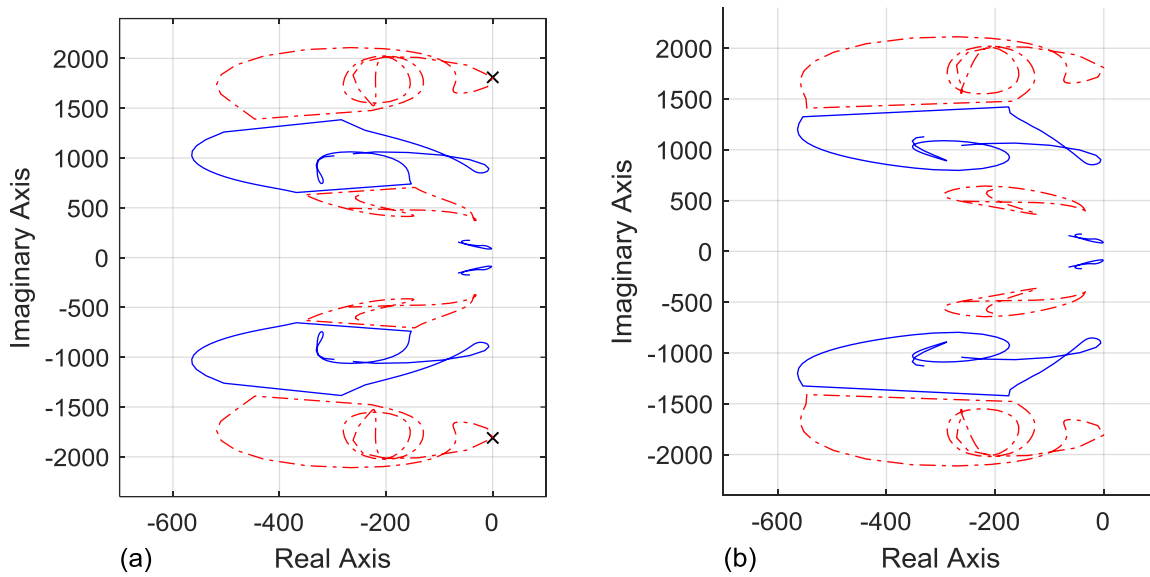
345 In the following investigations the states  $[q_1 \ q_2 \ \dot{q}_1]$  are used for control. This represents a fair compromise  
 346 between completeness of the solution and reduction of the structural deflection.

347 To assess the stability of the time-varying system  $(\mathbf{A}(t)-\mathbf{B}(t)\mathbf{k}(t))$ , where the proposed reduced order  
 348 controller is applied, its eigenvalues are calculated at certain time steps. Fig. 9 illustrates the course of the  
 349 first four resulting complex conjugate pole pairs. During the time the mass  $m=0.509$  kg travels with  $v=0.55$   
 350  $\text{ms}^{-1}$  over the beam the eigenfrequencies of the modes change, the poles circle in the negative left half  
 351 plane around the time-invariant poles (black crossed). The system stays stable for this parameter.



352  
 353 **Fig. 9** Time history of the poles of the time-varying controlled system, first and third mode (blue  
 354 continuous), second and fourth mode (red dashed), poles of the time-invariant system (black crossed),  $m=0.5$   
 355 kg,  $v=0.55$  m/s  
 356

357 Fig. 10(a) shows the poles of the simulated system with the reduced order controller and a traveling mass  
 358  $m= 0.509$  kg. For the increased traveling speed of 5.6 m/s one pole pair moves into the real half plane  
 359 causing instability. At this margin the full state controller (Fig. 10(b)) stays stable with all the poles in the  
 360 negative half plane. Higher velocities and weights cause also with the full-state control instability. Likewise  
 361 increasing the mass over  $m=6.5$  kg with a low speed of 0.55 m/s some poles will move into the real half  
 362 plane. In this way the theoretical stability margins of the system can be simulated. The additional actuator  
 363 pole located at -10000 on the real axes is not shown in the figures.



365

366

367 **Fig. 10** Comparison of the first four poles of the time-varying system with the reduced order controller (left)  
 368 and with the full state controller (right) instable poles (black asteriks) ,  $m = 0.5$  kg,  $v = 5.6$  m/s  
 369

370 In order to assess the reduction of the maximum deflection at sensor location  $x_{s2}$  depending on the used  
 371 control method three masses were tested at two speeds  $v = 0.3$  ms<sup>-1</sup> and  $v = 0.55$  ms<sup>-1</sup>. Five runs for each  
 372 mass were averaged for the calculation of maximum deflections. The relative maximum deflections in  
 373 Table 1 show a small reduction for the time-invariant control of around 3% for all the masses. The time-  
 374 varying control shows a better performance for all the tests with a deflection reduction from 12% for  
 375  $m=0.261$  kg to 17% for  $m= 0.509$  kg, with a higher reduction for higher masses.

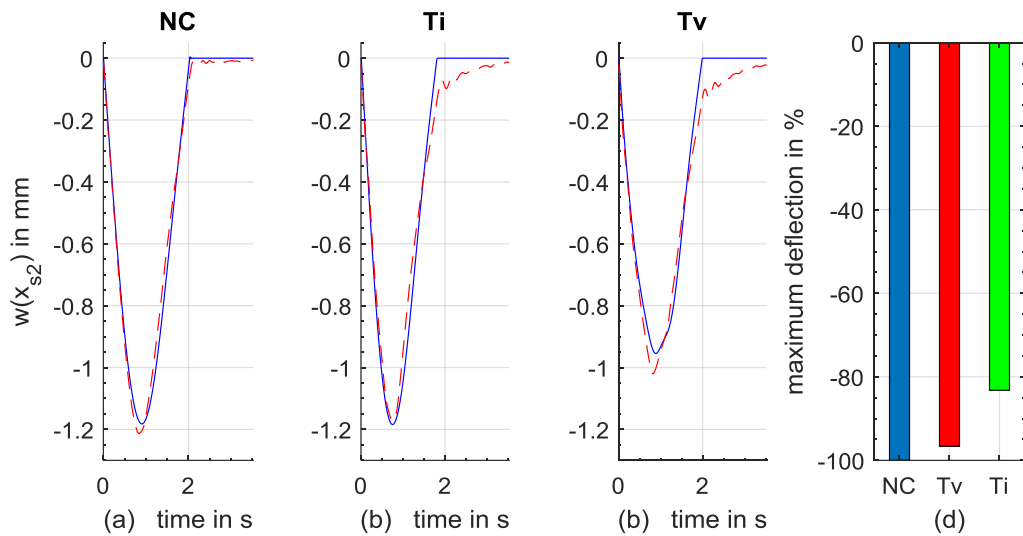
376

377 **Table 1** Relative maximum deflection at  $x_{s2}$  for different masses traveling at  $v = 0.3$  ms<sup>-1</sup> in percent.

mass m in kg	no control	time- invariant	time- varying
0.261	100	96.9	88
0.371	100	97.7	85.7
0.509	100	96.6	83.2

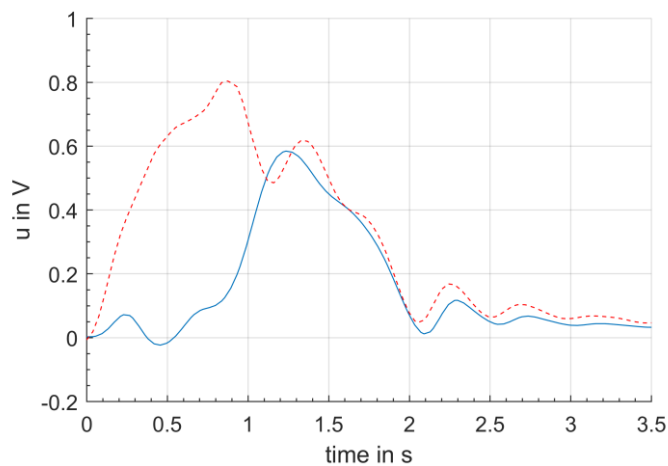
378

379 Fig. 11 illustrates the results obtained for mass  $m = 0.509$  kg with a traveling speed of  $v = 0.3$  ms<sup>-1</sup>. It also  
 380 shows a good agreement between the numerically calculated results and the experimentally measured  
 381 deflection  $w(x_{s2})$ . There is a small mismatch after the mass leaves the beam due to the not modelled back  
 382 electro-magnetic force (Back EMF) of the electromagnetic shaker [31].



383  
 384 **Fig. 11.** Mass  $m = 0.509$  kg moving with  $v = 0.3$  ms<sup>-1</sup>, comparison of the displacement  $w(x_{s2})$  for the  
 385 numerically calculated data (blue-continuous) and the experimentally measured data (red dashed), for the  
 386 case without control (a), with the time-invariant control (b), with the time-varying control (c) and the values  
 387 of the relative maximum deflection in percent (d).  
 388

389 Fig. 12 illustrates the time history of the experimental control inputs  $u(t)$  belonging to this example. It is  
 390 noticed that the time-variant control has a high actuation especially in the first half of the traveling time  
 391 whereas the time-invariant control is much less active in the first half.



392  
 393 **Fig. 12** Time-history of the experimentally measured control input, time-invariant (blue-continuous) and  
 394 time variant (red- dashed)  
 395

396 Table 2 shows the relative maximum deflections for three masses moving with a higher speed  $v = 0.55 \text{ ms}^{-1}$   
 397 <sup>1</sup>. The invariant control reduces the maximum deflection only by 1% for mass  $m = 0.261 \text{ kg}$  and by 8% for  
 398 mass  $m = 0.509 \text{ kg}$ . In contrast, the time-varying control achieves a reduction of approximately 18% for  
 399 mass  $m = 0.509 \text{ kg}$ . Again, it can be observed that the control is more effective for higher masses, as a  
 400 higher deflection results in higher actuation. The results are similar for the two investigated velocities.

401

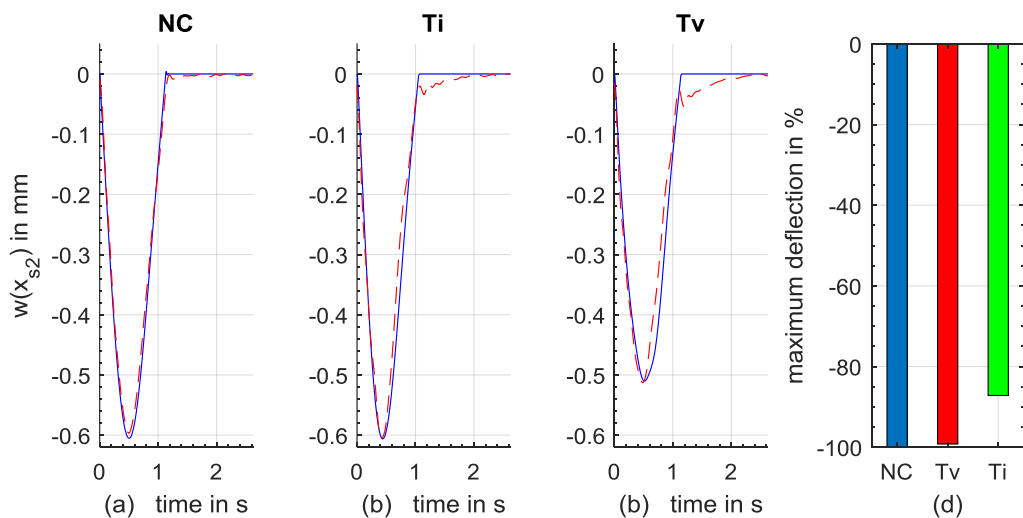
402 **Table 2** Relative maximum deflection measured at  $x_{s2}$  for different masses traveling at  $v = 0.55 \text{ ms}^{-1}$  in  
 403 percent.

mass in kg	no control	time-invariant	time-varying
0.261	100	99.2	87.2
0.371	100	97.1	83
0.509	100	92	82.1

404

405 Fig. 13 shows one example of the beam deflection at sensor location  $x_{s2}$  when mass  $m = 0.261 \text{ kg}$  moves  
 406 with velocity  $v = 0.55 \text{ ms}^{-1}$ . The measured deflections show a good match with the numerical model for all  
 407 the tests with the only discrepancy observed after the mass leaves the structure due to back EMF of the  
 408 electro-dynamic shaker. The time-varying controller reaches a reduction of 13%. A stable control with  
 409 reduction of the beam deflection is achieved for different masses traveling at different speeds.

410



411

412 **Fig. 13.** Mass  $m = 0.261 \text{ kg}$  moving with  $v = 0.55 \text{ ms}^{-1}$ , comparison of the displacement  $w(x_{s2})$  for the  
 413 numerically calculated data (blue-continuous) and the experimentally measured data (red dashed), for the



414 case without control (a), with the time-invariant control (b), with the time-varying control (c) and the values  
415 of the relative maximum deflection measured at  $x_{s2}$  in percent (d).

416

### 417 5.3 Experimental results for variable control gains

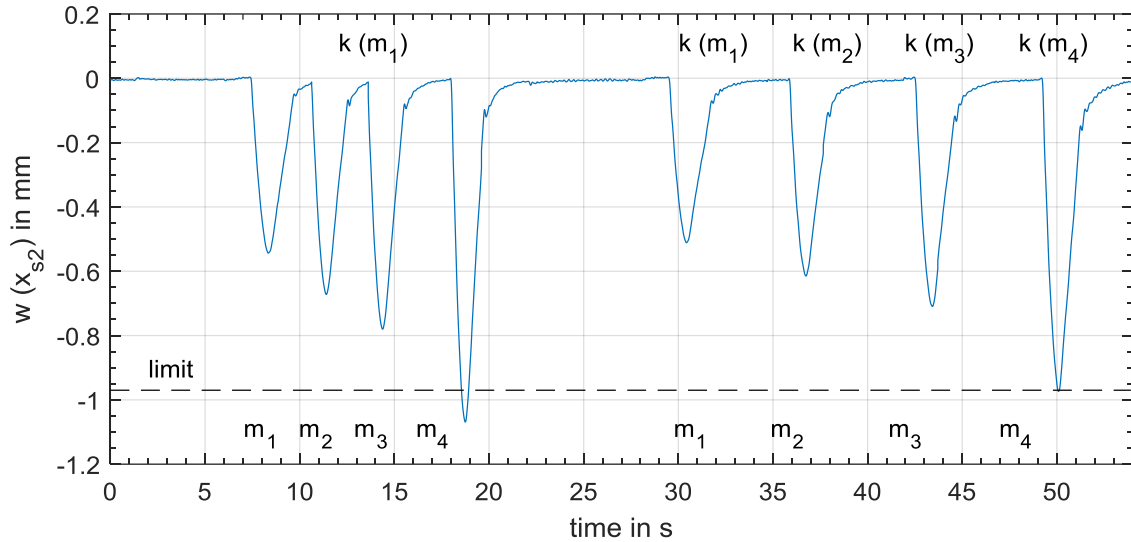
418 Following the experimental tests, it becomes clear that the performance of the controller depends on the  
419 weight of the moving mass. This means that a control gain that was designed to achieve a good reduction  
420 for a heavy mass may provide a too high control effort for a smaller mass whereas a controller gain designed  
421 for a small mass may not be enough to provide a good reduction of the deflection for a heavier mass.  
422 Therefore in terms of absolute deflection, the control effort required to achieve a prescribed absolute  
423 maximum deflection needs to change for the case when a small mass travels along the beam as compared  
424 to the case a heavier mass acts upon the beam.

425 In this respect a gain scheduling of the control gain either as  $\mathbf{k}(m)$  a function of mass or as  $\mathbf{k}(m, v)$  a function  
426 depending on both mass  $m$  and speed  $v$  is tested. The masses used are  $m_1=0.261$  kg,  $m_2=0.322$  kg,  $m_3=0.371$   
427 kg and  $m_4=0.509$  kg.

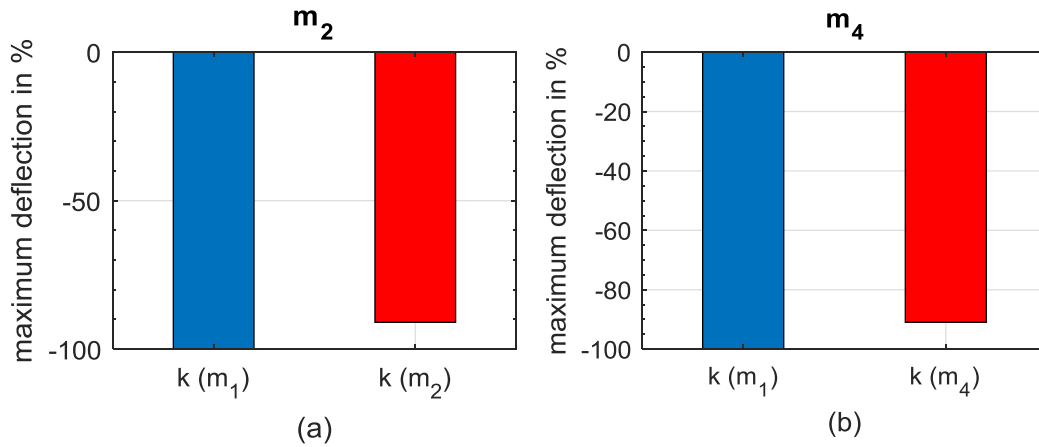
428 Fig. 14 shows the effect of using the specific scheduled time-varying gains  $\mathbf{k}(m_1)$ ,  $\mathbf{k}(m_2)$ ,  $\mathbf{k}(m_3)$  and  $\mathbf{k}(m_4)$ ,  
429 calculated taking into account every mass, compared with the time-varying gain  $\mathbf{k}(m_1)$  determined for mass  
430  $m_1$  and subsequently used for all masses. In this way the control switches to the specific control gain,  
431 therefore a heavier mass will have a higher control gain that will confine the deflection of the beam within  
432 a prescribed limit (in this case about 1 mm). Fig. 14 shows a gradual reduction of the deflection as the gain  
433 increases with the weight of the mass.

434 With this approach where the gains are scheduled taking into account the value of the mass, the relative  
435 maximum deflection is 10% lower compared to the unscheduled control using the gain of the first mass  
436  $\mathbf{k}(m_1)$  all over, see Fig. 15. The performance of this method can be improved if the gains are determined  
437 taking into account the moving mass into the system equation as an augmented system, introduced in [4].  
438 The gains can be scheduled based on deflection values in the first phase. On a real bridge-like structure,  
439 image processing or a scale can identify the actual load case of  $m$  and select the optimal gain for control.

440



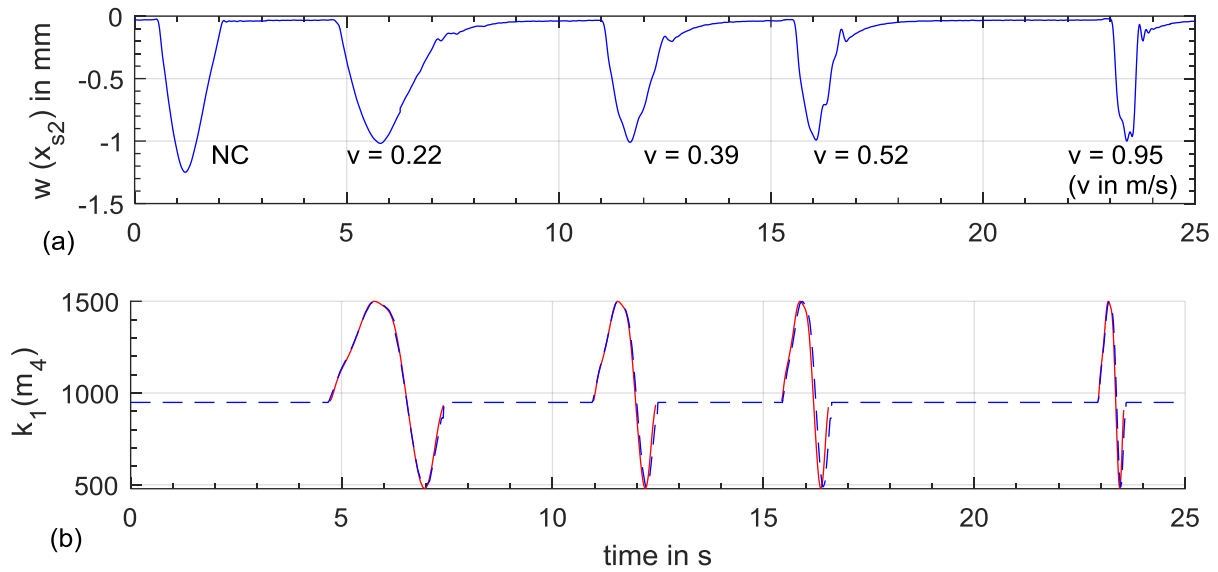
441  
 442 **Fig. 14.** Effect of using time-varying gain  $\mathbf{k}(m_1)$  (left) and scheduled for each mass specifically (right)  $\mathbf{k}(m_1)$   
 443  $-\mathbf{k}(m_4)$ .



444  
 445 **Fig. 15.** Relative maximum deflection for mass  $m_2$  (a) and mass  $m_4$  (b) using gain  $\mathbf{k}(m_1)$  (blue) in comparison  
 446 to using the specific gains  $\mathbf{k}(m_2)$  or  $\mathbf{k}(m_1)$  (red).  
 447

448 Another important factor of the proposed control strategy is the ability to adapt to different velocities of the  
 449 mass. The time-varying gain vector  $\mathbf{k}(t,m)$  is calculated beforehand for a predetermined velocity at equal  
 450 time steps and stored on the controller. By measuring the actual velocity in real time using two induction  
 451 sensors before the mass enters the structure, the leaving time  $t_f$  can be determined exactly. With the given  
 452  $t_f$  the control action is stretched or compressed towards the given traveling time of the mass. The gain is  
 453 then interpolated between the precalculated gain values for the actual position of the mass.

454 In Fig. 16 it can be seen how the control needs to adapt to different speeds ranging from  $v = 0.22 \text{ ms}^{-1}$  to  $v$   
 455  $= 0.95 \text{ ms}^{-1}$ . The gain  $k_1(m_4)$  calculated in real time coincides well with the numerically calculated gain.



456  
 457 **Fig. 16.** Deflection  $w(x_{s2})$ , no control action (NC) and with different speeds  $v$  (a); time-varying gain  $k_1(m_4)$   
 458 (b) calculated in real-time (blue-dashed) and numerically (red- continuous) .  
 459

## 460 6. Conclusion and future work

461 The present study extends numerical investigations into the problem of control of beam structures subjected  
 462 to a set of moving masses, and is concerned with the experimental implementation of the control solution  
 463 on a small-scale rig.

464 It presents and analyses the synthesis and implementation of an active controller on a small-scale test  
 465 structure. The structure is modelled as a simply supported beam, using displacement laser sensors and one  
 466 electromagnetic actuator located close to one of the supports. The importance of this study consists on  
 467 going beyond the theoretical solution to finding and validating solutions based on experimental data. In this  
 468 way the proposed solutions are one step closer to the relevant practical problem.

469 Due to the fast sampling rate of the data acquisition and control, a reduced order controller using estimated  
 470 modal displacements and velocities proves to be the best solution. The dynamics of the actuator was  
 471 simplified, and a first order model was used. Although the model proved correct while contact is  
 472 maintained, a small inaccuracy is observed when the mass leaves the beam.

473 As expected, due to the time-varying nature of the control system, it is shown both experimentally and  
 474 numerically that a control method based on a terminal-time optimal control solution provides better  
 475 performance than a time invariant optimal controller.

476 The possibility of using different moving masses travelling at different speeds also pointed toward a control  
477 solution that adapts the control effort, taking into account the type of load. Therefore, a simple gain-  
478 scheduling solution that makes a better use of the control effort is presented and proves to be the basis of  
479 further work and developments of the method.

480

481

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483

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486

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488

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