Asteroseismology Across the Hertzsprung–Russell Diagram

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Kurtz, Donald Wayne ORCID: 0000-0002-1015-3268 (2022) Asteroseismology Across the Hertzsprung–Russell Diagram. Annual Review of Astronomy and Astrophysics, 60 . ISSN 0066-4146 (Submitted)

ANNUAL REVIEWS

Annual Review of Astronomy and Astrophysics

Asteroseismology Across the Hertzsprung–Russell Diagram

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Annu. Rev. Astron. Astrophys. 2022. 60:31–71

The *Annual Review of Astronomy and Astrophysics* is online at astro.annualreviews.org

https://doi.org/10.1146/annurev-astro-052920-094232

Keywords

stars: oscillations (including pulsations), stars: interiors, stars: variables: general, stars: binaries: close

Abstract

Asteroseismology has grown from its beginnings three decades ago to a mature field teeming with discoveries and applications. This phenomenal growth has been enabled by space photometry with precision 10–100 times better than ground-based observations, with nearly continuous light curves for durations of weeks to years, and by large-scale ground-based surveys spanning years designed to detect all time-variable phenomena. The new high-precision data are full of surprises, deepening our understanding of the physics of stars.

- This review explores asteroseismic developments from the past decade primarily as a result of light curves from the *Kepler* and *Transiting Exoplanet Survey Satellite* space missions for massive upper main sequence OBAF stars, pre-main-sequence stars, peculiar stars, classical pulsators, white dwarfs and subdwarfs, and tidally interacting close binaries.
- The space missions have increased the numbers of pulsators in many classes by an order of magnitude.
- Asteroseismology measures fundamental stellar parameters and stellar interior physics—mass, radius, age, metallicity, luminosity, distance, magnetic fields, interior rotation, angular momentum transfer, convective overshoot, core-burning stage—supporting disparate fields such as galactic archeology, exoplanet host stars, supernovae progenitors, gamma-ray and gravitational wave precursors, close binary star origins and evolution, and standard candles.
- Stars are the luminous tracers of the Universe. Asteroseismology significantly improves models of stellar structure and evolution on which all inference from stars depends.

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1. INTRODUCTION

When I was an undergraduate at San Diego State University studying astronomy in the late 1960s, I had then on my bookshelf the classic work on stellar structure, *The Internal Constitution of the Stars* (Eddington 1926), which I had purchased used for a mere 50 cents. It is on my bookshelf to this day. Eddington (1920) is credited with being the first to propose H fusion as the power source for stars. He developed that idea further in *The Internal Constitution of the Stars*, although he was reluctant to admit a hydrogen fraction greater than 7%, which made his conclusions marginal as to whether there was enough hydrogen in the Sun to power it for the time required by Earth's geological record. He was reluctant despite Cecilia Payne (1925) having shown in her work at Harvard College Observatory that stars are composed primarily of hydrogen and helium, as presented in her Radcliffe College PhD thesis. She was English, but studied at Harvard, because Cambridge, where Eddington was, did not award PhDs to women at that time. Struve & Zebergs (1962, p. 220) declared her thesis to be "undoubtedly the most brilliant PhD thesis ever written in astronomy." Perhaps Eddington was influenced by Henry Norris Russell [the R in HR (Hertzsprung–Russell) diagram], who believed then that the stars had compositions similar to those of the Earth. He did not concede that Payne was correct until 1929.

In *The Internal Constitution of the Stars*, Eddington (1926, p. 1) began with these words:

> At first sight it would seem that the deep interior of the Sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?

Eddington's answer to this was theory. Nevertheless, in his third paragraph of that opening page, he warned: "We should be unwise to trust scientific inference very far when it becomes divorced from opportunity for observational test." This interesting contention both binds and divides astrophysicists across the breadth of our all-encompassing field.

It is more than 50 years since I first read Eddington's treatise, at a time when it was then only 40 years since he had written about such uncertainties in our understandings of the physics of the interior of stars. As I finished my undergraduate studies and moved to the University of Texas in 1970 to undertake my PhD, I heard from my fellow students about the solar five-minute oscillations. These were first noted in a broad report on the velocity fields in the solar atmosphere by Leighton et al. (1962). Then Ulrich (1970) and Leibacher & Stein (1971) correctly identified the five-minute oscillations as standing acoustic waves. The breakthrough came just a few years later when Deubner (1975, p. 371) resolved "three or four discrete stable modes" in a plot of wave number versus frequency. Many other studies followed, and Helioseismology was born. (See Christensen-Dalsgaard 2021 for a comprehensive history and account of solar structure and evolution, including helioseismology.) Eddington's appliance that could pierce through the outer layers of a star had been found, at least for the Sun.

Astronomers are often asked: What good is astronomy? That question comes particularly from funding agencies wanting to know what economic benefit or other "impact" there might be. This question reminds me of Howard Florey, who shared the 1945 Nobel Prize in Medicine for bringing penicillin into medical use, saving untold millions of lives. Talk about impact! Yet when asked about his work Florey famously said,

> People sometimes think that I and the others worked on penicillin because we were interested in suffering humanity. I don't think it ever crossed our minds about suffering humanity. This was an interesting

> scientific exercise, and because it was of some use in medicine is very gratifying, but this was not the reason that we started working on it.[1]

Three decades after helioseismology, the announcement of the birth of asteroseismology was made by Gough (2001) as a result of the discovery of solar-like oscillations in the G2V solar analogue α Cen A by Bouchy & Carrier (2001) and in the G2IV subgiant star β Hyi by Bedding et al. (2001). In the more than two decades since then, asteroseismology has blossomed, having had major impacts on our understanding of stellar pulsation, stellar structure, and evolution, and with wide applications in other fields of astrophysics. Nevertheless, in the spirit of Howard Florey, asteroseismologists are largely driven by a passion to understand the stars, although they are also gratified by applications of their results to other fields.

1.1. The Scope of This Review

The greatest success of asteroseismology is, of course, helioseismology. With spatially resolved observations, the interior structure of the Sun is constrained by measurements of the square of the sound speed to better than three parts per thousand from surface to core, and much better than that throughout most of the Sun. Nothing in asteroseismology yet competes with that. Nevertheless, there have been great successes in asteroseismic studies of stochastically driven pulsators—the solar-like and red giant oscillators—for which pulsations are driven by broad-spectrum white noise generated by convection. These studies have led to high-accuracy (less than a few percent) determinations of stellar mass and radius, knowledge that is critical to exoplanet studies. They have provided direct views—in the Eddington sense—of interior rotation and the extent of the convective core boundary, with impact on stellar structure and evolution models. They have made it possible to distinguish—all jumbled together in the HR diagram—which red giants are helium-core burning and which are hydrogen-shell burning. They have given model-dependent age determinations for single stars (cluster stellar age determinations are also model dependent) that are useful for galactic archeology (Miglio et al. 2017).

These successes are not discussed further here, as the solar-like oscillators have been thoroughly reviewed recently by Chaplin & Miglio (2013), García & Ballot (2019), and Jackiewicz (2021). Hekker & Christensen-Dalsgaard (2017) and Basu & Hekker (2020) have reviewed asteroseismic red giant discoveries, whereas Hon et al. (2021) have found over 158,000 pulsating red giants, an order of magnitude increase on the already huge number found by *Kepler* and K2. Aerts (2021) and colleagues (Aerts et al. 2019) examined results for interior rotation in over 1,200 giants and main sequence stars,[2] showing that these are consistent with white dwarf rotation rates. Interestingly, ? provided a unifying description of the transition from the stochastically driven solar-like and red giant oscillators, through stars with both stochastically driven and classical heat-engine-driven pulsations among the semiregular variables, to the classical pulsations of the Mira variables.

In this era of space photometry, this review looks at the observational discoveries and advances in asteroseismology for pulsating stars other than the solar-like and red giant stars, i.e., OBAF stars of the upper main sequence, subdwarfs, white dwarfs, and giant stars in the classical instability strip. The discussion here is based primarily on the vast, spectacularly successful data sets of the *Kepler* and *Transiting Exoplanet Survey Satellite* (TESS) missions. They were not the first, or only, asteroseismic satellites. Other missions have had significant successes. Both Bowman (2020a)

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[1] De Berg Collection in the National Library of Australia.

[2] A number that has increased by nearly 700 (Li et al. 2020b, for γ Dor stars; Tayar et al. 2019, for He-core burning red giants), showing how rapidly this field is expanding.

and Aerts (2021) discuss the asteroseismic space missions earlier than *Kepler*: WIRE (*Wide-Field Infrared Explorer*), MOST (*Microvariability and Oscillations of Stars*), and CoRoT (*Convection, Rotation & Planetary Transits*) as well as the contemporary nanosatellite cluster BRITE (*BRIght Target Explorer*).

In the **Supplemental Material**, I provide some personal stories and historical background, along with extensive detailed sections for various classes of pulsating stars studied asteroseismically.

Spherical harmonic quantum numbers, n, ℓ, m: n = radial overtone; ℓ = degree; m = azimuthal order

1.2. Asteroseismology: Terminology and a Pulsation HR Diagram

The basic data of asteroseismology are pulsation mode frequencies. Model frequencies are tuned by varying the internal physics to match observed frequencies in a wide spectrum of models with different input parameters, constraining the interior structure of the star. This is referred to as forward modeling, to distinguish it from inversion of the mode frequencies as is done for the Sun. Open-source stellar structure and evolution models from MESA (Modules for Experiments in Stellar Astrophysics; e.g., Paxton et al. 2019) in conjunction with the pulsation code `GYRE` (Townsend & Teitler 2013) are widely used in asteroseismology to constrain models of stellar structure, hence also evolution. The theory of asteroseismology has been reviewed in depth by Aerts (2021) and Aerts et al. (2010, their chapter 3). I provide here only the terminology necessary for this observational review.

The heat engine pulsators are stars for which pulsation driving is stronger than damping and for which the pulsation time is much less than the thermal time of the star. Driving is accomplished mostly by an opacity mechanism, the κ-mechanism, operating in an ionization zone of an abundant ion—usually H or He, but also, e.g., Fe–Ni in upper main sequence stars, and C–O in white dwarfs. The κ-mechanism can be accompanied by other mechanisms: variations in the adiabatic exponent, γ; tidal excitation; and, potentially, variation in the nuclear energy generation rate or ϵ-mechanism. The heat engine pulsators generally have mode lifetimes that are long compared to our observation times. Stars for which damping exceeds such driving can still be driven to observable pulsation by stochastic driving—as in the solar-like stars (giants and red giants that are not a topic in this review)—but also in some AF stars and all massive OB stars, a new asteroseismic discovery (Section 2.12).

There are five restoring forces: pressure (p modes), buoyancy (gravity or g modes), Coriolis force (inertial and Rossby or r modes), magnetic force (Alfvén), and tidal force. These forces can, and often do, operate together, but the main restoring forces are usually pressure and buoyancy. Coupled g modes and p modes are referred to as mixed modes. When the Coriolis force acts in concert with gravity, the modes are called gravito-inertial modes. When there is a strong, global magnetic field, its resulting Lorentz force along with the pressure force gives rise to magnetoacoustic modes. With the superb precision of the space photometry, it is now possible to observe the effects of these many restoring forces and to extract new asteroseismic inference from those effects.

To proceed with forward modeling, it is necessary to identify the modes of all frequencies modeled. Those are described in terms of spherical harmonics, even though those functions are only truly appropriate for perfectly spherical stars that are not rotating and have no magnetic fields. Nevertheless, the terminology is useful. See Aerts (2021) and Aerts et al. (2010, their chapter 1) for further introduction to this. For this review, the three quantum numbers describing the spherical harmonics are the radial overtone, n; the degree, ℓ; and the azimuthal order, m. Radial modes have $\ell = 0$, and nonradial modes have $\ell > 0$. The principal nonradial modes observed are dipole ($\ell = 1$) and quadrupole ($\ell = 2$) modes, because cancellation makes $\ell \geq 3$ modes undetectable in most cases. Modes with $m = 0$ are zonal modes; with $m = |\ell|$ are sectoral modes; and with

$0 < m < |\ell|$ are tesseral modes. The sectoral modes primarily, but potentially also the tesseral modes, are of great interest, because these are traveling waves with observed frequencies that depend on rotation in the mode cavities; hence, they provide the data on interior rotation, information that is unobtainable by any method other than asteroseismology.

Large frequency separation ($\Delta\nu$): the inverse of the sound travel time across the star; gives a measure of the mean density

Characteristic period (Π_0): the buoyancy travel time across the star

Note that p modes have largely radial displacement and g modes have horizontal displacement; p modes are primarily sensitive to the conditions in the outer envelope of the star, but g modes to the deeper interior; p modes have higher frequencies, g modes lower frequencies; p mode frequencies increase with increasing radial overtone n, g mode frequencies decrease with increasing n; high-overtone p mode frequencies asymptotically approach uniform spacing dependent on the large frequency separation, $\Delta\nu$, whereas high-overtone g mode periods asymptotically approach uniform spacing dependent on the characteristic period, Π_0.

The pulsation HR diagram in **Figure 1** gives an overview of the classes of pulsating stars. This diagram was initiated by Christensen-Dalsgaard (1998) and has been adapted many times (versions can be found in Aerts et al. 2010, Degroote 2010, Pápics 2013, Jeffery & Saio 2016, Kurtz et al. 2016, and Aerts 2021). This diagram is a useful guide to our discussion but is intentionally schematic, because the classes of the stars and their boundaries are fluid. Taxonomy is useful for organizing our thoughts, but species crossbreed, have subspecies, appear in the record, and go extinct. The sections of this review clarify the details.

1.3. Frequency Analysis

The observations made for asteroseismology are time series of stellar variability, either photometric variations in brightness, spectroscopic radial velocities, or line profile variations. The main observational goal is to extract the mode frequencies and identify the modes (n, ℓ, m) to then match with model mode frequencies. The emphasis in this review is on the space revolution, from which the frequencies are extracted from light curves by means of Fourier transforms, giving power spectra, power density spectra, amplitude spectra, frequency spectra, or periodograms. For the heat engine pulsators, the amplitude spectrum is most widely used, but for succinctness I use here the generic term FT (Fourier transform). See Aerts et al. (2010, their chapter 5) for an in-depth discussion.

Figure 2 shows examples of a light curve and its FT. Almost all primary publications in observational asteroseismology present FTs of the data under discussion. Aerts et al. (2010, their chapter 2) show examples of ground-based light curves and FTs of most of the classes discussed in this review, and I refer the reader to the referenced primary literature and abundant recent specialist reviews for the space-mission light curves and FTs for each class of pulsating stars.

1.4. Photometric Precision and Duty Cycle

To extract mode frequencies from FTs, we seek high signal-to-noise, high-frequency resolution, long time span, and continuous data. Under ground-based photometric conditions, noise is from instrumental variations, atmospheric transparency variations, photon statistics, and scintillation, the latter two of which are reduced with larger aperture telescopes. Space photometry eliminates transparency and scintillation noise. We seek longer time spans of observations for sharper frequency resolution, and we seek better duty cycles (the fraction of time on target) to minimize confusion from complex, overlapping spectral windows in the FT. For a thorough discussion of observational techniques and frequency analysis see Aerts et al. (2010, their chapters 4 and 5).

In the 1980s, I was making precise ground-based photometric measurements of stellar brightness, reaching photometric precision in pulsation amplitude of 10^{-4} in intensity, or 100 μmag. But I could only do that (*a*) when observing at night, so the time series had daily gaps, and the FTs

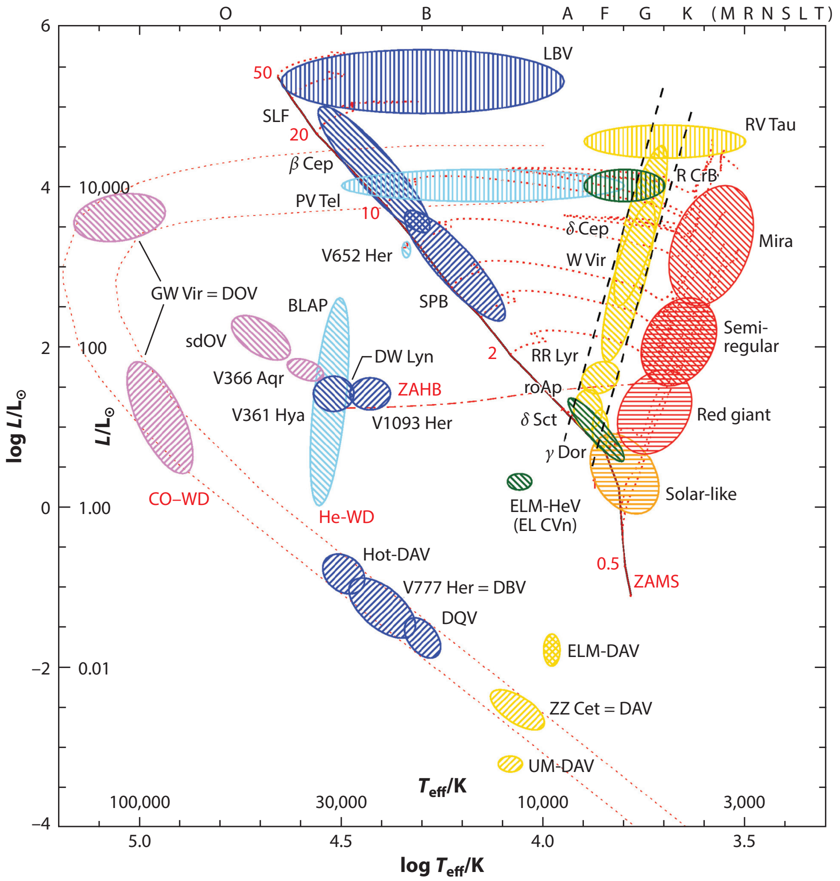

Figure 1

A pulsation, or asteroseismic, Hertzsprung–Russell diagram. This gives a schematic guide to classes of pulsating stars. The abbreviations for the names of many of the classes follow that of (Aerts et al. 2010, their chapter 2). Others are clarified in the relevant sections of the text. The solid red line is the zero-age main sequence, and standard evolutionary tracks are shown as dotted lines up to the tip of the first red giant stage; the red numbers along the main sequence are masses in units of solar mass. The cross-hatchings represent the primary mode types: acoustic *p* modes (\\\); gravity (buoyancy) *g* modes (///); stochastically driven pulsators (≡); and strange modes (|||). This figure is provided by Simon Jeffery, and is based on Jeffery et al. (2015, "Subaru and Swift observations of V652 Herculis: resolving the photospheric pulsation," their figure 1).

had daily aliases; (*b*) when the telescope was scheduled for me, leading to monthly aliases, because photometry of bright stars was scheduled for bright moon; (*c*) when the star was not too close to the Sun, leading to yearly aliases in long data sets; and (*d*) when the weather was good! Long-term weather averages for the superb photometric conditions needed for asteroseismology are typically about 50–70% for excellent observing sites.

In one case, the Whole Earth Telescope (WET), with 46 astronomers working over 35 days, obtained a 35% duty cycle and reached a photometric precision of 14 μmag (Kurtz et al. 2005)—but for only one star! Compare that to the *Kepler* main mission in which ∼150,000 stars were observed nearly continuously for 4 years, and ∼50,000 more were observed for at least 90 days, with a best precision of the order of 1 μmag–1 ppm. Compare it to the all-sky photometry of

Review in Advance first posted on April 8, 2022. (Changes may still occur before final publication.)

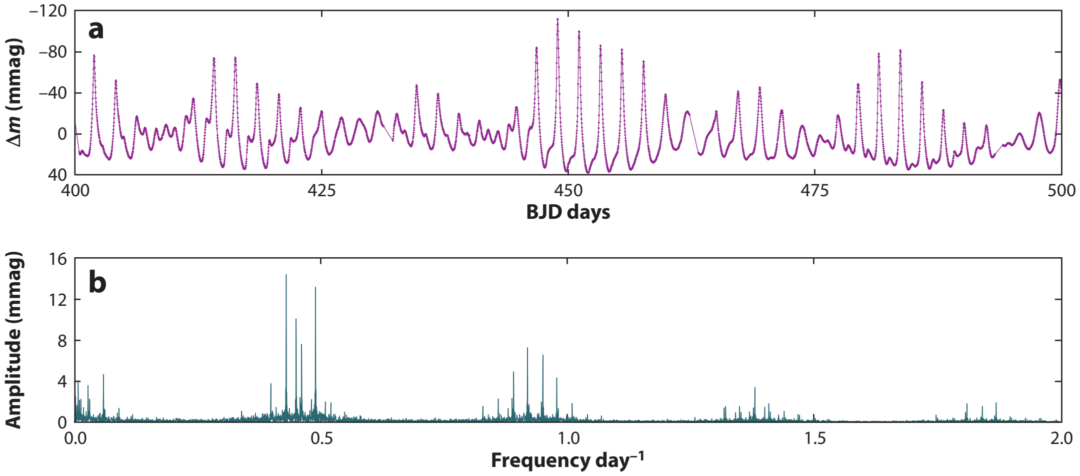

Figure 2

(*a*) A light curve for KIC 8113452, a strongly nonlinearly pulsating γ Dor star. The time is relative to BJD 2455000. This light curve is shown here as a clear example of the time series that are the fundamental observational data of asteroseismology. The *Kepler* and TESS data are so precise that for many kinds of multiperiodic pulsating stars the light curve can seem a jumble of points, even though at the scale of this plot the uncertainty in magnitude is 1,000 times smaller than the size of the data points. (*b*) A Fourier transform of this light curve in the low-frequency range of its *g* modes. The groups of frequencies around 0.4–0.5 day^{-1} are from the *g* modes. The other groups of frequencies are nonlinear combination frequencies that result from the coupled nonlinear pulsations. It is the job of the asteroseismologist to identify the correct mode frequencies for forward modeling from the plethora of combination frequencies. This is often done by pattern recognition, which humans are good at. But the big data of modern asteroseismology from space missions is overwhelming humans, and machine learning now usually makes the initial classification of the type of pulsator and flags objects of interest. Panel *b* adapted from Kurtz et al. (2015, "A unifying explanation of complex frequency spectra of γ Dor, SPB and Be stars: combination frequencies and highly nonsinusoidal light curves," their figure 3).

TESS with at least 27-day light curves at a precision of order 10 μmag. With an improvement in precision typically of a factor of 10–100 over excellent ground-based data, and with data sets that are nearly continuous for time spans of months to years, vast numbers of light curves hold waiting discoveries. Tycho Brahe's quest for ever higher astrometric precision was fundamental to Johannes Kepler's discovery of his laws of motion for the planets (which led to, much later, the naming of the *Kepler* mission). Kurtz et al. (2016), in the spirit of Tycho Brahe, called this quest for precision the Tychonic Principle, where a revolutionary improvement in precision inevitably leads to discovery.

For stars pulsating in *g* modes and *r* modes, time spans of at least months are needed to resolve the pulsation frequencies. That resolution is critical to mode identification and modeling of the stellar interior. For the stars that need these long time spans, the 4-year *Kepler* data are currently definitive. For stars for which shorter time spans are sufficient, the TESS mission data are the foundation of much current research. These space missions have revolutionized asteroseismology.

1.5. Summary: The Space Revolution

1. Space photometry has precision about 100 times better than ground-based photometry in the range of pulsation frequencies of most stars studied asteroseismically.
2. The space data are nearly continuous, greatly reducing—or even eliminating—confusion from spectral window aliases in the FT caused by gaps in the data.

3. The space data—particularly that of *Kepler*, and of TESS in the continuous viewing zone—have the high-frequency resolution that is needed for mode identification and to disentangle the FTs of many types of pulsating stars.
4. The space data have overwhelmed previous ground-based asteroseismology. For example, the number of δ Sct and γ Dor stars known prior to the space data was a few hundred. It is now many thousands. For solar-like and red giant pulsators it was dozens, now the number surpasses 100,000. For stars with known internal rotation it was one—the Sun. It is now thousands.
5. As a consequence of the discoveries from the space data, there has been a flood of recent reviews, showing the vibrancy of this field of research:
   - This review complements the theoretical asteroseismology review of Aerts (2021) and updates Aerts et al. (2010, their chapter 2).
   - Aerts et al. (2019) reviewed angular momentum transport in stellar interiors.
   - Chaplin & Miglio (2013), García & Ballot (2019), and Jackiewicz (2021) reviewed the stochastic solar-like oscillators.
   - Christensen-Dalsgaard (2021) reviewed solar structure, evolution, and helioseismology.
   - Hekker & Christensen-Dalsgaard (2017) and Basu & Hekker (2020) reviewed asteroseismology of red giants.
   - Bowman (2020a,b) reviewed OB stars and their pulsations.
   - Guzik (2021) reviewed *Kepler* and TESS observations of γ Dor stars and δ Sct stars.
   - Holdsworth (2021) reviewed *Kepler* observations of rapidly oscillating Ap (roAp) stars.
   - Plachy & Szabó (2021) reviewed *Kepler* observations of RR Lyrae stars.
   - Córsico et al. (2019a) reviewed pulsating white dwarfs, in general, and Córsico (2020) reviewed white dwarf asteroseismology with the *Kepler* telescope.
   - Heber (2016) reviewed sdB and sdO stars.
   - Lynas-Gray (2021) reviewed asteroseismology of hot subdwarf stars.
   - Guo (2021) reviewed asteroseismology of close binary stars, and Lampens (2021) reviewed eclipsing binaries with pulsating components.

γ Dor stars: late-A to early-F main sequence stars that pulsate in g modes with periods around 0.5–3 days

δ Sct stars: early-A to early-F main sequence stars that pulsate primarily in p modes with periods in the range of 20 min to 8 h; also usually pulsate in g modes

Rapidly oscillating Ap (roAp) stars: these stars pulsate in high-overtone magnetoacoustic modes with periods in the 4.7–25.8-min range; oblique pulsators

β Cep stars: early B stars that pulsate in p modes with periods in the range of 2–8 h; some also pulsate in gravito-inertial modes similar to SPB stars

Slowly pulsating B (SPB) stars: B stars that pulsate in gravitoinertial modes with periods around 0.5–4 days

2. PULSATION IN MAIN SEQUENCE OBAF STARS: γ Dor, δ Sct, roAp, SPB, AND β CEP STARS

The g mode and p mode pulsators γ Dor stars and δ Sct stars, respectively, are found along the main sequence in the AF range ($M = 1.3$–$2.5\ \mathrm{M}_\odot$). The theoretical instability strips for these two classes overlap. The high amplitude delta Scuti (HADS) stars and the older-population SX Phe stars are subclasses of the δ Sct class. In the same temperature range are also the high-overtone magnetoacoustic mode oblique pulsators, the rapidly oscillating Ap (roAp) stars. Among the OB stars are the hotter β Cep stars ($7 \leq M \leq 25\ \mathrm{M}_\odot$) and the cooler slowly pulsating B (SPB) stars[3] ($3 \leq M \leq 10\ \mathrm{M}_\odot$). All of these classes have core convection zones, as a consequence of CNO-cycle H burning. A number of OBAF stars fall outside of the theoretical instability strips, suggesting that the traditional pulsation classes are not pure. Nevertheless, it is convenient to use these classes as presented in **Figure 1**, recognizing that there is overlap.

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[3] See **Supplemental Text Section A.1** for stories of the naming of the SPB stars and discovery of the pulsation driving mechanism in B stars.

The δ Sct stars were defined traditionally as p mode pulsators driven by the κ-mechanism operating in the He II ionization zone in their envelopes. The γ Dor stars[4] were defined as g mode pulsators driven by convective blocking (Guzik et al. 2000, Dupret et al. 2005). It is clear now that these two classes are not distinct. Although there are pure g mode pulsators among the γ Dor stars, there are only a few pure p mode pulsators among the δ Sct stars; most also pulsate in g modes. In addition, it can now be seen in *Kepler* and TESS data that many γ Dor, δ Sct, and hotter B stars also pulsate in r modes (Saio et al. 2018), although this is not part of the classification of these stars. Xiong et al. (2016) have shown that the κ-mechanism and coupling between the pulsation and convection account for the driving and damping of both p modes and g modes in δ Sct and γ Dor stars.

λ Boo stars: late-B to early-F stars with underabundances of refractory elements

2.1. δ Sct Stars

These stars have a rich variety of pulsation behavior, ranging from rare singly periodic stars to stars with hundreds of frequencies spread across the 0–80 day^{-1} range, requiring g modes, r modes, p modes, mixed modes, rotationally split multiplets, nonlinear combination frequencies, and possibly more to understand. It is noteworthy that these most common of all main sequence heat engine pulsators are difficult to decipher because of the complexity of their behavior. The FTs of δ Sct stars have, until recently, defied mode identification for large numbers of their frequencies, a requisite for forward modeling. The reasons are that hundreds of modes can be excited in a given star, with radial, nonradial, and mixed modes appearing along with combination frequencies from nonlinear mode interaction. The δ Sct stars are mostly moderate to rapid rotators, so that rotationally split multiplets for nonradial modes are not equally split, and the frequency separation of the multiplets exceeds the mode frequency separation, leading to confusion in mode identification efforts.

Further complications are caused by both amplitude and frequency modulation. Bowman et al. (2016) studied 983 δ Sct stars with full 4-year data sets from *Kepler* and found more than 60% of these stars have frequencies that vary in amplitude. Besides presenting an interesting problem of how and why some pulsation modes vary in amplitude and others do not, this study makes us aware that spurious frequencies can be generated in frequency analysis, especially when it is automated. That can produce lists of hundreds, and even thousands, of frequencies where a large fraction of them are not pulsation frequencies. Because forward modeling requires mode frequencies with mode identifications, the δ Sct stars have not yet lived up to their full potential for asteroseismology.

Even with the large number of frequencies identified in the FTs of many δ Sct stars, pulsation models of A stars generally find an even larger number of mode frequencies excited. When only a few of those have confident mode identification, and many model frequencies are available to match, the chances of random matching increase, particularly where relatively large tolerances in the frequency matches are allowed. It is not known why so many more modes are found excited in models than are observed. Guzik (2021) has recently reviewed the δ Sct and γ Dor stars in the *Kepler* and TESS era, and Antoci et al. (2019), in their first-light paper on TESS sectors 1 and 2, give a broad background to the δ Sct and γ Dor stars, to the A star zoo of metal-strong Am and Ap stars, and to the metal weak λ Boo stars and SX Phe stars. I refer the reader to those papers for more background and discussion of successes in advancing our understanding of δ Sct stars. I

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[4] See **Supplemental Text Section A.2** for background on the discovery of the γ Dor stars.

highlight two of those successes here: The discovery of asymptotic spacing of high radial overtone *p* modes in δ Sct stars (Antoci et al. 2011, Bedding et al. 2020) in this section, and the discovery of tidally forced and tidally interacting *p* modes in δ Sct stars in binaries in Section 5.

A large part of the success of asteroseismology of stochastic pulsators, such as the solar-like oscillators and red giants, comes from the mode identification made possible by a long series of asymptotically spaced, high radial overtone *p* modes. For decades, a goal was to find such series of mode frequencies for δ Sct stars, but complications and confusion from spectral windows in the FTs of ground-based data (even multisite data) made this difficult. The first breakthrough in finding high-overtone asymptotically spaced *p* modes in a δ Sct star came from 20 days of *Kepler* data of HD 187547 (Antoci et al. 2011). This star shows many frequencies typical of δ Sct stars and, in addition, a series with a uniform spacing allowing a determination of $\Delta\nu$, the large separation, giving mode identification for modeling. Antoci et al. (2011) suggested that these high-overtone modes could be driven by acoustic noise in the surface convection zone, even though that zone is much thinner than that in cooler solar-like stars. This idea was tested by looking at the stability of the mode frequencies in HD 187547. For stochastically driven pulsation, the lifetimes should be short; they are only weeks to months in the Sun. As more data accumulated from the *Kepler* mission, Antoci et al. (2014) found from 960 days of *Kepler* data that the high-overtone mode frequencies in HD 187547 are stable—they have lifetimes longer than 960 days, as is evident from the sharp frequency peaks in the FT. Antoci et al. proposed a new mechanism, turbulent pressure, as the driving force of the high-overtone modes.

A bigger breakthrough came from the discovery of 60 δ Sct stars with regular frequency spacing, giving large separations, using TESS 27-day, 2-min cadence data sets and *Kepler* 1-min data sets of varying duration, but all of at least 30 days. Bedding et al. (2020) were able to make secure mode identifications and model these stars. They gave an example where their asteroseismic age of 150 Myr for the young Pisces-Eridanus stellar stream supports the 120 Myr age determined from gyrochronology of 101 low-mass members with TESS data. This newly discovered, nearby stellar stream spans 400 pc and was previously suggested to be 1 Gyr in age. Thus, the agreement in age from asteroseismology and gyrochronology is an important development in the study of this stellar association, and it supports the validity of both techniques.

Asteroseismic modeling of the 60 δ Sct stars by Bedding et al. also provides fundamental parameters for these stars—mass, radius, and age—that will have many applications: e.g., ages of young clusters, stellar streams, and moving groups that contain δ Sct stars, such as the age determination of 11 Myr for the Upper Centaurus–Lupus part of the Scorpius–Centaurus Association (Murphy et al. 2021; see Section 2.15.3). Knowledge of mass, radius, and age will help decipher the problem of mode selection in δ Sct stars. We do not yet know why some have hundreds of modes excited, some have only a few modes, or one mode, excited, and many stars in the instability strip do not pulsate at all. With many more δ Sct stars being observed by TESS, this form of asteroseismology is finally opening up for those most ubiquitous of upper main sequence pulsators.

2.2. Frequency Variability in δ Sct and Other Pulsating Stars

The question of frequency variability (or equivalently, period variability) in all pulsating stars is important primarily to observe evolutionary changes in the pulsation cavities, hence, to observe stellar evolution in real time. In the literature, this effect is usually quantified as a rate of period change, $\dot{P}/P$. Efforts have been made to do this in most types of pulsating stars, and it is commonly invoked in white dwarf asteroseismology as a measure of cooling time (Section 4.1). But the effort is fraught with difficulties, calling for caution.

Frequency variability can be driven externally by the Doppler effect for pulsating stars with companions, whether stars, planets, or even black holes. All that matters in this case is the orbital variation, and this is easily distinguished because all pulsation frequencies are affected (see **Supplemental Text Section D.1**).

Bowman et al. (2016) studied amplitude and phase modulation of frequencies in 983 δ Sct stars observed for 4 years by *Kepler* (pulsation phase modulation is equivalent to frequency modulation). They examined causes of the amplitude and phase variations, which include unresolved frequencies (whether mode frequencies or nonlinear combination frequencies), three mode resonances, variable driving and damping, and energy exchange between coupled modes.

Where frequency variations are detected with confidence, they can behave differently for different modes and even have opposite signs, i.e., some frequencies increasing and others decreasing in the same star. For δ Sct stars the observed frequency variability is often orders of magnitude greater than predicted from evolutionary models, and it also often has the wrong sign.[5] It is understandable evolutionarily that as mode cavities change, some modes will stretch in wavelength and others shrink to meet the boundary conditions of the cavities. Because the frequency changes may go either way, however, means that many mode frequencies in each star need to have measured frequency variability to begin to model evolutionary changes.

We often think of stellar evolution as being a smooth process as stars move along their evolutionary tracks in the HR diagram. But, as with all physical processes, at some level there are probably other effects that give rise to changes that are not smooth. I think of this as "evolutionary weather"; Breger et al. (2017) described it as temporary changes in the structure of the star. With the extremely high precision of frequency determinations being made with data sets that have durations of years and even decades, we now can see evolutionary weather in action.

For frequency variability that is obviously not evolutionary, the physical causes are not known. This is a case in asteroseismology in which the space data do not always dominate the studies, because a long time base is useful, and for small frequency changes, is needed. Note, e.g., the 160-year study of Polaris, one of the best-known stars in the sky. Polaris is a Cepheid that pulsates in the fundamental radial mode. Turner et al. (2005) examined data obtained from 1844–2004. They found a period change of $\sim$4.5 s year^{-1}, consistent with evolution during a redward first crossing of the instability strip. But they also found that the rate of change had a rapid decrease during 1963–1966, after which it continued as before. That was coupled with a steady pulsation amplitude for Polaris until 1963–1966, after which the amplitude was much smaller and somewhat erratic. This is what I call evolutionary weather. It gives us pause when interpreting pulsation frequency changes as evolutionary.

2.3. Summary: The δ Sct Stars

1. δ Sct stars are the most common heat-engine main sequence pulsators.
2. Hundreds of frequencies can be excited, and in models even more are predicted.
3. The mode selection mechanisms are not known.
4. Recent determinations of the large separations in many δ Sct stars have opened up asteroseismic inference for these stars: mass, radius, and age.
5. Precise asteroseismic ages agree well with gyrochronology, validating both methods.

2.4. *g* mode Pulsators: γ Dor, δ Sct, and SPB Stars

Observable *g* modes in upper main sequence stars are of fundamental importance because these modes have pulsation cavities that sample the conditions in the deep interior of the stars. It is

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[5] See **Supplemental Text Section A.3** for details of the cases of 4 CVn and KIC 5950759.

difficult to detect *g* modes in cooler stochastic pulsators, such as the Sun, because they cannot propagate in the thick outer convection zones, so that little of the signal reaches the surface. It was the advent of CoRoT, then *Kepler* and TESS, that led to an explosion of discovery—both observational and theoretical—from *g* modes.

We did not realize before these space missions, although it is in hindsight obvious, that the *g* mode frequencies are so closely spaced that typically 90 days or more of observations are needed to resolve them. Over that time span, gaps in ground-based observations cause a plague of aliases and tangled spectral window patterns in the FTs, so that the nearly continuous data of the space missions are required for studying these stars. It is the 4-year data sets of *Kepler* and the 1-year continuous viewing zone TESS data that have opened new fields of research with *g* modes in γ Dor, δ Sct, and SPB stars.

The new discoveries are the observations of deep interior rotation, allowing inference of angular momentum transport, of measurement of core overshoot at the convective core boundary, and of measurement of the mass and radius of the convective core, giving evidence of chemical mixing and extended stellar lifetimes. See Aerts et al. (2019) for an extensive review of our new knowledge of angular momentum transport in stars.

The exploitation of the long-duration space data sets for γ Dor stars is one of the outstanding achievements of asteroseismology in recent years. It has given us an observational view of these stars that could not have been obtained from the ground, and it has driven a plethora of theoretical developments to extract astrophysical inference from those data. In a study of 611 γ Dor stars with 4-year *Kepler* data sets, Li et al. (2020b) found that essentially all γ Dor stars pulsate in dipole *g* modes, 30% also pulsate in quadrupole *g* modes, and 16% in addition pulsate in *r* modes. The most visible modes are the prograde sectoral dipole modes ($\ell = 1, m = 1$). Those often have many consecutive radial overtones for which the frequencies are easily identifiable in an FT. As was first derived by Shibahashi (1979, his equation 23), in the asymptotic limit for *g* modes in a nonrotating, nonmagnetic star, the pulsation periods are roughly uniformly spaced by an amount that depends on the characteristic period spacing, Π_0, and mode degree, ℓ. Retrograde dipole *g* modes are also excited, whereas zonal dipole modes are less commonly seen.

These series of *g* mode overtones greatly facilitate mode identification. We now have new tools to measure internal rotation, and even differential rotation from core to surface, leading to the ability to calculate internal angular momentum transfer, and to measure the core mass, radius, and amount of convective overshoot, which affect mixing in the energy generating layers, hence main sequence lifetimes. These are major advances improving stellar structure and evolution models.

2.5. Core to Surface Rotation from Multiplets

KIC 11145123 is a δ Sct star with stunningly simple rotational multiplet patterns of dipole and quadrupole modes for both *g* modes and *p* modes that provided the first direct measure of both internal and surface rotation in an H-core burning star (Kurtz et al. 2014). **Figure 3** shows an FT for this star with higher resolution views of the *g* mode and *p* mode rotational multiplets. Measuring stellar rotation from frequency splitting of dipole and quadrupole frequency multiplets depends on the Ledoux constant, $C_{n,\ell}$ (e.g., see Aerts 2021, her equations 44–46), which asymptotically approaches 0.5 for high radial overtone dipole *g* modes and has a small value not much greater than zero for *p* modes. As a consequence, Kurtz et al. derived for KIC 11145123 a model-independent, near-core rotation period of 105.1 days, and a surface rotation period of 98.5 days. The surface rotates faster than the deep interior—a surprise at the time.

Simple arguments from stellar evolution and conservation of angular momentum lead to the expectation of the shrinking core spinning up and the expanding envelope spinning down with

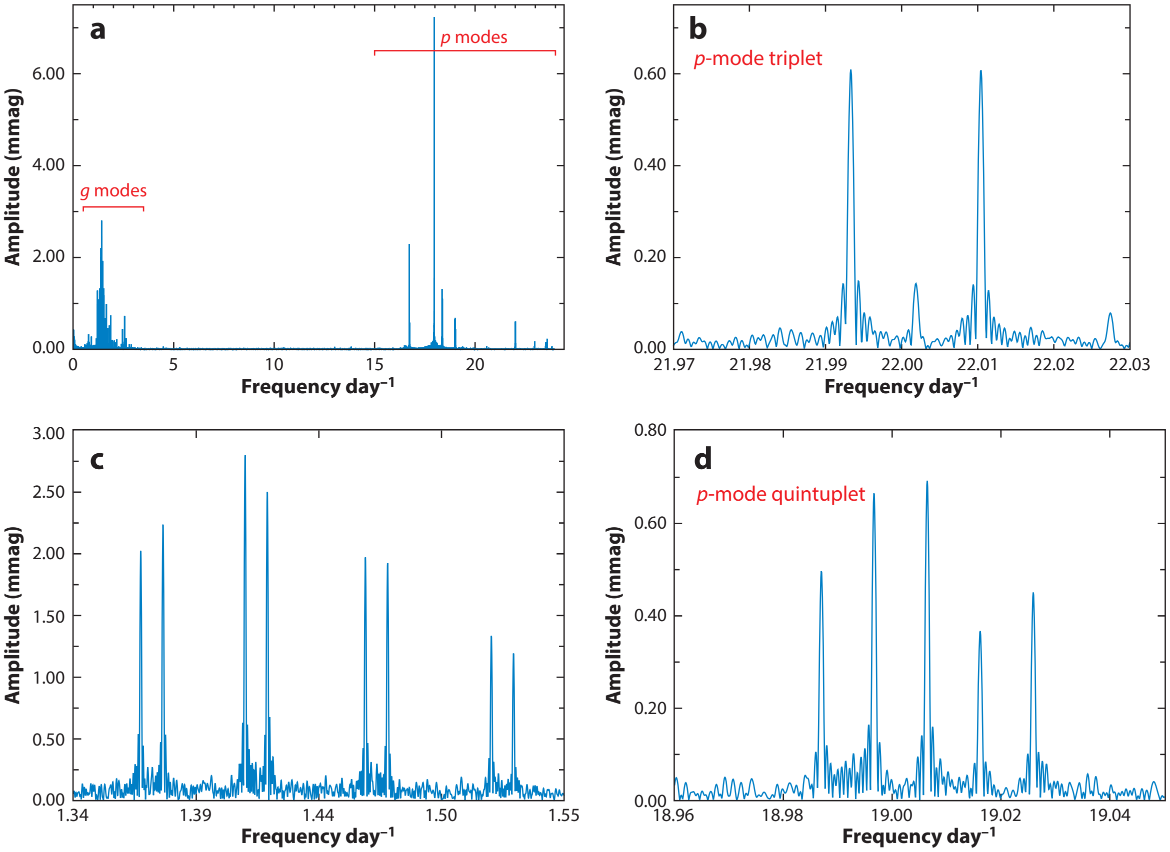

Figure 3

(*a*) The Fourier transform for 4 years of *Kepler* data for the A6 main sequence δ Sct–γ Dor star KIC 1145123, for which near-core to surface rotation was measured for the first time in a hydrogen-burning main sequence star. The frequency ranges of the g modes and p modes are marked. (*c*) An expanded view for the g modes, where four of many consecutive overtones are split into apparent doublets. Those are actually dipole rotational triplets with the central peak not visible at this scale. (*b*) A dipole rotational triplet and (*d*) a quadrupole rotational quintuplet for p modes. The asteroseismologist's tasks here are the recognition of the rotational multiplets and the g mode series and the identification of the overtones of the p modes by forward modeling. This star has a particularly clear view of the patterns as a consequence of its very slow rotation ($P_{\mathrm{rot}} \sim 100$ days). Panels adapted from Kurtz et al. (2014, "Asteroseismic measurement of surface-to-core rotation in a main sequence A star, KIC 11145123," their figures 1 and 3).

age. Clearly, a new view of interior angular momentum transfer in main sequence stars, as well as angular momentum gain from accretion or loss from mass loss, had become possible. This field has developed dramatically with interior rotation being observed in nearly 2,000 stars from the main sequence through red giants and on to white dwarfs. See Aerts et al. (2019) for a theoretical review and Aerts (2021, her figure 6) for an observational view of interior rotation versus $\log g$, hence, evolutionary stage.

Examining KIC 11145123 further, Hatta et al. (2019) used a detailed perturbative approach to obtain a 2D view of the rotation in this star. By studying mixed modes, which have some sensitivity to the convective core rotation, they derived a core rotation rate for $r/R < 0.05$ that is six times faster than the radiative exterior. Thus, there is a strong velocity shear at the convective core boundary that could impact mixing there, hence, main sequence lifetime. They also found marginally significant evidence of surface differential rotation, with higher latitudes rotating faster than the equatorial region, by exploiting the difference in the latitudinal dependence of dipole and quadrupole modes. Gizon et al. (2016) also used dipole and quadrupole *p* modes to calculate both polar and equatorial radii for KIC 11145123, remarkably finding the difference to be only 3 ± 1 km. This star is so round that some unknown factor is needed to suppress rotational oblateness even at its very slow rotation rate.

Other very slowly rotating stars were then found in the *Kepler* data and analyzed. Saio et al. (2015) found KIC 9244992 to be similar to KIC 11145123, with an interior rotation period of 64 days and a surface rotation period of 66 days; at least the surface is rotating more slowly than the interior in this case. Then Triana et al. (2015) found a core boundary rotation period of 71 days for the SPB star KIC 10526294 and suggested that the seismic data supported a counter-rotating envelope! That is amazing, if correct.

For δ Sct stars in which both *p* modes and *g* modes are observed, both deep interior and surface rotation can be measured from the multiplet splitting in the FTs. For pure γ Dor stars in which only *g* modes are observed, it is still possible to examine core and surface rotation rates by deriving the core rate from the period spacing of the *g* modes and constraining the surface rate from a determination of the radius and spectroscopically measured $v \sin i$. Murphy et al. (2016a) showed this for KIC 7661054, which has a core rotation period of 27 days.

But these few stars discussed here are all very slow rotators. Most A and early-F main sequence stars rotate much more quickly, with typical rotation periods of a day or two. For *g* modes in these stars the rotation period and pulsation periods are similar. To observe interior rotation and infer angular momentum transfer in upper main sequence stars, new techniques and new theoretical developments were needed. This has resulted in an explosive outpouring of research, both observational and theoretical, in this new field.

2.6. Internal Rotation from *g* Modes and *r* Modes in Upper Main Sequence Stars

A big breakthrough in determining internal rotation in γ Dor and SPB stars came when it was shown that, for a series of *g* modes with high radial overtones, the difference in the periods, ΔP, for consecutive overtones versus the periods, P, provides a direct measure of rotation in the pulsation cavity, with most sensitivity just above the convective core deep in the star. Bouabid et al. (2013), Van Reeth et al. (2016), and Ouazzani et al. (2017) showed how the slope of points in the $\Delta P - P$ diagram measures the interior rotation rate. The slopes differ for series of prograde sectoral, zonal, and retrograde sectoral dipole modes (Ouazzani et al. 2017, their figure 2), and all series are useful. The prograde modes are most common and in many cases dozens of consecutive radial overtone modes are detected, particularly in the 4-year *Kepler* data in which the frequency resolution is the best to date. Ouazzani et al. (2019, their figure 5) showed the good agreement between the internal rotation rates determined by Cunha et al. (2020) and Van Reeth et al. (2016), supporting the robustness of the two methods.

This new possibility was exploited immediately. Aerts et al. (2017) studied the internal rotation of 59 δ Sct/γ Dor stars plus 8 SPB stars covering a mass range of 1.4–5 $M_\odot$ for which they also had constraints on surface rotation from spectroscopic $v \sin i$. For rotation velocities up to 50% of breakup, they found that the stars deviate from solid body rotation only mildly. With comparison results for red giants, they concluded that core rotation rates must drop significantly in the evolutionary interval in which the stars shift from H- to He-core burning. It is clear that efficient mechanisms for the transport of interior angular momentum during the evolution of stars must be operational.

One proposed mechanism is internal gravity waves (IGWs), which are discussed in detail by Rogers et al. (2013) and Aerts et al. (2019). (IGWs should not be confused with gravity modes.)

The IGWs are generated by turbulence at the convective core boundary. As they travel out into the radiative envelope, where density and temperature are dropping, they can become shock waves, break and deposit energy and angular momentum, and contribute to chemical mixing.

Takahashi & Langer (2021) developed a formalism to model the evolution of a 1.5-$M_\odot$ star that includes the coupled effects of a global magnetic field and rotation. Their model settled to near-rigid rotation on the Alfvén timescale. In another model, angular momentum transport arises from magnetic torque caused by the Tayler instability (Fuller et al. 2019). Interestingly, Fuller & Ma (2019) examined that angular momentum transport mechanism and concluded that most single black holes, as well as many binary black holes, are born rotating slowly, and this has implications for future gravitational wave detections. Thus, asteroseismology even sheds light on gravitational wave astronomy.

Two new techniques for determining internal rotation have been developed that have the potential to be applied to many more γ Dor, δ Sct, and SPB stars across a range of rotation rates. These are the $\nu - \Delta\nu$ technique of Takata et al. (2020), and the exploitation of dips in the $\Delta P - P$ diagram by Ouazzani et al. (2020) and Saio et al. (2021). Details for these techniques are discussed in the **Supplemental Text Section A.4**.

2.7. Probing the Convective Core Boundary

For many years, a goal in studying the SPB and β Cep stars has been to determine the extent of core overshoot—which affects mixing and main sequence lifetime—and core rotation. Our crude understanding of the extent of core overshoot is one of the most uncertain parameters in stellar models. See Pápics (2013) for a historical review of the observational efforts. Until recently, the number of OB stars for which core overshoot or core rotation could be determined was limited to only 11 stars (Aerts 2013). That is now changing with analysis of *Kepler* data. TESS data in the 1-year continuous view zone also have sufficient duration. These studies will come to fruition with analysis of the TESS full frame images (FFI), which were originally 30-min cadence and are now 10-min cadence.

The β Cep stars are rarer than the SPB stars as a consequence of the stellar initial mass function and main sequence lifetimes. Because all of them have strong wind losses, and some of them are potential progenitors of core-collapse supernovae (SNe), they are of great interest in understanding their contribution to galactic chemical evolution. However, because of their scarcity, up to now only 12 of these stars have been modeled asteroseismically. Burssens et al. (2019) discussed some β Cep stars discovered in *Kepler* K2 data, and pointed out the asteroseismic potential of these stars once spectroscopic observations provide mode identifications. I refer the reader to the comprehensive reviews and discussions of OB stars and their pulsations by Bowman (2020a,b), who provides extensive background on asteroseismology, and discussion of ground-based and space photometry of these stars and their potential.

As a consequence of core nuclear burning, there is a strong gradient in the mean molecular weight—a μ-gradient—at the boundary of the convective core in upper main sequence stars. That boundary is a turning point for g modes; for each radial overtone g mode the standing wave must have a node at the boundary. As the radial overtone increases, the radial wavelength of the mode decreases while the mode cavity remains fixed. The mode wavelength adjusts to give an integral number of radial waves in the cavity, and the frequency adjusts with that. This produces cyclic dips in the $\Delta P - P$ diagram superposed on the rotational gradient. Miglio et al. (2008) first showed how these cyclic dips can be used to measure the radius of the convective core. That depends on core overshoot and mixing at the core boundary, because the introduction of fresh H fuel to depths where the temperature is high enough for CNO-cycle fusion modifies the convective core radius. That fresh fuel also extends the main sequence lifetime, so knowledge about core mixing is important in stellar evolution models. The cyclic behavior on the $\Delta P - P$ diagrams is evident in both calculated and observed diagrams (Bouabid et al. 2013, Van Reeth et al. 2016, Ouazzani et al. 2017).

This oscillatory character is also present in the period spacings of g modes in white dwarfs and has been used for decades to measure the extent of the surface layers that are the pulsation cavities in those stars. It is now used to determine the position of the μ-gradient boundary in γ Dor and SPB stars. Mombarg et al. (2019) combined asteroseismology and spectroscopy to estimate the core masses and convective overshooting in γ Dor stars to conclude that efficient angular momentum transport is already active for main sequence stars.

A breakthrough in modeling SPB stars has come from Pedersen et al. (2021), who selected 26 SPB stars out of a sample of 60 with *Kepler* long-cadence 30-min, 4-year data sets. They obtained spectra for 15 of these stars, giving boundary conditions of surface temperature and gravity, metallicity, and projected equatorial rotation velocity; Gaia astrometric data provided luminosities. With a grid of models with free parameters that include mixing at the core envelope boundary and internal rotation, Pedersen et al. modeled the g mode period spacings. They examined a number of envelope mixing sources, including convective envelope penetration (core overshoot), IGWs, vertical shear, and meridional circulation combined with vertical shear. The stars have a range of internal rotation frequencies up to 70% of the critical breakup frequency for 9 of them. Results show the fractional core mass to be 30% near the zero-age main sequence and only dropping to 6% (not zero) at the terminal-age main sequence where core H fusion finally ceases. They found internal mixing profiles of SPB stars that are radially stratified instead of constant.

All of these results are a major step in modeling stellar evolution of upper main sequence stars, with asteroseismic observations of interior conditions from core to surface. Higl et al. (2021) discussed their theoretical 2D hydrodynamical simulations to understand the mixing that occurs between the convective core and radiative layers above it in 1.3–3.5 $M_\odot$ stars. The results of Pedersen et al. (2021) and future expansion of these observational asteroseismic studies will allow constraints on the development of 3D hydrodynamical simulations.

2.8. Summary: Internal Rotation and the Convective Core Boundary

1. Prior to the photometric space missions, internal rotation was known essentially only for the Sun, and that was found to be completely different to what was predicted prior to helioseismology.
2. We now measure the internal rotation profiles in thousands of stars from the main sequence to white dwarfs, allowing direct observation of angular momentum transport with stellar evolution.
3. Strong angular momentum transport mechanisms are needed in stars.
4. Asteroseismology measures the size of the convective core, which impacts asteroseismic age determinations.

2.9. Atomic Diffusion

Atomic diffusion—radiative levitation and gravitational settling—has impact on stellar structure and evolution across the HR diagram. It is a small component of the standard model of the Sun, it significantly affects globular cluster ages determined from model isochrones, it is active in white dwarf atmosphere stratification, and it produces the atmospheric abundance anomalies of the metallic-lined Am and peculiar Ap stars, the most peculiar stars known (Section 2.15). See

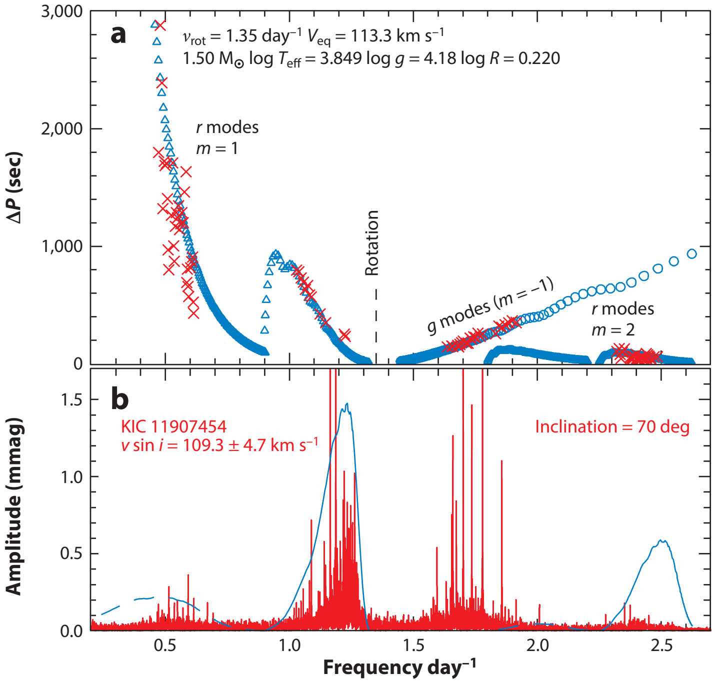

Figure 4

Observed period spacings (*red crosses*) and an FT for the early-F γ Dor star KIC 11907454 are compared with predicted period spacings (*blue triangles* and *circles*) of r and g modes (*a*), and the expected amplitude distribution of r modes for an inclination angle of 70 deg (*b*). Blue solid lines (for even modes) and dashed lines (for odd modes) show predicted amplitude distributions normalized to fit the hump. The adopted rotation frequency is indicated by a vertical dashed line (in panel *a*), which agrees with the observed rotation frequency 1.35 ± 0.02 day^{-1} obtained for KIC 11907454 by Van Reeth et al. (2016). This diagram shows the dense spacing of the r modes and g modes, highlighting the need for months to years of continuous light curves to decipher the FTs, with data only available from space missions. Figure and caption adapted from Saio et al. (2018, "Theory and evidence of global Rossby waves in upper main sequence stars: r-mode oscillations in many Kepler stars", their figure 5). Abbreviation: FT, Fourier transform.

Michaud & Richer (2020) for a conference review of the current status of atomic diffusion for Ap stars and Michaud et al. (2015) for a complete treatise on the subject. See **Supplemental Text Section A.5** for two case examples in asteroseismic models.

2.10. *r* Modes

Global Rossby waves, r modes, are inertial modes that are now observed in upper main sequence rotating stars, in which the toroidal motion of the r modes couples with the spheroidal motion of the Coriolis force, causing temperature variations that lead to visible light variations. The r modes are retrograde in the corotating frame, and they have mathematically an infinite set of eigenmodes, as do p modes and g modes. The r modes have frequencies that are so closely spaced that even with *Kepler* 4-year data sets they are not always fully resolved; hence, what is seen in the FT is a hump of amplitude with partially resolved peaks. **Figure 4** shows an example of an FT and period spacing for a γ Dor star in which the dense hump of closely spaced r mode frequencies is evident, as well as is a series of *g* mode frequencies. The model provides an acceptable match to the observations, confirming the *r* mode interpretation of the frequency humps.

Van Reeth et al. (2016) first recognized the signature of *r* modes in FTs of 10 γ Dor stars. Saio et al. (2018) then presented the theory of *r* modes, modeled the 10 stars studied by Van Reeth et al., and noted the signature of *r* modes in many stars with *Kepler* data, including γ Dor, δ Sct, and SPB stars; an outbursting B emission-line (Be) star (Section 2.13); and binary heartbeat stars (Section 5.1). Thus, *r* modes are ubiquitous across the upper main sequence.

Saio et al. (2018) found the main low-frequency hump in the FT to be from $m = 1$ *r* modes with frequencies just below the rotation frequency of the star. Often there is a sharper, generally unresolved peak at a slightly higher frequency than the hump, which they attributed to rotational variation caused by a faint spot or spots. The photometric amplitude for these spot variations is usually on the order of 10 ppm; hence, the spots have much less contrast than those in the Sun, where the ensemble of spots creates rotational variation up to 1 ppt. Saio et al. suggested the driving for the *r* modes comes from *g* modes, from deviated flows past spots, from mass outbursts (as in Be stars), and/or from tidal forces (as in heartbeat stars; Section 5.1). Some surface variation, such as spots, is needed for visibility, and possibly for driving, of the *r* modes. See **Supplemental Text Section A.6** for a short discussion of evidence for weak spots in upper main sequence stars.

Most γ Dor stars do not show the long series of sectoral or zonal modes needed for a $\Delta P - P$ diagram. The mode selection mechanisms in these stars are not understood, and it is also not known whether the stars that do show mode series for plotting $\Delta P - P$ diagrams are the same in their internal rotation as the majority of these stars that do not show such series. Li et al. (2019) inspected 1,593 *Kepler* stars in the temperature range 6,600–10,000 K, which encompasses the range of the δ Sct and γ Dor stars, finding 82 that have resolved period spacing patterns for both *g* modes and *r* modes, the largest sample yet. All of those stars are either in the γ Dor instability strip or only somewhat hotter than that. Li et al. determined near-core rotation rates and the characteristic periods, Π_0, for the 82 stars, noting that the *g* modes typically have radial overtones centered around $n \sim 55$, whereas the *r* modes have lower radial overtones in the $13 \leq n \leq 50$ range. The near-core rotation periods were in the range of 0.4–2.5 days. Using Π_0 is an indicator of stellar age, because it decreases over the main sequence lifetimes of γ Dor stars, they found no correlation between internal rotation and age, although that could be because of too small a range in ages. For the 5 out of 6 stars for which surface rotation signatures could be detected, those stars have uniform rotation.

Heartbeat stars: highly eccentric binary stars showing a regular pulse in brightness during periastron passage; more than 20% pulsate in tidally excited modes

Be stars: pulsating, rapidly rotating O to early-A stars that have outbursts and show emission lines from a decretion disk

Luminous blue variable (LBV): hot supergiants that generally show irregular light variations

α Cyg stars: BA supergiants showing nonradial pulsation with periods of weeks to months; some LBVs show periodic α Cyg variations when in quiescence

2.11. Summary: *r* Modes

1. *r* Modes are common in upper main sequence stars.
2. They provide a good measure of the surface rotation period.

2.12. Asteroseismology at the Top of the HR Diagram

Until very recently, asteroseismology of the most luminous blue stars was limited by a dearth of stable pulsation modes. Wolf–Rayet (WR) stars and luminous blue variables (LBVs), such as the iconic η Carinae, S Doradus, and P Cygni, have variability that is generally not periodic, especially during outburst, although some show pulsations in quiescence. There are also the α Cyg stars, which are supergiant B and A stars that pulsate with periods of days to weeks. The long periods have made it difficult to obtain sufficient data for asteroseismology of these stars.

These massive stars form near the Eddington limit, where their fierce radiation pressure puts an upper limit on the greatest mass a star can have. They are the precursors of core-collapse

Extreme He (EHe) stars: high L/M He stars evolved from merged white dwarfs

PV Tel stars: pulsating EHe stars

Stochastic low-frequency (SLF) variables: main sequence and supergiant stars that show stochastically excited low-frequency variability

SNe—hence, of neutron stars, black holes, and gamma-ray bursters—and the sources of detectable gravitational wave events. Their evolution is complex, with mass-loss rates, internal angular momentum transport, and mixing all uncertain. Models show blue loops, so that stars may be crossing the HR diagram from blue to red, or vice versa, making it challenging to distinguish their evolutionary state (Georgy et al. 2021).

Stars near the Eddington limit, including both massive stars and hydrogen-deficient extreme He (EHe) stars (Section 4.5), have high luminosity to mass ratios. For these stars highly nonadiabatic, opacity-driven pulsation modes with periods in the 10–100-day range may be confined to a layer in the outer envelope, where a density inversion provides the lower boundary to the thin pulsation cavity. In these cases, there is not a simple correspondence between the mode frequency and the radial overtone. These are the strange modes (see, e.g., Saio et al. 1998, Jeffery & Saio 2016). As seen in **Figure 1**, they occur across a number of pulsation classes, including the LBV, α Cyg, WR, PV Tel, and R CBr stars. It has been proposed that strange mode amplitudes may reach escape velocity and be implicated in LBV winds and outbursts (Glatzel et al. 1999), interestingly even for primordial, metal-free, supermassive stars at the beginning of the chemical evolution of the Universe (Yadav et al. 2018).

A related phenomenon to strange modes is surface trapped modes, in which the hydrogen partial ionization zone, rather than a density inversion, provides the lower pulsation cavity boundary. These have been found in models of RV Tau, BL Her, and W Vir Cepheids in different crossings of the instability strip (Smolec 2016). It is also possible that some strange modes are driven by convection (Shibahashi & Osaki 1981). Problems with mode identification, low numbers of modes, long periods, and mode instabilities have not been conducive to asteroseismology of strange modes yet.

However, now many of the most massive stars are amenable to asteroseismology. Bowman et al. (2019a, 2020) announced the discovery that essentially all OB stars from the main sequence to supergiants stochastic low-frequency (SLF) variables. SLFs are correlated with stellar mass and age, and with surface macroturbulence. With SLFs by themselves, and combined with cases in which there are coherent mode frequencies, this opens up asteroseismology for these rare, important, massive stars. The TESS mission is providing a wealth of observations of these stars.

2.13. Pulsating Be Stars

The Be stars range from O to early-A, and all, by definition, show emission lines in their spectra.[6] They are also all pulsating, primarily in *g* modes, and many show aperiodic outbursts that are detected both spectroscopically and photometrically. Labadie-Bartz et al. (2020) provide a review of Be stars characteristics, along with an analysis of 432 Be stars from the first year—the Southern Ecliptic Hemisphere—of TESS data. They found that nearly all have low-frequency variability typical of SPB stars, i.e., *g* modes; 17% show frequencies in the range of *p* modes; and many show evidence of *r* modes. We can now also expect SLF variations to be found in the hotter Be stars. They rotate with equatorial velocities that are greater than 80% of break-up velocity. Pulsation and rapid rotation are fundamental physical characteristics of these stars. During that first year of TESS data, 17% of the 432 Be stars had photometric outbursts—brightening events—and those outbursts, whether detected photometrically or spectroscopically, are another fundamental character of Be stars, although not exclusive to them. Van Beeck et al. (2021) found that a fraction of 38 SPB stars that they studied for nonlinear interactions show outbursts—without being Be stars.

Be stars have decretion disks—circumstellar disks that are supplied with material from equatorial mass loss during outbursts. There is no agreement about the physical mechanism for the mass

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[6] See **Supplemental Text Section A.7** for the early history of Be stars.

loss, but a leading proposal is that horizontal prograde g mode pulsation velocities from many modes can simultaneously add to the near break-up rotational velocities to give equatorial lift-off of material during outburst to populate the decretion disk, hence, provide gas in that disk to create the characteristic emission lines; this was first seen by Secchi (1866). TESS is observing almost all known Be stars. Asteroseismology promises to fill out our understanding of interior rotation and mixing in these stars and further illuminate the outburst mechanism.

Tidally tilted pulsators: obliquely pulsating stars in close binaries in which the tidal axis is the pulsation axis

2.14. Nonlinear Pulsation: Combination Frequencies—Opportunities and Pitfalls

Photometric time series—light curves—are the principal means by which we obtain pulsation mode frequencies for asteroseismic modeling. What we are observing is a change in the flux from the observed hemisphere, usually construed to be a change in luminosity, although that is not always the case, as for the tidally tilted pulsators (Section 5.2). The observed flux differences depend on changes in T_{eff} and changes in cross section. The latter depends directly on changes in R for radial modes, but may be more or less complex for nonradial modes. For example, dipole modes show no change in cross-section during pulsation; hence, light variations from them are purely the result of temperature changes.

For low-amplitude pulsators, in which the pulsation is only a perturbation to the structure of the star, to first order the luminosity variations can be linearized as a function of T_{eff}. Nevertheless, a sinusoidal variation in T_{eff} does not necessarily produce a sinusoidal light curve. Stars with multiple pulsation modes also experience coupling among the modes. Changes in the mode cavity of one mode affect the pulsation of other modes with different cavities. This mode coupling also produces nonsinusoidal light curves. Because we are thinking in terms of Fourier analysis when we extract pulsation mode frequencies, we naturally find frequencies in the FTs that are harmonics and combinations of the actual mode frequencies. Those are the Fourier description of the nonsinusoidal variations of the light curve. Because asteroseismology models pulsation mode frequencies, it is imperative to identify the harmonics and combination frequencies so as not to match them to model frequencies in forward modeling, unless, of course, they should resonantly excite a mode frequency.

However, it is possible to use the combination frequencies, i.e., the nonsinusoidal shape of the light curve, for asteroseismic inference. Montgomery (2005), building on the work of Wu (2001) and Brickhill (1992), showed how the nonsinusoidal light curves of white dwarf stars can be used to infer the thermal response timescale of the convection zone, hence, its depth. He pointed out that the technique can be applied to Cepheids and RR Lyr stars. See Aerts (2021, her section III.A.2) for further discussion and applications.

Observationally, we must recognize the combination frequencies. That seems straightforward, but there have been many stumbles in previous studies. Kurtz et al. (2015) examined the frequency groups often seen in FTs of γ Dor, SPB, and pulsating Be stars. Those frequency groups had previously been associated solely with rotation, and it is the case that g modes and r modes can produce frequency groups associated with rotation (Saio et al. 2018). But what Kurtz et al. showed for the extreme cases of stars with frequency groups is that there are only a few pulsation mode parent frequencies, and the rest of the many peaks that dominate the FTs are harmonics and combinations as a result of nonlinear interactions of the parent modes. They showed, both observationally and theoretically, that combination frequencies can have higher observed amplitudes than parent mode frequencies, calling for even greater care in identifying mode frequencies for asteroseismic modeling. They also found that combination frequencies in a Be star, both in outburst and between outbursts, support the interpretation of the star as a rapidly rotating SPB star. Van Beeck et al. (2021) make a plausible case that Be stars are complex SPB stars at the end of a range of outburst behavior for SPB stars. The two classes are not independent.

The technique for identifying combination frequencies has been to identify (or often assume) parent mode frequencies, then simply to calculate the combination frequencies and remove them from the FT by prewhitening, as well as disregard them in forward modeling. It is important to remove the combination frequencies during frequency analysis, because they can have high amplitudes (or even have the highest amplitudes) and their spectral windows can hide lower-amplitude mode frequencies. Lares-Martiz et al. (2020) recognized this and developed a self-consistent method to extract combination frequencies. Bowman et al. (2021) showed a plethora of combination frequencies and harmonics in the δ Sct star KIC 5950759, which Lares-Martiz et al. (2020) used to illustrate their method.

Am stars: metallic-lined main sequence AF stars with overabundances of Fe-peak and rare-earth elements, and deficiencies of Ca and Sc

Magnetic Ap stars: peculiar main sequence BAF stars with overabundances of rare-earth elements up to 10^6 solar, and having global, roughly dipolar magnetic fields with strengths of 1–34 kG

2.15. Chemically Peculiar Stars and Pulsation: Am, Ap, and λ Boo Stars

The B, A, and F stars on the upper main sequence are home to a zoo of spectroscopically peculiar stars; see Kurtz & Martinez (2000, their introduction) for a detailed guide to the numerous classes and arcane naming conventions. The three principal types I discuss here are the metallic-lined A (Am) stars, the magnetic peculiar A (Ap) stars, and the metal-weak λ Boo stars.

2.15.1. Am and Ap stars.

The Am and Ap stars have surface abundance anomalies that can reach overabundances of some rare-earth elements of a stunning factor of 10^6 in comparison with the Sun, whereas a few other elements or ions are deficient. The physical mechanism, atomic diffusion (Section 2.9), is most readily observable in the Am and Ap stars, hence, their wider interest in stellar astronomy. The Am and Ap stars are slow rotators, generally with $v \sin i \leq 100$ km s^{-1}, a necessary condition for atomic diffusion to produce observable anomalies in the atmosphere, as more rapid rotation generates faster meridional flows that become turbulent, keeping the outer layers mixed. The Am stars have no or very weak ($\sim$1 G) magnetic fields, whereas a subclass of the Ap stars, the magnetic Ap stars, have global magnetic fields with strengths of kiloGauss. The Ap stars constitute about 10% of main sequence A stars, whereas the Am stars are much more common, reaching a peak of about 50% of main sequence stars at A8.

It was once thought that Am stars do not pulsate as a consequence of He settling in the He II ionization zone where the κ-mechanism is the main driver of δ Sct pulsations (Kurtz 1976). Later, some Am stars were found to be δ Sct stars, and a large survey of Am stars with spectra from LAMOST (Large Sky Area Multi-Object Fibre Spectroscopic Telescope) by Smalley et al. (2017) found that while most Am stars do not pulsate, a fraction of them do in the temperature range 6, 900 $\leq T_{\mathrm{eff}} \leq$ 7,600 K at the cooler end of the δ Sct instability strip. They also found that the stars with the stronger abundance anomalies are less likely to pulsate. Smalley et al. proposed that the pulsations in these stars are driven by turbulent pressure (Antoci et al. 2014).

Murphy et al. (2020b) studied KIC 11296437, the first known roAp star (Section 2.15.2) that shows both high-overtone p modes typical of roAp stars and low-overtone p modes as in δ Sct stars. Their models showed how magnetic fields above about 1 kG damp low-overtone p modes, as in δ Sct stars, and also high-overtone g modes, as in γ Dor stars. Models with He depletion showed that a bump in the Rosseland mean opacity caused by the H-ionization edge allows the κ-mechanism to drive pulsations in Am stars, with a calculated instability strip that is in excellent agreement with that found observationally by Smalley et al. (2017). These models will be tested in the future with asteroseismology of pulsating Am stars.

2.15.2. The roAp stars.

The roAp stars are a rare subset of the magnetic Ap stars that pulsate in high-overtone magnetoacoustic modes with periods in the range of 4.7–25.8 min. They have strong, global, roughly dipolar magnetic fields with strengths in the range of 1–34 kG. The modes are largely confined to the outer layers of the star. In the visible photosphere and above, they are of striking complexity (Quitral-Manosalva et al. 2018).

I discovered this class (Kurtz 1982) and later discussed them in this journal (Kurtz 1990). These were the first stars found to have oblique pulsation axes in which the rotational deformation of the star did not dominate the physical selection of the pulsation axis. Many theoretical studies refined and improved the oblique pulsator model that I had proposed. Bigot & Dziembowski (2002) showed that the pulsation axis is not necessarily the magnetic axis but is still inclined to the rotation axis (see also Bigot & Kurtz 2011). Holdsworth (2021) has recently reviewed the roAp stars, and Holdsworth et al. (2021) give a detailed introduction to them in their report on TESS cycle 1 observations. I refer the reader to these two papers for the many references to the theoretical and observational literature on roAp stars.

The importance of oblique pulsation is that the pulsation mode is seen from varying aspect with rotation of the star, providing geometrical information that is not available for stars pulsating along the rotation axis. This has shown that roAp stars pulsate primarily in zonal distorted dipole and quadrupole modes ($\ell = 1, 2; m = 0$). The magnetoacoustic modes are distorted from pure dipole or quadrupole modes by the combined Coriolis and Lorentz forces and the distortion from spherical symmetry that the magnetic field and rotation cause. Holdsworth et al. (2018) reported four roAp stars with distorted quadrupole pulsations, along with models that include the effect of the magnetic field on the pulsations. Thus, the roAp stars provide an unusual laboratory to study the interaction of pulsation and magnetic fields on a global scale. The only other place where this physical interaction is observed is in local interactions of p modes with sunspots in the Sun.

The Ap stars are the most extreme laboratories where the results of atomic diffusion are observed (Sections 2.9 and 2.15.1). In the presence of stability against mixing and convection of the outer radiative zone with its strong magnetic field, ions with many absorption lines are radiatively levitated high into the atmosphere to continuum optical depths equivalent to the chromosphere in the Sun, as in the brightest roAp star, α Cir (Kochukhov et al. 2009). In particular, ions of rare-earth elements float isolated in cirrus-like clouds at various levels of the outer atmosphere far above optical depth $\tau = 1$, thus resolving the pulsation mode and the atmospheric structure in depth. These ions are also concentrated in spots, governed by the magnetic field configuration, giving surface resolution. Therefore, the roAp stars have the unique potential to map the atmospheric structure and pulsation geometry in 3D, although exploiting this potential is challenging.

It is possible to resolve the depth dependence of the pulsation mode by using radial velocity studies of ions that have line formation layers at different depths and studying radial velocity in line bisectors because the spectral line shape maps different depths. Many studies have done this, generally with observations from large telescopes, to meet the combined requirements of high spectral resolution, high signal-to-noise, and high time resolution. This work was pioneered by Baldry et al. (1998), Kochukhov & Ryabchikova (2001), and Mkrtichian et al. (2003).

Kochukhov (2006) studied the atmospheric depth behavior of pulsation in the photometrically well-studied, singly periodic roAp star HR 3831 with a spectroscopic study of line profile variations. He noted the superior diagnostic value of studies using radial velocity variation in spectral lines formed at different atmospheric depths compared to the blunter tool of photometric observations. In particular, he warned that the use of the amplitudes of frequency multiplets generated by rotation can only be used in the oblique pulsator model to infer a geometry of the pulsation mode at the depth of the line-forming region for a particular line and at the surface position of the spots for ions confined to those. These differ for different lines.

This problem has become clear in photometry from TESS observations of roAp stars, particularly HD 6532 (Kurtz & Holdsworth 2020), and also other stars (Holdsworth et al. 2021).

The amplitudes of the multiplet frequencies generated by oblique pulsation provide information about the mode geometry. However, these are very different for observations made through the TESS broad-band filter (0.6–1.0 μm) compared to ground-based observations of the same star typically through a Johnson *B* filter, which has an effective central wavelength 0.4353 μm that is much bluer than the TESS filter. Because the continuum opacity of the stellar atmosphere is wavelength dependent, observations through different filters sample different atmospheric depths. Interpretation of the geometry of the pulsation mode thus requires the same caution pointed out by Kochukhov (2006) from higher geometrical resolution spectroscopic observations of HR 3831.

Herbig Ae/Be stars: Early-B to early-F pre-main-sequence stars showing emission lines. Ages <10 Myr. Some are γ Dor, δ Sct, or SPB stars.

Supplemental Text Section A.8 gives two examples of high-resolution studies of roAp stars, discusses discoveries from TESS for roAp stars, and gives further details of the complexities of asteroseismology for these stars. Those high resolution spectroscopic studies stimulated the work of Quitral-Manosalva et al. (2018), who developed a theoretical model of roAp stars with coupling between the acoustic pulsations and the magnetic field. They showed in detail how the pulsation amplitude increases with atmospheric height and how the pulsation phase changes with height and viewing angle. The roAp stars are difficult. Their atmospheric abundances are stratified, the temperature gradient is not normal, the magnetic field concentrates some elements in surface spots that are difficult to model uniquely, and the pulsations are magnetoacoustic with strong changes in pulsation amplitude and phase in all dimensions. Understanding these complexities with further theory and full rotational high-resolution spectroscopic observations promises the most detailed view of pulsation and atmospheric abundance distributions for any stars other than the Sun.

2.15.3. λ Boo stars.

The λ Boo stars are generally rare, constituting only about 2% of late-B to early-F stars. Strikingly, however, they constitute more than 30% of Herbig Ae stars (Section 2.16), which host protoplanetary disks. The λ Boo stars are not slow rotators, having a mean $v \sin i \sim 160$ km s^{-1}, which is similar to that for spectroscopically normal A stars (i.e., stars that are not Am, Ap, or λ Boo). They show deficiencies down to 10^{-2} in the abundances of refractory elements (those with high melting points), such as Fe, Mg, and Si, whereas volatile elements (with low melting points), such as C, N, and O, have normal abundances. The definition of the class of λ Boo stars has been in flux, which Murphy et al. (2015) have clarified.

Hypotheses for the λ Boo phenomenon include accretion of interstellar or circumstellar gas. Kama et al. (2015) presented a model in which giant planets sweep up dust so that the star accretes gas that is depleted in refractory elements. They showed a correlation between λ Boo abundances in Herbig Ae stars and the presence of gaps in their protoplanetary disks and suggested that diversity in planet formation leads to the range of abundances observed in λ Boo stars.

About 80% of λ Boo stars are also δ Sct pulsators; hence, they can provide asteroseismic information, in particular age, mass, and metallicity. An example is HD 139614, a λ Boo/δ Sct star that has a protoplanetary disk with gaps. It is located in the Upper Centaurus–Lupus part of the Scorpius–Centaurus association, the nearest region to the Sun, at 140 pc, where there is massive star formation. There is a range of ages for the stars across the association, with a median age of 16 Myr and a 1σ spread of 7 Myr. Murphy et al. (2021) found an asteroseismic age for HD 139614 of 10.8 ± 0.8 Myr. This is the most precise age determination for a star in the association and is in good agreement with other age determinations, showing the power of asteroseismology for these pre-main-sequence stars. Murphy et al. also found a normal metallicity for their best model of HD 139614, which is the first determination that λ Boo stars have globally normal metallicity, as is expected from the accretion model for their surface abnormalities.

Bedding et al. (2020) found from asteroseismology and space motions that some of the 60 δ Sct stars for which they determined the large separation, $\Delta\nu$, and which they modeled, are young

(Section 2.1). They found an occurrence rate of 10% for λ Boo stars in their sample of 60 stars, supporting the hypothesis that many λ Boo stars are young and accrete from dust-depleted circumstellar disk material. Nevertheless, in a survey of Southern Hemisphere λ Boo stars, Murphy et al. (2020a) found only about 40% of them are young, with the rest further evolved along the main sequence. If the disk accretion model is correct, then the surface convection zone must not mix away the accreted chemical abnormalities over the main sequence lifetimes. More modeling is needed to determine if sufficient dust-depleted gas can accrete to provide the abundance anomalies through the entire surface convection zone, which has a thickness of only thousands of kilometers. Clearly, the origin of the λ Boo phenomenon still has its challenges. Asteroseismic ages and global metallicities for these stars will contribute to deeper understanding of them.

2.15.4. Summary: pulsation in chemically peculiar stars.

1. Pulsation in metallic-lined A (Am) stars is common with relatively low amplitudes. Stars with stronger abundance anomalies are less likely to pulsate due to depletion of He II driving.
2. Pulsation in rapidly oscillating Ap (roAp) stars is rare, even at the μmag detection level.
3. The oblique pulsator model shows the mode geometries in roAp stars to be complex as a function of atmospheric height. Spectroscopic line-by-line studies provide 3D information on atmospheric structure.
4. Young λ Boo stars appear to be accreting planetary disk material. They are usually δ Sct stars for which asteroseismic ages are in agreement with gyrochronology.

2.16. Pre-Main-Sequence Pulsators

Historically, pre-main-sequence evolution was thought of in terms of a star of a given mass having an evolution track across the HR diagram, first down a convective Hayashi track, then across toward the main sequence as the envelope becomes more radiative, until H fusion is ignited and the zero-age main sequence is reached. It has been clear for decades that pre-main-sequence evolution is much more complex. Stars form as seeds, or cores, in accretion envelopes. During their contraction phases they accrete mass, so that no single mass can be attributed to a star along its pre-main-sequence pathway in the HR diagram. See, e.g., Palla & Stahler (1991), Stahler & Palla (2004), and Kunitomo et al. (2017).

As these pre-main-sequence stars emerge from their cocoons at the stellar birthline, and appear observationally on the HR diagram, they still have circumstellar material, particularly an accretion disk. High-mass stars that are born as early-B stars do not emerge from their cocoons until about the time they ignite H fusion. Low-mass, $M \leq 1\,\mathrm{M}_{\odot}$, pre-main-sequence TT Tau stars may have stochastically excited modes, but those frequencies are generally lost in the FT because of the variations from the circumstellar material. Recently, attempts to extract the solar-like oscillation signal from the accretion noise have been made (Müllner et al. 2021), with some possible candidates found.

It is the stars between these high and low masses for which there is great promise for asteroseismology of pre-main-sequence stars. These are Herbig Ae and Be stars (Herbig 1960) with masses of $1.5 \leq M \leq 4\,\mathrm{M}_{\odot}$. As they contract, some of them appear as δ Sct, γ Dor, and SPB pulsators prior to their arrival on the zero-age main sequence. The opportunity to observe the interior of these stars asteroseismically to test and refine pre-main-sequence models is attractive and promising.

A problem in fulfilling that promise is that variable dust in the line of sight causes irregular light variations from which it can be difficult to extract pulsation signals. This is particularly true for γ Dor stars, which have low frequencies in the range where the noise from the disk variations is strong. The p mode frequencies, however, can be separated in the FT from the disk noise, as first shown by Kurtz & Marang (1995) for HR 5999 and in more detail for HD 142666 by Zwintz et al. (2009) with photometry from the MOST mission. While Zwintz (2008) listed only 36 pre-main-sequence pulsators and candidates, that number has now grown significantly. The goal with these stars is to distinguish asteroseismically their internal structure from the zero-age main sequence structure stars such as those they will soon become.

Some pre-main-sequence δ Sct stars have planet-forming disks that do not cause irregular light variations; hence, they have cleaner FTs at low frequency. An example is HD 139614, a λ Boo star (Murphy et al. 2021; Section 2.15.3). Steindl et al. (2021) have found in TESS data 16 new pre-main-sequence γ Dor, δ Sct, and SPB stars (or candidates) and initiated a modeling program for all known pre-main-sequence stars. They provide a good introduction to the current state of asteroseismology of these stars. See also Zwintz (2020) for a short review of this field.

3. THE UPPER CLASSICAL INSTABILITY STRIP: RR LYRAE STARS AND CEPHEIDS

The classical pulsators of the upper instability strip are fundamental to the galactic and extragalactic distance scales. These are, broadly, the RR Lyr stars and the Cepheid variables. Both classes have an array of subtypes, some of which are noted in **Figure 1**. The Cepheids are divided into type I (population I) and type II (Population II) Cepheids; anomalous Cepheids or BL Boo stars; BL Her and W Vir stars (also Population II); and RV Tau stars (also Population II). These types are based on light curves, on the period–luminosity relation, and on the mass, age, and evolutionary stage of the stars. The RR Lyr stars also have subtypes: RRab, RRc, RRd, and stars showing the Tseraskaya–Blazhko effect (see the sidebar titled The Tseraskaya–Blazhko Effect). See Aerts et al. (2010) for a guide to the nomenclature and evolutionary stages of the subtypes.

The period–luminosity relation, or Leavitt Law, for Cepheids was first noted by Henrietta Leavitt in 1908 and then announced with confidence in 1912 (Leavitt & Pickering 1912, pp. 1–2) when she commented, "A remarkable relation between the brightness of these variables and the length of their periods will be noticed. . .the brighter variables have the longer periods." The physical origin of this relation is now known from asteroseismology: The brighter stars have longer sound travel times across the star, hence longer pulsation periods, whereas the fainter stars have shorter sound travel times, hence shower periods, both as a consequence of radius and density. Sound travels more slowly and has farther to go in the brighter Cepheids. It is this Leavitt Law

THE TSERASKAYA–BLAZHKO EFFECT

The longest standing puzzle for RR Lyr stars concerns amplitude and period changes that have been called for over 60 years the Blazhko effect, based on an observation of RW Dra (Blažko 1907; RW Dra = Var. 87.1906 Draconis). Sergey Blazhko worked at the Moscow Observatory, where he later served as director. I note that in the opening sentence of this highly cited paper, Blazhko says that the observational changes in period were discovered by "Mrs. Ceraski." Lidiya Petrovna Tseraskaya (1855–1931) worked at Moscow Observatory (now the Sternberg State Astronomical Institute), where she discovered 219 variable stars. It was she who published under the name Mrs. W. Ceraski, taken from a transliteration from Cyrillic of her husband's form of their surname. Today, Mrs. W. Ceraski would be included as coauthor Lidiya Tseraskaya, as it was she who made the discovery. ADS (the Astrophysics Data System) lists 155 observing notes on variable stars published by her during 1879–1914. In 1908, she was awarded the prize of the Russian Astronomical Society, and crater Tseraskaya on Venus is named after her. I propose that what has previously been known as the Blazhko effect, based on this one paper, would more appropriately be known as the Tseraskaya–Blazhko effect.

that makes the Cepheids so useful as distance indicators, and it is primarily used on fundamental radial mode, or first-overtone mode, pulsation.

But Cepheids and RR Lyr stars are much more interesting asteroseismically than simple low-overtone radial mode pulsators. They are high-amplitude pulsators with strongly nonlinear light curves that challenge and test nonlinear pulsation theory. A significant fraction of them pulsate in two modes, or more, some of which are nonradial, providing the data for asteroseismic modeling. They show stable frequencies in some stars, frequency jitter in others, period doubling (alternating maxima in the light curves), the presence of subharmonics, and alternating minima in late evolutionary stage RV Tau stars (see, e.g., Plachy et al. 2021). Most, or all of these dynamical effects are related to mode resonances, providing additional asteroseismic constraints on the stellar models.

A large fraction of RR Lyr stars show the Tseraskaya–Blazhko effect—clear modulation of the light curves on timescales of tens to hundreds of days. The modulation is not simply periodic. Prudil & Skarka (2017), in a massive survey using OGLE-IV data, examined almost 8,300 RR Lyr stars in the Galactic Bulge, finding that over 40% of them show Tseraskaya–Blazhko modulation of their light curves. Although the K2 light curves for RR Lyr stars are not fully analyzed, they give a similar incidence. The Tseraskaya–Blazhko effect is thought to be the result of a resonance, and there is some connection to period doubling in some stars, supporting that idea. It is remarkable that after more than a century and with all of our powerful asteroseismic modeling tools, this phenomenon is not fully understood in stars of such fundamental importance.

RR Lyr stars have periods of about 0.5–1.2 days and Cepheids 1–100 days, making it possible to study these stars with ground-based survey projects, e.g., OGLE-IV (Udalski et al. 2015), among others. They will soon benefit further with upcoming data from the Zwicky Transient Facility (ZTF; see **Supplemental Text Section B.2**) and the Vera C. Rubin Observatory surveys. Recent literature on these classical variables is vast and growing, with newly observed behavior and new theoretical interest. Although the ground-based surveys dominate the data, with light curves for thousands of stars, space mission data from MOST, CoRoT, *Kepler*, TESS, and BRITE have illuminated our view of pulsation in these upper instability strip stars, as a consequence of long, uninterrupted high-precision light curves.

An outstanding tool for organizing the great variety of pulsation frequency behavior in Cepheids and RR Lyr stars is the Petersen diagram (Petersen 1973), which plots the period ratio of two frequencies against the frequency of the longer period mode. This diagram distinguishes, e.g., fundamental to first overtone radial mode pulsators from higher-overtone pulsators; nonradial pulsators; stars with period doubling; and a variety of as yet not understood pulsation behavior. Smolec et al. (2017) discussed the period-doubling phenomenon, first seen in *Kepler* data of RR Lyrae itself (Kolenberg et al. 2010). This phenomenon leads to alternating maxima in the light curves as a result of a frequency resonance, asteroseismically constraining models of these stars. Smolec et al. (2017) give an excellent short review entitled the "Petersen Diagram Revolution." Plachy & Szabó (2021) reviewed *Kepler* observations of RR Lyr stars. Their figure 8 further illuminates the wide variety of pulsation behavior seen in the Petersen diagram for these stars.

4. AFTER THE RED GIANTS: ASTEROSEISMOLOGY OF WHITE DWARFS, SUBDWARF O AND B STARS, AND EXTREME HELIUM STARS

4.1. Pulsating White Dwarfs

Many white dwarfs carry information about the cores of their progenitor red giant stages, including chemical changes from nuclear reactions and angular momentum transfer during early evolutionary stages. Some of them have a modicum of nuclear energy generation via hydrogen-shell burning, but on the whole they radiate leftover energy. That comes in several fascinating forms. Thermal energy remaining from the hot red giant core in the nondegenerate nuclei is a major source of white dwarf luminosity, but that is not always dominant. White dwarf cores are typically C–O, but also O–Ne among the ultramassive white dwarfs with $M \geq 1.05\ M_\odot$. When white dwarfs cool sufficiently, the cores crystallize, leading to a release of latent heat. As a result of Coulomb interactions, the nuclei are forced into a crystalline structure, unlike Earthly crystals, which are bound by electron sharing; unbound electrons cannot be shared in a degenerate gas. Some white dwarfs are now known to have cores that are more than 90% crystallized (Córsico et al. 2019b), an amazing thought: single crystals that are roughly the size of the Earth. Asteroseismic results support this (e.g., De Gerónimo et al. 2019).

In some white dwarfs the neutrino luminosity is the dominant radiation loss. If the neutrino has a magnetic moment—an important question for the standard model of particle physics—then an abundant source of neutrinos in white dwarf stars comes from plasmon decay of photons to neutrino–antineutrino pairs as a consequence of neutrino–photon interaction. White dwarf cooling times provide a direct measure of this process, and asteroseismology measures those cooling times. If axions exist, providing an explanation for the charge–parity problem in strong interactions, it is possible that white dwarf cooling times may also constrain axion mass, a quantity not predicted by theory. Asteroseismology measures white dwarf cooling times by determining evolutionary period changes to the pulsations (although, see my caution about evolutionary weather in Section 2.2). See Fontaine & Brassard (2008), Winget & Kepler (2008), and Córsico et al. (2019a) for extensive reviews of white dwarfs, including pulsations.

Aerts et al. (2010, their chapter 2) discussed in detail the arcane naming conventions for white dwarf stars. To briefly describe the conventions here, in order of decreasing temperature, the DO, DB, and DA white dwarfs have spectral features similar to those of O, B, and A main sequence stars, but not necessarily the same effective temperatures. Pulsating variables of those three spectral classes are the DOV (GW Vir), DBV (V777 Her), and DAV (ZZ Cet) stars that are the primary asteroseismic classes. There are also three additional classes of variable white dwarfs that are of asteroseismic interest: the DQV, hot DAV, and ultra-massive DAV stars.

Stars in close binary systems interact. In some cases, a companion strips away the outer envelope of a star, leaving behind a type of white dwarf that could never form as a consequence of single star evolution. Extremely low-mass (ELM) white dwarfs with He cores are exciting examples of this, and there are classes of ELM white dwarfs that pulsate and can be studied asteroseismically. Now, mostly in the past decade, we add to the DOV, DBV, and DAV classes the extremely low-mass variables (ELMVs) and pre-ELMVs, and some other rare (sometimes contested) classes. In other binary cases, material from a companion is accreted by a white dwarf, giving a born-again blue straggler. And, of course, almost the entire class of cataclysmic variables is made up of close interacting binary stars with one star being a white dwarf. Many of these are the progenitors of novae and dwarf novae. Some of those also pulsate and may be amenable to asteroseismic inference, e.g., GW Lib (van Zyl et al. 2004, Chote et al. 2021). Finally, in some binary pairs with two white dwarfs, orbital energy losses lead to mergers and the production of exotic stages in the lives of a few stars, such as the EHe stars (Section 4.5), a few of which pulsate spectacularly.

The *Kepler* main mission did not observe many white dwarf stars, as it was targeting a relatively distant part of the Galaxy just out of the plane, and white dwarfs in that direction are too faint. However, the extended *Kepler* mission, K2, did target white dwarf stars; see Córsico (2020) for a specific review of the *Kepler* results for asteroseismology of white dwarfs. Now TESS is also observing white dwarfs. Although both the *Kepler* and TESS missions are known for the high precision of their photometry, that is not greatly better than ground-based photometry for white dwarfs because the faintness of these stars and the small (by ground-based standards) apertures of the space telescopes mean that the noise is photon limited. Additionally, both the *Kepler* and TESS broadband filters do not incorporate blue to UV observations where most white dwarf stars have maximum pulsation amplitude. The advantages of the *Kepler* and TESS data sets come from their long time spans and continuity. *Gaia* has now detected hundreds of thousands of new white dwarfs and white dwarf candidates, some of which will be studied by TESS, and others, later in this decade, by the European Southern Observatory space mission PLATO (**https://sci.esa.int/web/plato**).

White dwarfs are fascinating in themselves, and they have importance beyond stellar astrophysics. White dwarfs with a mass near the Chandrasekhar limit[7] that orbit a donor companion in a close binary star are the progenitors of the type Ia SNe that are the standard candles of cosmology, fundamental to models with acceleration of the expansion rate and a need for Dark Energy. Single white dwarfs are excellent clocks to measure the age of galactic disk components and star clusters, given a knowledge of their cooling times, which asteroseismology can constrain. Hence, pulsating white dwarfs are of broad interest. See **Supplemental Text Sections B.2–B.6** for detailed discussions of asteroseismology of white dwarf classes DAV (ZZ Cet), DOV (GW Vir), DBV (V777 Her), ELMVs, and BLAPs (blue large amplitude pulsators), respectively. See **Supplemental Text Section B.7** for discussion of the DQV, hot DAV, and ultramassive DAV stars.

4.2. Summary: Asteroseismology of White Dwarfs

1. Asteroseismology of white dwarfs provides measurements of mass, radius, temperature, luminosity, age, rotation, and the extent of the nondegenerate surface layers, and it tests the physics of matter under extreme conditions.
2. White dwarfs are progenitors of type Ia SNe, the standard candles of observational cosmology.
3. Neutrinos are the dominant form of radiation from some white dwarfs, providing observational tests of neutrino physics.
4. Extremely low-mass white dwarfs are the stripped cores of red giants, allowing direct observations of red giant core physics.
5. White dwarfs in binaries are important in the study of cataclysmic variables, SN Ia precursors, blue stragglers, and exotic results of mergers, such as extreme He stars.

4.3. Extreme Horizontal Branch Stars: The Subdwarf O and B Variables

There is a population of He-core burning stars with thin H envelopes on the extreme or blue horizontal branch with typical masses around 0.5 $M_\odot$—the subdwarf B (sdB) stars. About half of them are in compact binaries; they are the stripped cores of red giants after common envelope ejection, or Roche lobe overflow. The single sdB stars probably originate from He white dwarf mergers, but possibly from other channels, including white dwarf—low-mass main sequence star mergers, and rarely mergers involving neutron stars (Wu et al. 2020). Some of them are also progenitors of the ELM white dwarfs (see **Supplemental Text Section B.5**). Most evolve to subdwarf O (sdO) stars when helium is exhausted in their cores and they begin He-shell burning, a stage with a lifetime much shorter than the $\sim 10^8$ years of He-core burning; hence, sdO stars are rarer than sdB stars.

The evolutionary relation of sdO and sdB stars is not completely clear. Most of these stars continue to evolve directly to the white dwarf stage without a return to the asymptotic giant branch. The exceptions are those with white dwarf companions that become type Ia SNe. The

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[7] See **Supplemental Text Section B.1** for a history of the Chandrasekhar–Eddington debate.

thin H atmospheres (10^{-2} $M_\odot$) of the sdB stars are He deficient as a consequence of gravitational settling (Section 2.9). The sdO stars, however, sometimes show He deficiency, sometimes not. These subdwarf stars are fascinating in their own right and important for our understanding of red giant and white dwarf evolution. See Heber (2016) for a comprehensive introduction to and review of these stars. See **Supplemental Text Section C** for a history of the discovery of the sdB stars and a detailed discussion of the asteroseismology of sdB and sdO stars.

4.4. Summary: sdBV Stars

1. Asteroseismology of sdBV stars has provided the fundamental parameters mass and radius, as well as structural parameters—the envelope mass, the core mass, and the core composition—illuminating their evolutionary origins.
2. Asteroseismic measures of the core O mass fraction in sdBV stars may ultimately constrain the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate in these stars.

4.5. Extreme Helium Stars, PV Tel Stars, and R CrB Stars

Not all pulsating stars are amenable to asteroseismology. Irregular variables provide only some astrophysical inference. Singly periodic variables—even when mode identification is certain—provide only a single constraint for forward modeling. Nevertheless, some of these stars are in such exotic evolutionary stages that they deserve mention in this review. These are the extreme He (EHe) stars, which have unusually high luminosity-to-mass ratios, little or no H, and enhanced C in their He atmospheres.

The evolutionary pathway to EHe stars is that of a binary merger of a typical 0.6-$M_\odot$ C-O white dwarf with a 0.3-$M_\odot$ low-mass He white dwarf. Ignition of a He-shell flash results in a supergiant with strong mass loss and a He-burning shell that eats its way inward in the core of the star (Saio & Jeffery 2002), remarkably in the opposite direction to normal shell burning. This pathway produces R CBr stars (Schwab 2019), with their episodes of dramatic drops and recoveries in optical light when C soot forms in their atmospheres and then is ejected by radiation pressure. Evolution of EHe stars is then blueward, eventually to a prewhite dwarf, then a white dwarf. These rare stars are inherently interesting for their novelty.

Among the EHe stars are variables known as PV Tel stars with generally irregular light curves with timescales of 0.5–25 days. The class is disparate; hence, despite small numbers, Jeffery (2008) divided it into three subtypes, the third (his type III) of which are the BX Cir stars comprising only two stars, V652 Her and BX Cir, both of which have regular pulsations with periods of 2.4 h.

V652 Her is the better studied star; hence, it is labeled in **Figure 1**, which shows that it lies somewhat below the main sequence. Its evolutionary future is to evolve through a series of loops to become a prewhite dwarf. At present, it is the remarkable "born-again rocket star" (Kurtz et al. 2016, p. 4.41; see also Jeffery et al. 2015), which accelerates pulsationally from 0–100 km s^{-1} in 15 min with a Mach-10 pulse (239 km s^{-1}) running through its atmosphere! This, thus, allows a look at extreme pulsation amplitude and supersonic velocity in a stellar atmosphere. At present, asteroseismology does not particularly exploit pulsation amplitude, as the nonlinear theory needed to understand fully what determines pulsation amplitude is difficult. (An exception to that is equipartition of driving energy in stochastic oscillators, which allows inference of viewing angle from relative amplitudes of the components of dipole multiplets.) In looking to a future when driving and damping may be modeled to explain successfully what determines mode amplitudes, the BX Cir stars will provide the extreme limits of pulsation amplitude to test the theory. Although there are only two such stars currently known, more should be found in TESS data.

Jeffery et al. (2020) studied two EHe stars observed with TESS, both of them irregular variables. They concluded that the variability timescale is of the same order as the radial fundamental mode. The reason for the irregularity is that the pulsations are nonlinear, nonadiabatic, and possibly not spherically symmetric—an interesting challenge!

5. TIDES: ASTEROSEISMOLOGY OF CLOSE BINARY STARS

Asteroseismology is simplest for stars that are approximately spherically symmetric. For slowly rotating single stars with a weak or no magnetic field, one-dimensional models are adequate. In recent years, two-dimensional models of stellar pulsation have been developed for the case of rapidly rotating stars (Reese et al. 2021), with potential application to the more rapidly rotating upper main sequence δ Sct, SPB, and β Cep stars. Pulsating stars in wide binaries are asteroseismically modeled as single stars. For pulsating stars in eclipsing binaries, the determination of fundamental parameters—particularly mass and radius—shows that model-dependent asteroseismic determinations of those quantities have excellent accuracies, attaining a few percent in the best cases. This gives confidence in other asteroseismically determined quantities, particularly age and global metallicity.

Upper main sequence stars mostly exist in binary or multiple star systems, with the incidence increasing with increasing mass to more than 90% for the hottest O stars (Moe & Di Stefano 2017). If we include the high incidence of planetary systems deduced from results of *Kepler* and TESS, then single stars with no stellar or planetary companion are rare. The stars of interest in this section are those that orbit closely enough that tidal forces cannot be neglected, for which the asteroseismic problem becomes more challenging and offers new inferences on the physics of close binary stars.

Using pulsating stars as stable frequency standards has led to new ways to study binary star orbits: the frequency modulation (FM; Shibahashi & Kurtz 2012, Shibahashi et al. 2015) and phase modulation (PM; Murphy et al. 2014, Murphy et al. 2016b) techniques. In addition, the orbital motion of the *Kepler* satellite allows the Nyquist limitation in frequency analysis of uniformly spaced data to be overcome in the super-Nyquist asteroseismology technique (sNa; Murphy et al. 2013). See **Supplemental Text Section D** for a discussion of these techniques and their impact.

Guo (2021) has given an extensive review of asteroseismology in close binary stars with particular emphasis on tides and the effects of mass transfer on pulsations and stellar structure. He examined both tidally excited and tidally perturbed, self-excited oscillations. Lampens (2021) has also reviewed eclipsing binaries, both wide and semidetached, with pulsating components, giving individual examples of δ Sct, γ Dor, and β Cep stars observed by *Kepler* and TESS. The reader is referred to these reviews for thorough discussions of this exciting new field of asteroseismic research.

Close binaries lead to mass loss, mass interchange, common envelope evolution, spin–orbit synchronization, orbital and rotational energy exchange with pulsation, and, of course, tides. They are the progenitors of type Ia SNe, the distance indicators of choice in the deep Universe; of binary neutron stars and black holes, the sources of measurable gravitation radiation, gamma-ray bursts, and many of the heavier elements; of cataclysmic variables; of blue stragglers; of X-ray binaries; of the newly discovered heartbeat stars and the tidally tilted pulsators; and of Doppler beaming and boosting, which have only become widely observable with the high precision of space photometry.

Doppler beaming and boosting has multiple components. The simple one is boosting where a component of a binary star appears brighter when approaching and fainter when receding simply because of the Doppler shift altering the energy in the photons. Add to that the special relativistic effects of beaming, time dilation, and photon arrival time and we expect all orbiting stars to be brighter when approaching and fainter when receding than they would otherwise have been without these effects. Prior to the precision of space photometry, Doppler beaming was difficult, or impossible, to observe as its amplitudes are typically only hundreds of μmag. A beautiful demonstration of it was given in the study of *Kepler* data of the eclipsing binary KPD 1946+4340, which consists of an sdB star and a white dwarf in a 9.7-h orbit (Bloemen et al. 2011). Clearly visible in the light curve are ellipsoidal variation from tidal distortion, the Doppler beaming modification of the ellipsoidal variability, the eclipses, and even the general relativistic effect of gravitational lensing by the white dwarf during primary eclipse. For space photometry of eclipsing binaries, Doppler beaming is now a standard part of light curve modeling. This is a case (see **Supplemental Text Section D.1** for another) for which it is now possible to measure radial velocities without the need for spectroscopy using the Doppler beaming signal. This saves expensive telescope time and even enables studies in which rotational broadening of spectral lines and/or faintness of the target preclude obtaining radial velocities spectroscopically.

Systematic searches for eclipsing binaries with pulsating components are ongoing. Gaulme & Guzik (2019) reported 303 pulsating eclipsing binaries from a systematic search of nearly 3,000 eclipsing binaries observed by *Kepler* (Kirk et al. 2016). IJspeert et al. (2021) reported over 3,000 eclipsing binaries and candidates from an automated search of about 190,000 upper main sequence stars in nearly all-sky data from TESS. EL CVn binaries have a bloated pre-He white dwarf in a short-period orbit about an AF star that in some cases is a δ Sct star. More than 100 of these systems are now known; in two of these, the white dwarf pulsates. See **Supplemental Text Section B.8** for further discussion of these fascinating binary systems. An explosion of discovery from pulsating binary stars is imminent.

5.1. Heartbeat Stars and Tidal Locking

Kumar et al. (1995) presented a theoretical study of tidal excitation of pulsation modes in eccentric binary systems, applying their results to two binary pulsars, although the relative faintnesses of those systems made it difficult to test the theory conclusively. They showed that the tidal force should excite pulsations with mode frequencies that are exact harmonics of the orbital frequency. There were observational indications of this behavior found in ground-based studies (e.g., Handler et al. 2002), but the breakthrough came with the discovery of the first heartbeat star, HD 187091, better known as Kepler object of interest (KOI)-54 (Welsh et al. 2011). This star shows sharp increases in brightness with a period of 41.8 days with commensurate pulsations superposed. The two highest-amplitude pulsations are *g* modes with frequencies that are exactly harmonics 90 and 91 of the orbital frequency. It was the continuity and high precision of the space data that made the tidally induced behavior obvious in the light curve.

Following the discovery of KOI-54, Thompson et al. (2012) announced the class of heartbeat stars in a study of 17 of these eccentric binaries in *Kepler* data. There are now 172 known in the *Kepler* Eclipsing Binary Catalog (KEBC; Kirk et al. 2016), and more than 200 have been discovered in TESS data. The regular brightening in the light curves of these stars is reminiscent of an electrocardiogram, hence, the heartbeat name; more prosaically, they are just highly eccentric binaries. For noneclipsing systems, the orbital variations are dominated primarily by the tidal distortion, with contributions from heating by reflection and Doppler beaming. The orbital periods are usually under 100 days, and eccentricities range from about 0.2 to 0.9! In the highest-eccentricity systems, the periastron passage is within a few stellar radii, with stunning tidal distortions that then relax as the stars separate in the outer parts of the orbit. See Guo (2021) and Cheng et al. (2020) for example light curves, FTs, and detailed discussion for some of these stars. Two results of the discovery of the heartbeat stars are that the previously untested theory of Kumar et al. (1995) provides excellent fits to the wide variety of the orbital light variations that result primarily from different viewing angles and eccentricity, and that strong astrophysical inference about tidally excited oscillations can be made in the more than 20% of these systems that show pulsations.

The tidal resonance between pulsation and orbital harmonics can be very finely tuned and of astrophysical importance. Hambleton et al. (2018) showed that the heartbeat star KIC 8164262 has an orbital period of $P_{\mathrm{orb}} = 87.5$ days, an eccentricity of $e = 0.87$, and a prominent pulsation at exactly 229 times the orbital frequency. Although there are other oscillation modes, this one very-high harmonic resonance dominates the pulsation in the light curve. In a companion paper, Fuller et al. (2017) discussed the theory for this particular heartbeat star, whereas Fuller (2017) gave a general theoretical examination of the physics of excitation and locking caused by dynamical tides. The heartbeat stars support the thesis that there is a positive feedback between the orbital evolution and stellar evolution that keeps the pulsation frequency resonant with a high harmonic of the orbital frequency. This is tidal locking. As the star and orbit evolve, the pulsation cavity and rotation of the star change, modifying the natural mode frequencies while tidal dissipation causes orbital decay and an increase in the orbital frequency. Feedback keeps the pulsation frequency locked to an orbital harmonic, giving a stable, high-amplitude, tidally excited oscillation, as in KIC 8164262.

Although tidal locking does not require high eccentricity—indeed, for circular orbits, ellipsoidal variability arises from a corotating quadrupolar tidal distortion—it is most easily seen in some pulsating heartbeat stars. In a study of tidal excitation in heartbeat stars, Cheng et al. (2020) found only 2 of 4 stars studied to exhibit tidal locking; i.e., not all do. The study of tidal locking in heartbeat stars informs other occurrences, such as migration in planetary and moon systems and inspiralling in white dwarfs. Burkart et al. (2013) examined double degenerate white dwarf binaries where gravitational radiation results in orbital decay. As in some heartbeat stars, they found resonance locking among the orbital frequency, spin frequency, and tidally excited *g* modes. This important result is much more difficult to test observationally in faint white dwarfs, but these are interesting connections.

Pulsating heartbeat stars are most commonly found among AF stars because of selection effects, although Beck et al. (2014) found and studied 18 heartbeat red giants with eccentricities 0.2–0.76 and with orbital periods up to 440 days. Pulsation in heartbeat stars is difficult to detect for stars less massive than about 1.5 $M_{\odot}$ because the *g* modes are not visible through the thick surface convection zone—a well-known problem in the Sun. Even though *g* modes are common in γ Dor, δ Sct, and SPB stars, the initial mass function and shorter lifetimes mean that the more massive stars are more uncommon. Kołaczek-Szymański et al. (2021) made a search of TESS sectors 1–16 for heartbeat stars among $M \geq 2$ $M_{\odot}$ stars, finding 20 systems, 7 of which show tidally excited oscillations. The most massive found is HD 5980 in the Large Magellanic Cloud, a young quadruple system with a total mass of 150 $M_{\odot}$. They also found self-excited pulsations in 9 heartbeat stars with β Cep, SPB, δ Sct, and γ Dor components. These will illuminate how tides interact with *p* modes, as previously observed in KIC 4544587 (Hambleton et al. 2013), but not yet probed for asteroseismic inference.

Guo et al. (2019) studied the eccentric binary KIC 4142768, which shows both self-excited *g* modes and *p* modes, as well as tidally excited modes. This combination can provide internal and surface rotation rates along with information on tidal dissipation, hence, the timescale of circularization and synchronization in eccentric binary stars. Surprisingly, it seems that spin–orbit tidal interaction might possibly go either way in some stars. Li et al. (2020a) studied 45 noneclipsing binaries found by the PM method (**Supplemental Text Section D.1**), for which 35 systems showed the *g* mode patterns needed to measure internal rotation. They found that, in general, tidal synchronism occurs for orbital periods of less than 10 days, with the exception of 3 stars for which an internal rotation rate was subsynchronous. They conjectured that tidally excited modes may be able to transfer internal angular momentum to the orbit, in a novel process they dubbed inverse tides.

Single-sided pulsators: tidally tilted pulsators in which the tidal distortion leads to pulsation predominantly on either the L_1 or L_3 side

5.2. Tidally Tilted Pulsators

When I discovered the roAp stars and proposed the oblique pulsator model (Kurtz 1982; Section 2.15.2), the clue to oblique pulsation was in frequency multiplets split by exactly the rotation frequency. In slowly rotating stars, rotationally perturbed multiplets, such as dipole triplets and quadrupole quintuplets, have frequency splittings that differ from the rotation frequency by a factor of $(1 - C_{n,\ell})$, where $C_{n,\ell}$ is the Ledoux constant, which is not zero. The splitting of the multiplet components by exactly the rotation frequency showed that the mode axis was inclined to the rotation axis as a consequence of the strong, roughly dipolar magnetic field. This suggested that other perturbations from spherical symmetry should also compete with rotation for the pulsation axis selection and that tidal deformation is an excellent candidate to do that.

Forty years later, obliquely pulsating close binary stars have now been discovered in which the pulsation axis coincides with the tidal axis. The first, HD 74423 (Handler et al. 2020), is composed of two similar-mass λ Boo stars (Section 2.15.3) in a 1.6-day orbit where one star is a δ Sct star pulsating in a single mode that shows 11 frequencies in the FT split by exactly the orbital frequency—the signature of oblique pulsation. A detailed analysis found a zonal ($m = 0$) distorted dipole pulsation mode with much higher amplitude on the L_1 side facing the companion; this led to the star being called a single-sided pulsator.

The second is CO Cam (Kurtz et al. 2020), a marginal Am δ Sct star in a 1.27-day orbit with an undetected G main sequence companion. It pulsates in four p modes with the pulsation axis aligned with the tidal axis, and three g modes that do not show this alignment. Pulsation modeling suggests that one of the p modes is the fundamental mode and that the others have a mixed p mode–g mode character. Mode identification independent of modeling is difficult because of the tidal distortion of the observable atmospheric layer. The best models require some core overshoot of 1% to 2% of the pressure scale height; models with no overshoot do not match the observed frequencies; hence, there is an asteroseismic constraint on core mixing in this close tidal binary.

The third is TIC 63328020 (Rappaport et al. 2021), a δ Sct star with a 1.1-$M_\odot$ companion in a 1.11-day orbit. There are two pulsation modes, wherein the primary one is a sectoral dipole ($\ell = 1, m = |1|$) mode. This star does not show the single-sided pulsation nature of the previous two; hence, we refer to the class as tidally tilted pulsators with the term singled-sided pulsator reserved for those with much higher amplitudes on one side of the star—either the L_1 or L_3 side.

Fuller et al. (2020) discussed the theory of tidal trapping, based on the above three tidally tilted pulsators. They examined how the tidal distortion affects the mode frequencies; how it causes alignment of the pulsation axis with the tidal axis; how it traps modes, largely confining them to the tidal poles or equator; and how it amplifies the mode amplitude at the tidal poles where the modes propagate closer to the stellar surface. Study of the orbital variations of HD 74423, CO Cam, and TIC 63328020 showed that they partially fill their Roche lobes to varying degree, such that with only three examples it is not clear why their single-sided pulsations differ.

The tidally tilted pulsators show asteroseismic promise. The oblique pulsation character helps with mode identification, a requisite for asteroseismology, even with the tidal distortion. The coupling of the surface p modes to interior g modes in a tidally distorted star requires further development of the models. The first three were discovered because their FTs are particularly simple, with only one or a few modes excited. Most δ Sct stars show many excited modes (Section 2.1), and in the cases of tidally tilted pulsators with many modes, there is complexity to be disentangled in the FT. For relatively slowly rotating stars, the tool to do that is the échelle diagram, and it is probable that many more of these stars will emerge from the TESS data. Further developments to the theory are also needed, including incorporating the Coriolis and centrifugal forces and possibly unanticipated surprises, as this new behavior is explored.

There is related behavior in other stars, e.g., the series of p modes separated by the orbital frequency, but which are not harmonics of that frequency, in the heartbeat star KIC 4544587 (Hambleton et al. 2013), and in the eclipsing binary U Gru (Bowman et al. 2019b). For the latter, Bowman et al. (2019b) discussed tidally excited and perturbed modes and also single-sided mode trapping. It is clear that the interaction of pulsation modes and tidal distortion in close binaries offers new asteroseismic inference, with a range of behavior culminating in the tidally tilted pulsators. The problem of mode selection and excitation is not solved for these stars; but then, it is not solved for any δ Sct stars. At least in this case of close binaries, when the mode frequencies are harmonics of the orbital frequency, there is a clear coupling to the binarity that can be exploited. An observational goal is to find tidally tilted pulsators in eclipsing binaries for which the eclipses add further measures of mass and radius, and the changes in the pulsation mode amplitude and phase during eclipse offers tomography of the mode geometry.

Yu et al. (2021) have theoretically studied the case of dynamical tides in close double degenerate white dwarfs with orbital periods of less than an hour and, in extreme cases, less than 10 min. These stars are, e.g., the double-degenerate interacting AM CVn stars, which are precursors of white dwarf mergers (Section 4.1) and SNe Ia, and are a source of gravitational waves that *Laser Interferometer Space Antenna* (LISA) is expected to detect. Yu et al. predicted that coupling of g modes and p modes can produce photometric variations up to 1%, which can be larger than the ellipsoidal variations. Furthermore, the pulsations generated by the dynamical tide have a phase shift of about 50 deg to the orbital variation, which will allow them to be distinguished from the ellipsoidal variability. Even for faint white dwarfs, TESS observations offer the possibility of detecting a 1% effect; hence, we can look forward to observations that will enable asteroseismic application of this new theory.

5.3. Summary: Asteroseismology of Close Binary Stars

1. Eclipsing binaries with pulsating components provide multiple measures of stellar fundamental parameters. Together they constrain mass, radius, age, and luminosity to the highest precision for application in exoplanet and galactic archeology studies.
2. The PM technique has opened up unexplored parameter space in the orbital periods of binary stars. It allows orbital solutions for binary stars without the need of spectroscopy to obtain radial velocities, saving vast amounts of telescope time and astronomers' time.
3. Doppler beaming also allows the measurement of radial velocities without the need of spectra.
4. Heartbeat stars and tidally tilted pulsators provide observations to test theories of stellar tides and their interactions with pulsation and with rotational and orbital angular momentum. They test mode trapping, tidal distortion of pulsation frequency and amplitude, and pulsation mode axis selection.
5. The heartbeat stars support the thesis of tidal locking where feedback among orbital, rotational, and stellar evolution changes keep the pulsation frequency resonant with the orbital frequency.

6. EPILOGUE: PAST AND FUTURE

I am fortunate that my career has spanned the time from the beginning of helioseismology in the 1960s through to this blossoming of asteroseismology today. This review has highlighted the results for stars across the HR diagram (except for the solar-like oscillators and red giants, as a consequence of other recent reviews of those iconic asteroseismic targets). All of this work uses asteroseismology to see inside the stars. Fundamental and other stellar parameters are extracted from asteroseismic inference: mass, radius, age, metallicity, atomic diffusion, luminosity, distance, magnetic fields, interior rotation, angular momentum transfer, convective overshoot, and core burning stage. In the spirit of Howard Florey, asteroseismologists are delighted to be exploiting stellar pulsation in more depth than previously possible and to be discovering new kinds of pulsation behavior, all improving our understanding of stellar structure and evolution, a bedrock of astrophysics, because stars are the luminous tracers of the Universe.

Space photometry and ground-based large-scale surveys have created an explosion of asteroseismic data. Big data techniques and machine learning are already active in exploring and categorizing the huge data sets, and this will expand in the future. Human labor cannot sift the large numbers of light curves and FTs as computers can. This is particularly true of the ground-based surveys such as ZTF and Rubin Observatory, the latter of which will produce 20 Tb of data per night! These ground-based surveys will enable and expand asteroseismology of many classes of stars discussed in this review, such as white dwarfs, BLAPs, EL CVn binaries, Cepheids, RR Lyr stars, and SLF O and B stars.

The photometric space missions have been designed for exoplanet studies, and they have been successful beyond expectation, with exoplanets now known in their thousands, soon to be tens of thousands. For these missions, we asteroseismologists initially rode on the coattails of the exoplanet community, but now that the usefulness of asteroseismology to exoplanets is known, asteroseismology is built into the science case for future missions. Nevertheless, hot stars are not the main hosts of exoplanets, and they are not expected to have inhabited planets, even if habitable planets orbiting them should be found, for the obvious reason of their short lifetimes. Hot main sequence stars are also rarer as a consequence of the initial mass function and those short lifetimes. But, as this review has shown, the massive upper main sequence stars, the white dwarfs and subdwarfs, the most luminous blue giants and supergiants, and the classical pulsators, are all of interest asteroseismically.

Most of the variability in hot pulsators comes from variations in $T_{\rm eff}$. As a simple consequence of their spectral energy distributions, the hot stars have much larger pulsation amplitudes in the UV and blue wavelength ranges. This is easy to see by picturing the changes in a blackbody curve with changes in temperature for stars for which the flux peaks in the UV or blue. The TESS bandpass is roughly 0.6–1.0 μm, that of *Kepler* was 0.4–0.9 μm; these were filters with broadly white bandpasses, but toward the redder end of the spectrum. That is the right choice for the study of exoplanet transits and of the stochastic pulsators such as the solar-like stars and red giants. But to obtain much higher signal-to-noise ratios for the hotter stars, a blue filter is needed. A dedicated space mission, which could be called the Blue Asteroseismic Survey Satellite (BASS), is needed to build on the revolutionary asteroseismic results obtained from the exoplanet surveys.

Asteroseismology informs the following in our understanding of hotter stars:

- stellar structure and evolution, including interior rotation, angular momentum transfer, and mixing;
- the most massive stars, precursors to core-collapse SNe, black holes and neutron stars, gamma-ray bursters, observable gravitational wave events, and sources of much of the chemical evolution of the galaxy.

Of cooler stars:

- galactic archeology;
- the fundamental parameters—mass, radius, and age—needed to characterize exoplanet host stars in our search for habitable planets;
- the red giant evolutionary core-burning state;
- the post-main-sequence giant to red giant stages, including interior rotation and magnetic fields.

Of compact stars:

- mass, radius, temperature, luminosity, age, rotation, and the extent of the nondegenerate surface layers in white dwarfs;
- physics under extreme conditions in white dwarfs and subdwarfs, and the end stages of the evolution of 97% of all stars;
- double and single degenerate binary stars, precursors of distance scale SNe Ia, and further sources of chemical evolution.

Of close binary stars:

- tides, tidal distortion of pulsation modes, and mode trapping;
- orbital and rotational angular momentum evolution and exchange;
- orbital circularization.

We have indeed found Eddington's "appliance" to "pierce the outer layers of a star and test the conditions within." He would have been amazed and gratified, as are we.

DISCLOSURE STATEMENT

The author is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS

I thank Professors Conny Aerts and Rob Kennicutt for their many scientific comments and their guidance in the preparation of this review. I also thank Professor Simon Jeffery for providing and discussing the pulsation HR diagram shown in **Figure 1**, adapted for this review.

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