

# Foreign and Domestic Bank Performances: An Ideal Decomposition of Industry Dynamics

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## Abstract:

The aggregate performance of the banking industry depends on the underlying micro-level dynamics within that industry – adjustments within banks, reallocations between banks, entries of new banks, and exits of existing banks. This paper develops a generalized ideal dynamic decomposition and applies it to the return on equity of foreign and domestic commercial banks in Korea from 1994 to 2000. The sample corresponds to the Asian financial crisis and the final stages of a long process of deregulation and privatization in the Korean banking industry. The comparison of our findings reveals that the overall performance of Korean banks largely reflects individual bank efficiencies, except immediately after the Asian financial crisis where restructuring played a more important role on average bank performance. Moreover, Korean regional banks started the restructuring process about one year before the Korean nationwide banks. Foreign bank performance, however, largely reflected individual bank efficiencies, even immediately after the Asian financial crisis.

**Key Words:** commercial banks, profitability, foreign banks and global advantage hypothesis

**JEL Classification:** E5, G2

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## 1. Introduction

Aggregate industry data hide important bank (firm or plant) level dynamics that collectively determine overall industry dynamics. That is, the performance of the aggregate industry reflects the accumulation of the underlying microeconomic dynamics within that industry -- that is, adjustments within banks, reallocations between banks, entry of new banks, and exit of existing banks. The availability of micro-level (establishment-level) data spawned a series of applied microeconomic research (especially for manufacturing industries).<sup>1</sup> That research effort reveals more heterogeneity among firms within the same industry than between industries.

The dynamic decomposition of industry dynamics typically adopts the method originally proposed by Bailey, Hulten, and Campbell (1992). They develop an algebraic decomposition of industry total factor productivity (TFP) growth into three effects – “within,” “between,” and “net-entry” effects. The within effect measures the contribution of surviving firms toward TFP growth. The between (or reallocation) effect measures the contribution of changing market share of surviving firms toward TFP growth, while the net-entry effect measures the contribution of firms entrants into and exits from the industry toward TFP growth. Haltiwanger (1997) extends Bailey, Hulten, and Campbell (1992) and separates the effects of firm entrants into and exit from the industry. Moreover, he also divides the between effect into two components – the “share” and “covariance” effects. The share effect measures the contribution of the changing share of firms while the covariance effect measures the contribution of the changing share of

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<sup>1</sup> McGuckin (1995) describes the Longitudinal Research Database (LRD) at the U.S. Bureau of the Census upon which this research relies. Scarpetta, Hemmings, Tressel, and Woo (2002) provide a more recent discussion firm-level databases in ten OECD countries.

firms times the changing TFP growth of firms toward TFP growth.

Jeon and Miller (2004a) extend that method and develop an ideal dynamic decomposition that they apply to the U.S. banking industry at the national and state-by-state levels of aggregation.<sup>2</sup> That extension began by noting that such decomposition methods share a common index-number problem – the choice of the base year. Bailey, Hulten, and Campbell (1992) and Haltiwanger (1997) choose the initial year as the base for their calculations. Thus, the within effect measures the change in TFP growth at the firm level between the initial and final years weighted by the initial year’s market share. Jeon and Miller (2004a) derive a decomposition of within, between (reallocation), entry, and exit effects where the within effect weights the change in TFP growth, actually the return on equity in their application, between the initial and final years for each firm by the firm’s industry share in the final year. Finally, Jeon and Miller (2004a) define an ideal dynamic decomposition by combining those two dynamic decompositions into a simple average. Thus, the weighting of the within, between (reallocation), entry, and exit effects all employ simple averages of the initial and final year weights. In the bargain, the ideal dynamic decomposition of the industry eliminates the covariance effect derived by Haltiwanger (1997).

We apply that decomposition analysis to the Korean banking industry – Korean nationwide and regional banks as well as foreign banks. As such, we provide the first analysis of the contributing factors to overall performance of banks in the Korean banking industry, measured by return on equity. That analysis also considers differences

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<sup>2</sup> Griliches and Regev (1995) employ the ideal decomposition method in their study of firm productivity in Israeli industry. Scarpetta, Hemmings, Tressel, and Woo (2002) briefly discuss the Griliches and Regev (1995) and Haltiwanger (1997) methods of decomposition, noting how they differ. Jeon and Miller (2004a), however, link the differences to the base-year weighting issue.

in performance between Korean banks and foreign banks as well as between Korean nationwide and regional banks. Further, the analysis examines differences in underlying causes of bank performance before, during, and after the Asian financial crisis.<sup>3</sup>

Several conclusions emerge. Changes in industry return on equity largely reflect changes in individual bank performance, the within effect, except for Korean nationwide and regional banks after the Asian financial crisis, where the reallocation and exit effects dominated. That is, Korean banks restructured response to the Asian financial crisis. Korean regional banks first experienced a large reallocation effect followed the next year by an increase in the exit effect. Korean nationwide banks responded to the Asian financial crisis one year later than the Korean regional banks and largely through a reallocation effect, with little change in the exit effect (too big to fail). Finally, the foreign banks did not experience similar reallocation and exit effects after the Asian financial crisis. Their performance continued to rely primarily on the within effect.

The paper unfolds as follows. Section 2 briefly reviews the differing views on how foreign banks affect the domestic economy and describes the structure of banking in Korea. Section 3 outlines the derivation of the ideal dynamic decomposition with more details provided in the Appendix. Section 4 applies the decomposition technique to the Korean banking industry, including foreign banks and Korean nationwide and regional banks. Section 5 concludes.

## **2. The Korean Banking Sector and the Asian Financial Crisis**

The Asian financial crisis underscores the importance of strong and stable financial markets for the maintenance of economic development. In this regard, some analysts

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<sup>3</sup> Jeon and Miller (2004a) examine how bank profitability differed and identify factors that explain some of those differences. Domestic Korean banks suffered more severely from the Asian financial crisis than foreign banks.

argue that foreign bank participation in domestic financial markets strengthens the domestic economy. Other analysts, however, contend that the financial service industry possesses public-good characteristics and that the unfettered private-markets should not completely control credit allocation decisions. An even more stringent view claims that state ownership and state-mandated credit allocation must send credit to those sectors most crucial for economic development.

The Korean banking system, as noted previously, includes three classes of institutions – nationwide, regional, and foreign banks. During the 1960s and 1970s, the major players in the Korean financial system were the nationwide banks. Regional banks, which operate only in their own provinces with a branch in Seoul, entered the scene in 1967 to encourage regionally based development. Plans to deregulate the financial system and place Korean nationwide commercial banks in the private sector began in the early 1980s. Deregulation in this period expanded the power of commercial banks.<sup>4</sup> The government's hand, however, still wielded a potent force, controlling interest rates on certain types of loans and deposits. Further, the government's informal credit policy continued to favor selected sectors.<sup>5</sup>

The first foreign bank, Chase Manhattan, entered the Korean economy in 1967. The participation of foreign banks grew at a good pace throughout the 1970s and 1980s, but stabilized in the 1990s and then fell somewhat after the Asian financial crisis.<sup>6</sup>

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<sup>4</sup> Over 1980 to 1994, Gilbert and Wilson (1998) calculate that nationwide banks experienced significant, large productivity improvement while regional banks experienced mixed results.

<sup>5</sup> Gilbert and Wilson (1998) argue that the Korean commercial banking system experienced a crisis in the mid-1980s with significant levels of bad loans. Yet, no Korean bank failed at that time. Bank of Korea (1994, 1998), Gilbert and Wilson (1998), and Jeon and Miller (2004b) provide more detailed information.

<sup>6</sup> From 1994 through 2000, 60 foreign banks operated in Korea – some for the full sample period, others for only parts. The 60 banks include 14 each from the U.S. and Japan, 6 from France, 4 each from Canada and

Foreign banks came to Korea during the 1970s and 1980s, partly because they received more favorable treatment in certain areas than domestic banks. In the mid-1980s, regulatory change not only whittled down the preferential treatment of foreign banks but also reduced barriers and restrictions on foreign bank activities in other areas. Thus, the playing field was basically leveled between foreign and domestic banks.

The Asian financial crisis hit the Korean economy and banking system near the end of this long process of deregulation and privatization. Although Korea experienced relatively high economic growth and low inflation in the early 1990s, some weaknesses existed in the financial sector: low international reserves, and poor government regulation and supervision of the banking system. The corporate sector overextended itself with too much investment and borrowing. When several *chaebols* (e.g. Kia, Hanbo, Haitai, and Sammi) went bankrupt and other *chaebols* defaulted on syndicated loans, foreign lenders reevaluated their positions in Korea. Foreign sources of funds quickly decreased. Regional and nationwide commercial banks overused short-term foreign lending as a source of funds. The lack of transparency of balance sheets, income statements, and management practices all led to a crisis of confidence in Korean institutions. In sum, the Asian financial crisis erected a roadblock across the path of deregulation and privatization. As financial conditions in banking sector deteriorated, the Korean government began searching for potential merger partners.

Did foreign bank operations promote stability or volatility during this crisis? Supporters of and detractors to foreign bank entry make several arguments (e.g., Claessens, Demirgüç-Kunt, and Huizinga 2001, Demirgüç-Kunt, Levin, and Min 1998,

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Singapore, 3 from the U.K., and 2 each from Australia, China, the Netherlands, and Switzerland, and 1 each from other countries.

Dages, Goldberg, and Kinney 2000, and Levin 1996). On the positive side, foreign banks provide an important channel for foreign capital inflows to finance a net expansion of domestic activities. Foreign banks can also increase competition in domestic markets that can improve the efficiency of domestic bank operations, lower the cost of providing financial services, reduce interest rates charged on loans, and increase the interest paid on deposits, thereby stimulating domestic saving and investment. Foreign banks can also promote improvements in government regulation and supervision of the financial system by importing business practices forged by more stringent home country regulations.

On the negative side, foreign banks increase the volatility of domestic financial markets. The foreign capital channel provided by foreign banks not only encourages an *inflow* of capital in good times but also expedites a rapid *outflow* of foreign capital when a financial crisis occurs. In addition, if foreign banks appear more stable than domestic institutions, they may attract the “best” domestic borrowers (higher-profit and lower-risk borrowers), putting domestic banks in the more precarious position of lending to less credit-worthy borrowers. Also, foreign banks can introduce unfamiliar business practices that domestic regulators may find difficult to evaluate and supervise. Rather than improving the regulatory or supervisory process, foreign banks can create complex problems for the domestic government.

Claessens, Demirgüç-Kunt, and Huizinga (2001) note that foreign banks, operating in developing countries, generally achieve higher profitability than domestic banks;<sup>7</sup> the opposite occurs in developed countries. They then offer several rationalizations for the differences between the profitability of foreign banks in

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<sup>7</sup>The Korean experience matches that observation over the 1994 to 2000 period. See Table 1.



developed and developing countries. First, low net-interest margins in developed countries may reflect participation in wholesale, rather than retail, markets with smaller net-interest margins. Second, the technical advantages for foreign banks in developed countries may be too small to cover informational disadvantages. Of course, those two explanations of low net-interest margins may reverse themselves in developing countries.

Berger, DeYoung, Genay, and Udell (2001) consider two alternative hypotheses that can explain differences in foreign and domestic bank performance – the home-field- and global-advantage hypotheses. The home-field-advantage hypothesis argues that domestic banks generally outperform foreign banks because of informational and cost advantages. The global-advantage hypothesis argues that banks from some countries possess sufficient efficiency gains, allowing them to overcome any home-field advantages accruing to domestic banks.

Various analysts suggest that foreign bank lending played a unique role in the Asian financial crisis vis-à-vis other similar events (Cho and Hong 2001, Kaminsky and Reinhart 2001, and Tornell 2001). Domestic banks supplied major quantities of credit to domestic firms. Domestic banks also came to rely more heavily on foreign bank lending. When the crisis hit, the supply of foreign lending evaporated quickly, confronting the domestic banks with a liquidity crisis. Moreover, some commentators indict the initial International Monetary Fund (IMF) rescue programs as worsening the liquidity crisis by requiring tighter credit (Radelet and Sachs 1998, and Stiglitz 2002).

Noland (2000) differentiates the Korean from other Southeast Asian crises, since the Korean investment boom occurred in the manufacturing sector, especially the *chaebols*, rather than in real estate and since investment growth was funded largely by

short-run capital inflows. That is, short-term capital controls were liberalized while the long-term controls were not.<sup>8</sup> In short, the financial crisis caused some important corporate borrowers to default on their loans to banks. That negative shock was reinforced and compounded by the loss of foreign lending to domestic banks. Impending bank failures necessitated the intervention by the central bank to assist in finding merger partners (possibly foreign) or to take over operations of the failed banks.

The Korean government's intervention into banking markets, however, did not limit itself to supervision and regulation. The government played a prominent role in allocating credit to "priority industries" as part of its economic development strategy. Banks granted "policy loans" to favored firms because the government directed them to do so. In some cases, the credit extended possessed little connection to the financial viability of the underwritten projects and contributed to loan quality problems experienced by Korean banks during the financial crisis.

Barth, Caprio, and Levine (2002) provide an interesting twist to the exiting literature. To wit, their paper, a part of a much larger research agenda using a new cross-country database on bank regulation and supervision, finds that banking crises positively correlate with limitations on foreign bank entry into domestic banking markets. Thus, merely reducing such limitations and easing the ability of foreign banks to enter the domestic banking market reduces the incidence of banking crises, even if foreign banks do not enter. In sum, their finding suggests that potential entry of foreign banks proves salutary on the stability of the domestic banking market.

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<sup>8</sup> Noland (2000) also argues that the initial IMF program exacerbated problems by confusing the Asian financial crisis with the earlier Latin American crisis. The Asian crisis differed according to Noland, because the corporate expansion was loan, not equity, based. Thus, the financial crisis raised interest rates, triggering a liquidity crisis. The IMF's prescription to tighten credit worsened the liquidity crisis.

The movement in recent decades toward more-open financial markets and the increased activity of foreign banks in domestic financial markets suggests that the proponents have currently won the day. The Asian financial crisis raises the issue of the role, if any, of foreign banks in creating or continuing the crisis. This paper examines the Korean experience.

### 3. Ideal Dynamic Decomposition

This section briefly outlines the steps necessary to generate the ideal dynamic decomposition.<sup>9</sup> The basic strategy involves decomposition using periods  $t$  and  $t-1$  as the base years and then combining the resulting decomposition into the ideal decomposition by computing their simple average.

The return on equity ( $R_t$ ) at time  $t$  is defined as net income ( $NI_t$ ) divided by equity ( $E_t$ ) as follows:

$$R_t = \frac{NI_t}{E_t} = \frac{\sum_{i=1}^{n_t} NI_{i,t}}{\sum_{i=1}^{n_t} E_{i,t}} = \sum_{i=1}^{n_t} \left( \frac{NI_{i,t}}{E_{i,t}} \right) \left( \frac{E_{i,t}}{\sum_{i=1}^{n_t} E_{i,t}} \right) = \sum_{i=1}^{n_t} r_{i,t} \theta_{i,t} ,$$

where  $r_{i,t} = \frac{NI_{i,t}}{E_{i,t}}$  and  $\theta_{i,t} = \frac{E_{i,t}}{\sum_{i=1}^{n_t} E_{i,t}}$ . That is,  $r_{i,t}$  equals net income divided by equity held

by bank  $i$  at time  $t$  and  $\theta_{i,t}$  equals its share of industry equity. Then, the change in return on equity between two periods equals the following expression:

$$\Delta R_t = R_t - R_{t-1} = \sum_{i=1}^{n_t} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}} r_{i,t-1} \theta_{i,t-1} .$$

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<sup>9</sup> More details in the derivations appear in the Appendix and in Jeon and Miller (2004a).

where  $n_{t-1}$  and  $n_t$  are the number of banks that exist at time t-1 and time t, respectively.

That is,  $n_t = n_{t-1} + n_t^{entry} - n_{t-1}^{exit}$ , where  $n_t^{entry}$  equals the number of banks that enter during time t and  $n_{t-1}^{exit}$  equals the number of banks that exit during time t-1. And also,  $n_t - n_t^{entry} = n_{t-1} - n_{t-1}^{exit} \equiv n_t^{stay}$ , where  $n_t^{stay}$  equals the number of banks staying at both t and t-1. Finally, we get that  $n_t = n_t^{entry} + n_t^{stay}$  and  $n_{t-1} = n_{t-1}^{exit} + n_t^{stay}$ .

**Proposition 1** *The change in return on equity over two periods decomposes into four different effects as follows:*

$$\begin{aligned}
\Delta R_t &= R_t - R_{t-1} \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \left( \frac{\theta_{i,t} + \theta_{i,t-1}}{2} \right) && \text{(i) within effect} \\
&+ \sum_{i=1}^{n_t^{stay}} \left[ \left( \frac{r_{i,t} + r_{i,t-1}}{2} \right) - \left( \frac{R_t + R_{t-1}}{2} \right) \right] \theta_{i,\Delta t} && \text{(ii) reallocation effect} \\
&+ \sum_{i=1}^{n_t^{enter}} \left[ r_{i,t} - \left( \frac{R_t + R_{t-1}}{2} \right) \right] \theta_{i,t} && \text{(iii) entry effect} \\
&- \sum_{i=1}^{n_{t-1}^{exit}} \left[ r_{i,t-1} - \left( \frac{R_t + R_{t-1}}{2} \right) \right] \theta_{i,t-1} && \text{(iv) exit effect}
\end{aligned}$$

where

$$r_{i,\Delta t} = r_{i,t} - r_{i,t-1} = \frac{NI_{i,t}}{E_{i,t}} - \frac{NI_{i,t-1}}{E_{i,t-1}}; \text{ and}$$

$$\theta_{i,\Delta t} = \theta_{i,t} - \theta_{i,t-1} = \left( \frac{E_{i,t}}{\sum_{i=1}^{n_t} E_{i,t}} \right) - \left( \frac{E_{i,t-1}}{\sum_{i=1}^{n_{t-1}} E_{i,t-1}} \right).$$

**Derivation:** See Appendix.

Proposition 1 incorporates the idea of the "Fisher Ideal Index". Two alternative methods can calculate the effects of a change in return on equity by each bank --

weighted by last year's or this year's equity share.<sup>10</sup> The existing literature typically uses last year's share.<sup>11</sup> When calculating the decomposition, isolate the terms for the exits and entrants. Relating them to some benchmark, add and subtract either the overall return on equity last year or this year. Then, multiply by the sum of the shares, which equals one by definition. Finally, break the summation apart to allocate the exits and entries as well as the banks that are staying in both years.

The existing literature decomposes the reallocation effect into two components -- a term that reflects changes in shares but relative to the first year's return on equity and a covariance term. This decomposition emerges from doing the decomposition only one way. The ideal decomposition identified in Proposition 1 does not include the covariance term (Jeon and Miller, 2004a). In addition, decompositions also exist for other portfolio variables such as return on assets, equity to assets, loans to assets, and so on.

#### **4. Application of Ideal Dynamic Decomposition**

Our data on banks in Korea come from Financial Supervisory Services (2001). Sixteen nationwide and 10 regional banks as well as 60 foreign banks enter our database for at least one year in the sample from 1994 through 2000. In addition, some bank entrances, mergers, acquisitions, and conversions occurred over the sample period. The within effect captures the performance of individual banks from one year to the next. The reallocation, entry, and exit effects provide an effective way to measure the consequences of bank restructuring.

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<sup>10</sup> Entry and exit complicate matters. When comparing numbers between two consecutive years, exits do not exist in the second year while entrants do not exist in the first year.

<sup>11</sup> Griliches and Regev (1995) provide an exception.

Table 1 shows that the average return on equity for all banks in Korea exceeded zero before 1997. The change in return on equity generally suffered negative changes, touching bottom in 1998 (see Table 2). More recently, though, the change in return on equity assumed positive values. The biggest effects occurred with the dramatic decrease from 1997 to 1998 and the near reversal from 1998 to 1999.

Table 1 also shows the differences across different bank types – foreign, and nationwide and regional Korean banks. Foreign banks did not experience the same dramatic swings in the return on equity during the Asian crisis, exhibiting positive values and reaching the maximum in 1997. Essentially no correlation exists between average return on equity for foreign banks and any grouping of Korean banks – nationwide, regional, or all Korean banks. Korean nationwide banks, however, set the trend for the all bank findings, since nationwide banks dominate the Korean banking industry. Regional Korean banks were greatly affected by the crisis. Moreover, even though they recovered somewhat in 1999, they experienced a relapse in 2000.

The information in Table 1 suggests that foreign banks benefited from two “global-advantage effects.” First, foreign banks exhibited a higher average return on equity than Korean banks – either nationwide, regional, or all Korean banks – for each and every year in our sample. Second, foreign banks succumbed less to the shock of the Asian financial crisis than did domestic Korean banks.

Table 2 provides an additional piece of information on foreign and domestic bank performance. To wit, both foreign banks and Korean banks exhibited a substantial decline in average return on equity between 1997 and 1998, where the foreign bank decline was lowest. The change in return on equity from 1998 to 1999, however, shows that Korean

banks outperformed foreign banks in recovering from the shock of the Asian financial crisis, reflecting, in large part, the intervention of the Korean government in response to the Asian financial crisis.

Table 3 shows that Korean banks exited the industry largely in 1997 and 1998. Also the number of banks, both domestic and foreign, decreased since 1995, partly because of bankruptcies, acquisitions, and mergers of Korean banks in 1997 and 1998 and partly due to the more frequent exit of foreign banks since 1998.

Table 4 reports the results of the decomposition analysis. For all banks, the within effect dominates movements in the return on equity, except for from 1995 to 1996 and 1998 to 1999. That is, the within effect moves in the same direction and with similar magnitude as the return on equity with a correlation of 0.86. The reallocation-effect, however, dominates events from 1995 to 1996 and 1998 to 1999, especially the latter. The exit effect from 1998 to 1999 also achieved a noteworthy level, adding to overall industry performance. That is, those banks that exited possessed, on average, a lower return on equity than the average for the industry, improving industry performance. More useful information emerges when we recomputed the decompositions by bank type – foreign, and nationwide and regional Korean banks.

The within-effect clearly dominates movements in the return on equity for foreign banks with a correlation of 0.99, which conforms to the findings of Jeon and Miller (2004a) for the U.S. banking industry. The reallocation effect's much smaller relative size indicates that foreign banks prefer to exit from the Korean market rather than to improve their efficiency when their businesses experience trouble. Six foreign banks with lower profitability entered in 1997 and 1998 while two newly entered banks in 1999 and

2000 caused positive entry effects. The exit effect generates a positive contribution to the change in return on equity (i.e., the exit of less-profitable banks, on average), except from 1998 to 1999 immediately after the Asian financial crisis.

For all Korean banks, the within effect, once again, dominates movements in the return on equity with a correlation of 0.84. The Asian financial crisis caused the within effect to experience large negative outcomes from 1996 to 1997 and 1997 to 1998. Then the reallocation effect dominated events from 1998 to 1999 when overall bank return on equity recovered. Finally, the within effect dominated the further improvement in return on equity from 1999 to 2000.

That pattern of events for all Korean banks more closely mirrors the findings for nationwide banks, than for regional banks, with correlations between the within effect and the change in return on equity for the regional and nationwide banks of 0.99 and 0.77, respectively. The positive reallocation effect observed from 1998 to 1999 for nationwide and all Korean banks emerges from 1997 to 1998 for regional banks. That is, regional banks began restructuring about a year ahead of nationwide banks. A large exit effect occurs from 1997 to 1998. The three regional banks that exit in 1997 exhibited above average performances while the two regional banks that exit in 1998 exhibited poorer performance, on average. Finally, the Korean regional banks experienced a dramatic reversal of fortunes from 1999 to 2000 while the Korean nationwide banks continued to improve their performance.

Because of their sheer size, changes in the Korean nationwide banks dominate the decomposition of movements in return on equity, no matter who else enters the group. In other words, the patterns observed in the decompositions of the change in the return on



equity for nationwide banks match more closely the patterns for all Korean banks or for all banks. The patterns observed for the foreign and Korean regional banks exhibited differences from the nationwide banks findings.

In sum, the within effects generally dominated the movements in the return on equity for banks operating the Korean. That is, the change of return on equity generally correlates highly with the within effect. In contrast, the Asian financial crisis precipitated a large restructuring that caused the reallocation effect to become much more important from 1998 to 1999 for nationwide banks and from 1997 to 1998 for regional banks. The recovery from the Asian financial crisis began in earnest in 1999 and continued in 2000, except for the poor performance of the Korean regional banks.

Foreign banks also fell victim to the Asian crisis, although not as significantly as domestic banks. Their performance, which is largely measured by the within effect, went down after 1997. Foreign banks did not face restructuring, as did Korean domestic banks. That is, foreign bank reallocation effects after the Asian crisis remained relatively small when compared to those of the Korean domestic banks. That last observation probably reflects the fact that foreign banks in Korea represent a small portion of banking sector in Korea as well as the consolidated operations of that bank's parent.

## **5. Conclusion**

The performance of domestic and foreign banks has engaged researchers in recent years. Should governments invite or allow foreign banks to operate within domestic financial markets? Can domestic banks compete with foreign banks on domestic soil? Our paper considers some of the issues in this debate, focusing on the events in Korea before, during, and after the Asian financial crisis.

Foreign banks performed uniformly better, on average, than domestic Korean banks. The evidence strongly suggests that foreign banks experienced a “global advantage” that overpowered any “home-field advantage” enjoyed by domestic banks. That global advantage by foreign banks reflected not only better performance on a year-by-year basis, but also better responses to the difficulties thrust on the Korean economy and financial sector by the Asian financial crisis. The quick intervention of the Korean government, however, to repair the damage allowed the domestic Korean banks to recover more vigorously than foreign banks over the 1998 to 1999 period.

The ideal dynamic decomposition reveals that the within effect generally dominated movements in return on equity. The Asian financial crisis did cause a dramatic restructuring of the Korean banking industry, excluding the foreign banks. As already mentioned, the Korean government played the major role in that restructuring, attempting to prevent the crisis from worsening.<sup>12</sup> As such, the reallocation effect of Korean banks increased dramatically during the period from 1997 to 1999. When the Asian financial crisis hit in 1997, the dramatic decrease in return on equity in Korean domestic banks, both nationwide and regional, entirely reflected the within effect. The banking system’s performance was buoyed from 1997 to 1998 by the reallocation effect, more so for the regional than the nationwide banks, as well as the exiting of banks with below average performance. That restructuring process continued from 1998 to 1999, with the nationwide banks participating more strongly than before. Finally, the regional banks experienced a reversal of fortunes and declining performance from 1999 to 2000.

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<sup>12</sup> U.S. regulators faced a financial crisis during the savings and loan debacle. The U.S. government decided to solve that crisis and not sweep the issues under the rug. Japan, on the other hand, has yet to address in any serious way the critical problems that it faces in its financial sector. Korea adopted the U.S. approach.

Two final issues deserve discussion. First, why did foreign banks outperform Korean domestic banks? One explanation may provide the bulk of the answer. Foreign banks, unlike domestic Korean banks, did not receive Korean government direction to offer bank credit to selected, favored industries. In that regard, foreign banks may have held more-diversified, less-vulnerable portfolios.<sup>13</sup> Another, possibly less-plausible, explanation exists, however. Foreign banks, since they are much smaller than Korean banks, even Korean regional banks, can more easily and quickly adjust to changing circumstances.

Second, did foreign-bank participation in the Korean economy affect domestic bank performance? Demirgüç-Kunt, Levin, and Min (1998) find that greater participation by foreign banks (i) reduces the probability of a banking crisis, (ii) improves the efficiency of domestic banks, and (iii) boosts economic growth indirectly by improving domestic bank efficiency. Further, the effects of foreign bank operations relate to the number of foreign banks and not the size of their operations. In Korea, the number of foreign banks exceeded the number of domestic Korean banks in each year of the sample (see Table 3), although foreign banks represent a small share of the Korean banking market. One can conjecture that the relative success of foreign bank operations in Korea provided an important “demonstration effect,” which encouraged the Korean government to restructure its own banking industry. We leave that conjecture for future investigation.

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<sup>13</sup> Before the Asian financial crisis, most foreign banks in Korea operated primarily as corporate bankers, focusing on fee-generated income from multinational companies and few Korean firms. Foreign banks did not foster a significant exposure to Korean *chaebols*. The corporate-oriented Korean banks, however, did rely on significant exposure to credit lending to *chaebols*. The failure of several *chaebols* during the Asian financial crisis damaged the financial health of corporate-oriented Korean banks. Retail-oriented domestic and foreign banks were less vulnerable to the crisis. Jeon, Miller, and Natke (2004) provide more discussion of the different business models for banks in Korea and examine whether foreign banks stabilized Korean financial markets.

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**Table 1: Return on Equity (Average)**

Year	1994	1995	1996	1997	1998	1999	2000
<b>All Banks</b>	6.37% (76)	4.66% (77)	4.72% (74)	-8.17% (79)	-38.76% (72)	-15.34% (63)	-5.80% (60)
<b>Foreign Banks</b>	8.26% (52)	7.96% (52)	10.70% (49)	29.58% (53)	9.38% (51)	6.31% (46)	12.15% (43)
<b>All Korean Banks</b>	6.09% (24)	4.19% (25)	3.80% (25)	-14.19% (26)	-52.48% (21)	-19.62% (17)	-9.47% (17)
<b>Korean Regional Banks</b>	5.73% (10)	5.63% (10)	5.41% (10)	-14.78% (10)	-87.28% (8)	-2.25% (6)	-22.00% (6)
<b>Korean Nationwide Banks</b>	6.17% (14)	3.91% (15)	3.49% (15)	-14.09% (16)	-48.59% (13)	-20.75% (11)	-8.58% (11)

**Note:** The numbers in parentheses equal the number of banks entering the average return on equity reported.

**Table 2: Change in Return on Equity (Average)**

Year	1994-1995	1995-1996	1996-1997	1997-1998	1998-1999	1999-2000
<b>All Banks</b>	-1.71%	0.06%	-12.89%	-30.59%	23.42%	9.54%
<b>Foreign Banks</b>	-0.30%	2.73%	18.88%	-20.20%	-3.07%	5.83%
<b>All Korean Banks</b>	-1.90%	-0.39%	-17.99%	-38.30%	32.86%	10.14%
<b>Korean Regional Banks</b>	-0.10%	-0.22%	-20.19%	-72.50%	85.03%	-19.75%
<b>Korean Nationwide Banks</b>	-2.26%	-0.41%	-17.58%	-34.50%	27.84%	12.18%

**Note:** These numbers appear also in Table 4 along with the decomposition.

**Table 3: Number of Staying, Entering, and Exiting Banks**

	Year	1994-1995	1995-1996	1996-1997	1997-1998	1998-1999	1999-2000
<b>All Banks</b>	<b>Stay</b>	76	74	73	71	62	59
	<b>Entry</b>	1	0	6	1	1	1
	<b>Exit</b>	0	3	1	8	10	4
<b>Foreign Banks</b>	<b>Stay</b>	52	49	48	50	45	42
	<b>Entry</b>	0	0	5	1	1	1
	<b>Exit</b>	0	3	1	3	6	4
<b>Korean Banks</b>	<b>Stay</b>	24	25	25	21	17	17
	<b>Entry</b>	1	0	1	0	0	0
	<b>Exit</b>	0	0	0	5	4	0
<b>Korean Regional Banks</b>	<b>Stay</b>	10	10	10	8	6	6
	<b>Entry</b>	0	0	0	0	0	0
	<b>Exit</b>	0	0	0	2	2	0
<b>Korean Nationwide Banks</b>	<b>Stay</b>	14	15	15	13	11	11
	<b>Entry</b>	1	0	1	0	0	0
	<b>Exit</b>	0	0	0	3	2	0

**Note:** Staying banks exist in both years. Entering banks exist in the second, but not the first, year. Exiting banks exist in the first, but not the second, year. For example, Korea saw one bank enter and 3 banks exit in 1996. See the 1995-1996 and 1996-1997 columns under the All-Banks category for entry and exit. As another example, Korea saw 6 banks enter and 8 banks exit in 1997.

**Table 4: An Ideal Decomposition of Industry Dynamics**

	Year	1994-1995	1995-1996	1996-1997	1997-1998	1998-1999	1999-2000	SUM
<b>All Banks</b>	<b>Within</b>	-1.94%	-0.21%	-14.54%	-38.10%	0.12%	11.76%	-42.91%
	<b>Reallocation</b>	0.10%	0.28%	1.28%	7.61%	16.04%	-2.33%	22.98%
	<b>Entry</b>	0.12%	0.00%	0.37%	0.05%	0.03%	0.47%	1.04%
	<b>Exit</b>	0.00%	0.01%	0.00%	0.14%	-7.24%	0.35%	-6.73%
	<b>ΔR</b>	-1.71%	0.06%	-12.89%	-30.59%	23.42%	9.54%	-12.16%
<b>Foreign Banks</b>	<b>Within</b>	-0.50%	2.19%	20.36%	-19.32%	-2.88%	3.52%	3.37%
	<b>Reallocation</b>	0.20%	0.02%	0.11%	-0.31%	-0.50%	0.71%	0.23%
	<b>Entry</b>	0.00%	0.00%	-1.88%	-0.16%	0.01%	0.88%	-1.15%
	<b>Exit</b>	0.00%	-0.52%	-0.29%	0.41%	-0.31%	-0.72%	-1.43%
	<b>ΔR</b>	-0.30%	2.73%	18.88%	-20.20%	-3.07%	5.83%	3.89%
<b>All Korean Banks</b>	<b>Within</b>	-2.14%	-0.56%	-20.00%	-42.16%	0.23%	13.43%	-51.20%
	<b>Reallocation</b>	0.07%	0.18%	1.05%	4.47%	23.92%	-3.29%	26.41%
	<b>Entry</b>	0.16%	0.00%	0.96%	0.00%	0.00%	0.00%	1.12%
	<b>Exit</b>	0.00%	0.00%	0.00%	0.61%	-8.71%	0.00%	-8.10%
	<b>ΔR</b>	-1.90%	-0.39%	-17.99%	-38.30%	32.86%	10.14%	-15.56%
<b>Korean Regional Banks</b>	<b>Within</b>	-0.11%	-0.39%	-20.74%	-89.50%	66.59%	-21.62%	-65.77%
	<b>Reallocation</b>	0.01%	0.17%	0.54%	22.17%	0.31%	1.88%	25.08%
	<b>Entry</b>	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	<b>Exit</b>	0.00%	0.00%	0.00%	5.17%	-18.12%	0.00%	-12.96%
	<b>ΔR</b>	-0.10%	-0.22%	-20.19%	-72.50%	85.03%	-19.75%	-27.73%
<b>Korean Nationwide Banks</b>	<b>Within</b>	-2.56%	-0.60%	-19.89%	-35.61%	-6.00%	15.86%	-48.81%
	<b>Reallocation</b>	0.09%	0.18%	1.18%	1.51%	26.05%	-3.68%	25.34%
	<b>Entry</b>	0.20%	0.00%	1.13%	0.00%	0.00%	0.00%	1.33%
	<b>Exit</b>	0.00%	0.00%	0.00%	0.39%	-7.79%	0.00%	-7.39%
	<b>ΔR</b>	-2.26%	-0.41%	-17.58%	-34.50%	27.84%	12.18%	-14.74%

**Note:** The change in return on equity equals  $\Delta R$  and also appears in Table 2. The within effect measures the increase in average return on equity due to higher return on equity in each bank. The reallocation effect measures the increase in average return on equity due to reallocation (restructuring) of equity between banks. And the entry and exit measure the increase in average return on equity due to entry and exit of banks. Remember that the exit effect enters with a negative sign so that a positive (negative) exit effect reduces (increases) overall average return on equity.



## APPENDIX

**DERIVATION OF PROPOSITION:** The change in return on equity can be rewritten as

$$\begin{aligned}
 \Delta R_t &= R_t - R_{t-1} \\
 &= \sum_{i=1}^{n_t} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}} r_{i,t-1} \theta_{i,t-1} \\
 &= \sum_{i=1}^{n_t^{stay} + n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{stay} + n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
 &= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{stay}} r_{i,t-1} \theta_{i,t-1} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \tag{A1}
 \end{aligned}$$

### **Decomposition 1**

Adding the term  $\sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t} - \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t}$  in the right hand side of (A1) then

$$\begin{aligned}
 \Delta R_t &= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{stay}} r_{i,t-1} \theta_{i,t-1} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} + \left[ \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t} - \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t} \right] \\
 &= \left[ \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t} \right] + \left[ \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t} - \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t-1} \right] + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
 &= \sum_{i=1}^{n_t^{stay}} (r_{i,t} - r_{i,t-1}) \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} r_{i,t-1} (\theta_{i,t} - \theta_{i,t-1}) + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
 &= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1}
 \end{aligned}$$

Note that  $\sum_{i=1}^{n_t} \theta_{i,t} = 1$  and  $\sum_{i=1}^{n_{t-1}} \theta_{i,t-1} = 1$ , which implies that

$$\sum_{i=1}^{n_t^{stay}} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} \theta_{i,t} = 1 \quad \text{and} \quad \sum_{i=1}^{n_{t-1}^{stay}} \theta_{i,t-1} + \sum_{i=1}^{n_{t-1}^{exit}} \theta_{i,t-1} = 1 \tag{A2}$$

Therefore, we have that

$$\begin{aligned}
\Delta R_t &= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
&\quad - R_t \left[ \sum_{i=1}^{n_t^{stay}} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} \theta_{i,t} \right] + R_t \left[ \sum_{i=1}^{n_t^{stay}} \theta_{i,t-1} + \sum_{i=1}^{n_t^{entry}} \theta_{i,t-1} \right] \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,\Delta t} - R_t \sum_{i=1}^{n_t^{stay}} \theta_{i,t} + R_t \sum_{i=1}^{n_t^{stay}} \theta_{i,t-1} \\
&\quad + \left[ \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - R_t \sum_{i=1}^{n_t^{entry}} \theta_{i,t} \right] - \left[ \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} - R_t \sum_{i=1}^{n_{t-1}^{exit}} \theta_{i,t-1} \right] \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,\Delta t} - \sum_{i=1}^{n_t^{stay}} R_t (\theta_{i,t} - \theta_{i,t-1}) \\
&\quad + \left[ \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_t) \theta_{i,t} \right] - \left[ \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_t) \theta_{i,t-1} \right] \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,\Delta t} - \sum_{i=1}^{n_t^{stay}} R_t \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_t) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_t) \theta_{i,t-1} \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} (r_{i,t-1} - R_t) \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_t) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_t) \theta_{i,t-1} \tag{A3}
\end{aligned}$$

## **Decomposition 2**

Adding the term  $\sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t-1} - \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t-1}$  in the right hand side of (A1) then

$$\begin{aligned}
\Delta R_t &= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t-1} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} + \left[ \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t-1} - \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t-1} \right] \\
&= \left[ \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t-1} \right] + \left[ \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t-1} - \sum_{i=1}^{n_t^{stay}} r_{i,t-1} \theta_{i,t-1} \right] + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,t} (\theta_{i,t} - \theta_{i,t-1}) + \sum_{i=1}^{n_t^{stay}} (r_{i,t} - r_{i,t-1}) \theta_{i,t-1} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t-1} + \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1}
\end{aligned}$$

With (A2), we have that

$$\begin{aligned}
\Delta R_t &= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} r_{i,\Delta t} \theta_{i,t-1} - \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} \\
&\quad - R_{t-1} \left[ \sum_{i=1}^{n_t^{stay}} \theta_{i,t} + \sum_{i=1}^{n_t^{entry}} \theta_{i,t} \right] + R_{t-1} \left[ \sum_{i=1}^{n_t^{stay}} \theta_{i,t-1} + \sum_{i=1}^{n_{t-1}^{entry}} \theta_{i,t-1} \right] \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t-1} - R_{t-1} \sum_{i=1}^{n_t^{stay}} \theta_{i,t} + R_{t-1} \sum_{i=1}^{n_t^{stay}} \theta_{i,t-1} \\
&\quad + \left[ \sum_{i=1}^{n_t^{entry}} r_{i,t} \theta_{i,t} - R_{t-1} \sum_{i=1}^{n_t^{entry}} \theta_{i,t} \right] - \left[ \sum_{i=1}^{n_{t-1}^{exit}} r_{i,t-1} \theta_{i,t-1} - R_{t-1} \sum_{i=1}^{n_{t-1}^{exit}} \theta_{i,t-1} \right] \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t-1} - \sum_{i=1}^{n_t^{stay}} R_{t-1} (\theta_{i,t} - \theta_{i,t-1}) \\
&\quad + \left[ \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_{t-1}) \theta_{i,t} \right] - \left[ \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_{t-1}) \theta_{i,t-1} \right] \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,t} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t-1} - \sum_{i=1}^{n_t^{stay}} R_{t-1} \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_{t-1}) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_{t-1}) \theta_{i,t-1} \\
&= \sum_{i=1}^{n_t^{stay}} (r_{i,t} - R_{t-1}) \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t-1} + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_{t-1}) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_{t-1}) \theta_{i,t-1} \quad (A4)
\end{aligned}$$

**Decomposition 3** (combining first two decompositions)

Add the decompositions in Case 1 and Case 2 together, (A3) and (A4). Thus,

$$\begin{aligned}
2\Delta R_t &= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t} + \sum_{i=1}^{n_t^{stay}} (r_{i,t-1} - R_t) \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_t) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_t) \theta_{i,t-1} \\
&\quad + \sum_{i=1}^{n_t^{stay}} (r_{i,t} - R_{t-1}) \theta_{i,\Delta t} + \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \theta_{i,t-1} + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_{t-1}) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_{t-1}) \theta_{i,t-1} \\
&= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} (\theta_{i,t} + \theta_{i,t-1}) + \sum_{i=1}^{n_t^{stay}} (r_{i,t-1} - R_t + r_{i,t} - R_{t-1}) \theta_{i,\Delta t} \\
&\quad + \sum_{i=1}^{n_t^{entry}} (r_{i,t} - R_t + r_{i,t} - R_{t-1}) \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} (r_{i,t-1} - R_t + r_{i,t-1} - R_{t-1}) \theta_{i,t-1} \\
\Delta R_t &= \sum_{i=1}^{n_t^{stay}} r_{i,\Delta t} \left( \frac{\theta_{i,t} + \theta_{i,t-1}}{2} \right) + \sum_{i=1}^{n_t^{stay}} \left[ \left( \frac{r_{i,t} + r_{i,t-1}}{2} \right) - \left( \frac{R_t + R_{t-1}}{2} \right) \right] \theta_{i,\Delta t} \\
&\quad + \sum_{i=1}^{n_t^{entry}} \left[ r_{i,t} - \left( \frac{R_t + R_{t-1}}{2} \right) \right] \theta_{i,t} - \sum_{i=1}^{n_{t-1}^{exit}} \left[ r_{i,t-1} - \left( \frac{R_t + R_{t-1}}{2} \right) \right] \theta_{i,t-1} \quad Q.E.D.
\end{aligned}$$