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Comparative Analysis of the Effective Income Tax Function: Empirical Evidence Using LIS Data

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Comparative Analysis of the Effective Income Tax Function: Empirical Evidence Using LIS Data

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Abstract

The effective income tax function is useful and practical methodology to analyze the relationship between income and tax amounts. It includes the measures of tax progressivity, the maximum effective tax rate, and horizontal inequity. We statistically estimated the effective income tax function using the seven countries out of LIS datasets and Korean data. We also estimated the R-S index and the Kakwani index so as to test the consistency among these indexes.

Our empirical results give us several implications. Four different indexes of tax progressivity reflect different aspects about the progressivity, implying that it requires policy planners to evaluate the income tax system with alternatives. The estimated maximum effective tax rate is usually less than or very close to its maximum statutory marginal tax rate, except for Norway and Korea. It implies that the estimation of the effective tax function is of great use and significance to evaluate the charateristics of the income tax law. The mean squared error from the effective income tax function can be used to represent the degree of the horizontal inequity as a 'quick' measure.

I. Introduction

Most countries have the progressive income tax system. Policy planners have been interested in examining the degree of the progressivity for the evaluation of the income tax system. In fact, many progressivity indexes have been suggested, however, these measures reflect different aspects about tax progressivity. Also, there are many ways to evaluate the income tax, for example, nominal tax rate, marginal effective tax rate, along with the amount and its upper limit by types of deductions, tax credits and income brackets specified in the laws.

Kiefer (1984) categorized the indexes of tax progressivity into two types,which are structural index and distributional index. The former is the index using a function of the relationship between the amount of income and the taxes imposed on it (the tax structure), while the latter measures a function of the tax structure and of the income distribution. Duclos and Tabi (1996) also divided the indexes of tax progressivity into two types which are a tax share view and a redistributive approach. They suggested that the Kakwani index and the Suits index belong to the tax share view and the redistributive approach can be applicable to the Musgrave and Thin, the Pechman – Okner index, and the Reynolds-Smolensky index. In addition to these categories, there are a lot of progressivity indexes, for example, Baum (1987, 1998), Aggarwal(1994), and etc.

Berliant and Gouveia (1993), Gouveia and Strauss (1999, 1994) developed an effective tax function to measuring the relationship between income and tax amounts by specific functional forms. This approach is conceptually simple and practical to apply, and also explains lots of characteristics about income tax system.

An effective income tax function includes several parameters, which are related to two indexes of progressivity. This function also can be used to show the difference in income tax systems. The effective income tax function has the several attributes. First, a maximum effective tax rate represents not only the incentive effect of the income tax on the labor supply, but also politically maximum feasible tax rate (Gouveia and Strauss, 1994). Second, the mean squared error (MSE) from the estimated function can be used as a 'quick' index based on the classical concept of horizontal inequity, which is unequal treatment of equals. Finally, both the elasticity of after-tax income with respect to economic income and the residual income elasticity as a tax progressivity can also be easily calculated.

The income tax system for each country has different aspects in tax rate, income bracket, income deduction, tax credit, etc. Thus it is very difficult to compare the income tax systems among different countries. The effective income tax function might give us one practical approach to compare the income tax systems among different countries.

The purpose of this study is to statistically estimate the effective income tax function for comparative analysis of income tax systems. We apply the effective income tax function to selective countries from Luxemboug Income Study (hereafter LIS) dataset and Korean Household Expenditure Survey Data. We show a degree of incentive effects of income taxes on the labor supply, the index of horizontal inequity, and two kinds of tax progressivity indexes. We also estimate the Reynolds-Smolensky index (hereafter, R-S index) and the Kakwani index to evaluate any difference of empirical results between the effective income tax function and popularly applied tax progressivity indexes.

This paper consists with four sections. Section 2 discusses the effective income tax function to be estimated, and its implications. Section 3 presents the empirical results for eight countries. Finally, Section 4 is summarized and concluded.

II. The Effective Income Tax Function

We discuss the derivation and characteristics of the effective income tax function. This function was originally derived from the equal sacrifice theory by Young (1988, 1990) and more generally by Berlaint and Gouveia (1993). It is also practically applied by Gouveia and Strauss (1999, 1994).

Now we briefly explain how the equation was derived. Young (1990) presented the tax functions constructed by applying the absolute version of the equal sacrifice principle to the isoelastic utility function as follows:

$$u = -c^{-p}$$

where u is the level of utility, c is the level of consumption, and p is a parameter. The rule defines the tax function that causes a sacrifice of 's' from economic income, y, as the solution to

$$-y^{-p} + (y - t(y))^{-p} = s$$

from which we find the total tax function

$$t(y) = y - (y^{-p} + s)^{-1/p}.$$

We derive the average tax function as follows:

$$\bar{t} = 1 - (s * y^{p} + 1)^{-1/p}.$$

This equation has the asymptotic marginal and average tax rates of 100 percent that might readily affect the willingness to work. The above function does not consider incentive effects of the income tax on labor supply. So Berliant and Gouveia (1993) propose the labor supply component in this framework by integrating the notion of equal sacrifice on optimal income taxation. Empirical studies were done by Gouveia and Strauss (1999, 1994) by adding a specification with one parameter 'b' as an approximation to incentive compatible to the equal sacrifice tax function as follows:

$$atr = b - b * (s * y^{p} + 1)^{-1/p} + e \qquad (1)$$

where

atr: average tax rate,

y: economic income

b, s, p: parameters to be estimated

e : an additive statistical disturbance

We use equation (1) for the estimation of the effective income tax function. Now we discuss the economic implications of parameter estimates from the effective income tax function.[1] First, the estimate of 'b' represents a maximum effective tax rate, which is interpreted as a weight given to incentives in the design of the effective income tax function. The parameter 'b' was interpreted as a maximum politically feasible tax rate (Gouveia and Strauss, 1994, p. 335). Also Gouveia and Strauss (1999) noted that a lower b represents more incentives to the willingness to work. So we display which countries have more incentive on labor supply of taxes with these values.

[1] The expression p+1 is the constant relative risk aversion coefficient and 1/(p+1) is the intertemporal elasticity of substitution assuming preferences take the form of additive utility functions (Gouveia and Strauss, 1999, p. 155). However, we don't deal with parameter 'p' in our study.

Second, the estimated mean squared error (MSE) from the effective income tax function can be used as a 'quick' index of the classical horizontal inequity. The classical notion of horizontal inequity means unequal treatment of equals. Even if there have been many indexes to measure the horizontal inequity (see Kaplow, 1989, Aronson et al., 1994), MSE, which is a by-product of the effective tax function estimation, can be used as another measure to represent the horizontal inequity.

Third, the elasticity of after-tax income with respect to economic income evaluated at a given point, which is called as residual income elasticity, may be a local measure of distributional effects of the income tax (Jakobssen, 1976). But, we follow Pfingsten (1986), who proposed the average of individually calculated residual income elasticities as a global measure. We estimate the mean of a global measure proposed by Pfingsten (1986) as the residual income elasticity using the estimates of the effective income tax function like Gouveia and Strauss (1994) as follows:

$$\frac{dx}{dy}\cdot\frac{y}{x}=\sum_{i=1}^{N}\frac{1}{N}\cdot\frac{1-t'(y_i)}{1-\bar{t}(y_i)},$$

where $t'(y_i)$ is a marginal tax rate for individual i, $\bar{t}(y_i)$ is an average tax rate for individual i, and N is the number of taxpayers. The elasticity which has smaller than one represents a progressive tax system. The lower the elasticity is, the larger the equalizing effects on the income distribution is.

Fourth, we measure the elasticity of the tax revenue with respect to income. We use the aggregate of the elasticity, which is a weighted average of the individual elasticities where weights are tax payments. Its expression is as follows:

$$\frac{dR}{dy}\cdot\frac{y}{R}=\sum_{i=1}^{N}\frac{E_{(t,y_i)}}{N}\frac{t(y_i)}{R}$$

where $E_{(t,y_i)} = \frac{dt(y_i)}{dy} \cdot \frac{y}{t(y_i)}$, R is the tax revenue, and $\frac{t(y_i)}{R}$ is weights. Its value is greater than one for the progressive tax system. This elasticity can be easily computed with estimates of the effective tax function.

The economic income is used as all sources of taxable income.[2] We define the economic income as total gross income minus non-taxable income like means-tested cash benefits. Let us give an example of non-taxable income with Belgium. There are noncash property income, family or child allowances, social assistance, old age assitance, other means-tested allowances, near cash housing benefits, near cash education benefits, and so on. The average tax rate is calculated by dividing income taxes by economic income. The income variables of LIS datasets are shown in Appendix.

III. Empirical Results

We applied our model to seven countries from the LIS dataset for practical adaptibility and Korean micro data. We got the convergent results of effective income tax functions from Australia (1994), Belgium (1992, 1997), and the Canada (1991, 1997, 1998), Germany (1994), Israel (1992, 1997), Norway (1991, 1995), USA (1991, 1994, 1997, 2000). However, we select one specific year for each country to discuss the difference in income tax systems. Table 1 shows our empirical estimates for eight countries including Korea (1996).

[2] Taxable incomes used for the estimation in each country can be found in the lissification tables of LIS homepage.

Table 1: Empirical Results from the Effective Tax Functions

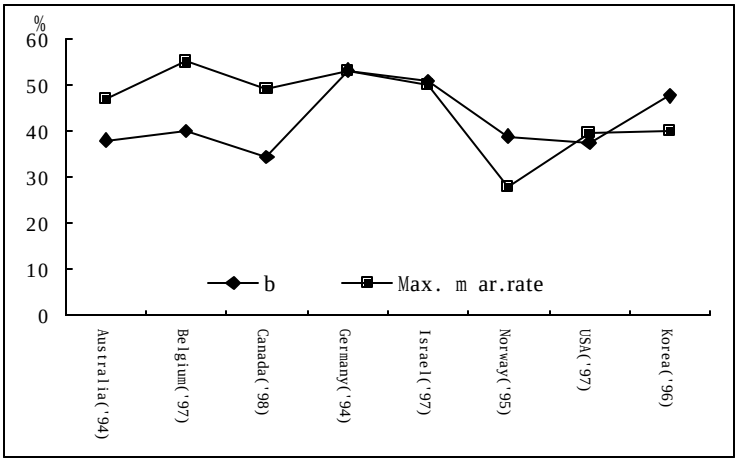
Country	Sample Size	b (%)	Max. marginal tax rate (%)	P	S	RIE	RE1	ERI	MSE
Australia (1994)	4624	38.0 (0.0093)	47.0	1.925 (0.0681)	7.50E-09 (0.000)	0.8470	0.153	1.5660	0.00213
Belgium (1997)	3710	40.0 (0.0085)	55.0	3.489 (0.1279)	1.45E-20 (0.000)	0.8047	0.195	1.6524	0.00241
Canada (1998)	25611	34.3 (0.0043)	49.0	1.915 (0.0348)	6.53E-09 (0.000)	0.8668	0.133	1.5512	0.00289
Germany (1994)	4382	53.2 (0.0248)	53.0	1.348 (0.0363)	2.29E-07 (0.000)	0.8648	0.135	1.8287	0.00141
Israel (1997)	3464	50.9 (0.0244)	50.0	1.492 (0.0535)	2.96E-08 (0.000)	0.8587	0.141	1.7580	0.00305
Norway (1995)	9035	38.9 (0.0145)	28.0	1.204 (0.0420)	3.92E-07 (0.000)	0.8769	0.123	1.5500	0.00186
USA (1997)	39508	37.6 (0.00364)	39.6	1.367 (0.0125)	5.51E-07 (0.000)	0.8944	0.106	1.5938	0.00172
Korea (1996)	24290	47.6 (0.0274)	40.0	1.203 (0.0222)	7.85E-07 (0.000)	0.9484	0.115	1.9295	0.00197

(Note) 1. RIE: Residual Income Elasticity,
ERI: Elasticity of Tax Revenue with respect to pre-tax Income,
RE1 = 1- RIE.
2. The maximum marginal tax rate of Canada adds the federal rate to the provincial/territorial taxes. Norway has the flat tax rate of 28%.
3. Standard error in parentheses.

We interpret our estimates of tax progressivities and various properties about the income tax systems for each country. First, the residual income elasticity (hereafter RIE) and the elasticity of tax revenue with respect to pre-tax income (hereafter ERI) show considerable variations in the countries for study (see also Figure 1). Ranking the countries in descending order with RIE, Belgium is followed by Australia, Israel, Germany, Canada, Norway, Korea, and USA. Also, ranking the countries in the same way as the above indexes for ERI, Korea is followed by Germany, Israel, Belgium, USA, Australia, Canada, and Norway.

Second, there are country-specific fluctuations of a maximum effective tax rate in all the countries.

Figure 1: Maximum Effective or Marginal Tax Rate



As we explained, the lower b indicates the more incentive. In this context, Canada has the least disincentive effect on work in income tax system, which means that Canadian income tax system has a relatively small influence on the labor supply, even if the maximum marginal tax rate is not low relative to other countries. By the same token, Germany has the most disincentive on the willingness to work. The intersting finding is that the estimated maximum effective tax rate is less than or very similar to its maximum marginal tax rate except for Norway and Korea. In the cases of Israel and Germany, the maximum effective tax rates are almost the same as the maximum statutory marginal tax rates.

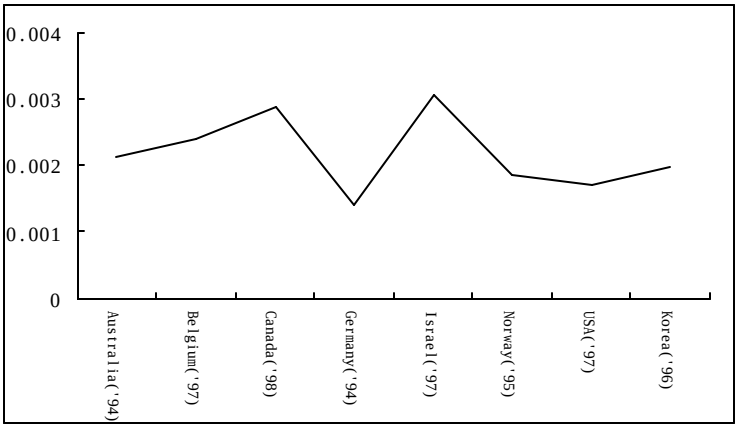
Third, from RE1 in Table 1 to represent the redistributive effect of income tax system, Belgium is followed by Australia, Israel, Germany, Canada, Norway, Korea, and USA. Belgium has the most redistributive effect of income tax system, while Korea and USA have the least redistributive effect. In general, the tax progressivity can be interpreted as the vertical effect, so we choose a measure of vertical effect as the

elasticity of tax revenue with respect to pre-tax income to find the implications with the horizontal inequity.

Fourth, for the 'quick' index of horizontal inequity which is a by-product from the effective income tax function, Israel is followed by Canada, Belgium, Australia, Korea, Norway, USA, and Germany (see Figure 2). This index shows that the income tax system violates the horizontal equity due to deductions, tax credits, and other items. Comparing horizontal inequity with vertical inequity, Germany has a high effect of vertical equity and a low horizontal inequity. Korea, which has the highest vertical effect, shows a medium level of the horizontal inequity. Israel, whose horizontal inequity is the highest, demonstrates the third ranking in the vertical effect. There seems to be no correlation between the vertical and horizontal effects. This comparison implies that the evaluation of the income tax systems requires us to consider both the vertical and horizontal equity. However, it needs more analysis to support this assertion, as the MSE is a 'quick' index unlike the other horizontal inequity index.

In this context, it can be said that the estimates of the effective income tax function are of great use in evaluating the income tax systems for different countries.

Figure 2: Horizontal Inequity Index



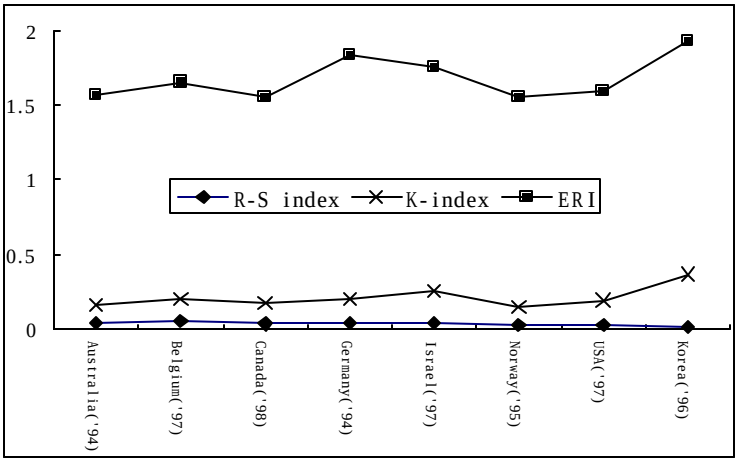
Now we calculate the measures of the R-S index and the Kakwani index so as to supplement the progressivity values of the effective income tax function. The comparison of these indexes checks to see if each index tells the different stories about the progressivity from the same dataset. The R-S index as a tax progressivity index is calculated by subtracting the post-tax Gini coefficient from the pre-tax one, while the Kakwani index is measured as a concentration index for the income tax minus the pretax Gini coefficient. The RE2 is the percentage change in the Gini coefficients of pre- and post-tax income.

Table 2: Income Inequality and Other Progressivity Indexes

Country	Gini Coefficients			Progressivity		RE2 (%)
	Pretax	Post tax	Concentration Index for tax	R-S index	Kakwani index	
Australia (1994)	0.281	0.242	0.444	0.038	0.163	13.6
Belgium (1997)	0.266	0.216	0.465	0.050	0.199	18.9
Canada (1998)	0.291	0.257	0.463	0.034	0.172	11.6
Germany (1994)	0.266	0.221	0.471	0.045	0.205	17.1
Israel (1997)	0.335	0.294	0.588	0.041	0.253	12.2
Norway (1995)	0.235	0.209	0.384	0.027	0.148	11.5
USA (1997)	0.369	0.339	0.561	0.030	0.192	8.2
Korea (1996)	0.370	0.353	0.736	0.017	0.366	4.6

Note: RE2 = R-S index/pre-tax Gini coefficient *100

Figure 3: Comparison of Tax Progressivities Indexes



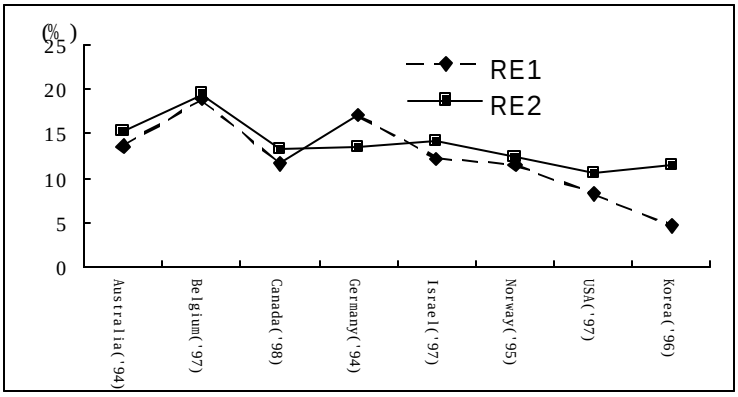
We get several interesting results from these comparisons. First, the international comparison of the conventional inequality indexes shows that there are relatively much differences in the degrees of pre-tax and post-tax income inequality. However, these differences are relatively higher in Korea and USA than in other countries. We also find

that the trend of differences between the pre-tax and post-tax Gini coefficients corresponds to that of the R-S index.

Second, ranking countries in descending order of progressivity using the R-S index, Belgium is followed by Germany, and USA. Ranking the country-specific elasticities of tax revenue with respect to income, Korea is followed by Germany, Israel, Belgium, USA, Australia, Canada, and Norway. These results definitely illustrate that each index tells different stories about the progressivity from the same dataset (see Figure 3). Besides, empirical results about tax progressivity clearly indicate slight difference across countries in the degree of four indexes about tax progressivities.

Third, comparing the RE1 with the RE2 as a measure of the redistributive effect, the two values appear to display a relatively different trend as can be seen in Figure 4.

Figure 4: Comparison of Redistributive Effects



The findings from these comparisons justify the necessity for policy planners to use several alternatives of a tax progressivity index together in order to correctly evaluate the income tax system. The effective income tax function has various advantages. Our

empirical results illustrate that it is reasonable to consider several measures to properly evaluate the income tax system.

IV. Conclusions

We statistically estimated the effective income tax function using the seven countries dataset from LIS and Korea data, to measure the tax progressivity, the maximum effective tax rate, and the horizontal inequity. We also estimated the R-S index and the Kakwani index so as to supplement the evaluation of the income tax system with the index values from the effective income tax function.

Our empirical results give us several implications. Each index in four types of tax progressivity tells the different stories about the progressivity as expected, implying that it requires policy planners to evaluate the income tax system with alternatives. The estimates of maximum effective tax rate are usually less than or very close to its maximum statutory marginal tax rate, except for Norway and Korea. It implies that the estimation of the effective income tax function is of great use and significance to evaluate the charateristics of the income tax system. The MSE, which is a by-product from the estimate of the effective income tax function, represents the degree of the horizontal inequity as a 'quick' measure.

In conclusion, the effective income tax function can be said to be a useful and appropriate criterion to evaluate the charateristics of the income tax system, despite some differences in the degree of the tax progressivity and redistributive effect from existing indexes.

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Appendix: LIS Summary Income Variables

	Variable Definition	Variable Name			
+	Gross wages and salaries	V1	=SELFI		
+	Farm self-employment income	V4			
+	Non-farm self-employment income	V5			
=	Total Earnings	EARNING			
+	Cash property income	V8			
=	Factor Income	FI			
+	Private pensions	V32	=PENSIONI		
+	Public sector pensions	V33			
=	Market Income	MI			
+	Social retirement benefits	V19		=SOCI	SOCTRANS
+	Child or family allowances	V20			
+	Unemployment compensation	V21			
+	Sick pay	V16	=OTHSOCI		
+	Accident pays	V17			
+	Disability pay	V18			
+	Maternity pay	V22			
+	Military/vet/war benefits	V23			
+	Other social insurance	V24			
+	Means-teste cash benefits	V25	=MEANSI		
+	Near-cash benefits	V26			
+	Alimony or Child Support	V34	=PRIVATI		
+	Other regular private income	V35			
+	Other cash income	V36			
=	Total Gross Income	GI			
-	Mandatory contributions for self-employed	V7	=PAYROLL		
-	Mandatory employee contribution	V13			
-	Income tax	V11			
=	Disposable Income	DPI			

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Note: www.lisproject.org