Effectiveness and practical implicability of motivational low-threshold support methods on the performance of building factual knowledge of students with learning difficulties in math

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Acknowledgments

When I started my work at the University of Cologne in 2017, I was highly motivated to teach and guide future teachers into their practical professional career. However, a vision of me as a scientist was still a little tangible. Yet, I felt something incredibly more life-changing was happening and a great opportunity was in front of me. I grabbed it and a new part of myself has emerged: an inquisitive scientist, finding another vocation in life besides being a teacher.

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“Perhaps you have felt already, from the tone of my letter,
that I am more than ever now the bride of science […].”
(Ada Lovelace cited in Toole, 1992, p. 269)

Abstract

This dissertation presents four studies published in peer-reviewed journals focusing on improving the development of mathematical factual knowledge of students with learning difficulties. Mastering fluency building with basic math facts is considered a strong predictor to achieve sufficient or above mathematical competencies. Struggling with this development can already lead to students' demotivation, frustration and avoiding further engagement. Therefore, all papers were conducted with the aim to provide access to methods that help overcome this hurdle with different motivational support for those struggling students and into a deeper engagement of mathematical competency acquisition.

One study focused on the impact of enhancing students' motivation by implementing a combined multicomponent motivational system (MMS). The second study used a specially designed combination of the previously examined motivational methods that math racetracks enhanced. After this examination, again an extension was made by a combination of the previous examined methods with peer tutoring. All three papers were evaluated regarding their effectivity concerning the development of math fact fluency. The final paper evaluated the effectiveness of a response card intervention on the participation of low-performers during their math class.

For the purpose of the research, single-case designs were applied and the studies were conducted in elementary and secondary grades in regular and special schools. This alone led to participants with very heterogeneous conditions and further research will be necessary to underline these studies' findings. Still, all four papers' results indicated that all interventions concerning motivation, racetracks and response cards are highly effective and therefore could provide students and teachers alike with an opportunity to enrich the classroom through a significant increase of students' performance and engagement into mastering mathematical basic facts fluency. Providing teachers with effective, easy-to-implement methods in their classroom independent of special needs or not as well as of grade is an aim any research should try to meet more often, and which, in this case, can be considered successfully achieved.

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List of Abbreviations

CCSS	Common Core Standards
DI	Direct Instruction
HRT 1-4	Heidelberger Calculation test
IQ	Intelligence Quotient
LD	Learning Disabilities
MBD	Mean Baseline Difference
MD	Mathematic Difficulty
MMS	Multicomponent motivational system
NAEP	National Assessment of Educational Progress
NAP	Non-Overlap of All Pairs
NCES	National Center for Educational Statistics
NMAP	National Mathematics Advisory Panel
OtR	Opportunities to Respond
PAND	Percentage of All Non-Overlapping Data
PEM	Percentage Exceeding the Median
PET	Percentage exceeding the trend
PND	Percentage of non-overlapping Data
RC	Response Cards
RtQ	Response to Question
SCRIBE	Single-Case Reporting Guideline in BEhavioural Interventions
STEM	Science Technology Engineering Mathematics

1. Introduction

"We will always have STEM with us. Some things will drop out of the public eye and will go away, but there will always be science, engineering, and technology.
And there will always, always be mathematics"
(Katherine Johnson cited in Wood, 2021, p. 2)

My personal motivation to leave school and return to university was to evaluate methods for students with learning difficulties to improve their academic performance. As a math teacher, my experience was that students with learning difficulties preferred math because it follows clear rules and an individual can be successful if they know and can apply them. Still, all my efforts in class never led to having enough resources to meet every student's needs and frustrations in learning led to experiences such as demotivation or failure in terms of content for individual students. Therefore, the aim of this dissertation is to point out the relevance of implementing effective methods to increase basic mathematical competence development. This includes the aspect of struggling students' motivation to engage themselves into mathematical learning. Another aim is to increase students' active participation in educational learning opportunities. This development should be in focus not only regarding an academic perspective, but also with a wider view on the relation to participation in society as well as providing workable approaches to effectively support students' increasing performance. In addition to the relevance of successful participation in everyday school life, ensuring participation in the narrower and broader sense is a particular challenge for teachers of students with any intensity of learning difficulties. Those students need expedient support to successfully find and shape their place in society, both professionally and privately. School education largely shapes the cornerstone for success or failure in later life.

One crucial aspect of this academic growth and success is acquiring mathematical competencies. Foundations are laid at the beginning of school-supported mathematical development for more complex academic facts and operations, which are already predictors for students' entire (mathematical) academic careers, combined with the fact that quite a few students have difficulties with those exact elementary basic skills and their challenges are likely to remain stable even into higher class levels (Bryant et al., 2015a). The National Center for Educational Statistics (NCES) states that 22% of the adults remain in the mathematical skills of eighth grade (Geary et al., 2013). The National Assessment of Educational Progress (NAEP) reported in 2019, that only 41% of the fourth graders, 34% of the eighth graders and 37% of students in 12th graders scored "at or above proficient" on the mathematics assessment test (NCES, 2019). For students with disabilities, the situation is even worse: More than 90% students with learning disabilities (LD) in the fourth and eighth grade perform below the determined level of proficiency in their overall mathematical development (Horowitz et al., 2017). Struggling to master basic proficiency in particular, the basic math skills that are predictive for further academic careers—and here especially in accuracy and speed—are common for children with low mathematics achievement as well as those classified as learning disabled (Garnett & Fleischner, 1983; Geary, 2004; Jordan et al., 2003). Still, there already are different evidence-based ways to foster exactly students' above-described mathematical engagement, such as methods to increase students' motivation as well as participation in class.

This dissertation explores the effectiveness of easy-to-implement methods for students to improve performance through deep engagement into learning as well as participation in class on building this crucial mathematical factual knowledge for students with learning difficulties. To this aim, the research has been divided into four different studies that were published in four separate peer-reviewed papers. The studies build on each other, thus providing an encompassing picture of various methodological aspects to fully address the research question from different competent angles.

2. Problems in acquiring mathematical competencies

"We need to go back to the discovery, to posing a question, to having a hypothesis and having kids know that they can discover the answers and can peel away a layer."
(Shirley Jackson cited in *Greatest Blacks Ever: Top 100 Blacks Who Changed the World for Peace. Progress. Prosperity. Pleasure*, 2017, p. 108)

The relevance of mathematical competence development is evident not only in terms of academic education in the classroom, but also with regard to social participation in general and it can—if successfully acquired—lead to better conditions in future employment (Geary et al., 2012; National Mathematics Advisory Panel [NMAP], 2008; Rivera-Batiz, 1992). Fortunately, lately mathematics in K-12 education has increasingly become the focus of scientific attention through introducing the Common Core Standards (CCSS; Porter et al., 2011). At the unsuccessful attempt to attain mathematical proficiency, students' inability to achieve fact fluency plays a decisive role (Bryant et al., 2015a; Kanive et al., 2014).

This targeted mathematical proficiency includes five interconnected strands, which are not independent but rather they influence each other and thereby lead to deeper understanding (Mathematics Learning Study Committee, 2001). Those five strands are: understanding mathematics, computing fluency, applying concepts to solve problems, reasoning logically and engaging with mathematics by realizing the problems to be solved as sensible, useful and doable (Mathematics Learning Study Committee, 2002). After the need for developing conceptual understanding of mathematical concepts, accuracy and computational fluency through purposeful practice is required. When those competencies are acquired and strategic flexibility through strategies is developed, improving reasoning and the ability to solve more complex mathematical problems by using all those skills is possible (Mathematics Learning Study Committee, 2001). On the other hand, the progress through higher-level mathematics is difficult without conceptual understanding and computational fluency (Geary, 2011; 2013).

Three general types of knowledge are involved in developing mathematical proficiency with declarative, procedural and conceptual knowledge (Goldman & Hasselbring, 1997). While declarative knowledge are the facts about mathematics, procedural knowledge includes rules and procedures utilized to solve mathematical problems. Conceptual knowledge is the connected information and linked relationships about information. Developing declarative knowledge lays the foundation for computational competence in solving procedural and conceptual problems. Finally, declarative knowledge allows more capacity of working memory to be used for those processes (Cozad & Riccomini, 2016).

Sweller (2005) developed the theory of cognitive load with regard to working memory, which assumes that learning is associated with this cognitive load. Regardless of the ongoing ambiguity in the scientific community about the individual components attributed to the working memory, the cognitive load is of greater importance for this work. Science from various research fields continues to discuss different models of working memory (Chai et. al., 2013). Nevertheless, some have demonstrated the relationship between mathematical learning and working memory (Alloway & Alloway, 2012; Klescewski et al., 2018). As there is no gold standard for which model to use, for this thesis, Baddeley's (1986, 1996, 2012) domain-general model will be used. This multicomponent working memory model includes the components of phonological loop (also called verbal working memory), visuospatial sketchpad (also called visual-spatial working memory) and the central executive involving the attentional control system. Because the capacity of working memory (completely regardless of the model choice) is limited in humans, and for struggling learners in particular, it is important that overload does not occur. This enables acquiring concepts and procedures and thus effective learning. When cognitive resources are overloaded, comprehension and memory problems can occur (LeFevre et al., 2005). In the context of mathematical competency development, this means that, on the one hand, the learning load should be kept as low as possible so that as much capacity is possible, and on the other hand, still remains available for developing conceptual and procedural knowledge.

With regard to the aforementioned NCES (2019) and NAEP (2008) estimates that only 41% of the fourth graders, 34% of the eighth graders and 37% of students in 12th grade scored "at or above proficient" on the mathematics assessment test, and in addition the results from the National Longitudinal Transition Study-2 (2009) that 14% to 27% of students with LD performed 2 standard deviations or more below the mean scores of students their age without

LD within the mathematics-related tests, the special needs of all those struggling students should be taken into account (NCES, 2019; Newman et al., 2010). Further, specific LD, such as dyscalculia or mathematics LD in particular, are often not formally diagnosed in their early elementary grades (O'Connor et al., 2013), and the terms describing difficulties in acquiring mathematical competencies vary from region to region. This circumstance makes it hard to compare research findings and quite necessary to expand the group of students taken into consideration as struggling students, students at risk of developing a mathematics LD, and being diagnosed with dyscalculia or having a diagnosed LD. This extension allows a greater understanding of students who struggle in mathematical learning. Therefore, in this dissertation, as well as in the included articles, the term of mathematic difficulty (MD) is used to describe students with persistent low mathematical performance with or without a disability diagnosis.

Those students usually struggle with deficits in attention and working memory, problem solving, conceptual understanding of foundational mathematics skills, coordinating the steps during the problem-solving process such as inadequate use of strategy knowledge and using retrieval-based skills to solve computations (Geary, 2003; Stevens et al., 2018). Therefore, those components are as vital to achieve the goal of learning mathematics for academic success, but they have also become vital for enabling participation in today's employment world (Hudson & Miller, 2006).

As previously mentioned, fluent recall of basic facts plays a decisive role in developing more complex mathematical skills by reducing the requisition of working memory and it enables students to focus on their mathematical problems' overall purpose and therefore achieve better understanding (Burns et al., 2012; Burns et al., 2016; Johnson & Street, 2013; Sweller et al., 2011). In comparison, fluent peers are more likely to respond to more advanced mathematical tasks, are more willing to engage in extra effort and have an increased motivation for engagement (Codding et al., 2009; McCallum et al., 2006). This should be seen as a further incentive to explore more detailed findings on the effectiveness of methods that address the significant issue to support progress in development, and thus, willingness and increased motivation.

The definitions for students with or are at risk for LD, learning difficulties or learning differences greatly vary in national and international literature (Kraemer et al., 2021). Further, within research, there is a perceived lack of clarity about the attributions affecting the group of people considered as "learning disabled": what phenomena they share, how to identify them and therefore how to classify and define them (Büttner & Hasselhorn, 2011; Heward, 2012; Lloyd et al., 2007). Those with LD or those at risk for LD have in common a higher risk of failure in academic development in general; in addition—and this is of particular interest for this dissertation—they show the most severe challenges in proficient mathematical competency achievement compared to other students (Glago et al., 2009; Milsom & Glanville, 2010).

The common term of LD will be used in this dissertation for students who were assigned a learning disability according to the criteria established in Germany, which includes a slightly reduced intelligence quotient (IQ), academic failure over time as well as over different subjects and is accompanied by special needs (Gruenke & Cavendish, 2016). Those students show a lack in developing knowledge, skills and self-regulation, which overall might lead to a struggle in meeting curricular requirements and therefore achieving below-average grades (Dombrowski et al., 2004).

These difficulties apply to students who have already been diagnosed with an LD, yet students who exhibit these phenomena due to other conditions and therefore are called "at risk" of an LD also need special support from the educational side to learn successfully because they too have an increased risk of failing in the school system at some point (Glago et al., 2009; Grigorenko et al., 2020; Milsom & Glanville, 2010; Ritchie & Bates, 2013). Therefore, because struggling students with or without official diagnoses do need deeper support, in this paper, the term "learning difficulties" will be used uniformly for those students who are challenged by the demands in their academic careers in general. As those students experience severe and repeated failures in academic achievements, a tendency to avoid engagement is common for students with learning difficulties, which subsequently is decisive for enhancing their own performance (Allday et al., 2011; Carini et al., 2006; Neff et al., 2005).

Regarding all the aforementioned aspects of potential failure experience, non-improvement and unwillingness to engage further, the suggestions of research to face these multiple challenges should be taken into consideration. This aspect is addressed in this work through the selection of participants in the studies, the methods offered throughout the intervention of each study and the overall goal of achieving an improvement in mathematical performance.

3. Paper 1: Fostering motivation during math instruction

"Time passes rapidly when you are having fun. Have a goal that you really care about. Don't be afraid of hard work. Nothing worthwhile comes along easily. Don't let others discourage you or tell you can't do it. In my day I was told women didn't go into chemistry. I saw no reason why we couldn't."

(Gertrude Belle Elion cited in Avery, 2000, p. 25)

As mentioned above, students with or at risk of disabilities may develop feeling incompetent because of experiences of failure and frustration in mathematics, which can lead to decreased motivation, especially if students fear further negative experiences (Weiser, 2014). Motivation is of central importance in any subject, but motivational barriers, up to and including anxiety, are of unique relevance especially in the acquisition of mathematical competencies (Dowker et al., 2016) extended by the fact that students assume it has no value for their own lives and is not of personal interest (Peterson & Hyde, 2017). Still, experiencing success and improving in mathematic competencies might be followed by an increase in motivation, which Irvin et al. (2007) describe as a reinforcing cycle of engaging, succeeding academically and motivation. Willingness to exert effort to accomplish tasks, development of strategies for effective self-regulation, persistence in overcoming challenging problems, and better performance are more often found in motivated students (Pintrich & De Groot, 1990; Renninger & Hidi, 2019).

Although attendance and participation are critical key examples for learning students within their behavioral engagement, as hereby they position themselves, to receive and process new knowledge (National Research Council & Institute of Medicine, 2009), their affective engagement is related to their individual intrinsic motivation and emotions, such as positive feelings about learning (Christenson et al., 2012). The intrinsic motivation, that achieving mathematical competencies is useful and that this achievement is possible through effort, leads to the development of believing that mathematics can be understood, learned and utilized, and students are able to figure this out for themselves (Kilpatrick et al., 2001).

Concerning motivation, a wealth of knowledge in different disciplines is cultivated, from which for the purpose of this dissertation is that of education as well as the disciplines influencing it, should all be considered more closely. Still, the uniqueness and validity of motivational theories of learning are fairly limited regarding the wide diversity of behavior as well as the wide variety of learning types (Cheng & Yeh, 2009; Weiler, 2005). Motivation is defined as the relation between a person's will to embrace or be involved in a task, or on the other hand, an action that can lead to an individual's pursuit to engage one action or avoid another (Schunk et al., 2008; Weiner, 1992). Furthermore, wishes and fears linked to the term "affection" became part of the theoretical and scientific discussions about motivational theories. Wishes represent a desired outcome becoming less intense after reaching it, while fear stands for an undesired outcome combined with uncomfortable feelings (McClelland, 1985). Beside this classification, another considers motivation as being both conscious and unconscious: the conscious and explicit motivation leads to focus on concrete and immediate goal achieving while unconscious and implicit motivation is a primary driver over time of one's behavior (McClelland, 1985; Mikulincer & Shaver, 2005). Moreover, operant conditioning describes people's will to avoid punishment and their preferences in reward for behavior and its history especially in education. Research in many disciplines have focused on the perspective of a cognitive approach, including education with an emphasis on perceived abilities and linkages between achievement and effort as being vital for motivation (Trautwein et al., 2012). This focus on goals leads to Locke’s (2006) goal-setting theory, which states that specific conditions related to a gap between the current state and desired situation need to exist to motivate an individual into performing.

Research in recent decades has focused on intrinsic versus extrinsic motivation and has described rewards and punishment as short-term outcomes of extrinsic motivation with the extent to motivate for developing competencies. However, on the other hand, research has considered intrinsic motivators as the most powerful, because as people become active, they have the inner desire to do so (Deci et al., 2001; Ryan & Deci, 2000). Because most educational systems worldwide strive for lifelong learning, and despite the more effective influence of intrinsic motivation on learning processes (Ryan & Deci, 2000; Spinath & Steinmayr, 2012; Sungur, 2007), a good balance between the two needs to be considered. A “both and” could be the current answer simply because intrinsic motivation is less accessible to teachers during class than extrinsic motivation.

Motivation has four different expressions. The expressions of physiological reactions, e.g., hormones, heart rate and respiratory system, as well as the self-reported reactions, describes people talking about their motivation when asked. Both expressions are relevant, but not so much in focus in this paper that they should be described in more detail. The expression of behavior refers to, e.g., attention, effort, probability of responding that indicates weak or intense motivation within a person (Reeve, 2012). The second expression of interest for this thesis is engagement, which is defined as actively being involved in a learning situation. It is a multidimensional construct consisting of behavioral, cognitive and emotional engagement as well as voice. Behavioral engagement relates to visible aspects, such as effort, participation and rule compliance (Reeve, 2008). Cognitive engagement describes managing the learning process while material mastery and understanding are occurring (Reeve, 2008). Feelings such as happiness, frustration, joy and anger are linked to the emotional engagement. Being emotionally engaged leads to positive and enthusiastic feelings toward the learning situation (Reeve, 2008). The last of the four highly inter-correlating dimensions is the voice, relating to actual participation and the chance to express, e.g., opinions, desires and needs (Reeve, 2008).

With a focus on students’ motivation, the relations refer to several theory-based constructs including goals, self-efficacy, attributes, interests, competency beliefs, task value and self-determination (Buehl & Alexander, 2009; Pintrich, 2000). In general, self-efficacy, as one of those constructs, relates to an individual’s judgement about being capable or not to accomplish a task (Bandura, 1986; 1997; Marsh et al., 2019; Pajares, 1997), while related to students’ perception concerning their mathematical skills, it refers to intrinsic motivation (Middleton & Spanias, 1999).

As the most direct prediction for student's achievement, a student's expectation for success and task value such as described in Eccles-Parsons' (1983) expectancy-value theory refers to how much a student suspects that they will succeed in solving a task (Simpkins et al., 2006). This theory provides a categorization for the three reasons a task is viewed as important: attainment value (perceived fit of a task to an individual's personality), intrinsic value (interest in a task) and utility value (relation to personal life and future goals) (Eccles-Parsons et al., 1983; Rosenzweig et al., 2019). All those theories lead to how students' motivation is vital for engaging in learning. Therefore, teachers should promote motivating learning opportunities to support students upkeep or recovery of their motivation and accordingly also their performance.

Well-grounded interventions focusing on increasing students' motivation to engage in learning without being dependent on a learning content are, e.g., *explicit timing, immediate and corrective feedback on performance, adaptive reattribution* including *positive reinforcement through self-scoring* and *visualization* in a *line diagram* as well as through a *personal high score*. All of these interventions aim to increase intrinsic motivation in the second step through implementing an intervention aimed to, e.g., build factual knowledge or improve performance. The aim is to release again a value of learning, the expectation into personal (mathematical) achievement such as competency beliefs by using extrinsic motivation like a broken-down car needing a jump-start. Methods clearly related to motivation used in this dissertation are briefly presented here. These include: explicit timing, immediate feedback on performance and casual attribution.

Explicit timing sets a time limit for exercises, intending to make the time interval most effective. Current research's findings have evaluated this to be effective at increasing students' performance in various school settings if they met the prerequisite before, but did not demonstrate proficient accuracy and speed thus far (Duhon et al., 2015; Grays et al., 2017; Gruenke et al., 2017; Haydon & Kroeger, 2016; Haydon et al., 2012; Rhymer & Morgan, 2005; Rhymer et al., 2002; Van Houten & Thompson, 1976; Van Houten et al., 1974).

Immediate feedback on performance offers (corrective) feedback instantly after finishing a task and is considered one of the greatest influences on students' performance (Hattie, 2009) without the need of receiving it necessarily through the teacher, but also through the means of self-control. Using *positive reinforcement* can even increase this impact via implementing a line diagram to visualize an individual's performance or via further visualizing a personal high score, which research considers to be even more proficient (Wells et al., 2017).

Causal attribution implies that replacing the self-damaging attribution of causes for high or low performance through motivational methods helps students to no longer trace failure back to stable characteristics (e.g., lack of talent), but variable causes (e.g., insufficient effort), while success should be linked to internal factors (e.g., special talents or engagement) (Gonzálvez et al., 2018; Lohbeck et al., 2017; Weiner, 1972).

Gruenke, M., Karnes, J., & Hisgen, S. (2019). The effects of a multicomponent motivational intervention on math performance of elementary school students with learning disabilities. *Insights into Learning Disabilities*, *16*(1), 23–35.

Introduction

In some studies, the aforementioned reviewed methods of explicit timing, feedback through self-scoring, adaptive reattribution through verbal praise and display of high score, and operant conditioning have already been successfully implemented, thus increasing students' performance in different contexts, such as writing instruction (e.g., Gruenke et al., 2017; Leko, 2016) and reading (e.g. Gruenke et al., 2019; McDaniel et al., 2013). Nevertheless, only a few studies have reviewed the efficacy of such a multicomponent motivational system (MMS) on math fact recall. Some research approaches this topic through investigating benefits in isolation (e.g., Duhon et al., 2015; Grays et al., 2017; Rhymer et al., 1999), but there is no study that considers connecting those approaches to check whether the increase in motivation can be further intensified by linking several motivating elements, and what influence this has on students' performance.

Therefore, the purpose of Paper 1 was to examine the efficacy of an MMS consisting of explicit timing; immediate feedback through self-scoring, goal setting and displaying high scores; and positive reinforcement on automating basic math facts of third graders with an LD. The expectation of this was an immediate slope in motivation to engage as well as in the participants' performance.

Method

A multiple-baseline design over persons was applied using an ABA plan to evaluate the training's effects within 15 daily probes. Therefore, a baseline phase (A1) is followed by the intervention phase (B) with a randomly determined beginning and ending of the intervention for each participant to increase internal validity (Dugard et al., 2012, Tate et al., 2016). For making long-term effects visible, the treatment was withdrawn (A2) to see if the effects remain or returned to the baseline (Riley-Tillmann & Burns, 2009).

The probe consisted of three third graders (Anna, Ben and Collin[1]). They were eligible for participating in this study due to their understanding of the concept of two-digit addition and subtraction, but still performed the last 20% in a standardized math test compared to students their age. Further, they were willing to take part in the study.

[1] Names are changed across all papers for anonymity.

The number of correctly solved math items (SMI) on 15 different worksheets was used as the dependent variable. Each worksheet consisted of 20 two-digit addition and 20 two-digit subtraction tasks in random order. Furthermore, students had a time limit of 10 minutes to finish the worksheet in their probe on each day of the study.

During baseline conditions (A1 and A2), each participant was taken out of class and asked to work as hard as they could on the math problems with the interventionist measuring the time covered. After 10 minutes, the participants were asked to stop working and were brought back to class without further feedback. As soon as the intervention started, a timer was placed in front of the student to be able to monitor the time because they were given the time restriction information. In addition, a line diagram, visualizing their performance over time was implemented, a positive reattribution for their efforts and success was given and an index card displaying the students' personal high score was introduced and adjusted when exceeded.

Collected data was statistically analyzed using descriptive analysis, non-overlap indices and a piecewise regression analysis.

Results

Five different most common non-overlap effect sizes were calculated to compare phases A1 and A2 to phase B: percentage of non-overlapping data (PND), percentage of all non-overlapping data (PAND), percentage exceeding the median (PEM), percentage exceeding the trend (PET) and non-overlap of all pairs (NAP) (Alresheed et al., 2013), which equaled the maximum value for each participant across all effect sizes. Moreover, the conducted randomization test for ABA multiple baseline designs (Dugard et al., 2012) to analyze the conjoint effects of all cases reached statistical significance (*p<.001)* regarding the mean differences.

Finally, a piecewise regression analysis for each participant (see Huitema & McKean, 2000) was carried out, with significant level effects for two participants. The third participant even fell below 5% with the beginning of the intervention. In conclusion, all participants showed a significant drop in level once the intervention ended.

Discussion

Thus study examined the effects of the multicomponent motivational intervention using explicit timing, immediate feedback through self-scoring, goal setting (and displaying high scores) and operant conditioning on math fact recall of three third graders with LD. The results show that the number of correctly solved math facts increased greatly during the intervention. Therefore, it is permissible to conclude that the automation of the participants' basic facts increased noticeably with easy-to-use motivation-increasing methods.

It should be mentioned restrictively that findings of single-case studies cannot be generalized due to the small sample size, and in this case in particular the number of measurement times was relatively low. Furthermore, information about participants could have been worked out in more detail, which makes replicating this study difficult. Another limitation is that one of the methods, namely explicit timing, does not for sure fit for all students, as it could expand performance pressure, stress or even anxiety (Rhymer et al., 1999). Finally, this study's results can only be interpreted for the combination of all used methods in a collective manner and no conclusion is possible as to what extent each element was causative for the effects of the treatment.

Still, this study's findings show noticeably how students with an LD can easily be motivated to successfully engage in a basic math fact recall forming process, which they might even have avoided before the training began.

Because students with an especially negative academic self-concept and who experience failure develop verse feelings toward school (Gottfried & Kirksey, 2017), this study, notwithstanding its limitations in validity and generalizability, supplies an additional valuable perspective to interrupt this negative development by enabling students to engage once more in the learning content and experience self-efficacy through their increased performance.

The possibility for students to approach a supposedly unattractive learning content by using motivational methods and then to master it successfully due to their own effort represents a relevant part of overcoming mathematical learning hurdles. The necessity of expanding empirical findings on using motivational methods with students with learning difficulties results from findings such as those of this study. Thereby, on the one hand, those students can be supported even more effectively while, on the other hand, teachers can be offered promising and scientifically well-grounded methods to choose from to achieve that goal.

The summarized presentation of the first paper showed that by adding some easy-to-implement methods, participants increased their performance due to implementing explicit timing, positive reattribution, visualization of performance on a line diagram as well as of their personal high score. After this MMS was applied and results showed a significant increase for all participants, the question arises whether these effects can also be shown through a more challenging as well as attractive character, such as through integrating them into a game situation. *Direct instruction flashcards* (DI flashcards) as well as *racetracks* are both evidence-based methods with exactly this character as they both offer highly frequented repetition of tasks and provide challenging a character through creating a game situation with speed.

Further, in paper 1, the focus was on accuracy and performance speed, which is defined as computing fluency. To master this, it is necessary, to automate basic math facts, which can be achieved through repeating highly frequented non-automated math facts to be able to memorize them.

4. Methods to foster students' fluency building

"What's the use of doing all this work if we don't get some fun out of this?"

(Rosalind Franklin cited in Veenvliet, 2014, p. 236)

There are a variety of methods for teaching basic facts and promoting accuracy. One of those methods is using DI flashcards. DI flashcards introduce facts with immediate and corrective feedback and ensure highly frequent repetition of not automated facts throughout the training (e.g., Lund et al., 2012; Pfaff et al., 2013; Stein et al., 2017; Walker et al., 2012). Another empirically and evidence-based method to reach this aim is using *racetracks*, where students are asked to solve problems to move from cell to cell on a game board that resembles a common racetrack (as known, e.g., from Formula 1). This method engages the student in highly frequent practical activity with the learning content, offers immediate corrective feedback and includes repetition of the targeted tasks as well as some type of speed character (e.g., Beveridge et al., 2005; Erbey et al., 2011; Lund et al., 2012; Pfaff et al., 2013; Skarr et al., 2014; Standish et al., 2012; Walker et al., 2012). Both methods provide a motivating game-based approach, which can be expanded by further motivation elements such as those already discussed in paper 1's intervention. This is particularly relevant because increasing motivation is crucial for further engagement especially for students who are at risk or already left behind and need to catch up with their peers.

As far as the Agreement of the United Nations regarding the structurally desired change toward an inclusive society is concerned—an agreement that has also found its way into the processes of educational institutions with the overall aim to promote equal opportunities as well as equal rights in society (United Nations Convention on the Rights of Persons with Disabilities, 2006)—all 193 member States committed to ensuring participation in the general education system and providing the necessary support measures for this purpose. This leads to teachers in various school facilities being challenged with all forms of diversity and heterogeneity of students (Gavish & Shimoni, 2011), to a scale often unknown prior the Convention's implementation. Therefore, there is a great need for effective strategies, interventions, materials supporting individual learning necessities and with as close a fit as possible with respect to the individual level of performance that needs to become the focus of research. Thus, current research is increasingly attempting to address differentiated considerations as well as conception and evaluation.

In the area of acquiring mathematical skill, and specifically in developing math fact fluency, several meta-analyses have been published recently that examine the effectiveness of various interventions for students with difficulties in adequate mathematical development (e.g., Bowman et al., 2019; Cozad & Riccomini, 2016; Gersten et al., 2016; Marita & Hord, 2017; Mulcahy et al., 2016; Stevens et al., 2018). One of the advantages that methods aiming to support the development of computational fluency need to address, is to be flexibly adapted to each students' or small group's learning levels. This allows the possibility to reach students who show massive delays and thus cannot or hardly find any motivation to further engage with a learning content. Furthermore, there are some students who do not experience sufficient support from their social environment. In summary, this group in particular often has difficulties keeping up with their peers in terms of content within inclusive schooling (Mitchell & Sutherland, 2020).

Math fact fluency is defined as performance including accuracy as well as speed in the four basic operations, resulting after an acquisition phase, where students require modeling, prompting with scaffolds, directly guided practical phases and immediate feedback (Bryant, et al., 2015b; Burns et al., 2010; Duhon et al., 2012; Johnson & Layng, 1996; Musti-Rao & Plati, 2015). After mastering basic understanding, students need to become accurate and then move into fluency-building activities (Rhymer et al., 1998). The aim of mathematics fluency interventions is to propose opportunities for students to engage into practice the known facts until fluency in the forms of both accuracy and speed are reached (Burns et al., 2010).

According to the outlined parameters of What Works Clearinghouse, students should practice for about 10 minutes each day after guided instruction, such as using technology, flashcards or other fact retrieval facilitating material in the targeted facts (Gersten et al., 2009). This fluency practice should include presenting a fact, giving the student appropriate time to respond, focusing on automatic recall and providing immediate and corrective feedback (Gersten et al., 2009). Lindsley (1990) expands these requirements for successful methods to include graphing performance to show sensitivity to changes in students' performance with measures derived from operationally defined and measured results.

5. Paper 2: Racetracks intervention to master math fact fluency

Karnes, J., Barwasser, A. & Gruenke, M. (2021). The effects of a math racetracks instruction on the single-digit multiplication facts fluency of four struggling elementary school students. *Insights into Learning Disabilities*, *18*(1), 1–25.

Introduction

The aim of paper 2 was to examine the easy-to-implement intervention of racetracks' effectivity on automated basic math fact recall with single-digit multiplication tasks of four struggling elementary students. Automation of those facts is vital, as they cannot be solved by finger-counting, and neither can the following and more complex mathematical problems be solved properly without this development. It therefore represents a crucial role in students' mathematical development (Burns et al., 2016; Pólya, 2002; Stein et al., 2006). Fluent arithmeticians can solve tasks faster by relying on automated knowledge as well as use mental algorithms to find solutions to a task (Lerner, 2003; Logan et al., 1996). Frequency-building procedures are necessary to practice individual items using drill and practice, timed repetition with immediate feedback and sufficient effective learning time to give students the opportunity to master automation (Burns, 2005; De Visscher et al., 2018; Greene et al., 2018). Using motivational methods to increase students' engagement, especially if they have been trying to overcome this hurdle for a long time without success, can increase the general learning process and even more specifically in the field of mathematics, where many students show insecurities about exams or even regular math classes. Therefore, their need to experience success to increase their motivation for further engagement is appreciable (e.g., Duhon et al., 2015; Hattie, 2009; Jaffe, 2020). DI flashcards, immediate feedback, positive reinforcement and their effectivity were explained earlier in this dissertation. To implement them in a playful way, using the racetrack method is a repeatedly evaluated method for a variety of mathematical skills (e.g., Beveridge et al., 2005; Erbey et al., 2011; Lund et al., 2012; Pfaff et al., 2013; Skarr et al., 2014; Standish et al., 2012; Walker et al., 2012). On a racetrack game board, which is designed like a circuit (e.g. known from Formula 1) with a determined number of cells, students need to solve tasks to move onto the next cell. There are many ways to play the racetrack, such as using a die or moving from cell to cell and adding a stopwatch to measure the time needed to complete the racetrack.

The purpose of this research was to evaluate the effects of a math racetrack intervention enhanced by an MMS consisting of DI flashcards, immediate feedback, and positive reinforcement, all to address four struggling elementary students' challenges on their basic single-digit multiplication fact recall. The aforementioned components in combination were used as independent variables and the number of single-digit multiplication tasks correctly solved orally within two seconds were defined as the dependent variables.

Method

A single-case multiple-baseline design (AB extension) across participants was conducted (Horner et al., 2005) with a baseline phase (A) without intervention and the treatment phase (B) where the MMS and the racetracks were implemented, consisting of three 20-minute training sessions over six weeks and a maintenance phase after a three-week break (E). A demonstration of long-term effects of an intervention can be supported by collecting maintenance data (Riley-Tillman & Burns, 2009).

Four third graders (aged 8 to 9 years old) from an inclusive elementary school attending two different classes met the following inclusion criteria and therefore were chosen as participants for this study (based on the information the respective teacher provided): basic understanding of single-digit multiplication tasks, no to rudimentary automation of single-digit multiplication tasks from 1×1 to 10×10, regular school attendance in the previous two months, no fostering automation of single-digit multiplication tasks in the current math lessons and participation in more than 80% of the intervention, and finally, motivation to participate in the training according to their own orally given statements.

As the dependent variable, 26 tasks that the participants did not know (13 different items and their reversals) were presented randomly allocated on flashcards. The interventionists noted correct answers on the record sheet that were orally given within two seconds. The independent variable was the MMS containing DI flashcards, immediate feedback, and positive reinforcement that the racetrack method enhanced.

The training was conducted with three 20-minute individual sessions each week. With the beginning of the treatment, measurement was realized in the beginning (such as during baseline) and followed by one session using the DI flashcards to train the pool of not yet automated items and finished with a minimum of one round of the racetrack. With a correct answer, students could move onto the next cell; if incorrect, immediate feedback was given via modeling the correct answer, then asking the student for repetition. Meanwhile, the present task was put in the third position in the deck of items and the student remained in the current cell. A stopwatch was used to measure time and after the racetrack was finished, the time was noted in a line diagram. If the participant reached a new high score, it was updated. Furthermore, students received feedback concerning their cooperation including positive reinforcement. Treatment fidelity was implemented with a checklist and observations ≥ 35% of the sessions. After the final training, students were led through a social validity questionnaire.

For data analysis, the first descriptive analysis for each case was analyzed applying some commonly used non-overlap indices (PEM, NAP and Tau-U) (Lenz, 2013; Parker & Vannest, 2009; Parker et al., 2011a; 2014), as well as mean baseline difference (MBD) (Campbell, 2003). Additionally, a piecewise regression analysis was conducted on the individual level (level 1) and across all four cases (level 2) (Van den Noortgate & Onghena, 2008).

Results

Regarding the descriptive data for each participant, all participants' performance generally improved compared to the baseline phase. Still, with a closer look at the applied non-overlap effect sizes, students' scores ranged from moderate to large effects for PEM, NAP and Tau-U. Further, MBD was calculated to demonstrate the increase (Campbell, 2003; O'Brien & Repp, 1990) with a range from 136.45% as the lowest to the maximum increase of 833.33%.

In addition, a piecewise regression analysis at the individual level (level 1) was conducted as well as across all four cases (level 2) (Van den Noortgate & Onghena, 2008). In three cases, a significant slope effect thereby became visible with respect to the comparison between phases A and B. On level 2, there was a significant slope effect across all participants, but similar to the level 1 analysis, no baseline trends were found. Students increased the correctly solved tasks by 1.128 per intervention.

In summary, all students' performance improved concerning the number of correctly solved single-digit multiplication tasks within two seconds, and therefore, the intervention had a remarkable influence on the dependent variable. Concerning the answers given in the social validity questionnaire, all students unanimously stated to have enjoyed the treatment. No participant answered any of the posed questions in a negative way.

In this paper, the effects of a math racetrack game with an MMS, including DI flashcards, immediate feedback through self-scoring as well as positive reinforcement through visualizing a personal high score on solving single-digit multiplication fact recall of four struggling elementary students were examined.

Results indicate that the intervention increased students' learning progress and therefore can be considered one promising way of adequately supporting students to overcome individual learning hurdles.

Even if previous findings have already described similar conclusions, this paper's new approach consists of combining all components to one intervention without losing practicality for implementation. Therefore, this study adds value to support this easy-to-implement intervention to be designated evidence-based according to single-case research standards found in the Single-Case Reporting Guideline in Behavioural Interventions (SCRIBE) (Tate et al., 2016).

Yet, the research's limitations also need to be pointed out. As the most limiting aspect beside the sample size again being too small to draw generalized conclusions, two participants had to be excluded due to only attending 73% of the measurement. Further, only two different baseline durations could be applied even though random allocation of the baseline's duration was conducted. Another important limitation is that data do not allow to determine with certainty the source of positive effects because the motivational aspects were conducted altogether and not separate. To draw more specific conclusions, an alternating-treatment design comparing the different motivational elements or racetracks and DI flashcards would be helpful. Finally, the time students needed to answer was controlled by using a regular stop watch and accuracy of data might not be highly precise. Therefore, a digital measurement supported by technology, such as PowerPoint, could provide more accurate data.

Future research should aim for effects concerning digitalized racetracks to enhance motivation as well as practicability. Moreover, the methods' influence on the outcome could be examined. One weakness of the multicomponent motivational system applied in this study is that some of the methods require a good accompaniment within the learning process, which is why future research should also try to point out if math racetracks combined with peer tutoring deliver similarly promising results.

In summary, combining the racetrack with an MMS is not only easy -to implement, but an enjoyable way to foster struggling students. Effective methods such as this can be implemented in a simple and quick way and are therefore of great importance for today's learning environments in school.

The summaries of paper 1 and 2 show a variety of easy-to-implement motivating methods to significantly increase students' performance overall. and in paper 2, this was enhanced by a game-based approach to engage students into mastering math fact fluency with basic math facts. Both papers' results indicate that those methods provide an effective way to support struggling students to reach that goal. Even if those methods are all easy -to implement and therefore should be attractive for teachers to use, especially the immediate feedback as a necessary step to avoid saving incorrect results, the implementation on an everyday basis might be challenging on an individual level during class and could tie up a lot of capacity. As a result of this challenge, the question arises: Can a more developed student in computing fluency be instructed to support the struggling student during this process?

6. Paper 3: Peer tutoring to enhance increasing performance

The question that follows from paper 2's promising results is whether these methods can be implemented in such a way that, on the one hand, students show even more willingness to do their best, and on the other hand, to ensure that the teacher has the capacity available to address the individual needs of more than one student in class. One well-grounded approach is peer tutoring, which provides individualized instruction, practice, immediate corrective feedback, and repetition by employing peers to function as one-on-one teachers (Utley et al., 1997). As the peer-tutoring method led to an increase in students' performance in different academic outcomes such as reading (Fuchs & Fuchs, 2005) or math (Fuchs et al., 2001), independent of having or not having an LD (Fuchs et al., 1995), the combination of an MMS using racetracks in combination with this well-grounded method of peer tutoring takes researchers a step further in answering the question of effective easy-to-implement methods to foster struggling students in automating basic math facts.

Karnes, J. & Gruenke, M. (2021). The effects of a multicomponent motivational system intervention using peer- tutoring for implementation on the automation of single-digit addition tasks of four struggling elementary school students. *Education in Sciences*, *11*(265), 1–14.

Introduction

Close to paper 4's aim, this study examined the effectivity of an easy-to-implement MMS combined with the racetrack method on developing math fact fluency. However, this research expanded its focus by a peer-tutoring approach. As mentioned before, the importance of achieving math fact fluency for students struggling to develop basic fact recall, as well as ways to foster math fact fluency, have already been considered within the other presented papers. Peer tutoring is a meaningful way to implement effective interventions under real-life conditions, as teachers still are challenged by implementing methods that provide sufficient time to engage students into practical activity while being able to respond and give immediate feedback. The peer-tutoring approach defines one student into the role of a teacher (trainer) and another one as tutee to be instructed by the trainer. Students can have a different or the same academic level and the approach can be implemented in class-wide or one-on-one settings as well as for different subjects (Delquardi et al., 1986; Dufrene et al., 2010; Fuchs et al., 1997; Greenwood et al., 1993; Lo & Cartledge, 2004; Mitchell, 2014; Utley et al., 1997). A major benefit for students—besides being actively engaged in academic learning (Alegre et al., 2019; Bowman-Perrott et al., 2013)—consists in the opportunity of a high frequency of immediate feedback regarding their performance (Bowmann-Perrott et al., 2013).

Therefore, paper 3's research question was to examine the effects of a peer-tutoring intervention combined with the use of racetracks, DI flashcards, explicit timing, positive reinforcement through self-scoring and immediate feedback on building math fact fluency for the single-digit addition tasks of four struggling elementary students.

Method

Similar as in paper 2, again an ABE multiple-baseline design across subjects was conducted (Tate et al., 2016) with a staggered, random start of intervention to increase internal validity. Treatment included 19 measurements with an average of three probes per week over a 10-week period.

Inclusion criteria for students to be trained as tutees were basic understanding of single-digit addition tasks (a standardized math test, the *Heidelberger Calculation test/ HRT1-4* [HRT 1-4] by Haffner et al. [2005] was conducted), less than 10% automated single-digit addition tasks, regular school attendance over the last six months, willingness to take part in the intervention and social capability of independently working with a partner without needing constant adult attention. Tutors were chosen on the basis of their results in the HRT1-4 with at least the 50th percentile as outcome and further according to the teachers' assessment as being socially capable of taking the responsibility for accompanying a fellow student. Tutees and tutors were assigned partners on the basis of their results in the pre-test and ruling out either a close friendship or a problematic relationship with one another. Tutees, whose results were the only ones evaluated in this study, were aged 7 to 8 years and visited an inclusive elementary school in a so-called family class together with students from the first through fourth grade.

As the dependent variable, the common intersection of 27 not-automated, single-digit addition tasks among the students was determined. For the measurement randomly allocated for each measurement, as well as randomization between the record sheets for each data point, was implied to improve internal validity. The independent variable was using racetracks combined with the MMS in a peer-tutoring setting.

Materials were close to the material described in paper 2, enhanced by the measurement being conducted via PowerPoint to ensure as accurately as possible that the answers were recorded within two seconds. At the beginning of the treatment, tutors and tutees received instruction with the procedure from the interventionist (intensively instructed graduate students—see reference for further information: paper 3 in Appendices). After that the interventionist modelled the onset of treatment, then the procedure was very close to that from paper 2.

After the last treatment session, the interventionists interviewed the four participants (the tutees) and completed the questionnaire concerning the treatment's social validity. Tutants were only asked orally if they enjoyed the intervention, but questionnaires were not handed out.

Results

A visual analysis was carried out describing the scores for each participant's correctly solved addition tasks. MBD (Campbell, 2003; O'Brien & Repp, 1990) was applied with an average varying from 222.5% to 627.5% and an average of 471%. NAP was calculated, being significant at the .01 level for all participants as well as Tau-U, which was significant at the .01 level in two cases, significant at the .05 level in one case and showed no significance in the last case (Lenz, 2013; Parker & Vannest, 2009; Parker et al., 2011b; Vannest & Ninci, 2015).

In accordance with Tate et al.'s (2016) standards for single-case research, a piecewise regression analysis on level 2 (across all participants) was conducted (Van den Noortgate & Onghena, 2008) to substantiate the aforementioned indices. A significant level effect from phase A to B on the .05 level as well as a slope effect on the .01 level was thereby evident.

Three of the four participants said to have enjoyed the training a lot, while one only participant liked it a little. Still, they all stated the math racetrack helped them to solve the tasks, they performed better through the intervention and they enjoyed both the immediate feedback and working with a partner.

Discussion

This paper aimed to examine the effects of a peer-tutoring intervention using math racetracks, DI flashcards, explicit timing, positive reinforcement and immediate feedback on the fact recall of non-automated, single-digit addition tasks of four struggling elementary students.

All previously presented results indicate that the intervention can be a helpful approach to help struggling students overcome hurdles through implementation by peers in the role of tutors. Therefore, results once again underline previous findings for both peer tutoring (Bowmann-Perrott et al., 2013; Ginsburg-Block et al., 2006; Madrid et al., 2007) and math racetracks (Irvin et al., 2007; Skarr et al., 2014) on students' increased performance.

Still, this research is unique due to combining both easy-to-implement methods into one approach combined with further motivational components, and in that sense, is substantial for teachers, as students increased their capacities significantly in a short time.

In regard to the most limiting aspects of this study, the lack of generalizability due to the small sample has to be noted again. This study should be considered as a prelude for further single-case research to be able to call the combination of the methods evidence-based according to standards, such as SCRIBE (Tate et al., 2016). Further, no data were collected concerning neither the exact influence of each of the two main methods, nor each motivating method's individual influence on students' performance. Subsequent research should focus on the individual effects of each of the MMS's aspects as well as the extent of the racetracks' influence compared to peer tutoring. Further, the aim to meet more standards for single-case research, such as SCRIBE (Tate et al., 2016), should be considered to be able to call the method evidence-based. More relevant than ever, a digital version of the racetrack method should be investigated because there is a great need for digital support services concerning math fluency interventions (Cozard & Riccomini, 2016) and the racetracks method could be a promising approach to be offered via a digital approach.

In sum, the combination of the MMS with racetracks and peer tutoring is encouraging for students to work together, easy to implement in everyday school life and is a helpful way to overcome hurdles. Methods such as these are necessary to relieve teachers' workload and support students' development successfully at the same time.

All papers presented to this point indicated strong effects over all participants to improve their mathematical performance. They all were able to increase their accuracy and speed and to extend the numbers of automated basic math facts and therefore also accuracy and speed to master basic math fact fluency.

Even if those performances were not pure mental work, as they needed to answer orally at least in papers 2 and 3, a next level of development for low-performing students could be to share their knowledge and performance in class. Still, participation in class can be challenging for struggling students, which is why the question arises of whether there is another easy-to-implement method to support students to engage into academic activity in class and not only in one-on-one settings, as seen in the papers 1 through 3.

7. Paper 4: Active participation in math lessons

"A ship in port is safe, but that's not what ships are for. Sail out to sea and do new things."
(Grace Hopper cited in Parinos, 2015, p. 5)

Participation and class attendance are major factors for students' learning outcomes, e.g., in quiz scores, exam performance or achieving higher grades (Clump et al., 2003; Corbin et al., 2010; Gump, 2010; Kupczynski et al., 2011; Launius, 1997). Effective participation in class can be defined as the highest engagement to academic activity added by students who are actively involved in the learning process (Christenson et al., 2012). Multi-dimensional motivating processes such as students' needs, expectations, beliefs and goals strengthen their engagement in classroom activities (Lee & Reeve, 2012). In addition, students who are eager to learn are more willing to participate in class, repeat knowledge, relate to existing knowledge and ask questions (Schunk, 2009). Participation further contributes to students' academic competence, achievement, socialization and life satisfaction (Bost & Riccomini, 2006; Harris, 2008; Lewis, 2010).

The importance for students to actively engage during class is a given because the cognitive event of learning requires an interactive process between the teacher sharing information with students and in addition ensure whether the students understood this knowledge (Parsons et al., 2014). To enable teachers to accompany students' learning during the teaching process, teachers are dependent on repetitive active participation from students' side. Therefore, the extent to which teachers are able to motivate students to engage in the teaching process is a vital issue to success.

One definition of students' participation involves different dimensions of student participation such as the social experience of engagement in a meaningful and shared activity in combination with talking, thinking, feeling and a sense of belonging (Rogoff et al., 2003; Wenger, 1998). Another definition is the classification of the dimensions as behavioral engagement (e.g., complying with behavioral norms such as attendance and involvement), emotional engagement (e.g., experiencing affective reactions such as interest or sense of belonging), and cognitive engagement (e.g., investing in learning such as going beyond requirements) (Skinner et al., 2009; Wang et al., 2014).

In the context of a classroom, there exists an influence in both directions: activities that are considered meaningful as well as becoming part of an activity (Thomas et al., 2012). When an approach for participation in class is implemented, it can be helpful for some students, but might exclude others, who would prefer different ways to participate. Therefore, multiple options should be offered to actively participate in class, thus enabling the group of students to widen their participation options (Dallimore et al., 2004). These could be different opportunities to respond (OtR), which offers practice and rehearsal to help students organize and store information, asking questions, verifying their understanding and therefore also monitoring their work, including correcting errors through offered corrective feedback for students with and without disabilities (Haydon et al., 2009; Sutherland et al., 2003). When offered teacher-directed instruction with frequent OtR, immediate corrective feedback with a predictable number of questions, which allows each student to answer the equal number of questions, enhances struggling students' academic capacities as well as behavioral response frequency and thereby helps students to stay focused during class (Cavanaugh et al., 1996; Christle & Schuster, 2003). Using response cards (RC) is an evidence-based method that is easily implemented in class and provides engagement to learners requiring an active role during instruction (Adamson & Lewis, 2017; Christle & Schuster, 2003; Gardner et al., 1994; Kellum et al. 2001; Marmolejo et al., 2004).

Müllerke, N., Duchaine, E. L., Gruenke, M. & Karnes, J. (2020). The effects of a response card instruction on the active participation in math lessons of five seventh graders with learning disabilities. *Insights into Learning Disabilities*, *16*(2), 107–120.

Introduction

This study aimed to examine the impact of a single RC intervention on students' engagement to participate during math classes. Thus, in contrast to the first article, this study was primarily concerned with participation in co-curricular math activities, and here, of course, primarily with low-performers, who in this case were adolescents in a seventh grade class with LD.

As academic learning is a cognitive event to be considered as an interactive process, it requires not only sharing teachers' information with their students, but also ensuring that students have understood and internalized this knowledge (Parsons et al., 2014). Therefore, students' active engagement is vital on the one hand, but on the other, there is evidence that students neither enter the classroom with the same knowledge nor with the same engagement in performance and active participation (Parsons et al., 2014). One secure and nearly errorless learning opportunity to provide students with the correct skills and contents is through interactive direct instruction, which provides a concrete introduction of information, linked to ongoing practice with immediate feedback (Brophy & Good, 1986; Engelmann, 2017; Engelmann & Carmin, 2016; Watkins, & Slocum, 2004).

Repetitive active participation is required, as is teachers' feedback, to make the learning process as profitable as possible. Various opportunities exist to demand all students' responses to questions (MacSuga-Gage & Simonsen, 2015; Menzies et al., 2017; Twyman & Heward, 2018). In contrast to oral answers, the RC approach uses visualized reactions on prepared cards with different answer options such as right/wrong, fact/opinion, multiple choice, numbers, etc. (Duchaine et al., 2011; Owiny et al., 2018). Therefore, the RC method is nearly limitless and not only provides an enormous variety of possibilities, but also is an easy-to-use tactic for teachers (Twyman & Heward, 2018).

Therefore, the purpose of this research was to replicate Christle and Schuster's (2003) study on the RC intervention as a means of responding simultaneously during direct (math) instruction.

Method

Similar to paper 1, a single-case multiple-baseline design (ABA) across participants was conducted (Horner et al., 2005) with a baseline phase (A1) without intervention and the treatment phase (B), where RC were implemented and a return-to-baseline phase (A2). Adding a second baseline phase allowed more reliable positing of a cause-and-effect relationship between the intervention and the improvement (Riley-Tillman & Burns, 2009).

The five participants of this study attended the seventh-grade of a school with special learning needs and were preselected based on the main teacher's observations concerning the frequency of hand raising to respond in math class over past weeks. All students had been diagnosed with an LD, were tested with an IQ below average (range 49 - 74) using the Kaufman Assessment Battery for Children (KABC-II; Kaufman & Kaufman, 2004) and were determined in first grade to fifth grade level concerning their mathematical competencies by using a standardized test (Moser Opitz et al., 2010).

As the dependent variable, the extent of active student participation in classroom activities was measured; therefore, an observation scale that documented how frequent participants raised their hands or held up their RC was used over 15 probes. Further quizzes were implemented to determine the increase of performance as a result of an increase in participation.

Results

Already visual analysis showed an impressive increase in the number of responses to the interventionists' questions (RtQ) during baseline, treatment and back-to-baseline phases. Similar to paper 1, the SCAN package for R by Wilbert and Lueke (2019) was used. All participants increased their number of hand raises during treatment phase and all of them returned to lower scores after treatment ended.

Comparing the RtQs from baseline to treatment phase, student 1 increased his performance about 211.76%, student 2 by 483.33%, student 3 increased from continuous 0 raises in baseline and an average of 13.60 RtQs during intervention, student 4 increased his performance by about 5.740 and student 5 by 3.600%. The decrease of all students in A2 (back-to-baseline condition) had a range from the least decrease (student 2) 64.14% to the highest decrease for student 4 with 94.52%. Still, all participants remained far above the values from the baseline.

Further, four of the most common non-overlap effect sizes comparing baseline phase with treatment phase were calculated: PND, PEM, PET and NAP (Alresheed et al., 2013). All participants reached the highest possible outcome in each of the effect sizes of 100%.

In addition, a piecewise regression analysis was applied to each participant (Huitema & McKean, 2000). For all participants, a significant level effect from A1 to B as well as phase B to A2 for all students was visible, with the exception of student 2, who shortly missed statistical significance with his performance (p=.07). On level 2 analysis (aggregating the cases into one), there were very clear level effects between the phases.

Finally, the quizzes were evaluated in terms of gains within participants' mathematical performance. Student 2 missed completing the quizzes, but the rest achieved all six. For all four students, the results lead to the assumption that their growth in learning was the highest while using the RC during class.

Discussion

This study aimed to examine the effects of an RC intervention based on principles of interactive direct instruction on the participation during math classes of five unengaged students from a seventh grade class who were with diagnosed an LD. As the results indicate, students' RtQs increased in an impressive manner with the beginning of implementing the RC. For all cases, all calculated non-overlap indices reached statistical significance with a maximum of 100%. The moment the treatment ended, performance dropped for all cases. Furthermore, students achieved a higher growth in learning during the treatment phase using the RC.

As meaningful as these results are, the small sample size and particular school subject taught during the lessons lower the results' generalizability. Preselecting the sample was only based on the main teachers' impression and no further clear-cut criteria were accessed. Further, the observers knew the study's aim and were not blind its purpose. In addition, the level of difficulty was tried to be kept equal across the performance quizzes and no testing to the extent that the goal of learning was achieved. Finally, the ABA design could have been extended for another B phase so as to underline the comparably distinction in the differences from the other phases. As the method of using RC was new, a possible habituation effect could have emerged with continuous changes of phases such as in an ABAB or even an ABABAB design.

Still, this study's practical implications clearly show that using this RC method during instructive learning opportunities increased students' participation substantially and led to augmented mathematical performance. All participants were interested in participating the moment they were offered this way of unison responding through the RC. Therefore, this method's outstanding strength is to not only motivate the low-performing students to engage into active participation, but at the same time being able to give the high-performing students their space to do the same.

Future research should address the high-performing students and RC's influence on their participation as well as their mathematical performance to allow for research on the investigation of the effect RC might have on their performance compared to the results on their effect on low-performing students.

The last paper of this dissertation focuses on the participation and active engagement in joining activity during class. While high-performing students are still able to perform using the RC method, the low-performing students are activated to share their knowledge actively at the same time. This does not only provide students more possibilities to engage in participating, but also gives the teacher an opportunity to observe all students' performance levels. The results of this research were impressive, which indicates that this method ranks among the top support options presented in this dissertation for these struggling students who are challenged in their learning process to successfully develop mathematical skills.

8. Overall conclusion and implications

The results of the last presented paper, which examined the effects of an RC intervention on the interactive direct instruction's principles on participation during math classes of five unengaged seventh graders diagnosed with LD, are in line with the previous papers' findings With the beginning of the intervention, students' RtQs increased in an impressive way, wherefore the findings all reached statistical significance with a maximum of 100%. Therefore, the results can be interpreted to be caused by an effective method to increase the participants' motivation to engage in the offered learning opportunities. This circumstance is in line with the previously presented studies' results.

Despite the conducted research's limited generalizability, all data collected showed that using motivational methods has positive effects on engagement in math performance immediately and shortly thereafter as well for all students who were participants in the four studies. Paper 1 in particular showed how students were able to increase their performance only by adding the implemented motivational methods.

As expected, the struggling students in paper 2 were able to at least interrupt or even overcome their demotivation and negative academic self-concept hurdles because of partly years of failure and maybe even averse feelings toward school (Gottfried & Kirksey, 2017). They became deeply engaged into a supposedly unattractive and boring process of automating still unknown math facts and afterwards performed better and even experienced self-efficacy through their progression. Paper 3 enhanced those findings by adding peer tutoring, which further augments students' motivation to engage into the content and perform as well as possible, while simultaneously relieving the teacher so as to have more time to support other struggling students.

All these findings support current researches' statements on the methods of positive reattribution, explicit timing, positive reinforcement through self-scoring and visualization of an individual's own performance in a line diagram, as well as a personal high score, immediate corrective feedback with or without combination with a game-based intervention such as DI flashcards or racetracks (at this point, I am referring to the previously mentioned research on the methods), while at the same time they create with an actual strength due to the simplicity of adaption to the mathematical conditions of each research sample concerning their individual learning content.

Outstandingly, paper 4's finding make it possible to even draw conclusions about an intervention under almost real school conditions because it was conducted within the whole class, which underlines the method's relevance for everyday school life. However, paper 3 also offers the possibility to implement the methods from papers 1 and 2 within a whole class, as it combines the motivational aspects with peer tutoring. Through this, the teacher receives capacities to use these methods within the whole class and still pursue individual learning goals with each student.

In summary, combining the motivating aspects with highly frequented repetition of unknown facts such as DI flashcards or racetrack provide active engagement into math class by using RC methods, which are all easy to implement. Each method individually or in combination with the others is an enjoyable way to foster struggling students from both the teachers' and students' perspectives. Effective methods such as these are necessary for today's learning environments in school because of their practical implementation and their possibility to easily adapt to individual needs.

8.1. Answering the research question

This dissertation was devoted to the question: What kind of feasible interventions are there that support struggling students' basic math competency development and can increase students' performance, including their motivation to engage in mathematical learning situations and participate in educational learning opportunities? This question was raised by presenting the results of four papers, which were based on four studies that build on each other and reflected an analysis including different methodological aspects and perspectives. Following the four papers' combined findings, an intervention should include the aspects of motivation to engage (even after repeated experience of failure) into a learning content, high frequency of repetition of unknown basic math tasks and opportunities to participate actively during class, all ideally combined. Interventions focusing on motivational methods as seen in papers 1 to 3, encourage students to use individual necessary prior knowledge needed to master their tasks. Therefore, the focus is to enhance students' enthusiasm even after having experienced negative experiences of failure. In addition, increasing performance that concerns automated basic math facts could enable struggling students to increase their academic self-efficacy and venture into more complex tasks in the future because the capacities for this are now available and are no longer needed for basic mathematical calculations.

As the findings of papers 1 to 3 show, participants improved their math fact recall of, until then, non-automated basic math facts through motivation and high frequency of repetition, including immediate corrective feedback to avoid incorrect memorizing. The motivating aspects of explicit timing, the racetracks' game-based method combined with DI flashcards, positive reinforcement through self-scoring and visualization of a personal high score, combined with the feedback, led to an increase in automated basic math facts for the participants.

Paper 1's findings showed the maximum value for each participant across all calculated effect sizes as well as statistical significance. Paper 2's results show improvement for all students' performance concerning the number of correctly solved single-digit multiplication tasks within two seconds. In paper 3, the scores of correctly solved addition tasks for each participant led to an MBD with a range from 222.5% to 627.5%. NAP and Tau-U were calculated and showed significance at the .01 level for all participants (NAP), significance at the .01 level (for two participants), significance at the .05 level (one participant) and no significance could be demonstrated in one case (Tau-U).The findings in paper 3 are very encouraging because peer-assisted learning is implemented. This leads to the opportunity to decrease teachers' workload, e.g., in giving immediate corrective feedback. In paper 4, participants' RtQs increased impressively immediately after the intervention with the RC began and dropped with the return to baseline conditions. All participants reached the highest possible outcome across all effect sizes. Four of the five participants' results showed statistical significance and the fifth participant missed it shortly. Along with this increase, students even achieved better performance in math classes during the intervention through using RC.

Further, this representative experience of being able to master a development through engagement in effort could be vital to stop an otherwise (probable) vicious circle of failing and a decrease of self-efficacy. For the students, experiencing success, which is contrary to what has been experienced so far, could thus be the kick-off for developing even intrinsic motivation to open up to future engagement in mathematical learning.

Therefore, by using those methods, struggling students with or without LD have the opportunity to improve their performance in automating basic math facts. This can be done through engaging deeply in a learning content, through the implemented motivating methods, resulting in positive learning experiences and connecting them to their engagement and mathematics to catch up and achieve proficiency.

As I already mentioned in the beginning of my thesis, my personal aim to do research in the scientific field of education and especially at the chair for which I do research with the title “Conception and Evaluation of School Support with Focus on Learning” is motivated by my experiences as a teacher. The ambition to make school life and learning more efficient in an attractive way for students and teachers is my scientific driving force. My studies’ results described in this dissertation represent an important contribution to this personal and professional aim.

8.2. Implications and future research

Obviously, there is no direct link in the presented papers between the motivating or game-based methods and the RC. Still, both methods could be used supplementary in class, even if this was not part of one of the presented studies. Future research should examine the effects of an overall combination on students’ performance as well.

Further, none of the studies used digital offers such as a digital version of the racetrack game, which does not exist yet, or some digital method to implement RC, e.g., on the platform www.plickers.com. The methods presented in of this dissertation are effective to support students in learning, memorizing and quickly retrieving basic math facts on an individual level. Still, the periodic and precise diagnosis of the known and unknown facts, as well as preparing the material tailored to each student, could be quite complex. To increase the implacability of those on their own easy-to-implement methods to fit everyday classes, a digital version supporting both mentioned steps could be valuable for teachers. Same applies for immediate feedback with self-control and positive reinforcement. Therefore, future research could also conduct a study regarding the effects of a computer-based intervention with these elements.

As all studies focused on students’ measurable benefits concerning their performance, future research should also try to point out the benefits for teachers using those methods in class, as research finds them easy to implement, but there is no data that underline that assumption. To examine their impressions of the implacability and benefits for themselves as well as their students, it would be advisable to gather teacher feedback in the form of an interview or questionnaire.

The multicomponent-motivational system consists of each of the implemented methods as evidence-based, which in the conducted research, were further increasing students' performance in combination. The next step of research should be to examine which of the methods, if used in a combined manner, have what kind of exact effect on performance.

In sum, the aim of future research should certainly be to identify further methods that are helpful in increasing students' motivation to engage in participation in class and increasing the automating basic math facts. Additionally, all elements are vital for further mathematical engagement and development, especially when students already have experienced failure.

Regardless of the papers presented in this dissertation, there is a deficit when it comes to anchoring scientific findings such as these in a teacher's school day. Methods such as RC, DI flashcards, racetracks, peer tutoring, explicit timing, immediate corrective feedback, adaptive reattribution, positive reinforcement through self-scoring and visualization of a personal high score still need to find their way into most classrooms as regular tools. A concern of the utmost importance for future research should be how this gap between scientific findings and implementing them in school life could be filled because students mastering their mathematical development adequately is vital for not only students of all school types, but also worldwide to enable active participation in today's society and therefore to improve students' chances in the labor market later on in life.

To summarize this dissertation in its entirety on a personal and professional scientific level, I end with a quote from Margaret Hamilton, which applies to me and my work as much as to the content I have presented here, the effort of teachers and that of students:

"It is always great when people take interest in your work."

8. References

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Appendix

Appendix A: Paper 1 (*peer reviewed*)

Grünke, M., Karnes, J., & Hisgen, S. (2019). The effects of a multicomponent motivational intervention on math performance of elementary school students with learning disabilities. *Insights into Learning Disabilities*, *16*(1), 23–35.

Abstract

The purpose of this single case study was to evaluate the effects of a multi-component motivational intervention that consisted of explicit timing, immediate feedback through self-scoring, goal setting, and positive reinforcement on the arithmetical skills of three third graders with learning disabilities. An ABA reversal multiple-baseline across-participants design was applied to establish a functional relationship between the intervention and the expected outcome. Immediately after the motivational system was implemented, the participants solved a higher number of tasks. The effect ended abruptly once the treatment was terminated. Results show that even for struggling students with learning disabilities, motivation to solve math tasks can be notably increased with relatively little effort. The paper ends with a discussion of current literature and the experiment's limitations, as well as the practical use of the findings.

Keywords: nonverbal learning disabilities in written language, multicomponent motivational system, explicit timing, self-scoring, positive reinforcement

Introduction

Basic mathematical computational competences are absolutely essential for many areas of children's current and future life (Casey, McLaughlin, Weber, & Everson, 2003; Haring, Lovitt, Eaton, & Hansen, 1978; Lloyd, 1978). Capabilities to manage time properly, to handle financial matters, or to purchase daily goods are just a few examples of how basic mathematical skills are used in one's daily routine (Brown & Snell, 2000).

Sufficient arithmetic fluency is an essential, yet not sufficient, prerequisite for being able to solve complex mathematical problems. One must be in a position to retrieve math facts quickly and effortlessly to attend to more sophisticated tasks. Otherwise, one's working memory might be overstrained and not in the place to meet the requirements posed by a particular intricate problem. The risk of getting stuck in the use of counting strategies and the overall failure frequency increase for children who lack these capacities (Miller & Heward, 1992; Pieper, 1983). Finger-counting strategies are inadequate when multiplication and division tasks or even more complex math problems have to be solved (Casey et al., 2003;

Silbert, Carnine, & Stein, 1990; Stein, Silbert, & Carnine, 1997). This emphasizes the relevance of promoting arithmetic facts in struggling children.

Regardless of the considerable variance in the numerical development in children, most of them arrive at the same “place” academically by the end of their elementary education and possess sufficient math skills to successfully tackle formidable word problems. However, a considerable share of up to a quarter do not acquire basic arithmetic competencies before moving on to secondary school (Salend, 1998; Wendt et al., 2016; Wood, 1992).

Students with a learning disability are especially at risk for developing severe math difficulties (Goldman, Pellegrino, & Mertz, 1988). They usually experience serious problems understanding number-related concepts or using symbols or functions needed for calculating (Swanson, Olide, & Kong, 2018). Fortunately, there are a number of effective interventions that can help these children and adolescents to meaningfully improve in their academic abilities (Dennis, Sharp, & Chovanes, 2016; Jitendra, Lein, & Im, 2018; Stevens, Rodgers, & Powell, 2018). Approaches aimed at enhancing math fact fluency involve daily drills and practice (Anthony, Rinaldi, Hern, & McLaughlin, 1997), flash cards (Ashbaugh & McLaughlin, 1997), Say All Facts One Minute Each Day Shuffled (Eshleman, 1985; McDade, Austin, & Olander, 1985), and daily charting (Abba & McLaughlin, 1995; Casey et al., 2003; Lindsley, 1991; West, Young, & Spooner, 1990).

One technique that has received little attention in recent years, but appears to be very promising in this context, is called explicit timing (Van Houten & Thompson, 1976). It is a procedure that alerts students to a time limit while they are completing an assignment. The mere fact that learners are made aware of how long they have been working on a task and how long there is still to go seems to help them focus on a given challenge and perform better (Grays, Rhymer, & Swartzmiller, 2017). Explicit timing is often combined with immediate feedback through self-scoring (Gross & Duhon, 2013), goal setting (Codding, Lewandowski, & Eckert, 2005), and operant conditioning (Freeland & Noell, 1999). Using these techniques conjointly appears to accelerate the effects of explicit timing. When applying immediate feedback through self-scoring, students keep track of their performance at the end of each learning session (Light, McKeachie, & Lin, 1988). They set a goal to not fade next time but to beat their own high score. Oftentimes, the respective high score is prominently displayed on a poster on the wall of the classroom or on the front of a student’s workbook. The teacher acknowledges the children’s effort and accomplishments by providing rewards in the form of vouchers, toys, sweets, and the like (Archer & Hughes, 2011; Prater, 2018).

Explicit timing in combination with immediate feedback through self- scoring, goal setting, displaying high scores, and operant conditioning has been shown to be effective in a variety of different contexts, such as reading (e.g., Grünke, Karnes, & Hisgen, 2019; McDaniel, Jolivette & Ennis, 2013) and writing instruction (e.g., Grünke, Knaak, & Hisgen, 2018; Grünke, Sperling, & Burke, 2017). However, such a multicomponent motivational system has not been evaluated very often in connection with enhancing students' math fact fluency. A number of researchers have reviewed the benefits of explicit timing on arithmetic skills in isolation (e.g., Duhon, House, Hastings, Poncy, & Solomon, 2015; Grays et al., 2017; Rhymer, Henington, Skinner, & Looby, 1999), but not in connection with accompanying approaches aimed at additionally boosting motivation and performance.

Thus, the purpose of the present study was to examine the efficacy of a multicomponent motivational system consisting of explicit timing, immediate feedback through self-scoring, goal setting and displaying high scores, and positive reinforcement on the math fact fluency of learning-disabled third graders. We expected that the intervention would elicit an immediate boost in performance, which would disappear as soon as the treatment ended.

Method

Participants and Setting

Participants included three third graders (Anna, Ben, and Colin; names changed for anonymity) from an inclusive elementary school in a major city in Northrhine-Westfalia (Germany). All of them were referred to the first author by their main teacher due to their low motivation to engage in math activities. The school and the three authors of this paper have been cooperating for a number of years, trying to jointly find ways to better support especially challenging students to achieve basic reading, writing, and math proficiency by the end of their elementary education. To be eligible for this experiment, children had to understand the concepts of two-digit addition and two-digit subtraction but perform within the last 20% of their age bracket in a standardized math test. Furthermore, despite their low math motivation, they had to be willing to take part in the study.

To select suitable participants for this experiment, we conducted the subtests "Addition" and "Subtraction" of the Heidelberg Math Test 1–4 (HRT 1–4) by Haffner, Baro, Parzer, and Resch (2005) with the whole class and asked each student to finish a DIN-A-4 worksheet containing 10 two-digit addition and 10 two-digit subtraction problems, taken from Klauer (1994). It was arranged in a way that addition and subtraction items alternated. The children were granted as much time as they needed to complete the worksheet. We considered the prerequisite for basic comprehension of addition and subtraction concepts to be fulfilled if the students were able to solve at least 80% of the problems.

One girl and two boys were identified as eligible for the study. All of them had been diagnosed with a learning disability by a specialist. The first participant was 10-year-old Anna. She was born to German parents and spoke German at home. According to her main teacher, she was generally eager to learn but had developed a very negative math self-concept. Anna had trouble working on basic arithmetic problems for longer than 5 minutes. She usually became tired and halfhearted after a very short time, subsequently engaging in daydreaming.

According to the HRT-4, Anna achieved a percentile of 4 in her ability to correctly solve addition problems, as well as in her ability to solve sub- traction problems. However, she ended up with 85% correctly solved items on the worksheet. Nine-year-old Ben (male) was the second participant. Like Anna, Ben did not have an immigrant background. His main teacher characterized him as a boy who frequently needed breaks due to his short attention span. Ben's percentile in the HRT-4 addition subtest was 17, and his subtraction percentile was 4. He performed well on the worksheet, solving 87% of problems correctly. The third and last participant was 10-year-old Colin. His teacher reported that he stood out due to his tendency to get distracted. She ranked his potential far higher than his actual performance. Colin achieved a percentile of 2 in the HRT-4 addition subtest and 1 in the subtraction subtest. However, he was able to solve all items on the worksheet correctly.

Interventionist

A female graduate student of special education executed the study together with the treatment. Because of her side job as a private tutor and her practical school training as part of several internships, she was used to working with low-performing elementary school children. During four 45-minute meetings, the interventionist received extensive briefings by the first author on how to conduct the experiment. We used a checklist that contained every central feature of the assessment and the treatment that she was supposed to adhere to. The interventionist and the first author stayed in contact via e-mail each week of the experiment to make sure that everything went according to plan.

Dependent Variables and Measurement

We used the number of correctly solved math items (SMIs) on 15 different worksheets as the dependent variable. These materials were arranged like the ones we used to appraise the comprehension of basic addition and subtraction concepts (see above). Each worksheet contained 20 two-digit addition and 20 two-digit subtraction problems. Again, the items were taken from Klauer (1994), and the types of tasks alternated. The time limit was set to 10 minutes. On each day of the study, the participants were handed one of the 15 sheets in random order. However, it was ensured that they were never given the same set of items twice.

Experimental Design and Procedures

An ABA plan was implemented to evaluate the effects of the intervention with 15 daily probes. In this design, a baseline period (A1) is followed by a treatment phase (B). To test if the effects return to the baseline without intervention, the treatment is then withdrawn (A2; Riley-Tillman & Burns, 2009). To increase the internal validity of the study (see Dugard, File, & Todman, 2012; Tate et al., 2016), the beginning and the end of the intervention were determined randomly for every case within the constraint that each phase had to consist of at least three measurements. Thus, the B phase could have started any time between the 4th and the 9th and ended anywhere between the 7th and 12th probe. A random drawing of all possible options for each participant resulted in an arrangement whereby the treatment began for Anna after the 5th and ceased after the 10th; for Ben, it started after the 3rd and finished after the 8th; and for Colin, it launched after the 6th and concluded after the 11th measurement.

During baseline conditions (A1 and A2), the children were individually taken out of their class during the second period each day to a resource room of the school and seated at a table. The order in which they accompanied the interventionist varied. Once the participants had settled, the graduate student asked them to work on the math problems and to try as hard as they could to achieve the best results they were capable of. She measured the time coveredly with a wrist watch and asked them to stop as soon as 10 minutes were up. At the end of each session, Anna, Ben, and Colin filed their worksheets in a plastic folder. No feedback was given whatsoever.

During the B phase, the interventionist placed a 7x7-inch timer on the table for the children to see. She explained to them that they had exactly 10 minutes to work on the problems as quickly and assiduously as possible. In addition, she presented them with a line diagram, depicting their performance during all previous baseline and intervention sessions. The interventionist pointed out that the children had already delivered respectable results and encouraged them to keep trying hard. Each folder had a cellophane window on the front cover. With the first B-phase session, an index card with the hitherto existing SMI high score was placed in this window for the participants to see. After the interventionist introduced the timer, the line diagrams, and the index card, she set the timer to 10 minutes and the children worked on the math problems until the time was up. Subsequently, the student, with the assistance of the interventionist, counted the number of SMIs and recorded it on the line diagram. If the high score was beaten, the card in the cellophane window was replaced. Finally, the worksheet was filed in the folder. Every time the children reached at least their previous high scores, the interventionist rewarded them with a sticker.

Results

Figure 1 shows the number of math problems correctly solved by each participant in 10 minutes.

Anna was in the baseline condition (A1) for 5 days and averaged 33.80 SMIs (range = 31–37). The measurements during this phase can be considered relatively stable. The intervention (B) was introduced on the sixth day, coinciding with an immediate achievement gain. In fact, Anna's mean SMI improved by 38.46% to 46.80 (range = 44–51). On Days 7 and 8, she reached 45 SMIs each time. Apart from that, each subsequent score in the B phase always exceeded the previous one. The return to baseline (A2) led to an immediate change in level:

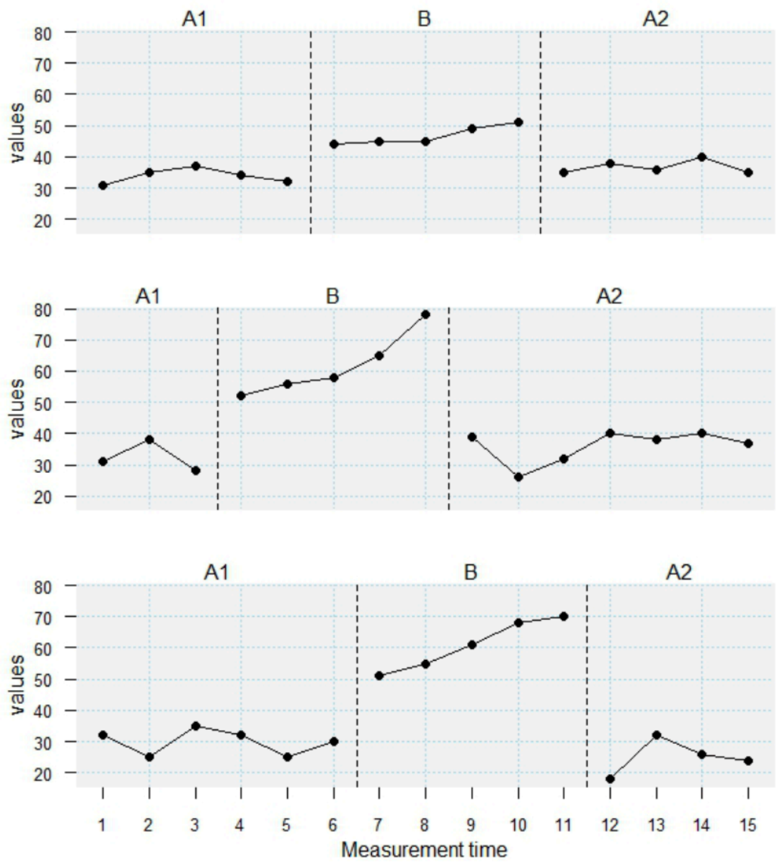

Figure 1. **Number of SMIs for Anna, Ben, and Colin in the three phases.**

The average SMI subsided by 21.37% to 36.80 (range = 35–40). Calculating five of the most common non-overlap effect sizes comparing phases A1 and A2 to phase B—percentage of non-overlapping data (PND), percentage of data exceeding the median (PEM), percentage of data exceeding the median trend (PEM-T), non-overlap of all pairs (NAP), and percentage of all non-overlapping data (PAND; Alresheed, Hott, & Bano, 2013)—resulted every time in the highest outcome of 100%.

Ben was in the baseline condition for only three measurements and averaged 32.33 SMIs (28–38). With the start of the intervention, his performance rose from 28 on Day 3 to 52 on Day 4. He continuously improved during the B phase until he reached 78 on Day 8. His mean value for SMIs during the intervention equaled 61.80 (range = 52–78), which corresponded with a 91.15% increase. After the treatment was suspended, his average achievement dropped by 41.75% to 36.00 (range = 26–40). Like in the case of Anna, Ben’s non-overlap effect sizes (PND, PEM, PEM-T, NAP, and PAND) all equaled the maximum value.

During six days of the baseline condition, Colin averaged 29.83 (range = 25–35) with relatively stable data. Introduction of the intervention was ac- companied by a performance gain in SMIs from 30 on Day 6 to 51 on Day 7. The mean value for the measurements during the B phase was 61.00 (range = 51–70), which parallels an impressive 104.49% increase. Again, Colin demonstrated a consistent boost in SMIs over the course of the treatment. The grave decline in output between his last measurement during phase B (70) and his first one in phase A2 (18) is remarkable. His average performance after his return to baseline conditions equaled 25.00 (18–32), which reflects a 59.02% decrease. All effect sizes (PND, PEM, PEM-T, NAP, and PAND) reached a peak outcome of 100%.

To analyze the conjoint effect of all cases, we conducted a randomization test for ABA multiple baseline designs (Dugard et al., 2012) using the SCAN package for R by Wilbert (2018). This statistical technique is robust against serial dependent data and provides probability values for generalizing the results (Grünke, Boon, & Burke, 2015). The randomization test was set up in accordance with the design, allowing for at least three measurements per phase. As expected, the mean differences between the phases did reach statistical significance ($p < .001$).

Finally, we carried out a piecewise regression analysis for each participant (see Huitema & McKean, 2000), again applying the SCAN package by Wilbert (2018). The results are depicted in Table 1.

Table 1. ***Piecewise Regression Model for Number of SMIs***

	b	*SE*	*t*	*p*	R^2
	Anna				
Intercept	33.50	2.36	14.21	>.01**	
Trend	0.10	0.71	0.14	0.89	0.00
Level Phase B	7.40	2.93	2.52	.03*	0.06
Level Phase A2	-14.20	2.93	-4.84	>.01**	0.22
Slope B	1.70	1.01	1.69	0.13	0.03
Slope A2	-1.60	1.01	-1.59	0.15	0.02
	Ben				
Intercept	35.33	7.67	4.61	>.01**	
Trend	-1.50	3.55	-0.42	.68	0.00
Level Phase B	12.67	6.98	1.81	.10	0.03
Level Phase A2	-42.00	5.76	-7.30	>.01**	0.43
Slope Phase B	7.60	3.89	1.95	.08	0.03
Slope Phase A2	-5.10	1.85	-2.76	.02*	0.06
	Colin				
Intercept	31.13	4.15	7.50	>.01**	
Trend	-0.37	1.07	-0.35	.74	0.00
Level Phase B	16.80	5.69	2.95	.02*	0.04
Level Phase A2	-49.20	6.47	-7.61	>.01**	0.28
Slope Phase B	5.47	1.77	3.09	>.01**	0.05
Slope Phase A2	-3.90	2.44	-1.60	.15	0.01

Note: * Significant at the 5% level; ** significant at the 1% level.

As the findings indicate, Anna and Colin demonstrated a significant level effect from phase A1 to phase B. However, only in the case of Colin did the changes in slope fall below a *p*-level of 5% upon the onset of the intervention. As the treatment came to a halt, all three children showed a significant drop in level. However, comparing phases B and A2, only Ben’s slope turned out to be different, with a probability of less than 5%.

Discussion

Main Findings

In the present study, we examined the effects of a multicomponent motivational intervention, consisting of explicit timing, immediate feedback through self-scoring, goal setting (and displaying high scores), and operant conditioning on the math fact fluency of three third graders with learning disabilities. The results show that the number of SMIs was greatly increased by the treatment. Visual inspection, effect size indices, a randomization test, and piecewise regression analyses all suggest that the intervention was very effective in increasing participants’ performance. The data indicate that the math fact fluency of learning-disabled third graders can be significantly improved even by very simple means. In the A phases, achievement was considerably lower for all students than during the B phase. From this, it can be concluded that the target behavior was not transferred to situations in which the intervention was not implemented. Overall, the results of the present study confirm the findings from the previous research works cited above.

Critical Reflections

Despite these positive results, some limitations of the experiment need to be considered. As with any single-case analysis, the findings cannot be generalized due to the small sample size. In addition, it is critical to note that only a relatively short period of time was available for executing the study. The internal validity of the single-case analysis could have been increased by including a larger number of measurement times. A second B phase would have served the same purpose. Nonetheless, an ABA reversal design, as used in the present study, is already considered very meaningful when trying to quarry valid findings (Riley-Tillman & Bruns, 2009).

Another point of criticism is that in this work, only scarce information was provided about the participants. No details were given on the individual backgrounds of the students (such as a description of their previous school career or their IQ). This lack of specific information makes replicating the study difficult.

In addition, the results could only be determined based on the effects of the combination of different motivational methods. To what extent which element of the approach was responsible for the treatment effects cannot be specified. However, this would have been necessary to be able to appraise the benefits of the different components of the motivational system on which this study focused.

It should also be noted that setting a time limit is not necessarily helpful with all kinds of tasks and all kinds of students. What might be useful in quickly retrieving math fact fluency might not be at all advantageous when solving elaborate word problems. Furthermore, some learners might not take too well to time limits. It could elicit performance pressure, causing stress, uptightness, or even anxiety (Rhymer et al., 1999). Thus, it might be appropriate to provide high-strung or jumpy students with interventions other than explicit timing.

Practical Implications

Despite the described limitations, the present study provides valuable insights into how learning-disabled students with problems in retrieving basic math facts quickly can be easily motivated to engage in arithmetic problems and how fluency in this respect can be increased. The results show that a multicomponent motivational intervention has the potential to have a tremendous positive impact on the performance of elementary school children and their willingness to get involved in tasks that they previously avoided.

Girls and boys with learning disabilities frequently demonstrate severe difficulties in mathematics. It is often extremely challenging for them to live up to even very basic expectations in this area. Initial problems accumulate over time and lead to regular experiences of failure, a negative academic self-concept, and feelings of aversion toward school in general (Gottfried & Kirksey, 2017). To interrupt this negative spiral, it is important to enable the affected students to experience self-efficacy. The approach described in this study offers a chance to do just that. Through the presentation of each solved task, the children are shown how much they have already accomplished and how much they have improved. Displaying the high scores and providing frequent praise contribute to students' beliefs of having successfully mastered ambitious tasks. Of course, executing a simple intervention for a couple of days is not enough to make up for many experiences of failure that some students might have accumulated. Nevertheless, the approach can be considered a serviceable means to reduce fear of failure and build motivation.

Notwithstanding the smallness of the contribution that this study is able to make to the body of research in the field of fostering basic math skills, it has demonstrated that it is undoubtedly possible to enhance arithmetic performance with rather plain means in a very effective way. All techniques applied in this experiment—explicit timing, immediate feedback through self-scoring, goal setting, and operant conditioning—are extremely simple to implement and require very little time and effort. In addition, the process is so frugal that it does not require in-service training for teachers before they can incorporate the motivational system into their daily teaching routine. Thus, the intervention can be considered very user friendly under conditions of everyday life at school. Especially in view of increased heterogeneity among students in an era of inclusion, this aspect is of particular importance. The majority of teachers simply do not have the ways and means to constantly attend to the needs of every child. Hence, simple techniques like the multicomponent motivational system described in this paper are needed more than ever.

Conclusions

The present single-case experiment has shown that basic arithmetic fact fluency can be easily enhanced with very little means. Insights such as the ones based on the findings of this study have the potential to prepare learning-disabled or otherwise struggling students for the demands they face during math instruction. This could contribute to combatting the problem that many children leave elementary school without meeting fundamental numeracy skills. Equipping them with solid abilities to perform basic math operations would certainly help them when they have to successfully master the transition from elementary to secondary school. Of course, there are still some significant blind spots in research concerning the efficacy of multicomponent motivational systems to promote numeracy skills in children. Nonetheless, this study can be seen as at least a small contribution to shed more light on the benefits that an intervention like the one described in this paper can have on learning-disabled or otherwise low-performing children.

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Appendix B: Paper 2 (*peer reviewed*)

Karnes, J., Barwasser, A. & Grünke, M. (2021). The effects of a math racetracks instruction on the single-digit multiplication facts fluency of four struggling elementary school students. *Insights into Learning Disabilities*, *18*(1), 1–25.

Abstract

There is a significant need for easy-to-implement interventions in the early grades, especially in mathematics, as many children have difficulties in their first years of school with automated basic math facts recall and, therefore, fall behind. Automation of single-digit multiplication tasks represents an important developmental step, as it can neither be accomplished by finger counting nor can more complex operations be mastered without it. Previous research has supported math racetracks as an effective intervention for increasing early math skills of elementary school students. The present study sought to replicate previous findings on the positive effect of a racetrack intervention extended by a multicomponent motivational treatment. Further, we tried to closely comply with the single-case reporting guidelines by Tate et al. (2012) for the purpose of establishing racetracks as evidence-based treatment and continue the work of Lund et al. (2012), Walker et al. (2012), Skarr et al. (2014), and Rivera et al. (2014). Four female elementary-school third graders who faced problems with basic math facts received 9-10 individual training sessions over 7 weeks. Visual analysis of results indicates a level increase in multiplication facts solved correctly from baseline to intervention for all four participants, with an average MBD of 410.45 (range 136.45 to 833.33). Limitations and future directions are discussed.

Keywords: Automation, Basic Math Facts Fluency, Learning Difficulties, Math Racetracks, DI Flashcards

Introduction

Significance of Numeracy Skills

Mathematical abilities are vital for students to be successful in school, to handle many daily tasks (e.g., reading the time or managing allowances), and later to be able to find employment (Brown & Snell, 2000; Cihak & Foust, 2008; Watts et al., 2014). Therefore, gaining proficiency in skills like early numeracy or addition and subtraction as well as multiplication and division is critical for achieving the benchmarks for mathematical competence at all grade levels (Mullis et al., 2016; National Mathematics Advisory Panel, 2008; Organization for Economic Co-Operation and Development [OECD], 2014).

Acquisition of elementary skills in this area is a reliable predictor of further mathematical development (e.g., Duncan et al., 2007; Watts et al., 2014). However, as the complexity of mathematical problems in future grades increases, students who do not master basic skills concerning numeracy, addition, subtraction, multiplication, and division have more difficulty mastering those future mathematical concepts and, therefore, are at higher risk of school failure with respect to mathematics (Cai, 2007).

Multiplication Fact Fluency

Fluency in multiplication, including automation and generalization, plays a decisive role in mathematical development as it is the first operation in the higher number space that requires the use of strategies beyond finger counting (Burns et al., 2015; Pólya, 2002; Stein et al., 2006). Grasping the concept of this operation by understanding and interpreting, for example, the symbols 4 x 5 to mean four copies of five, children need time to learn basic facts and automate the basic single-digit multiplication facts recall (Park & Nunes, 2001; Stein et al., 2006; Thornton, 1989). As a fluent arithmetician, a student solves faster by automated recall than by calculating the mathematical fact and using a mental algorithm to find the solution to the task (Lerner, 2003; Logan et al., 1996). Stein et al. (2006) set a benchmark of less than two seconds for an answer to be considered automated. Automation of basic facts recall relieves the burden on working memory with its very limited capacities, thereby allowing the student to concentrate on other components, such as subtasks, story problems, and accuracy (Geary, 2007; Stein et al., 2006; Stood & Jitendra, 2007).

Frequency-Building Procedures

To increase the time students need to complete fundamental multiplication tasks, it is necessary to practice individual items, provide drill and practice besides frequency-building procedures that combine timed repetition with feedback as learning opportunities. Both drill and practice and frequency-building are well-grounded and highly effective strategies (Burns, 2005; De Visscher et al., 2018; Greene et al., 2018). Supporting struggling students in their mathematical development seems to be most successfully performed in one-on-one settings, with a significant amount of effective learning time using non-curriculum-based methods that focus on the current achievement level of the individual student without reference to the competencies the child is expected already to have achieved according to the curriculum (Stevens et al., 2017).

In adding to strategy training, peer tutoring, and schema-based learning, direct instruction and intense drill and practice are evidence-based tools that individually, and combined, have proven particularly helpful and necessary for successfully teaching struggling student's basic skills and improving their automation processes (Boon et al., 2019; Butler et al., 2001; Stevens et al., 2017).

Use of Motivational Methods to Increase Student Engagement

Motivational factors influence learning processes and can increase or decrease students' performance. Specifically, in the field of mathematics, many children show signs of insecurity concerning math exams or even math courses in their everyday school life (Devine et al., 2018; Hattie, 2009; OECD, 2013). If students experience failure, their motivation to deeply engage with learning decreases, whereas success increases their motivation to engage with further content (e.g., Duhon et al., 2015; Hattie, 2009; Jaffe, 2020). Thus, for students experiencing failure in mathematics, additional motivational factors might help to turn around the natural tendency for motivation to decrease.

Direct instruction (DI) flashcards, positive reinforcement by using students' high scores in addition to visual representation of results in a line diagram and immediate feedback through self-scoring are all well-grounded methods found in current research in different combinations. Combining all those methods into a multicomponent motivational system holds particular promise for increasing the motivation of otherwise struggling students (Jaspers et al., 2017; Montague, 2010; Prater, 2018).

Direct Instruction Flashcards

DI flashcards consist of a predetermined set of targets, such as basic math facts, in a flashcard format. If the student can provide an answer within two seconds, the card is considered processed and given to the student. If not, the card is placed at the third place in the pile of cards so the incorrectly answered question can be presented again quickly (e.g. Hopewell et al., 2011; Skarr et al., 2014).

Immediate Feedback and Positive Reinforcement

Immediate feedback through self-scoring combined with positive reinforcement using high scores and entering scores into a line diagram also are effective ways to improve students' motivation and, therefore, help them tackle learning emotional or subject-specific challenging contents (e.g., Duhon et al., 2015; Eckert et al., 2006; Grays et al., 2017; Van Houten et al., 1974; Wells et al., 2017). This technique enables students themselves to monitor whether a given answer is correct or incorrect and, therefore, to correct errors. When using personal high scores, students compare their personal results with those they achieved the last time they trained. That is, they battle against their own previous results, which decreases anxiety around failure and increases their motivation to perform even better (Duhon et al., 2015). Finally, entering their personal scores into a line diagram shows students visually their individual performance and increases in chronical order, and is easy to interpret by students as well as teachers.

Racetracks

All the above-mentioned methods are designed to increase students' motivation to deeply engage with individually challenging learning. To implement them in a playful way, the use of the racetrack method has been documented repeatedly as being enjoyable to students as well as a highly effective direct instruction drill-and-practice and frequency-building method for a variety of mathematical skills (e.g., Beveridge et al., 2005; Erbey et al., 2011; Lund et al., 2012; Pfaff et al., 2013; Skarr et al., 2014; Standish et al., 2012; Walker et al., 2012). In addition to increases in acquisition of the content to be learned, studies detected positive results across settings and participants (i.e., students of varying ages, with and without learning disabilities, attention deficit hyper- activity disorder, emotional and behavioral disorders). Of particular interest to the current study, the research around automation of single-digit multiplication tasks by Lund et al. (2012), Rivera et al. (2014), Skarr et al. (2014), and Walker et al. (2012) focused on automation of multiplication tasks by using racetracks combined with DI flashcards.

Skarr et al. (2014), for example, added DI flashcards to math racetracks for three struggling elementary students, using a single-subject multiple-baseline design across three sets of unknown multiplication facts. During baseline, the students received 15 math facts in random order for measurement. Afterwards, they were presented with the racetrack and the DI flashcards with known math facts to get used to the procedure. On the flashcards 6-7 unmastered and 7-8 mastered math facts were printed. Shown a task, the student had to answer within two seconds by saying the entire statement. If the answer was wrong, the interventionist modeled the correct answer, the student repeated it, and the card was put back in the pile, but three cards back for fast repetition. In addition, the interventionist motivated students by telling them to do their best and praising them if they were able to answer correctly within two seconds.

On the racetrack game board, 28 cells were divided into 14 mastered math facts and 5-7 unknown math facts, which were at least repeated twice on the board. Baseline lasted for 3 to 20 measurement points for a total of 23 days of measurement. According to Skarr et al. (2014), their results clearly indicate the relationship between both applied methods and the mastery of the before unknown math facts.

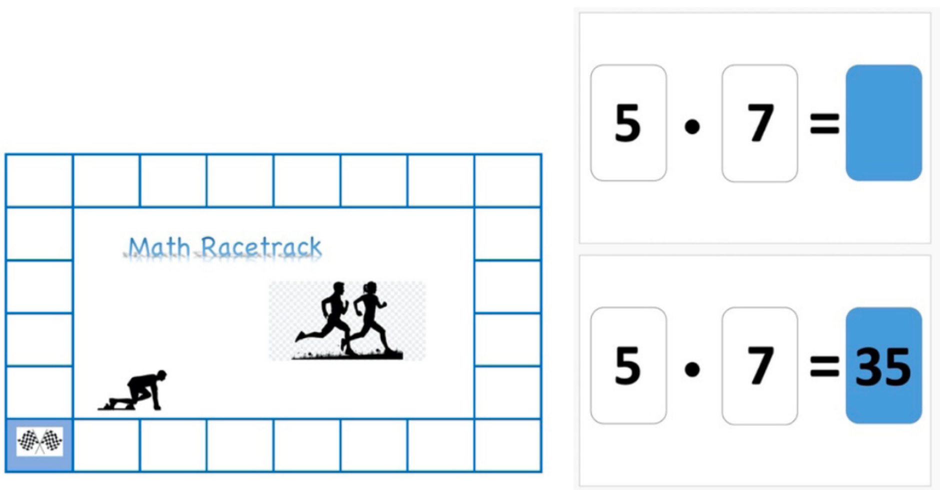

Figure 1. ***Racetrack Game Board and DI Flashcard Example Showing Front and Back***

Using the racetrack method requires a game board that is designed to look like a circuit (e.g., known from Formula 1) with a predefined number of cells. There are many ways to implement a racetrack, such as with a die, by moving from cell to cell, or by using a stopwatch to measure the time needed to complete the racetrack. To play a racetrack, a list of individually unknown math problems or words are written on cards, and each card is placed in one of the cells on the game board. If the student can provide an answer to the card presented within two seconds, then he or she can move on to the next cell or roll the die again, depending on the arranged rules. If the student struggles to answer, corrective feedback is given, and the card is placed back in the cell for the content to be repeated on the next round.

Purpose of the Present Research

Although all the aforementioned publications are single-case experiments reporting medium to large effects and therefore seem to provide a well- grounded technique to foster struggling students, especially concerning their replicability and therefore the methodologically description. Referring to Mulcahy et al. (2016), most research concerning the effects of interventions fostering mathematical skills does not satisfy even the basic standards. This reported lack of high-quality interventions is to be considered as well with previous DI and racetrack research.

The Single Case Reporting Guideline in Behavioral Interventions (SCRIBE; Tate et al., 2016), a common research tool to improve the quality of single-case designs, specifies 26 items that should be considered in a single-case study while planning, preparing a manuscript, and reporting results. Items are clustered into the following sections: title and abstract, introduction, method, results, discussion, and documentation. As the centerpiece of research is planning and conducting the methods in detail, while being embedded later into a detailed introduction and a critical interpretation of the results, the reported standards in the method section will be described here in detail, too. To the above cluster, Tate et al. (2016) added a description of the design with all phases and procedural changes, any planned replication and randomization, inclusion criteria for the selection of the participants and their characteristics, ethic approvals, measurement and equipment, intervention, including procedural fidelity, and analysis.

As Horner et al. (2005), Kratochwill et al. (2010), as well as Tate et al. (2016) noted, a set of criteria has been established for determining evidence- based treatments, one of the purposes of this study. Those criteria are as follows: (a) a minimum of five methodologically strong research reports, (b) conducted by at least three different research teams at three different geographical locations, and (c) with the combined number of cases being at least 20.

As the conditions in this study differ from those of Lund et al. (2012), Rivera et al. (2014), Skarr et al. (2014), and Walker et al. (2012), it was not a replication of these studies even though it aimed to add to their findings. The present study was performed in a different setting than the previous research, which was conducted by a research team from the USA, with the aim of meeting more of the aforementioned standards with an additional survey of social validity with respect to the participants.

Referring to successively increasing mathematical complexity in school, on the one hand, and the associated risk of failure correlated with decreasing motivation, on the other hand, the purpose of this research was to evaluate the effects of a math racetrack procedure enhanced by the addition of a multicomponent motivational system consisting of DI flashcards, immediate feedback, and positive reinforcement to address the challenges of four struggling elementary students related to basic single-digit multiplication fact recall. It was hypothesized that the intervention would lead to an overall increase in basic facts computational fluency. Therefore, the aforementioned components in combination were used as the independent variable, whereas the number of single-digit multiplication tasks correctly solved orally within two seconds were defined as the dependent variable.

Method

Participants and Setting

Subjects were six third graders (aged 8 to 9 years old) attending two different classes of an inclusive elementary school with approximately 200 students between grades 1 and 4 in a major city located in North Rhine Westphalia (Germany). The two third-grade classroom teachers proposed 12 students as a preliminary selection of eligible students based on teachers' impression of the students' mathematical competences.

Specifically, the preselection of the students was based on the following inclusion criteria: (a) basic understanding of single-digit multiplication tasks, (b) no to rudimentary automation of single-digit multiplication tasks from 1 x 1 to 10 x 10 according to the teacher, (c) regular school attendance in the previous two months according to the teacher, (d) motivation to be participate in the training according to their own orally given statements, (e) math lessons of the previous two months in class do not include any of the methods of the intervention nor fostering of the automation of single-digit multiplication tasks, and (f) participation in more than 80% of the intervention.

With 12 students meeting the inclusion criteria (a) through (e), we conducted the Multiplication subtest of the Heidelberg Math Test 1-4 (HRT 1-4; Haffner et al., 2005), which was developed to describe computational skills of basic mathematical operations regardless of grade. This instrument was standardized with a calibration sample of N = 3354 for the first to fourth grade from 2002-2004. Internal consistency of the HRT 1-4 is r = -.67 for math grade, re-test reliability is with r = .69- .93. Further, the test can be considered with objectivity concerning implementation, evaluation, and interpretation (Haffner et al., 2005). According to the test manual, students with a percentile between 11th and 25th are specified to be at the borderline of competency, while those between the 0th and 11th percentiles show a marked weakness. To be eligible for the study, students had to reach a percentile between 0th and 25th.

Further, a DIN-A-4 worksheet with 72 single-digit multiplication tasks was administered without a time limit. The items were arranged randomly and excluded tasks with the factors 0, 1, and 10. To be selected for the study, students had to solve at least 50% of the tasks correctly with a processing time ≥ 15 minutes to ensure that multiplication as operation was well understood, but the tasks were automated only to a small extent as the aim of this study was to explicitly show how the single-digit multiplication fact recall increases through drill and practice on the basis of a previously established understanding of the operation. To determine how many of the 72 items had already been automated, every student had two seconds to provide an oral answer to the orally and visually presented tasks. According to Stein et al. (2006), an answer given in less than two seconds can be considered to be automated.

Finally, two of the six remaining children had to be excluded later due to the number of days they were not at school during the intervention. Of these two students, data could only be collected at 73% of the measurement points across baseline and intervention. Ultimately, only four female participants met all the criteria (a) to (g). Their names were changed for this study to ensure confidentiality.

The first participant was Anna (9 years old), who was born into a native German-speaking household. Based on the HRT1-4, her multiplication skills showed a “marked weakness,” with a percentile of 4 on the Multiplication sub- test. Further, she only reached 56% of solved multiplication items (SMI) in 18 minutes, which was the lowest result of all the participants in this study. Anna continued to use her fingers to solve even simple multiplication problems for some tasks. However, she told the teacher and two graduate students who served as interventionists (see below) that she was eager to participate in class. Like all the participants, Anna had no diagnosed disability concerning behavior, learning, or acquisition of mathematical competence.

The second student was Dilara (8 years old), who was of Greek and Turkish migration background but spoke German fluently according to the interventionists. Based on the HRT1-4, she reached a percentile of 8, which was at the “marked weakness” level. On the worksheet with multiplication tasks, she reached 68% SMI in 16 minutes. Dilara claimed to be motivated to participate in the training.

The third participant, Eda (8 years old), also had a Turkish migration background but also spoke German fluently according to the interventionists. She reached a percentile of 10 on the HRT1-4, which was the transition from “marked weakness” to “risk area,” She reached greater than 90% SMI on the worksheet after working with her fingers for 33 minutes. She also claimed to be motivated to train with the interventionists.

Betül (8 years old) was also born into a family with a Turkish migration background and spoke fluent German like the other participants. In the Multi- plication subtest from the HRT1-4, she reached a percentile of 24 and was able to solve more than 90% of the items from the worksheet. However, like some of the other students, it took her more than 30 minutes. According to the teacher, she needed special attention because she was easily distracted.

For all participating students, the legal guardians or parents were in- formed by the teacher about the intervention in a personal meeting and a writ- ten consent was obtained before the beginning of diagnostic procedure. Further, a contract was drawn up with the school that the study could take place within regular school hours and premises.

Three female graduate university students in special education for children with learning disabilities were chosen as interventionists who conducted the training. They had attended math classes at the university and were close to graduating with a Master of Education for special needs. Prior to the study, they were extensively trained in four 60-minute sessions in a personal lecture-type format by the first author on how to perform the intervention. Additionally, the graduate students, who were experienced in working with children in school set- tings accompanied with interventions using single-case designs, were instructed to use a detailed script for the implementation. During the study, the interventionists and the first author had at least one phone conversation per week and stayed in contact via email over the entire period of the study.

Dependent Variable and Measurement

To preselect the common intersection of unautomated single-digit multiplication tasks among the participants as the dependent variable for the study, flashcards containing a pool of 72 items (excluding the reversals along with tasks with factor 1) were presented. One of the graduate students presented an item, while another recorded the time required for the answer using a hidden stop- watch to ensure that the participant would not feel pressured to answer quickly but was still motivated to solve a task, even if she exceeded the two-second time limit. The answer was noted on a recording sheet with the following categories: “correct within two seconds,” “counted correct (more than two seconds),” “wrong,” “wrong with correction,” and “no answer.” From the pool of 72 tasks, the intersection of the 26 (13 different tasks and their reversals) unknown tasks was filtered out. Those 26 tasks were randomly allocated to prepare 18 record sheets for each measurement over all data point from baseline until maintenance. Further, those record sheets were assigned randomly to each data point per participant to improve the internal validity of measurement.

Experimental Design

A single-subject concurrent multiple-baseline design (AB extension) across participants was applied to evaluate the effects of the training (Horner et al., 2005). In this design, the baseline phase (without treatment) (A) was immediately followed by a treatment phase (with racetracks and the multicomponent motivational system) (B), which consisted of three 20-minute training sessions each followed by measurement. The training was conducted on Monday, Wednesday, and Friday over the entire period of the six-week intervention. To support the demonstration of the long-term effects of the intervention, maintenance data were collected three weeks after the end of the study (Riley-Tillman & Burns, 2009). The treatment onset for each participant was staggered randomly with a baseline phase duration of five to nine days to increase internal validity by controlling history and maturation (Dugard et al., 2012; Tate et al., 2016). This design resulted in Anna and Dilara having 5 probes in Phase A and 10 in Phase B and Eda and Betül having 6 days in the baseline condition and 9 scheduled treatment sessions. For all the participants, Phase E (maintenance without treatment) lasted three days. The two children to have had a baseline length of seven days and an intervention length of eight sessions were excluded due to many missing data. Therefore, implementation of the study underwent procedural changes from the original plan, and only two different baselines can be reported in the results.

Further randomization was used besides allocation of participants to baselines like a randomly allocated set per measurement per student, order of tasks during each training session, treatment integrity observation per student and interventionist, including allocation of interventionists observing treatment integrity.

Procedures

The study was conducted over six weeks with three 20-minute individual sessions each week. Implementation among the three interventionists across the students was alternated to ensure that the effects were independent of the interventionist. For each session, the participants were brought to a separate room by one of the graduate students and led back again after the training.

Baseline Conditions

During measurement, each student was tested by one of the interventionists, who used the flashcards in a prepared order, the corresponding record sheet, and a hidden stopwatch. Feedback was given with constant interjections, independent of the accuracy of the students' answers. To ensure the conditions were comparable to those in the intervention, the students had to read short stories for 15 minutes after they were tested and, therefore, received a similar amount of attention from the interventionist at the same time a comparable effort regarding their ability to concentrate as during Phase B. Texts were chosen that currently were available in the classroom and met students' reading level according to teacher statements.

Treatment

Measurement during the treatment phase occurred the same way as throughout baseline conditions at the beginning of each session. Training sessions occurred in the same separate room and were structured according to a specially designed manual. The instructions included incorporating every item at least once in the beginning of the session using the DI flashcards, followed by a minimum of one round of the racetrack. On the racetrack, participants were presented turn by turn with one of the 26 items. With a correct answer, their figure could move on to the next cell. If the answer was incorrect, immediate feedback was given, and the interventionist and student repeated the correct solution in unison, the student remained in the current cell, and the math problem was moved into the third position in the deck of flashcards to be repeated fairly quickly. To move to the next cell, the next task was taken from the deck and had to be solved correctly. The stopwatch was used to measure time. After finishing the racetrack, students' possibly new high score concerning the time needed to finish the racetrack and the line diagram with the scores from the actual measurement were updated. Further, the student received feedback concerning cooperation during treatment with positive reinforcement through stickers for good working attitude.

Maintenance

After a three-week break following the last training session, maintenance data were collected. In this phase, the same measurement conditions were applied as in the baseline, but without reading after collecting the data.

Materials

In addition to the personal lecture-type format training by the first author, the graduate students received a manual explaining the implementation in detail and created a DIN-A-3 racetrack field with 26 cells along with DIN A-7 flashcards consisting of the identified 26 target tasks, as seen in Figure 1. On the front side of the card, the task was printed, on the backside the task including the solution was printed. The flashcards were used for the training itself as well as the measurement afterwards. For each of the 18 measurement points, a record sheet (see above) was created, and the interventionists used a stopwatch to record reaction time. In addition, a folder was created for each participant containing their personal high score on the front page. The folder was also used to increase the participants' motivation along with a line diagram to record and visualize their improvement and to organize the record sheets in order. Further, the graduate students developed a token system to give the participants feed- back concerning their cooperation at the end of each session supplemented with stickers for good cooperation.

A checklist comprising 20 items to consider treatment integrity was created with the categories setting, schedule, materials, procedure, measurement and feedback, dealing with students' behavior as well as space for notes for each session. Finally, a five-item social validity questionnaire was developed to get an impression of the students' acceptance of the intervention.

Treatment Fidelity and Social Validity

The intervention was implemented by the graduate students alternating during the research period so each observed ≥ 35% of the sessions to consider treatment integrity (Noell et al., 2002). The mean correct implementation of the items described in the checklist was 97.3% (range 94 to 100%) across the baseline, intervention, and maintenance conditions. During the interventionists' weekly contact with the first author, steps for improvement of the procedural fidelity could immediately be considered.

After the final training, the interventionists led the students through the social validity questionnaire to get in impression of the acceptance of the intervention by the participants. The interventionists read the questions aloud, and each student answered them orally. The questionnaire consisted of the following statements: "I enjoyed the math racetrack," "I would like to perform the math racetrack again," "It was easy for me to remember the tasks," "I felt comfort- able," "I always look forward to the math racetrack," and "I enjoyed entering my scores onto the line diagram." The students could answer each question with "yes," "a little bit," or "no."

Data Analysis

In addition to the descriptive analysis of the graphed data and some prespecified non-overlap indices, a piecewise regression analysis was conducted at the individual level (level 1) as well as across all cases (level 2). For visual analysis, at the same time as statistical analysis, the SCAN package for R (Wilbert & Lueke, 2019) was used to create the plots for each case in addition to computing the non-overlap indices. All effect sizes measurements aim to represent the strength of association between the outcomes and the implemented intervention and, therefore, support the detection of small differences in data between the treatment phases (Vannest & Ninci, 2014). A major argument for using a regression analysis is that only this method adequately takes both level and trend changes into account (Huitema & McKean, 2000).

Results

Table 1 presents the descriptive data of the children for all three phases. The mean scores show the overall improvement between baseline and intervention phases. The maintenance dates show a further increase in data across the participants. As illustrated, Anna displayed the greatest gain in the number of solved multiplication items from baseline to treatment and also in the maintenance sessions, while Dilara showed the least gain when comparing all three phases.

Table 1. Descriptive Data for Each Participant

	n (A)	*n* (B)	*n (E)*	*M (A) SD*	*M (B) SD*	***M (E) SD***
Dilara	5	10	3	0.80 (1.30)	4.25 (3.33)	5.67 (1.53)
Anna	5	10	3	1.20 (0.84)	11.20 (5.88)	21.00 (1.00)
Betül	6	9	3	4.17 (2.14)	9.86 (0.90)	15.33 (0.58)
Eslem	6	9	3	4.00 (1.90)	13.62 (6.05)	22.33 (0.58)

Note. M = mean, SD = standard deviation, A = Phase A, B = Phase B, E = maintenance, n = measurements.

Figure 2 demonstrates the number of SMIs for each participant. As illustrated, under the treatment conditions, the students' performance generally improved compared to the baseline phase. Phase A for Dilara and Eslem shows a steady baseline, while Betül and Anna show a slightly positive tendency.

A remarkable trend can be seen for Anna, which is a decreasing trend in the maintenance measurements. The other probes partly increased after the treatment ended, and some students seemed to be able to continue to advance their progress. Dilara's maintenance measurements display a level effect, and her data are not as stable as the other participants'. In all cases, the variability of the data is quite large, but the improvement is obvious for all four participants.

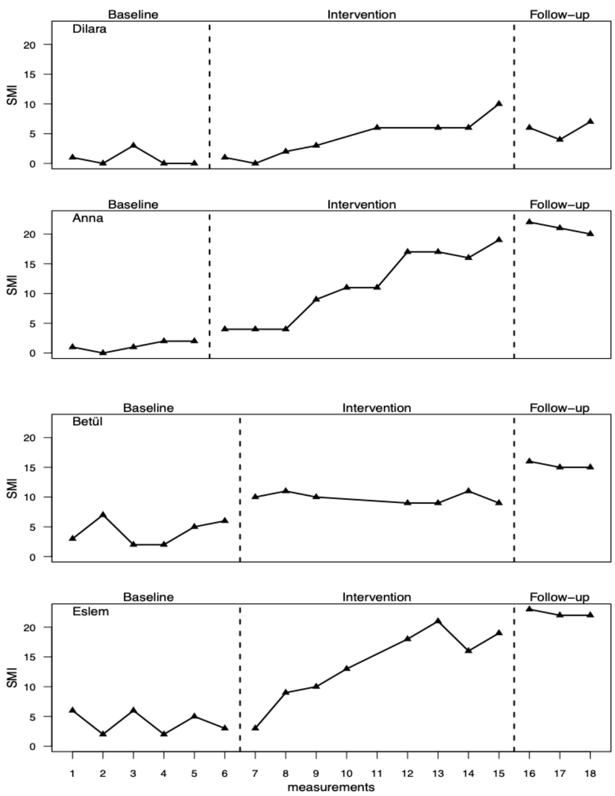

Figure 2. ***Dependent Variable in Phases A, B and Maintenance for Each Participant***

In addition to visual analysis and descriptive data, Table 2 summarizes the results of some non-overlap indices commonly used in single-case research to illustrate the extent of improvement from Phase A to Phase B. The percentage exceeding the median (PEM), non-overlap of all pairs (NAP), Tau-U (Lenz, 2013; Parker et al., 2009; Parker et al., 2011; Vannest & Ninci, 2014), and mean baseline difference (MBD) (Campbell, 2003) were applied. For calculating Tau-U, the option of correcting for the baseline trend (A vs. B + trendB + trendA) was used.

Table 2. Effect Sizes for the Number of Solved Multiplication Items

	Tau-U	*p*	NAP	*p*	PEM	MBD
Betül	0.32	.090	100	.002**	100	136.45%
Dilara	0.51	.008**	84.00	.03*	87.50	431.25%
Eslem	0.63	.001***	93.00	.005**	87.50	240.75%
Anna	0.76	.000***	100	.001***	100	833.33%

Note. NAP = nonoverlap of all pairs, PEM = percentage exceeding the median, MBD = mean baseline difference. *significant at the .05 level, ** significant at the .01 level, ***significant at the .001 level .

As the students' scores ranged from small/questionable effects to very large effects, the data should be considered more closely. One strength of the PEM is its insensitivity to fluctuations. Values between .70 and <.90 can be considered moderate, and values >.90 can be considered to be strong effects (Ma, 2006). Dilara and Eslem showed moderate effects, while the results for Anna and Betül can be categorized as strong effects.

The NAP compares each data point of one phase with each point of the other phase and, thus, is relatively insensitive to outliers. The benchmarks for moderate effects are .32 to .84; .85-1 can be considered to be a large effect (Parker et al., 2011). While Dilara reached a moderate effect of 84.00 ($p< .05$), Anna (100; $p\leq.001$), Betül (100; $p<.01$), and Eslem (93.00; $p<.01$) achieved large effects.

To underline the effects shown through the non-overlap methods, Tau- U was calculated to represent a significant correlation and differences between Phases A and B. The common benchmarks are .2 to .6 (moderate), .6 to .8 (large), and >.8 (very large) effects (Vannest & Ninci, 2014). While Betül's (.32; $p = .09$) and Dilara's (.51; $p<.01$) results can be considered to be moderate, Eslem's (.63; $p\leq.001$) and Anna's (.76; $p = <.001$) improvements resulted in large effects. Therefore, after implementing the racetrack procedure, all the pupils improved their performance.

Additionally, the mean baseline difference (MBD) was calculated to demonstrate the increase in automated SMIs from the baseline (Campbell, 2003; O'Brien & Repp, 1990). The MBD ranged from 136.45% as the lowest improvement (Betül) to the maximum increase of

833.33% shown by Anna. Nevertheless, Dilara (MBD = 431.25%) and Eda (MBD = 240.75%) also showed remarkable improvement in their performances.

Further, a piecewise regression analysis was conducted at the individual level (level 1, see Table 3) and across all four cases (level 2, see Table 4) (Van den Noortgate & Onghena, 2008). First, no significant baseline trend for any of the students was found. For three of the students, there was a significant slope effect with respect to the comparison between Phases A and B. Dilara showed a significant slope effect ($p<.05$) and improved, on average, by 1.130 in the intervention. Eslem also showed a significant increase ($p<.01$), with an average improvement of 1.205 scale points, with Anna showing similar values, with a significant increase ($p<.05$) and an improvement of 1.467 per intervention session. Only Betül showed no statistical slope effect ($p = .414$), but a statistically significant level effect ($p<.01$) means that there has been a direct improvement from the start of the intervention.

Piecewise regression analysis on the second level reveals a significant slope effect across all participants ($p<.05$); however, no baseline trends were found. In summary, the students increased the number of solved multiplication items by 1.128 per intervention. Thus, the intervention seems to have had a perceivable impact on the dependent variable.

In the social validity questionnaire, all four participants stated unanimously that they enjoyed playing the racetracks and entering their scores on the line diagram. They would also like to continue playing and were looking forward to playing the racetracks again during the intervention phase. Eslem stated that remembering the facts was “a little bit” easy and helped her “a little bit” to solve the tasks given with the flashcards. Anna, Dilara, and Betül gave positive answers to both questions, saying it was easy and it helped in solving the tasks. No one answered “no” to any question, nor did anyone want to add anything to supplement the questions on the questionnaire.

Table 3. **Piecewise regression model for the number of solved multiplication items (Level 1 analysis)**

	B	*SE*	*t*	*p*
		Betül		
Intercept	3.267	1.498	2.180	.054
Trend	0.257	0.385	0.668	.519
Level	5.647	1.700	3.321	.008**
Slope	-0.374	0.438	-0.853	.414
		Dilara		
Intercept	1.400	1.447	0.967	.356
Trend	-0.200	0.436	-0.458	.657
Level	-1.146	1.439	-0.797	.444
Slope	1.130	2.463	2.440	.035*
		Eslem		
Intercept	5.000	2.238	2.235	.047
Trend	-0.286	0.575	-0.497	.629
Level	1.006	2.481	0.405	.693
Slope	2.152	0.653	3.296	.007**
		Anna		
Intercept	0.000	1.524	0.000	1.000
Trend	0.400	0.459	0.871	.401
Level	-1,067	1.501	-0.711	.491
Slope	1.467	0.487	3.015	.011**

*significant at the .05 level, **significant at the .01 level.

Table 4. **Piecewise Regression Model for Number of Solved Multiplication Items (Level 2 Analysis)**

	B	SE	*t*	p
		Overall		
Intercept	2.332	2.258	1.033	.306
Trend	0.052	0.423	0.124	.902
Level	0.932	1.647	0.566	.574
Slope	1.128	0.463	2.438	.018*

*significant at the .05 level.

Discussion

Main Findings

This study examined the effects of a math racetrack game with a multicomponent motivational system, including DI flashcards, immediate feedback through self-scoring and positive reinforcement through visualization of a personal high score, on the single-digit multiplication fact recall of four struggling elementary students. As the results indicate, the treatment increased the participants' learning progress; therefore, it can be considered to be a way of supporting students to overcome their individual challenges.

Previous studies of individual components of the training describe similar conclusions, but in the present research all those components are combined to one intervention that is still easy to implement for teachers in everyday school life and is an attractive way to encourage demotivated learners to deeply engage in learning otherwise fatiguing content. Although the baseline data were not totally stable, the students seemed to have benefited from the intervention to different extents. As the intervention aims to lead demotivated students back on track and to be a tool that is easy for teachers to implement, even with the limitation of variable data, the added value was considerable.

All students' performance improved substantially. Under the baseline conditions, their mean score ranged from 0.80 to 4.17, while during the intervention the range of the mean value was from 4.00 (the lowest) to 15.25 (the highest). The non-overlap indices also support the descriptive analysis showing that the intervention had positive effects on the dependent variable. In addition, the results of the piecewise linear regression analysis of all cases confirm these findings, presenting a statistically significant slope effect in three cases and a statistically significant level effect in one case when comparing Phases A and B. On the second level, a significant slope effect across all participants is displayed by the data, and an increase of 1.13 SMIs per intervention for the students was found.

The maintenance data were collected after a three-week break before the summer holidays. Our data indicate that the effects maintained after the end of the intervention. However, a variability in the data is present, given that Dilara's maintenance data show a level effect while the data of the other students are more stable. Nevertheless, the intervention seemed to improve the performance of all the participants.

Further, responses on the social validity questionnaire give an indication of a high degree of acceptance of the intervention for all students who participated in the study. No one commented negatively on the instruction. Only Eslem answered two questions with more reserve; however, his answer was "a little bit" as opposed to "no."

Overall, the results of this research are compatible with those from the previous studies, all conducted in the USA, by Lund et al. (2012), Rivera et al. (2014), Skarr et al. (2014), and Walker et al. (2012) focusing on the effects of math racetracks on the number of math facts recalled by struggling students with and without disabilities. Therefore, this research adds value in support of designating this easy-to-implement intervention as evidence-based according to the single-case research standards found in SCRIBE (Tate et al., 2016) and establishing a functional relationship (Kazdin, 2010; Ledford & Gast, 2018) between the math racetrack game combined with the multicomponent motivational system.

As reported throughout this paper, this study not only tried to add valuable findings concerning the effectivity of the racetrack intervention itself, but further had the aim to meet as many standards listed in SCRIBE as possible. Clearly, the methods section is the most detailed and the most “vulnerable” part, as this research is taking place under real-life conditions. Still, the design could be clearly identified and described the phases as the phase sequences, procedural changes during the course were noted, the aspects of replication were designated, and randomization, including methods along with the elements that were randomized, are found in the section on the design. Further, the selection of participants is based on the description of inclusion criteria and the method of recruitment. Each participant’s demographic characteristics and features relevant to the research question ensuring anonymity are reported as is the setting and location where the study was conducted.

The measurement is explained in detail, and the target behavior and outcome measures, including how and when measurement was applied, are reported. Moreover, the equipment and materials used to measure the target behavior were taken into account. In addition, the intervention itself, baseline condition, and the maintenance are described in sufficient detail to enable replication of the study, and the treatment fidelity is reported in detail, too. The section on the methods concludes with the explanation of the way data analysis was planned and conducted.

The results are reported for each participant, including the score for each session, raw data for the target behavior. Finally, a summary of the findings and interpretation are to be found in the context of current evidence. The concluding section presents the limitations of the study along with applicability and implications.

As mentioned in the beginning of this paper, the impact of motivation on the learning process can both increase and decrease students’ performance, and failure in the mathematical development can lead to math anxiety (Devine et al., 2018; Hattie, 2009; OECD, 2013). The findings of this study show how easily struggling students can be engaged into learning contents like automation of single-digit math facts recall if motivational factors are added to instruction that otherwise can have boring and tiring effect on students. As such, the study extends the findings of previous research employing math racetrack combined with DI flashcards and further motivational components.

Limitations

Despite the promising results of the study, a number of limitations deserve mention. First, the third tier in the multiple baseline design is missing as two students had to be excluded due to the fact that they only attended 73% of the measurement time. Although the outcome does not meet the standards of three baseline durations, the procedure itself did so as participants were allocated randomly to the duration of a baseline between five and nine days. Any study that is conducted in a school environment is vulnerable to unintentionally violate best-practice standards, as seen in this case. This underlines the importance of trying to meet current standards while designing further research.

In addition, the sample is too small to draw general conclusions about the effects. However, the findings should not be seen as isolated from previous investigations concerning the effectiveness of math racetracks on the number of automated math facts achieved. On the contrary, one of the main objectives of this study was to substantiate the results obtained to date considering the standards for single-case research and interpreting them in a broader context of previous findings. Additionally, it was not possible for this study to meet all the standards described in Tate et al.'s SCRIBE (2016), such as the absence of interobserver agreement for the measurement and some changes in procedure, which makes it hard to consider the findings as a true replication of previous research. However, many standards were met and allow future research teams to replicate this study due to the precision of the description of all methods and materials. Even with this limitation, this study confirms the effectivity of the intervention.

Another limitation is the lack of data about incorrect responses, which might otherwise have provided a more detailed interpretation of the existing results concerning the correctly solved target tasks. Additionally, the classroom teachers knew about the treatment, and it is possible, therefore, that they unconsciously influenced the intervention and the measurement.

Given that this was a multicomponent motivational intervention, it is not possible to determine with certainty the source of the specific positive effects. To be able to draw conclusions about the change in data due to the exposure to math problems and corrective feedback over baseline and intervention, an alternating-treatments design comparing racetracks and flashcards, for example, would have been helpful. Another limitation is that the interventionists conducted the treatment integrity and performed the measurements, which, according to Podsakoff et al. (2003), might have led to inflated scores.

Finally, the measurement was performed orally using flashcards. The students' time to answer was controlled using a hidden stopwatch. Therefore, the accuracy of the data might not be highly precise, as it leaves room for interpretation. If an answer is correct or goes beyond the two-second time limit that proves automation of knowledge. Using a digital measurement supported by technology, such as PowerPoint, where slides change after two seconds, would have been more accurate. However, the interventionists were instructed by the first author exactly on how to measure the time, control the answers, and complete the protocol sheet; therefore, the results of their measurements can be considered to be comparable.

Practical Implications and Future Research

This investigation provides teachers with further arguments for using this intervention to increase the number of automated mathematical facts among struggling students. Since the results of this study are comparable to the findings obtained by a research team in the USA, it can be seen as progress, and the method can be regarded as slightly more evidence-based. In our case, there were some differences from the previous research concerning the effects of the math racetracks due to greater compliance with the standards. Additionally, the present study included more motivational aspects, such as positive reinforcement through the individual high score, entering scores onto the line diagram, implementing a token system, and providing immediate feedback.

One of the strengths of this economic method is its ease of implementation in everyday school life. Due to its simplicity, peers or parents should be motivated to use this method in addition to teachers. Another strength is the flexibility of the learning content, which can be easily adapted to individual needs by making changes, for example, to the mathematical facts on the flash- cards. That is, if a student needs to learn to automate basic addition facts instead of multiplication tasks, this would be simple to change.

Future research should aim to fulfill more of the standards for single- case studies, such as SCRIBE, with the purpose of being able to call the method evidence-based. In addition, further research should evaluate the effects of using math racetracks in a peer tutor setting more closely. It would be interesting to determine whether students would show the same improvement when trained by a classmate. With reference to the multicomponent nature of the intervention, especially regarding the motivational aspects, one could investigate to what extent these aspects influence the course of learning when using racetracks.

Additionally, there is great need for further research concerning digitalized racetracks for struggling students. A digital approach could increase students' motivation and could be easier for a teacher to implement. In their review of literature about digital-based math fluency interventions, Cozard and Riccomini (2016) determined a great need and pointed out that even less research is to be found that meets scientific standards. Developing a digital version of the racetrack should not be too challenging. A digital version of the racetrack could help, for example, when selecting the items to be learned and during the instruction, where the immediate feedback on each math fact would be provided automatically by the program, freeing up the class teacher's time to provide other needed help.

In summary, the racetrack combined with a multicomponent motivational system is simple to implement, and this study showed that the children had fun and considered the intervention useful. Effective methods like this that can be implemented easily and quickly are of great importance in today's school environment to add to teachers' toolbox while supporting student development.

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Appendix C: Paper 3 (*peer reviewed*)

Karnes, J. & Grünke, M. (2021). The effects of a multicomponent motivational system intervention using peer- tutoring for implementation on the automation of single-digit addition tasks of four struggling elementary school students. *Education in Sciences*, *11*(265), 1–14.

Abstract

Derived math fact fluency becomes more imperative across all mathematical content areas during a students' mathematics development. However, many of them struggle to automate the most basic math facts sufficiently and therefore are not able to deal with more complex mathematical problems. This leads to the fact that many of them are already left behind in the early years of their school careers whether they have diagnosed learning disabilities or not. In this single-case research project, we evaluated a peer-tutoring approach designed to extend the number of automated single-digit addition tasks for four struggling elementary students through a multicomponent motivational system including immediate correction of errors, graphical feedback on performance, positive reinforcement, direct instruction flashcards and a racetrack game.

A multiple-baseline design (ABE) across subjects was applied to assess the effects of the treatment. Results indicate significant and large effects of the intervention on the number of automated math facts for the participants. This substantiates the assumption that the math-fact recall performance of struggling students can be improved through the method of peer-tutoring even with the limited resources available in everyday school life.

Keywords: automation, basic math fact fluency, learning difficulties, peer-tutoring, math racetracks, DI flashcards

Introduction

The Importance of Addition Fact Fluency

Good basic math skills are essential for both school and personal life. They are indispensable for succeeding in various subjects (e.g. science, social studies and of course – math), and we need them to handle many life tasks such as managing our finances. However, the significance of basic math skills goes beyond the immediately obvious: they help us to develop our logical and critical thinking in general. Analyzing and solving problems would not be possible without this ability. Calculation skills bring order to almost every aspect of life and enable us to orient ourselves in a complex society [1–4].

Unfortunately, far too many individuals struggle with math from childhood all the way into adulthood. This holds them back in many ways. For example, they do not earn the formal qualifications during their lives that they would otherwise have received and often enter low-paying careers [2,4–6]. The problems begin early in life and usually become evident during the first year of school. Currently, a large percentage of elementary and secondary students does not meet minimal standards in the area of math. They struggle even with basic arithmetic. According to the 2019 National Assessment of Educational Progress, about 40% of all students score below the benchmark set for defining fundamental calculation skills [7]. Here, a more nuanced look is necessary at how the struggle of not being able to make adequate progress in developing more complex concepts, such as overcoming counting-on, is a substantial predictor for less computational flexibility in particular as well as mathematical achievements in general [8,9]. This circumstance underlines the relevance of classroom instruction in overcoming this hurdle in students' early school years.

Many of the problems these young people demonstrate concern their lack of knowledge of basic math facts [10,11]. Students need to show sufficient fluency in this respect to be able later to tackle more complex mathematical operations successfully [12,13]. Math fact fluency is defined as the ability to answer simple facts in addition, subtraction, multiplication and division accurately and rapidly [14]. According to Steel and Funnell [15], students who have not reached adequate fluency entering sixth grade will not be likely to catch up in this backlog. The reason is that automated retrieval eases the very limited capacities of the human working memory [6]. Automation enables learners to use their cognitive resources for solving more complex math problems later in their development [16–18]. Therefore, acquiring these skills to an advanced level by the end of elementary school is decisive for their continuing school careers.

Of the four basic arithmetical operations, addition is most primary. Understanding the underlying concept presupposes different insights: it begins with a deep comprehension of what a number represents and what its connection is with other numbers. Later, children pass through different strategies: count-all, count-on, maximum addend, minimum addend. As soon as an awareness of the basal principles of this operation has sunk in, automatic fact retrieval can be built up [19]. However, this last step seems to be particularly challenging for many elementary school children [20].

Ways to Foster Math Fact Fluency

Luckily, research has identified some effective strategies to foster math fact fluency and thus pave the way for promising school careers without severe problems in arithmetic [3,4,21–25]. Some of the most effective approaches involve (1) immediate correction of errors, (2) graphical feedback displaying previous performance, (3) verbal praise that attributes high performance to effort and low performance to various reasons, (4) direct instruction flashcards, and (5) racetrack games.

Immediate correction of errors prevents students from committing a mistake to memory. Different findings demonstrate that such feedback needs to occur immediately after an accurate response, not after completing an entire work sheet [26,27]. The graphical feedback displaying previous performance gives a student the chance to monitor his or her own performance individually. This self-monitoring has been demonstrated in different findings to be highly motivating due to the fact that low performers are finally able to compete successfully by doing so against their own previous performance [28–30]. Using verbal praise that attributes high performance to effort and low performance to various reasons is a well-grounded approach to maintain or increase learners' motivation to engage as it divides performance into internal stable attribution (e.g. results are due to my engagement, my strategies, my preparation) and into external and unstable attribution (e.g. the weather had a bad influence, there has been an exhausting class before the performance). This attribution supports the emotional connection of personal effort and augmented competencies [31,32]. The direct instruction flashcards provide a procedure for struggling students to automate and memorize certain information and therefore benefit in recalling this content [33,34]. Finally, the racetracks method uses a game board with a number of cells. To move forward and reach the goal of the game board, the student must solve, for example, a math task (or e.g. read a word aloud, or spell a word depending on the learning content) [34–36].

In conclusion, the aforementioned methods do not focus on building an understanding of mathematical operations but rather presuppose this already. They focus on the subarea of automation in order to achieve increases in performance and therefore in motivation to further engage in learning with a positive attitude toward mathematics and in addition relief of the working memory. This achieves the long-term goal of creating more capacity for more complex tasks [25,37,38].

Peer-Tutoring as a Means to implement Effective Interventions Under Real-Life Conditions

Even though these elements seem simple and plausible, teachers still face challenges as they try to implement them in their daily classroom routines. It is not easy to provide sufficient time for engaging students in practice activities while providing enough opportunities for responding and immediate feedback. Complicating matters, Burns et al. [37] mention that students have different rates for acquiring tasks as well as initial ratios of known and unknown facts varying from student to student over time to achieve math-fact fluency. Therefore, individualized instruction and methods that take the special needs of students into account are vital to enabling professionals to teach more effectively.

This is where peer-tutoring can come to the rescue. Peer-tutoring is defined as the process of one student helping another to learn and master some aspect of the curriculum. One is taking on the role of a teacher, instructing another either at the same academic level or lower [38,39]. Peer-tutoring can be implemented in a Peer Assisted one-to-one setting [42] or as class-wide or total-class peer tutoring [43,44]. The impact on students' performance is, among other aspects, linked to the time actively engaged with the academic task [45,46], as well as to the frequency with which immediate feedback regarding students' performance is possible [46].

Current research generally describes peer-tutoring as effective for tutor and tutee, across settings such as general and special education or alternative education, for students with and without disabilities as well as students with different native languages [47,48]. It has already been examined and proven effective for different learning contents such as reading, writing, as well as arithmetic [49–51].

Research Question

Several factors altogether lead to the aim of this study: the aforementioned aspects concerning struggling students' need for instruction and close monitoring; their lack of time actively engaged in academic tasks; the need to automate the basic math facts to successfully develop more complex computing competencies; and the results of the current research concerning the increase of student performance using racetracks, direct instruction flashcards, immediate feedback, explicit timing and positive reinforcement as well as those concerning the effectivity of the peer-tutoring method. Therefore, this study examines the effects of a peer-tutoring intervention combined with the use of racetracks, direct instruction flashcards, and explicit timing in combination with immediate feedback and positive reinforcement on the automation of single-digit addition tasks of four struggling elementary students.

Method

Participants and Setting

The study was conducted in an inclusive elementary school in a large city in North Rhine-Westphalia. In it, children are taught in so-called family classes together with students from the first to the fourth grade. Following the class teacher's suggestion, a preselection of possible participants was tested to identify suitable participants. The following inclusion criteria were determined as the basis for this selection: a) basic understanding of single-digit addition tasks, b) automation of the single-digit addition tasks was less than ten percent, c) regular school attendance over the last six months, d) students had to be willing to take part in the intervention, and e) socially capable of independently working with a partner without needing constant attention from an adult.

To find students eligible for the study, a class-wide standardized assessment of math operation skills (Heidelberg Math Test 1-4 [HRT 1-4] by Haffner et al.) [52] was applied, different from a non-standardized paper-pen assessment concerning the automation of the single-digit addition tasks with all second to fourth-grade students. Despite weaknesses concerning the automation of the single-digit addition tasks (range from zero to three tasks), four students reached a percentile between 20 and 43 in the HRT 1-4 concerning their addition skills, which means they may be described as at the borderline of competency. Therefore, they were chosen to be trained as tutees, according to the fact that they met all other inclusion criteria as well. Those students chosen as tutees were tested concerning known but not automatized single-digit addition tasks and answered the German version of the standardized Math Anxiety Questionnaire by Thomas and Dowker [53] to determine whether they were eligible for the training.

The tutors were chosen on the basis of their results in the HRT 1-4, where they had to reach at least the 50th percentile. Additionally, the teacher had to consider them socially capable of taking responsibility for accompanying a fellow student. Those four tutors were then assigned to a partner from the tutees on the basis of their results in the pretest as well as their social connections with the tutees, which should be neither a close friendship nor a problematic relationship. All four tutees were born and raised in Germany and had no diagnosed learning disabilities. As this study focuses on the results for the tutees, the tutors will not be described further.

The first team was Anton (tutee, age 7) and René (tutor) (all names have been changed to comply with data protection regulations). Anton's teacher described him as curious and motivated to participate in the training. On the other hand, he was easily distracted, but it was not difficult to turn his attention back to his tasks. He reached the 20th percentile in the HRT 1-4 test, and his scores in the Math Anxiety Questionnaire were the lowest of all participants, but still did not mark him as having math anxiety. Anton was able to solve 69.44% of the single-digit paper pen test, but like all participants solved zero tasks in the oral test within two seconds.

The second team consisted of Berta as the tutee (age 7) and Barbara as tutor. According to her teacher, Berta behaved introvertedly but still participated willingly during training sessions. She was focused on her tasks and did not distract herself noteworthily. She achieved the highest results in the HRT 1-4 in the 43rd percentile and was able to solve 75% of tasks correctly in the paper pen test.

Celina (age 7) worked together in a team with Stephanie. Her teacher characterized her as lively and open. She was always looking forward to the next training session. Celina was highly engaged in improving her timing when playing racetracks. Like Berta, she was focused and not easily distracted. Celina reached the 20th percentile in the HRT 1-4 and 84.72% in the paper pen test.

The fourth and last team consisted of Diana (age 8) as tutee and Monika as tutor. Her teacher viewed her as introverted and quiet. However, she was still willing to make an effort, stayed focused and was not easy to distract. She had 97.22% correct answers on the paper pen test and reached the 35th percentile in the HRT 1-4.

Experimental Design

An ABE multiple-baseline design was conducted across subjects [54]. The data were collected in a period of 10 weeks with 19 measurements in total and an average of three probes per week. The ABE design allows control of internal validity such as maturation or history [55].

Taking the SCRIBE guidelines by Tate et al. [56] into account, the start of the intervention was staggered randomly to increase internal validity. In addition to that, each phase in the design had to include at least three measurement points. Accordingly, the training started between the 6th and the 9th probe. The assignment via random dragging with put-back resulted in Celina and Anton having 5 days, Diana 6 and Berta 8 days' duration in the baseline. Consequently, Celina and Anton received 11, Diana 10, and Berta 8 training sessions. There were three weeks between the last treatment and the follow-up phase, which lasted three days for all cases.

Dependent Variable and Measurement

As dependent variable, the common intersection of unautomated single-digit addition tasks among the participants was determined through preselection. To this purpose PowerPoint slides were presented containing a pool of 55 single-digit addition tasks (excluding 0 and reverse tasks). Each slide with one task was displayed for two seconds and then crossfaded by a neutralizing slide. This happened to keep the participant motivated to solve the missing tasks but without having further pressure due to the tracking of their response time. The oral answers were noted on a protocol sheet using the following classification of the possible given answers: “correct within two seconds”, “counted correct (more than two seconds)”, “wrong”, “wrong with correction”, and “no answer.” From this pool of 55 items, the intersection of 28 different single-digit tasks without their reversals remained as not automated among overall students. The 27 tasks remaining from the pool of 55 items reviewed could be assumed to be automated by at least one participant, which, if included in the grant, could potentially skew the results. Therefore, these 27 tasks were not included in the training. Moreover, the measurements over all data points consisted of those 27 tasks randomly allocated for each measurement. Additionally, randomization of assignments between record sheets for each data point and participant was applied to improve the internal validity.

Materials and Procedures

Three graduate students created an instruction manual together with the first author in order to obtain a detailed description for every step of the intervention. For each training session, a PowerPoint presentation with a set consisting of the intersection of non-automated single-digit addition tasks over all participants was prepared in random order of the tasks as measurement (randomization per tutee as well as session). The random assignment was conducted with respect to the order of the tasks per set, the assignment to the measurement, as well as with respect to the student at which the test was performed. Each slide with a task disappeared after two seconds for the interventionists to control whether the answer was given in an automated way or the participant was able to solve the task by counting. As in the determination of the non-automated facts described before, the same neutralizing slide was placed between two slides as well as a protocol sheet with the same five categories as described before for each of these tests was conducted.

The items were printed on flashcards (half the size of a postcard) on thick obscure paper with the task itself on the front and the task including the sum on the back. In addition to that, the racetrack was printed on thick paper twice the size of a worksheet. This racetrack included 28 cells from the starting point to the finishing cell and had running figures for identification and motivation imprinted on them. Furthermore, a stop watch was used to document the duration for one round on the racetrack. In addition, a line diagram was provided for students to monitor their own performance during the intervention. Together with the line diagram, the personal high score was noted on a small sheet by the tutee and put on top of each participant's individual folder, which they received at the beginning of the intervention.

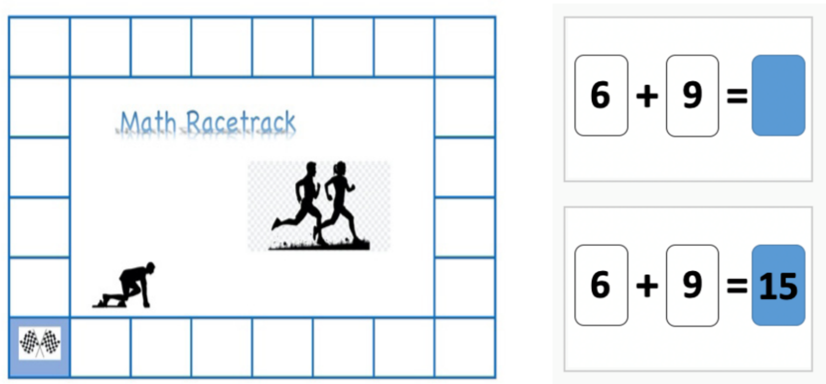

Figure 1. Racetrack Game Board and DI Flashcard Example Showing Front and Back.

During the baseline (A-phase), each tutee was taken out of the classroom and into a separate room, where two measurements (and later, two interventions) had to be conducted simultaneously. This was due to school organization issues, yet all students managed to work properly and without remarkable disturbance. The number of correctly orally-solved, single-digit addition tasks within two seconds functioned as the dependent variable and was noted by the interventionist each day. Therefore, the measurement via PowerPoint presentation of not-automated 28 single-digit addition tasks for the tutee was carried out, where the math facts had to be solved as quickly and correctly as possible, while the interventionist wrote the results down on the protocol sheet. Afterwards, the tutor also entered the room, and tutee and tutor read texts for fifteen minutes to control the impact of attention through the interventionists and to keep conditions of both phases as similar as possible. No further treatment was implemented at that point.

With the beginning of the intervention, the procedure stayed the same, and the training sessions started with the measurement of the tutees. Differing from conditions in the baseline, the line diagram was presented to increase the students' motivation and make their individual progress visible. Further, the interventionist gave feedback in relation to previous results, which was either attributed to effort (in case of an increase in the students' performance) or to external influences such as bad weather or bad general conditions on that day (in case of stagnation or even deterioration). After that, the tutor was brought to the room, where the training using the direct instruction flashcards, the racetrack and the procedure for tutee and tutor were implemented by the interventionist. After a demonstration of the procedure, the interventionist observed the tutors and the tutees training and corrected immediately whenever they did not adhere exactly to the agreement. Tutees were told that they were competing with themselves, being as fast as possible at the destination of the playing field through solution of the math tasks as quickly and at the same time as correctly as they could.

The deck of 28 flashcards was placed in front of the tutor, concealed by hand. The tutor then raised one flashcard after the other to the tutee with the side on which the task was shown. On the back, the tutor could see the task itself as well as the correct answer, while the tutee had to name the task and result correctly. In that case, the tutor gave immediate positive feedback and handed the card to the tutee. If the task was solved incorrectly, the tutor presented the correct task and its solution, directly instructing as a model, and replaced the flashcard to the third place in the deck of flashcards, thus to be repeated soon. Meanwhile, the tutee was asked to repeat the task and the correct answer, thereby building the problem-answer association. When a task appeared again, the tutor increased the tutee's focus by saying something like "Let's see if you still remember that task." Training ended when the tutee had received all flashcards. This procedure reinforced near-term repetition after direct instruction, including repetition, and therefore supports students in trying to remember the solution to the task.

After all the items were recalled and trained by means of the flashcards, the participants played the racetrack game. Thus, they were again asked to solve the tasks, but this time with a more inviting game character supplemented with time measurement, which increased the motivation of students to answer as quickly and correctly as possible. For playing the racetrack game, the flashcards were reshuffled, and again the tutor covered them by hand. The game board was directed to the tutee, including the playing piece on the starting field. As soon as the participant was ready, the tutor started the timing and showed the tutee all flashcards one after the other. Again, the tutee had to state the task and solution correctly. If he or she succeeded in doing so, they could move the playing piece one field forward. If they answered incorrectly, the tutor asked them to have a closer look at the task. If they still could not answer correctly, the tutor named the correct task and solution, and the tutee had to repeat. This procedure was implemented to further engage participants not only to move as fast as possible but also to answer correctly. After this, the playing pieces were also moved one field forward. The timing ended when the participant had processed all 28 tasks and reached the target field.

The tutor noted the measured lap time on the protocol sheet with positive feedback. If the time was a personal best, it was noted as a new high score for the participant, accompanied by further positive feedback. The interventionist finished the session after about 15 minutes, giving the tutor and tutee feedback concerning their cooperation. After that, all children returned to their classes.

Treatment Fidelity and Social Validity

Three 60-minute training sessions were conducted to instruct three graduate students in the correct implementation of the intervention. Furthermore, they received a 24-item checklist to ensure the quality of implementation during the treatment (list is available on request). All aforementioned factors to be considered were included in this checklist. The assistants randomly observed the interventions distributed over all participants for more than 80% of all measurements during the implementation. If an aspect was not fulfilled during the training session, the observing assistant corrected the course following the instructions from the manual after the treatment session ended.

After the last intervention, the interventionists interviewed each participant individually to collect data in the form of a questionnaire concerning the social validity of the treatment. Therefore, the participants were asked if a) they enjoyed the training with the math racetrack, b) the math racetrack helped them solve the tasks, c) they could solve single-digit addition tasks better now, d) they liked receiving immediate feedback on their performance, e) they were looking forward to working with the math racetracks, f) they would like to proceed in working with the math racetracks, and g) they liked working with a partner and found it helpful. The interventionists supported the students in filling out the questionnaire by reading the questions aloud.

Results

Visual Analysis

As can be seen in the graphs created with the SCAN package in R by Wilbert [57] in Figure 2 as well as in Table 1 concerning the descriptive scores for correctly solved addition tasks of each participant, all participants increased their number of correctly solved single-digit addition tasks within two seconds compared to the number of the same addition tasks correctly counted before the intervention. In all cases the scores improved visibly after the onset of the intervention. With the exception of Celina, no score during intervention fell below the ones during phase A. Still, over all scores in the B phase variability had to be recorded for all cases, and no participant finished the intervention with their personal high score. Anton, Berta and Diana show a change of level, while in Celina's case a slope is to be mentioned with implementation of the training. The mean baseline difference [58] as an index, which does not calculate the non-overlap effect sizes, was applied, where an average increase of about 471% was recorded. Celina benefitted the most with a mean baseline of 627.5% and Diana the least but still considerably with 222.5%. Even if an increase in performance was evident for all participants, Anton, Berta and Celina demonstrated the most impressive enhancements concerning the mean baseline difference. Regarding the maintenance data, no further increase in performance took place in three cases. Only Celina scored her maximum during this phase. The other participants all reached their high scores from the B phase also during maintenance or were slightly below it.

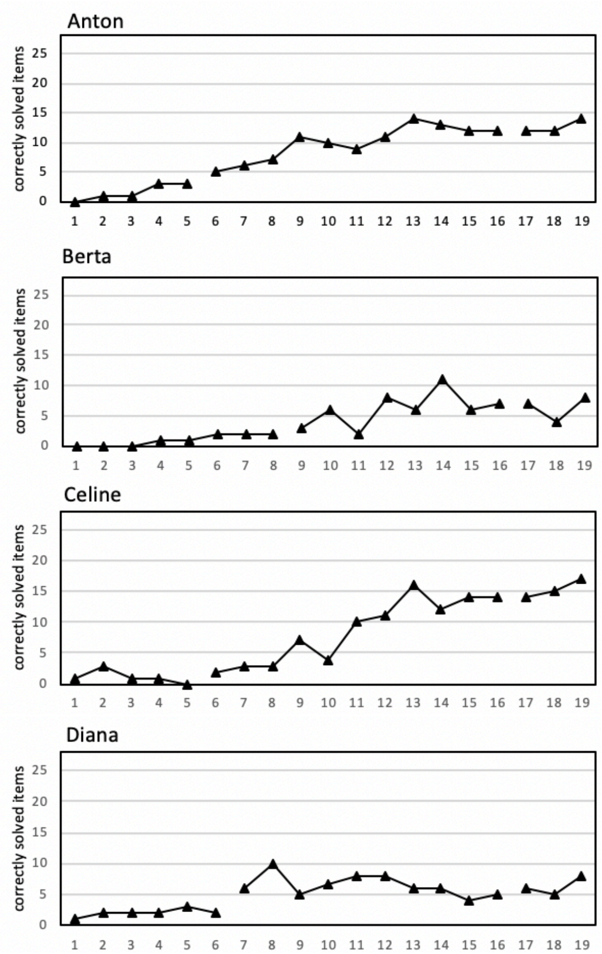

Figure 2. Number of correctly solved single-digit addition items in phases A, B and E for each participant.

Quantitative Analysis

Table 2 presents the descriptive statistics for the number of correctly solved single-digit addition facts within two seconds for each participant. Some of the most common and reliable non-overlap effect sizes were calculated again using the SCAN package by Wilbert [57] with non-overlap of all pairs (NAP), which compares each data point of the intervention phase with each data point of the baseline and therefore provides a reliable effect size and Tau-U. This in turn has the ability to analyze data independently for several phases showing contrasts in a single-case design and is able to combine non-overlap and trend [59]. The treatment can be documented as effective due to both indices. From the A phase to the B phase, all participants showed great improvement. Diana showed the least improvement. Still, NAP as well as Tau-U showed significant differences between phases A and B for all students.

Table 2. Effect sizes for the number of correctly solved items.

	Tau-U$_{corrected}$	*p*	NAP	*p*	MBD
Anton	0.63	.001**	100	.001**	525.0%
Berta	0.43	.019*	97.66	.000**	512.5%
Celina	0.84	.000**	96.36	.002**	627.5%
Diana	0.27	.144	100	.001**	222.5%

[1] *Note.* NAP = non-overlap of all pairs, MBD = mean baseline difference.
*significant at the .05 level, ** significant at the .01 level, ***significant at the .001 level.

To substantiate the aforementioned indices, and in accordance with the standards for single-case research by Tate et al. [56] calling for a broader examination of the outcomes than visual inspection and the calculation of the effect sizes, a piecewise regression analysis on level 2 (across all participants) as to be seen in Table 3 was also conducted. This allows more reliable statements if the data follow different trends over different participants by pointing out connections between the data concerning the change in data from baseline to intervention as well as if there is a slope or level effect to be mentioned. As seen in these results, there was a significant level effect from phase A to B on the 0.05 level, as well as a significant slope effect on the 0.01 level.

Table 3. Piecewise regression level 2 analysis.

	B	*SE*	*t*	*p*
Intercept	2.18	2.10	1.04	0.30
Trend	0.15	0.38	0.40	0.69
Level Phase B	1.44	1.54	0.94	0.53
Level Phase E	13.39	5.03	2.66	0.01
Slope B	1.01	0.42	2.58	0.01
Slope E	−0.25	1.11	−0.23	0.82

Anton, Berta, and Celina stated that they enjoyed the training with the math racetrack a lot, whereas Diana liked it only a little. They all gave the feedback that the math racetrack helped them solve the tasks and that they could solve single-digit addition tasks better than before. Furthermore, they all liked receiving immediate feedback on their performance and liked working with a partner, which they also stated unanimously was helpful. Again, Diana was the only one who was looking forward only a little to working with the racetracks, whereas the others strongly looked forward to it during the period of intervention.

Discussion

Main Findings

The purpose of this single-case study was to examine the effects of a peer-tutoring intervention using math racetracks, direct instruction flashcards, and explicit timing, all combined with immediate feedback and positive reinforcement on the number of automated single-digit addition tasks of four struggling elementary school students. The results facilitated the interpretation that the intervention can be considered an appropriate way to help struggling students successfully automate basic facts through implantation by peers in the role of tutors. The remarkable magnitude of increase in performance went from 222.5% as the lowest to 627.5% as the highest. On an individual basis, the different utilized effect sizes were notable for all participants, and the used indices stated large to very large effects. Finally, a regression analysis at level 2 confirmed the previous results and therefore underlined the benefits of using peer tutoring on the performance of the participants.

Regarding Diana's less-notable results in comparison to the other participants, it should be mentioned that she and her tutor did not harmonize well and that she was dealing with difficult circumstances in her private environment, which she reported during the treatment. Another reason for the different rate of improvement across a multiple baseline design could be found in the suggestion that some students who are over-reliant on counting-base strategies might improve less than others [60]. Still, she was able to increase her performance, albeit to a lesser extent.

Moreover, these results fit well with previously described effects for both peer tutoring [49–51] as well as math racetracks [34–36] on the performance of struggling students. Still, this study is to be considered unique in combining both easy-to-implement methods in one intervention added to further motivational components. These findings are also substantial for future implantation by teachers because it did not take long for the students to increase their performance significantly.

In particular, the students stated in the social validity questionnaire that they enjoyed their own improvement. Overall, the responses from the participants on the social validity questionnaire indicated a high degree of acceptance of the intervention using the math racetrack in a peer-tutored setting. No negative comment on the instruction was given, and only Diana twice answered in a more reserved way with "a little bit," and none of the answers of any participant was "no." According to the class teacher, they even seemed to act more confident in math classes after the intervention.

Limitations

Regardless of the aforementioned promising results, there are also limitations to this study. First, large-scale generalizability is not given due to the small sample examined in single case studies. Following the standards for single-case research by Tate et al. [56], this limitation can be countered by replications of the study. According to the prescription of these standards, for an intervention to be considered evidence-based, it requires at least five methodologically sound case reports with positive effects and at least 20 participants across all studies. Therefore, this study can only be considered a prelude for further single-case research examining the effects of peer-tutoring intervention combined with the use of math racetracks to increase student performance. Nevertheless, the results should not be considered isolated from the positive effects described in the aforementioned research to both peer tutoring as well as math racetracks and have to be regarded as an important supplement to the previous findings.

In addition, no data were collected that would allow any conclusions to be drawn as to the exact influence that each of the two methods had on the performance itself. Moreover, the additionally implemented motivational aspects such as use of a personal high score, corrective feedback, and a line diagram cannot be analyzed separately concerning their individual impact on the results. To determine the respective source of the positive outcomes more specifically, an alternating treatment design comparing, for example, the racetracks, the flashcards, and the peer tutoring could give more detailed information about the degree of influence of each aspect of the multicomponent motivational intervention. Besides the missing data concerning each method itself, no data were collected on the impact the intervention had on the tutors' performance. Although these were selected in advance based on their higher performance, a closer look at the effects of the treatment would also have been a desirable addition. The selection, assignment, and methodical monitoring of the tutors was not checked more closely, which could have provided important information about the success of the teams, the tutees, and the tutors, as well. Furthermore, the data sets could have been presented more precisely, including the comparison of the total number of correctly solved items to those correctly solved within two seconds. The operation skills concerning addition itself, however, were already in the inclusion criteria, as were the items examined as facts known by all participants but not yet automated by any of them. In addition to that, the time needed for the students to finish their racetracks had been stopped but not recorded as further data to underline the students' improvement. Future research should focus on the extent of the impact the training has on the time needed to finish the game board.

Finally, missing data concerning the incorrect answers is an important limitation as it does not allow a more accurate interpretation of the students' performance. This could have been given, for example, by using a digital version of the racetrack, collecting data about which tasks were solved incorrectly and which were solved correctly, or by the interventionists collecting these data in a hidden way using a protocol sheet as they did during measurement.

Practical Implications and Future Research

The results of this research support efforts to offer teachers additional reasons to use these methods to increase the number of automated single-digit addition facts in low-performing students. Even if the results cannot join the ranks of previous studies due to the expansion to include peer tutoring to bring them closer to being evidence- based related to the standards described in SCRIBE [56], they still should be used as supplementation to previous results, as the use of peer tutoring provides further clarity on the method’s suitability in challenging situations where teachers try to accommodate every level of learning.

This also represents one of the greatest advantages of the implemented economic method of the racetracks, as it is simple for teachers to implement, and—as this study highlighted—it also encourages peers to work together. Besides the simplicity of the implementation, the content is easily customizable to any learning content needed through, for example, small changes in the use of mathematical facts. Thus, if a struggling student needs to automate multiplication facts instead of single-digit addition tasks, this method is easily adaptable to his or her individual needs.

Follow-up research should investigate the extent of each aspect of this multicomponent motivational intervention more closely as well as the particular influence of the racetracks themselves compared to the peer-tutoring setting. In addition, further research should try to meet more of the standards for single-case studies, such as SCRIBE [56], so that researchers may finally be able to call the method evidence-based. In addition, the effects on the tutors’ performance should be considered in order to determine whether and to what extent they could also benefit from training in tasks they were already more familiar with than the tutees. Not only should future researchers take into account a tutors’ math competencies, but they should take a closer look at the impact on social skills, taking into account a methodical, continuous monitoring and training of the tutors with the aim of discovering whether it is possible to maximize profit for them as well.

Finally, but more relevant and current than ever, a digital version of the racetracks should be developed. This should not be difficult to achieve, as the digital approach is a promising one for increasing student motivation even more than the existing, implemented motivational aspects. In the area of digital support services concerning math fluency interventions, there is great need for further offerings on one hand and even more need for research meeting scientific standards on the other [20]. Besides the fact that digital learning seems to have a motivating effect, the need to support the development of digital skills in all students is an obvious one, with digitalization increasing in all sectors of society.

All in all, the racetrack, in combination with the multicomponent motivational system, is not only easy to implement, but also encourages peers to work together. Tutees as well as tutors enjoyed it, and it was a helpful way for the tutees to overcome hurdles. Methods such as this will not only relieve teachers in the classroom, but at the same time they will easily support children’s development.

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visualization, J.K.; supervision, J.K. All authors have read and agreed to the published version of the manuscript.

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Institutional Review Board Statement: Ethical review and approval were waived for this study, due to that no personal nor sensitive information that could allow conclusions to be drawn about the persons involved can be found in the study. In addition, there existed no risk but only the opportunity of benefit for the participants.

Informed Consent Statement: Patient consent was waived due to the fact that there is a contract between the cooperating school and the University of Cologne with permission to use data from interventions. Further, no diagnostics were used that do not take place in regular school life. The informed consent process therefore was appropriate and comprehensive for participants as well as custodians.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

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Appendix D: Paper 4 (*peer reviewed*)

Müllerke, N., Duchaine, E. L., Grünke, M. & Karnes, J. (2020). The effects of a response card instruction on the active participation in math lessons of five seventh graders with learning disabilities. *Insights into Learning Disabilities*, *16*(2), 107–120.

Abstract

In this single-case study, we examined the impact of a simple response card intervention on student engagement during math lessons. An ABA reversal across-subjects design was used to establish a causal relationship between the treatment and the expected outcome. Five adolescents with learning disabilities from a seventh-grade classroom were observed during hand-raising and response-card conditions to determine the effects of response cards on student responding and test scores. Results indicated that the intervention increased both participation and performance. The paper ends with a critical discussion of the results and future research challenges.

Keywords: Student Participation, Learning Disabilities, Response Cards, Math Instruction, Single- Case Research

Introduction

Instruction is defined as the process of transmitting skills and/or knowledge in such a way that students learn. In today's classrooms, this means integrating grade-level standards throughout the curriculum, teaching, and assessment (Engelmann & Carmin, 2016). Academic learning is a cognitive event. It is an interactive process that requires teachers not only to share information with students but also to ensure that they have grasped the knowledge (Parsons, Nuland, & Parsons, 2014). Because teaching and learning are interactive, instruction must include active engagement not only from the teacher, but also from the students (Brophy & Good, 1986). That is, teachers share information and students are expected to respond, including practicing. Complicating matters is the fact that all students do not enter a lesson with the same base of knowledge. Thus, educators are required to create lessons using evidence-based instructional practices to teach students at various achievement levels within the same classroom (Parsons et al., 2014). Skillfully designed lessons are critical in meeting the needs of students in classrooms worldwide.

Interactive Direct Instruction (Engelmann, 2017) is a scientific approach to teaching that enables educators to be more effective and efficient in conveying skills and knowledge to students across grade levels. Direct Instruction is often mistaken for teacher-based lecture accompanied by little student interaction. In reality, Direct Instruction means providing a concrete introduction of information followed by ongoing brisk-paced practice that receives immediate feedback (Watkins & Slocum, 2004). This almost errorless learning approach sets students up for success as they interact with new information until they reach mastery (Brophy & Good, 1986; Engelmann & Carmin, 2016; Watkins & Slocum, 2004).

The key goal of Direct Instruction is to provide students with the correct skill and/or content and then immediately involve them in the cognitive process of understanding and remembering. This, in turn, requires repetitive active participation followed by teacher confirmation of correct responses and/or corrective feedback to make learning as seamless as possible. During both whole- class and small-group instruction, students interact with the content by having multiple opportunities to respond (OtR) together, known as unison responding.

Research supports the use of various methods of unison responding, whereby all students respond to questions or prompts, simultaneously allowing them to practice and the teacher to assess their understanding before going on to the next learning target (MacSuga-Gage & Simonsen, 2015; Menzies, Lane, Oakes & Ennis, 2017; Twyman & Heward, 2018).

Choral responding and response cards are two well-researched methods of using unison responding to increase opportunities for students to respond. Both methods are evidence-based and support active participation and achievement, as well as high levels of time on task (Haydon, Marsicano, & Scott, 2013; Owiny, Spriggs, Sartini, & Mills, 2018).

In choral responding, students verbally answer the teacher's questions together when prompted. This approach is commonly applied across grade levels and content areas whether using scripted or unscripted Direct Instruction. When using response cards, students visually answer the teacher's questions together when prompted. That is, students present a response to the teacher using write-on or preprinted cards.

The format of response cards is almost limitless. Cards may be small white boards that allow students to use erasable markers to write their answers on before holding them up, or they may be preprinted cards with various options for choosing predetermined responses such as true/false, fact/opinion, multiple choice (A, B, C, D), numbers, math symbols, and so forth (Duchaine, Green, & Jolivette, 2011; Owiny et al., 2018). The various formats provide a

great deal of flexibility for students to participate. In short, response cards are an "easy-to-use teaching tactic derived from applied behavior analysis" (Twyman & Heward, 2018, p. 78), as repeatedly demonstrated in the literature across types of students, subjects, and grade levels. For example, the research supports using response cards for students with and without special education needs in inclusive classrooms (Duchaine, Jolivette, Fredrick & Alberto, 2018; Haydon, Richmond Mancil, & Van Loan, 2009; Narayan, Heward, Gardner, Courson, & Omness, 1990) and in both special classrooms and special schools for students with disabilities (Blood, 2010; Bondy & Tincani, 2018; Christle & Schuster, 2003; Davis & O'Neill, 2004; George, 2010). In addition, response cards have been found to be effective at both the elementary (Bondy & Tincani, 2018; Christle & Schuster, 2003) and the secondary level

(Adamson & Lewis 2017; Blood, 2010; Duchaine et al., 2018; George, 2010). The flexibility of response cards is demonstrated by their use in math (Adamson & Lewis, 2017; Christle & Schuster, 2003; Duchaine et al., 2018), science (Duchaine et al., 2018), social studies (Blood, 2010; George, 2010), and writing (Davis & O'Neil, 2004).

Purpose of the Present Study

The purpose of the present study was to replicate the Christle and Schuster's (2003) research on the use of response cards as a means of unison responding during Direct (math) Instruction. Specifically, we implemented response cards during math lessons using an ABA reversal across-subjects design to investigate the effect on student participation, specifically the number of student responses to teacher questions and performance on weekly quizzes. The teacher taught math in accordance with Direct Instruction principles and added response cards as an intervention.

Method

Participants and Setting

The study took place in a seventh-grade classroom of a rural school for students with special learning needs on the outskirts of a large metropolitan area in Western Germany. The main teacher selected the participants based on her observations of how intensively they had engaged in math lessons over past weeks, as measured by how frequently they raised their hands to respond in class. She identified five students (three males and two females) whom she deemed to be extraordinarily passive during math lessons as the target group.

Three of the students had a migrant background; one had only lived in Germany for a little over two years. All participants had been diagnosed with a learning disability (LD) by a multi-professional team. The diagnoses were based on a conception of LD aligned with the criteria outlined by Grünke and Morrison Cavendish (2016), who describe students with LD as those who "fail to develop the knowledge, skill, will, and self-regulation necessary to succeed in key subject areas" (p. 1), thus, including students with an IQ below average. In our case, intelligence level was measured using the Kaufman Assessment Battery for Children (KABC-II; Kaufman & Kaufman, 2004). The level of math proficiency was determined by scores on a standardized test (Moser Opitz et al., 2010).

All participants attended the same class in the aforementioned school. According to their teacher, their inactivity during math lessons was not due to a lack of language comprehension. Table 1 gives an overview over important participant characteristics.

Table 1. ***Demographic Characteristics of the Participants***

Name	Gender	Age	IQ	In Germany	Math Competence	Ethnicity
Student 1	male	12	74	for 2;5 years	class 5	Mongolian
Student 2	male	15	49	for 4;8 years	class 1	Serbian
Student 3	male	13	56	since birth	class 4	Russian
Student 4	female	13	56	since birth	class 2	German
Student 5	female	14	60	since birth	class 1	German

Our experiment was implemented in a highly structured and low arousal classroom where distractions were kept to a minimum in order to help everyone focus on learning. The students sat at tables of two in three consecutive rows, facing forward towards the desk and the board. The rows of tables were divided by an aisle.

Design

A single-subject multiple-baseline design (ABA) across participants was used (Horner et al., 2005) consisting of a baseline phase (without intervention) (A1), a treatment phase (using the response cards) (B), and a return-to-baseline condition (A2). A simple AB design does not allow for positing a cause-and- effect relationship. However, adding a second A phase (A2) and observing an increase in behavior only during the treatment phase strengthens the argument that it was the intervention that was responsible for the improvements (Riley- Tillman & Burns, 2009).

Materials

White 5.8 x 8.3 inch cards were used as response cards. They were laminated so the students could write on them with non-permanent markers. Students received markers and wipes to erase answers between questions. To capture students' participation in classroom activities, we designed an observation scale, on which any attempt to give an answer to a question was recorded. We also prepared six different exercise sheets consisting of 10 questions or math problems each. The format of the quizzes was kept identical. We also tried to keep the level of difficulty constant across the six sheets. Five of the questions focused on repetition, five on new teaching content, and five questions aimed at securing knowledge transfer to everyday contexts. The six sheets consisted of three pairs, each focusing on certain content that was supposed to be taught during one particular week. We administered one test at the beginning and one test at the end of each week (i.e., before the first math lesson and after the last math lesson of the week). For every fully correct answer, the students received one point. The lessons followed a carefully prepared plan, focusing on volumes and weights.

For each session, we created 15 questions that always required a particular solution to a math problem as a response and that were verbally posed to the students. (All materials are available from the first author upon request.)

Measures

The extent of active student participation in classroom activities was used as the key dependent variable. We used the aforementioned observation scale to document how often participants raised their hand or held up their response card to answer a question. In addition, we used the results on the quizzes to determine whether increases in participation led to increases in performance. For each week, we calculated the proportionate increase (in percent) between pre- and post-measurement. Which of the two test versions for each week was administered first to a particular participant was determined by chance. The observation scales were independently filled out by the main teacher and a graduate student of special education, who both sat at the back of the room. They also administered and scored all quizzes. Interrater reliability equaled 100% for both.

Procedures

Instruction was alternately provided by three female graduate students of special education. The experiment extended over a period of three weeks with five weekly lessons of 30 minutes each. On Monday, the instruction started at 9:15 am, on every other day of the week, it started at 10:20 am. Each session was systematically structured in accordance with basic Interactive Direct Instruction principles so students were able to build up their skills, with questioning being used to help them to make sense of a given task. The interventionists posed each of the prepared 15 questions to the class during each lesson such that every short sequence of instruction was separated by a question.

During baseline conditions, the interventionists motivated the students to actively participate. That is, at the beginning of each lesson, they encouraged the students to try to answer each question that they would ask during the next 30 minutes and to raise their hands often. Before the first lesson of the B phase, the interventionists instructed the whole class on how to use the response cards, as follows: (a) write down the answer, (b) hold up the card, (c) erase the answer, and (d) put down the card and marker. This process was practiced for 5 minutes. Then the interventionists again encouraged the students to actively engage in classroom activities, only this time they were asked to raise their completed response cards instead of their hands. The conditions during the A2 phase resembled the ones of the A1 phase.

Participation in classroom activities was documented by counting the number of responses to questions (either by raising a hand or a response card) during each of the 15 lessons. Proficiency level was assessed before each of the three Monday sessions and after the end of each of the three Friday sessions. Even though we were only interested in how the five target students performed, we administered the test to the whole class.

Results

Figure 1 shows the number of indications to respond to the interventionists' questions (RtQ) during the three phases. For all figures and statistical analyses, we used the SCAN package for R by Wilbert (2019).

As illustrated, four of the participants had rather stable baselines, while Student 1 showed a trend in A1. All of them improved their performance during intervention and returned to lower scores when response cards were not used any more (A2).

Student 1 averaged 6.80 RtQs (range = 2-10) in A1. The measurements during this phase showed a clear upward trend. However, as soon as the intervention was implemented, performance not only continued to improve, but the data indicated a significant leap. That is, on the first two days of phase B, student 1 responded to every single question that the interventionist posed to the class. In fact, RtQs reached a mean of 14.40 during treatment, which corresponded with an average increase of 211.76% (range 13-15). Regardless of the trend in the A1, each score in the B phase exceeded those of the two A phases. The return to the second baseline phase (A2) coincided with a change in level, with the average RtQ decreasing by 77.78%, to 11.20 (range 9-12).

Student 2 scored an average of 3.00 RtQ (range 2-6) in A1. The intro- duction of the intervention was accompanied by a performance leap from 2 on day 5 to 15 on day 6. His mean value of RtQs during the B phase equaled 14.50 (range 14-15), which parallels an increase of 483.33%. After returning to baseline conditions (A2), his mean achievement dropped by 64.14%, to an average of 5.20 RtQs (range 1-10).

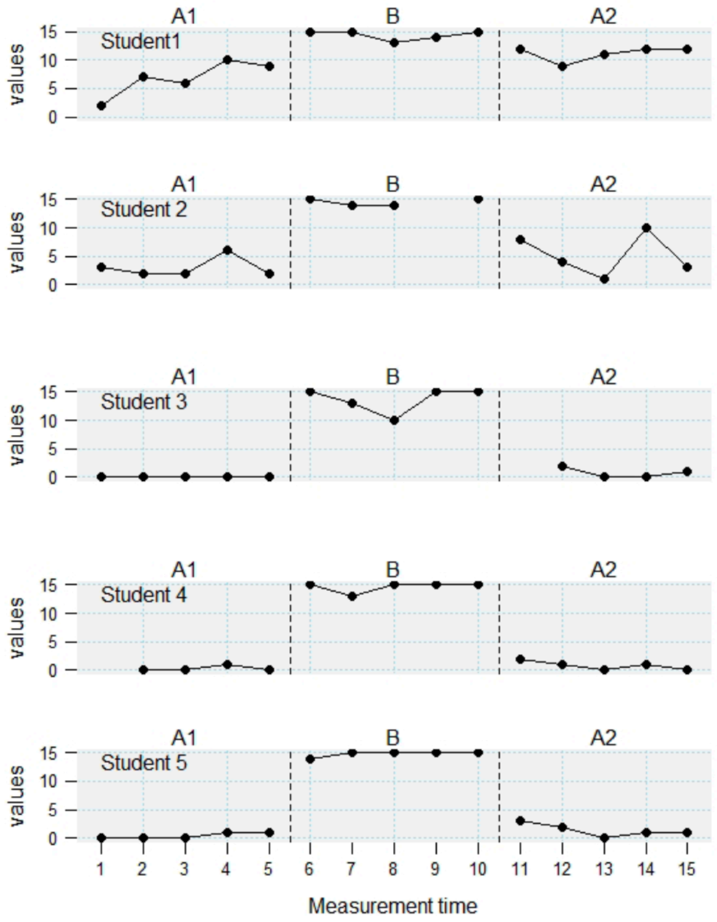

Figure 1. **Number of RtQs by the five participants in the three phases.**

Student 3 did not show any attempt to participate in classroom activities during the first baseline condition (A1), but with the start of the intervention, her performance increased from 0 to the maximum value of 15. She reached 13.60 RtQs, on average, during phase B (range 10-15) (a percentage increase could not be calculated due to an average value of 0 during the baseline phase). Just as remarkable as the increase in value from A1 to B, there was a distinct performance drop from B to A2, with an average performance of 0.75 (range 0-2), which corresponds to a decrease by 94.49%.

For Student 4, the mean RtQ value in A1 was 0.25 (range 0-1), which increased to a mean score of 14.60 (range 13-15) during B (this parallels an impressive percentage increase of 5,740). After the treatment stopped, his achievement dropped to a mean score of 0.80 (range 0-2) (which equals a decrease of 94.52%).

Student 5 started in A1 with an average RtQ value of 0.40 (range 0-1) and – like all the other students – showed an immediate increase in level with the beginning of the treatment. She rose from 1 RtQ on day 5 to 14 on day 6. Her mean value during intervention showed an increase from 0.40 to 14.80 (range 14-15) (which corresponds to a leap of 3,600%). With the end of the treatment phase, her performance dropped by 90.54%, to an average score of 1.40 (range 0-3).

Four of the most common non-overlap effect sizes comparing phases A1 and A2 to phase B were calculated: PND (percentage of non-overlapping data), PEM (percentage of data exceeding the median), PEM-T (percentage of data exceeding the median trend), NAP (non-overlap of all pairs) (Alresheed, Hott, & Bano, 2013). In each case, the participants received the highest possible outcome of 100%.

Next, a piecewise regression analysis was applied to each participant (Huitema & McKean, 2000). The results of this analysis are presented in Table 2.

Table 2. ***Piecewise Regression for Number of RtQs***

	B	*SE*	*t*	*p*	*R²*
		Student 1			
Intercept	1.70	1.52	1.11	0.29	
Trend	1.70	0.46	3.71	0.01**	0.15
Level Phase B	4.50	1.89	2.38	0.04*	0.06
Level Phase A2	-3.90	1.89	-2.06	0.07	0.05
Slope B	-1.80	0.65	-2.78	0.02*	0.08
Slope A2	0.40	0.65	0.62	0.55	0.00
		Student 2			
Intercept	2.40	3.01	0.8	0.45	
Trend	0.20	0.91	0.22	0.83	0.00
Level Phase B	10.94	3.75	2.92	0.02*	0.18
Level Phase A2	-8.23	3.98	-2.07	0.07	0.09
Slope Phase B	-0.14	1.33	-0.11	0.92	0.00
Slope Phase A2	-0.46	1.33	-0.34	0.74	0.00
		Student 3			
Intercept	0.00	1.70	0.00	1.00	
Trend	0.00	0.51	0.00	1.00	0.00
Level Phase B	13.00	2.18	6.14	0.00**	0.17
Level Phase A2	-12.20	2.95	-4.14	0.00**	0.08
Slope Phase B	0.20	0.73	0.28	0.79	0.00
Slope Phase A2	-0.50	0.89	-0.56	0.59	0.00
		Student 4			
Intercept	-0.10	1.26	-0.08	0.94	
Trend	0.10	0.34	0.29	0.78	0.00
Level Phase B	13.60	1.03	13.23	0.00**	0.16
Level Phase A2	-13.00	1.00	-13.01	0.00**	0.16
Slope Phase B	0.10	0.42	0.24	0.82	0.00
Slope Phase A2	-0.60	0.34	-1.75	0.12	0.00
		Student 5			
Intercept	-0.50	0.65	-0.78	0.46	
Trend	0.30	0.19	1.54	0.16	0.00
Level Phase B	13.20	0.80	16.47	0.00*	0.16
Level Phase A2	-12.30	0.80	-15.35	0.00*	0.14
Slope Phase B	-0.10	0.28	-0.36	0.72	0.00
Slope Phase A2	-0.70	0.28	-2.55	0.03*	0.00

Note: * Significant at the 5% level; ** significant at the 1% level.

In summary, the analyses yielded significant level effects from A1 to B for all participants, and from B to A2 for all except Student 2, whose values slightly failed to reach statistical significance ($p = 0.07$). However, aggregating the five cases into one as part of a level 2 analysis resulted in very clear level effects between phases (see Table 3).

Table 3. ***Piecewise Regression Model for Number of RtQs***

	B	*SE*	*t*	*p*
Intercept	0.63	1.54	0.41	0.68
Trend	0.48	0.35	1.37	0.18
Level Phase B	11.02	1.38	8.01	0.00**
Level Phase A2	-9.77	1.45	-6.76	0.00**
Slope B	-0.35	0.48	-0.73	0.47
Slope A2	-0.44	0.48	-0.91	0.37

Note: ** Significant at the 1% level.

Finally, we considered possible gains in math performance. Student 2 was not able to complete the quizzes, because he was otherwise engaged. The rest of the participants attended all six testing sessions. Figure 2 depicts the proportionate pre-/post-improvements of each student for Week 1 (A1), Week 2 (B), and Week 3 (A2)

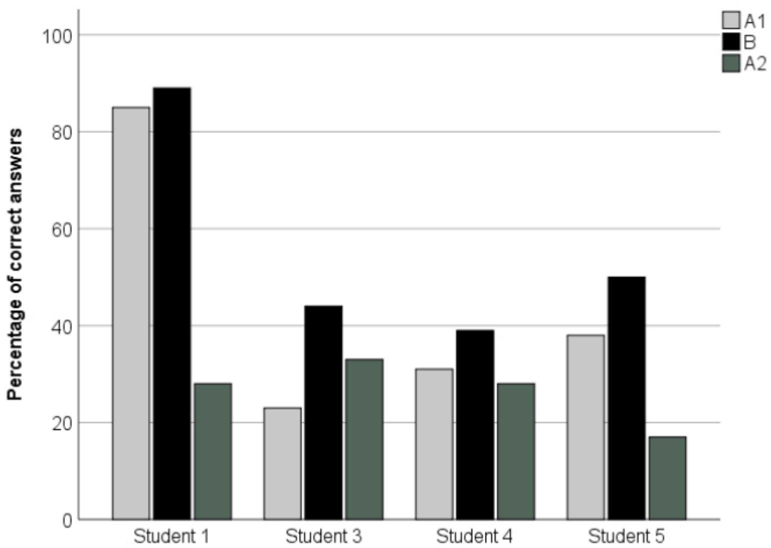

Figure 2. Proportionate increase in math competence during the three phases for Students 1, 3, 4, and 5.

As illustrated, the performance increase in Week 2 was always larger than in any other week. This confirms the assumption that there was always growth in learning, but the gains were never as large as when the response cards were used.

Discussion

This study examined the effects of a response card intervention based on Interactive Direct Instruction principles on the engagement in classroom activities during math lessons of five typically unengaged seventh graders with LD. The results indicate that in all cases the number of RtQs increased strikingly as soon as the cards were introduced. Improvements from the baseline condition to the treatment phase reached statistical significance in all five cases with non-overlap indices reaching their maximum value of 100%. The drops in performance were equally striking as soon as the response cards were no longer in use. In addition, we tested students' performance level at the beginning and the end of each week. When the response cards were used, the students achieved a higher growth in learning than if they were just encouraged to actively participate by raising their hands.

Despite these impressive results, the study is subject to certain limitations. First, the small sample size and the fact that all lessons were geared toward teaching a particular topic limit the generalizability of the results. Second, the selection of the participants was left to the discretion of the main teacher. No clear-cut criteria were used. This makes replication of the results difficult. Third, the observers were not blind to the purpose of the study. They knew what the implementation of the response card intervention was aiming at. Fourth, we tried to keep the level of difficulty equal across the performance quizzes. However, we have not tested to what extent we achieved that goal. Finally, we used a reversal design (ABA) with only one treatment phase. Even though student engagement increased greatly from A1 to B, and subsided equally marked from B to A2, we cannot be sure if the differences would have been comparably distinct if we had incorporated another B phase. The participants were without a doubt very responsive to the intervention. However, part of that may be due to the fact that the response cards were new and unfamiliar to them. Thus, it is possible that a habituation effect would have set in if we had continued with the changes of phases (e.g. by applying an ABAB or an ABABAB design).

Giving these limitations, the practical implications of this study nevertheless support the systematic use of response cards during instruction to increase OtR and classroom participation, which in turn results in increased content mastery. Based on our participants, who presented as uninterested or uncertain about their ability to respond to questions in class, when given the OtR within the safety of unison responding, each demonstrated an interest in participating. This is of no small importance. Many teachers struggle greatly as they try to involve all of their students in classroom activities and be mindful of students who are shy and reluctant to raise their hands in class.

All too often, educators pose questions to the whole class and create situations in which the more able and outgoing learners feel encouraged to respond, whereas the more timid stay in the background. Response cards seem to be an excellent way of involving even the most diffident students to participate. Holding up a piece of cardboard along with everyone else in the class does not seem very intimidating. However, encouraging learners to do so at every given OtR seems to get them more into a lesson and to acquire more of the curriculum content that is being taught. Thus, response cards offer an easy-to-implement and low-cost solution to the challenge of engaging even the most reluctant students.

Our findings replicate those of Christle and Schuster (2003) and add to the growing body of research on the use of response cards with students with special needs (e.g., Cakiroglu, 2014; Didion, Toste, & Wehby, in press; Good- night, Whitley, & Brophy-Dick, in press; Rao, 2018). Even though our study sheds some light on this quick-and-easy way to increase student response rates in lessons, a number of research questions still need to be addressed in order to widen the knowledge base on this kind of intervention. For example, future studies may consider collecting data over a longer span of time and with more reversal phases to either support or dispute the possibility that increased participation may be the result of the novelty of using response cards for the first time. Another consideration for future research is to include a sample of students who regularly participate. This will allow researchers to investigate the effect response cards have on the participation and performance of students perceived to ready be active participants.

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Appendix E: Declaration of Performance

Paper 1 (*peer reviewed)*

Grünke, M., Karnes, J., & Hisgen, S. (2019). The effects of a multicomponent motivational intervention on math performance of elementary school students with learning disabilities. *Insights into Learning Disabilities*, *16*(1), 23–35.

Conceptualization and elaboration of the research idea	With Co-authors
Literature research & evaluation	With Co-authors
Creating the research design	With Co-authors
Instrumentation	With Co-authors
Selection of instruments	With Co-authors
Construction of new instruments/procedures	With Co-authors
Data collection & /-management	With Co-authors
Data Analysis	With Co-authors
Selection of statistical tests/analyses	With Co-authors
Performance of the statistical tests/analyses	With Co-authors
Interpretation of the statistical tests/analyses	With Co-authors
Writing the publication	With Co-authors
First draft	With Co-authors
Revision & finalizing	With Co-authors

Paper 2 (*peer reviewed)*

Karnes, J., Barwasser, A. & Grünke, M. (2021). The effects of a math racetracks instruction on the single-digit multiplication facts fluency of four struggling elementary school students. *Insights into Learning Disabilities*, *18*(1), 1–25.

Conceptualization and elaboration of the research idea	Overall control
Literature research & evaluation	With Co-authors
Creating the research design	With Co-authors
Instrumentation	Overall control
Selection of instruments	Overall control
Construction of new instruments/procedures	With Co-authors
Data collection & /-management	Overall control
Data Analysis	With Co-authors
Selection of statistical tests/analyses	With Co-authors
Performance of the statistical tests/analyses	With Co-authors
Interpretation of the statistical tests/analyses	With Co-authors
Writing the publication	With Co-authors
First draft	With Co-authors
Revision & finalizing	Overall control

Paper 3 (*peer reviewed)*

Karnes, J. & Grünke, M. (2021). The effects of a multicomponent motivational system intervention using peer- tutoring for implementation on the automation of single-digit addition tasks of four struggling elementary school students. *Education in Sciences*, *11*(265), 1–14.

Conceptualization and elaboration of the research idea	Overall control
Literature research & evaluation	Overall control
Creating the research design	Overall control
Instrumentation	Overall control
Selection of instruments	Overall control
Construction of new instruments/procedures	Overall control
Data collection & /-management	Overall control
Data Analysis	Overall control
Selection of statistical tests/analyses	Overall control
Performance of the statistical tests/analyses	Overall control
Interpretation of the statistical tests/analyses	With Co-author
Writing the publication	Overall control
First draft	Overall control
Revision & finalizing	With Co-author

Paper 4 (*peer reviewed)*

Müllerke, N., Duchaine, E. L., Grünke, M. & Karnes, J. (2020). The effects of a response card instruction on the active participation in math lessons of five seventh graders with learning disabilities. *Insights into Learning Disabilities*, *16*(2), 107–120.

Conceptualization and elaboration of the research idea	With Co-authors
Literature research & evaluation	With Co-authors
Creating the research design	With Co-authors
Instrumentation	With Co-authors
Selection of instruments	With Co-authors
Construction of new instruments/procedures	With Co-authors
Data collection & /-management	With Co-authors
Data Analysis	With Co-authors
Selection of statistical tests/analyses	With Co-authors
Performance of the statistical tests/analyses	Overall control
Interpretation of the statistical tests/analyses	With Co-authors
Writing the publication	With Co-authors
First draft	With Co-authors
Revision & finalizing	With Co-authors

Appendix F: Declaration of Independence

Declaration of independence (according to §11 (1) 8)

“Ich versichere eidesstattlich, dass ich die von mir vorgelegte Dissertation selbständig und ohne unzulässige Hilfe angefertigt, die benutzten Quellen und Hilfsmittel vollständig angegeben und die Stellen der Arbeit einschließlich Tabellen, Karten und Abbildungen, die anderen Werken im Wortlaut oder dem Sinn nach entnommen sind, in jedem Einzelfall als Entlehnung kenntlich gemacht habe; dass diese Dissertation noch keinem anderen Fachbereich zur Prüfung vorgelegen hat; dass sie noch nicht veröffentlicht worden ist, sowie dass ich eine solche Veröffentlichung vor Abschluss des Promotionsverfahrens nicht vornehmen werde. Die Promotionsordnung ist mir bekannt. Die von mir vorgelegte Dissertation ist von Prof. Dr. Matthias Grünke betreut worden.”

"I affirm in lieu of an oath that I have prepared the dissertation submitted by me independently and without unauthorized assistance, that I have fully indicated the sources and aids used, and that I have identified in each case as borrowed those passages of the work, including tables, maps, and illustrations, which are taken from other works in wording or meaning; that this dissertation has not yet been submitted to any other department for examination; that it has not yet been published, and that I will not undertake such a publication before completion of the doctoral examination procedure. I am aware of the doctoral regulations. The dissertation I have submitted has been supervised by Prof. Dr. Matthias Grünke."

Köln, den 17.09.2021

Jennifer Karnes

Appendix G: List of publications

*Karnes, J., & Grünke, M. (2021). The effects of a multicomponent motivational system intervention using peer-tutoring for implementation on the automation of single-digit addition tasks of four struggling elementary students. *Education Sciences, 11*(265), 1–14.

*Hisgen, S., Klöpfer, C., **Karnes**, J., & Grünke, M. (2021). Fachbeitrag: Die Einflüsse motivierender Methoden auf das Verfassen von Texten von Schüler/innen der Sekundarstufe mit Förderschwerpunkt Lernen. *Vierteljahresschrift für Heilpädagogik und ihre Nachbargebiete*.

***Karnes**, J., Barwasser, A., & Grünke, M. (2021). The effects of a math racetrack intervention on the single-digit multiplication facts fluency of four struggling students. *Insights in Learning Disabilities: A Contemporary Journal*, *18*(1), 93–110.

*Müllerke, N., Duchaine, E. L., Grünke, M., & **Karnes**, J. (2019). The effects of a response card intervention on the active participation in math lessons of five seventh graders with learning disabilities. *Insights into Learning Disabilities*, *16(2),* 107–120.

*Grünke, M., **Karnes**, J., & Hisgen, S. (2019). Effects of explicit timing on the reading fluency of third graders with learning challenges. *Journal of Education and Training Studies, 7*, 1–9.

*Grünke, M., Urton, K., & Karnes, J. (2018). The effects of a brief touch point intervention for children with intellectual developmental disabilities (IDD). *Journal of Educational and Developmental Psychology*, *8*(2), 187–196.

peer-reviewed