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Impedance Control for a Flexible Robot Enhanced with Energy Tanks in the port-Hamiltonian Framework

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***Abstract*— In modern robotics, the manipulators are no longer isolated under fully controlled conditions but rather conceived to work in unconstrained environments. Under these operations, compliant control and passivity properties of the robot are of great importance, and thus the system's energy function plays a crucial role in the control design. In this work, we propose a new design of cartesian impedance control for a flexible robot whose dynamics is represented within the port-Hamiltonian framework. To improve the performance of the system and maximize the capabilities of the robot, the robotic control system is enhanced with energy tanks that allow for temporarily non-passive operations, but ensure the passivity of the extended system. In addition, a secondary controller is designed using the port-Hamiltonian approach to cover the case of redundant robotic manipulators. The performance of the full control system is tested via simulations of the Kuka iiwa manipulator in closed loop with the proposed passivity-based controller. The results show a satisfactory performance of the control system for set-point regulation, external forces, time-varying reference trajectories, and parametric uncertainty.**

I. INTRODUCTION

Advances in robotics have led to new and more challenging tasks to be performed, both from the industrial and research points of view. Modern applications require robotic manipulators with the capability to work in a shared unconstrained environment, where unexpected contact forces are part of the regular scenario. This is in contrast with a classical assumption where the robot works in an isolated cage. This change of paradigm has led to the development of lighter, more flexible robots that, equipped with joint torque sensors, are well suited to dynamically interact with the environment [1].

The described scenario imposes, from the control perspective, the need for compliant controllers to consider those interaction forces acting on the robot. A classical strategy is the impedance control proposed by Hogan in [2] and still used nowadays in modern robots, such as in [3] where an impedance controller is used to react to collisions on the robot. In some cases, the lack of previous knowledge of the task to be accomplished can be mitigated by using time-varying parameters in the control law. For instance, in [4], the authors implemented a variable impedance control along with a redundancy handling strategy to improve collaborative tasks. Another important aspect of the new lightweigthed robots, such as KUKA LWR IV+ or KUKA LWR iiwa, is that they have shown to possess a non negligible joint mechanical flexibility, which increase the difficulty of the controllers design. In [5], the authors proposed a framework for control design of this flexible manipulators, and analysed the application of different type of controllers when the motor dynamics and joint flexibility are considered. Then, the particular case of impedance control was studied in-depth in [6]. Recently, a variable impedance control law for flexible joint robots was proposed in [7].

The concept of passivity has become a standard property for safety when the robot operates in unknown environments [8]. Nevertheless, this condition might be a limitation for some controllers, specially when the robot parameters vary over time. Recently, the concept of energy tanks was introduced as a formalism to overcome the performance limitation of previous passive approaches [9], [10], [8], [11], [12]. By storing the dissipated energy in a virtual tank, non-passive actions can be performed, as long as there is enough energy in the tank. Then, if there are no interactions with the environment, the total energy of the extended system will not increase, and thus passivity of the extended system will be preserved. In the context of teleoperation, the energy tank approach was used in [9]. The authors studied the lost of passivity due to the data loss that might produce jumps in the position reference. As the end effector is attached through a virtual spring to this reference, this jumps in the trajectory might result in a non-passive operation. Therefore the authors proposed a framework named *passive set-position modulation* to attach the other side of the spring (also connected to the robot position) to a new virtual reference. This reference is generated based on the original desired trajectory, but modulated by the energy stored in a reservoir, so to ensure that the system remains passive.

A formalization of the energy tank idea was developed in [10] and applied in the context of physical human-robot interaction. The objective was to adjust the inertia matrix of the admittance controller to avoid oscillations, which results in potential non-passive operations. Then, the authors proposed to include the energy tank concept to ensure passivity of the control system. In a previous work, the authors also used the energy tank approach to adjust the stiffness matrix of the controller [8]. Recently, along with the variations of the stiffness matrix, the authors in [11] considered the effect of the null space controller in the passivity of the system. As they considered a redundant manipulator, the loose of passivity could appear due to variations in the main controller but also due to the null space controller. In their work, the energy tank approach is used as a solution to preserve the

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passivity of the extended control system.

In [12], the authors used a similar concept to ensure the passivity in a force-impedance control. In that work, the impedance controller is enhanced with a force tracking algorithm, which might result in a non-passive operation as the force reference might change over time. This problem is addressed by applying the force tracking control action only when there is enough energy in the tank, otherwise only the impedance controller is used, and thus passivity is preserved.

A common aspect in all previous works is the important role of the system's energy function and the power exchange with the environment. For these reasons, the port-Hamiltonian (PH) formalism seems attractive as PH models are described in term of energy properties [13]. Also, PH models are well-suited for control design based on passivity properties for general physical system and in particular for mechanical systems, see e.g. [14], [15]. In [16], the PH framework was used in a teleoperation problem to describe the desired dynamics of the cartesian behaviour, but the tank dynamics was not integrated in the PH formulation.

In this work, we propose a novel approach to represent a flexible robot in closed loop with a new cartesian impedance control as a port-Hamiltonian system. The passivity of the closed-loop system is proved for set-point tasks. Thereafter, we consider time variable references that might result in the lost of passivity of the system. Then, the addition of energy tanks in the PH dynamics allows passivity to be ensured even for time varying references. The proposed controller is also extended to consider redundant robotic manipulators. In addition, the full closed-loop dynamics is written in the PH form, so that the passivity of the control system can be readily ensured.

The paper is organised as follows. Section II introduces basic concepts of PH systems and flexible robot dynamics. Section III presents the cartesian impedance controller developed within the PH formalism. Section IV extends the control design by using energy tanks. An additional secondary controller for redundant robots is designed in V. Section VI shows the performance of the controller for a 7-DoF robot in simulations. Conclusions and perspectives of this work are discussed in Section VII.

II. PRELIMINARIES

A. Port-Hamiltonian Systems

Many physical systems can be described as PH systems, whose dynamics has the form [17]:

$$\begin{cases} \dot{x} = [J(x) - R(x)]\nabla H(x) + g(x)u \\ y = g^T(x)\nabla H(x). \end{cases} \tag{1}$$

where x, u an y are the state, input and output vectors, respectively. The matrix $J(x) = -J^T(x)$ describes the power-preserving interconnection structure of the system, the matrix $R(x) = R^T(x) \geq 0$ represents the dissipation and the function $H(x)$ is the Hamiltonian, which represents the total energy of the system.[1] An important property of the PH systems is that they are passive with input u, output y and storage function $H(x) \geq 0$ [17].

B. Flexible joint manipulators

In this paper, we consider robotic manipulators where the flexibility of the joints and the apparent inertia of the motor cannot be neglected, and we use the model proposed by Spong in [18]:

$$\begin{aligned} M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + \nabla_q V(q) &= \tau_a + \tau_e, \\ B\,\ddot{\theta} + \tau_a &= \tau_m \\ \tau_a &= K(\theta - q) + D(\dot{\theta} - \dot{q}) \end{aligned} \tag{2}$$

where q is the link coordinate vector, θ is the motor angle vector, the control input τ_m is the vector of motor torques, and τ_e is the joint torques due to external forces. The dynamics of the link is characterised by the inertia matrix $M(q)$, the Coriolis matrix $C(q,\dot{q})$, and the potential function $V(q)$ whose gradient represents the gravity torque vector. The visco-elastic phenomena in the joints is described by the torque τ_a, with stiffness and damping matrices K and D, respectively, and the inertia of the motors is represented by the matrix B.

The Euler-Lagrange dynamics (2) can be equivalently written in the PH form

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{p} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & I_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & I_{n\times n} \\ -I_{n\times n} & 0_{n\times n} & -D & D \\ 0_{n\times n} & -I_{n\times n} & D & -D \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_\theta H \\ \nabla_p H \\ \nabla_s H \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tau_e \\ \tau_m \end{bmatrix}, \tag{3}$$

where $p = M(q)\dot{q}$, $s = B\dot{\theta}$, and Hamiltonian

$$\begin{aligned} H(q,\theta,p,s) = &\frac{1}{2}p^T M^{-1}(q)p + \frac{1}{2}s^T B^{-1} s \\ &+ \frac{1}{2}(\theta - q)^T K(\theta - q) + V(q). \end{aligned} \tag{4}$$

III. CARTESIAN IMPEDANCE CONTROL FOR A FLEXIBLE MANIPULATOR

In this section, we propose an impedance control using the nonlinear model of the flexible manipulator (2). The objective is to approximate a target behaviour (i.e. the one of an ideal mass-spring-damper system) and preserve passivity for the input-output pair given by external forces and link velocities. Notice that the control input and the external forces are non-collocated, which complicates the design. We follow the approach in [19] and we use the control law

$$\tau_m = K_F\left[\tau_e - C(q,\dot{q})\dot{q} - \nabla_g V(q)\right] - K_G\,\tau_a + K_H\,\tau_u, \tag{5}$$

with the non-linear gains are defined as:

$$K_F = -BK^{-1}(K_e - K)M(q)^{-1} \tag{6}$$

$$K_G = [BK^{-1}K_e B_e^{-1} - K_F - I_n] \tag{7}$$

$$K_H = BK^{-1}K_e B_e^{-1}, \tag{8}$$

[1]We denote with the symbol ∇ to the gradient operator, i.e. $\nabla H(x) := \frac{\partial H(x)}{\partial x}$, which is a column vector in this paper.

where K_e and B_e are the tuning parameters representing desired stiffness and inertia matrices for the closed loop dynamics, while I_n is the identity matrix of appropriate dimensions. The new control input τ_u will be used later to design an impedance controller in the cartesian space. As shown in [19], the dynamics of the system (3) in closed loop with the controller (5) can be written in the PH form

$$\begin{bmatrix}\dot{q}\\ \dot{\varphi}\\ \dot{p}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}0 & 0 & I & 0\\ 0 & 0 & 0 & I\\ -I & 0 & -D_e & D_e\\ 0 & -I & D_e & -D_e\end{bmatrix}\begin{bmatrix}\nabla_q\mathcal{H}\\ \nabla_\varphi\mathcal{H}\\ \nabla_p\mathcal{H}\\ \nabla_z\mathcal{H}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \tau_e\\ \tau_u\end{bmatrix}, \quad (9)$$

with Hamiltonian

$$\begin{aligned}\mathcal{H}(q,\varphi,p,z) &= \frac{1}{2}p^T M^{-1}(q)p + \frac{1}{2}z^T B_e^{-1} z\\ &\quad + \frac{1}{2}(\varphi - q)^T K_e(\varphi - q) + V(q),\end{aligned} \quad (10)$$

with desired damping $D_e = DK^{-1}K_e$ and a change of coordinates defined as:

$$\begin{aligned}\varphi &= K_e^{-1}(K_e - K)q + K_e^{-1}K\theta\\ z &= B_e K_e^{-1}(K_e - K)M^{-1}(q)p + B_e K_e^{-1} K B^{-1} s\end{aligned} \quad (11)$$

The details and proof can be found in [19]. The controller (5) can reduce the apparent motor inertia and, eventually, the stiffness of the joint can also be adjusted by a suitable selection of the controller gains.

We now use the available control input τ_u and propose a new cartesian impedance controller that is more appropriate for interaction tasks at the end effector level than the joint space controller, as contact forces are usually described in the cartesian space. The following proposition summarises the result.

Proposition 1: Consider the system (9) in closed loop with the controller

$$\tau_u = -J(\varphi)^T K_x(f(\varphi) - x_d) - J(\varphi)^T D_x J(\varphi)\dot{\varphi} + \tau_{u_2}, \quad (12)$$

where K_x and D_x are positive definite matrices, τ_{u_2} is the new available control input that will be used later to design an additional control for manipulators with redundancy, $f(\varphi)$ is the forward kinematics that transforms from φ into cartesian positions x, x_d are the desired cartesian postions, and the Jacobian of the manipulator is defined as $J(\varphi) := \frac{\partial^T f(\varphi)}{\partial \varphi}$.

Then, the following statements hold true:

1) The closed loop dynamics can be represented in the PH form

$$\begin{bmatrix}\dot{q}\\ \dot{\varphi}\\ \dot{p}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}0 & 0 & I & 0\\ 0 & 0 & 0 & I\\ -I & 0 & -D_e & D_e\\ 0 & -I & D_e & -D^*\end{bmatrix}\begin{bmatrix}\nabla_q\mathcal{H}_1\\ \nabla_\varphi\mathcal{H}_1\\ \nabla_p\mathcal{H}_1\\ \nabla_z\mathcal{H}_1\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \tau_e\\ \tau_{u_2}\end{bmatrix}, \quad (13)$$

with $D^* = D_e + J(\varphi)^T D_x J(\varphi)$, and closed-loop Hamiltonian

$$\begin{aligned}\mathcal{H}_1(q,\varphi,p,z) &= \frac{1}{2}p^T M^{-1}(q)p + \frac{1}{2}z^T B_e^{-1} z\\ &\quad + \frac{1}{2}(\varphi - q)^T K_e(\varphi - q) + V(q)\\ &\quad + \frac{1}{2}(f(\varphi) - x_d)^T K_x(f(\varphi) - x_d)\end{aligned} \quad (14)$$

2) The closed-loop dynamics (13) is passive with inputs (τ_e, τ_{u_2}), outputs $(\dot{q}, \dot{\varphi})$ and storage function (14).

Proof: To prove the first statement, we notice that the first row of (9) and (13) are

$$\dot{q} = M^{-1}(q)p, \quad (15)$$

which shows the exact matching of these equations. Similarly, the second and third rows of (9) and (13) are

$$\dot{\varphi} = B_e^{-1}(q)z, \quad (16)$$

and

$$\begin{aligned}\dot{p} &= -\nabla_q[\frac{1}{2}p^T M^{-1}(q)p] + K_e(\varphi - q)\\ &\quad - D_e M^{-1}(q)p + D_e B_e^{-1} z + \tau_e,\end{aligned} \quad (17)$$

which shows that the equations match.

Then, the last row of (9) is

$$\begin{aligned}\dot{z} &= -\nabla_\varphi\mathcal{H} + D_e\nabla_p\mathcal{H} - D_e\nabla_z\mathcal{H} + \tau_u\\ &= -K_e(\varphi - q) + D_e M^{-1}(q)p - D_e B_e^{-1} z + \tau_u,\end{aligned}$$

in which we use (12) to obtain

$$\begin{aligned}\dot{z} &= -K_e(\varphi - q) + D_e M^{-1}(q)p - D_e B_e^{-1} z\\ &\quad - J(\varphi)^T K_x(f(\varphi) - x_d) - J(\varphi)^T D_x J(\varphi)\dot{\varphi} + \tau_{u_2}\end{aligned}$$

Finally, using $z = B_e\dot{\varphi}$ and the definition of the Jacobian matrix, it follows that

$$\begin{aligned}\dot{z} &= -K_e(\varphi - q) + D_e M^{-1}p - D_e B_e^{-1} z\\ &\quad - \frac{\partial f(\varphi)}{\partial \varphi}K_x(f(\varphi) - x_d) - J(\varphi)^T D_x J(\varphi) B_e^{-1} z + \tau_{u_2}\\ &= -K_e(\varphi - q) - \frac{\partial f(\varphi)}{\partial \varphi}K_x(f(\varphi) - x_d) + D_e M^{-1}p\\ &\quad - D^* B_e^{-1} z + \tau_{u_2}\\ &= -\nabla_\varphi\mathcal{H}_1 + D_e\nabla_p\mathcal{H}_1 - D^*\nabla_z\mathcal{H}_1 + \tau_{u_2},\end{aligned}$$

which matches the last row of the closed-loop dynamics in (13), which proves the first statement of the proposition.

To prove the passivity of the system, we use the Hamiltonian function (14) as storage function and we compute its time derivative as follows

$$\begin{aligned}\dot{\mathcal{H}}_1 &= [\nabla_q\mathcal{H}_1]^T\dot{q} + [\nabla_\varphi\mathcal{H}_1]^T\dot{\varphi} + [\nabla_p\mathcal{H}_1]^T\dot{p} + [\nabla_z\mathcal{H}_1]^T\dot{z}\\ &= [\nabla_q\mathcal{H}_1]^T[\nabla_p\mathcal{H}_1] + [\nabla_\varphi\mathcal{H}_1]^T[\nabla_z\mathcal{H}_1]\\ &\quad + [\nabla_p\mathcal{H}_1]^T(-\nabla_q\mathcal{H}_1 - D_e[\nabla_p\mathcal{H}_1] + D_e\nabla_z\mathcal{H}_1 + \tau_e)\\ &\quad + [\nabla_z\mathcal{H}_1]^T(-\nabla_\varphi\mathcal{H}_1 + D_e[\nabla_p\mathcal{H}_1] - D^*\nabla_z\mathcal{H}_1 + \tau_{u_2})\\ &= -\begin{bmatrix}[\nabla_p\mathcal{H}_1]^T & [\nabla_z\mathcal{H}_1]^T\end{bmatrix}\begin{bmatrix}D_e & -D_e\\ -D_e & D^*\end{bmatrix}\begin{bmatrix}\nabla_p\mathcal{H}_1\\ \nabla_z\mathcal{H}_1\end{bmatrix}\\ &\quad + \dot{q}^T\tau_e + \dot{\varphi}^T\tau_{u_2} \le \dot{q}^T\tau_e + \dot{\varphi}^T\tau_{u_2},\end{aligned}$$

which proves the passivity of the closed-loop system (13). ■

IV. TIME VARYING REFERENCE TRAJECTORY

A. Problem statement

In the previous section, the desired trajectory x_d was considered constant. However, in many scenarios the trajectory of interest is time varying, which complicates the control design.

We consider the desired time-varying trajectory $x_d(t)$ and we compute the time derivative of the Hamiltonian (14) as follows

$$\begin{aligned}\dot{\mathcal{H}}_1 = &- \left[[\nabla_p \mathcal{H}_1]^T \; [\nabla_z \mathcal{H}_1]^T\right] \begin{bmatrix} D_e & -D_e \\ -D_e & D^* \end{bmatrix} \begin{bmatrix} \nabla_p \mathcal{H}_1 \\ \nabla_z \mathcal{H}_1 \end{bmatrix} \\ &+ [\nabla_{x_d} \mathcal{H}_1]^T \dot{x}_d + \dot{q}^T \tau_e + \dot{\varphi}^T \tau,\end{aligned} \tag{18}$$

which shows that since the term $[\nabla x_d \mathcal{H}_1]^T \dot{x}_d$ is non sign defined, then the passivity property cannot be ensured directly as done in the *Proposition 1*. To overcome this problem, we propose a design using energy tanks.

B. Energy tank approach

Following the approach in [10], we augment the dynamics of the system by including the state of the tank $n(t)$, whose dynamics is

$$\dot{n} = \frac{\alpha(t)}{n(t)} P_D(t) - \frac{1}{n(t)} \left| P_X(t) \right|, \tag{19}$$

where $P_D(t)$ is the power dissipated by the robot and $P_X(t)$ the power injected by the time varying reference x_d. The absolute value is used so that the energy variations due to a change of the reference will always drain the tank and thus it avoids compromising the passivity of the system. From (18), we define $P_D(t)$ and $P_X(t)$ as follows

$$P_D(t) = \left[[\nabla_p \mathcal{H}_1]^T \, [\nabla_z \mathcal{H}_1]^T\right] \begin{bmatrix} D_e & -D_e \\ -D_e & D^* \end{bmatrix} \begin{bmatrix} \nabla_p \mathcal{H}_1 \\ \nabla_z \mathcal{H}_1 \end{bmatrix}, \tag{20}$$

$$P_X(t) = [\nabla x_d \mathcal{H}_1]^T \dot{x}_d \tag{21}$$

The variable $\alpha(t) \in \{0, 1\}$ allows the dissipated power to fill the tank. The tank level can be limited by setting $\alpha(t) = 0$ when the level reaches a maximum values. The minimum level of the tank can be limited by either using another variable as $\alpha(t)$ or rendering $P_X(t) = 0$, which will prevent draining the tank below its minimum level. The energy stored in the tank is defined as

$$T(n) = \frac{1}{2} n^2, \tag{22}$$

and its time derivative is

$$\dot{T}(n) = n \, \dot{n} = \alpha(t) P_D(t) - \left| P_X(t) \right|, \tag{23}$$

which satisfies the balance of power. As discussed in [10], $n(t)$ could become zero if the tank is completely drained, leading to a singularity in (19), therefore the minimum level of the tank should be set above zero.

C. PH representation of the augmented system

In this section, we describe the dynamics of the system augmented by the tank in the PH form. The preservation of the Hamiltonian structure is desirable because it simplifies the passivity analysis and readily evidences the energy of the system and power exchange between the states that represent the tank and the robot. The next proposition shows the dynamics of the extended system in the PH form.

Proposition 2: Consider the system (13) with the storage energy (14) and the dynamic of the tank (19), with the modification to ensures that the power injected by P_X will always absorb energy from the tank. Then, the following statements hold true.

1) The closed-loop dynamics of the augmented system can be written in the PH form

$$\begin{bmatrix} \dot{q} \\ \dot{\varphi} \\ \dot{p} \\ \dot{z} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ -I & 0 & 0 & 0 & d_1 \\ 0 & -I & 0 & 0 & d_2 \\ 0 & 0 & -d_1^T & -d_2^T & -d_3 \end{bmatrix} \begin{bmatrix} \nabla_q \mathcal{H}_2 \\ \nabla_\varphi \mathcal{H}_2 \\ \nabla_p \mathcal{H}_2 \\ \nabla_z \mathcal{H}_2 \\ \nabla_n \mathcal{H}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \tau_e \\ \tau_{u_2} \\ 0 \end{bmatrix}, \tag{24}$$

with

$$d_1 = \frac{\alpha}{n}(-D_e \nabla_p \mathcal{H}_2 + D_e \nabla_z \mathcal{H}_2) \tag{25}$$

$$d_2 = \frac{\alpha}{n}(D_e \nabla_p \mathcal{H}_2 - D^* \nabla_z \mathcal{H}_2) \tag{26}$$

$$d_3 = \frac{1}{n^2(t)} \left| P_X(t) \right| = \frac{1}{n^2(t)} \left| [\nabla_{x_d} \mathcal{H}_2]^T \dot{x}_d \right|, \tag{27}$$

and augmented Hamiltonian function

$$\begin{aligned}\mathcal{H}_2(q, \varphi, p, z, t) = &\frac{1}{2} p^T M^{-1}(q) p + \frac{1}{2} z^T B_e^{-1} z + V(q) \\ &+ \frac{1}{2}(\varphi - q)^T K_e (\varphi - q) + \frac{1}{2} n^2 \\ &+ \frac{1}{2}(f(\varphi) - x_d(t))^T K_x (f(\varphi) - x_d(t))\end{aligned} \tag{28}$$

2) The augmented system (24) is passive with inputs (τ_e, τ_{u_2}), outputs $(\dot{q}, \dot{\varphi})$ and storage function (28).

Proof: To prove the augmented dynamics can be written in the PH form (24), we first notice that the state equations for q and φ in (13) and (24) are the same. Second, we will show that the equations for $\dot{p}$ and $\dot{z}$ in (13) and in (24) match. Indeed, from the third row of (13), we obtain

$$\begin{aligned}\dot{p} &= -\nabla_q \mathcal{H}_1 - D_e \nabla_p \mathcal{H}_1 + D_e \nabla_z \mathcal{H}_1 + \tau_e \\ &= -\nabla_q \mathcal{H}_1 + \frac{1}{n}(-D_e \nabla_p \mathcal{H}_1 + D_e \nabla_z \mathcal{H}_1) n + \tau_e \\ &= -\nabla_q \mathcal{H}_2 + d_1 \nabla_n \mathcal{H}_2 + \tau_e,\end{aligned}$$

which matches the third row of (24). From the fourth row of (13), we obtain

$$\begin{aligned}\dot{z} &= -\nabla_\varphi \mathcal{H}_1 + D_e \nabla_p \mathcal{H}_1 - D^* \nabla_z \mathcal{H}_1 + \tau_{u_2} \\ &= -\nabla_\varphi \mathcal{H}_1 + \frac{1}{n}(D_e \nabla_p \mathcal{H}_1 - D^* \nabla_z \mathcal{H}_1) n + \tau_{u_2} \\ &= -\nabla_\varphi \mathcal{H}_2 + d_2 \nabla_n \mathcal{H}_2 + \tau_{u_2},\end{aligned}$$

which matches the fourth row of (24). Finally, using the dynamic of the tank defined in (19), we obtain

$$\begin{aligned}\dot{n} &= \frac{\alpha}{n}P_D - \frac{1}{n}|P_X| \\ &= \frac{\alpha}{n}[-[\nabla_p\mathcal{H}_1]^T D_e[\nabla_p\mathcal{H}_1] + 2[\nabla_p\mathcal{H}_1]^T D_e[\nabla_z\mathcal{H}_1] \\ &\quad - [\nabla_z\mathcal{H}_1]^T D^*[\nabla_z\mathcal{H}_1]] - \frac{1}{n}|P_X| \\ &= -d_1^T\nabla_p\mathcal{H}_2 - d_2^T\nabla_z\mathcal{H}_2 - d_3\nabla_n\mathcal{H}_2,\end{aligned}$$

which is exactly the last row of (24).

To prove the passivity property of (24), we use the Hamiltonian (28) as storage function and compute its time derivative as follows

$$\begin{aligned}\dot{\mathcal{H}}_2 &= [\nabla_q\mathcal{H}_2]^T\dot{q} + [\nabla_\varphi\mathcal{H}_2]^T\dot{\varphi} + [\nabla_p\mathcal{H}_2]^T\dot{p} \\ &\quad + [\nabla_z\mathcal{H}_2]^T\dot{z} + [\nabla_n\mathcal{H}_2]^T\dot{n} + [\nabla x_d\mathcal{H}_2]^T\dot{x}_d \\ &= -\left[[\nabla_p\mathcal{H}_2]^T \;\; [\nabla_z\mathcal{H}_2]^T\right]\begin{bmatrix} D_e & -D_e \\ -D_e & D^*\end{bmatrix}\begin{bmatrix}\nabla_p\mathcal{H}_2 \\ \nabla_z\mathcal{H}_2\end{bmatrix} \\ &\quad + n\left[-d_1^T\nabla_p\mathcal{H}_2 - d_2^T\nabla_z\mathcal{H}_2 - d_3\nabla_n\mathcal{H}_2\right] \\ &\quad + [\nabla x_d\mathcal{H}_2]^T\dot{x}_d + \dot{q}^T\tau_e + \dot{\varphi}^T\tau \\ &= -\left|[\nabla_{x_d}\mathcal{H}_2]^T\dot{x}_d\right| + [\nabla x_d\mathcal{H}_2]^T\dot{x}_d + \dot{q}^T\tau_e + \dot{\varphi}^T\tau \\ &\leq \dot{q}^T\tau_e + \dot{\varphi}^T\tau,\end{aligned} \tag{29}$$

which shows that the system (24) is passive. ■

Notice that the structure of the PH dynamics (24) was built such that the energy dissipated by the mechanical system fills the tank up, which can be noticed by the fact that the terms d_1 and d_2 appear in the interconnection matrix instead of the dissipation matrix. This energy is stored in the tank and it will be dissipated when required to preserve the passivity of the augmented system. Also, when the tank reaches a minimum level then the desired trajectory should be adjusted so that $P_X(t)$ becomes zero, meaning that no more energy is injected into the system.

V. REDUNDANCY HANDLING

For redundant manipulators, it is possible to include a signal in the control that actuates on the null space of the Jacobian matrix and, therefore, does not affect the motion of the end effector. A generalized expression of the control law for redundant manipulators has the form

$$u = J^T(q)u_c + [I_n - J^T(q)\bar{J}^T(q)]u_{ns}, \tag{30}$$

where $J(q)$ is the Jacobian of the manipulator, u_c is the primary control law for the end effector's task, $\bar{J}(q)$ is the generalized inverse of the Jacobian matrix, and u_{ns} is the secondary law, which can be used for example to stabilize the motion in the null space or to avoid singularities. A particular choice for the generalized inverse is

$$\begin{aligned}\bar{J}(q) &= M^{-1}(q)J^T(q)\Lambda(q) \\ &= M^{-1}(q)J^T(q)[J(q)M^{-1}(q)J^T(q)]^{-1},\end{aligned} \tag{31}$$

which minimizes the instantaneous kinetic energy of the manipulator [20]. It is important to note that this choice of the inverse (using the virtual cartesian inertia matrix $\Lambda(q)$) might present singularities. In addition to the problem of computing the matrix inverse, undesired high joint velocities might appear in the surroundings of singularities. In this work, we consider the robot far from singularities.

In the following proposition, we present a secondary control that minimises the joint velocities, which might be important from a safety point of view as it avoids unnecessary movements.

Proposition 3: Consider the system (24) and the input τ_{u_2} given by the secondary control law defined as follows:

$$\tau_{u_2} = -[I_n - J^T(q)\bar{J}^T(q)]k_{ns}M(q)\dot{\varphi}, \tag{32}$$

where k_{ns} is a positive constant, and the generalized inverse of the Jacobian matrix defined as in (31). We assume that, the parameter k_{ns} is selected to satisfy

$$D_x + k_{ns}M(q) - k_{ns}\Lambda(q) \geq 0. \tag{33}$$

Then, the following statements hold true.

1) The closed-loop dynamics of the new full system with the secondary control law can be written in the PH form with the same structure as (24), energy function $\mathcal{H}_3$ identical to (28), and the function d_2 redefined as follows

$$d_2 := \frac{\alpha}{n}(D_e\nabla_p\mathcal{H}_3 - [D^* + D_{ns}]\nabla_z\mathcal{H}_3), \tag{34}$$

with

$$D_{ns} = k_{ns}M(q) - k_{ns}J^T\Lambda^T J \tag{35}$$

2) The closed-loop system is passive with input (τ_e), outputs $(\dot{q}, \dot{\varphi})$ and storage function (28).

Remark: The structure of the tank dynamics with the addition of the secondary control has the form (19). However, since the redefinition of d_2 in (34) includes the additional dissipation term D_{ns}, injected by the secondary controller, the function $P_D(t)$ takes the form

$$P_D(t) = \left[[\nabla_p\mathcal{H}_3]^T\,[\nabla_z\mathcal{H}_3]^T\right]\begin{bmatrix} D_e & -D_e \\ -D_e & (D^* + D_{ns})\end{bmatrix}\begin{bmatrix}\nabla_p\mathcal{H}_3 \\ \nabla_z\mathcal{H}_3\end{bmatrix} \tag{36}$$

Proof: To show that the structure of the dynamics is in the form (24), we notice that the only difference is the redefinition of d_2, which appears in the state equations of z and n, and the remaining equations do not change. The dynamics of the tank given by $\dot{n}$ is modified according to the redefinition of the function d_2 in (34). Then, it is sufficient to show that the secondary control (32) renders the dynamics of z as in the fourth row of (24) but with d_2 redefined as in (34). From (24) we obtain

$$\begin{aligned}\dot{z} &= -\nabla_\varphi\mathcal{H}_2 + d_2\nabla_n\mathcal{H}_2 - [I_n - J^T(q)\bar{J}^T(q)]k_{ns}M(q)\dot{\varphi} \\ &= -\nabla_\varphi\mathcal{H}_2 + \frac{\alpha}{n}(D_e\nabla_p\mathcal{H}_2 - D^*\nabla_z\mathcal{H}_2)\nabla_n\mathcal{H}_2 \\ &\quad - \left[k_{ns}M - k_{ns}J^T\Lambda^T J\right]\nabla_z\mathcal{H}_2 \\ &= -\nabla_\varphi\mathcal{H}_3 + \frac{\alpha}{n}[D_e\nabla_p\mathcal{H}_3 - (D^* + D_{ns})\nabla_z\mathcal{H}_3]\nabla_n\mathcal{H}_3\end{aligned}$$

which matches the desired dynamics with the new definition of d_2 in (34). The passivity property follows *vis-à-vis* the proof in Proposition 2, that is using Hamiltonian as storage function and exploiting the structure of the PH dynamics. ■

VI. CONTROL SYSTEM PERFORMANCE

In this section we present simulation results to assess the performance of the proposed control system. We apply the full control law given by (5), (12), (32) to the system including the tank (equivalent to Proposition 3). We simulate the model of the Kuka iiwa manipulator described in [21] with the joint flexibility, damping, and motor inertia obtained from its predecessor robot, the Kuka LWR IV+ in [22][2].

The parameters of the proposed controller were selected to reduce the influence of the motor inertia and adapt the visco-elastic phenomena: $K_e = 10\,K$, $B_e = 0.01\,B$ and $D_e = 10\,D$. Also, the gain of the redundancy control is $k_{ns} = 5$ and the initial condition of the tank is $n(0) = 2$. The desired impedance behaviour in the cartesian space is characterised by a mass-spring-damper (MSD) system with stiffness $K_x = diag\{60, 60, 60, 60, 60, 60\}Nm^{-1}$ and damping $D_x = diag\{20, 20, 20, 20, 20, 20\}Nm^{-1}s$. Since the goal of the cartesian impedance control is to make the closed-loop system behave as a MSD system, we performed a comparative simulation with the ideal MSD system. Also, we compare the proposed controller with a state of the art impedance controller presented in [23], which is used as baseline. This baseline controller is designed to exactly cancel the robots dynamics and impose a target MSD linear dynamics. The baseline controller was selected to have a damping and stiffness matrices D_x and K_x, respectively, and the inertia was selected such that the damping ratio coefficient is $\zeta = 0.7$.

Figure 1 shows the time responses of the closed-loop systems subject to set-point changes. It can be seen that the response of the controller proposed in this work is very close to the response of the desired MSD system. In comparison with the baseline controller, our control system shows less oscillations and faster rate of convergence. The reason for the difference can be attributed to the fact that the baseline controller is highly dependent on the model due to the exact nonlinear cancellation, hence, not considering the robot's flexibility degrades significantly the results. The proposed control system, whilst not being able to modify the apparent inertia of the end-effector, seems to be best suited for a flexible robot and less dependent on the model errors. The small differences between our control system and the target MSD system can be due to the fact that the robot flexibility makes it impossible for the robot to behave exactly as the MSD system. Notice also that the Coriolis and centrifugal terms are not cancelled, which makes the controller less sensitive to model errors and enhances the robustness of the closed loop.

The second simulation shows the manipulator performing a repetitive circular movement in the Y-Z plane, which emulates for example a polishing task. It is important to notice that the trajectory described by the robot does not follow perfectly the reference, as the parameters of the impedance controller were chosen to provide a compliant

[2]To the best of the authors' knowledge, no full identification of the motor dynamics and joint flexibility of the Kuka iiwa is available in the literature.

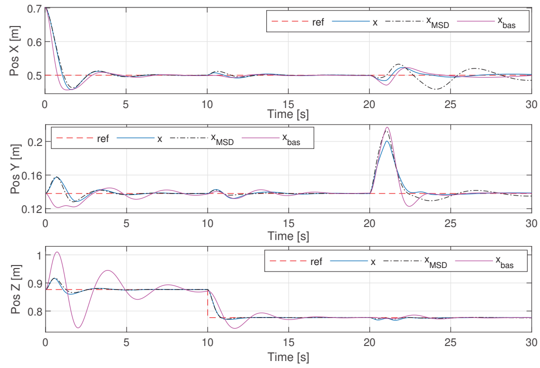

Fig. 1. Cartesian positions for the robot's end effector: proposed controller (blue), desired MSD system (black, dash-dot), baseline controller (magenta). There is a set-point change in Z at $t = 10s$, and an external force of 4 N is applied in the Y direction at $t = 20s$ and for $1s$.

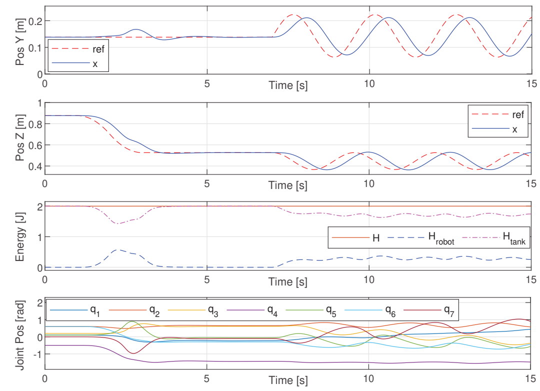

Fig. 2. Position of the end-effector in Y and Z axis and the desired trajectories (first and second plot). The total system energy (orange), the robot energy (dashed blue) and the tank energy (dash dotted magenta) are shown in the third plot. The joint positions are shown in the fourth plot.

behaviour for the robot, while accomplishing the task. As the sinusoidal reference for the movement in each axis varies over time (top two images of Fig. 2), the energy of the robot increases, but the tank dissipates its own energy in such a way that the total energy of the system is preserved. This behaviour can be seen in the third plot of Fig. 2, as the the total energy of the system and the energy stored in the robot and the tank are represented. It is evident how the variation of the robot's energy is reflected in the variation of the tank's energy. The last plot of Fig. 2 shows the joint positions. The joint velocities are minimized by the secondary controller, which is clear in the interval before the circular motion task ($t = 3s$ to $t = 7s$). Notice that during the dynamic scenario the controller is only able to reduce the joint velocities in the null space but some velocity drift remains. A possibility would be to use a secondary law to make the robot return to a given configuration. It is also worth noticing that the tank should not be emptied at any time. To avoid that, different approaches can be considered, from disconnecting the non-passive part of the system when a minimum is reached as in [12] and [8], or an optimization of the parameter adaptation

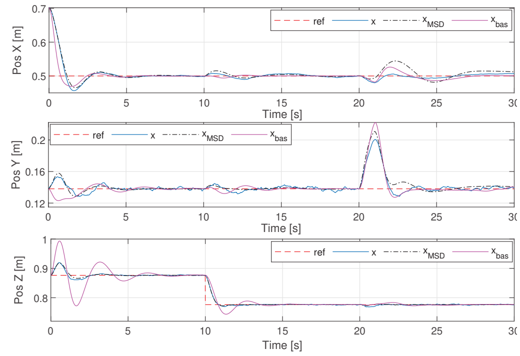

Fig. 3. Cartesian positions for the robot's end effector: proposed controller (blue), desired MSD system (black, dash-dot), baseline controller (magenta). Similar experiment conditions than Fig. 1 but with errors in the parameters and noise on the joint torque measurement.

as in [10] and [9].

Finally, as the proposed controller uses parameters such as joint flexibility, the inertia matrices of the motor and links, we repeated simulations of Fig. 1 but considering that the parameter K was overestimated by a 50% while B, D and inertia of the robot were underestimated also by a 50% of their real value. Also, zero mean Gaussian white noise with a variance of $0.2Nm$ was added in the torque τ_a in (5) to represent the measurement noise of a joint torque sensor or coming from the measurement of the angles. For the baseline controller we also considered the same parameter uncertainty. Results are shown in Fig. 3, which shows that, despite the adverse conditions, the closed-loop system exhibits a close behaviour to the MSD system. Also, the performance, although slightly different compared to the results without parameter uncertainty, is satisfactory. The small oscillations are due to the added noise, which could be suppressed by adding an appropriate filter.

VII. CONCLUSIONS AND PERSPECTIVES

In this paper we designed a cartesian impedance controller for robots with flexible joints. We further augmented the design using energy tanks to allow for time varying trajectories and a redundancy controller to minimise the joint velocities. The full controller has been developed in the port-Hamiltonian framework, which simplifies the analysis of passivity properties. The simulation results show a satisfactory performance of the proposed design and a good approximation to the ideal behaviour of the target MSD system. Another advantage of the port-Hamiltonian formalism is that it readily shows the fundamental role of the energy exchange in the passivity of the system.

In future works, we will explore different methods to ensure that the tank level is always above a minimum value to then evaluate our approach on a real robot manipulator.

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