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Least Square Modification of Stokes Formulae with Additive Corrections Estimator for Klang Valley Geoid Modeling

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Abstract. Klang Valley is a fast-growing area and its development shall be equivalent with precise measurements for a precise vertical reference. Thus, existing vertical reference with 3 centimetres (cm) is inadequate and processed with complicated remove-compute-restore (RCR) procedure. Apart from this, areas such as Klang Valley should better than one (1) centimetre level vertical reference. Meanwhile processing method for vertical reference should be simplified and easy tasking. Because of that, methodology for this study is by employing the least squares modification of Stokes formula with additive corrections (KTH). This approach fully uses anomalies rather than residuals which it is more complicated. At the same time, the additive corrections estimator introduced combining the direct and indirect computations method. Datasets used in this study were refined rigorously prior to the gridding scheme in cross validation, free air anomalies, as well as anomaly correction. The KTHKVGM2020 gravimetric and geometric geoid models are evaluated from the reference position using GNSS levelling. It found that KTHKVGM2020 Geoid model is better than one (1) centimetre for Klang Valley area with efficiency processing method. Therefore, the study is an essential in future to develop high-precision geoid model with efficient methods particular for urban and rapidly developing areas.

1. Introduction

Demand of using geoid model increasing as well as it is a vertical reference datum measurement for engineering works. Recently, high-capacity engineering works such as high-speed railways construction and other construction activities required accurate information, low costing for construction work and faster information. Meanwhile measurement task for previous geoid model is expensive and inefficient. For example, levelling data for mountain area requires lot of data collection, more workforce and expenses. In addition, processing procedure to produce a quality geoid model is difficult task to understand. Geoid model such as MyGEOID [1] and Klang Valley High Modernization System (HMS) previously were produced by RCR method [2] and involve with complicated processing [3]. Recently, many researchers and scientists from various countries introduce much simpler the geoid modelling and computation in producing a good quality of the geoid models. One of the familiar methods is least square modification stokes formula with additive corrections or called KTH method [4]. KTH method with additive corrections was introduced by

Professor E Lars Sjoberg from KTH Royal Institute of Technology, Sweden. Procedures using KTH method are easier to develop a quality geoid model and not complicate to handle.

Currently, geoid model in Malaysia developed using RCR method involves with large areas study. Similarly, geoid model produced by researchers and scientists from University of Technology Mara (UiTM) involve with regional area [5]. Therefore, it is appropriate to produce a small or local area size geoid model for special purpose use [6] in this study. According to a study conducted at Mediterranean that the appropriate area to produce a geoid model is not more than 50 kilometers square [7]. KTH method are involved with direct and indirect of combinations correction for topographic and atmosphere, atmosphere as well as ellipsoid correction. KTH additive corrections are implementing with full anomalies without reduction rather than using residual.

The aim of this study is to identify the KTH method with additive corrections estimator better results with classical method (i.e. RCR) particular for a small area. This study will conduct and engage high, medium and short wavelength frequency data acquisition as well. It is including with global geopotential model (GGM) data, digital elevation model (DEM), terrestrial and airborne gravity data as well as GNSS leveling data. All acquisition data are rigorous refined in particular to determine the degree of variance of the signal and noise to GGM, best method for cross validation and gridding scheme as well. Meanwhile free air of gravity anomalies and bouguer for gravity data as well as additive corrections are determined. In other hand, the KTHKVGM2020 gravimetric and geometric geoid model validated with local vertical datum using GNSS leveling as well. The studies area located at $2^o \geq \phi \leq 4^o$ and $100^o30 \geq \lambda \leq 102^o$ including the cap size.

2. Methodology

Geoid height determination is based on the original Stokes formula [8] an expressed in equation (1).

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma o} S(\Psi)\, \Delta g\, d\sigma \quad (1)$$

where;

R	=	mean earth radius
γ	=	normal gravity on ellipsoid reference
$S(\Psi)$	=	stokes function
Δg	=	gravity anomaly on geoid
$d\sigma$	=	surface element of unit sphere σ

The studies engage with multi sources of acquisition data with high-pass filtering. Least square modification of stokes formula is seeking to revoke and minimize potential of coefficients as well as gravity anomaly into a spherical harmonic in term of boundary value problem (BVP). The studies involve additive corrections estimator in the term of direct and indirect combine correction. The least square modification of stoke formula with additive corrections as expressed in equation (2) [4],[5],[9],[10],[11].

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma o} S^L(\Psi)\Delta g d\sigma + \frac{R}{2\pi}\sum_{n=0}^{M}(Q_n^L + S_n)\Delta g_n^{GGM} + \delta N_{comb}^{T} + \delta N^{DWC} + \delta N_{comb}^{a} + \delta N^{e} \quad (2)$$

where;

Δg_n^{GGM}	=	Laplace harmonic of degree n for GGM
b_n	=	$S_n + Q_n^L$; $2 \leq n \leq M$
S_n	=	modification of parameter
Q_n^L	=	Molodensky truncation coefficients of limited L
δN_{comb}^{T}	=	combined topographic effect
δN^{DWC}	=	downward continuation effect

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δN^{a}_{comb} = combined atmosphere effect
δN^{e} = ellipsoidal effect

3. Results and Discussion

3.1 Data Quality

The acquisition data are engaged from short, medium or long wavelength frequency. However, data multi discipline sources need proper engage in order to minimal the bias [12]. The acquisitions data engaged are GGM, DEM, terrestrial and airborne gravity data as well as GNSS leveling data.

GGM model is highly concerned in this research study and best GGM model much related with estimate errors of GGM refer to the location and method be used. The studies evaluated GGM03C, GIF48, DGM-1S, GOGRA02S and EGM2008 model base on 1,276 benchmarks. Table 1 shows analysis of selected EGM2008 GGM model is the best model to used and shows 0.566 meter better than other models in term of root mean square.

Table 1. Statistical Analysis of GGM model

GGM Model	GGM03C	GIF48	DGM-1S	GOGRA02S	EGM2008
Max	-3.681	-3.833	-3.987	-4.044	-3.074
Min	-6.014	-6.273	-5.996	-6.155	-4.500
Mean	-4.702	-4.972	-4.924	-5.021	-3.781
RMSE	0.772	0.776	0.726	0.726	0.566

[Unit: Meters]

The quality of DEM in horizontal and vertical accuracy depends on grid size [11]. Table 2 shows analyses of statistics DEM model of SRTM is the best model to evaluate with 3.024 meter better than other models in term of root mean square error.

Table 2. Statistical Analyses of DEM model

DEM Model	SRTM	GTOPO	GLOBE
Max	66.782	44.980	36.998
Min	-38.981	-54.657	-65.882
Mean	5.299	7.332	6.455
RMSE	3.024	3.611	3.592

[Unit: Meters]

There are 1,276 terrestrial gravity data and 458 airborne gravity data are selected to engage with this study. The study provided free air gravity anomaly for offshore gravity and bouguer gravity anomaly for onshore gravity determine as well as obtains actual gravity value along the Earth surface. Free air gravity anomaly, g_{FA} formula for this study expressed with equation 3. The residuals of free-air gravity anomalies, Δg_{FA} illustrated in histogram Figure 1. Gravity data show in Figure 1 that good quality and condition because of difference residuals with EGM2008 and not exceed than 10 mGal [11],[13].

$$g_{FA} \quad = g_{obs} - \gamma + \frac{\partial g}{\partial h}\Delta h \tag{3}$$

where;
g_{obs} = gravity observation
Δh = differential of elevation height

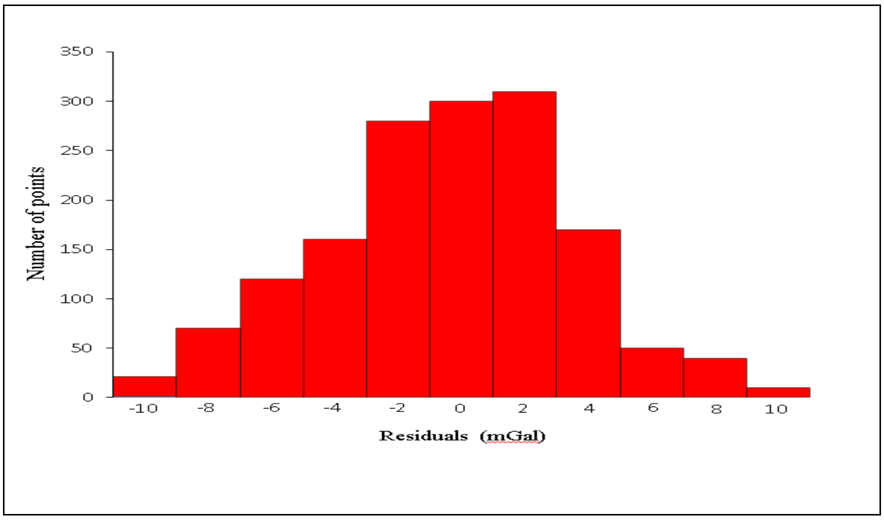

Figure 1. Histogram of residuals free-air anomalies, Δg_{FA} for terrestrial and airborne gravity with EGM2008

Free air elevation correction need to downward continuation to geoid surface in order to equivalent with mean sea level and eliminate the difference gap. All gravity data are incorporated with global dataset and associated with free air anomaly using the second order approximation [14]. Therefore, ellipsoid free air anomaly or gravity disturbance, δg_{FA} denoted at equation (4) with GNSS technique [8].

$$\delta g_{FA} \quad = g_{ob} - \gamma + \delta\gamma\,(h) \tag{4}$$

Meanwhile bouguer anomaly is hidden by underground mass and removing from masses influence. Terrestrial gravity data implemented to reduce erroneous on geoid surface. GPS data are implemented and seeking for ellipsoid bouguer anomaly appropriate as height on reference ellipsoid. To accomplish the elevation and bouguer anomaly corrections will be combined denoted at equation (5) [15].

$$g_B \quad = g_0 + 0196\Delta h \tag{5}$$

On the other hand, gravity value calculated by gravity standard formulae usually not match with each other because of invisible anomalous mass involved with direct or indirect. It is call gravity anomaly, γ expressed on equation (6) [16].

$$\gamma \quad = g\,(1 + \beta\sin^2\phi - \beta'\sin^2 2\phi) \tag{6}$$

where;
g = standard gravity value
ϕ = latitude
β = $\frac{5}{2}\frac{\omega^2 a}{g} - \varepsilon - \frac{17}{14}\frac{\omega^2 a}{g}\varepsilon$

β' = $\frac{\varepsilon}{8}\left(\frac{5\omega^2 a}{g} - \varepsilon\right)$
ω = 7292115.10^{-11} rad s^{-1}
ε = $\frac{E}{a}$
E = $\sqrt{a^2 - b^2}$
a = major radius of earth
b = polar radius of earth

3.2 Combined Topographic Correction, δN^{T}_{comb}

Topographic correction involves with indirect and direct effects correction. Combine topographic correction of Stoke modification be lot of benefits and depend on terrain effects [17],[18]. This study area almost flat topographic surface, gravity anomaly of stoke integral are not much effect with discretization error and equation (7) [19]. The study using standard different topographic density,ρ 2.67 g/cm³ [8] and this studies involve 5%, 10% and 20% density differ respectively as shows on the Table 3.

$$\delta N^{T}_{comb} = \delta N_{dir} + \delta N_{indir} \approx - \frac{2\pi G\rho}{\gamma}\left[H^2 + \frac{2}{3r}H^3\right] \quad (7)$$

where;
r = R + H

Table 3. Statistical Analysis of Combine Topographic Effect on % Density Mass Differ

	MIN	MEAN	MAX	Std. Dev
Standard Density (2.67g/m³)	-0.034	-0.010	0	0.028
Density Differ 5%	-0.039	-0.013	0	0.021
Density Differ 10%	-0.033	0.011	0	0.02
Density Differ 20%	-0.029	0.009	0	0.017

[Unit: Meters]

Table 3 shows that 17, 20 and 21 millimeters with different topography density respectively in term of standard deviation. It is also to shows that combine topographic correction with 20% density topographic differs is better results than others. Apart from this, high resolution to handle maximize interpolation of acquisition data is essential [20],[21]. Figure 2 shows result distribution of topographic geoid surface after combine topographic correction and with negative height when implementation equation (7).

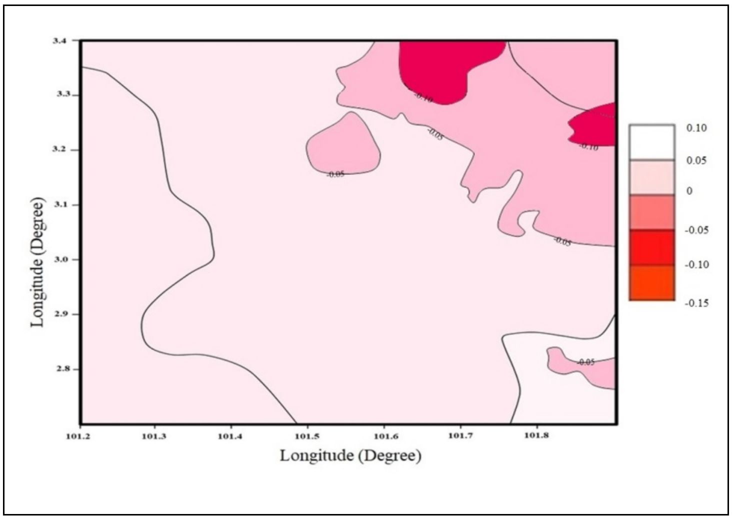

Figure 2. Combine Topographic Correction Effects
[Unit: Meters]

3.3 Downward Continuation Effect, δN_{DWC}

Klang Valley geoid estimation surface needs downward continuation to geoid surface layer after combine topographic correction, in order to fulfill BVP of spherical integral [22]. It is to minimum bias from spherical surface and equivalent with mean sea level (MSL) when fitting to a local datum. Best method based on least square modification parameters denoted at equation (8). Spherical effect on equation (8) is refers to vertical along earth surface and consult with height anomaly, ζ as well as surface divided by normal gravity at normal height,γ [23].

$$\delta N_{DWC} = \delta N^{(1)}_{DWC} + \delta N^{L1,Far}_{DWC} + \delta N^{L2}_{DWC} \tag{8}$$

where;

$$\delta N^{(1)}_{DWC} = \frac{\Delta g}{\gamma} H + 3\frac{\zeta^0}{r} H - \frac{1}{2\gamma}\frac{\partial \Delta g}{\partial r}\Big| H^2$$

$$\delta N^{L1,Far}_{DWC} = c\sum_{n=2}^{M}(S^*_n + Q^L_n)\,[(\frac{R}{r})^{n+2} - 1]\Delta g_n$$

$$\delta N^{L2}_{DWC} = \frac{c}{2\pi}\iint_{\sigma o} S^L(\Psi)\left(\frac{\partial \Delta g}{\partial r}\Big|\right)(H - H_Q)\,d\sigma_Q$$

$$\zeta^0 \approx \frac{c}{2\pi}\iint_{\sigma o} S^L(\Psi)\Delta g d\sigma + c\sum_{n=2}^{M}(S_n + Q^L_n)\Delta g^{GGM}_n$$

$$\frac{\delta \Delta g}{\delta r}\Big| = \frac{R^2}{2\pi}\iint_{\sigma o}\frac{\Delta g_Q - \Delta g}{\iota^3_0}\, d\sigma_Q - \frac{2}{R}\Delta g$$

$$\iota_o = 2R\sin\frac{\Psi_Q}{2}$$

Ψ_Q = spherical distance between computation point and Q in Stokes formula

In order to essential implementation of DWC in this study, equation $\delta N^{(1)}_{DWC}$ and $\delta N^{L2}_{DWC}$are significant for short wavelength data acquisition geoid height estimation as well as δN^{L2}_{DWC} more significant for surrounding of point Q. Meanwhile ($\delta N^{L1,Far}_{DWC}$) is significant for long wavelength data acquisition. Figure 3 depicting for DWC of the studies area.

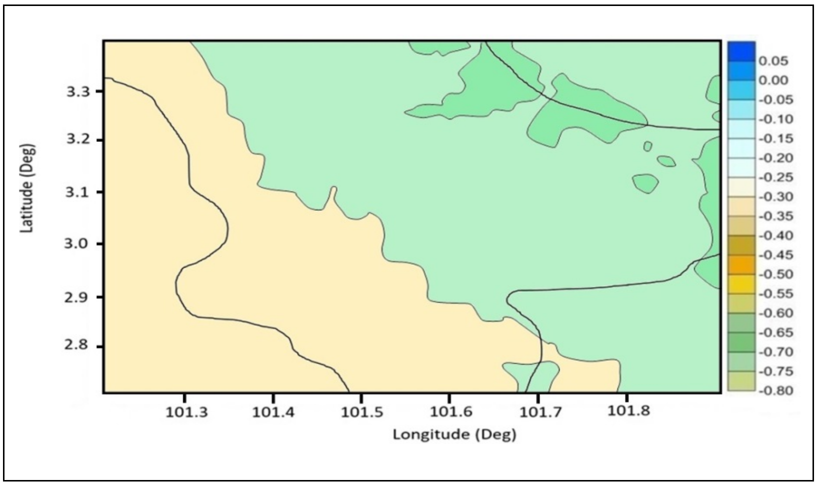

Figure 3. Downward Continuation Correction (δN^{DWC}) Effects
[Unit: Meters]

Table 4 shows statistical analysis of DWC for geoid height on each component correction respectively. Geoid height of DWC effects total is in range between -0.511 to -0.743 meter and 0.229 meter in term of standard deviation.

Table 4. Statistical Analysis of Downward Continuation Effect on Geoid Height

	MIN	MEAN	MAX	Std. Dev
$\delta N^{(1)}_{DWC}$	-0.702	-0.684	-0.667	0.031
$\delta N^{L1,Far}_{DWC}$	-0.287	-0.193	-0.162	0.097
δN^{L2}_{DWC}	-0.233	-0.144	-0.109	0.022
δN^{DWC}	-0.743	-0.657	-0.511	0.229

[Unit: Meters]

3.4 Combined Atmospheric Effect, δN^{a}_{comb}

Apart from topography and downward continuous correction, atmosphere correction plays direct correction relationship for geoid height estimation. The study is using long wavelength frequency GGM data and layering by atmospheric and topographic as well. Its might be disturbing spherical potential in term of BVP. In this regards, atmosphere masses much direct effect with gravity anomaly and essential limit with cap size [24]. The atmosphere effect refers to modification coefficient of S_n and upper limit of M in term of spherical harmonic as mention in equation (9).

$$\delta N^{a}_{comb} = -\frac{2\pi R\rho_0}{\gamma}\sum_{n=2}^{M}\left(\frac{2}{n-1} - S_n - Q^L_n\right)H_n - \frac{2\pi R\rho_0}{\gamma}\sum_{n=M+1}^{M}\left(\frac{2}{n-1} - \frac{n+2}{2n+1}Q^L_n\right)H_n \tag{9}$$

where;

ρ_0 = 1.23 x 10^{-3} gcm^{-3} (density of sea level radius)
H_n = $\sum_{m=-n}^{n} C_{nm} Y_{nm}$
C_{nm} = coefficients of GGM with degree n and order m
Y_{nm} = spherical harmonics
M = maximum degree of GGM

The KTH approach using combined atmosphere effect, δN^{a}_{comb} and approximate with orthometric height, H denoted as equation (9) more competent as well as no truncation error will be committed [25]. Figure 4 shows total combine atmosphere effects. Table 5 shows statistical analysis of various topographic spherical harmonic upper limits for the studies area and only 1.3 mm error in term of standard deviation. Nevertheless it is a small value and can be neglected. The studies still consider on its in order to achieve sub millimeters geoid modeling.

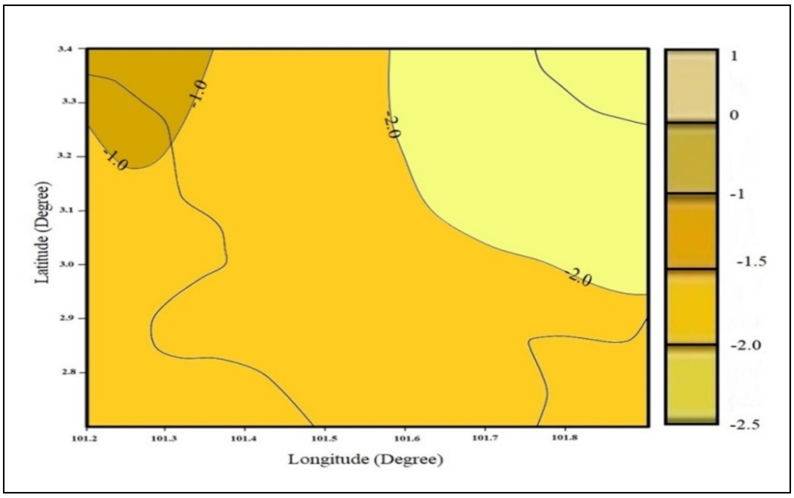

Figure 4. Combine Atmosphere Correction Effects
[Unit: Millimeters]

Table 5. Statistical Analysis of Various Topographic Spherical Harmonic Upper Limits Atmosphere Effect

Global Topographic Spherical Harmonic Upper Limits	MIN	MEAN	MAX	Std. Dev
360	-0.0023	-0.0009	-0.0011	0.0013
720	-0.0023	-0.0009	-0.0011	0.0013
1400	-0.0023	-0.0009	-0.0011	0.0013

[Unit: Meters]

3.5 Ellipsoid Effect, δN^{e}

It's not impossible to implement ellipsoidal correction for a small area local geoid for a specific purpose with better accuracy [7]. Ellipsoid correction refers to ellipsoidal surface of actual shape for a spherical harmonics. Series of spherical request order of first eccentricity, e^2 ellipsoidal reference. Within these studies, ellipsoidal correction refers to combined data in between land gravity anomalies and GGM. GGM with stokes modification formula limited with integration cap size. A practice computation with Laplace harmonics of ellipsoidal corrections denoted at equation (10) [26].

$$\delta N^{e} = \frac{R}{4\pi\gamma}\iint_{\sigma} S(\Psi)\left(k\Delta g + \frac{a}{R}\delta g^{e}\right) d\sigma \quad (10)$$

where;

k = (a-R)/R (Scale factor)
δg^{e} = Laplace harmonics of ellipsoidal correction to gravity anomaly

Ellipsoidal correction requests spherical harmonics on a BVP depend with coefficient, S_n and refer to cap size. Figure 5 shows an overview of ellipsoidal correction at studies area with cap size 2°. Table 6 shows analysis of ellipsoid height correction. Ellipsoidal correction for the studies area is in range -1.99 to 4.11 millimeter (mm) with 1.5 mm in term of standard deviation. Result shows small value of correction and need to count table because of small area studies with precise vertical reference.

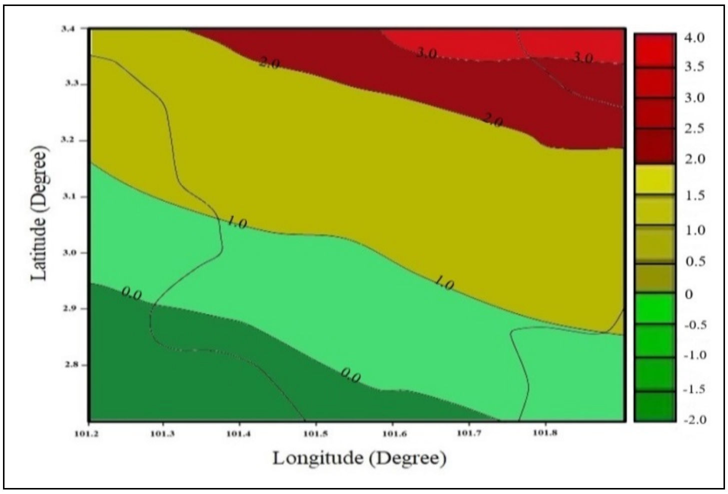

Figure 5. Ellipsoidal Correction Effects
[Unit: Millimeters]

Table 6. Analysis of Ellipsoid Height Correction

	MIN	MEAN	MAX	Std. Dev
Height Correction	-0.00199	0.00147	0.00411	0.0015

[Unit: Meters]

3.6 Klang Valley Geoid Model

The Klang Valley gravimetric geoid model ($KTHKVGM2020_{Grav}$) before fitting to local vertical datum need to certify with orthometric height using GNSS. Equation (11) shows relationship in between orthometric, ellipsoid and geoid height [25].

$$H - h - N = 0 \tag{11}$$

Where;
H = orthometric height
h = ellipsoid height
N = geoid height.

The $KTHKVGM2020_{Grav}$ geoid model evaluated by parameter model and correlated surface in order fitted quasi geoid with GNSS leveling denoted as equation (12) [27].

$$h_i - H_i - N_i = a^n{}_i x + \varepsilon \tag{12}$$

where;
x = unknown paramaters
a_i = coefficients
ε_i = residual of random noise term

$a^n{}_i x$ = parameter model
x = $(A^T A)^{-1} A^T \Delta N$
ΔN = $Ax - \varepsilon$

Table 7. Statistical Analysis of Parameters Model of ε Before Fitting Geoid

	ε of 4 Parameter	ε of 5 Parameter	ε of 7 Parameter
Min	1.3351	1.3401	1.3197
Mean	1.3622	1.3621	1.3723
Max	1.4032	1.4012	1.4091
SD	0.0147	0.0205	0.0197

[Unit: Meters]

The study conducted with 132 GNSS Leveling observations from 289 collocation and existing benchmark (BM) or standard benchmark (SBM) and well known position at Klang Valley areas. $KTHKVGM2020_{Grav}$ geoid model validated with best parameters model with EGM2008 model as presented as Table 1 [13]. Table 7 shows residual of parameters model and 4 parameters model is best to fit with EGM2008 at 1.47 cm in term of standard deviation. Based on 132 GNSS leveling after filtering with least square approach of 1 dimensional fitting model, δ with basic formulae denoted as equation (13) and only 28 stations were conducted. It is due to much difference of orthometric height, δH. Figure 6 shows Klang Valley geoid model after fitting to local vertical datum. Meanwhile Table 8 shows statistical analysis for $KTHKVGM2020_{fit}$ fitted geoid model base on 4 parameter model.

$$\Delta = Mx + C \tag{13}$$

where;
M = coefficients
C = constant

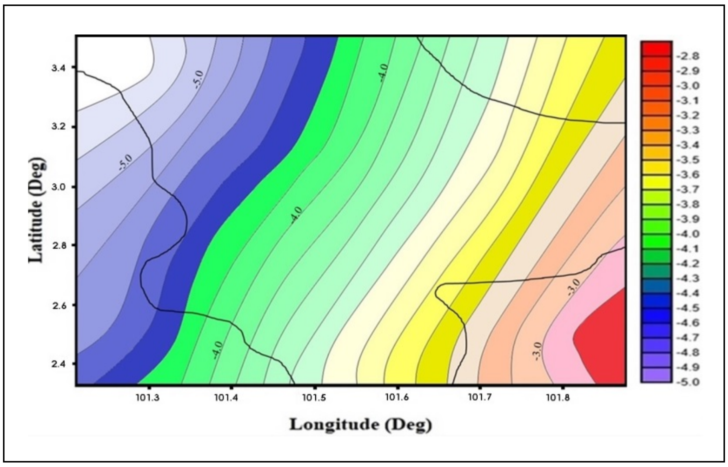

Table 8. Statistical Analysis Geoid Height of KTHKVGM2020$_{Fit}$ Fitted Geoid Model

GEOID MODEL	MIN	MEAN	MAX	Std. Dev
KTHKVGM2020$_{Fit}$	1.3107	1.3363	1.3496	0.0071

[Unit: Meters]

KTHKVGM2020$_{Fit}$ fitted geoid model compared with RCR method and shows the differences. Table 9 shows differences between RCR and KTH approach. KTH approach shows better accuracy with 7.1 mm than 31.1 mm with RCR approach. The study shows that KTH approach available obtains better than one (1) centimeter geoid model with a small area as well.

Table 9. Statistical Analysis of geoid height in between KTH and RCR Method

METHOD	MIN	MEAN	MAX	SD
RCR	-4.332	0.355	4.332	0.0311
KTH	-4.311	0.331	4.315	0.0071

[Unit: Meters]

4. Conclusion

KTH is an easy-to-understand processing method and seeking a high quality geoid modeling. In this study, KTH approach able to produce KTHKVGM2020$_{Fit}$ model better than one (1) centimeter compared with HMS geoid model previously, used RCR method. It is shown that KTH approach available to engage with small area, data acquisition refine rigorously prior to gridding scheme, in cross validation and fully size implement combine of additive correction as well. Meanwhile geoid modeling fitted to a local vertical datum with prober parameter model or surface correlated.

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