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Block Matching Algorithms for the Estimation of Motion in Image Sequences: Analysis

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Abstract

Several video coding standards and techniques have been introduced for multimedia applications, particularly the h.26x series for video processing. These standards employ motion estimation processing to reduce the amount of data that is required to store or transmit the video. The motion estimation process is an inextricable part of the video coding as it removes the temporal redundancy between successive frames of video sequences. This paper is about these motion estimation algorithms, their search procedures, complexity, advantages, and limitations. A survey of motion estimation algorithms including full search, many fast, and fast full search block-based algorithms has been presented. An evaluation of up-to-date motion estimation algorithms, based on several empirical results on several test video sequences, is presented as well.

Keywords: Type your keywords here, separated by semicolons ;

1. Introduction

At present, online videos play a significant role in everyday life and video technology has become the future of content marketing. The basic task of video coding is to reduce the huge amount of raw data in a video sequence by removing spatial and temporal redundancies in video data. Motion estimation technique plays an important role in

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the video coding process by removing temporal redundancy of video signal. The simple and efficient motion estimation technique is the block-based motion estimation (BBME) technique, which has been adopted in many video coding standards such as h.26x series and MPEGx series [1] –[6]. In real-time video processing, the Full-Search (FS) algorithm demands enormous computations. The huge computational cost of the FS algorithm has laid the foundations for broad and deep research in motion estimation. The research has given many fast block matching algorithms. These algorithms scan roughly be categorized as fast search [7]–[50] and fast full-search [51]–[59] block matching algorithms.

In this paper, an overview of selected algorithms in the last forty years and a comprehensive comparison of some well-known algorithms in terms of computational complexity and error distortion are presented. The rest of the paper is organized as follows. In section 2, the brief analysis of fast search and fast full-search block-based motion estimation algorithms are presented. The section-3 gives a comparison of some well-known algorithms. Finally, the conclusions are presented in section 4.

2. Block-Based Motion Estimation Algorithms

The key goal of block-based motion estimation algorithms is to find out the magnitude and direction of motion (motion vector) between a macro-block of the current frame and the best-matched candidate block of the reference frame. The most commonly used matching criterion which measures the error distortion between the macroblock of current fame and candidate blocks in the reference frame is the sum of absolute difference (SAD). The SAD between an M×N size macroblock with a top-left corner at (p, q) and an M × N size candidate block with a top-left corner at (p + x, q + y) is defined in the eq (1).

$$SAD\ (x,y) = \sum_{i=0}^{M-1}\sum_{j=0}^{N-1} |I(p+i,q+j) - R(p+x+i,q+y+j)\ | \tag{1}$$

where I (.,.) and R (.,.) denote current frame and reference frame pixel values. The coordinates of motion vectors 'x' and 'y' are defined in the eq (2).

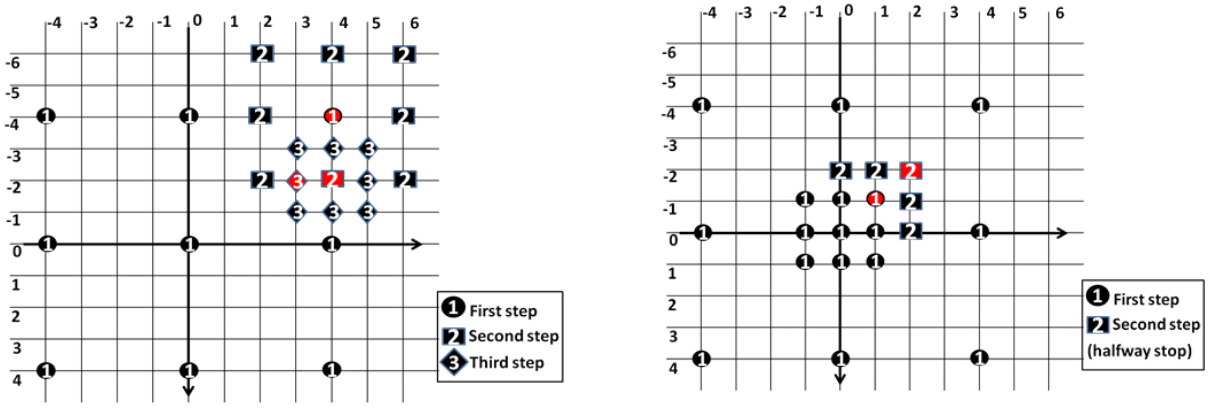
$$(x,y) = arg \min_{(\hat{x},\hat{y})\in R} SAD\ (\hat{x},\hat{y}) \tag{2}$$

where R={$(\hat{x},\hat{y})$ | -s≤$\hat{x}$, $\hat{y}$≤ d} and 'd' represent the search range. It is obvious from eq (2) that the *SAD* criterion involves (M×N)−1 addition operations, M×N absolute operations, and M×N subtraction operations i.e.one *SAD* computation requires 3×M×N operations approximately.

2.1. Fast Search Block-Based Motion Estimation Algorithms

To reduce the huge computational cost of the FS algorithm, many fast search block-based motion estimation algorithms [7]–[50] have been presented at the cost of a slight reduction in error distortion given by peak signal-to-noise ratio (PSNR). These algorithms may be classified into the following categories: reduction in several search points [7]–[27], predictive motion estimation [28]–[34], adaptive search pattern switching strategy [35]–[38], multi-resolution motion estimation [39]–[45] and fractional-pixel interpolation [46]–[50]. Present fast search block-based motion estimation algorithms belong to any one of them or utilize a combination of the above categories. In general, the fast search block matching algorithms which belong to a reduction in many search points category [7]–[27] are mainly developed with an assumption that the error between a macroblock and a candidate block increases monotonically as the search point moves away from the optimal search point. In the early 1980s, some fast search block-based motion estimation algorithms such as the Three-Step Search (TSS) [7], two-dimensional logarithmic search (TDL) [8], the Conjugate Directional Search (CDS), and its simplified version one-at-a-time search (OTS) [9], etc. were proposed.

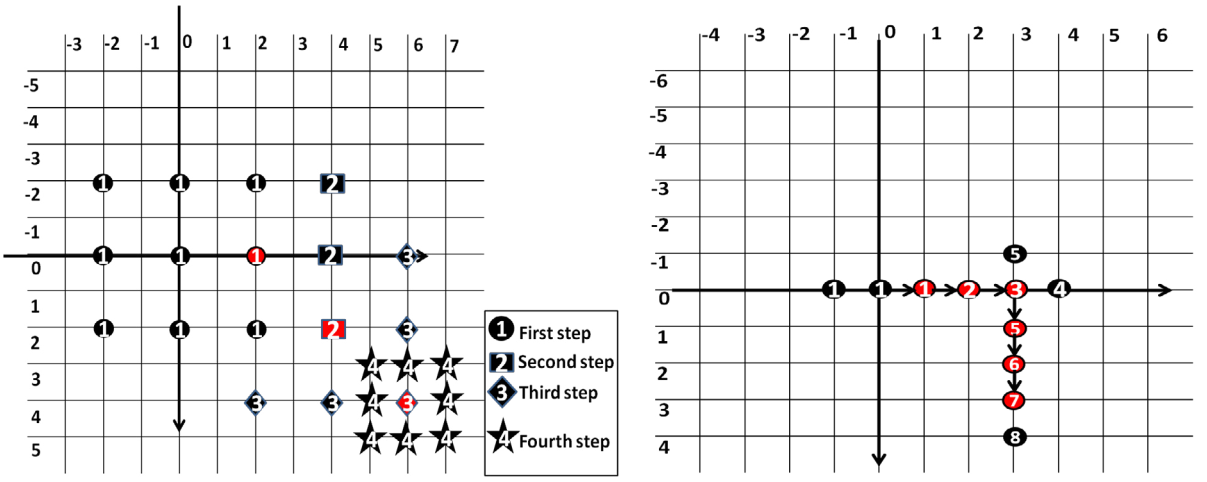
In the TSS algorithm, the search procedure employs a rectangular-shaped search pattern which consists of nine search points including the center at each step. Initially, the step size is taken as ceil (s/2) and is reduced by a factor of two in the subsequent steps, where s is the search range. The search stops when the step size is reduced to 1. Fig. 1(a) shows an example of the TSS search procedure to find a motion vector at (3, -2). The total number of steps and the total number of checking points are given by log2(s + 1) and 1 + 8*[log2(s + 1)] respectively. The NTSS algorithm, proposed by Ren Xiang Li et al., performs better than TSS in terms of motion prediction quality and computational complexity while retaining the regularity and simplicity of the TSS algorithm. The NTSS algorithm is developed mainly with an assumption that the motion vector distribution of most real-world video sequences is center biased. Therefore, besides the original search points of TSS, NTSS checks eight additional search points around the search center at the first step (total 17) as shown in Fig. 1(b). Furthermore, the NTSS quickly identifies stationary and quasi-stationary blocks by applying a halfway stop technique. In the first step, the minimum BDM point may occur at the search window center, at any one of the eight search points around the search center, or at any one of the remaining eight search points. In the first case, the block is considered a stationary block, and the search stops. In the second case, the block is considered as a quasi-stationary block and the search stops after checking eight search points around the minimum BDM. In the final case (if the block is neither stationary nor quasi-stationary), the search follows the complete TSS procedure.



(a) (b)

Fig.1 (a) An example of a search procedure of the TSS algorithm for finding motion vector (3, −2). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point, (b) An example of a search procedure of NTSS for finding motion vector (2, −2). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point.

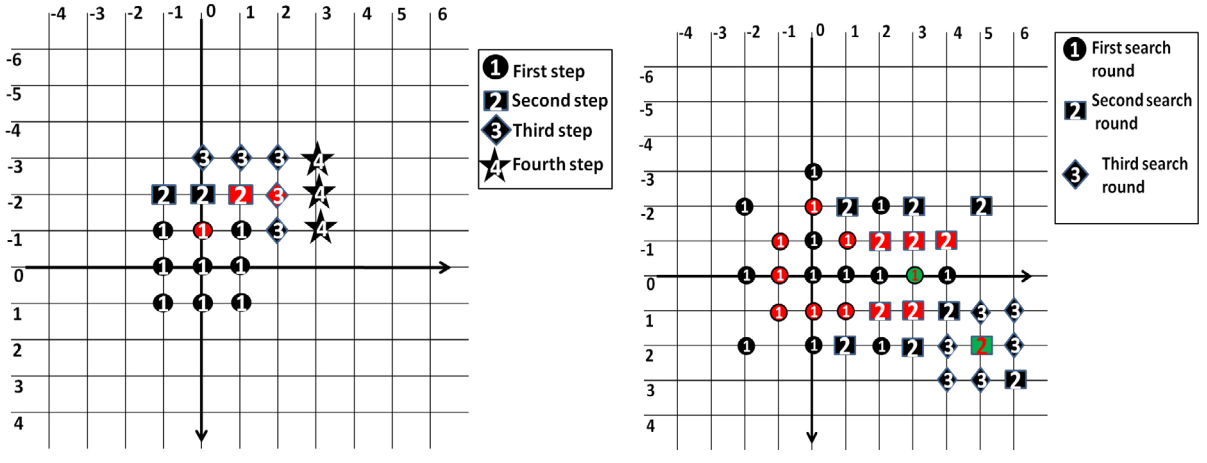
In [13], a Four-Step Search (4SS) algorithm has been proposed for motion estimation [13]. This algorithm includes a half−way stop technique and center-biased motion vector distribution characteristic similar to NTSS. However, the number of block matches of 4SS in the worst case is 27 when the maximum search range is ±7. With the maximum search range of ±7, the 4SS employs two different search patterns with 5×5 and 3×3 square window sizes. For the first three search steps, if the minimum BDM search point is positioned at the center the search goes directly to the fourth search step. An example of a search procedure to find a motion vector at (6, 4) is shown in Fig. 2(a). One-at-a-time search (OTS) [9] is a 1-D gradient descent search algorithm. At first, OTS searches along the horizontal search direction until the minimum BDM value lies between two higher BDM values. Then, the search direction changes to the vertical direction until the minimum BDM value is found in the vertical direction. The OTS search path to locate motion vector (3, 3) is shown in Fig. 2(b). Several OTS-based motion estimation algorithms such as block-based gradient descent search (BBGDS) [14] and directional gradient descent search (DGDS) [15] algorithms have been developed. The BBGDS is a 2-D gradient descent search motion estimation algorithm that searches for the minimum BDM block along the block-based gradient descent direction. At each search step, it applies a square search pattern which consists of nine search points. The eight search points surround the search center independently perform motion estimation in all the possible eight directions from the search center.



(a) (b)

Fig.2(a). An example of a search procedure of 4SS for finding motion vector (6, 4). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point, (b) An example of a search procedure of OTS for finding motion vector (3, 3). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point.

The search continues until the minimum BDM search point is positioned at the search center. An example of a BBGDS search path to locate a motion vector at (2, -2) is shown in Fig. 3(a). The DGDS independently applies the OTS strategy in eight directions of the search center to find eight directional minimum search points. Among these eight directional minimum search points, the minimum one becomes the search center for the next search step. At any search step, if the least among eight directional minimum search points in the search center, the search stops with the search center as the motion vector. The DGDS search path to locate the motion vector (5, 2) is shown in Fig. 3(b).

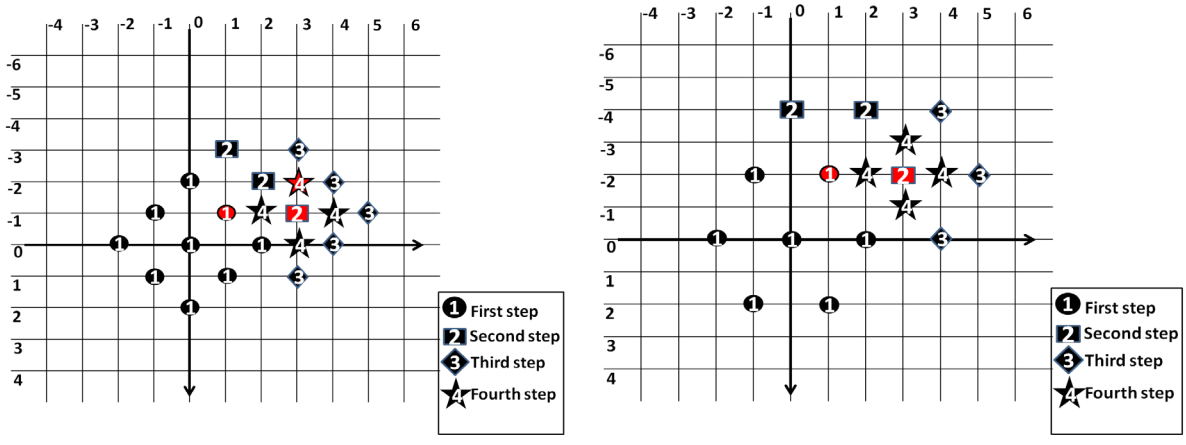


(a) (b)

Fig. 3(a). An example of a search procedure of BBGDS for finding motion vector (2, -2). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point, (b) An example of a search procedure of DGDS for finding motion vector (5, 2), each search point is indicated by its search round number, red-colored points are the directional minimum search points and the green-colored point is the least of directional minimum search points.

The diamond search (DS) algorithm [16]–[17] locates a small area of global minimum by applying a large diamond search pattern (LDSP) and then traces the global minimum in the located small area by applying a compact

small diamond search pattern (SDSP). An example of a search procedure to find a motion vector at (3, -2) is shown in Fig. 4 (a). DS starts the search by checking 9 search points of LDSP positioned at the search window center. A new SDSP or LDSP is centered at a minimum BDM point depending on whether the minimum BDM point is search center or not. The search continues until the new SDSP is centered and the minimum BDM point of SDSP will be the final motion vector. The Hexagonal Search (HS) algorithm with circle approximated search pattern is proposed in [18]. The search procedure of HS is the same as that of DS except that the HS performs a coarse search by using a large hexagon search pattern that is close enough to a circle. An example of an HS search path to locate a motion vector at (3, -2) is shown in Fig. 4 (b).



(a) (b)

Fig. 4(a) An example of a search procedure of DS for finding motion vector (3, -2). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point, (b)An example of a search procedure of HS for finding motion vector (3, -2). Each search point is indicated by its search step number and the red-colored point is the minimum BDM point.

The modifications of HS [19] - [21] are developed for reducing computational cost against the HS algorithm. These algorithms essentially focus on methods to improve the inner search procedure of HS. An enhanced hexagonal search (EHS) algorithm [19] reduces the search points by employing the six-side-based fast inner search method. The EHS algorithm calculates the group-sum distortion to predict a part of the inner search that has to be examined. In [20], and enhanced hexagonal search using point-oriented inner search (EHS-POIS) [20] applies to mean internal distance to calculate the normalized group distortions of the large hexagon. Then it checks only two inner search points which are associated with minimum normalized group distortions. An enhanced hexagonal search using direction-oriented inner search (EHS-DOIS) [21] forms a pseudo-points prediction pattern from the large hexagon. EHS-DOIS calculates the group distortions of these pseudo-points to select one inner search point.

The algorithms belong to the predictive motion estimation category [28]–[34] reduces the computational cost considerably by using the temporal and/or spatial correlation among motion vectors. In [31], the Motion Vector Field Adaptive Search technique (MVFAST) efficiently uses adjacent blocks motion information for performing motion estimation effectively. Before starting a search at each macroblock, MVFAST calculates the city block lengths of the adjacent motion vectors. This city block length classifies the motion content of the current macroblock as high, medium, or slow motion. According to motion activity, the search strategy and search center of the current macroblock are determined. Furthermore, a halfway-stop technique is included in MVFAST such that it terminates the search early by checking (0, 0) predictor. The search performance of MVFAST is further improved in the predictive motion vector field adaptive search technique (PMVFAST) [32] with median predictor and collocated block's motion vector. The PMVFAST employs an adaptively early search termination technique, unlike MVFAST where a fixed early search termination technique is used. Enhanced predictive zonal search (EPZS) [34] improves the search performance of PMVFAST by using additional higher probable predictors, and with improved threshold

calculations. The algorithms belong to the search patterns switching category [35]–[38] employ an adaptive switching strategy i.e. the algorithms dynamically apply various search patterns according to the motion activity. Consequently, the number of search locations is reduced drastically. An adaptive search pattern switching algorithm was proposed in [38]. This algorithm predicts the motion activity of a block and then uses an appropriate search pattern for performing motion estimation. For small motions, center-based search patterns such as NTSS, DS, and BBGDS are used. The non-center-biased search patterns such as TSS and 4SS are used for large motions. The motion content of a block is determined by an error descent rate (EDR). This EDR is calculated from block distortions of the search window center and its four neighboring search points. This EDR is defined as EDR = D_B/D_A where D_A represents a distortion of the block at the center of the search window and D_B represents a minimum distortion of the four neighboring blocks of the search window center.

The algorithms belong to multiresolution techniques [39]–[45] to represent the reference and current frames by pyramidal structure with various levels. Each level of this representation is a reduced resolution representation of the lower level and is obtained by sub-sampling and spatial low-pass filtering the lower level. The motion field estimated at the present coarser resolution level is interpolated to form the initial solution for the motion field at the next finer resolution level as this initial solution is more likely to be near the global minimum point. Therefore, the search at each resolution level is restricted to a smaller search range than the actual search range at the finest resolution level. Consequently, the total computational cost is less than the computational cost demanded in the finest resolution directly. The algorithms belong to fractional-pixel motion estimation (FPME) techniques [46]–[50] to achieve further reduction in bit rate i.e., improvement in video quality by applying fractional-pixel interpolation (FPI) algorithms.

2.2. Fast Full-Search Block-Based Motion Estimation Algorithms

The fast full-search algorithms minimize the computational complexity of the motion estimation process while preserving the same PSNR performance as the full-search algorithm. Many fast full-search algorithms have been proposed in the last four decades. Some eminent algorithms are successive elimination technique-based algorithms [51]–[58], partial distortion elimination-based algorithms [75]–[82], winner-update approach-based algorithms [69]–[70], and projection techniques-based algorithms [71]–[74]. The most popular of these algorithms is the successive elimination algorithm (SEA) [51].

SEA finds the optimal motion vectors like the full-search algorithm, but with less computational cost. The SEA rejects the search points which may not be the best possible search points before computing full distortion measure for those search points. SEA skips these impossible search points by examining if the current minimum SAD (SAD_{min}) is less than the partial distortion measure. In [52], the block sum pyramid algorithm (BSPA) skips the non-best candidate blocks by calculating partial errors hierarchically at every candidate block before computing the rigorous full distortion. In [53], the multilevel successive elimination algorithm (MSEA) rejects a greater number of candidate blocks than those of SEA by using additional boundary levels. MSEA obtains these boundary levels by partitioning blocks into four equal-sized sub-blocks continually until a 2×2 sub-block arrives at MSEA has shown search speed improvement against SEA by applying these boundary levels sequentially to skip some highly impossible search points which could not be rejected by the SEA boundary.

In MSEA, very large gaps exist between two contiguous boundary levels. Because of such large gaps, the effectiveness of MSEA is undermined. In [54], a fine granularity successive elimination (FGSE) is proposed to make up for this inefficiency of MSEA. FGSE algorithm reduces the gaps between two contiguous boundary levels by increasing the number of boundary levels. So, highly impossible search points are filtered out earlier in the FGSE algorithm than in MSEA. In [55], an adaptive MSEA (AdaMSEA) divides the search area based on the homogeneity of the macroblock. To increase the possibility of skipping impossible search points in the early stage, the blocks with large variances are partitioned into sub-blocks first. A winner update algorithm with an integral image (WUI) is proposed in [59]. This algorithm replaces the hierarchical pyramid structure of the matching block with an integral image. This integral image facilitates the evaluation of partial block sum norms dynamically and therefore, WUI reduces the computational complexity of motion estimation.

3. Results

This section presents the simulation results about the motion prediction quality and computational complexity of various up-to-date and famous motion estimation algorithms such as DS, CDS, DGDS, EHS-DOIS, ARPS-2, DASp, SEA, MSEA, AdaMSEA, and WUI. Ten test video sequences with different motion contents and different video formats (HD, CIF, and QCIF) have been used to analyze the performance of these algorithms. Ten test videos contain various motion contents and have different resolutions. Kirsten-Sara and Akiyo test videos contain low-motion content i.e., maximum blocks are stationary blocks. Suzie, Mobile, and Flower are the test videos that consist of medium motions with stationary and quasi-stationary blocks. Mobile is a typical test video in which the local and global motions are complex. Rocket launch, Cricket, and Foreman test videos have large motions. Rhinos and Robot boat test videos consist of complex motions with fast camera zooming and panning. The search ranges ±63 and ±15 are used for HD test video sequences (Rocket launch and Kirsten-Sara) and the remaining (QCIF and CIF) video sequences respectively. Block size set to 16 x 16. In the comparison of various algorithms, PSNR is used as a measure for motion prediction quality and an average number of operations per block measures the computational complexity. The average numbers of operations per block in each algorithm are summarized in Table 1. The degree of motion prediction quality of every algorithm with respect to the full search algorithm is shown in Table 2. It is very clear from these tables that the fast search algorithms (DS, CDS, DGDS, C, ARPS-2, and DASp) reduce the computational complexity significantly but degrade the PSNR performance when compared to the full search algorithm. Whereas, the fast-full search algorithms (SEA, MSEA, AdaMSEA and WUI) obtain same PSNR of full search but with high computational complexity.

Table 1. The average number of operations per block in each algorithm.

Video sequence	FS	Fast search motion estimation algorithms						Fast full-search algorithms			
		DS	CDS	DGDS	EHS-DOIS	ARPS-2	DASp	SEA	MSEA	Ada MSEA	WUI
Foreman	601520	13203	12398	14334	8178	7288	6532	175080	29301	25612	25301
Mobile	668516	8253	7877	8919	5948	5181	4174	270890	30927	24153	24039
Rhinos	668516	29327	23440	26515	12990	11662	10378	329652	65321	50602	49305
Robot boat	668516	26494	22016	25413	12447	11067	11943	363925	72109	66944	66001
Suzie	601520	9627	7993	8587	5768	4805	4496	113280	14317	12209	12152
Akiyo	601520	6327	6314	6054	5155	4305	3551	24608	6877	6205	6109
Cricket	668516	13911	12734	12599	8344	7508	6379	106799	28641	22943	21513
Flower	668516	10072	9323	9968	6729	6194	5322	104960	18303	16112	16001
Kirsten-Sara	10813330	7981	7410	7537	5839	5201	4218	216852	63928	54883	53697
Rocket launch	10813330	13364	12149	12964	8278	7403	6752	284621	82504	73864	73359

From Table 1, it is obvious that DASp demands a smaller number of operations when compared to other algorithms. ARPS-2 is better than DS, CDS, EHS-DOIS, and DGDS in terms of the number of operations. With respect to video sequences (Akiyo and Kirsten-Sara) that have small motion content, all the algorithms including DASp and ARPS-2 have the approximately same number of operations. However, DASp and ARPS-2 require a smaller number of operations irrespective of motion activity in video sequences. It is clear from Table 2 that the DGDS obtains better average PSNRs than those of DS, CDS, DASp, ARPS-2, and EHS-DOIS in all the video

sequences. On average, DGDS obtains 0.304dB better PSNR than that of CDS. However, CDS requires a smaller number of operations when compared to that of DGDS. It is very clear from table 1 that EHS-DOIS finds motion vectors with less computational cost when compared to that of DGDS and CDS. However, EHS-DOIS gives the least PSNR performance among all the algorithms (refer to Table 2). On the whole, in terms of the average number of operations per block as the indicator for computational complexity, DASp is certainly the best. Simultaneously, with reference to PSNR as an indication for the quality of video, the DASp is also apparently better than the DS, CDS, EHS-DOIS and ARPS-2 algorithms and comparable to the DGDS. Among fast full-search algorithms (SEA, MSEA, AdaMSEA, and WUI), WUI has faster search performance.

Table 2. The degree of motion prediction quality of every algorithm with respect to the full search algorithm

Video sequence	FS or Fast full-search algorithms	Fast search motion estimation algorithms					
		DS	CDS	DGDS	EHS-DOIS	ARPS-2	DASp
Foreman	28.89	28.15	28.03	28.28	26.70	28.22	28.05
Mobile	24.29	23.52	23.85	23.87	22.71	23.63	23.83
Rhinos	30.23	27.62	27.81	28.40	27.66	27.34	28.35
Robot boat	30.62	29.21	29.10	29.54	28.83	29.32	29.05
Suzie	35.90	35.02	35.10	35.25	33.87	35.11	35.13
Akiyo	44.16	44.16	44.16	44.16	43.25	44.16	44.16
Cricket	35.95	33.66	33.95	34.99	33.19	33.94	34.67
Flower	33.69	33.02	33.19	33.35	31.47	33.28	33.17
Kirsten-Sara	44.74	44.18	44.21	44.39	42.45	44.10	44.17
Rocket launch	38.95	37.53	37.69	37.90	36.20	37.66	37.89

4. Conclusions

In the last four decades, multimedia research involves in the development of efficient block matching algorithms to decrease the computational cost of motion estimation. These algorithms have been categorized into fast search and fast full search algorithms. This paper has presented basic search procedures of well-known fast search and fast full search algorithms. A comprehensive analysis of famous and state-of-the-art algorithms in terms of their computational costs and motion prediction qualities is presented.

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