# An Instantaneous Impact Point Guidance for Rocket with Aerodynamics Control

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**Abstract:** This paper aims to propose a new guidance algorithm for a rocket with aerodynamics control for launch operations, based on the concept of the instantaneous impact point (IIP). In this study, the rocket with aerodynamics control is considered with the purpose of reducing dispersion of the impact point after separation of the rocket for safety reasons. Since a very limited aerodynamic maneuverability is typically allowed for the rocket due to the structural limit, a guidance algorithm producing a huge acceleration demand is not desirable. Based on this aspect, the proposed guidance algorithm is derived directly from the underlying principle of the guidance process: forming the collision geometry towards a target point. To be more specific, the collision-ballistic-trajectory where the instantaneous impact point becomes the target point, and the corresponding heading error are first determined using a rapid ballistic trajectory prediction technique. Here, the trajectory prediction method is based on the partial closed-form solutions of the ballistic trajectory equations considering aerodynamic drag and gravity. And then, the proposed guidance algorithm works to nullify the heading error in a finite time, governed by the optimal error dynamics. The key feature of the proposed guidance algorithm lies in its simple implementation and exact collision geometry nature. Hence, the proposed method allows achieving the collision course with minimal guidance command, and it is a desirable property for the guidance algorithm of the rocket with the aerodynamics control. Finally, numerical simulations are conducted to demonstrate the effectiveness of the proposed guidance algorithms.

**Keywords:** Instantaneous Impact Point (IIP) Guidance, Optimal Error Dynamics, Rocket Landing Guidance, Aerodynamic Control

## 1. INTRODUCTION

In space launch vehicle operations, reducing the impact point dispersion of separated rocket stages is important for flight safety reasons. The separated rocket typically conducts a free-fall flight after the separations. Thus, the flight trajectories and impact points significantly vary due to external disturbances such as wind, aerodynamic, and uncertainty in engine cutoff time. As a result, the impact point dispersion for existing systems is relatively huge, and it becomes an obstacle for flight safety operations.

To reduce the impact point dispersion caused by external disturbances, it is required to provide a control mean and guidance capability to a separated rocket in order to correct its flight path direction so that the impact point reaches the desired target point. In this context, one of the possible options for providing guidance capability would be a separated rocket system with aerodynamic control fins. In this system, only very limited aerodynamic maneuverability is typically allowed due to the structural load limit. Therefore, a guidance algorithm producing a minimal acceleration demand is required, and the instantaneous impact point (IIP)-based guidance algorithm is desirable to meet this requirement [2], [1].

By definition, the IIP of a rocket is a touch-down point when it is assumed that it undergoes a free-fall flight [3]. The basic idea of the IIP guidance is to alter the current IIP to the desired target point by steering the flight path angle. Originally, the IIP guidance has been pro-

posed for the upper stages of rockets to compensate for burnout parameter errors [4]. Recently, the IIP guidance has also been applied to reusable rocket systems in the boost-back-burn guidance phase [1]. However, few studies applying the IIP guidance to a separated rocket with aerodynamic control with the purpose of reducing the IIP dispersion are available in the open literature. In addition, the existing method did not consider the aerodynamic effect while determining the IIP. Therefore, in the endo-atmospheric region, the predicted IIP has less accuracy, and it results in unnecessary acceleration demand in a vicinity of a target point.

Motivated by these observations, this paper aims to propose a guidance algorithm for a rocket with aerodynamic control to guide the rocket towards the desired dive point while minimizing the acceleration demand, based on the concept of the IIP guidance with consideration of the aerodynamic effect. To be more specific, the proposed method is derived from the underlying principle of shaping the collision geometry towards the desired dive point. The collision-ballistic-trajectory, where the instantaneous impact point becomes the target point, and the corresponding heading error are first determined using the partial closed-form solutions of the ballistic trajectory equations considering aerodynamic drag and gravity. Then, by utilizing the optimal error dynamics [5], the proposed guidance command nullifying the heading error in a finite time is determined. The key feature of the proposed guidance algorithm lies in its simple implemen-

tation and exact collision geometry nature. Hence, the proposed method allows achieving the collision course with minimal acceleration demand, which is desirable for an aerodynamically controlled rocket. Finally, numerical simulations are conducted to show the effectiveness of the proposed guidance algorithm.

This paper is organized as follows. In Section 2, the problem to be solved is defined. Section 3 provides the proposed guidance algorithm in detail. Simulation results are provided in Section 4. Lastly, concluding remarks are offered in Section 5.

## 2. PROBLEM DEFINITION

In this section, the guidance problem considered in this study is formulated. First, the guidance kinematics for a rocket with aerodynamic control is derived. The guidance goal is then stated.

### 2.1 Kinematic Equations

As guidance for a rocket with aerodynamic control is typically conducted in the vertical plane in the endo-atmospheric region, we consider a two-dimensional guidance geometry over a flat earth, as shown in Fig. 1. In this
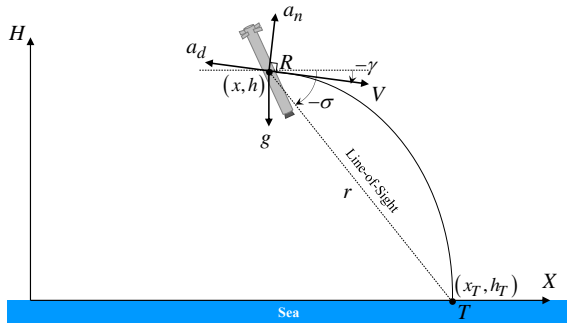


Fig. 1. Guidance geometry.

figure, $(H, X)$ denotes the inertial reference frame. The notations $R$ and $T$ are the rocket and the target point. In this study, the target point can be considered as the desired dive point. The variables $r$ and $\sigma$ represent the relative range and the line-of-sight (LOS) angle between the rocket booster and the target point, respectively. The variables $V$ and $\gamma$ represent the velocity and the flight path angle. Additionally, the normal acceleration and the axial acceleration can be denoted by $a_n$ and $a_d$, respectively. The normal acceleration is acting perpendicular to the velocity vector, and it contributes to altering the velocity direction (or changing the flight path angle). The axial acceleration (i.e., drag acceleration) is acting in the opposite direction to the velocity vector, and it mainly contributes to decreasing the magnitude of velocity. In the guidance geometry given in Fig. 1, the kinematic equations can be

written as

$$\dot{h} = V \sin \gamma, \tag{1}$$
$$\dot{x} = V \cos \gamma, \tag{2}$$
$$\dot{V} = -a_d - g \sin \gamma, \tag{3}$$
$$\dot{\gamma} = \frac{a_n}{V} - g \cos \gamma. \tag{4}$$

where the variables $h$ and $x$ represent the height and the downrange for the rocket, respectively. In the above equation, $a_d$ and $a_n$ are produced by the aerodynamics forces, and they are given by

$$a_n = \frac{\rho V^2 S C_L}{2m}, \quad a_d = \frac{\rho V^2 S C_D}{2m} \tag{5}$$

where $\rho$, $S$, and $m$ represent the air density, the reference area, and the mass, respectively. $C_L$ and $C_D$ are the lift and drag coefficients. Here, $a_d$ can be rewritten in the term of the ballistic coefficient $\beta$ as

$$a_d = g \beta V^2, \quad \text{where} \quad \beta = \frac{\rho S C_D}{2mg} \tag{6}$$

In Eq. (5), as $C_L$ can be controlled by the aerodynamic control fins attached to the rocket booster, $a_d$ can be considered as the control input for the kinematic equations.

### 2.2 Guidance Goal

The ultimate goal of guidance considered in this study is to drive the rocket towards the desired dive point $(x_T, h_T)$ with the purpose of reducing the impact point dispersion on the sea for safety reasons. If the impact time $t_f$ is defined as the time when the height of the dive point is equal to the height of the rocket (i.e., $h(t_f) = h_T$), the requirement for achieving the above guidance goal can be expressed as

$$x_f - x_T = 0 \tag{7}$$

where $x_f = x(t_f)$ represents the impact point at the final time.

## 3. PROPOSED GUIDANCE ALGORITHM

This section describes the proposed guidance algorithm. First, the proposed guidance concept, which is based on the instantaneous impact point (IIP), is explained. Next, the prediction method for the instantaneous impact point considering the aerodynamic force is then discussed. Finally, the proposed guidance algorithm is determined by utilizing the optimal error dynamics [5].

### 3.1 Guidance Concept

As a very limited aerodynamic maneuverability is generally allowed for the rocket because of its structural load limit, a guidance algorithm generating a huge acceleration demand is not desirable for this system. Therefore, in this study, a guidance algorithm based on the concept of IIP is considered to produce a minimal acceleration

demand. The IIP is defined as the touch-down point of a rocket under the assumption that the rocket immediately ends the propelled flight without corrective maneuver [3]. Accordingly, by definition, the IIP in Fig. 2 is the achieved position of the rocket at the target height $h_T$ under the given initial conditions $(V_0, \gamma_0)$ with $a_n = 0$.
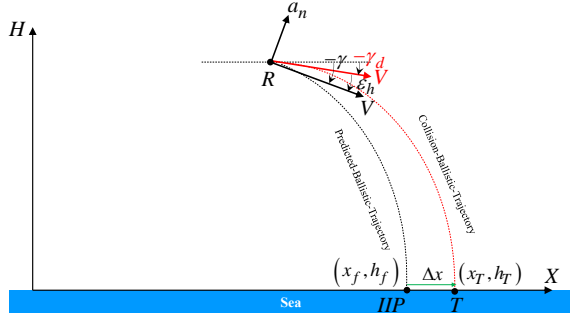


Fig. 2. Definition of IIP and CBT.

In general, the IIP is not equal to the desired dive point at the beginning of the guidance phase because of the initial guidance error. Therefore, the impact point error is usually non-zero as follows.

$$\Delta x = x_f - x_T \neq 0 \tag{8}$$

This error should be nullified in a finite time in order to achieve the guidance goal, and the proposed guidance algorithm will work in a way to change the current IIP to the desired point.

The achieved IIP usually depends on the initial conditions such as the initial position $(x_0, h_0)$, the initial velocity $V_0$, and the initial flight path angle $\gamma_0$. Among these initial conditions, the flight path angle can significantly affect the achieved IIP. Suppose that the IIP and the desired dive point match when the initial flight path angle becomes $\gamma_d$. In that case, the rocket can reach the target point along a certain ballistic trajectory without any maneuver. This specific trajectory is called the collision-ballistic-trajectory (CBT), and the corresponding initial flight path angle leading to the CBT (i.e., $\gamma_d$) is called the desired flight path angle in this study. Since the collision course is achieved when $\gamma \to \gamma_d$, the difference between the current flight path angle and the desired flight path angle can be defined as the heading error as follows.

$$\varepsilon_h = \gamma_d - \gamma \tag{9}$$

From the kinematic relationship as given in Eq. (4), it can be readily predicted that the flight path angle can be altered by imposing the normal acceleration. Therefore, the satisfaction of the guidance goal with a minimal acceleration demand can be accomplished by producing appropriate acceleration command in a way to nullify the heading error. In the next section, the proposed guidance algorithm will be determined based on the guidance concept mentioned above.

## 3.2 Prediction Method for Instantaneous Impact Point

To realize the proposed guidance concept, the IIP should be rapidly predicted. Additionally, the aerodynamic force should be taken into account when predicting the IIP in order to improve prediction accuracy. By definition, the kinematic equations for predicting the IIP can be determined by imposing the condition $a_n = 0$ to Eq. (4) as

$$\dot{h} = V \sin \gamma, \tag{10}$$
$$\dot{x} = V \cos \gamma, \tag{11}$$
$$\dot{V} = -g\beta V^2 - g \sin \gamma, \tag{12}$$
$$\dot{\gamma} = -g \cos \gamma. \tag{13}$$

An integral of these equations is required to determine the IIP. However, as these equations are highly nonlinear, it is difficult to get complete closed-form solutions. Instead, numerical integration needs to be performed, but it requires a lot of computation burden to determine the IIP.

Therefore, we use Chudinov's equation approach [6] in which the computational burden does not significantly increase while providing accurate prediction result. The main idea of this approach is that the flight path angle is used as an independent variable instead of time. Then, the nonlinear kinematic equations can be converted into a more simplified form that is easy to handle. Under the ballistic trajectory, the flight path angle monotonically decreases. Accordingly, if the flight path angle is discretized with a step size $\Delta\gamma$, the flight path angle at the $k$-th step can be written as

$$\gamma_{k+1} = \gamma_k - \Delta\gamma, \quad \text{where} \quad k = 0, 1, 2, \tag{14}$$

When the step size $\Delta\gamma$ is chosen as a small value, the ballistic coefficient and the gravity can be approximated as constant values during the interval $[\gamma_k, \gamma_{k+1}]$. Under the approximation, the kinematic equations can be reformulated with respect to the flight path angle as

$$V' = V \tan \gamma + \beta_k V^3 \sec \gamma, \tag{15}$$
$$t' = -(V/g_k) \sec \gamma, \tag{16}$$
$$x' = -(V^2/g_k), \tag{17}$$
$$h' = -(V^2/g_k) \tan \gamma. \tag{18}$$

where $(\cdot)'$ represents the derivative with respect to the flight path angle $\gamma$. From Eq. (15), the closed-form solution of the velocity during the interval $[\gamma_k, \gamma_{k+1}]$ is determined as

$$V_{k+1} = \frac{V_k \cos \gamma_k}{\cos \gamma_{k+1} \sqrt{1 + \beta_k V_k^2 \cos^2 \gamma_k (\varphi_k - \varphi_{k+1})}} \tag{19}$$

where

$$\varphi_k = \frac{\sin \gamma_k}{\cos^2 \gamma_k} + \ln \left[ \tan \left( \frac{\gamma_k}{2} + \frac{\pi}{4} \right) \right] \tag{20}$$

Based on the trapezoidal method with Eq. (19), the approximated closed-form solutions for the remaining variables are determined as

$$t_{k+1} = t_k + \frac{2\left[V_k \sin \gamma_k - V_{k+1} \sin \gamma_{k+1}\right]}{g_k\left(2 + \mu_k\right)}, \quad (21)$$

$$x_{k+1} = x_k + \frac{V_k^2 \sin 2\gamma_k - V_{k+1}^2 \sin 2\gamma_{k+1}}{2g_k\left(1 + \mu_k\right)}, \quad (22)$$

$$h_{k+1} = h_k + \frac{V_k^2 \sin^2 \gamma_k - V_{k+1}^2 \sin^2 \gamma_{k+1}}{g_k\left(2 + \mu_k\right)}. \quad (23)$$

where

$$\mu_k = \beta_k\left(V_k^2 \sin \gamma_k + V_{k+1}^2 \sin \gamma_{k+1}\right) \quad (24)$$

Based on the above iterative solutions, the IIP can be determined. The detail of the IIP prediction method is summarized in Algorithm 1.

**Remark 1**. As the state variables at the next step are computed by utilizing the partial closed-loop solutions in the proposed method, a relatively large $\Delta\gamma$ is allowed while guaranteeing a satisfactory prediction accuracy. In this way, the proposed method can determine the IIP with less computational burden compared to existing numerical integration methods.

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**Algorithm 1** IIP Prediction Algorithm
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**Input:** initial conditions $x_0$, $h_0$, $V_0$, $\gamma_0$, $t_0$ and terminal condition $h_T$
    **while** $h_k > h_T$ **do**
        determine trajectory solutions using Eqs. (19) to (24)
    **end while**
    **Return** $x_k$
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### 3.3 Computation of Collision-Ballistic-Trajectory

The next step is to determine the desired flight path angle that forms the collision-ballistic trajectory. For convenience, let us define a function $f\left(\cdot\right)$ as the predicted IIP for a given initial flight path angle $\gamma_0$ by utilizing Algorithm 1.

$$x_f = f\left(\gamma_0\right) \quad (25)$$

If $g\left(\cdot\right)$ is defined to be the inverse function of $f\left(\cdot\right)$, the desired flight path angle can be written as

$$\gamma_0 = g\left(x_T\right) \quad (26)$$

However, the function $f\left(\cdot\right)$ is given by the algorithm, determining the inverse function $g\left(\cdot\right)$ is intractable. Thus, the iterative gradient method [2] is used in this study.

The changes in the IIP with respect to the initial flight angle can be written as

$$\frac{\partial x_f}{\partial \gamma_0} \approx \frac{\Delta x_f}{\Delta \gamma_0} = \frac{f\left(\gamma_0 + \Delta \gamma_0\right) - f\left(\gamma_0\right)}{\Delta \gamma_0} = \alpha\left(\gamma_0\right) \quad (27)$$

where the parameter $\Delta\gamma_0$ represents a small perturbation of the initial flight path angle. If there is the impact point

error as shown in Eq. (8), the correction of the initial flight path angle for reducing this error can be made as follows.

$$\gamma_0 \leftarrow \gamma_0 + \frac{1}{\alpha\left(\gamma_0\right)}\Delta x \quad (28)$$

If this process is repeated until $\Delta x$ meets the predetermined convergence criterion, the desired flight path angle for the collision-ballistic-trajectory can be determined. This procedure can be summarized as Algorithm 2.

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**Algorithm 2** $\gamma_d$ Determination Algorithm
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**Input:** initial flight path angle $\gamma_0$ and convergence criterion $\varepsilon_{\text{tol}}$
    **while** $|\Delta x| > \varepsilon_{\text{tol}}$ **do**
        update the initial flight path using Eq. (28)
    **end while**
    **Return** $\gamma_0$
---

### 3.4 Guidance Command

Hereafter, the proposed guidance command for nullifying the heading error in a finite time is discussed. From Eq. (9), it is assumed that the desired flight path angle $\gamma_d$ is slowly varying. Under this assumption, the time-derivative of the heading error can be approximated as

$$\dot{\varepsilon}_h \approx -\dot{\gamma} \quad (29)$$

It is worth noting that the gravity term is already taken into account when determining the IIP in the kinematic relationship as given in Eq. (4). Therefore, from Eqs. (4) and (29), the effective heading error dynamics can be written as

$$\dot{\varepsilon}_h = -\frac{a_n}{V} \quad (30)$$

This equation implies that the heading error can be controlled by imposing the normal acceleration.

According to the previous study [5], any tracking error can converge to zero in a finite time while minimizing the control effort under the specific form of the error dynamics as

$$\dot{\varepsilon}_h + \frac{K}{t_{go}}\varepsilon_h = 0 \quad (31)$$

where $t_{go} = t_f - t$ represents the remaining time of impact (or time-to-go), which can be determined by Algorithm 1. The parameter $K$ is a positive constant, and it can be considered the design parameter that decides the convergence pattern of $\varepsilon_h$. As the parameter $K$ increases, the heading error $\varepsilon_h$ rapidly decreases.

Finally, the proposed guidance command can be obtained by combining Eqs. (30) and (31).

$$a_n = \frac{KV\varepsilon_h}{t_{go}} \quad (32)$$

**Remark 2.** As the error dynamics shown in Eq. (31) is given by the Cauchy-Euler equation, the closed-form solution can be determined as

$$\varepsilon_h = \frac{\varepsilon_{h,0}}{t_f}t_{go}^K \quad (33)$$

From Eq. (33), it can be readily observed that the heading error $\varepsilon_h$ decreases as the time-to-go $t_{go}$ approaches zero. Therefore, finite time convergence is guaranteed in the proposed method.

## 4. SIMULATION STUDY

In this section, numerical simulations are performed to investigate the characteristics of the proposed method. The rocket model used in this study refers to [7]. The rocket model parameters are given as $m = 55,000 kg$ and $S = 12.54 m^2$. The simulation conditions are provided in Table 1 [7]. In this simulation, other design parameters are chosen as $K = 3$, $\Delta\gamma = -0.1\ deg$, $\varepsilon_{tol} = 1.0\ m$. Those are considered as the default values.

Table 1. The simulation conditions used in this study.

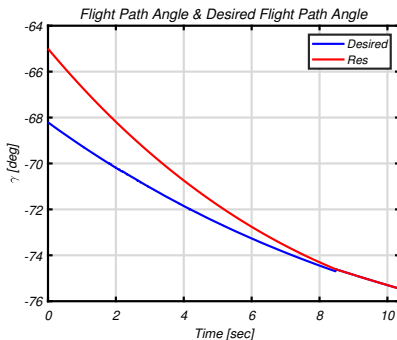| Parameters | Values |
|---|---|
| Initial Point, $(x_0, h_0)$ | $(0.0, 3.0)\ km$ |
| Dive Point, $(x_T, h_T)$ | $(1.0, 0.0)\ km$ |
| Initial Flight Path Angle, $\gamma_0$ | $-65\ deg$ |
| Initial Velocity, $V_0$ | $280\ m/s$ |
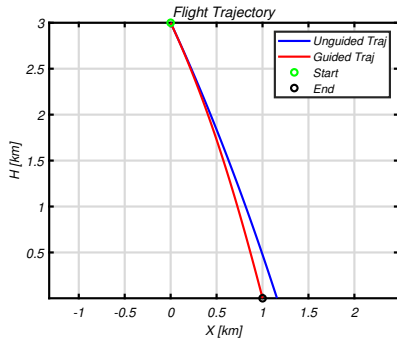


Fig. 3. Flight trajectory profile.



Fig. 4. Flight path angle profile.

Fig. 3 shows the flight trajectory. It can be readily observed that the rocket touches down the desired dive point successfully under the proposed guidance algorithm. Fig. 4 provides the flight path angle and the desired flight path angle determined by Algorithm 2 respectively. As shown in the result, the flight path angle approaches the desired flight path angle by the proposed method. Accordingly, the heading error decreases
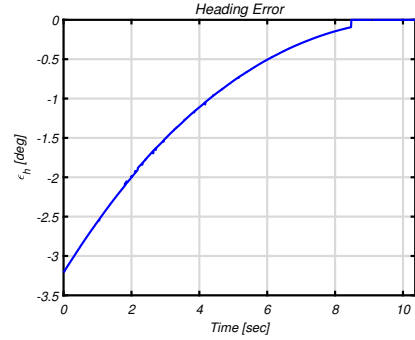


Fig. 5. Heading error profile.
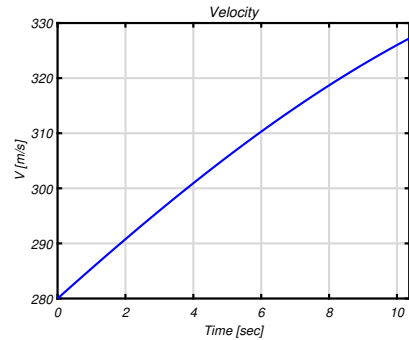


Fig. 6. Normal acceleration profile.



Fig. 7. Velocity profile.

as the rocket approaches the desired dive point, as shown in Fig. 5. The guidance command profile can be observed in Fig. 6. The result obtained indicates that the proposed guidance algorithm produces a small acceleration demand for achieving the guidance goal, which desirable property for the rocket system. Fig. 7 shows the velocity profile. We can observe that the velocity increases during the flight because of gravity.

Figs. 8 and 9 show the simulation results with various design parameter $K = 3, 4, 5$. As shown in Fig. 8, the magnitude of guidance command increases as the design parameter $K$ increases. As a result, the heading error rapidly converges to zero as the design parameter $K$ increases. Figs. 10 and 11 represent the simulation results with various dive points $x_T = 0.5, 1.0, 1.5\ km$. As shown in Fig. 10, the rocket can successfully reach the desired dive point by the proposed method even though the dive point changes. However, more acceleration is required depending on the dive points, as shown in Fig. 11.
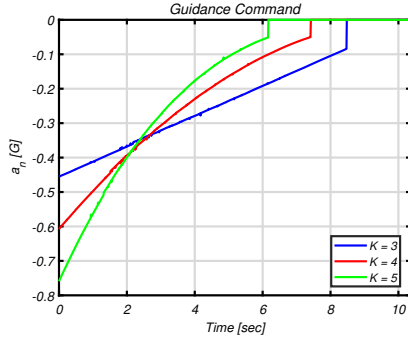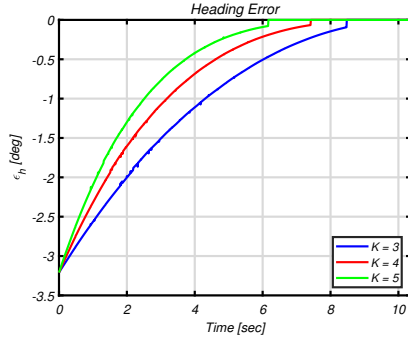
Fig. 8. Normal acceleration profile with various $K$.
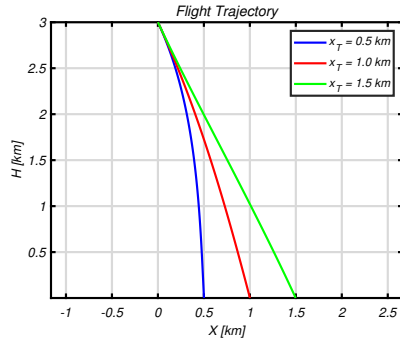


Fig. 9. Heading error profile with various $K$.
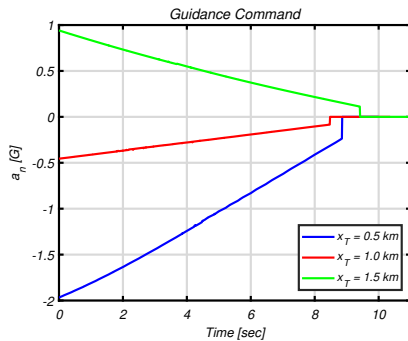


Fig. 10. Flight trajectory with various $x_T$.



Fig. 11. Normal acceleration with various $x_T$.

## 5. CONCLUSION

This paper studies a new instantaneous impact point (IIP) guidance algorithm for a rocket with aerodynamic control for flight safety operations. To this end, we developed a rapid IIP prediction method considering the aerodynamic drag and gravity based on the partial closed-form solutions of the ballistic trajectory equations. In addition, the computing algorithm for the collision-ballistic-trajectory was proposed. Under the collision-ballistic-trajectory, the IIP reaches the desired target point without any correction maneuver. The proposed guidance algorithm was then realized in a way to nullify the heading error by utilizing the optimal error dynamics. Finally, numerical simulations were performed to verify the performance of the proposed method. The results obtained indicate that the proposed method can successfully guide the rocket towards the desired dive point with a minimal acceleration demand, even in the presence of the aerodynamic effect.

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