RESULTS FROM A COMPARISON OF APPROXIMATE ANALYTICAL SOLUTIONS WITH A DETAILED NUMERICAL INVERSION ANALYSIS TO DETERMINE THE THERMAL CONDUCTIVITY OF THE REGOLITH AT THE MARS INSIGHT LANDING SITE USING DATA FROM HP³ HEATING EXPERIMENTS. P. Morgan[1], M. Grott[2], T Spohn[2], J. Knollenberg[2], J. P. Murphy[3], S. Nagihara[4] and S. D. King[3], [1]Colorado Geological Survey, Colorado School of Mines, Golden, CO 80401, USA, morgan@mines.edu, [2]Institute of Planetary Research, German Aerospace Center (DLR), Rutherfordstraße 2, 12489 Berlin, Germany, [3]Department of Geosciences, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA, [4]Department of Geosciences, Texas Tech University, Lubbock, TX 79409, USA.

Introduction: A direct measurement of the regolith thermal conductivity at the Mars InSight landing site (4.50°, 132.62°E) was made by heating experiments using the physical properties package (HP³) of the Mars InSight mission [1]. Temperature and time data from these heating experiments, after removal of background temperature variations, were analyzed using a finite element model for which Monte Carlo simulations were run varying regolith thermal conductivity, density, thermal contact conductance between the probe and the regolith to determine parameter combinations that best fit the heating curve [1,2]. In terms of simulating details of heating experiment this data reduction and numerical inversion is as complete as possible within the current constraints of the experiment. However, no information was included in the model concerning regolith thermal conductivity variations radial to the probe caused during penetration of the probe.

The probe data may also be interpreted using analytical methods, such as the line- and cylinder-source solutions. These solutions are not capable of reproducing structural and thermal property details in the probe and they assume that the probe is infinite in length. However, they are simple and quick to calculate. They have an additional advantage that as heating time increases, the solutions are influenced by thermal conductivities with greater radial distance from the axis of the probe.

Analytical Solution Theory: For a continuous, constant, infinitely long line-source, initially at zero temperature, started at time t = 0, the temperature $T(t)$ in the medium is given by [3,4]:

$$T(t) = \frac{q}{4\pi K}\int_{r^2/4\kappa T}^{\infty}\frac{e^{-u}du}{u}$$

where q is the strength of the heat source per unit length per unit time, K is thermal conductivity, and κ is thermal diffusivity. Then

$$T(t) = -\frac{q}{4\pi K}\,\mathbf{Ei}\left(\frac{r^2}{4\kappa t}\right)$$

where r is the perpendicular distance from the axis of the heat source, and

$$-\mathbf{Ei}\,(x) = \int_x^{\infty}\frac{e^{-u}du}{u}$$

is the exponential integral.

For small values of x, **Ei**(x) may be expanded as

$$-\mathbf{Ei}\,(-x)) = -\gamma - \ln x - \frac{x^2}{2\cdot 2!} + \frac{x^3}{3\cdot 3!} - \frac{x^4}{4\cdot 4!} + \cdots$$

where γ = 0.577216… is Euler's constant. The boundary conditions for the line source are $t = 0$, $r \neq 0$, $T(0) = 0$; $t > 0$, $r = \infty$, $T(t) = 0$; and $t > 0$, $r \to 0$, q = constant = $-2\pi rK\,dT/dr$.

When $r^2/4\kappa t$ is small compared to one, corresponding to temperatures at points close to the line heat source and/or large values of time, the solution reduces to

$$T(t) = \frac{q}{4\pi K}\left(-\gamma - \ln\frac{r^2}{4\kappa t}\right).$$

For a fixed radius r, the temperature rise from T_1 to T_2 from times t_1 to t_2 is given by

$$T_2 - T_1 = \frac{q}{4\pi K}\,ln\left(\frac{t_2}{t_1}\right)$$

or the thermal conductivity is given as

$$K = \frac{q}{4\pi}\left(\frac{\ln(t_2/t_1)}{T_2 - T_1}\right) = \frac{q}{4\pi}\,Slope$$

where *Slope* is the gradient of the linear section of the plot of temperature vs. ln(time).

Consider a continuous, constant, infinitely long cylinder-source, made of a perfect conductor, initially at zero temperature, started at time t = 0, with outer radius $r = a$ and heat capacity per unit length of S. The medium outside the cylinder has a density ρ, specific heat c, thermal conductivity K, and diffusivity $\kappa = K/\rho c$. The temperature in the medium is $T(t)$ at radius r from the axis of the cylinder and is equal to $Ta(t)$ on the surface of the cylinder. Starting at t = 0 heat is supplied to the surface of the cylinder at the rate of q per unit length per unit time. There is a thermal contact resistance between the perfectly conducting cylinder and the medium of $1/H$ per unit are. The following solution is taken primarily from [3, 5 & 6]. Heat flux across the outer surface of the cylinder is

$$H\big(T(t) - T_a(t)\big)$$

The solution is expressed in terms of three dimensionless parameters, i.e.,

$$\tau = \kappa t/a^2 \qquad \alpha = 2\pi a^2\rho c/S \qquad h = K/aH$$

For large values of time, the temperature rise of the cylinder is given by

$$T(t) = \frac{q}{4\pi K}\left[\frac{2K}{aH} + \ln 4\tau - \gamma - \frac{h-\beta}{2\beta\tau} + \frac{\beta-2}{2\beta\tau}(\ln 4\tau - \gamma)\right]$$

For large values of time (and large values of τ) the temperature rise becomes directly proportional to the logarithm of time.

Intermediate steps have been omitted because of requirements of brevity, but for material used in probes (cylinders) to determine thermal conductivity of insulating materials, such as earth materials, for large values of time ($t >> a^2/\kappa$ or τ) the equation for the temperature rise of the probe reduces to

$$T(t) = \frac{q}{4\pi K}\left(\ln\frac{4\kappa t}{a^2} - \gamma + \frac{2K}{aH}\right)$$

By analogy with the temperature rise at a fixed radius for the line-source solution

$$T_2 - T_1 = \frac{q}{4\pi K} ln\left(\frac{t_2}{t_1}\right)$$

or the thermal conductivity is given as

$$K = \frac{q}{4\pi}\left(\frac{\ln(t_2/t_1)}{T_2 - T_1}\right) = \frac{q}{4\pi} Slope$$

Thus, the equation from which to determine thermal conductivity from a plot of temperature versus ln(t) is the same for a cylinder-source probe as for a line-source probe. The reason that the analytical line- and cylinder-source solutions are the same for long times is that as time increases the heating depends on the increase in temperature of increasingly large radii cylinders of the enclosing medium surrounding the probe. For short time scales the rate of heating is controlled by the geometry of the probe and the medium immediately adjacent to the probe. As time increases, however, probe geometry and the media close to the probe decrease in significance and the rate of heating depends only on the far-field geometry and the medium in the far-field, which are identical for both probe geometries.

InSight Landing Site Results: As of January 2022, four successful HP3 heating experiments have been executed to determine regolith thermal conductivity at the landing site. These experiments were performed over a period of approximately 273 Sols (Sol 798 to Sol 1070 relative to landing). Thermal conductivity varied in response to atmospheric pressure changes affecting gas heat transfer in the pore spaces of the regolith, as shown in Figure 1 and Table 1. Results are shown for both the detailed numerical analysis and the analytical cylinder solution. The mismatch in these results is certainly due to the infinite probe length assumption in the analytical solution. The error in this assumption was calculated to be slightly <1% for $L/a = 25$, where L is the probe half-length and a is its radius and negligible for typical experimental measurements for $L/a = 30$ [7]. For the HP^3 probe $L/a \approx 7.5$ indicating a significant error in the infinite length assumption. A commercial finite element program, COMSOL®, has been used to calculate the difference in thermal conductivities calculated using the analytical cylinder solution for data sets created for probes of L/a ratios. This work is continuing but preliminary results suggest that the correction for the HP^3 probe should be ~-0.007 W/(m K). This preliminary correction has been applied to the analytical results in Table 1 and brings the analytical results to within 0.002 W/(m K) of the detailed numerical results.

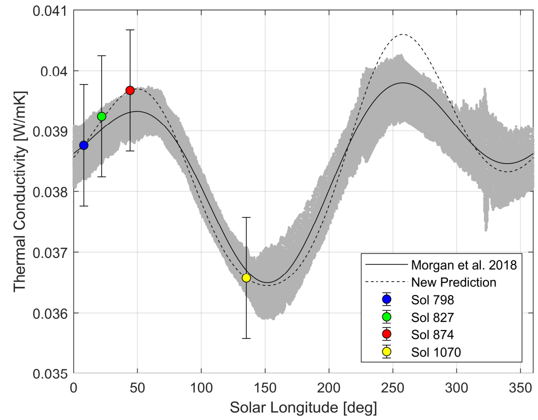

Figure 1. Plot of measured thermal conductivity versus solar longitude. Results shown are from detailed numerical analysis.

Table 1. Comparison of thermal conductivities derived from detailed numerical analysis (K_{comp}) and analytical cylinder solution without (K_{cyl}) and with ($K_{cylcorr}$) finite probe length correction.

InSight Sol	Solar Lon. °	Thermal Conductivity, W/(m K)				
		K_{comp}	err	K_{cyl}	$K_{cylcorr}$	diff
798	8	0.0388	0.0011	0.047	0.04	0.0013
827	22	0.0392	0.0010	0.047	0.04	0.0010
874	44.2	0.0397	0.0010	0.048	0.041	0.0008
1070	135.3	0.0366	0.0010	0.042	0.035	-0.0017

References: [1] Grott, M. et al. (2019) *Earth & Space Sci., 6*, 2556-2574. [2] Grott, M. et al. (2021) *JGR, 126*, e2021JE006861. [3] Carslaw, H. S. & J. C. Jaeger (1959) Conduction of Heat in Solids, Oxford, **10.4** II 510 pp. [4] Wechsler, A. E. (1992) in K. D. Maglić K. D. et al., eds., Compendium of Thermophysical Property Measurement Methods, v. 2, Springer Science+Business Media, New York, 643 pp. [5] Blackwell, J. H. (1954) *J. Appl. Phys., 25*, 137-144. [6] Jaeger, J.C. (1956) *Aust. J. Phys., 9*, 167-179. [7] Blackwell, J. H. (1956) *Can. J. Phys., 34*, 412–417.