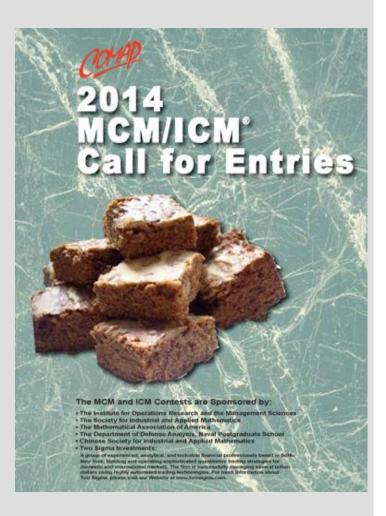
Mathematical Modeling of Highway Traffic Policies

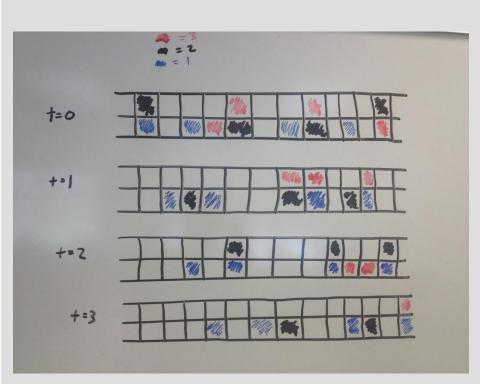
Ben Squire, Nate Minor, John-Paul Mann

Mathematical Competition in Modeling

- 96-hour International Math Competition organized by COMAP
- Prompt:
 - Model and analyze the efficiency and safety of the Stay-Right-Except-to-Pass highway traffic policy
- Over 8000 teams
- Awarded 'Meritorious' (top 10%)



Approach

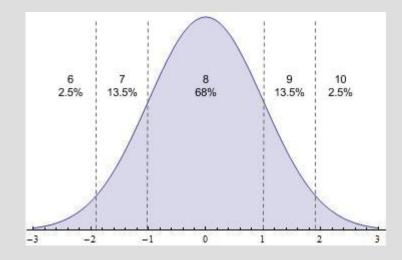


- Macroscopic Model: Differential Equations
 - Fluid Dynamics: Convection-Diffusion Equation
 - Lighthill-Whitman-Richards Model
- Microscopic Model: Agent-Based
 - Proto-Model

Assumptions & Definitions

- No Exit or Entry Ramps (Conservation of Cars)
- Identical Vehicles (length, driving ability, braking ability)
- Lane width, road curvature, shoulder width, and weather are not considered in our analysis
- Only Side-Swipe and Rear-End Accidents
- Accidents are only counted, they do not actually happen during the computer simulation
- Normal distribution of initial velocities
- Uniform random initial positions of vehicles

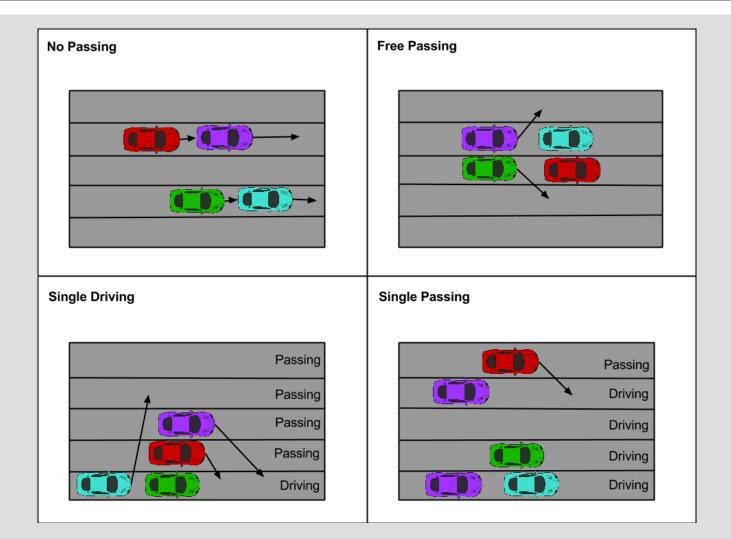
Simulation Scaling



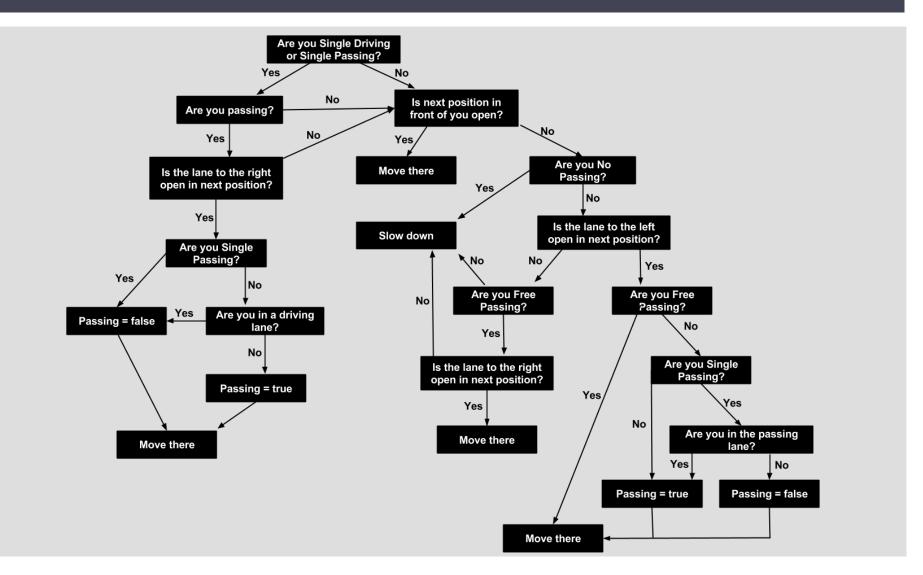
- Road Length: 10,000 tiles
- High Density: 75% tiles occupied by vehicles
- Low Density: 25% tiles occupied by vehicles
- 500 iterations per simulation (approx. 8 minutes)

Simulation Speed	Real Speed		
Tiles/Iteration	(MPH)	(KPH)	
6	54	86	
7	63	101	
8	70	115	
9	80	130	
10	90	144	

Road Rules

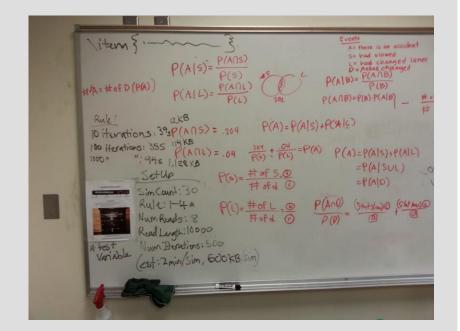


Vehicle Interaction Decision Tree



Experimental Methods

- Test 4 passing rules on 8 different road types (high/low density, 2-5 lanes)
- Run 30 computer simulations per passing rule
- Compare the average vehicle speed, average traffic flow, and average safety rating for each rule paired with a road type (i.e.,compare free passing to single driving for a high-density 3-lane road)



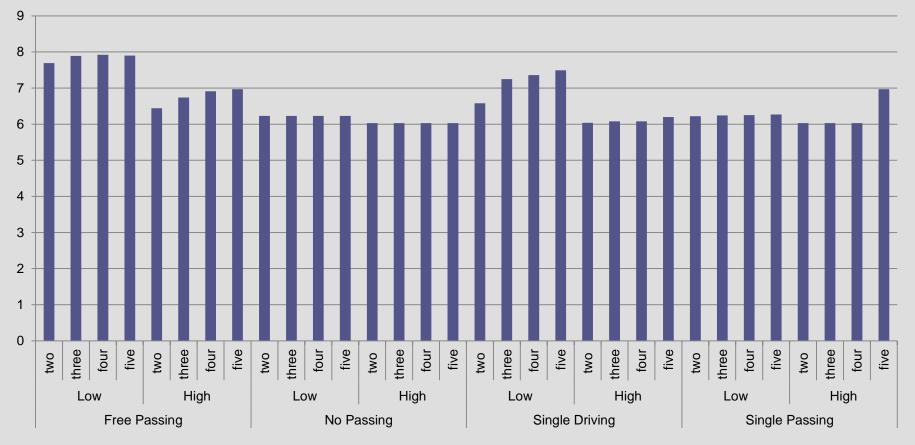
Simulation Data

20 Road Oila density (dash density (dash density (dash density) (dash density (dash density) (dash density (dash density) (dash density (dash density) (dash density (dash density) (dash density (dash density)	V V. Z D Z Z V. Z D Z Z D Z Z V V V V V V V V V V V V V V V V V V V	And	$\begin{array}{c} \overline{g}_{1}, \overline{p}_{1}, \overline{g}_{1}, \\ \overline{g}_{2}, \overline{p}_{1}, \overline{g}_{1}, \\ \overline{g}_{1}, \overline{g}_{2}, \overline{g}_{2}, \\ \overline{g}_{1}, \overline{g}_{2}, \overline{g}_{2}, \\ \overline{g}_{1}, \\ \overline{g}_{2}, \\ \overline{g}_{2$	A) P(A(15) P(A(12) S) 	$= \overline{\rho} \overline{\gamma}_{n} = 25 \frac{3}{20} \frac{\overline{\gamma}_{n}}{30}$ $= \frac{2}{2} (q_{n} - \frac{q_{n}}{2})$ $= \frac{2}{2} (28\overline{\gamma}_{n} - \overline{\gamma}_{n})^{2}$ $= \frac{2}{2} (q_{n} - \overline{\gamma}_{n})^{2}$ $= \frac{2}{2} (q_{n} - \overline{\gamma}_{n})^{2}$
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- Collected per Iteration:
 - Average vehicle speed
 - Number of decisions
 - Number of lane change
 - Number of slow downs
- Calculated at the end of each Simulation:
 - Average vehicle speed
 - Average number of decisions
 - Average number of lane changes
 - Average number of slow downs

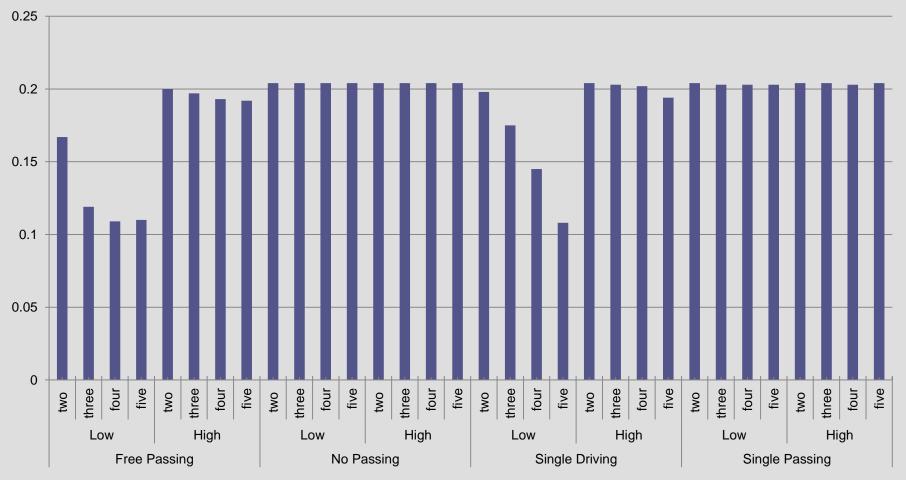
Simulation Results

Average Vehicle Speed (tiles per iteration)



Simulation Results

Average Safety Rating



Conclusion

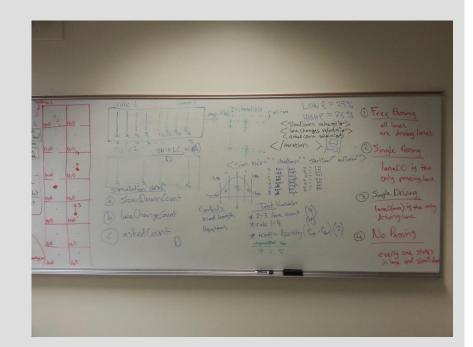
Free Passing: highest average traffic flow, highest average vehicle speed, & lowest average safety rating for all traffic densities and lanes counts.

Free Passing vs. Single Driving					
Lane Count : Density	Tests Flow (z-Score)	Tests Speed (z-Score)			
2: Light	45.88	45.88			
2: Heavy	26.74	26.74			
3: Light	51.51	167.08			
3: Heavy	43.04	108.69			
4: Light	44.69	44.69			
4: Heavy	87.26	87.26			
5: Light	50.45	50.45			
5: Heavy	22.45	121.39			

Challenges & Solutions

Challenges

- Exporting XML to Excel without Macros
- Interpreting results
- Solutions
 - Perform summary statistics in Java
 - Review statistical analysis step-by-step



Further Research

- Simulation data moved to a database
- Graphical User Interface (GUI) for 'playing-back' simulation data
- Individualized Vehicle Rules
- Variations in Driver Psychology
 - Oncoming Traffic
 - Condition of Road
 - Weather
 - Following Reaction Time

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