ESSAYS IN CONTEST THEORY

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Introduction

A contest is a game in which participants can exert costly effort to improve their chances of winning a valuable prize. Such an allocation mechanism describes many real-life situations: In election campaigns, political parties spend resources to sway voters in their favour. Interest groups try to influence political decisions by organising meetings with politicians and drafting competing legal bills. Battling factions buy weapons and hire soldiers to win violent confrontations. Firms invest in research to obtain patents before their competitors do. Contests are also deliberately designed, for example in the form of innovation and design contests, admission competitions for job positions, colleges, and grants, and sports tournaments.

In this thesis, I use game-theoretical models to analyse important aspects of real-world contests that have so far either not been considered or looked at from a different angle in the literature: In Chapter 1, I modify the model on group contest so that individuals are a member of more than one group. In Chapter 2, I allow for the endogenous formation of such "non-exclusive" groups. And in Chapter 3, I investigate how the ability of players to copy their opponent's effort affects innovation contests.

The literature on contest theory can be traced back to a number of seminal contributions. Tullock (1980) studies rent seeking and formulates a contest success function with noise—that is, the contestant who exerts the highest effort does not necessarily win. This type of contest is usually referred to as "Tullock contest" and sometimes as "lottery contest". Hillman and Riley (1989) study a contest in which the contestant who exerts the highest effort wins with certainty, usually referred to as an "all-pay auction". This model is applied to the selection of contestants in a lobbying contest by Baye et al. (1993). The equilibrium in the all-pay auction with complete information is fully characterised by Baye et al. (1996). Hirshleifer (1989) introduces a contest success function in which a contestant's winning probability depends on the difference between her effort and the other contestants' efforts.

Throughout this thesis, I use the Tullock contest success function. It has been axiomatised for single players by Skaperdas (1996) and for groups by Münster (2009). This means that the Tullock contest success function can be derived from a set of intuitive rules a contest allocation mechanism should fulfil. It can also be underpinned with a microeconomic foundation: Hirshleifer and Riley (1992) demonstrate that it can be derived departing from an all-pay auction with noise, a property I make use of in my analysis of copying in contests in Chapter 3. Baye and Hoppe (2003) show that a Tullock contest is strategically equivalent to a patent race and an innovation tournament under certain conditions.[1] Analytically, the Tullock contest has the nice property that it allows for the derivation of Nash equilibria in pure strategies, both with single as well as group contestants, for a wide range of parameters.

The contest literature has been extended to incorporate many important aspects of real-world competitions. For instance, dynamic contests have been thoroughly investigated by Rosen (1986), Gradstein and Konrad (1999), Moldovanu and Sela (2006), Konrad and Kovenock (2009b), and Groh et al. (2012). Pre-commitment and sequential moves in contests have been analysed by Dixit (1987) and Perez-Castrillo and Verdier (1992). Contestants may not be fully informed about the nature of the game—for example, the value of the prize—and not all participants in a contest may share the same information. Variants of such settings have been investigated by Glazer and Hassin (1988), Amann and Leininger (1996), Krishna and Morgan (1997), Moldovanu and Sela (2001), and Wärneryd (2003). The fact that it is not uncommon to have more than one prize allocated in a contest is reflected in the work of Clark and Riis (1998a) and Moldovanu and Sela (2001). Roberson (2006), Clark and Konrad (2007), and Kovenock and Roberson (2010) analyse settings in which players compete in multiple contests. The theoretical predictions developed in the contest theory literature have also been extensively tested in laboratory experiments. Important contributions include Millner and Pratt (1989, 1991), Davis and Reilly (1998), Potters et al. (1998), and Sheremeta (2010). A comprehensive overview of the contest literature can be found in Konrad (2009). For a more recent survey of the contest literature in general, see Corchón and Serena (2018), and for a survey of experimental research on contests in particular, consult Dechenaux et al. (2015).

One important aspect of contests is that they are often carried out between groups: interest groups and political parties consist of individuals with similar goals, wars are often fought between alliances, and firms may join forces in research contests. Such a group contest, in which effort contribution has a public-good character for fellow group members, has been analysed by Katz et al. (1990). In their model, groups fight

[1] The Tullock contest success function can additionally be derived from an optimal-design perspective, from incomplete information and search based foundations, and from Bayesian foundations. See Jia et al. (2013).

in a Tullock contest over a prize which is a public good to the winning group. Due to linear effort costs, only the players with the highest valuation of the prize exert effort, and group size is irrelevant. Nitzan (1991a) considers a group contest in which a private prize is partly distributed equally among members of the winning group and partly dependent on effort contributions. Baik (1993) considers a more general contest success function and allows for players who have different prize valuations. In Riaz et al. (1995), players have budget constraints and additionally consume a private good, which leads to increasing marginal effort costs and thus makes group size "matter". A majority of the literature on contests between groups uses the Tullock contest success function or a more generalised success function with similar properties. One exception is Baik et al. (2001), who show that a Nash equilibrium in pure strategies does not exist in an all-pay auction between groups. Esteban and Ray (2001) analyse contests between groups over a prize with a varying mix of public and private characteristics. They find that, if the prize is sufficiently public and the elasticity of the marginal effort cost sufficiently high, an increase in a group's size increases its probability to win the prize. This finding qualifies the "group size paradox" described by Olson (1965) according to which larger groups are less effective in furthering their interests. The impact of the size of a group on its effectiveness in a contest is further investigated by Nitzan and Ueda (2009, 2011). Baik (2008) considers contests between an arbitrary number of groups and investigates how individual budget constraints affect free-riding behaviour. In the majority of the group contest literature, as well as in this thesis, efforts by members of a group are treated as subsitutes. Alternatives are investigated by Lee (2012), Chowdhury et al. (2013), and Kolmar and Rommeswinkel (2013). There is a growing literature that tests theoretical predictions on group contests in experiments. See, for example, Nalbantian and Schotter (1997), Abbink et al. (2010), Sheremeta and Zhang (2010), and Chowdhury et al. (2016), and for a recent survey, consult Sheremeta (2018). If the prize is not a pure public good for the winning group, the question arises how the prize is allocated within this group. One approach is to analyse potentially endogenous sharing rules, which is done for example by Nitzan (1991a,b), Davis and Reilly (1999), and Nitzan and Ueda (2011). If such sharing rules do not exist or cannot be enforced, members of the winning group might enter an additional intra-group contest. Notable contributions on this form of multiple-stage contest are Katz and Tokatlidu (1996), Wärneryd (1998), Konrad (2004), Münster (2007b), and Choi et al. (2016). Note that the question of intra-group prize allocation does not arise in Chapter 1 since I consider a public-good prize, and in Chapter 2, I assume a simple egalitarian sharing rule for simplicity.

In real-life group contests, individuals are often members of more than one group. For instance, firms can support different lobby groups who fight for industry-specific subsidies. In distributional conflicts, individuals naturally belong to multiple interest groups, such as "rich", "young", and "urban". In such contests between what I call *non-exclusive groups*, players have multiple channels to win the prize, and must decide which of their groups to support with effort. However, non-exclusive group-membership has so far been absent from the contest literature. I try to fill this gap in Chapter 1 and Chapter 2 of this dissertation.

In Chapter 1, I introduce a model of Tullock group contest in which individuals are partitioned into two groups in two dimensions each.[2] Individuals can exert effort for both of their groups and are indifferent which of their groups wins and provides the public-good prize. Their marginal effort cost is increasing in effort. Individual efforts for a group are summed up and in the baseline model enter a concave impact function. In this framework, I show that the additional partition dimension does not alter the level of aggregate and individual effort in equilibrium compared to the canonical model in which individuals belong to only one group. This means that also equilibrium utilities are equivalent. This equivalence result supports the validity of the model of group-contest that has been predominantly used in the literature. Additionally, I investigate asymmetries in group size and effort cost. Non-exclusive group membership allows players to shift their effort between groups. This leads to non-monotonic effects of asymmetries and qualifies the results on group-effectiveness by Esteban and Ray (2001). I further show that it is beneficial for individuals to be a member of additional groups if others are not. It provides them with additional channels to exert effort and allows them to free-ride on the effort of group-members who do not have access to additional groups. The main result of equilibrium equivalence to the model of exclusive group membership carries over to a setting with an arbitrary number of symmetric groups in an arbitrary number of dimensions. Finally, I show that this equivalence also holds to a certain extent if we loosen the concavity restriction on the group impact function.

The question how groups in contests form endogenously has received significant attention in the literature. Baik and Shogren (1995a) analyse a contest in which players decide simultaneously whether to join a group which distributes the prize

[2] A version of Chapter 1 has been published as: Send, J. (2020). Conflict between non-exclusive groups. Journal of Economic Behavior & Organization, 177, 858-874.

according to an endogenous sharing rule. Baik and Lee (1997) allow for inter-group mobility before the contest. Esteban and Sákovics (2003) investigate group formation in contests in the light of subsequent intra-group contest. Konrad and Kovenock (2009a) make the point that budget constraints can make groups more attractive to form. Other important contributions on the formation of groups include Skaperdas (1998) and Garfinkel (2004a,b). For surveys of the literature on endogenous group formation in contests see Bloch (2012) and Konrad (2014). The topic has been analysed experimentally by Herbst et al. (2015) and Ke et al. (2013, 2015). A natural extension of Chapter 1 is to ask how group formation in contests is affected by non-exclusive membership and whether such non-exclusivity can arise endogenously.

To answer these questions, in Chapter 2, I analyse a stylised model of group contest in which the leaders of two groups can decide over two dimensions of membership-exclusivity: whether an additional member is allowed to join their group at all, and whether this member is allowed to join the other group as well. The potential additional member does not have access to the contest on her own. Following the literature on the group-size paradox, I consider a prize with a private and public component and an effort cost that is convex of a variable degree. If the prize is mostly private, group leaders do not offer membership in equilibrium. If the prize is mostly public or the elasticity of marginal effort cost high, they offer exclusive membership. Non-exclusive membership is never offered in this baseline setting. However, I show that membership commitment, membership fees, and the introduction of a third group all lead to the emergence of equilibria with endogenous non-exclusive membership. This underlines the importance to investigate non-exclusive membership in contests. Commitment by the potential member to join non-exclusive groups rather than exclusive ones harms group leaders. If group leaders can charge a membership fee, they offer non-exclusive membership and extract the member's surplus if groups are less likely to win the contest—less effective—than singletons. This mitigates conflict and is beneficial to group leaders. Otherwise, group leaders compete for the member by offering potentially negative fees for exclusive membership. This is harmful to group leaders compared to the baseline without fees. The introduction of a third group to the baseline model makes it more likely for the member to be able to join a group and allows for an equilibrium with non-exclusive membership if the prize is highly public. The latter is due to the fact that, with three groups, a group leader cannot make all groups effectively exclusive on her own. I also look at the game from the perspective of a contest designer interested in maximising effort, and show that she would like to prohibit non-exclusive membership and allow exclusive membership only if groups are more effective than singletons. The results are robust to a higher number of groups and a higher number of potential members.

In many contests, exerting effort may not be the only way for participants to increase their chances of winning. Before a contest starts, contestants may try to enhance their performance, which has been investigated, among others, by Berentsen (2002), Haugen (2004), and Kräkel (2007). Contestants can try to sabotage their opponents' efforts, an activity that has been studied by Konrad (2000), Chen (2003), and Münster (2007a). Especially in innovation contests, contestants may benefit from spying on their opponents and copying their ideas. Examples of such behaviour have been documented in the race to develop the first atomic bomb (Haynes and Klehr, 2000), the space race between the United States and the Soviet Union (Wesley, 1967), and in Formula 1 (Solitander and Solitander, 2010). However, such a copying of contest effort has so far not been studied in the theoretical literature.

In Chapter 3, I try to fill this gap and analyse a contest between two players who can pay a fixed cost for the ability to copy their opponent's effort and add it to their own.[3] I allow one player to be more productive than the other. I characterise the unique Nash equilibrium of this game in dependence on the cost of copying and the stronger player's productivity advantage. In many cases, players play a mixed strategy and copy their opponent's effort only with a certain probability. Intuitively, players copy more often if their relative productivity or the cost of copying declines. However, also a number of surprising effects emerge: First, if the cost of copying is low, the weaker player is more likely to win the prize in equilibrium. Second, the more productive player's utility can decrease in her productivity advantage. This implies that a government who wants to increase a domestic firm's profit may not want to subsidise this firm's innovative effort, even if the subsidy were costless. Third, the aggregate effort players exert can also decrease in the stronger player's productivity advantage. Hence, in contrast to the baseline without copying, a contest designer who would like to maximise aggregate innovative effort in a contest between two players may want to exclude a more productive contestant in favour of a weaker one. Finally, I show that the expected winner's effort—potentially including effort copied from an opponent—is generally increasing in the cost of copying. The designer of an innovation contest would thus like to make copying of effort prohibitively costly, even though

[3] A version of Chapter 3 has been published as: Send, J. (2022). Contest Copycats: Adversarial Duplication of Effort in Contests. Defence and Peace Economics.

copying allows players to have access to both their own as well as their opponent's effort.

Chapter 1

Conflict between Non-exclusive Groups

1.1 Introduction

Political conflict over scarce resources often takes place between many non-exclusive groups. For example, a society can be divided into urban and rural, and wealthy and poor residents, into atheist and religious, and liberal and conservative communities, or into different ethnic and cultural groups. Individuals usually are members of more than one of these groups and can also support more than one of these groups at the same time.

For instance, consider a wealthy individual living in a rural area who lobbies for a subsidy for the construction of a public yacht haven and at the same time for the public extension of digital infrastructure to the countryside. Both projects are financed from the same public budget. The first one only benefits the wealthy who can afford a yacht, whereas the second one only benefits rural residents. In the spatial dimension, the rural group might oppose an urban group lobbying for urban public transportation. In the class dimension, the wealthy might oppose the poor, who want to secure public resources to build a community centre. Hence, four groups are in a conflict over the same prize to finance a group-specific public good, and individuals can support multiple groups. Figure 1.1 illustrates this example.

Real-world instances include wealthy donors supporting multiple candidates in political races, both within and across parties; companies organising in multiple interest groups to lobby on different, at times conflicting issues; and individuals being members of various activist groups that want to implement different projects financed as part

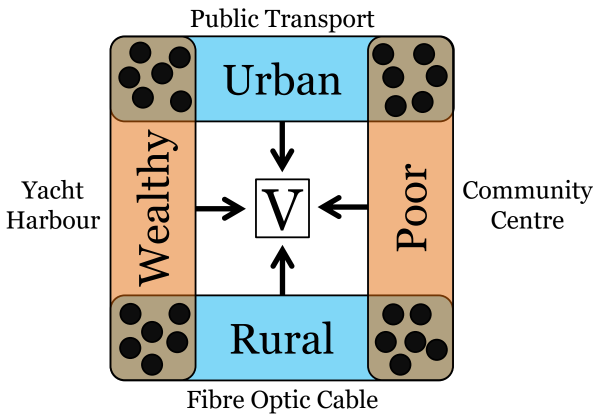

Figure 1.1: A society divided into two groups in two dimensions

of a finite public budget.[1]

I capture conflict between multiple non-exclusive groups in a novel theoretical framework: Individuals are partitioned in two dimensions into two groups as in Figure 1.1. All groups compete in a Tullock contest over a single public prize, where I assume convex effort cost and a Tullock exponent r smaller than 1. New to the literature, individuals can exert multiple efforts for all groups they are a member of simultaneously.

If groups have the same size and individuals face the same effort cost, aggregate effort and expected individual utility is the same as in the standard Tullock group contest model with only one partition dimension. All groups win the prize with the same probability. Group-size asymmetry in one dimension leads members of the smaller group to focus their effort on that small group. Members of the larger group on average shift some of their effort to the groups in the other dimension. If a group becomes sufficiently large, its members' win probability and expected utility rise, while members of the small group see their win probability and expected utility decline. An asymmetry in effort cost has non-monotonic effects: At first, it benefits members of the 'weak' group while having a negative effect on individuals in the 'strong' group. If the asymmetry is high enough, it benefits individuals with low effort cost while hurting those with high effort cost.

If only one of the two groups in the second dimension can take part in the contest over the prize, its members will direct their effort to this additional effective channel. In their first-dimension group, they free-ride to at least some extent on the effort of individuals who cannot support an additional group. This is beneficial for individuals

[1]The potential influence on group policy in these examples goes beyond the scope of this paper. But they show that non-exclusivity of groups is a wide-spread phenomenon in conflicts.

who have access to two groups and harms individuals who are not a member of a group in the second dimension.

In a general setting with an arbitrary number of symmetric groups in an arbitrary number of dimensions, aggregate effort is independent of the number of dimensions in which individuals are partitioned into groups. The existence of a unique Nash equilibrium on the level of group-level effort, and thus individual utility, is not ensured under $r \geq 1$. Irrespective of r, expected individual utilities of existing equilibria are not altered by the addition of new group dimensions.

In Section 1.2, I review the related literature. I introduce a two-by-two model in Section 1.3 and solve for its unique Nash equilibrium under symmetry and under group-size and effort-cost asymmetry, respectively, in Section 1.4. In Section 1.5, I analyse only one additional group in the second dimension and a more general model with symmetry. In Section 1.6, I sum up my contribution.

1.2 Related literature

The first to formulate group contests were Katz et al. (1990) and Ursprung (1990), both in the context of rent seeking for public goods and both using the success function introduced by Tullock (1980). Whereas Katz et al. (1990) focus on heterogeneous prize valuations and risk aversion, Ursprung (1990) investigates endogenous prize value in the setting of political candidate competition. In both approaches, the number of group members does not matter for the aggregate expenditure of effort if effort cost is linear. Nitzan (1991a) considers a contest between groups in which part of a private prize is distributed on egalitarian grounds and the rest is distributed according to relative effort. Baik (1993) also considers players who have heterogenous prize valuations. Riaz et al. (1995) formulate a more generalised model with a budget constraint and private good consumption and conclude that collective rent seeking over a public good is increasing in group size and wealth levels. Nti (1998) investigates per-capita payoffs in an asymmetric two-group contest over a public good. He shows that payoff per capita for a group increases with its own valuation and decreases with the opposing group's valuation and that per capita payoff for a group increases with its size. In an important contribution, Esteban and Ray (2001) study a collective contest over a prize with a varying mix of public and private characteristics and with nonlinear effort cost. They find that, if the prize is sufficiently public and the elasticity of the marginal effort cost sufficiently high, an increase in group size increases group effectiveness. Nitzan and Ueda (2009, 2011) further analyse group-size effectiveness in contests by allowing for endogeneous usage of and sharing rules for the prize. Baik (2008) considers contests between an arbitrary number of groups with individual budget constraints. Münster (2009) axiomatises the group contest success function. Related to my work, Cherry and Cotten (2011) consider a setting in which a group competes with a subset of it, but individuals can only support one group. Since I consider a contest over a public good, I do not consider in-group fighting. For the latter, see e.g. Konrad (2004) for hierarchies in contests, Hausken (2005) and Münster (2007b) for simultaneous between- and within-group fighting, and Choi et al. (2016) for within-group power asymmetry and complementarity in group members' efforts. Refer to Ke et al. (2013) for an experimental approach.

Multiple group membership offers individuals multiple channels to exert effort. In a sense, my model thus offers individuals multiple 'arms' or 'activities'. For work on multi-activity contests without the strategic aspects of group contest see Epstein and Hefeker (2003) and Arbatskaya and Mialon (2010), who consider different activities as multiplicative strategic complements, Hausken et al. (2020) who analyses additive efforts, and Rai and Sarin (2009), who treat one effort as fixed. Closely related to my work, but focussing on two individuals fighting alone, Osório (2018) considers a framework in which two issues are disputed and a single prize is awarded. For a comprehensive introduction into various aspects of contest design, see Konrad (2009). For a recent survey of the contest literature see Corchón and Serena (2018).

My work is in contrast to the concept of multiple identities and their shifting salience by Sen (2007) in the sense that group membership is fixed and the valuation of their success is equal and fixed as well. Further, any in-group altruism or out-group spite is absent from my model.

My work is further related to Esteban and Ray (2008). They consider a society split into cross-cutting economic and ethnic groups who can compete in a Tullock contest over a group-specific public good. However, there is either conflict between classes or between ethnicities, but not both at the same time; the same is true for Robinson (2003).

In the network literature, externalities of conflict effort have been modelled for example by Chowdhury and Kovenock (2012), Franke and Öztürk (2015), König et al. (2017) and Bozbay and Vesperoni (2018). To my knowledge, there is no framework in which individuals simultaneously exert effort for multiple groups involved in the same contest.

1.3 A two-by-two model

There are $N = 2n, n \geq 2$, risk neutral individuals and a set of four groups $\Gamma = \{W, P, U, R\}$. A group is defined as a non-empty set of individuals who can bundle their contest effort and benefit from the same public good as specified below. Each individual i is a member of two groups, $i \in c$ and $i \in l$, where $c \in \{W, P\}$ and $l \in \{U, R\}$. We have $W \cap P = U \cap R = \emptyset$ and $|W \cup P| = |U \cup R| = N$. All groups compete in the same contest over a public-good prize valued at $V > 0$ by all individuals $i \in \{1, ..., N\}$. See again Figure 1.1 for an illustration. The contest technology which determines the probability p_g with which group $g \in \Gamma$ wins the prize is the general Tullock function

$$p_g = \frac{(X^g)^r}{\sum_{j\in\Gamma}(X^j)^r},$$

where the group-level effort X^g is the sum of individual efforts of its members on behalf of it, $X^g = \sum_{i\in g} x_i^g$. Efforts from different individuals are perfect substitutes. If no group exerts any effort, $\sum_j X^j = 0$, the prize is randomly allocated, $p_g = 1/4$. I assume that $r \in (0, 1)$. A concave group-level impact function is intuitive: The first unit of money or time spent lobbying will be more effective than the hundredth. For example, the first contact to a politician leads to the latter knowing of one's cause in the first place and is thus very effective. The hundredth meeting will hardly introduce additional arguments to sway the politician's opinion. For a brief investigation of the case $r \geq 1$, see Subsection 1.5.3.

Expected utility of individual i who is a member of $c \in \{W, P\}$ and $l \in \{U, R\}$ is

$$u_i = \frac{(X^c)^r + (X^l)^r}{\sum_{j\in\Gamma}(X^j)^r} V - \frac{(x_i^c + x_i^l)^{1+\alpha}}{1+\alpha}, \tag{1.1}$$

where effort cost is convex, $\alpha > 0$. Convexity in the cost function is intuitive: the first unit of money or time spent on lobbying should have lower opportunity cost than the last penny of one's budget or the last minute of one's day. Effort for both groups is provided in the same unit and enters the cost function as a sum. This is an important simplifying assumption. If we consider political lobbying, it is intuitive since lobbying groups are mostly supported with money. I briefly come back to this assumption when I discuss the symmetric equilibrium in Subsection 1.4.1. The first term of (1.1) is a sum of the two Tullock terms for the two groups i is a member of. The individual derives the same utility of a group-specific public good irrespective of which group provides it.

Individuals have two decision variables: effort for each of the two groups they are a member of, x_i^c and x_i^l. I denote the sum of individual effort as total individual effort $x_i = x_i^c + x_i^l$ and the set of groups that i is a member of as $G_i = \{c, l\}$. Individuals simultaneously choose their two effort levels to maximise their expected utility. It can never be an equilibrium to have $x_i = 0 \ \forall \ i$, since then any individual could win the prize for one of her groups for sure with an infinitesimal effort. Taking the first derivative of (1.1) with respect to x_i^g, $g \in G_i$, yields the first order conditions

$$\frac{\partial u_i}{\partial x_i^g} = \frac{r(X^g)^{r-1} \sum_{j \in \Gamma \setminus G_i} (X^j)^r}{(\sum_{j \in \Gamma} (X^j)^r)^2} V - (x_i)^\alpha \overset{!}{\leq} 0, \tag{1.2}$$

If any group that i is not a member of has strictly positive group-level effort, $\exists \ X^j > 0, j \in \Gamma \setminus G_i$, for finite x_i the left-hand side of (1.2) tends to infinity as X^g tends to zero. Hence, in any equilibrium, all group-level efforts must be strictly positive, $X^j > 0 \ \forall \ j \in \Gamma$ and thus also all individual efforts, $x_i > 0, \ \forall \ i$. We ensure concavity of the objective function by looking at the second order conditions

$$\frac{r(X^g)^{r-2} \sum_{j \in \Gamma \setminus G_i} (X^j)^r \left((r-1) \sum_{j \in \Gamma} (X^j)^r - 2r(X^g)^r\right)}{(\sum_{j \in \Gamma} (X^j)^r)^3} V - \alpha (x_i)^{\alpha - 1} < 0. \tag{1.3}$$

Since $r < 1$ and $\alpha > 0$ by assumption, the second order conditions hold for any strictly positive group-level efforts. Thus, any interior solution we find to the system of first order conditions (1.2) constitutes a Nash equilibrium in pure strategies. I show bellow that such a solution exists and yields unique expected individual utility.

1.4 Equilibrium

I consider a symmetric setting before investigating non-monotonic effects of asymmetries in group size and effort cost.

1.4.1 Symmetric benchmark

Each of the four groups has $n = N/2$ members, as in Figure 1.1.[2] Since $r < 1$, the marginal impact of group effort is decreasing. Hence, individuals have an incentive to exclusively provide effort to their group with the lowest total effort. I show in Proof 1 in Appendix 1.A that this, together with symmetric group size, implies symmetric

[2]Symmetric size of subgroups—the intersections of two groups—is not a necessary assumption.

group-level effort in any equilibrium, $X^{g*} = X^*/4 \; \forall \; g \in \Gamma$. The first term of (1.2) must then be identical across groups and individuals. Thus, in equilibrium all first order conditions must bind. Since $\alpha > 0$, individual effort must be identical across individuals, $x_i^* = X^*/N \; \forall \; i \in \{1, ..., N\}$. We can rewrite (1.2) as

$$\frac{rV}{2X} - \left(\frac{X}{N}\right)^{\alpha} = 0. \tag{1.4}$$

The left hand side of this condition is defined for any positive aggregate effort X and monotonically decreasing. It tends to infinity as X tends to zero and to negative infinity as X tends to infinity. Hence, condition (1.4) uniquely pins down X^* in equilibrium and we can formulate:

Proposition 1 (**Solution to the symmetric benchmark**). *Suppose individuals are partitioned into two groups of equal size in two dimensions each. Unique aggregate effort in equilibrium is then*

$$X^* = \left(\frac{rN^{\alpha}V}{2}\right)^{1/(1+\alpha)},$$

which equals aggregate effort in a setting with only one partition dimension.

The symmetric corresponding group-level effort is $X^{g*} = X^*/4$ and symmetric individual effort is $x_i^* = X^*/N$. Because individual effort on behalf of the two groups enters the cost function as a sum, we cannot determine how much an individual contributes to each group. Both aggregate effort X^* and individual effort x_i^* are identical to the equilibrium that arises when two exclusive groups of size n compete over a prize V. The effect of additional effective channels to win the prize—and thus a higher marginal impact of effort—is offset by the same technology available to all other individuals and the negative externality of effort that non-exclusive membership introduces: Exerting effort for one of your groups increases this group's chance of winning. But at the same time, it also decreases the chance of winning for the other group you are a member of. This finding relies on the assumption that the efforts an individual exerts for her groups enter a single cost function as a sum. Whether it generalises to multiple types of effort with separate cost functions, for example time and money, depends on the modelling choices. If all groups benefit from a sum of all effort types, additional group dimensions will not alter aggregate effort. If new groups can be supported with new types of effort that enter separate cost terms, additional group dimensions will increase aggregate effort. The fact that equilibrium utilities and aggregate efforts are

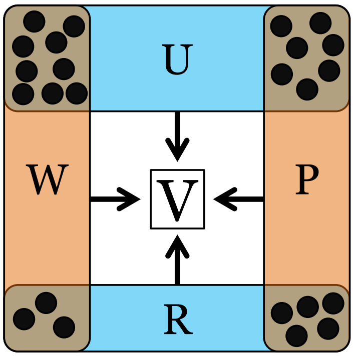

Figure 1.2: Group U has more members than group R

equivalent to the canonical contest model with only one partition dimension does not depend on the assumption that the prize is public. Moreover, this equivalence result does not depend on the assumption that there is no in-group fighting if the in-group fighting is symmetric within all groups.[3]

1.4.2 Group-size asymmetry

Suppose individuals move from the countryside into the cities, increasing the size of the urban group U and reducing that of the rural group R—see Figure 1.2 for an illustration. We can use our model to investigate the consequences of this migration for the equilibrium conflict outcome.

Groups in the second partition dimension are of potentially different sizes, namely $|U| \equiv n_U \geq |R| \equiv n_R$.[4] Group-size symmetry is still given in the first dimension, $n_W = n_P$. The total number of individuals remains N. Suppose that the size of the group does not lead to an efficiency loss in the public good provision, i.e. that prize valuation is independent of group size.[5] We can formulate:

Proposition 2 (**Effects of group-size asymmetry**). *Suppose group U is potentially larger than group R, $n_U \geq n_R$. As long as $n_U \leq 3n_R$, individual, group-level, and*

[3]Naturally, in the following sections in which I allow for asymmetries in group size and effort cost, these assumptions become more important.

[4]The findings are also valid if the asymmetry of group sizes goes into the other direction or is in the other dimension. The same is true for the later considered effort cost asymmetry and the formation of an additional group.

[5]We will see below that large enough groups also win the prize with higher probability. Thus, this assumption leads to an increase in expected aggregate utility. This is valid if the prize is truly public—for example, if the prize is the avoidance of a public bad, such as the storage of nuclear waste, and all residential areas have enough capacity for all residents. In other contexts, independence of prize valuation from group size can be more problematic—for example, public transport systems might have finite passenger capacity.

aggregate effort as well as expected individual utility do not change relative to the symmetric case characterised in Proposition 1. If $n_U \geq 3n_R$, members of the small group R only provide effort to R. If the size of U further increases at the expense of R, individual total effort decreases for members of the majority group U, and increases for members of the minority group R. Expected utility increases for the majority and decreases for the minority. Aggregate effort decreases.

I show in Proof 2 in Appendix 1.B that all groups must exert the same effort in equilibrium if $n_U \leq 3n_R$. All first order conditions must bind. Symmetric utility functions imply $x_i^* = X^*/N \ \forall \ i$, where X^* is the same as in Proposition 1. For $X^{g*} = X^*/4 \ \forall \ g \in \Gamma$ and $x_i^* = X^*/N \ \forall \ i$ to hold, any increase in $n_U < 3n_R$ must on average lead effort from members of the larger group U to increasingly crowd out effort provided by members of R to groups W and P. At $n_U = 3n_R$, individual equilibrium effort x_i^* is symmetric and all first order conditions still bind, but members of R only exert effort for group R. Unchanged effort levels imply unchanged equilibrium utilities.

If $n_U > 3n_R$, even if members of R only contribute effort to R, group-level and individual effort cannot be symmetric at the same time.[6] Thus, only the first order conditions with respect to x_i^R are necessarily binding in equilibrium. I show in Proof 3 in Appendix 1.B that groups W, P, and U must receive symmetric total effort in equilibrium, $X^W = X^P = X^U$. This implies symmetric first order conditions, $\partial u_i/\partial x_i^W = \partial u_i/\partial x_i^P = \partial u_i/\partial x_i^U \ \forall \ i \in U$ and $\partial u_i/\partial x_i^R \ \forall \ i \in R$, respectively. It follows that individual total effort x_i is symmetric within U and within R, respectively. Then $X^U = n_U x_i/3 \ \forall \ i \in U$ and $X^R = n_R x_i = (N - n_U)x_i \ \forall \ i \in R$. Divide the first order condition that does necessarily bind for members of R, $\partial u_i/\partial x_i^R, i \in R$ by $\partial u_i/\partial x_i^U, i \in U$ to get

$$\frac{2}{1 + (X^R/X^U)^r} \left(\frac{3(N - n_U)}{n_U} \right)^{\alpha} = \left(\frac{X^R}{X^U} \right)^{1+\alpha-r}. \tag{1.5}$$

It follows that $X^R < X^U$ and $x_i > x_j \ \forall \ i \in R, j \in U$ if $n_U > 3n_R$. The total effort of a sufficiently small group is lower than that of the larger groups. Its members exert more effort than members of the large group.

If we define $\rho \equiv X^R/X^U$ and treat ρ and n_U as continuous variables, we can derive the total differential of (1.5) to show that $d\rho/dn_U < 0$. It follows that $dp_i/dn_U > 0 \ \forall \ i \in U$ and $dp_i/dn_U < 0 \ \forall \ i \in R$. The win probability increases for members

[6] If there is group-size asymmetry in both dimensions, additionally, some first order conditions cannot bind in equilibrium if there exists a subgroup which is larger than half of the whole population.

n_U	X	X^U	X^R	x_i	x_j	u_i	u_j
10	6.06	1.51	1.51	0.30	0.30	4.89	4.89
15	6.06	1.51	1.51	0.30	0.30	4.89	4.89
16	6.01	1.55	1.37	0.29	0.34	5.00	4.77
17	5.93	1.58	1.20	0.28	0.40	5.12	4.61
18	5.80	1.60	0.99	0.27	0.50	5.28	4.40
19	5.58	1.62	0.71	0.26	0.71	5.50	4.01

Table 1.1: Numerical example of group-size asymmetry

For $i \in U$, $j \in R$; parametric specification: $r = 2/3$, $\alpha = 1/2$, $V = 10$, $N = 20$

of the large group U while it decreases for members of the small group R. Further, if we write x_i as an implicit function of ρ, we can show that $dx_i/dn_U < 0 \; \forall \; i \in U$ and $dx_i/dn_U > 0 \; \forall \; i \in R$. Individual effort increases for members of the small group and decreases for those of the large group as the size of U increases. It follows that $du_i/dn_U > 0 \; \forall \; i \in U$ and $du_i/dn_U < 0 \; \forall \; i \in R$. We can show that aggregate utility $\sum_{i \in \{1,\dots,N\}} u_i$ is increasing in n_U if $n_U > 3n_R$. Aggregate effort X decreases in n_U. See Proofs 4 and 5 in Appendix 1.B for the derivations of $d\rho/dn_U$, dx_i/dn_U, $d\sum u_i/dn_U$, and dX/dn_U.

As long as the urban and rural groups are somewhat similar in size, namely $n_U \leq 3n_R$, migration from the countryside into the cities has no utility implications. Rural individuals will on average shift more and more of their effort to group R, while urban individuals will shift on average some of their effort towards W and P. When enough people have moved into the cities and $n_U \geq 3n_R$, rural individuals will only provide effort to R; small enough minorities only support their minority group. Any further shift of individuals from R to U will make members of R worse off and members of U better off.[7] Table 1.1 illustrates a numerical example of group-size asymmetry with $r = 2/3$, $\alpha = 1/2$, $V = 10$, and $N = 20$. Up to $n_U = 3n_R = 3N/4 = 15$, all relevant values remain the same. If the size of U grows further, we see the discussed effects.

These findings add to the debate started by Olson (1965) and advanced markedly by Esteban and Ray (2001) on the group-size paradox: If groups are overlapping and the prize is public, a change in group sizes may only shift individual effort from group to group while holding win probabilities and individual utilities constant. Only

[7] If group membership were to some degree endogenous, individuals within R would have an incentive to retain members while individuals within U would try to lure mobile members of R into the cities. However, with fully endogenous group membership, in the context of public goods the trivial equilibrium entails only one group per dimension and no conflict. For work on endogenous group membership in the context of private-good prizes, see e.g. Baik and Shogren (1995a), Baik and Lee (1997, 2001), Konrad and Kovenock (2009a), and Herbst et al. (2015).

if the group-size asymmetry grows large enough do we see the common finding of a large-group advantage to win a public prize reconfirmed.

1.4.3 Effort-cost asymmetry

Assume group- and subgroup-size symmetry for simplicity. Suppose the cost function for the wealthy takes on the form $(1-c)(x_i)^{1+\alpha}/(1+\alpha)\ \forall\, i \in W$, whereas for members of P effort cost is $(1+c)(x_i)^{1+\alpha}/(1+\alpha)$, where $c \in [0,1)$. The unweighted average effort cost is unchanged by a change in c. At the same effort level, the effort cost for wealthy individuals is lower than for poor individuals. This could simplistically model the wealthy having a higher income to spend on effort. Define $\gamma \equiv (1+c)/(1-c)$. We can formulate:

Proposition 3 **(Non-monotonic effects of effort-cost asymmetry).** *Individual total effort of members of W and aggregate effort is increasing in c. Individual total effort of members of P is decreasing in c. If $\gamma \leq 3^\alpha$, win probabilities remain unchanged and if $\gamma < 3^\alpha$ expected utility increases for members of P and decreases for members of W in c. If $\gamma \geq 3^\alpha$, members of P only provide effort to P. Expected utility decreases for members of P when c further increases and increases for members of W.*

For an individual of the wealthy group W, who is also a member of group $l \in \{U, R\}$, the first order condition with respect to $x_i^g, g \in \{W, l\}$ is

$$\frac{r(X^g)^{r-1}((X^P)^r + (X^{-l})^r)}{(\sum_{i\in\Gamma}(X^j)^r)^2} V - (1-c)(x_i)^\alpha \overset{!}{\leq} 0, \tag{1.6}$$

where $-l = \{U, R\} \setminus l$. The second order conditions hold for any strictly positive group-level efforts. I show in Proof 6 in Appendix 1.C that in any equilibrium groups W, U, and R must exhibit the same total effort. Thus, both first order conditions must bind for all members of group W.

For a member of the poorer group P and group $l \in \{U, R\}$, the first order conditions are

$$\frac{r(X^g)^{r-1}((X^W)^r + (X^{-l})^r)}{(\sum_{i\in\Gamma}(X^j)^r)^2} V - (1+c)(x_i)^\alpha \overset{!}{\leq} 0, \tag{1.7}$$

where $g \in \{P, l\}$. The second order conditions hold for any strictly positive group-level efforts. The first order condition with respect to x_i^l does not necessarily bind in equilibrium. Members of W might already exert so much effort for l that $X^W = X^U = X^R > X^P$ and members of P do not have an incentive to provide effort for l at all.

Suppose group-level effort is symmetric, $X^g = X/4$. Then, all first order conditions are symmetric for members of W and P, respectively, and must bind in equilibrium. Divide (1.6) by (1.7) to get

$$\frac{x_i}{x_j} = \left(\frac{1+c}{1-c}\right)^{1/\alpha} = \gamma^{1/\alpha}. \tag{1.8}$$

for $i \in W, j \in P$. Under group-level-effort symmetry, individual effort by members of W can be up to three times as large as individual effort by members of P and still fulfil the group-effort condition $X^g = \sum_{i \in g} x_i^g$. Hence, the equilibrium with symmetric group-level effort exists as long as $\gamma \leq 3^\alpha$. If $\gamma > 3^\alpha$, there is no feasible solution to binding equations (1.6) to (1.8) and in any equilibrium we must have that $X^W = X^U = X^R > X^P$. Then, individual effort of members of group W must be more than three times as large as that of members of group P, who only support P.

If $\gamma < 3^\alpha$, equation (1.8) pins down the equilibrium and group-level effort is symmetric. Plugging $X = nx_i(1 + 1/\gamma^{1/\alpha}), i \in W$ into the first order conditions, we can show that $dx_i/dc > 0 \; \forall \; i \in W$ and $dx_i/dc < 0 \; \forall \; i \in P$. As c increases, individual effort of members of W increases while individual effort of members of P decreases. Further, aggregate effort X increases in c. Win probabilities remain the same as long as group-level effort remains symmetric. Expected utility of members of P increases in c. For members of W, we can show that expected utility is decreasing in c. The fact that an increase in c leads members of the weak group P to free-ride more intensely in groups U and R harms members of W despite their lowered effort cost. Moreover, we can show that $du_i/dc = -du_j/dc$ if $\gamma < 3^\alpha$. Hence, aggregate utility is unaffected by cost asymmetry if it is sufficiently small. At $\gamma = 3^\alpha$, equation (1.8) still pins down the equilibrium with symmetric group-level effort, but members of P only exert effort for group P.

If $\gamma > 3^\alpha$, individuals within W must in equilibrium exert more than three times as much effort as members of group P and $X^W = X^U = X^R > X^P = nx_i, i \in P$. Then, (1.7) binds only for $g = P$ but not for $g = l$. Similar to the previous subsection about group-size asymmetry, 'minorities' with relatively higher effort cost only support their minority group with effort. Using $X^W = nx_i/3, i \in W$ we can divide the first order condition that does bind for members of P, (1.7) with $g = P$, by (1.6) to get

$$\frac{2}{1 + (X^P/X^W)^r} \frac{3^\alpha(1-c)}{1+c} = \left(\frac{X^P}{X^W}\right)^{1+\alpha-r}. \tag{1.9}$$

c	γ	X	X^W	X^P	x_i	x_j	u_i	u_j
0.0	1.00	6.06	1.51	1.51	0.30	0.30	4.89	4.89
0.1	1.22	6.12	1.53	1.53	0.37	0.25	4.87	4.91
0.2	1.50	6.31	1.58	1.58	0.44	0.19	4.85	4.93
0.3	1.86	6.61	1.68	1.58	0.50	0.16	4.88	4.90
0.6	4.00	8.40	2.42	1.15	0.73	0.11	5.38	4.42
0.9	19.00	18.29	5.90	0.60	1.77	0.06	6.06	3.77

Table 1.2: Numerical example of effort-cost asymmetry

For $i \in W$, $j \in P$; parametric specification: $r = 2/3$, $\alpha = 1/2$, $V = 10$, $N = 20$

If we define $\rho \equiv X^P/X^W$, we can show that $d\rho/dc < 0$. It follows that $dp_i/dc > 0 \ \forall \ i \in W$ and $dp_i/dc < 0 \ \forall \ i \in P$. As c increases, the win probability increases for members of W and decreases for members of P. Formulating individual effort x_i as implicit function of ρ, we can show that $dx_i/dc > 0 \ \forall \ i \in W$ and $dx_i/dc < 0 \ \forall \ i \in P$. Aggregate effort again increases in c.

Formulating expected individual utility u_i as an implicit function and taking the total differential yields $du_i/dc > 0 \ \forall \ i \in W$ and $du_i/dc < 0 \ \forall \ i \in P$. If the effort cost asymmetry is high enough to fulfil $\gamma \geq 3^\alpha$, any further increase in c increases expected utility of members of W and decreases expected utility of members of P.[8] This can intuitively be explained by the fact that members of P cannot further increase the extent to which they free-ride on the effort exerted by members of W. Similar to the case of group-size asymmetry, the cost asymmetry has to be large enough to confirm previous findings in the literature. Lastly, aggregate utility increases in c if $\gamma > 3^\alpha$. I refer the interested reader to Proofs 7 to 10 in Appendix 1.C for derivations of $d\rho/dc$, dx_i/dc, du_i/dc, $d\sum u_i/dc$, and dX/dc. Table 1.2 illustrates a numerical example of effort-cost asymmetry, with $r = 2/3$, $\alpha = 1/2$, $V = 10$, and $N = 20$. Up to $\gamma = 3^{(1/2)} \approx 1.73$, group-level effort remains symmetric and utility increases for the poor and decreases for the wealthy. Beyond that threshold, the effect of an increase in c is flipped.

[8] The effort cost asymmetry investigated here has a similar effect on members of group P as a higher prize valuation by members of W, irrespective of which group provides the prize, would have. If members of W only valued the prize more highly if it were provided by group W—for example, due to efficiency/productivity reasons—an increasingly asymmetric prize valuation would have a monotonic negative effect on members of P.

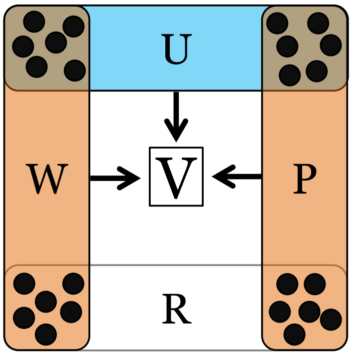

Figure 1.3: One additional group in the second dimension

1.5 Extensions

In the model presented above, individuals are partitioned into two groups in two dimensions. I now first turn to a setting in which only one group in the second dimension, the urbanites, can influence the prize allocation. Second, I generalise the model to an arbitrary number of groups of the same size in an arbitrary number of dimensions. Third, I discuss issues with linear and convex group-level impact functions.

1.5.1 Additional group

Consider two exclusive groups W and P of equal size n that are in conflict over a public prize of value V. Expected utility of an individual $i \in c$, where $c \in \Gamma = \{W, P\}$ is

$$u_i = \frac{(X^c)^r}{\sum_{j \in \Gamma}(X^j)^r} V - \frac{(x_i^c)^{1+\alpha}}{1+\alpha}. \tag{1.10}$$

The symmetric Nash equilibrium in pure strategies is then uniquely given by the individual total effort $x_i^* = (rV/(2N))^{1/(1+\alpha)}\ \forall\ i$. Both groups have the same win probability.

Now, assume half of the members of both W and P form an additional group U which also participates in the conflict. The other half of both groups, whom I call again R, does not have the opportunity to form their own group. This setup is exemplified in Figure 1.3. We can formulate:

Proposition 4 (Additional group). *The formation of an additional group U in the second dimension that participates in the same conflict as groups W and P benefits members of U and harms members of the inactive group R. Individual effort increases for members of R and decreases for members of U. The winning probability increases* *for members of U and decreases for members of R. Aggregate effort increases if $\alpha < 1$, remains the same if $\alpha = 1$, and decreases if $\alpha > 1$.*

After the formation of the additional group U, the utility function for members of R remains the same as in (1.10) where now $\Gamma = \{W, P, U\}$. Expected utility for members of group U is

$$u_i = \frac{(X^c)^r + (X^U)^r}{\sum_{j\in\Gamma}(X^j)^r} V - \frac{(x_i^c + x_i^U)^{1+\alpha}}{1+\alpha},$$

where $c \in \{W, P\}$. The first order conditions are derived as before. The second order conditions again hold for strictly positive group-level efforts. Due to $r < 1$ and group-size symmetry, in equilibrium we must have $X^W = X^P$. Moreover, it must hold that $X^U \leq X^c$, $c \in \{W, P\}$. Otherwise, members of U would have an incentive to shift some of their effort from U to their other group $c \in \{W, P\}$, which cannot be an equilibrium. Additionally, from the first order conditions and group-size symmetry it is clear that $x_i < x_j \ \forall\ i \in U, j \in R$. This is intuitive since members of U gain from two groups and thus have less incentive to exert effort.

In Proof 11 in Appendix 1.D, I show that if $\alpha < 1$, it follows that $X^U < X^W = X^P$. If $\alpha = 1$ then $X^U = X^W = X^P$. Hence, if $\alpha \leq 1$, members of U only contribute effort to group U. If $\alpha > 1$ members of U also contribute to W and P. Moreover, I show in Proof 12 in Appendix 1.D that the introduction of an active group U always increases individual effort x_i for $i \in R$ and decreases individual effort x_i for $i \in U$. Thus, the introduction of an additional group U leads to a higher expected individual utility for all members of U and to a reduction of expected individual utility for all members of R. Aggregate effort X increases if $\alpha < 1$, remains the same if $\alpha = 1$ and decreases if $\alpha > 1$; if the effort cost is sufficiently convex, the reduction in effort by members of U is not offset by an increase in effort by members of the inactive group R.

The formation of U gives its members another channel to exert effort to win the prize. Since $r < 1$, this additional channel makes effort exerted by individuals within U more effective. Consequently, members of U free-ride—the extent depends on α—on the effort of members of R within their classes W and P and by exerting effort for U increase their win probability. This harms members of R who do not have an additional channel for their effort.

1.5.2 A general model with symmetry

What happens if there are more than two partition dimensions and more than two groups per dimension? For example, a society could be split into an urban, suburban, and rural population, an upper, middle, and lower class, and three different ethnicities who have additional conflicting interests. Let us extend the symmetric model to more dimensions and more groups per dimension.

Consider $N = mn$ identical individuals who are partitioned into $m \geq 2$ groups of n members in $d \geq 2$ dimensions. Denote the set of md groups by Γ. Each individual i is a member of exactly d groups, one in each dimension. Denote this set of groups by $G_i \subset \Gamma$. A subgroup h is the intersection of d groups, one group per dimension. Denote the set of non-empty subgroups by Ψ.

All groups in Γ compete over the same prize to finance a group-specific public good, valued at V across all groups and individuals. The technology of contest which determines the win probability $p_g(X^{g_1^1}, ..., X^{g_m^k})$ of group $g \in \Gamma$ is the general Tullock function,

$$p_g(X^{g_1^1}, ..., X^{g_m^k}) = \frac{(X^g)^r}{\sum_{j\in\Gamma}(X^j)^r},$$

where group effort X^g is the sum of individual efforts of its members on behalf of it, $X^g = \sum_{i\in g} x_i^g$.

Expected utility of an individual i, who is a member of d different groups $G_i \subset \Gamma$, is

$$u_i = \frac{\sum_{j\in G_i}(X^j)^r}{\sum_{j\in\Gamma}(X^j)^r}V - \frac{(\sum_{j\in G_i} x_i^j)^{1+\alpha}}{1+\alpha},$$

where $\alpha > 0$. The individual maximisation problem of choosing individual effort for group $g \in G_i$ yields the first order condition

$$\frac{\partial u_i}{\partial x_i^g} = \frac{r(X^g)^{r-1}\sum_{j\in\Gamma\setminus G_i}(X^j)^r}{(\sum_{j\in\Gamma}(X^j)^r)^2}V - (x_i)^\alpha \overset{!}{\leq} 0. \tag{1.11}$$

Equation (1.11) is identical for all members of any subgroup $h \in \Psi$. The second order conditions are still given by (1.3) and hold for any strictly positive group-level efforts.

Since $r < 1$, there are d idiosyncratic first order conditions for all subgroups $h \in \Psi$. I prove in Appendix 1.A that all of these conditions must bind in any equilibrium since all groups have to exert the same effort in any equilibrium, $X^g = X/(md)\ \forall\ g \in \Gamma$. The following proposition characterises the Nash equilibrium in pure strategies:

Proposition 5 (**Nash equilibrium in the general symmetric setting**). *Suppose individuals are partitioned into m groups in d dimensions each. Unique aggregate effort in equilibrium is*

$$X^* = \left(\frac{r(m-1)N^\alpha V}{m}\right)^{1/(1+\alpha)}, \tag{1.12}$$

and the corresponding symmetric group-level effort $X^{g*} = X^*/md$. Conflict effort is independent of the number of dimensions d along which individuals are partitioned into groups.

The corresponding symmetric individual equilibrium effort is $x_i^* = X^*/N$. Note that there would naturally be multiple equilibria on the individual level if effort cost were linear, $\alpha = 0$. The result from Subsection 1.4.1 that the higher effectiveness of additional effort channels is offset by the same 'technology' being available to all other conflict parties holds also in the general symmetric case. Individual group-level effort x_i^g is indeterminate since there is no discrimination between groups in the individual cost function. Aggregate effort in equilibrium is independent of the number of partition dimensions d, and increasing in the Tullock exponent r, the value of the prize V, the number of groups per partition dimension m, and the number of individuals N. The effect of cost parameter α on aggregate equilibrium effort X^* is ambiguous: it is positive if $r(m-1)V < mN$ or alternatively $x_i < 1$, zero if $x_i = 1$ and negative if $x_i > 1$. If individual effort is smaller than one, an increase in the convexity of the cost function decreases the marginal cost and thus gives incentive to raise effort.

1.5.3 Linear and convex group-level impact functions

The assumption $r < 0$ is reasonable in many contexts, but it might not always hold. For example, if overlapping interest groups develop different project proposals whose quality can be quite easily assessed, $r = 1$ or even $r > 1$ might be a better reflection of reality. What happens if impact functions are not concave?

If $r = 1$, there is only one idiosyncratic first order condition per subgroup. Then $\partial u_i/(\partial x_i^g) > 0$ whenever $\sum_{j\in\Gamma\setminus G_i} X^j > 0$ and $x_i = 0$ and thus all first order conditions must bind in equilibrium:

$$\frac{\sum_{j\in\Gamma\setminus G_i} X^j}{(\sum_{j\in\Gamma} X^j)^2} V - (x_i)^\alpha \stackrel{!}{=} 0, \tag{1.13}$$

which is identical for each individual within any subgroup h, respectively. The second order condition holds for any positive individual effort.

The symmetric solution dictated by a concave group-level impact function with aggregate effort (1.12) also characterises a Nash equilibrium in pure strategies in the linear case. However in general, the system of first order conditions does not have a unique solution since there are more unknowns than equations.

Due to group-size symmetry, we can sum (1.13) over all individuals to get

$$\sum_i (x_i)^\alpha \sum_i (x_i) = N\frac{m-1}{m}V.$$

The right-hand side of this equation is a constant. The left hand side is homogeneous of degree $1+\alpha$ in symmetric individual effort. Thus, any equilibrium whose aggregate effort differs from the one characterised by (1.12) cannot be symmetric on the individual level.

If $\alpha \neq 1$, aggregate effort X in equilibrium does not necessarily equal aggregate effort in the symmetric equilibrium $X^{*S} = ((m-1)N^\alpha V/m)^{1/(1+\alpha)}$. If $\alpha \in (0,1)$, then $X^* \geq X^{*S}$ in any equilibrium and, if $\alpha > 1$, then $X^* \leq X^{*S}$; aggregate effort may be larger or smaller than in the one-dimensional group conflict case, dependent on the convexity of the cost function.

The case $\alpha = 1$ allows us to infer that any Nash equilibrium in pure strategies has the same unique aggregate-effort level. If $\alpha = 1$, due to symmetry we can sum up (1.13) over all individuals to see that

$$X^* = \left(\frac{(m-1)NV}{m}\right)^{1/2},$$

which is a special case of the formula derived under concavity (1.12). Due to convex effort cost, individual effort is equal within subgroups. Equilibrium effort on the individual or group level is not uniquely defined, however.

If $r > 1$, Nash equilibria in pure strategies might not be unique on the aggregate-effort level: If the group-level impact function is convex and thus the marginal return to group-level effort increasing in group-level effort, individuals would like to provide all their effort to the group with the highest total effort. This mechanism introduces a complex coordination problem and the potential for multiple equilibria with varying utilities. In any equilibrium in pure strategies that does exist, there is at least one group $g \in \Gamma$ with effort $X^g \geq X^j \; \forall \, j \in \Gamma$ and $X^g - x_i \geq X^{g'} \; \forall \, g \cap g' \neq \emptyset, i \in g \cap g'$, and this group will receive the total effort from all its members.[9] Symmetric group-level effort can never be an equilibrium. Individuals within a subgroup must only provide effort to one of their mutual groups. Moreover, solutions that can be derived from first order conditions might not constitue Nash equilibria due to known problems with potentially negative expected utility that arise with a Tullock exponent r larger than 1.[10]

Regardless of the specification of $r > 0$, if we consider any existing Nash equilibrium, the later addition of a new group-partition dimension does not alter expected individual utility. If $r \geq 1$, no individual has an incentive to shift their effort to a newly existing group or alter the effort provided to their old groups. If $r < 1$, individuals shift some of their effort to their new groups to equalise group-level efforts. But as is clear from Proposition 5, this will not change individual total effort levels or individual win probabilities.

In the future, it should be of interest to analyse how equilibrium outcomes are affected if conflict between non-exclusive groups is combined with weakest-link and best-shot contest success functions. For the former, see for example Lee (2012), for the latter Chowdhury et al. (2013), and for a combination Chowdhury and Topolyan (2016).

1.6 Conclusion

I introduce a conflict model in which individuals are members of and can contribute effort to two groups simultaneously. Group impact functions are concave and effort cost is convex. Group-size asymmetry in one partition dimension crowds out effort by individuals of the smaller group provided to their groups in the other dimension. Up to a threshold, group-size asymmetry does not affect expected individual utility. Beyond that threshold, it hurts members of the minority and benefits members of the majority. Asymmetric effort cost has an inverted U-shaped effect on the utility of individuals with the higher relative effort cost and a U-shaped effect on the utility of individuals with the lower relative cost. In general, individuals who form a minority in one dimension focus their conflict effort on this minority group. The formation of one additional group in a standard two-group setting benefits individuals who have access

[9] If $X^g - x_i < X^{g'}$ for any $i \in g \cap g'$, i would have an incentive to change her behaviour and provide her effort to the other group due to increasing marginal returns.

[10] See e.g. Perez-Castrillo and Verdier (1992) Baye et al. (1994), Cornes and Hartley (2005), and Lee (2015).

to this new group as an additional channel to win the prize. It harms those who have only one group membership.

With multiple groups of symmetric size in an arbitrary number of partition dimensions, only a concave group-level impact function ensures the existence of a unique Nash equilibrium in pure strategies. Then, conflict effort is independent of the number of partition dimensions. The later addition of additional group dimensions does not alter expected utilities of existing equilibria. This equivalence result supports the validity of the canonical model of group contest with only one partition dimension.

The model makes a couple of predictions that can be policy relevant. First, the introduction of additional contentious points that split society into additional interest groups does not increase the intensity of political conflict. An increase in the number of lobbying groups only aggravates conflict if it is due to smaller groups, not if it is due to groups in more dimensions. Second, members of a minority group, be it due to group-size or strength, focus their efforts on this minority group. This does not reflect a higher valuation of this particular group's success but simply the lack of support by other individuals. Third, group-size and effort-cost asymmetries, if large enough, facilitate social efficiency by increasing aggregate utility.

There are a number of avenues for future research. One is endogenous group membership. In practice, players can often to some extent choose to which groups they want to belong to: individuals may join or leave activist groups; companies can team up with other companies in lobby groups or fight alone. A second one is to investigate whether the effort and utility equivalence under non-exclusivity of group membership also holds with weakest-link or best-shot contest technologies. A third one is the modelling of influence on a group's policy goals, which is often inherent in real-world examples. Lastly, to gain insights into recent issues of political polarisation, it might be fruitful to analyse in-group altruism in combination with varying overlap of non-exclusive groups. In such a framework, one would expect increased 'redundancy' of group membership (e.g. all urbanites are wealthy and vice versa) to increase conflict intensity.

Appendices

1.A Appendix – Symmetric benchmark

Proof 1 (Group-level effort symmetry). Individuals are partitioned into $m \geq 2$ groups of equal size in $d \geq 1$ dimensions each. Suppose that not all groups exert the same level of effort in equilibrium. Then, $\exists\ L, H \in \Gamma : X^H > X^L, X^H \geq X^g \geq X^L\ \forall\ g \in \Gamma$.

Define a *multidimensional group cluster* $\Gamma_L \subset \Gamma$ where $L \in \Gamma_L$, $X^j = X^L\ \forall\ j \in \Gamma_L$, and if $g_1 \in \Gamma_L$, there is a 'chain' of intersecting groups $g_1, ..., g_k, k \geq 2, g_k = L$ such that $g_j \cap g_{j+1} \neq \emptyset\ \forall\ j \in \{1, ..., k-1\}$. Thus, all groups within the cluster Γ_L have the same effort in equilibrium, $X^j = X^L$. Moreover all groups within this cluster share at least one member with at least one other group in this cluster, and all groups can be 'connected' via intersecting groups with group L. Due to $r < 1$, $\partial u_i/\partial x_i^g$ is strictly decreasing in X^g. Hence, in any equilibrium individuals will only exert effort for their groups with the lowest total effort. Then, any individual who is a member of any of the groups in cluster Γ_L only supports groups within this cluster since she cannot be a member of any other group with lower or equal total effort. Otherwise, this group would be a part of the cluster as well. The effort for any individual i_L who is a member of any of the groups in cluster Γ_L is given by the first order condition

$$\frac{r \sum_{j \in \Gamma \setminus G_{i_L}} (X^j)^r}{(X^L)^{1-r} (\sum_{j \in \Gamma} (X^j)^r)^2} V = (x_{i_L})^\alpha\,. \tag{1.14}$$

Denote by $\overline{x_L}$ the average effort of members of Γ_L. Members of the cluster Γ_L can at most support d groups within Γ_L. Then,

$$X^L \geq n\overline{x_L}/d. \tag{1.15}$$

Now, define a group cluster $\Gamma_H \subset \Gamma$ where $H \in \Gamma_H$, $X^j = X^H\ \forall\ j \in \Gamma_H$, and if $g_1 \in \Gamma_H$, there is a 'chain' of intersecting groups $g_1, ..., g_k, k \geq 2, g_k = H$ such that $g_j \cap g_{j+1} \neq \emptyset\ \forall\ j \in \{1, ..., k-1\}$. Due to concavity in the group-level impact function, individuals who are a member of Γ_H only support groups within this cluster if all the groups they are a member of are within this cluster. The individual effort of any of these individuals i_H is given by

$$\frac{r\sum_{j\in\Gamma\setminus G_{i_H}}(X^j)^r}{(X^H)^{1-r}(\sum_{j\in\Gamma}(X^j)^r)^2}V = (x_{i_H})^\alpha\,. \tag{1.16}$$

Denote by $\overline{x_H}$ the average effort of members of Γ_H that support the groups within this cluster. Groups within Γ_H have at most n active supporters who support d groups within Γ_H. Then,

$$X^H \leq n\overline{x_H}/d. \tag{1.17}$$

We know that

$$\sum_{j\in\Gamma\setminus G_{i_L}}(X^j)^r = \sum_{j\in\Gamma}(X^j)^r - \sum_{j\in G_{i_L}}(X^j)^r$$

and

$$\sum_{j\in\Gamma\setminus G_{i_H}}(X^j)^r = \sum_{j\in\Gamma}(X^j)^r - d(X^H)^r.$$

Since any individual i_L is the member of at least one group exerting effort X^L and all her other groups cannot exert a higher effort than X^H by assumption,

$$\sum_{j\in\Gamma\setminus G_{i_L}}(X^j)^r > \sum_{j\in\Gamma\setminus G_{i_H}}(X^j)^r$$

for all i_L, i_H. Looking at (1.14) and (1.16), this means that

$$\overline{x_L} > \overline{x_H}. \tag{1.18}$$

However, combining (1.15), (1.17), and (1.18) yields

$$X^L \geq n\overline{x_L}/d > n\overline{x_H}/d \geq X^H.$$

This is a contradiction of the starting assumption. Hence, group-level effort must be symmetric in any equilibrium, $X^g = X/(md)\ \forall\ g \in \Gamma$. □

1.B Appendix – Group-size asymmetry

Proof 2 (Group-level effort symmetry if $\mathbf{n_U \leq 3n_R}$). Suppose that $\exists\ L, H \in \Gamma$: $X^H > X^L, X^H \geq X^g \geq X^L\ \forall\ g \in \Gamma$. If an individual exerts positive effort for group H, all her groups must have the same effort X^H. There are thus three cases: There are two groups in different dimensions with effort X^H, and either two groups with effort X^L or one group with effort X^L and one group with effort $X^M, X^H > X^M > X^L$. Or there are three groups with effort X^H and one group with effort X^L. Denote the average effort of individuals who exert positive effort for H as $\overline{x_H}$ and the average effort of individuals who exert effort for L as $\overline{x_L}$. Since $n_U \leq 3N/4$, $n_R \geq N/4$, and $n_W = n_P = N/2$ we can write in all three cases

$$X^H \leq \frac{N\overline{x_H}}{4}$$

and

$$X^L \geq \frac{N\overline{x_L}}{4}.$$

However, we can derive from the first order conditions that $\overline{x_L} > \overline{x_H}$, which means that $X^H < X^L$. This is a contradiction. Hence, $X^j = X/4 \; \forall \; j \in \Gamma$ in equilibrium if $n_U \leq 3n_R$. □

Proof 3 (**Group-level effort symmetry $\mathbf{X^W = X^P = X^U}$ if $\mathbf{n_U > 3n_R}$).** Suppose $X^U > X^W$. Then, the subgroup $W \cap U$ only exerts effort for group W. The subgroup $P \cap U$ must exert effort for U for $X^U > 0$ to hold, from which follows $X^U \leq X^P$ and $X^W < X^P$. Further, it must hold that $X^R \leq X^P$ since otherwise R would have no supporters. But by the group-size assumptions and the first order conditions we can write $X^W > Nx_i/4 > Nx_j/4 \geq X^P, i \in W \cap U, j \in P \cap U$, which is a contradiction.

Now, suppose $X^U < X^W$. Then, the subgroup $W \cap U$ only supports U. This means that $W \cap R$ exerts effort for W and thus $X^W \leq X^R$. But then $X^U < X^R$, which implies by the first order conditions that $x_i > x_j \; \forall \; i \in W \cap U, j \in W \cap R$. Subgroup-size asymmetry implies $X^U > x_iN/4 > x_jN/4 > X^W, i \in W \cap U, j \in W \cap R$, which is a contradiction.

Thus, $X^U = X^W$ in equilibrium. The same reasoning yields $X^U = X^P$. In any equilibrium $X^W = X^P = X^U$ and all first order conditions bind for all individuals in the larger group U. □

Proof 4 (**$dx_i/dn_U < 0 \; \forall \; i \in U$ and $dx_i/dn_U > 0 \; \forall \; i \in R$ if $n_U > 3n_R$).** The total differential of (1.5) is

$$\frac{d\rho}{dn_U} = \frac{-(2/(1+\rho^r))\alpha(3(N-n_U)/n_U)^{\alpha-1}3N/n_U^2}{2r\rho^{r-1}/(1+\rho^r)^2(3(N-n_U)/n_U)^\alpha + (1+\alpha-r)\rho^{\alpha-r}} < 0. \tag{1.19}$$

If $n_R = N - n_U \leq n_U/3$, we can plug $X^R = n_R x_i, i \in R$, $X^W = X^P = X^U =$

$x_i n_U/3, i \in U$, and $\rho = X^R/X^U$ into the first order conditions to write individual effort x_i as implicit functions of ρ and n_U:

$$x_i = \left(\frac{r(1+\rho^r)V}{(n_U/3)(3+\rho^r)^2}\right)^{1/(1+\alpha)}, i \in U \tag{1.20}$$

and

$$x_i = \left(\frac{r\rho^r 2V}{(N-n_U)(3+\rho^r)^2}\right)^{1/(1+\alpha)}, i \in R. \tag{1.21}$$

We can derive the total differential of (1.20) to get

$$\frac{dx_i}{dn_U} = \frac{1}{1+\alpha}\left(\frac{n_U/3(3+\rho^r)^2}{r(1+\rho^r)V}\right)^{\alpha/(1+\alpha)} \frac{3rV}{n_U(3+\rho^r)^2}\left(\frac{r\rho^{r-1}(1-\rho^r)}{3+\rho^r}\frac{d\rho}{dn_U} - \frac{1+\rho^r}{n_U}\right) < 0$$

for $i \in U$. Individual effort in the larger group U is decreasing in the group size of U if $n_U > 3n_R$.

The total differential of (1.21) is

$$\frac{dx_i}{dn_U} = \frac{1}{1+\alpha}\left(\frac{(N-n_U)(3+\rho^r)^2}{r\rho^r 2V}\right)^{\alpha/(1+\alpha)} \frac{2r\rho^r V}{((N-n_U)(3+\rho^r))^2}\left(\frac{r(3-\rho^r)(N-n_U)}{\rho(3+\rho^r)}\frac{d\rho}{dn_U} + 1\right)$$

for $i \in R$. The first three terms are positive. Hence, the sign of $dx_i/dn_U, i \in R$ is the sign of the term in the last bracket. Plugging the formula for $d\rho/dn_U$, (1.19), into this bracket and rearranging yields

$$1 - \frac{3-\rho^r}{3+\rho^r}\frac{2/(1+\rho^r)\alpha N/n_U}{2\rho^r/(1+\rho^r)^2 + ((1+\alpha)/r - 1)\rho^{1+\alpha-r}(n_U/(3(N-n_U)))^\alpha}.$$

Making use of equation (1.5), we can simplify this to

$$1 - \frac{\alpha N/n_U}{(3+\rho^r)/(3-\rho^r)((1+\alpha)/r - 1/(1+\rho^r))}. \tag{1.22}$$

Since $r < 1$ and $\rho^r \geq 0$, we know that $((1+\alpha)/r - 1/(1+\rho^r)) > \alpha$. Moreover, since $\rho \leq 1$ and $x_i \geq x_j, i \in R, j \in U$ we know that $\rho^r \geq \rho = 3(N-n_U)x_i/(n_U x_j) \geq 3(N-n_U)/n_U, i \in R, j \in U$ and thus

$$\frac{3+\rho^r}{3-\rho^r} \geq \frac{3+3(N-n_U)/n_U}{3-3(N-n_U)}.$$

It follows that (1.22) is strictly larger than

$$N/n_U - 1 \geq 0.$$

Hence, $dx_i/dn_U > 0, i \in R$. Individual effort of individuals in the smaller group R is increasing in the group size of U if $n_U > 3n_R$. □

Proof 5 **($d\sum u_i/dn_U$ and dX/dn_U if $n_U > 3n_R$).** Using (1.20) and (1.21), we can write aggregate utility as

$$\textstyle\sum_{i\in\{1,\dots,N\}} u_i = n_U u_k + (N-n_U)u_j = \left(n_U \frac{2}{3+\rho^r} + (N-n_U)\frac{1+\rho^r}{3+\rho^r}\right)V - \frac{rV}{1+\alpha}\frac{3(1+\rho^r)+2\rho^r}{(3+\rho^r)^2}$$

for $k \in U$ and $j \in R$. Then,

$$\frac{d\sum u_i}{dn_U} = \left(\frac{1-\rho^r}{3+\rho^r} - (2n_U - N)\frac{2r\rho^{r-1}}{(3+\rho^r)^2}\frac{d\rho}{dn_U}\right) - \frac{rV}{1+\alpha} r\rho^{r-1}\frac{d\rho}{dn_U}\frac{9-5\rho^r}{(3+\rho^r)^3} > 0$$

since $d\rho/dn_U < 0$ and $\rho^r < 1$ if $n_U > 3n_R$.

Using (1.20) and (1.21), we can write aggregate effort X as

$$X = n_U\left(\frac{r(1+\rho^r)V}{(n_U/3)(3+\rho^r)^2}\right)^{1/(1+\alpha)} + (N-n_U)\left(\frac{r\rho^r 2V}{(N-n_U)(3+\rho^r)^2}\right)^{1/(1+\alpha)}.$$

Taking the total derivative of this term gives

$$\begin{aligned}\frac{dX}{dn_U} = &\left(\frac{V}{(3+\rho^r)^2}\right)^{1/(1+\alpha)}\frac{\alpha}{1+\alpha}\left(\left(\frac{3(1+\rho^r)}{n_U}\right)^{1/(1+\alpha)} - \left(\frac{2\rho^r}{N-n_U}\right)^{1/(1+\alpha)}\right) + \\ &\frac{d\rho}{dn_U} r\rho^{r-1}(rV)^{1/(1+\alpha)}\frac{1}{1+\alpha}\left((3(n_U)^\alpha)^{1/(1+\alpha)}\left(\frac{1+\rho^r}{(3+\rho^r)^2}\right)^{-\alpha/(1+\alpha)}\frac{1-\rho^r}{(3+\rho^r)^3} + \right.\\ &\left.(2(N-n_U)^\alpha)^{1/(1+\alpha)}\left(\frac{\rho^r}{(3+\rho^r)^2}\right)^{-\alpha/(1+\alpha)}\frac{3-\rho^r}{(3+\rho^r)^3}\right) < 0.\end{aligned}$$

To see this, recall that $3(1+\rho^r)/n_U \leq 2\rho^r/(N-n_U)$ and $d\rho/dn_U < 0$. □

1.C Appendix – Effort-cost asymmetry

Proof 6 **(Binding first order conditions with effort cost asymmetry).** Suppose $X^W > X^U$. The subgroup $W \cap U$ then only exerts effort for group U. The subgroup $W \cap R$ must exert effort for W for $X^W > 0$ to hold, from which follows that $X^W \leq X^R$.

But then $X^U < X^R$, which implies by the first order conditions that $x_i > x_j \; \forall \; i \in W \cap U, j \in W \cap R$. Subgroup-size symmetry means $X^U \geq x_i n/2 > x_j n/2 \geq X^W$. This is a contradiction.

Now, assume $X^W < X^U$. Then, the subgroup $W \cap U$ only supports W. It follows that $P \cap U$ exerts effort for U and thus $X^U \leq X^P$. But then $X^W < X^P$, which implies $x_i > x_j \; \forall \; i \in W \cap U, j \in P \cap U$, especially since members of W face lower effort cost by assumption. Subgroup-size symmetry means that $X^W \geq x_i n/2 > x_j n/2 \geq X^U$, which is a contradiction.

Thus $X^W = X^U$ in equilibrium. The same reasoning yields $X^W = X^R$. It follows that in any equilibrium $X^W = X^U = X^R$ and all first order conditions hold for all individuals in group W. □

Proof 7 ($dx_i/dc > 0 \; \forall \; i \in W$, $dx_i/dc < 0 \; \forall \; i \in P$, and $dX/dc > 0$ if $\gamma < 3^\alpha$). Suppose $\gamma < 3^\alpha$. Plugging $X = nx_i(1+1/\gamma^{1/\alpha}), i \in W$ and $X^j = X/4 \; \forall \; j \in \Gamma$ into the first order condition (1.6), we can write

$$x_i = \left(\frac{rV}{2n(1+1/\gamma^{1/\alpha})(1-c)} \right)^{1/(1+\alpha)}, i \in W. \tag{1.23}$$

Then,

$$\frac{dx_i}{dc} = \frac{x_i}{(1+\alpha)(1-c)} \left(1 + \frac{2/\alpha \left((1-c)/(1+c)^{1+\alpha}\right)^{1/\alpha}}{1+1/\gamma^{1/\alpha}} \right) > 0. \tag{1.24}$$

Individual effort of individuals in W is increasing in c if $\gamma < 3^\alpha$.

Plugging $X = nx_i(1+\gamma^{1/\alpha}), i \in P$ and $X^j = X/4 \; \forall \; j \in \Gamma$ into the first order condition (1.7) yields

$$x_i = \left(\frac{rV}{2n(1+\gamma^{1/\alpha})(1+c)} \right)^{1/(1+\alpha)}, i \in P. \tag{1.25}$$

Then,

$$\frac{dx_i}{dc} = -\frac{x_i}{(1+\alpha)(1+c)} \left(1 + \frac{2/\alpha \left((1+c)/(1-c)^{1+\alpha}\right)^{1/\alpha}}{1+\gamma^{1/\alpha}} \right) < 0. \tag{1.26}$$

Individual effort of individuals in P is decreasing in c if $\gamma < 3^\alpha$.

Due to group-size and individual symmetry $dX/dc = n(dx_i/dc + dx_j/dc), i \in W, j \in P$. To determine how aggregate effort changes in response to effort cost asymmetry, we can look at whether $-(dx_i/dc)/(dx_j/dc), i \in W, j \in P$ is smaller or larger than one. Dividing (1.24) by (1.26) yields

$$-\frac{dx_i/dc}{dx_j/dc} = \frac{\alpha(1+\gamma^{1/\alpha})(1+c)+2}{\alpha(1+\gamma^{1/\alpha})(1-c)/\gamma^{1/\alpha}+2} > 1,$$

where we make use of $\gamma \geq 1$ and $\alpha > 0$. It follows that $dX/dc > 0$ if $\gamma < 3^\alpha$. □

Proof 8 ($du_i/dc < 0\ \forall\ i \in W$ and $du_i/dc > 0\ \forall\ i \in P$ if $\gamma < 3^\alpha$). Using (1.23), we can write expected utility for $i \in W$ as

$$u_i = \frac{V}{2} - \frac{rV}{2n(1+1/\gamma^{1/\alpha})(1+\alpha)}.$$

The total differential is

$$\frac{du_i}{dc} = -\frac{(x_i)^{1+\alpha}}{1+\alpha}\frac{2((1-c)/(1+c)^{1+\alpha})^{1/\alpha}}{\alpha(1+1/\gamma^{1/\alpha})} < 0. \tag{1.27}$$

Expected utility for members of the wealthy group W is decreasing in c if $\gamma < 3^\alpha$.

Using (1.25), we can write expected utility for $i \in P$ as

$$u_i = \frac{V}{2} - \frac{rV}{2n(1+\gamma^{1/\alpha})(1+\alpha)}.$$

The total differential is

$$\frac{du_i}{dc} = \frac{(x_i)^{1+\alpha}}{1+\alpha}\frac{2((1+c)/(1-c)^{1+\alpha})^{1/\alpha}}{\alpha(1+\gamma^{1/\alpha})} > 0. \tag{1.28}$$

Expected utility for members of the poor group P is increasing in c if $\gamma < 3^\alpha$. To see that $du_i/dc = -du_j/dc$, plug (1.23) and (1.25) into (1.27) and (1.28), respectively. □

Proof 9 ($dx_i/dc > 0\ \forall\ i \in W$, $dx_i/dc < 0\ \forall\ i \in P$, and $dX/dc > 0$ if $\gamma > 3^\alpha$). Assume $\gamma > 3^\alpha$. Then W, U, and R are only supported by members of W and have symmetric total effort. Individuals in P only support P and $X^P < X^W$. Using $\rho \equiv X^P/X^W$ and $X^W = nx_i/3, i \in W$, we can write individual effort as

$$x_i = \left(\frac{3r(1+\rho^r)V}{n(3+\rho^r)^2(1-c)}\right)^{1/(1+\alpha)}$$

for $i \in W$. The total differential is

$$\frac{dx_i}{dc} = \frac{x_i}{(1+\alpha)(1-c)}\left(1+(1-c)\frac{d\rho}{dc}\frac{r\rho^{r-1}(1-\rho^r)}{(1+\rho^r)(3+\rho^r)}\right). \tag{1.29}$$

The total differential of (1.9) is

$$\frac{d\rho}{dc} = \frac{-(4/(1+\rho^r))3^\alpha/(1+c)^2}{(2r\rho^{r-1}/(1+\rho^r)^2)3^\alpha(1-c)/(1+c)+(1+\alpha-r)\rho^{\alpha-r}} < 0. \tag{1.30}$$

If we plug (1.30) into (1.29) and simplify using (1.9), we get

$$\frac{dx_i}{dc} = \frac{x_i}{(1+\alpha)(1-c)}\left(1-\frac{2(1-\rho^r)/1+c)}{(3+\rho^r)(1+((1+\alpha)/r-1)(1+\rho^r)/\rho^r)}\right) > 0. \tag{1.31}$$

Individual effort of individuals in W is increasing in c if $\gamma > 3^\alpha$.

For $i \in P$, we can write individual effort as

$$x_i = \left(\frac{2r\rho^r V}{n(3+\rho^r)^2(1+c)}\right)^{1/(1+\alpha)}.$$

Using (1.30) and (1.9), the total differential can be written as

$$\frac{dx_i}{dc} = \frac{x_i}{(1+\alpha)(1+c)}\left(1+\frac{2(3-\rho^r)(1+\rho^r)/((1-c)\rho^r)}{(3+\rho^r)(1+((1+\alpha)/r-1)(1+\rho^r)/\rho^r)}\right) < 0. \tag{1.32}$$

Individual effort of individuals in P is decreasing in c if $\gamma > 3^\alpha$.

To determine how aggregate effort changes, we can again look at whether $-(dx_i/dc)/(dx_j/dc), i \in W, j \in P$ is smaller or larger than one. Dividing (1.31) by (1.32) and making use of $x_i/x_j = 3/\rho$ yields

$$-\frac{dx_i}{dx_j} = \frac{3(1+c)}{\rho(1-c)}\left(\frac{(3+\rho^r)(1+((1+\alpha)/r-1)(1+\rho^r)/\rho^r)+2(1-\rho^r)/(1+c)}{(3+\rho^r)(1+((1+\alpha)/r-1)(1+\rho^r)/\rho^r)+2(3-\rho^r)(1+\rho^r)/(1-c)}\right).$$

Making use of $\rho \in (0,1], c \in (0,1)$, and $r < 1$, it is enough to show that $(4+c(3+\rho^r))/r > (1+\rho^r)(3-\rho^r)/\rho^r$ to conclude that $-(dx_i/dc)/(dx_j/dc) > 1$ and thus $dX/dc > 0$ if $\gamma > 3^\alpha$. □

Proof 10 ($du_i/dc > 0\ \forall\ i \in W$, $du_i/dc < 0\ \forall\ i \in P$, and $d\sum u_i/dc > 0$ if $\gamma > 3^\alpha$). Expected utility for members of group W can be written as

$$u_i = \frac{2}{3+\rho^r}V - \frac{3r(1+\rho^r)}{n(3+\rho^r)^2(1+\alpha)}V.$$

The total differential is

$$\frac{du_i}{dc} = -\frac{d\rho}{dc}\frac{r\rho^{r-1}V}{(3+\rho^r)^2}\left(2+\frac{3r(1-\rho^r)}{n(3+\rho^r)(1+\alpha)}\right) > 0. \tag{1.33}$$

Expected utility of individuals in W is increasing in c if $\gamma > 3^\alpha$.

Expected utility for members of group P can be written as

$$u_i = \frac{1+\rho^r}{3+\rho^r}V - \frac{r\rho^r 2}{n(3+\rho^r)^2(1+\alpha)}V.$$

The total differential is

$$\frac{du_i}{dc} = \frac{d\rho}{dc}\frac{2r\rho^{r-1}V}{(3+\rho^r)^2}\left(1 - \frac{r(3-\rho^r)}{n(3+\rho^r)(1+\alpha)}\right) < 0. \tag{1.34}$$

Expected utility of individuals in P is decreasing in c if $\gamma > 3^\alpha$.

Combining (1.33) and (1.34), we can write the total differential of aggregate utility as

$$\frac{d\sum u_i}{dc} = -\frac{d\rho}{dc}\frac{3r^2\rho^{r-1}(3-\rho^r)V}{(3+\rho^r)^3(1+\alpha)} > 0.$$

Aggregate utility is increasing in c if $\gamma > 3^\alpha$. □

1.D Appendix – Additional group

Proof 11 (Group-effort contribution). In the case with three groups W, P, and U, we can derive three first order conditions:

$$(x_i)^\alpha = \frac{r(X^c)^{r-1}((X^{-c})^r + (X^U)^r)}{(\sum_{j\in\Gamma}(X^j)^r)^2}V \tag{1.35}$$

for $i \in c \cap R$, $c \in \{W, P\}$ and $\Gamma = \{W, P, U\}$ and where $-c = \{W, P\} \setminus c$. Moreover,

$$(x_i)^\alpha = \frac{r(X^U)^{r-1}((X^{-c})^r)}{(\sum_{j\in\Gamma}(X^j)^r)^2}V \tag{1.36}$$

and

$$(x_i)^\alpha \geq \frac{r(X^c)^{r-1}((X^{-c})^r)}{(\sum_{j\in\Gamma}(X^j)^r)^2}V, \tag{1.37}$$

for $i \in c \cap U$, $c \in \{W, P\}$.

Now, suppose members of U do not contribute to W or P. Then, $x_i = X^U/n \;\forall\, i \in U$ and $x_i = 2X^W/n \;\forall\, i \in R$, where $X^W = X^P$ due to subgroup-size symmetry. We can divide (1.35) by (1.36) and rearrange to obtain

$$2^\alpha \left(\frac{X^W}{X^U}\right)^{(1+\alpha)} = \left(\frac{X^W}{X^U}\right)^r + 1. \tag{1.38}$$

It follows that $X^U < X^W$ and that (1.37) is non-binding if $\alpha < 1$. If $\alpha = 1$, $X^U = X^W = X^P$ and all first order conditions bind. Thus, if $\alpha \leq 1$, members of U only contribute effort to group U.

If $\alpha > 1$ all conditions bind since there is no feasible solution for equation (1.38). In this latter case, members of U also contribute to W and P. □

Proof 12 (Individual and aggregate effort). Let $\rho = X^W/X^U$. For $\alpha \leq 1$ we can rearrange (1.35) as

$$x_i = \left(\frac{2rV\rho^r(1+\rho^r)}{n(2\rho^r+1)^2}\right)^{1/(1+\alpha)},$$

$\forall\, i \in R$. In equilibrium with only two groups, individual effort is $x_i^* = (rV/(4n))^{1/(1+\alpha)}$. Since $\rho \geq 1$, $x_i > x_i^* \;\forall\, \alpha \leq 1, i \in R$.

We can write the total individual effort of individuals in U as

$$x_i = \left(\frac{r\rho^r V}{n(2\rho^r+1)^2}\right)^{1/(1+\alpha)}, \tag{1.39}$$

which is smaller than individual effort in the setting with only two groups.

If we use (1.39) and $\rho = X^W/X^U$, we can express aggregate effort as

$$X = \left(\frac{rVn^\alpha\rho^r(2\rho+1)^{1+\alpha}}{(2\rho^r+1)^2}\right)^{1/(1+\alpha)} \tag{1.40}$$

Dividing (1.40) by aggregate effort in the case with only two groups, $(rV(2n)^\alpha/2)^{1/(1+\alpha)}$, yields

$$\left(\frac{2^{1-\alpha}\rho^r(2\rho+1)^{1+\alpha}}{(2\rho^r+1)^2}\right)^{1/(1+\alpha)}. \tag{1.41}$$

Using numerical simulations and the condition $2^\alpha\rho^{1+\alpha} = \rho^r + 1$, we can show that this ratio is always larger than one if $\alpha < 1$. This ratio is equal to one if $\alpha = 1$.

Moreover, if $\alpha > 1$, we can divide (1.35) by (1.36) to show that $x_i = 2^{1/\alpha}x_j, \forall\, i \in R, j \in U$, where we made use of the fact $X^W = X^P = X^U$. For $\alpha > 1$, we can rewrite (1.35) as

$$x_i = \left(\frac{2rV}{3n(1+1/2^{1/\alpha})}\right)^{1/(1+\alpha)},$$

which is larger than x_i^*. Thus, the formation of an active group U always makes members of R exert more effort. Following the same logic, it is straightforward to show that members of U always exert less effort if they alone get the choice between two groups.

Aggregate effort can be written as

$$X = \left(\frac{rVn^{\alpha}(2^{1/\alpha} + 1)^{\alpha}}{3} \right)^{1/(1+\alpha)},$$

which, using $\alpha > 1$, can be shown to be smaller than aggregate effort in the setting with only two groups. □

Chapter 2

Exclusivity of Groups in Contests

2.1 Introduction

Many settings in real life resemble contests. Examples include tendering for public contracts,[1] design contests for construction projects, lobbying to sway politicians, election campaigns, and implicit and explicit innovation competitions. Often, there are groups or players with restricted access to these contests, due to domain-specific know-how, licences, or other formal or informal constraints. These groups may decide whether they allow other players to join them and, potentially, share the prize. Importantly, groups have power over two dimensions of exclusivity: whether additional members may join at all and whether they are allowed to join other groups simultaneously.

Group membership in contests is often *exogenously* non-exclusive; for instance, distributional conflicts may arise between the rich, the poor, the young, and the old, and individuals are members of two of these groups simultaneously. But it remains an open question whether non-exclusive membership is ever an optimal *endogenous* choice from a group leader's perspective. For example, should political parties allow their members to also join other parties? Should companies with access to public tenders join forces with other companies and allow them to do so with other competitors as well? Or should lobby groups allow donations only conditional on the donor not supporting other competing groups? If non-exclusive membership arises endogenously in a contest, is this in the interest of a contest designer who wants to maximise the aggregate effort contestants exert? Unfortunately, the existing literature has little to say about exclusivity decisions of groups in contests.

I devise a stylised Tullock contest model with two group leaders and one potential

[1] For examples of (sometimes non-exclusive) subcontracting in public procurement, see Marion (2015) and Moretti and Valbonesi (2015).

member to investigate these questions. In this three-stage model, group leaders first decide simultaneously whether they allow group entry and whether group membership is exclusive. The potential member then decides which group(s) to join. The member only has access to the contest if she joins a group. In the third stage, groups compete over a prize which is shared equally within the winning group. I solve for the equilibria in this game dependent on the degree of publicness of the prize and the elasticity of marginal effort cost, two parameters that have been of particular importance in the literature on the "group-size paradox" in group contests (see Esteban and Ray (2001)).

If the prize is mostly private, group leaders do not offer membership in equilibrium. If the prize is mostly public or the elasticity of marginal effort cost high, group leaders offer exclusive membership. Non-exclusive membership is never offered in the baseline setting. However, I present several relevant extensions to the model which lead to the emergence of equilibria in which groups offer non-exclusive membership: If the potential member can commit to join non-exclusive groups rather than exclusive ones, she may 'force' group leaders to make their groups non-exclusive. This harms group leaders. If group leaders can charge a membership fee, they offer non-exclusive membership and extract the member's surplus if groups are less likely to win the contest—less effective—than singletons. This mitigates conflict and is beneficial to group leaders. The introduction of a third group to the baseline model makes it more likely for the member to be able to join a group and allows for an equilibrium with non-exclusive membership if the prize is highly public. The latter is due to the fact that, with three groups, a group leader cannot make all groups effectively exclusive on her own. A contest designer interested in maximising aggregate effort, such as a university running an innovation competition, would always like to prohibit non-exclusive membership. The same contest designer prefers allowing exclusive membership only if groups are more effective in the contest than singletons.

After a brief literature review in Section 2.2, I introduce and solve the baseline model in Section 2.3. Section 2.4 contains extensions to membership commitment, membership fees, and a third group. I conclude with a summary of my results and a brief discussion of limitations and avenues for future research in Section 2.5.

2.2 Related literature

The existing literature on endogenous group membership in contests considers players who participate in a contest either alone or as a member of only one group. In Baik and Shogren (1995a), players decide simultaneously whether to join a group which distributes the prize according to an endogenous sharing rule. Baik and Lee (1997) allow for inter-group mobility before the contest. A considerable share of the literature uses group stability as an equilibrium concept. Prominent examples include Skaperdas (1998), Baik and Lee (2001), Noh (2002), Esteban and Sákovics (2003), and Garfinkel (2004a,b). Bloch et al. (2006) analyse secession and group formation in a general contest model. Sánchez-Pagés (2007) considers two modi of group formation: in the first, players simultaneously announce groups to be formed, while in the second, groups are formed sequentially. For an overview of endogenous group formation in contests see Bloch (2012) and Konrad (2014), and for experimental analyses Herbst et al. (2015) and Ke et al. (2013, 2015). For a broader perspective on coalition formation in strategic games see Ray and Vohra (2015).

Related to my work is the analysis of cross-shareholdings in contests by Konrad (2006) and Heijnen and Schoonbeek (2020). In their models, players' preferences for who wins the contest are to some extent aligned, reducing effort expenditure. In the present paper, a member who is a member of more than one group simultaneously also generally exerts less effort—for each group, as well as in total—than the group leaders. Two overlapping effects are at work: First, the member has a lower incentive to support one of her groups with effort since this decreases the winning probability of her other group. Second, the member's opportunity costs are higher because in contrast to the group leaders, she also has the option of providing effort to a second group. These effects link my model to the literature on public good provision following Bergstrom et al. (1986), in which players with lower marginal benefit or higher opportunity cost typically contribute less.

I find that a contest designer interested in maximising aggregate effort would like to allow exclusive membership only if groups are more effective in the contest than singletons. The threshold at which this change in effectiveness happens has been investigated by Esteban and Ray (2001).[2] The effectiveness of groups also influences the exclusivity decisions of group leaders. If they can charge fees for membership, the threshold described by Esteban and Ray (2001) separates existing equilibria.

None of the related studies looks at non-exclusive groups in contests. The exception is Send (2020),[3] whose focus is on exogenous non-exclusive group membership by all players instead of endogenous group membership, however.

[2]The analysis of the "group size paradox" is furthered by Nitzan and Ueda (2009, 2011).

[3]Chapter 1 is based on this paper.

2.3 The baseline model

There are three risk neutral players: two group leaders, l_A and l_B, and one potential member, m. Group leader l_g is the only member of group $g \in \{A, B\}$ at the start, but cannot be a member of group $h \neq g$. A group is defined as a non-empty set of individuals who can bundle their effort in a Tullock contest and benefit if their group wins a prize as specified below. At the start of the game, m is not a member of any of the two groups. The game has three stages. In the first stage, group leaders simultaneously choose the membership menu they offer the potential member. The membership menu is characterised by two dimensions of group exclusivity: First, a group leader can decide whether to allow m to enter her group. Second, the group leader can make group membership conditional on m not being a member of the other group. A group leader l_g's strategy space in the first stage is thus $M_g \in \{N, E, O\}$. N denotes not allowing the member to join the group. E represents the decision to make membership exclusive: m may join the group, but only if she does not join the other group. If a group leader chooses O, a member can join her group non-exclusively. In the second stage, m observes both groups' membership menus and decides which group(s) to join. If both group leaders play O in stage 1, she may join both groups simultaneously. In the third stage, the two groups enter a Tullock contest, which is described in more detail in Section 2.3.1. The winning group receives a prize with value $V > 0$ with degree of publicness $\lambda \in [0, 1]$. The prize is shared equally within the winning group and I abstract from within-group conflict. I briefly come back to this important assumption in Section 2.5. For a given prize value V, the utility a player receives from winning the prize with a group of size n is $(\lambda + (1 - \lambda)/n)V$.[4] An example for a fully private prize is fixed prize money which has to be shared and which contestants care about exclusively. A more public prize can for instance represent reputation that all contestants can attain by winning. All players can support all groups they are a member of with contest effort, which is chosen simultaneously. If m is not a member of any group, she has no access to the contest. In what follows, I solve for the Nash equilibrium in pure strategies of the game by backward induction and offer a more formal description of each stage.

[4]This closely follows Esteban and Ray (2001), who additionally allow for different valuations for the public and private component of the prize.

2.3.1 Stage 3: contest

At this stage, group leaders have chosen their membership menus, m has decided which group(s) to join, and group membership is fixed. The contest between the two groups is modelled as a Tullock contest. The expected utility of group leader $l_g, g \in \{A, B\}$ is

$$u_{l_g} = \frac{\sum_{i\in g} x_i^g}{\sum_{i\in g} x_i^g + \sum_{i\in h} x_i^h}\left(\lambda + \frac{1-\lambda}{n_g}\right)V - \frac{(x_{l_g})^{1+\alpha}}{1+\alpha}, \tag{2.1}$$

where $h \neq g$, n_g is the number of members of group g, and $\alpha \geq 0$ is the elasticity of marginal effort cost. The effort player i provides to group g is denoted x_i^g. For group leader l_g, who can only be a member of one group, the effort she provides to her group g equals her total effort, $x_{l_g}^g = x_{l_g}$. A group's aggregate effort is the sum of its members' efforts in support of it. The first fraction in (2.1) is the probability p_g with which group g wins the prize. If all efforts are zero, the prize is allocated at random and $p_g = 1/2$.[5] The expected utility of m who is a member of group(s) $G_m \subseteq \{A, B\}$ is

$$u_m = \frac{\sum_{g\in G_m}\sum_{i\in g} x_i^g}{\sum_{i\in g} x_i^g + \sum_{i\in h} x_i^h}\left(\lambda + \frac{1-\lambda}{2}\right)V - \frac{(x_m)^{1+\alpha}}{1+\alpha}.$$

If m is not a member of any group, the first term simplifies to zero. If m is a member of both groups, the first term reduces to one. Additionally, if m is a member of both groups, her individual effort is the sum of the two efforts she exerts for her two groups, $x_m = x_m^A + x_m^B$.

There are three cases: m is not a member of any group, m is a member of one group, and m is a member of both groups. I analyse these three cases in the following subsections.

Singleton conflict

If m is not a member of any group, the contest reduces to a singleton contest between group leaders l_A and l_B. With one player per group, the publicness of the prize does not matter and its value simplifies to V. The Nash equilibrium in pure strategies of this subgame is characterised by symmetric individual effort $x^s = (V/4)^{(1/(1+\alpha))}$ for

[5] This contest function based on Tullock (1980) is the most widely used specification in the literature on group contest. I restrict my analysis to its simplest form to retain tractability. It is axiomatised in its general form in Münster (2009). There is inherent uncertainty in the contest; the group whose members exert the highest effort does not necessarily win. This is intuitive if we consider for example design, lobbying, or election contests, in which efforts might not translate one-to-one into effective impact or in which the winning conditions might not be fully transparent.

group leaders. Their expected equilibrium utility is

$$u^s = \frac{1+2\alpha}{4(1+\alpha)}V. \tag{2.2}$$

Player m has no access to the contest and thus exerts zero effort and has zero utility.

Group versus singleton

Suppose m is a member of group $g \in \{A, B\}$ but not of group $h \neq g$. Then, players l_g and m can both exert effort for group g. If they win, they share the prize and receive $V(1+\lambda)/2$. Publicness of the prize matters in this case. Player l_h fights alone. From the first order conditions follows that members of group g exert symmetric effort in equilibrium if $\alpha > 0$. For simplicity, I assume symmetry also for linear cost, $\alpha = 0$. Denote individual equilibrium effort by members of g by x^{2v1} and effort by l_h by x^{1v2}. We can write the first order conditions as

$$\frac{x^{1v2}}{(2x^{2v1}+x^{1v2})^2}\frac{V(1+\lambda)}{2} = (x^{2v1})^\alpha \tag{2.3}$$

for members of group g and

$$\frac{2x^{2v1}}{(2x^{2v1}+x^{1v2})^2}V = (x^{1v2})^\alpha \tag{2.4}$$

for player l_h. Second order conditions hold for strictly positive efforts. By dividing (2.3) by (2.4) and rearranging, we can write

$$x^{2v1} = ((1+\lambda)/4)^{(1/(1+\alpha))}x^{1v2}. \tag{2.5}$$

In Nash equilibrium, members of g exert less individual effort than the sole member of h. Intuitively, this effect decreases in the publicness of the prize λ and the elasticity of marginal cost α. We can use equation (2.5) to solve for individual equilibrium effort in group g as

$$x^{2v1} = \left(\frac{(1+\lambda)((1+\lambda)/4)^{1/(1+\alpha)}}{2(1+2((1+\lambda)/4)^{1/(1+\alpha)})^2}V\right)^{1/(1+\alpha)}.$$

The leader of group h exerts

$$x^{1v2} = \left(\frac{2((1+\lambda)/4)^{1/(1+\alpha)}}{(1+2((1+\lambda)/4)^{1/(1+\alpha)})^2}V\right)^{1/(1+\alpha)}.$$

Individual equilibrium utilities are thus

$$u^{2v1} = \frac{2((1+\lambda)/4)^{(2+\alpha)/(1+\alpha)}(1+2\alpha+4(1+\alpha)((1+\lambda)/4)^{1/(1+\alpha)})}{(1+\alpha)(1+2((1+\lambda)/4)^{1/(1+\alpha)})^2}V \tag{2.6}$$

for the members of group g and

$$u^{1v2} = \frac{1+\alpha(1+2((1+\lambda)/4)^{1/(1+\alpha)})}{(1+\alpha)(1+2((1+\lambda)/4)^{1/(1+\alpha)})^2}V. \tag{2.7}$$

for the leader of group h, who fights alone.

Group versus group

If m enters both groups, she wins the contest with one of these groups with probability one. She will thus not exert any effort, and her utility is $u^b = V(1+\lambda)/2$. The subgame effectively becomes a contest between the two group leaders over a prize valued at $V(1+\lambda)/2$. In equilibrium, individual efforts of group leaders are $x^{2v2} = (V(1+\lambda)/8)^{1/(1+\alpha)}$. These result in the expected utilities

$$u^{2v2} = \frac{(1+\lambda)(1+2\alpha)}{8(1+\alpha)}V \tag{2.8}$$

for the two group leaders.

2.3.2 Stage 2: member decision

The member's decision in stage 2 is trivial: she joins as many groups as she can. This becomes clear by comparing the member's expected utilities in the three possible cases. If m does not join any group, her utility is zero. Further, it is clear that her utility if she joins one group, (2.6), is larger than zero. The member's utility if she joins both groups, $V(1+\lambda)/2$, is in turn larger than (2.6). The membership menus chosen by both group leaders in stage 1 are $(M_A, M_B) \in \{N, E, O\}^2$. Then, m cannot join any group if $(M_A, M_B) = (N, N)$. If $(M_A, M_B) \in \{(N, E), (E, N), (N, O), (O, N)\}$, m joins the group she is allowed to. Due to symmetry, m picks one group to join at random if $(M_A, M_B) \in \{(E, E), (E, O), (O, E)\}$. Finally, m joins both groups if both groups allow non-exclusive membership, $(M_A, M_B) = (O, O)$. The potential member can only join both groups if both groups allow non-exclusive membership.

		M_A		
		N	E	O
M_B	N	u^s u^s	u^{2v1} u^{1v2}	u^{2v1} u^{1v2}
	E	u^{1v2} u^{2v1}	$(u^{2v1}+u^{1v2})/2$ $(u^{2v1}+u^{1v2})/2$	$(u^{2v1}+u^{1v2})/2$ $(u^{2v1}+u^{1v2})/2$
	O	u^{1v2} u^{2v1}	$(u^{2v1}+u^{1v2})/2$ $(u^{2v1}+u^{1v2})/2$	u^{2v2} u^{2v2}

Table 1: Expected utilities for group leaders in stage 1

2.3.3 Stage 1: membership menu

In stage 1, group leaders can anticipate the potential member's decision in stage 2 and the resulting contest equilibria in stage 3. The group leaders' expected utilities given their membership menu decisions are depicted in Table 1. If both group leaders offer group membership, but at least one of them exclusively, the potential member joins a group at random. Hence, the expected utility of a group leader is $(u^{2v1} + u^{1v2})/2$ in this case.

A two-dimensional membership menu $(M_A, M_B) \in \{N, E, O\}^2$ constitutes a Nash equilibrium in pure strategies if none of the group leaders has an incentive to deviate. Figure 1 illustrates the Nash equilibria in stage 1 dependent on the elasticity of marginal effort cost α and the publicness of the prize λ. If λ is large enough, i.e. the price public enough, we have $(u^{2v1} + u^{1v2})/2 \geq u^{1v2}$ and it becomes attractive to fight together with an additional member. Then, both group leaders offer group membership, at least one of them exclusively, $(M_A^*, M_B^*) \in \{(E, E), (E, O), (O, E)\}$. Vice versa, if λ is small, we have $u^s \geq u^{2v1}$ and it is attractive to fight alone, and both group leaders do not offer any membership, $(M_A^*, M_B^*) = (N, N)$. An increase in the elasticity of marginal effort cost α makes fighting together with an additional group member more attractive. If α is small, there is a parametric region in which we have $u^s \leq u^{2v1}$ and $(u^{2v1} + u^{1v2})/2 \leq u^{1v2}$ and the Nash equilibrium is asymmetric. One group leader offers group membership, and the other leader keeps her group fully exclusive, $(M_A^*, M_B^*) \in \{(N, E), (N, O), (E, N), (O, N)\}$. If α is large, there is a region in which both the fully exclusive equilibrium (N, N) as well as the exclu-

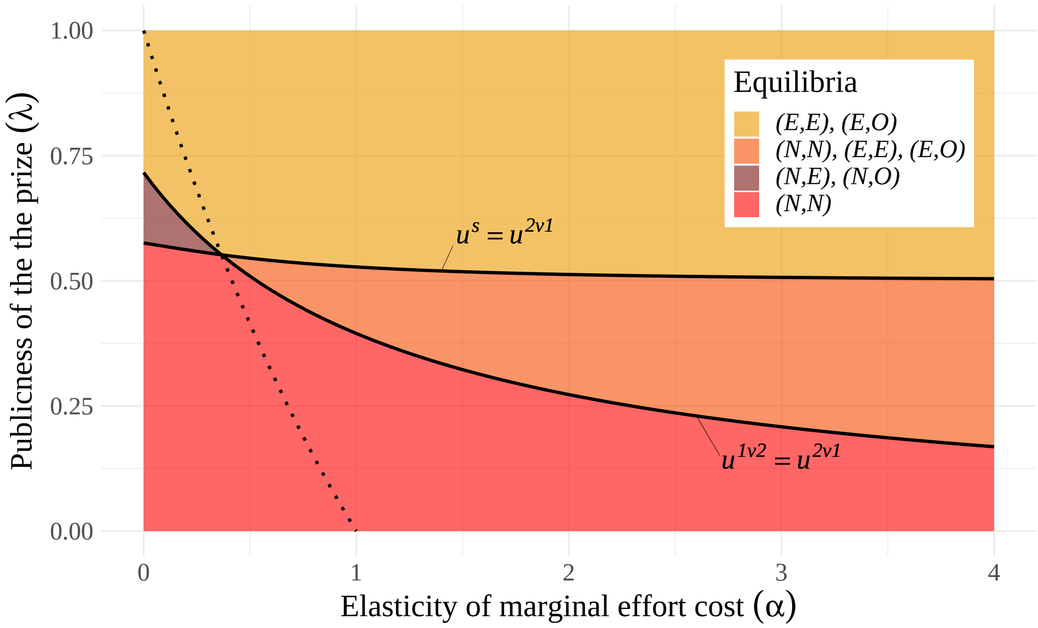

Figure 1: Nash equilibria in stage 1.

For the sake of brevity, the legend shows only one possible constellation of asymmetric equilibria.

sive group membership equilibrium $\{(E,E),(E,O),(O,E)\}$ exist since $u^s \geq u^{2v1}$ and $(u^{2v1}+u^{1v2})/2 \geq u^{1v2}$. If we use payoff dominance as refinement concept, only (N,N) survives as equilibrium in this region since there $u^s > u^{2v1} > (u^{2v1}+u^{1v2})/2$. Strategy O is weakly dominated by E since $(u^{2v1}+u^{1v2})/2 > u^{2v2}$, which I briefly show in Appendix 2.A. In the baseline model, there is no equilibrium in which both group leaders offer non-exclusive membership simultaneously.

Figure 1 is based on numerical simulations. We can confirm analytically what happens along the margins, i.e. $\alpha = 0$ and/or $\lambda = 0$ or $\lambda = 1$. Moreover, both equalities $u^s = u^{2v1}$ and $u^{1v2} = u^{2v1}$ define a continuous implicit function

$$F(\alpha, \lambda) = 0$$

which makes λ a continuous function of α and uniquely pins down the threshold at which equilibrium existence changes.[6] However, we cannot find closed-form equilibrium solutions for stage 1 for the whole α-λ-plain.

If neither (N,N) nor $\{(E,E),(E,O),(O,E)\}$ or both of them constitute a Nash equilibrium, there is a Nash equilibrium in mixed strategies in which both group leaders play N with symmetric probability $p = (u^{2v1} - u^{1v2})/(u^s - (u^{2v1} + u^{1v2})/2)/2$

[6] See Appendix 2.A for more detail.

and E with probability $(1-p)$.[7] Only strategies that are not strictly dominated can be played in a mixed-strategy equilibrium. If (N, N) is a pure-strategy equilibrium, but not $\{(E, E), (E, O), (O, E)\}$, N strictly dominates both E and O. Vice versa, if $\{(E, E), (E, O), (O, E)\}$ are pure-strategy equilibria, but (N, N) is not, E strictly dominates N. To find the symmetric mixed-strategy equilibrium in the case where both N and E are not strictly dominated, we solve the indifference equation $\sigma u^s + (1-\sigma)u^{1v2} = \sigma u^{2v1} + (1-\sigma)(u^{2v1} + u^{1v2})/2$ for σ, which gives us p. For simplicity, I restrict the rest of my analysis to pure strategies.

If we use payoff dominance, we can also make some welfare judgements. The switch from (N, N) to (N, E) or (N, O), which happens at the line $u^s = u^{2v1}$ but below the line $u^{1v2} = u^{2v1}$, is beneficial to both group leaders and thus automatically welfare increasing. The switch from (N, N) to (E, E) however is harmful to both group leaders, while again being welfare increasing since there $2u^{2v1} + u^{1v2} > 2u^{2v1} = 2u^s$. Since the switch from (N, E) or (N, O) to (E, E) happens at a point where $u^{1v2} = u^{2v1}$, there is no discontinuous jump in utility of group leaders or overall welfare. If we look at the game from the perspective of a contest designer who wants to maximise aggregate effort, we can confirm numerically that left to the dotted line in Figure 1 she has an incentive to enforce full exclusivity, while to the right of it aggregate effort is higher if m is allowed to join one group. This line is where both groups have the same probability of winning the prize and is given by $\lambda(\alpha) = 2^{1-\alpha} - 1$. For the singleton player, it does not matter whether she competes against another singleton or a group. Naturally, if her opposing group exerts combined effort x^s, her best response is the same as in the singleton case, effort x^s as well. To the left of this line, the singleton group is more likely to win—more 'effective'—and to the right of it, the 2-member group is more likely to win. This threshold is the focal point of Esteban and Ray (2001).[8]

[7] In this case, there are also infinite payoff-equivalent asymmetric mixed-strategy equilibria in which one leader plays the weakly dominated strategy O instead of E with some probability $\epsilon(1-p)$, $\epsilon \in [0, 1]$. Naturally, also when the equilibrium in pure strategies (E, E) exists, there is an infinite number of mixed-strategy equilibria in which one of the leaders sometimes plays O instead of E.

[8] In our case of a switch from singleton players to a two-player group, the threshold derived by Esteban and Ray (2001) is given by the equation $\lambda = 1 - \alpha$. Above their threshold, groups are always more effective. Below it, group effectiveness depends on the starting group size. In our case, the exact threshold at which a two-player group is more effective than a singleton is $\lambda = 2^{1-\alpha} - 1$, which we get by solving $2x^{2v1} = x^{1v2}$ for λ, and which we can confirm is below $\lambda = 1 - \alpha$. The finding that a contest designer in many cases prefers to allow exclusive membership is in line with the fact that firms are often legally allowed to bid jointly in public procurement if a firm does not have access as a solo bidder; see for example Albano et al. (2009).

		M_A		
		N	E	O
	N	u^s u^s	u^{2v1} u^{1v2}	u^{2v1} u^{1v2}
M_B	E	u^{1v2} u^{2v1}	$(u^{2v1}+u^{1v2})/2$ $(u^{2v1}+u^{1v2})/2$	u^{2v1} u^{1v2}
	O	u^{1v2} u^{2v1}	u^{1v2} u^{2v1}	u^{2v2} u^{2v2}

Table 2: Expected utilities for group leaders in stage 1 with commitment

2.4 Extensions

2.4.1 Membership commitment

In the baseline game above, there is never an equilibrium in which both group leaders offer non-exclusive membership. Thus, in equilibrium the potential member m is never a member of both groups simultaneously. However, if group leaders prefer the group-versus-group contest to fighting alone against a group, there is a way for m to improve her outcome: she can commit to join the group whose leader offers non-exclusive membership. Is this commitment credible? In the described model, the answer is ambiguous: in stage 2, m is indifferent between the two groups. However, even a minuscule reputation cost or the slightest chance of a repeated game would make the commitment credible. For the sake of analysis, I will assume that the commitment is credible without introducing an additional aspect to the model that would unambiguously make it so.

If the potential member m credibly commits to join the group whose leader has chosen O in stage 1, the expected-utility matrix of stage 1 changes to Table 2. Now, it does make a difference to group leaders whether they choose E or O whenever the other group leader chooses E. In this case, a group leader can 'convince' m to join her group for sure by offering non-exclusive membership.

This change in the game leads to the existence of the additional Nash equilibrium in pure strategies in stage 1 with non-exclusive membership offered by both group leaders, (O, O). This equilibrium arises if the prize is highly public and effort cost convex. See Figure 2, which is again based on numerical simulations: instead of

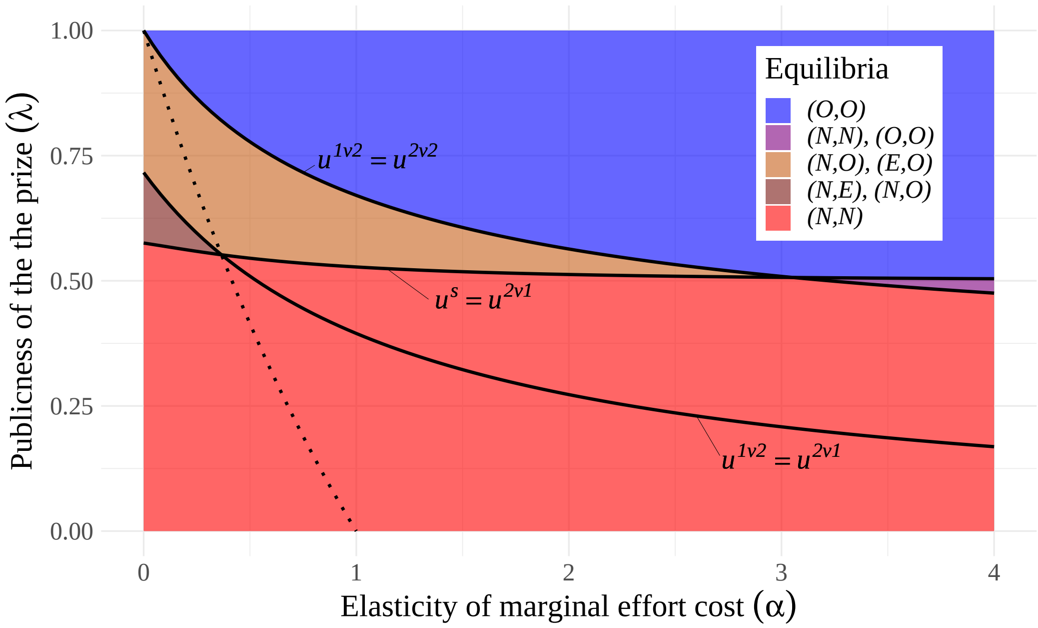

Figure 2: Nash equilibria in stage 1 with commitment

For the sake of brevity, the legend shows only one possible constellation of asymmetric equilibria.

two regions '$(E, E), (E, O)$' and '$(N, N), (E, E), (E, O)$' we get four regions '(O, O)', '$(N, O), (E, O)$', '$(N, N), (O, O)$', and an additional region where (N, N) is the only equilibrium in stage 1. If we use payoff dominance as refinement concept, in the region '$(N, N), (O, O)$' only (N, N) survives as equilibrium because $u^s > u^{2v2}$ if $\lambda < 1$. In addition to the comparisons that are relevant in the baseline game, we also need to compare group leaders' utility fighting alone against two players u^{1v2} and their utility in case m is a member of both groups u^{2v2}. See Appendix 2.B for additional detail. As before, for all relevant boundaries we can define continuous implicit functions of the marginal elasticity of effort cost α and the publicness of the prize λ, $F(\alpha, \lambda) = 0$, making λ a continuous function of α.

With commitment by m to join non-exclusive groups, (E, E) can only be an equilibrium in stage 1 if $u^{1v2} = u^{2v1}$. Given that the other group leader chooses E, a leader only has an incentive to also play E instead of N if $u^{1v2} \leq u^{2v1}$. But if $u^{1v2} < u^{2v1}$, a leader would like to make sure that m joins her group, which she can by choosing O instead of E.

If we think of the commitment decision by player m as stage 0 that comes before stage 1, we can determine her equilibrium strategy in this initial stage. For any (α, λ) tuple for which $u^{1v2} \leq u^{2v2}$ and $u^s \leq u^{2v1}$ holds, member m is better off in expectation by committing to join non-exclusive groups. Below the line $u^{1v2} = u^{2v1}$ as well as in the region '$(N, O), (E, O)$' in Figure 2, m is indifferent between commitment and no commitment. In the region between the line $u^s = u^{2v1}$ and the line $u^{1v2} = u^{2v1}$, the existence of multiple equilibria makes the member's decision ambiguous. If we use payoff dominance as equilibrium refinement concept in stage 1, m is indifferent there and thus in general never worse off by committing.

If the member m can commit to join groups who offer non-exclusive membership, in contrast to the baseline game above there is a wide range of parameters in which the Nash equilibrium in stage 1 is (O, O). For the two group leaders, this outcome is worse than (E, E). However, it can be shown numerically that it is welfare increasing. Moreover, if $u^{1v2} \leq u^{2v1}$, $u^s \leq u^{2v1}$ and $u^{1v2} \geq u^{2v2}$, the equilibrium (E, E) is replaced by the asymmetric equilibria (N, O) and (E, O). This has no impact on the member's utility or overall welfare, but benefits the group leader offering non-exclusive membership and harms the other one. A contest designer would like to rule out non-exclusive membership, since we can show numerically that aggregate effort in the 2-versus-1 scenario is always higher than in the 2-versus-2 scenario. As in the baseline game, left to the dotted line in Figure 2 the contest designer has an incentive to enforce full exclusivity.

2.4.2 Membership fee

Access to a group often comes with a price tag. In our setting, we can think of entry fees as a third dimension of exclusivity. I model this by allowing group leaders $l_g, g \in \{A, B\}$ in stage 1 in addition to deciding the degree of membership exclusivity to simultaneously choose a (potentially negative) fee $f_g \in \mathbb{R}$ that member m has to pay if she joins the group. Naturally, setting f_g is only meaningful if $M_g \in \{E, O\}$. I abstract from any potential membership commitment discussed in Section 2.4.1. I again restrict my analysis to pure-strategy equilibria.

Stage 3 again remains the same as before. Stage 2 now becomes more intricate. If both group leaders allow no membership, m has no choice but to not join any group. If only one group $g \in \{A, B\}$ offers membership, member m joins this group g if $f_g \leq u^{2v1}$ and otherwise is better off by not joining any group. If the group membership menu decided in stage 1 is $(M_A, M_B) \in \{(E, E), (E, O), (O, E)\}$ and it is cheaper to join one group than it is to join the other, $f_g < f_h, g \neq h$, m joins the cheaper group g if $f_g \leq u^{2v1}$ and no group otherwise. If access to both groups cost the same, m chooses to join one of them at random if again $f_g \leq u^{2v1}$ and no group otherwise. If both groups offer non-exclusive membership, $(M_A, M_B) = (O, O)$, and one group g offers cheaper membership, m joins the cheaper group if $0 \leq u^{2v1} - f_g$ and $u^b - (f_g + f_h)) < u^{2v1} - f_g$, joins both groups if $\max(0, u^{2v1} - f_g) \leq u^b - (f_g + f_h)$, and does not join any group otherwise. If both groups offer non-exclusive membership for the same fee, m joins one of them at random if $0 \leq u^{2v1} - f_g$ and $u^b - 2f_g < u^{2v1} - f_g$, both groups if $\max(0, u^{2v1} - f_g) \leq u^b - 2f_g$, and no group otherwise. I assume here that m joins both groups if she is indifferent between joining one and both groups.

For stage 1, we cannot write down a simple payoff matrix as before since f_g is a continuous decision variable. Rather, we can go through each potential membership-menu profile, consider which associated fee structure is stable, and again use a mixture of numerical and analytical approaches to determine whether and when these strategies constitute a Nash equilibrium.

Consider the strategy profile (N, N) and $f_g \in \mathbb{R}$. If one group leader l_g deviates, she can charge $f_g = u^{2v1}$ and thus have utility $2u^{2v1}$. The stage-1 strategy profile (N, N) and $f_g \in \mathbb{R}$ thus constitutes a Nash equilibrium if $u^s \geq 2u^{2v1}$. You can see in Figure 3, which is based on numerical simulations, that this only holds if the marginal elasticity of effort cost α and the publicness of the prize λ are both close to zero.[9] All equilibria in pure strategies in which one leader plays N can be substituted with the same leader playing E or O but setting a prohibitively high fee that makes the group effectively fully exclusive.

For the profiles (N, E) and (E, N), the only fee for group g that allows membership that can constitute a Nash equilibrium which is not equivalent to (N, N) is $f_g = u^{2v1}$. Why? If group leader l_g sets a lower fee, she would have an incentive to deviate and set a higher fee at which m still joins, thus increasing the utility of l_g. If $f_g > u^{2v1}$, m does not have an incentive to join the group and the strategy profile is equivalent to (N, N). Given this optimal fee $f_g = u^{2v1}$, two conditions need to hold for (N, E) and (E, N) to constitute a Nash equilibrium: $u^s \leq 2u^{2v1}$ and $u^{1v2} \geq 2u^{2v1}$. To understand the latter, note that the player l_h playing N has expected utility u^{1v2}. She could deviate to playing E or O and either set fee $f_h = u^{2v1}$ to secure $(2u^{2v1} + u^{1v2})/2$ or choose $f_h = u^{2v1} - \epsilon, \epsilon > 0$ to secure $2u^{2v1} - \epsilon$.

For the profiles (N, O) and (O, N), the optimal fee set by the player l_g choosing O is again $f_g = u^{2v1}$. The additional condition (compared to (N, E) and (E, N)) that must hold for these profiles to constitute a stage-1 Nash equilibrium is $u^{2v2} + (u^b - u^{2v1}) \leq u^{1v2}$, where $u^b - u^{2v1}$ is the fee the leader playing N can extract from m by switching

[9] All lines shown in Figure 3 are again defined by implicit continuous functions $F(\alpha, \lambda) = 0$. See Appendix 2.C.

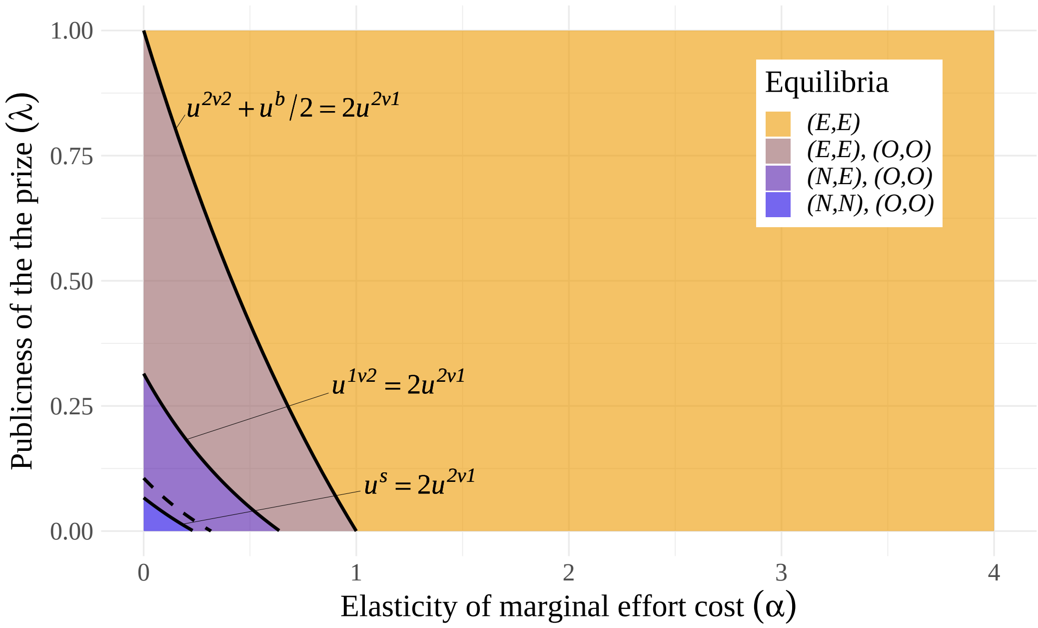

Figure 3: Nash equilibria in stage 1 with membership fees

For the fee profiles that correspond to each equilibrium membership menu, see the text. Instead of playing N, group leaders can also set prohibitively high fees. Below the dashed line, we have $u^{1v2} > u^{2v2} + u^b/2$, a condition required for equilibrium refinement by payoff dominance. For the sake of brevity, the legend shows only one possible constellation of asymmetric equilibria.

to playing O as well. I briefly show in Appendix 2.C that this condition never holds.

For the profile (E, E), the only stable fee profile which is not equivalent to at least one leader choosing N is $f_g = f_h = u^{1v2} - u^{2v1}$. It guarantees group leaders the utility $(u^{1v2} + u^{2v1} + (u^{1v2} - u^{2v1}))/2 = u^{1v2}$. If one group leader raises the fee, m always joins the other group, and her utility becomes u^{1v2}, the same as before. If one group leader decreases the fee to join her group by ϵ, m joins her group for sure, making her new utility $u^{1v2} - \epsilon < u^{1v2}$. It follows that the condition for this strategy profile to be a Nash equilibrium which is not equivalent to (N, N) is $u^{1v2} \leq 2u^{2v1}$, since otherwise m will not join any group. You can see in Figure 3 that (E, E) is the only Nash equilibrium if α and/or λ get large.

To constitute a Nash equilibrium , the profiles (E, O) and (O, E) require the additional condition (compared to (E, E)) that $u^{2v2} + (u^b - u^{2v1}) \leq u^{1v2}$, where $u^b - u^{2v1}$ is the maximum fee the leader playing E can extract from m if she deviates to O. Recall that this condition never holds.

Lastly, consider the strategy profile (O, O). If $u^b > 2u^{2v1}$, the optimal fee set by both group leaders is $f_g = u^b/2$,[10] at which m is indifferent between joining both

[10] Or any fee schedule with $f_A + f_B = u^b$ and $u^{2v1} - \min(f_A, f_B) \leq 0$, $u^{2v2} + \min(f_A, f_B) \geq 2u^{2v1}$,

groups and not joining any group. Given this optimal fee, two conditions need to hold for (O, O) to be a Nash equilibrium in stage 1. First, $u^{2v2} + u^b/2 \geq 2u^{2v1}$, where the right hand side is the utility a group leader l_g can get by deviating to E and charging the lower fee $f_g = u^{2v1}$.[11] Second, $u^{2v2} + u^b/2 \geq u^s$, where the right hand side is the utility a group leader gets by deviating to N (or a fee higher than $u^b/2$). It is straightforward to show that this latter condition always holds. If $u^b \leq 2u^{2v1}$, the optimal fee for the profile (O, O) which is not equivalent to a group leader playing N is $f_g = u^b - u^{2v1}$ for both group leaders. If one group leader sets a higher fee, m only joins the other group. If one group leader sets a lower fee, she loses utility while m still joins both groups. For this optimal fee, there are three conditions for (O, O) to be a Nash equilibrium in stage 1: $u^{2v2}+u^b-u^{2v1} \geq (u^b+u^{1v2})/2$, $u^{2v2}+u^b-u^{2v1} \geq u^{1v2}$, and $u^{2v2} + u^b - u^{2v1} \geq u^b - \epsilon, \epsilon > 0$. The last condition reduces to $u^{2v2} > u^{2v1}$. We always have that $u^{2v2} < u^b/2$. Since we consider the parametric range where $u^b \leq 2u^{2v1}$, it is clear that $u^{2v2} \leq u^{2v1}$. Hence, the third condition does not hold and (O, O) cannot be a Nash equilibrium if $u^b \leq 2u^{2v1}$. Consequently, (O, O) is a Nash equilibrium if $u^{2v2} + u^b/2 \geq 2u^{2v1}$, which is the case if neither α nor λ is too large.

Figure 3 shows that the introduction of membership fees completely alters the membership menus that are offered by group leaders compared to the baseline model's depicted in Figure 1. Remember that instead of playing N in equilibrium, group leaders can also play E or O and setting a prohibitively high fee, which is not depicted in Figure 3. Numerical simulations show that the equilibrium (O, O) is payoff dominant in stage 1 whenever it exists, except within the region '$(N, E), (O, O)$' below the dashed line where $u^{1v2} > u^{2v2} + u^b/2$.

If we compare the payoff dominant equilibria to those of the baseline model, the introduction of membership fees can have different effects on players' utility: If a group is less effective than a singleton, the region to the left of the line $u^{2v2} + u^b/2 = 2u^{2v1}$ in Figure 3,[12] the member is indifferent if $u^s >= u^{2v1}$ and worse off otherwise. In both cases, the group leader(s) whose group(s) she joins extract her full utility via fees, whereas in the latter case she could receive u^{2v1} without fees. If a group is more effective than a singleton, the member is unambiguously better off after the introduction of fees, since her utility is then $2u^{2v1} - u^{1v2}$. Using numerical simulations,

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and $u^{2v2} + \min(f_A, f_B) \geq u^s$. I restrict my analysis to the symmetric case.

[11] If $u^{2v2} + u^b/2 \geq 2u^{2v1}$ holds, it is clear that also $u^b > 2u^{2v1}$ holds since $u^{2v2} < u^b/2$.

[12] As noted above, if $2x^{2v1} + x^{1v2} = 2x^s$, we also have that $2x^{2v1} = x^{1v2} = x^s$. From our formulas for effort follows that then $\lambda = 2^{1-\alpha} - 1$. If we plug these conditions into our formulas for u^{2v2}, u^b and u^{2v1} we can see that $u^{2v2} + u^b/2 = 2u^{2v1}$ holds.

we can make additional welfare judgements: Irrespective of asymmetric equilibria in stage 1 in both scenarios, group leaders are better off with fees if groups are less effective than singletons and worse off otherwise. In the latter case, their competition to lure the member into their exclusive groups harms them compared to the baseline. The introduction of membership fees leads to an outcome that is weakly more efficient. As above, a contest designer would like to always rule out non-exclusive membership and enforce full exclusivity if groups are less effective than singletons.

2.4.3 Three groups

Another natural way to extend our baseline model is to allow for more groups and see how this affects equilibrium outcomes. For simplicity, I consider three groups and relegate a brief analysis of more groups to Appendix 2.D.

Stage 3

Consider again our baseline model from Section 2.3 with the sole addition of a third group leader l_C. Nothing changes in stage 3 except that there are now three groups, each with one or two members, who compete over the prize. It follows that, dependent on which groups m joins in stage 2, there are four scenarios: singleton conflict, a group of two competing against two singletons, two groups of two competing against one singleton, and three groups of two competing against each other.

If m is not a member of any group, the contest reduces to a singleton contest between the three group leaders l_A, l_B, and l_C. With one player per group, the publicness λ of the prize does not matter and its value simplifies to V. The Nash equilibrium in pure strategies of this subgame is characterised by symmetric individual effort $x^{s'} = (2V/9)^{(1/(1+\alpha))}$ for group leaders. Their expected equilibrium utility is

$$u^{s'} = \frac{1+3\alpha}{9(1+\alpha)}V.$$

Player m has no access to the contest and thus exerts zero effort and has zero utility.

If m is a member of one group g, we cannot generally get closed-form solutions for efforts and utilities. However, we can use first order conditions to arrive at a numerical equilibrium solution. If we assume that under constant marginal cost, that is if $\alpha = 0$, both members of g exert the symmetric effort x^{2v11}, which follows from the first order conditions if $\alpha > 0$, we can write its members' first order condition as

$$\frac{2x^{1v21}}{(2x^{2v11}+2x^{1v21})^2}\frac{1+\lambda}{2}V = (x^{2v11})^{\alpha}, \tag{2.9}$$

where x^{1v21} is the symmetric effort exerted by the two group leaders competing on their own. These two leaders' first order conditions are

$$\frac{2x^{2v11}+x^{1v21}}{(2x^{2v11}+2x^{1v21})^2}V = (x^{1v21})^{\alpha}. \tag{2.10}$$

Second order conditions hold for strictly positive efforts. If we divide (2.9) by (2.10) and define $\rho = x^{2v11}/x^{1v21}$, we get

$$\frac{2}{2\rho+1}\frac{1+\lambda}{2} = \rho^{\alpha}. \tag{2.11}$$

If $\alpha = \lambda = 0$, we get $x^{2v11} = 0$ and $x^{1v21} = V/4$. Otherwise, we can numerically solve (2.11) for ρ and use that to derive efforts and consequently utilities. Denote the utility of m and the leader of the group she joins as u^{2v11} and the utility of the two group leaders competing alone as u^{1v21}.

If m enters two groups, we can again not get closed-form solutions for efforts and utilities. If we assume that m divides her effort x^{22v1} symmetrically between the two groups she is a member of,[13] we can write the first order condition for m as

$$\frac{x^{1v22}}{(x^{22v1}+2x^{2v21}+x^{1v22})^2}\frac{1+\lambda}{2}V \leq (x^{22v1})^{\alpha}, \tag{2.12}$$

where x^{2v21} is the effort of both group leaders who have m as a member—which we can assume to be symmetric in any equilibrium due to symmetry—and x^{1v22} is the effort of the group leader competing alone. The first order condition for the leaders of the groups m is a member of are

$$\frac{x^{22v1}/2+x^{2v21}+x^{1v22}}{(x^{22v1}+2x^{2v21}+x^{1v22})^2}\frac{1+\lambda}{2}V = (x^{2v21})^{\alpha}. \tag{2.13}$$

[13] We can assume this to be the payoff dominant equilibrium for m. Suppose m is a member of groups A and B but not C and exerts symmetric efforts for both of her groups in equilibrium. Unless $\alpha = 0$, if m shifts some of her effort from group A to group B, in the new equilibrium the leader of B reduces her effort by more than the leader of A increases hers, in turn reducing the member's aggregate win probability.

For the group leader who competes on her own, the first order condition is

$$\frac{x^{22v1} + 2x^{2v21}}{(x^{22v1} + 2x^{2v21} + x^{1v22})^2} V = (x^{1v22})^{\alpha}. \tag{2.14}$$

If $\alpha = 0$, (2.12) and (2.13) cannot bind at the same time and we have that $x^{22v1} = 0$. Then we can derive that $x^{2v21} = 2((1+\lambda)/(5+\lambda))^2 V$ and $x^{1v22} = (3-\lambda)/(1+\lambda)x^{2v21}$ which gives us the expected utilities for all players. If $\alpha > 0$, we can divide (2.12), which now binds, by (2.14) to get

$$x^{1v22} = \left((x^{22v1})^{\alpha} (x^{22v1} + 2x^{2v21}) \frac{2}{1+\lambda} \right)^{1/(1+\alpha)}. \tag{2.15}$$

Dividing (2.12) by (2.13) yields

$$\frac{1}{(x^{22v1}/2 + x^{2v21})/x^{1v22} + 1} = \left(\frac{x^{22v1}}{x^{2v21}} \right)^{\alpha}. \tag{2.16}$$

For positive efforts it follows that we must have $x^{22v1} < x^{2v21}$ and naturally $x^{22v1}/2 < x^{2v21}$ in equilibrium. We see that in general a player who is a member of multiple groups exerts less effort for each group, as well as in total, than the group leaders. Two effects are at work here: First, the member has a lower incentive to support one of her groups with effort since this decreases the winning probability of her other group. Second, the member's opportunity costs are higher because in contrast to the group leaders, she also has the option of providing effort to a second group. If effort cost is linear, $\alpha = 0$, m does not exert effort when she is a member of more than one group. If m is a member of all groups, the first effect makes her generally abstain completely from exerting any effort. If we plug (2.15) into (2.16) and define $\rho = x^{22v1}/(2x^{2v21})$, we can write

$$\frac{2}{((1+1/\rho)^{\alpha}(1+\lambda)/2)^{1/(1+\alpha)} + 2} = (2\rho)^{\alpha}. \tag{2.17}$$

Solving (2.17) numerically for ρ enables us to derive efforts and thus utilities. Denote the utility of m as u^{22v1}, the utility of the leaders of the two groups m joins as u^{2v21} and that of the group leader competing alone as u^{1v22}.

If m joins all three groups, she wins the contest with probability one. She will thus not exert any effort, and her utility is $u^a = V(1+\lambda)/2$. The subgame effectively becomes a contest between the three group leaders over a prize valued at $V(1+\lambda)/2$. In equilibrium, individual efforts of group leaders are $x^{2v22} = (V(1+\lambda)/9)^{1/(1+\alpha)}$. For the three group leaders, these result in the expected utilities

$$u^{2v22} = \frac{(1+\lambda)(1+3\alpha)}{18(1+\alpha)}V.$$

Stage 2

It can be shown numerically that the decision of the potential member in the second stage remains trivial: she will join as many groups as she can. A situation is now possible in which two group leaders have chosen O in stage 1 and one group leader has chosen N or E. Then, m will join the two groups which allow for non-exclusive membership. If no or only one leader chooses O in stage 1, m will again randomly pick one of the groups she is allowed to join.

Stage 1

For the first stage of the game with three groups, if we neglect which exact group leader offers which membership menu, there are still ten strategy profiles that are candidates to constitute a Nash equilibrium. I refer the interested reader to Appendix 2.D for a short case-by-case analysis and present the Nash equilibria that arise in dependence of the marginal elasticity of effort cost α and the publicness of the prize λ in Figure 4. The results are based on numerical simulations.

The introduction of a third group into the game lowers the λ-threshold above which all group leaders offer exclusive membership. Additionally, unless α is small, it lowers the λ-threshold below which groups remain fully exclusive. Interestingly, a third group allows for a Nash equilibrium in stage 1 in which all group leaders offer non-exclusive membership, resulting in m joining all three groups at once. Whereas in the two-group baseline, a group leader can deviate from (O, O) and play E to still have an even chance to have m only join her group, in the three-group setting, a deviation from (O, O, O) results in a leader competing on her own against two two-player groups.[14] For large α and λ, this is not an attractive outside option, making (O, O, O) a Nash equilibrium. Payoff dominance eliminates the equilibrium (O, O, O) in general, and the equilibria (E, E, E) and (E, E, O) if the equilibrium (N, N, N) exists.[15] In Appendix 2.D, I consider $k \geq 3$ groups and use numerical simulations to derive the existence of

[14] This mechanism is similar to the commitment by m to join non-exclusive groups discussed in Section 2.4.1, which also lead to the emergence of non-exclusive equilibria.

[15] I restrict the notation of asymmetric equilibria to one of their constellations in the three-group case for the sake of brevity.

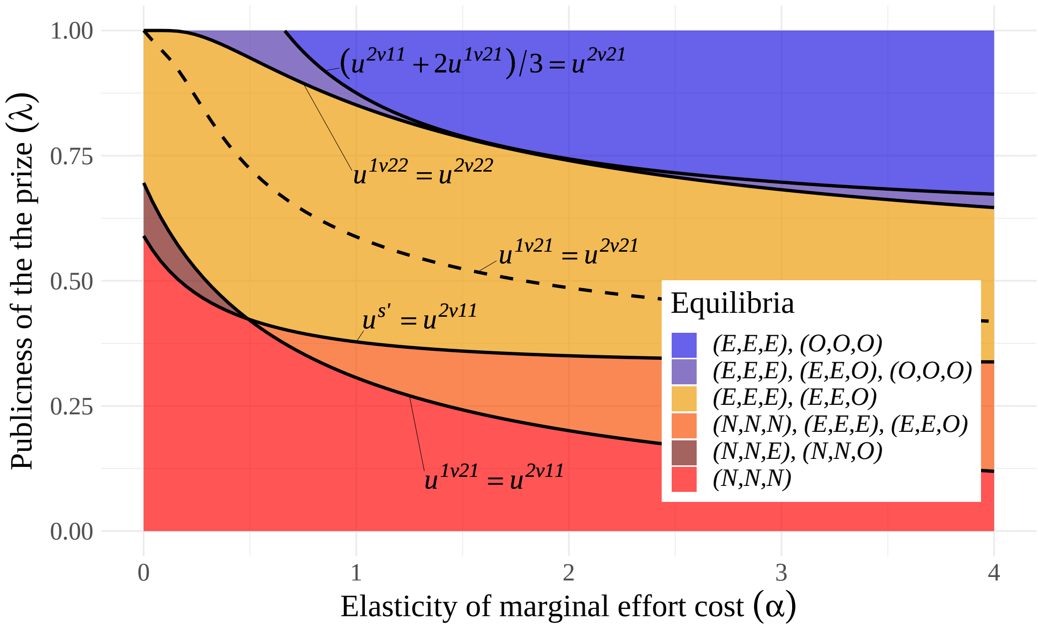

Figure 4: Nash equilibria in stage 1 with three groups

For the sake of brevity, the legend shows only one possible constellation of asymmetric equilibria. There are additional equilibria (N, E, E) *and* (N, E, O) *and all their other constellations on the line* $u^{2v11} = u^{1v21}$. *The lines* $u^{1v22} = u^{2v22}$ *and* $(u^{2v11} + 2u^{1v21})/3 = u^{2v21}$ *never intersect.*

symmetric stage-1 equilibria with five and ten groups to illustrate how the results from this section generalise. It appears that they do with two caveats: First, if α is very small, a higher number of groups appears to increase the λ above which all groups offer exclusive membership. Second, the region in which groups offer non-exclusive membership shrinks if additional groups are added.

2.5 Conclusion

In order to understand how groups in contests manage their membership exclusivity, I devise a stylised theoretical framework with two group leaders with access to a contest and one potential member without direct access. This model predicts that groups choose full exclusivity if the prize is mostly private—as we could expect to be the case in public tenders and explicit design contests—and exclusive membership otherwise—for example in lobbying or election campaigns, where a win can benefit a number of agents simultaneously. Moreover, groups tend to bar additional members if the elasticity of marginal effort cost is low; this might correspond to a setting in which effort represents monetary payments in a complete financial market. If the elasticity of marginal effort cost is high—as might be the case if effort represents invested time—groups tend to offer exclusive membership.

I present three extensions to the model in which non-exclusive membership emerges endogenously. The member may improve her utility by committing to join non-exclusive groups and forcing groups to offer non-exclusive instead of exclusive membership. This harms group leaders. If groups can charge a membership fee, they offer non-exclusive membership if singletons are more effective than groups and exclusive membership otherwise. Group leaders are better off with fees in the former case and worse off in the latter. For the potential member, this effect is reversed. The addition of a third group allows for an equilibrium in which groups offer non-exclusive membership—without commitment or fees.

A contest designer who wants to maximise effort, such as a government that aims to enhance competition in public tenders, would always like to prohibit non-exclusive membership. Moreover, such a contest designer wants to allow exclusive membership only if groups are more effective than singletons. Naturally, in settings in which the contest designer perceives contest effort as a loss, the opposite applies.

The complexity of endogenous, non-exclusive group membership in contests has imposed a number of restrictions. First, although I am interested in 'group' exclusivity, decisions about it are made by single group leaders. However, modelling a number of symmetric decision makers for each group would not yield much additional insight but add more complexity. Only heterogeneous group leaders would generate potentially novel results, but add even more complexity. Second, I assumed that the prize is shared equally within the winning group without intra-group conflict or sharing-rule endogeneity. Introducing intra-group conflict will most likely lead to fewer incentives for group leaders to offer membership. However, in settings like public tenders and design contests where a certain level of legal certainty can be assumed, this omission should not be too severe. It could be interesting to investigate whether competition among group leaders via contractual sharing rules leads to similar results as the competition via membership fees that I consider. Third, my model has rather tight restrictions on possible group structures: group leaders cannot join each other's groups and the potential member cannot found its own group with access to the contest. While these restrictions are informed by some of the described applications and are not straightforward to lift, it might be fruitful to further investigate this issue in the future. Fourth, my analysis of the first stage and most of the three-group extension is to a varying extent based on numerical solutions. This does not curb the validity of my results, but might make it difficult to generalise the framework or apply it to other settings. I leave it to future research to develop a more tractable model or further analyse the present one. Lastly, throughout my analysis I have only considered one potential member. However, I show in Appendix 2.E that up to five members who join groups in stage 2 sequentially and in a forward-looking manner do not substantially change the membership menu offered in my baseline model.

Appendices

2.A Appendix – Stage 1: membership menu

In order to know when (N, N) is a Nash equilibrium in pure strategies, we need to know when $u^s \geq u^{2v1}$. If $\lambda = 0$, we can compare (2.2) and (2.6) to show that $u^s > u^{2v1}$ by simplifying this condition to $1 + 2y + 2\alpha + 4\alpha y > 4y^2$ where $y = (1/4)^{1/(1+\alpha)} \leq 1$. So see this, note that $4y^2 \geq 1$ only if $\alpha \geq 1$ but then $4\alpha y \geq 4y^2$. If $\lambda = 1$, we can show that $u^s < u^{2v1}$ by simplifying to $1 + 2\alpha < 12y^2 + 8\alpha y^2$ where $y = (1/2)^{1/(1+\alpha)} \geq 1/2$. Moreover, it is straightforward to show that $du^s/d\lambda = 0$. Taking the total derivative of (2.6) with respect to λ gives us $du^{2v1}/d\lambda > 0$. Both expected-equilibrium-utility functions u^s and u^{2v1} are continuously differentiable functions on their domain $\alpha \geq 0$, $\lambda \in [0, 1]$, and $V > 0$. Define the implicit function $F(\alpha, \lambda) = 0$ by the equality $u^s = u^{2v1}$. By the Implicit Function Theorem, this assignment makes λ a continuous function of α. This means that there is a continuous boundary $\lambda(\alpha)$ implicitly defined by the equation $u^s = u^{2v1}$, separating the region where the equilibrium (N, N) exists and the region where it does not. As α tends to infinity, this boundary converges to $\lambda = 1/2$, since u^s converges to $V/2$ and u^{2v1} converges to $V(1 + \lambda)/3$.

The Nash equilibrium (E, E) exists if $u^{1v2} \leq u^{2v1}$. If we compare (2.6) with (2.7) at $\lambda = 0$, we can show that $u^{1v2} > u^{2v1}$. We do this by simplifying to

$$1 - y\left(\frac{1}{2} + 2y\right) + \alpha y(1 + 2y)\left(\frac{1}{y} - 1\right) > 0,$$

where $y = (1/4)^{1/(1+\alpha)}$, and noting that this is true for $\alpha \leq 1$ and that this is also true for $\alpha > 1$ since $\alpha(1/y - 1) > 1$. It follows that the equilibrium (E, E) does not exist if $\lambda = 0$. If $\lambda = 1$ however, $u^{1v2} < u^{2v1}$, a condition that simplifies to $1+\alpha < y+4y^2+4\alpha y^2$ where $y = (1/2)^{1/(1+\alpha)} \geq 1/2$. Further, since $d(u^{2v1}/u^{1v2})/d\lambda > 0$, there must be a continuous boundary $\lambda(\alpha)$ implicitly defined by $u^{1v2} = u^{2v1}$ between the region where the equilibrium (E, E) exists and the region where it does not. As α tends to infinity, this boundary converges to $\lambda = 0$, since u^{1v2} converges to $V/3$ and u^{2v1} converges to $V(1+\lambda)/3$.

The asymmetric Nash equilibria $\{(N, E), (E, N)\}$ exist above $u^s = u^{2v1}$ and below $u^{1v2} = u^{2v1}$. For $\{(N, O), (O, N)\}$ to constitute equilibria, we must additionally have that $u^{1v2} \geq u^{2v2}$. Below $u^{1v2} = u^{2v1}$ and above $u^s = u^{2v1}$ we have that $u^{1v2} \geq u^s$. Moreover, it is easy to show that $u^s \geq u^{2v2}\ \forall\ \alpha \geq 0, \lambda \in [0, 1]$. Therefore, the equilibria $(N, O), (O, N)$ also exist above $u^s = u^{2v1}$ and below $u^{1v2} = u^{2v1}$.

Finally, for (O, O) to be a Nash equilibrium in the first stage, we must have that $u^{2v2} \geq \max(u^{1v2}, (u^{2v1} + u^{1v2})/2)$. We can show that $u^{2v2} < (u^{2v1} + u^{1v2})/2\ \forall\ \alpha \geq 0, \lambda \in [0, 1]$: Assume the opposite, $u^{2v2} \geq (u^{2v1} + u^{1v2})/2$. Plugging in (2.6), (2.7), and (2.8) and simplifying yields

$$y\left(1 + 2y^{1/(1+\alpha)} - 4y^{2/(1+\alpha)}\right) + 2\alpha y + 4\alpha y^{(2+\alpha)/(1+\alpha)} > 1 + \alpha + 2\alpha y^{1/(1+\alpha)},$$

where $y = (1+\lambda)/4 \leq 1/2$. Since $2\alpha y \leq \alpha$ and $4\alpha y^{(2+\alpha)/(1+\alpha)} \leq 2\alpha y^{1/(1+\alpha)}$, we must have that $1 + 2y^{1/(1+\alpha)} - 4y^{2/(1+\alpha)} > 2$. This implies $2y^{1/(1+\alpha)}(1 - 2y^{1/(1+\alpha)}) > 1$ which is a contradiction since $x(1-x) < 1\ \forall\ x \in \mathbb{R}$. Hence, (O, O) can never be an equilibrium in the baseline game.

2.B Appendix – Membership commitment

To determine equilibrium existence under membership commitment, we need to additionally compare u^{2v2} to u^{1v2}. Only if the former is equal to or larger than the latter does the equilibrium (O, O) exist. If $\alpha = 0$ and $\lambda = 1$, it is easy to verify that $u^{1v2} = u^{2v2} = V/4$. It is straightforward to show that $u^{1v2} > u^{2v2}$ if $\lambda = 0$ and that $u^{1v2} \leq u^{2v2}$ if $\lambda = 1$. Moreover, if $\alpha = 0$ and $\lambda < 1$, we have that $u^{1v2} > u^{2v2}$. Since $d(u^{2v2}/u^{1v2})/d\lambda > 0$, there is a boundary $\lambda(\alpha)$, running through the point $\alpha = 0, \lambda = 1$, separating the region where (O, O) is a Nash equilibrium and the region where it is not. This boundary converges to $\lambda = 1/3$ as α tends to infinity since u^{1v2} converges to $V/3$ and u^{2v2} converges to $V(1+\lambda)/4$.

2.C Appendix – Membership fee

The stage-1 equilibrium candidates (N, O) and (O, N) require that $u^{2v2} + (u^b - u^{2v1}) \leq u^{1v2}$. Assume this holds. By plugging in our utility formulas and simplifying, we can show that this condition can only hold if $y(1 + 4y^{2/(1+\alpha)} + 8\alpha y^{2/(1+\alpha)}) < \alpha/2$ where $y = (1 + \lambda)/4 \geq 1/4$. Since $1 + 4y^{2/(1+\alpha)} \geq 5/4$ we must have that $\alpha > 5/8$. But then $8\alpha y^{(3+\alpha)/(1+\alpha)} > \alpha/3$ which further implies $\alpha > 30/16$ for $\alpha/6 > 5y/4$ to hold. But this in turn means $8\alpha y^{(3+\alpha)/(1+\alpha)} > \alpha/2$ which is a contradiction.

The Nash equilibrium (N, N) (with arbitrary membership fees) exists if $u^s \geq 2u^{2v1}$. It is clear that for this to hold, $u^s \geq u^{2v1}$ must hold, a condition we investigated in the baseline setting. Moreover, it is easy to show that if $\lambda = 0$ and α grows large, $u^s < 2u^{2v1}$. Since $d(2u^{2v1}/u^s)/d\lambda > 0$, $u^s = 2u^{2v1}$ implicitly defines a continuous boundary $\lambda(\alpha)$.

For $u^{1v2} \geq 2u^{2v1}$ to hold, it is clear that $u^{1v2} \geq u^{2v1}$ has to hold, a condition we looked into for the baseline model. It follows that $u^{1v2} \geq 2u^{2v1}$ does not hold if α grows large. Since $d(2u^{2v1}/u^{1v2})/d\lambda > 0$, $u^{1v2} = 2u^{2v1}$ implicitly defines a continuous boundary $\lambda(\alpha)$.

If $u^b > 2u^{2v1}$, for (O, O) to constitute a Nash equilibrium in stage 1 we first must have that $u^{2v2}+u^b \geq u^s$. It is straightforward to show that this always holds. Second, it must hold that $u^{2v2}+u^b/2 \geq 2u^{2v1}$. If $\alpha = 0$, this condition simplifies to $11 \geq 6\lambda+5\lambda^2$, which always holds and binds if additionally $\lambda = 1$. Since $d((u^{2v2}+u^b/2)/(2u^{2v1}))/d\lambda < 0$, $u^{2v2} + u^b/2 = u^s$ implicitly defines a continuous boundary $\lambda(\alpha)$ that intersects the α-axis at $\alpha = 1$.

Since $d(u^{1v2}/(u^{2v2}+u^b/2))/d\lambda < 0$, $u^{1v2} = u^{2v2}+u^b/2$ implicitly defines a continuous boundary $\lambda(\alpha)$.

2.D Appendix – Three groups (and more)

In stage 1, with three groups there are ten potential strategy profiles to constitute Nash equilibria (ignoring which exact group leader plays which strategy). For (N, N, N) to be an equilibrium, we must have that $u^{s'} \geq u^{2v11}$. (N, N, E) requires that $u^{s'} \leq u^{2v11}$ and $u^{1v21} \geq (u^{2v11} + u^{1v21})/2$ which simplifies to $u^{1v21} \geq u^{2v11}$.[16] (N, N, O) is an equilibrium strategy profile if additionally to the conditions for (N, N, E) the condition $u^{1v21} \geq u^{2v21}$ holds. For (N, E, E) to be a stage-1 equilibrium we need that $u^{1v21} \geq u^{2v11}$ and $u^{1v21} \leq u^{2v11}$, reducing to $u^{1v21} = u^{2v11}$. The fully asymmetric equilibrium (N, E, O) requires $u^{1v21} = u^{2v11}$, $u^{1v21} \geq u^{2v21}$, and $u^{2v21} \leq (u^{2v11} + u^{1v21})/2$. Given that the first condition holds, the second and the third conditions are equivalent. The third condition can be shown numerically to always hold. Thus (N, E, O) effectively

[16] As in the main body, I restrict the notation of asymmetric equilibria to one of their constellations in the three-group case for the sake of brevity.

requires the same as (N, E, E): $u^{1v21} = u^{2v11}$. For (N, O, O) to be an equilibrium, we need $u^{1v22} \geq u^{2v22}$, $u^{1v21} \leq u^{2v21}$, and $u^{2v21} \geq (u^{2v11} + u^{1v21})/2$. The last condition never holds and thus (N, O, O) can never be an equilibrium. For (E, E, E) we simply require $u^{1v21} \leq u^{2v11}$. (E, E, O) in addition requires $(u^{2v11} + 2u^{1v21})/3 \geq u^{2v21}$. For (E, O, O) to be an equilibrium, we must have that $u^{1v22} \geq u^{2v22}$, $u^{1v21} \leq u^{2v21}$, and $(u^{2v11} + 2u^{1v21})/3 \leq u^{2v21}$. Finally, (O, O, O) simply requires that $u^{1v22} \leq u^{2v22}$.

Now, consider our game with $k \geq 3$ groups and one member m. In order to know whether the three symmetric strategy profiles for stage 1, in which all group leaders play N, E, or O, respectively, constitute Nash equilibria, we need the group leaders' utilities in stage 3. Specifically, we need to look at the singleton case, the case in which m only joins one group, the case in which m joins all but one group, and the case in which m joins all groups.

In the singleton contest, individual efforts by group leaders are $x^{s''} = ((k-1)V/k^2)^{1/(1+\alpha))}$ which leads to expected utilities $u^{s''} = (1 + k\alpha)V/(k^2(1 + \alpha))$.

Suppose m joins one group. Denote her effort, as well as that of the group leader whose group she joins, as x_2. Here, symmetry is only an assumption if $\alpha = 0$ and otherwise follows from first order conditions. Denote the symmetric effort of group leaders who compete on their own as x_1. The two first order conditions are

$$\frac{(k-1)x_1}{(2x_2 + (k-1)x_1)^2} \frac{1+\lambda}{2} V \leq x_2^\alpha \tag{2.18}$$

and

$$\frac{2x_2 + (k-2)x_1}{(2x_2 + (k-1)x_1)^2} V = x_1^\alpha. \tag{2.19}$$

If $\alpha = 0$ and $(k-1)(1+\lambda)/(2(k-2)) \leq 1$, we have that $x_2 = 0$ and $x_1 = (k-2)/(k-1)^2 V$. Otherwise, we can divide (2.18) by (2.19) and define $\rho = x_2/x_1$ to get

$$\frac{k-1}{2\rho + k - 2} \frac{1+\lambda}{2} = \rho^\alpha.$$

We can solve numerically for ρ to calculate efforts and utilities.

If m joins all but one group and we assume m spreads her effort symmetrically across her groups, we can denote her effort as x_m, the effort of group leaders whose groups she joins as x_2, and the effort of the group leader who competes alone as x_1. The first order conditions are

$$\frac{x_1}{(x_m + (k-1)x_2 + x_1)^2} \frac{1+\lambda}{2} V \leq x_m^\alpha \tag{2.20}$$

for m,

$$\frac{(k-2)(x_m/(k-1)+x_2)+x_1}{(x_m+(k-1)x_2+x_1)^2}\frac{1+\lambda}{2}V \leq x_2^\alpha \tag{2.21}$$

for group leaders whose groups m joins, and

$$\frac{x_m+(k-1)x_2}{(x_m+(k-1)x_2+x_1)^2}V \leq x_1^\alpha \tag{2.22}$$

for the group leader who competes on her own. If $\alpha = 0$, we must have that $x_m = 0$ and can derive that $x_2 = (k-1)((1+\lambda)/(2k-1+\lambda))^2 V$ and $x_1 = (k-(k-2)\lambda)/(1+\lambda)x_2$, from which we can calculate utilities. Otherwise, we can divide (2.20) by (2.22) to write $\rho = x_m/x_2$ and get $x_1 = x_m(2/(1+\lambda)(1+(k-1)/\rho))^{1/(1+\alpha)}$. If we divide (2.20) by (2.21) and plug in our formula for x_1 we get

$$\frac{1}{(k-2)/(k-1)((1+(k-1)/\rho)^\alpha(1+\lambda)/2)^{1/(1+\alpha)}+1} = \rho^\alpha,$$

which we can solve numerically for ρ to get efforts and utilities.

Finally, if m joins all groups, she will not exert any effort and we can solve for group leaders' symmetric individual efforts as $x = ((k-1)(1+\lambda)V/(2k^2))^{1/(+\alpha)}$ and their resulting utilities as $u = (1+k\alpha)(1+\lambda)V/(2k^2(1+\alpha))$.

For simplicity, I make the reasonable assumption that m again joins as many groups as she can in stage 2. In stage 1, all group leaders playing N is a Nash equilibrium if their utilities in the singleton contest is higher than their utility if m only joins their group. All group leaders allowing exclusive membership E is an equilibrium if the utility of having m only joining their group is higher than competing alone against $(k-2)$ other singletons and one group of two. Lastly, all leaders offering non-exclusive group membership O is an equilibrium in stage 1 if the utility of having m join all groups simultaneously is higher than competing alone against $(k-1)$ groups which m all joins. Figure 2.D.1 illustrates symmetric stage-1 equilibria for five and ten groups, respectively. You can see that a higher number of groups, unless α is small, decreases the λ above which groups offer exclusive membership and that the region in which groups offer non-exclusive membership shrinks.

2.E Appendix – Additional potential members

Let there be $n \geq 1$ potential members. Groups cannot discriminate: if they allow exclusive or non-exclusive membership, all potential members can make use of it.

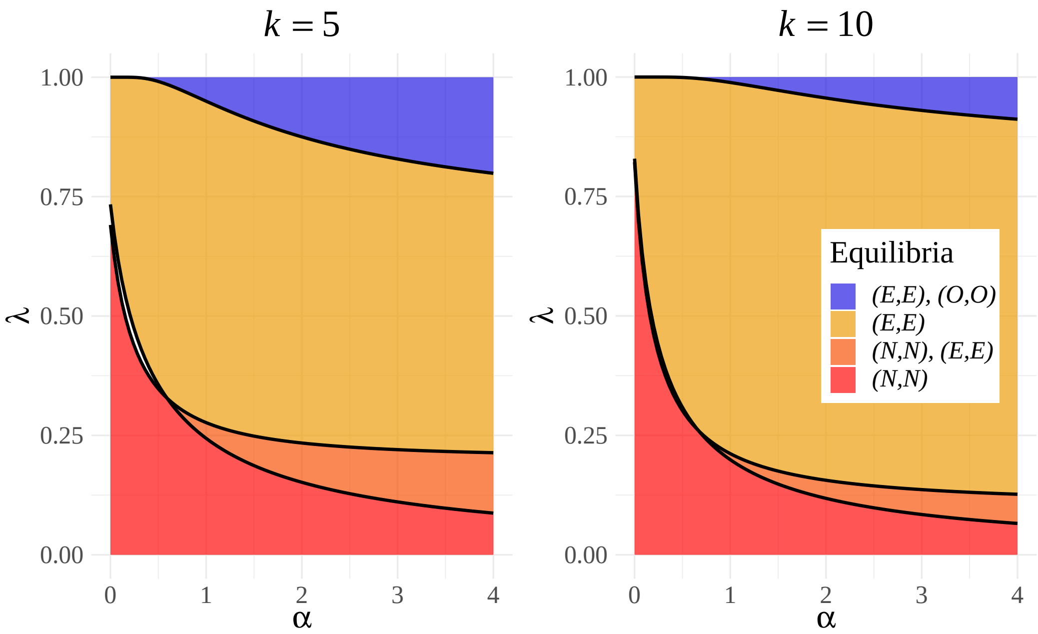

Figure 2.D.1: Symmetric Nash equilibria in stage 1 with five and ten groups

In the second stage, members are sorted randomly and sequentially decide which group(s) to join, taking into account how members that may come after them will decide. Membership decisions are final. Stage-3 outcomes change dependent on how many potential members play the the game. The strategy space in stage 1 remains the same. I apply sub-game-perfect Nash equilibrium as solution concept and solve the game for up to five potential members. If both groups offer non-exclusive group membership, all members join both groups. The members' choices in stage 2 for two, three, four, and five members in the case of both groups offering non-exclusive membership are depicted in Figure 2.E.1. The resulting stage-1 equilibria are shown in Figure 2.E.2. While I analytically derived utilities for all group structures in stage 3, stage-1 and stage-2 results are based on numerical simulations. Equilibria in stage 1 are not qualitatively changed by the introduction of additional potential members.

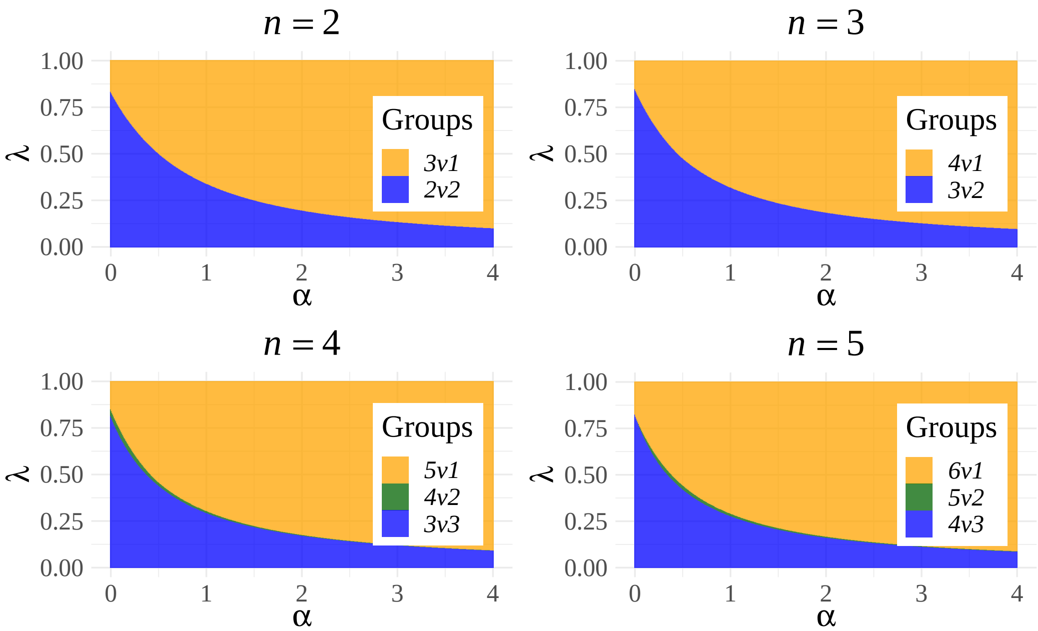

Figure 2.E.1: Members' stage-2 choices with two to five potential members in the case of both groups offering non-exclusive membership

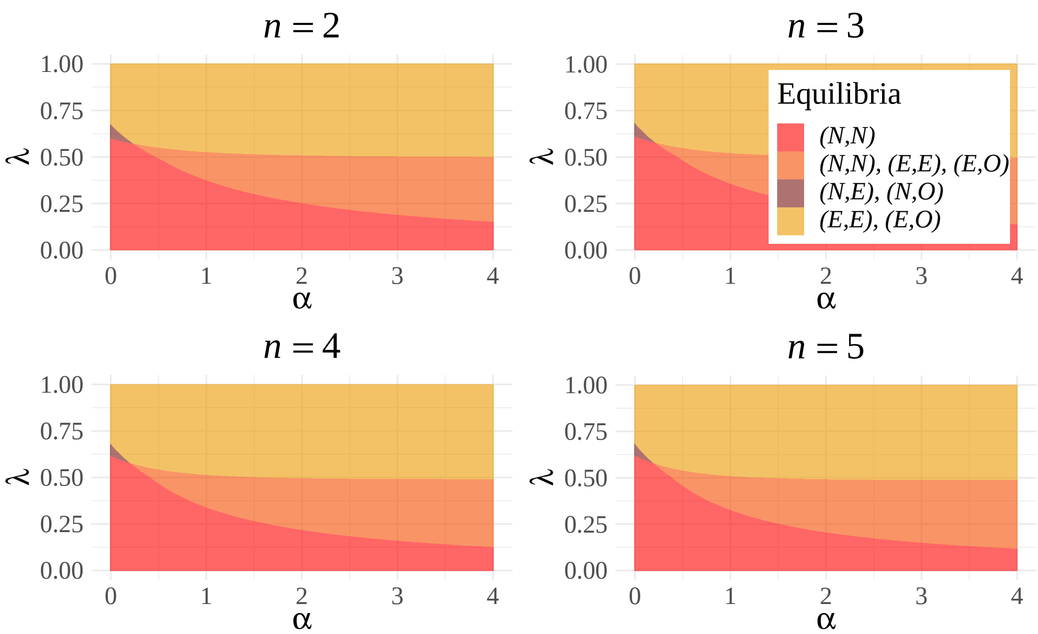

Figure 2.E.2: Nash equilibria in stage 1 with two to five potential members

For the sake of brevity, the legend shows only one possible constellation of asymmetric equilibria.

Chapter 3

Contest Copycats: Adversarial Duplication of Effort in Contests

3.1 Introduction

In innovation contests, participants may try to increase their chance of winning by spying on their opponents and copying their ideas. For example, the space race between the United States and the Soviet Union was accompanied by constant espionage, which went as far as "kidnapping" a Soviet lunar spacecraft from an exhibition for a thorough inspection by the CIA (Wesley, 1967). In the race to develop the first atomic bomb, the Soviet Union made significant efforts to gain access to the Manhattan Project (Haynes and Klehr, 2000). Attempts to steal other contestants' ideas have also been documented in Formula 1 (Solitander and Solitander, 2010). In 2004, back when it was a world leader in wireless technology, the Canadian company Nortel was hacked and had a large amount of its reports, design details, and top-secret source code stolen. While some have suspected an involvement of the Chinese government and the Chinese company Huawei, which subsequently gained a large share of the global wireless market, the hackers have never been identified (Pearson, 2020). In general, evidence on economic espionage is scarce since firms who are spied on often are either ignorant of their situation or reluctant to report on it. But the US Intellectual Property Commission (2017) estimates the annual cost of IP theft to the US economy to be between \$225 billion and \$600 billion.

To enhance our understanding of how spying on other participants' ideas affects contests, I analyse a stylised model of a Tullock contest between two players. Both players can exert effort—which I interpret as generating ideas—to increase their chance of winning the prize. I allow one player to be more productive in exerting contest effort.

Both players can pay a fixed cost for the ability to copy their opponent's effort and add it to theirs—representing stealing the other's ideas and combining it with one's own. Let me briefly illustrate this concept: Without copying, if player 1 and player 2 exert efforts x_1 and x_2, their respective probabilities of winning the contest are $x_1/(x_1 + x_2)$ and $x_2/(x_1 + x_2)$. If player 2 copies the effort that player 1 exerts, her effective effort becomes x_1+x_2, and the probability that player 2 wins the prize becomes $(x_1 + x_2)/(2x_1 + x_2)$, while player 1's win probability decreases to $x_1/(2x_1 + x_2)$. I characterise the unique Nash equilibrium in this game and show how copying behaviour depends on the cost of copying and the stronger player's productivity advantage.

If the cost of copying is low, the weaker player is more likely to win the prize in equilibrium. Both the more productive player's utility as well as the aggregate effort players exert can decrease in the stronger player's productivity advantage. This is in contrast to the baseline without copying. It implies that a government who wants to increase a domestic firm's profit may not want to subsidise this firm's effort, even if the subsidy were costless. It also means that a contest designer who would like to maximise aggregate effort in a contest may in some circumstances want to exclude a more productive contestant in favour of a weaker one. Moreover, I show that the expected winner's effort—potentially including effort copied from an opponent—is generally increasing in the cost of copying. The designer of an innovation contest would like to make copying of effort prohibitively costly.

In Section 3.2, I review the related literature. I introduce and motivate a two-player contest model with copying of effort in Section 3.3. I characterise the unique Nash equilibrium in Section 3.4 and discuss the model's comparative statics and its implications in Section 3.5. In Section 3.6, I briefly sum up my contribution and present promising future avenues of research.

3.2 Related literature

In my model, I interpret contest effort as generating ideas, and the sum of ideas in turn to determine the quality of a player's innovation. I assume a noisy contest, in which the player wins whose effort multiplied by an independent random variable is the highest. Following Hirshleifer and Riley (1992), this contest, including copying decisions, can be formulated as a standard lottery contest introduced by Tullock (1980). The resulting contest success function was axiomatised by Skaperdas (1996) and Clark and Riis (1998b). Baye and Hoppe (2003) discuss two other microfoundations of innovation contests from which the Tullock contest can be derived. They show that under certain conditions, there is a strategic equivalence between innovation tournaments in the sense of Fullerton and McAfee (1999), patent races in the tradition of Loury (1979), and the Tullock rent-seeking game. The copying mechanism I introduce is not directly compatible with the framework by Fullerton and McAfee (1999), but can be integrated in the model discussed by Loury (1979); see Section 3.3.

If players copy their opponents' effort, this introduces an additional effect of effort, which relates my work to that by Baye et al. (2012) on spillovers in contests, although they focus on a rank-order contest, the effort spillover does not enter the contest function, and the spillover is exogenously given. My paper has direct applications to the field of innovation contests, which is surveyed in Adamczyk et al. (2012).

Whereas I focus on the adversarial duplication, or copying, of another player's effort, others have analysed harmful behaviour such as cheating, doping, and sabotage in contests. Enhancing one's own performance is for example treated by Eber and Thépot (1999), Berentsen (2002), Haugen (2004), Konrad (2005), Kräkel (2007), and Gilpatric (2011). The concept of sabotage—reducing the other's effective effort—was introduced into contests by Konrad (2000) and subsequently analysed by Münster (2007a), Kräkel (2005), Gürtler (2008), and Gürtler and Münster (2010). Related to the notion of sabotage in contests is the work on negative campaigning by Skaperdas and Grofman (1995) and Chaturvedi (2005). Baumol (1992) considers sabotage in innovation processes. An earlier consideration of sabotage in a competitive structure can be found in Lazear (1989). For an overview of the literature on sabotage in contests, see Chowdhury and Gürtler (2015).[1] Spying on another player's ability in contests with uncertainty about the latter has been studied by Baik and Shogren (1995b), though issues with inconsistent beliefs in their work have been pointed out by Bolle (1996). Chen (2019) analyses a setting in which contestants can spy on their opponent's valuation of the prize. In my model, players do not spy on contest productivity or private prize valuation, but rather copy their opponent's exerted effort.

Since the main field of application of this paper is innovation contests, my work is related to innovation tournaments investigated by Taylor (1995), Fullerton and McAfee (1999), and Che and Gale (2003), and patent races in the sense of Loury (1979), Dasgupta and Stiglitz (1980), and Lee and Wilde (1980). A player copying her opponent's effort can be interpreted as a form of spillover or imitation. Seminal

[1]Note that copying of effort is strategically equivalent to "full sabotage"—reducing the opponent's effort to zero—if the contest success function is of the "all-pay" or "difference" form. For the Tullock contest considered here, the effects differ, however.

contributions on spillovers in innovation races are Spence et al. (1984) and d'Aspremont and Jacquemin (1988). There is a large literature on imitation in innovation races and its impact on economic growth and consumer welfare, with influential contributions by Scherer (1967), Reinganum (1982), Katz and Shapiro (1987), Grossman and Helpman (1991), Segerstrom (1991), Helpman (1993), and Aghion et al. (2001). Gallini (1992) considers costly innovation in the form of "innovating around". The copying of effort that I consider is a more active mechanism than passive spillovers, and more immediate and more immediately targeted against another player than imitation. Moreover, in my model players can copy and exert effort themselves simultaneously.

The act of copying of another player's contest effort which I consider is a form of espionage. Cozzi (2001) and Cozzi and Spinesi (2006) conceptualise espionage in patent races as stealing an innovation from an innovator on their way to the patent office. They focus on the implications of this spying on economic growth. Whereas I consider the direct duplication of an opponent's ideas, spies in their model have the opportunity to potentially steal all the ideas which are produced in a given economy. Moreover, I focus on effects on player's utilities and aggregate innovative effort, and on the effects of copying costs and productivity asymmetries. In a related paper, Grossman (2005) analyses the creation, protection, and pirating of ideas and focusses on the choice of players to either be an inventor or a "pirate".

A small game-theoretical literature on espionage models the ability to spy on other players' strategies (Matsui, 1989; Solan and Yariv, 2004; Alon et al., 2013; Barrachina et al., 2014), characteristics (Ho, 2008; Wang, 2020; Barrachina et al., 2021), or private signals (Kozlovskaya, 2018; Pavan and Tirole). In the literature on espionage in oligopolistic competition, there are some exceptions, in which players can duplicate their opponents' technologies. In the work by Whitney and Gaisford (1996, 1999), firms (or their governments) can steal their opponent's production technology before entering a Cournot competition. Chen et al. (2016) builds on this and endogenises additional innovation. Similarly, in Billand et al. (2010), firms can spy on other competitors to improve their product before competing in oligopolistic markets, and Marjit and Yang (2015) model imitators who can steal the production technology of an innovator in a duopoly model with binary choices. Grabiszewski and Minor (2019) analyse a game in which a foreign firm can duplicate the effort a domestic firm exerts to innovate. Unfortunately, their model is not analytically tractable and leads to multiplicity of equilibria. In contrast, I investigate a contest in which both players can exert effort and copy their opponent's effort and derive closed-form solutions.

3.3 The model

Two risk neutral players, $i \in \{1, 2\}$ engage in a costly contest to win a prize of value $V > 0$. This prize may represent the value of being the first to patent a new invention, winning a research competition, or putting the first man on the moon. Both players simultaneously choose effort $x_i \geq 0$, with effort cost defined as $(1 - \alpha_i)x_i$. Player 1 might have a productivity advantage: $\alpha_1 = \alpha \in [0, 1)$ and $\alpha_2 = 0$. Effort costs are common knowledge. I interpret effort as directly translating into ideas whose sum determine the quality of a player's innovation; the more effort a player exerts, the more ideas she generates, and the higher is the quality of her innovation.

In addition, players have the option to pay the fixed cost β to copy their opponent's effort and add it to theirs. This cost represents, for example, required expenditures—such as hiring hackers and spies—to be able to copy an opponent's ideas, or expected future costs, such as potential legal fees and penalties.[2] Copying decisions are made simultaneously and at the same time as efforts are chosen.[3] Denote a player's choice to copy by $c_i \in \{0, 1\}$, where $c_i = 1$ means player i copies and $c_i = 0$ means she refrains from copying. Denote a player's effective effort as $y_i = x_i + c_i x_j, i \neq j$. Additionally to exerting effort and thus generating ideas, a player can also copy their opponent's ideas and add it to their own.

That the copying of an opponent's effort is successful with certainty is a simplifying assumption. While copying might be highly successful in contests with low protection barriers, such as innovation contests within a specific company, this might not be the case in other settings: attempts to hack into a competitor's IT system might be thwarted, and spies might be captured. Unfortunately, modelling copying success as uncertain in the presented framework would not allow me to solve for all equilibrium candidates. However, we can expect the model with certain copying success to be a good approximation for settings in which the probability of copying success is high. Moreover, in settings in which this probability is low, copying is not a very viable option unless its cost is sufficiently low too, making it a less relevant aspect of the contest anyway. I thus argue that certain copying success is not only a necessary

[2]See Crane (2005) for examples of such legal fees and penalties.

[3]This corresponds to effort and copying decisions being made sequentially if the effort decision by the other player is unknown at the time of the copying decision. This is intuitive: the level of the effort is unknown to the opposing player before it is copied. Another form of sequentiality is copying decisions being made before efforts are chosen. Again, this is congruent with the simultaneous model if players are uninformed about the other player's choices. This reflects a reality in which espionage is often hard to detect.

assumption, but also a reasonable one.

Let player $i \neq j$ win the prize if $\theta_i y_i > \theta_j y_j$, where θ_i and θ_j are independent draws from an exponential distribution $F(\theta) = 1 - e^{-\lambda\theta}$ with $\lambda > 0$. The more ideas a player has, the higher is the quality of her innovation, captured by her effective effort y_i. However, there is randomness involved in the contest: The submissions in a research competition might be hard to evaluate. In the race to the moon, the more innovative contestant might face random setbacks such as accidents and illnesses. Following Hirshleifer and Riley (1992), it is straightforward to show that player i's probability to win this contest can be formulated as

$$p_i = \frac{y_i}{y_i + y_j}, i \neq j, \tag{3.1}$$

which corresponds to the standard Tullock contest success function; for a brief derivation, see Appendix 3.A.

There are other ways to derive a Tullock contest from the microfoundations of an innovation contest. Namely, Baye and Hoppe (2003) show that under certain conditions, there is a strategic equivalence between innovation tournaments in the sense of Fullerton and McAfee (1999), patent races in the tradition of Loury (1979), and the Tullock rent-seeking game. However, in the case of Fullerton and McAfee (1999), this equivalence is not robust to the introduction of copying in the form that I use, since they consider a model in which a single drawn idea wins the game.[4] In the framework of Loury (1979), effort yields a hazard rate at which an innovation arrives. If we allow effort to be copied in order to simply increase one's own hazard rate in this framework, and if we assume a discount rate of zero, this game is strategically equivalent to the model I present here.

I make the usual assumption that $p_1 = p_2 = 1/2$ if $y_1 = y_2 = 0$. One way to motivate it is to interpret the status quo as a split prize. For example, if no competitor innovates, firms continue to share the market. If no country makes it to the moon, no country gains, but at the same time, no country loses.[5] Related is the fact that in

[4]In Fullerton and McAfee (1999), effort translates into random draws of ideas and the player with the best idea wins. In this framework, if we assume that a winner is picked at random if both players have access to the (same) best idea, the success function with copying becomes $p_i^{alt} = \frac{(1+c_i)x_i + c_i(1+c_j)x_j}{(1+c_i)(1+c_j)(x_i+x_j)}, i \neq j$. However, many innovations or achievements are a combination of a number of ideas rather than a single one, especially the most significant ones. The first iPhone combined innovations in display technology, user interface, design, and many more. The moon landing required, among others, innovations in rocket and computer technology and the development of new fabrics and other materials. The model reflects this interpretation.

[5]In the model, the issue of zero total effort only arises in equilibrium with symmetric players. In

the model, the level of effort does not play a role in the sense that the contest success function (3.1) is homogeneous of degree zero in effective efforts. In a game like the race to the moon, where a higher overall level of innovative effort can be expected to lead to a player winning the game earlier, this implies a discount rate of zero (which is the assumption required for the models' equivalence with the Loury (1979) framework).

If player i copies the effort x_j of player $j \neq i$, player i's effective effort becomes $y_i = x_i + x_j$. This assumes that efforts are additive. If we interpret effort as translating into innovative ideas, this means that these ideas are original. However, simultaneous research effort may sometimes lead to redundant innovations.[6] If a player copies an idea that she already has, this should arguably not increase her chance of winning as much as gaining access to an original idea. To test the importance of the assumption of additive efforts, I also analyse the model with redundant effort, where formally effective effort is $y_i = \max(x_i, c_i x_j), i \neq j$, and show that my findings generalise; see Appendix 3.C. Ideas and innovations (and the ways by which they are generated) are rarely exactly the same. For instance, the different Covid-19 vaccines that have recently been developed most likely have different strengths and weaknesses, and may even be more effective when used in combination. It is highly unlikely that two active contestants in an innovation contest can learn nothing from each other. The assumption of additive efforts is thus both intuitive, as well as uncritical to my findings.

Given the players' effort and copying decisions, a player's expected utility is

$$u_i(x_i, x_j, c_i, c_j) = \frac{x_i + c_i x_j}{(1 + c_j)x_i + (1 + c_i)x_j} V - (1 - \alpha_i)x_i - \beta c_i, \tag{3.2}$$

where $i \neq j$. Denote by $q_i = P(c_i = 1) \in [0, 1]$ the probability with which player i decides to copy. Additionally, denote by $x_{i|c_i=0} = x_i^n$ the effort a player chooses if she does not copy and by $x_{i|c_i=1} = x_i^c$ the potentially different effort she chooses if she does. If a player chooses not to copy, her first order condition with respect to exerted

reality, productivity of players is often heterogeneous. Furthermore, in some contests, such as for example Kaggle machine-learning challenges, there is often a benchmark publicly available whose submission might represent exerting no effort, but cannot easily be ruled out as a winning approach by a contest designer. For an example of explicit modelling of draws, see Blavatskyy (2010).

[6] Famous historical examples include the independent formulation of calculus by Newton and Leibniz, the development of the theory of natural selection by Darwin and Wallace, and the invention of the telephone by Gray and Bell. See Ogburn and Thomas (1922) for an early and fascinating account of almost 150 multiple inventions and discoveries. It is noteworthy that Gray and Bell ended up in a controversy over who had been first to invent the telephone, a controversy which did not remain free of accusations of copying the other's ideas.

effort x_i^n is

$$\frac{\partial u_i}{\partial x_i^n} = \left((1-q_j)\frac{x_j^n}{(x_i^n + x_j^n)^2} + q_j \frac{x_j^c}{(2x_i^n + x_j^c)^2} \right) V - (1-\alpha_i) \overset{!}{\leq} 0, \tag{3.3}$$

where $i \neq j$. If player i chooses to copy, her first order condition with respect to exerted effort x_i^c is

$$\frac{\partial u_i}{\partial x_i^c} = (1-q_j)\frac{x_j^n}{(x_i^c + 2x_j^n)^2} V - (1-\alpha_i) \overset{!}{\leq} 0. \tag{3.4}$$

If both players copy, effort is wasted and thus has no marginal benefit. If player j always copies, $q_j = 1$, player i optimally exerts zero effort when copying. Lastly, if player i mixes, $0 < q_i < 1$, it must hold in equilibrium that she is indifferent between not copying and copying:

$$\begin{aligned} \left((1-q_j)\frac{x_i^n}{x_i^n + x_j^n} + q_j \frac{x_i^n}{2x_i^n + x_j^c} \right) V - (1-\alpha)x_i^n \overset{!}{=} \\ \left((1-q_j)\frac{x_i^c + x_j^n}{x_i^c + 2x_j^n} + q_j \frac{1}{2} \right) V - (1-\alpha)x_i^c - \beta. \end{aligned} \tag{3.5}$$

It cannot be an equilibrium to have both players play a pure strategy and always copy, i.e. $q_1 = q_2 = 1$. This would imply zero efforts and $u_1 = u_2 = V/2 - \beta$. It is clear that a player would have an incentive to deviate and not copy and receive $V/2$ for sure. This means that at least one player at least sometimes does not copy in any equilibrium. Moreover, it cannot be an equilibrium for a player to exert zero effort in expectation. To see this, assume $x_i^n = x_i^c = 0$ in equilibrium. If $q_i < 1$, player $j \neq i$ optimally responds by not copying, $c_j = 0$, and exerting infinitesimal effort $x_j = \epsilon$ where $\epsilon \to 0$ cannot be pinned down.[7] If $q_i = 1$, player j's best response is then to not copy $c_j = 0$ and exert zero effort $x_j = 0$. But then, i would have an incentive to deviate to $q_i = 0$ and $x_i = \epsilon$ with $\epsilon \to 0$. This means that in any equilibrium, both players must exert strictly positive effort in expectation. The second order conditions of equation (3.3) and, if $q_j < 1$, equation (3.4) hold for any non-zero effort of the opposing player j and on the whole domain $x_i \geq 0$. Hence, players' best reply functions with respect to effort are single peaked given a strictly positive expected effort by their opponent.[8]

[7] Even if we could fix a very small ϵ, player i would then have an incentive to deviate and exert strictly positive effort.

[8] This also holds if the opponent were to play a mixed strategy with respect to effort, given her copying decision. It follows that players, given their copying decision, do not play a mixed strategy with respect to the effort they exert in equilibrium.

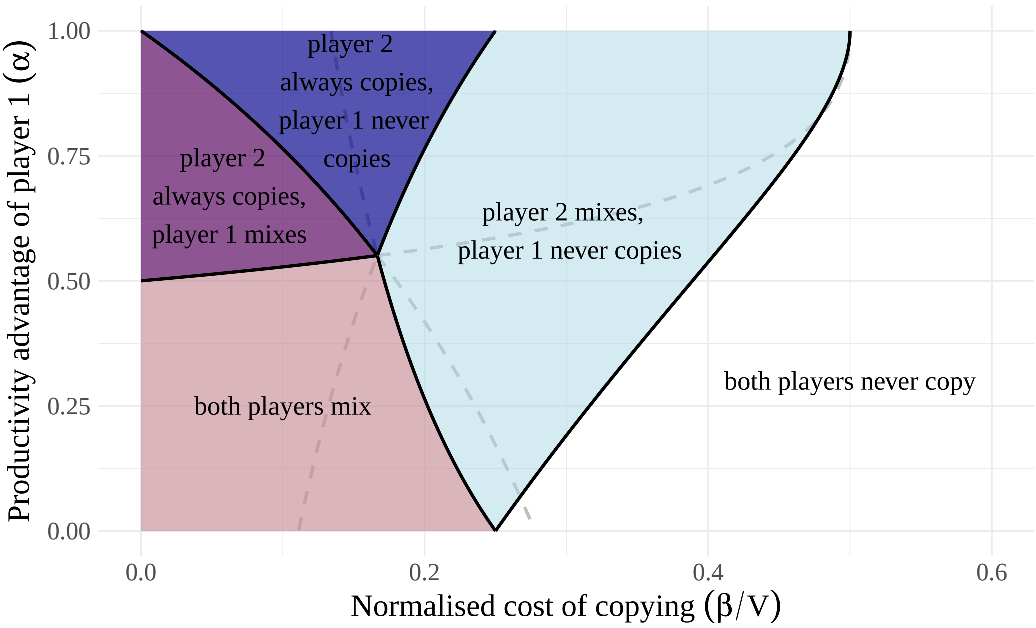

Figure 1: Nash equilibria in dependence of the normalised cost of copying β/V and player 1's productivity advantage α

In the following section, I derive the unique solution to equations (3.3) to (3.5) for all combinations of productivity advantage $\alpha \in [0, 1)$ and copying cost $\beta > 0$, which pins down the unique Nash equilibrium.

3.4 Equilibrium

Both players can never copy, mix between copying and not copying, or always copy. As stated above, it cannot be an equilibrium for both players to always copy. Moreover, I show in Appendix 3.B that it cannot be an equilibrium for player 1 to copy with a higher probability in equilibrium than player 2. This leaves us with five combinations as equilibrium candidates: both players never copy, player 1 never copies and player 2 mixes, player 1 never copies and player 2 always copies, player 1 mixes and player 2 always copies, and both players mix.

Figure 1 anticipates the results of this section and illustrates in which parametric regions the five equilibrium candidates exist. In what follows, I characterise these equilibria and derive the boundaries which separate them. I briefly sum up the results in Section 3.4.6 and formulate the first proposition.

3.4.1 Both players never copy

Start by assuming that both payers never copy, $c_1 = c_2 = 0$. This case corresponds to the standard model without copying. The first order condition (3.3) reduces to

$$\frac{\partial u_i}{\partial x_i} = \frac{x_j}{(x_i + x_j)^2} V - (1 - \alpha_i) \overset{!}{\leq} 0.$$

We can solve this for equilibrium efforts $x_1^* = V/(2-\alpha)^2$ and $x_2^* = V(1-\alpha)/(2-\alpha)^2$ which yield equilibrium utilities $u_1^* = V/(2-\alpha)^2$ and $u_2^* = V((1-\alpha)/(2-\alpha))^2$. Intuitively, as player 1's productivity advantage α increases, her utility increases as well, while the utility of player 2 decreases.

This equilibrium exists if no player has an incentive to deviate by copying the effort of the other player. If player 2 deviates and copies, $c_2 = 1$, her best-response effort is $x_2^c = 0$. This gives her the utility $V/2 - \beta$. Thus, player 2 has an incentive to copy if player 1 never copies if

$$\frac{V}{2} - \beta > V \left(\frac{1-\alpha}{2-\alpha} \right)^2. \tag{3.6}$$

If player 1 deviates and copies, her best-response effort is $x_1^c = \alpha V/(2-\alpha)^2$, yielding the utility $V(1 + (1-\alpha)^2)/(2-\alpha)^2 - \beta$. This means that player 1 has an incentive to copy given that player 2 never copies if

$$\frac{1 + (1-\alpha)^2}{(2-\alpha)^2} V - \beta > \frac{V}{(2-\alpha)^2}. \tag{3.7}$$

Since $((1-\alpha)/(2-\alpha))^2 \leq 1/4$, condition (3.6) always holds if condition (3.7) holds. Thus, both players never copying is a Nash equilibrium if

$$\alpha \leq \frac{1 - 2\sqrt{1/2 - \beta/V}}{1 - \sqrt{1/2 - \beta/V}}, \tag{3.8}$$

which follows from (3.6). The term $1/2 - \beta/V$ is the utility a player receives if she copies and does not exert effort, normalised by the value of the prize.[9] You can see in Figure 1 that this holds if the cost of copying, normalised by the value of the prize, β/V, is sufficiently large. If player 1's productivity advantage α is high, player 2 is willing to pay a higher price for the ability to copy.

[9] The term $\sqrt{1/2 - \beta/V}$ is only well defined if $\beta/V \leq 1/2$. If $\beta/V > 1/2$, copying of effort is never optimal for any player.

3.4.2 Player 1 never copies, player 2 mixes

Consider an equilibrium in which player 1 never copies, $q_1 = 0$ and thus $x_1 = x_1^n$, and player 2 plays a mixed strategy, $q_2 \in (0,1)$. This means that player 2 copies the effort of player 1 with probability q_2 and refrains from doing so with probability $(1-q_2)$. For player 2, condition (3.3) reduces to

$$\frac{\partial u_2}{\partial x_2^n} = \frac{x_1}{(x_1 + x_2^n)^2} V - 1 \overset{!}{\leq} 0,$$

and condition (3.4) becomes

$$\frac{\partial u_2}{\partial x_2^c} = \frac{x_1}{(2x_1 + x_2^c)^2} V - 1 \overset{!}{\leq} 0. \tag{3.9}$$

If $x_1 > x_2^n$, we must have that $x_2^c = 0$, and $x_2^c = x_2^n - x_1$ otherwise. Since the marginal cost of effort is constant, a player would like to have the same equal marginal benefit of effort, regardless whether she copies or not.

First, assume $x_1 > x_2^n$. Then, condition (3.9) cannot bind and in equilibrium we must have $x_2^{c*} = 0$. We can solve for the probability that player 2 copies and get $q_2^* = 1 - (1-\alpha)(1/\sqrt{1/2 - \beta/V} - 1) < 1$. Equilibrium efforts are $x_1^* = (1 - \sqrt{1/2 - \beta/V})^2 V$ and $x_2^{n*} = (\sqrt{1/2 - \beta/V} - (1/2 - \beta/V))V$. This equilibrium can only exist if $\beta/V < 1/2$. Additionally, we must have that $q_2^* > 0$. This is the case if $\alpha > (1 - 2\sqrt{1/2 - \beta/V})/(1 - \sqrt{1/2 - \beta/V})$, which corresponds to (3.8). Further, $x_1 > x_2^n$ is true if $\beta/V > 1/4$. Player 1 never has an incentive to deviate and copy if these conditions hold.[10]

Now, assume $x_1 \leq x_2^n$. First order conditions in this case yield equilibrium efforts $x_1^* = \beta$, $x_2^{n*} = \sqrt{\beta V} - \beta$, and $x_2^{c*} = \sqrt{\beta V} - 2\beta$. Win probabilities are independent of whether or not player 2 copies, and player 2 is (weakly) more likely to win. Player 2 copies with probability $q_2^* = 1/\sqrt{\beta/V} + \alpha - 2 \geq 0$ in this equilibrium. For $x_1^* \leq x_2^{n*}$ to hold, we must have that $\beta/V \leq 1/4$. For $q_2^* < 1$ to hold, we must have that

$$\alpha < 3 - \frac{1}{\sqrt{\beta/V}}. \tag{3.10}$$

Again, an increase in player 1's productivity advantage α increases player 2's inclina-

[10] Note that 1/4 is the effort players exert if they are symmetric and do not copy, normalised by the value of the prize. If players are symmetric, they start to copy if the normalised cost of copying β/V falls below this threshold. See Figure 1.

tion to copy. Player 1 does not have an incentive to deviate and copy if

$$\alpha \geq \frac{1/2 - \sqrt{\beta/V}}{\beta/V}. \tag{3.11}$$

Intuitively, an increase in her productivity advantage makes player 1 less inclined to copy. The light blue region in Figure 1 depicts the combinations of the normalised cost of copying β/V and player 1's productivity advantage α that make player 1 never copy and player 2 mix between copying and not copying in equilibrium.

3.4.3 Player 1 never copies, player 2 always copies

Consider an equilibrium in which player 1 never copies, $q_1 = 0$ and $x_1 = x_1^n$, and player 2 always copies, $q_2 = 1$ and $x_2 = x_2^c$. Efforts are then $x_1^* = V/(3-\alpha)^2$ and $x_2^* = (1-\alpha)V/(3-\alpha)^2$. Player 1 has no incentive to deviate and copy if her productivity advantage is large enough:

$$\alpha \geq 3 - \sqrt{\frac{2}{1/2 - \beta/V}}. \tag{3.12}$$

Player 2 does not have an incentive to deviate and not copy if $\alpha \geq 3 - 1/\sqrt{\beta/V}$, which corresponds to (3.10). The region on the β/V-α-plane where this Nash equilibrium in asymmetric pure strategies exists is shown in dark blue in Figure 1.

3.4.4 Player 1 mixes, player 2 always copies

Assume that in equilibrium, player 1 mixes between copying and not copying, $q_1 \in (0,1)$, and player 2 always copies, $q_2 = 1$ and $x_2 = x_2^c$. If player 1 copies, she has no incentive to exert effort since she wins the price with probability 1/2 in any case, $x_1^{c*} = 0$. We can solve for the probability that player 1 copies as $q_1^* = 1 - (\sqrt{2/(1/2 - \beta/V)} - 2)/(1-\alpha) < 1$ and for her equilibrium effort if she does not copy as $x_1^{n*} = (1 - q_1^*)/(2 + (1-\alpha)(1-q_1^*))^2$. Moreover, $x_2^* = (1-\alpha)(1-q_1^*)x_1^{n*}$. We have that $q_1^* > 0$ if $\alpha < 3 - \sqrt{2/(1/2 - \beta/V)}$, which corresponds to (3.12). Further, player 2 has no incentive to deviate and not copy if her opponent's productivity advantage is large enough:

$$\alpha \geq \frac{1/2 - \sqrt{(1/2 - \beta/V)/2}}{\beta/V}. \tag{3.13}$$

These conditions hold if the normalised cost of copying β/V is low and player 1's productivity advantage α is high, but not too high, as illustrated by the violet region in Figure 1.

3.4.5 Both players mix

Lastly, consider an equilibrium in which both players play a mixed strategy, $q_i \in (0,1)\ \forall\ i \in \{1,2\}$. We can solve for equilibrium efforts as $x_1^{n*} = \beta$, $x_1^{c*} = 0$, $x_2^{n*} = \beta(1/2 + \alpha\beta/V)/(1/2 - \alpha\beta/V) \geq x_1^{n*}$, and $x_2^{c*} = \beta(2\alpha\beta/V)/(1/2 - \alpha\beta/V) = x_2^{n*} - x_1^{n*}$. The equilibrium probabilities with which players copy are given by $q_1^* = 1 - (\beta/V)/(1/2 - \alpha\beta/V)^2 < 1$ and $q_2^* = (1/2 - \beta/V)/(1/2 - \alpha\beta/V)^2 - 1$. Note that $q_1^* \leq q_2^*$. We have that $q_1^* > 0$ if $\alpha < (1/2 - \sqrt{\beta/V})/(\beta/V)$, which corresponds to (3.11). Further, $q_2^* < 1$ if $\alpha < (1/2 - \sqrt{(1/2 - \beta/V)/2})/(\beta/V)$, which corresponds to (3.13). Both players mixing between copying and not copying is an equilibrium if the normalised cost of copying β/V and player 1's productivity advantage α are both sufficiently low. This region is depicted in orange in Figure 1.

3.4.6 Summary

As is evident in Figure 1, the five described equilibria are mutually exclusive and cover the entire area where the game is defined. This means that for player 1's productivity advantage $\alpha \in [0,1)$ and the normalised cost of copying $\beta/V > 0$, the Nash equilibrium of the contest with copying exists and is unique. Moreover, the transition between the different types of equilibria is "smooth": for example, at the switch from player 2 mixing to copying, her probability of copying converges to zero as α approaches the threshold characterised by (3.8). This means that there are no discontinuous jumps in the decision and outcome variables. Some intuitive patterns emerge: In general, players copy more often if the cost of copying, normalised by the value of the prize, β/V is lower. Additionally, players copy more often if the relative productivity with which they exert effort declines.

Having solved for the equilibrium for all combinations of normalised copying cost β/V and productivity advantage α of player 1, we can formulate the first proposition. The expected probability of winning the contest for player 1 in equilibrium is $p_1^* \in [0,1]$. We have:

Proposition 1 (**Probability of winning**)**.** *If $\beta/V < 1/4$ and $\alpha > 0$, $p_1^* < 1/2$. If the normalised cost of copying is sufficiently low, player 1 is less likely to win the contest in equilibrium if she has a productivity advantage. If $\beta/V > 1/4$ and $\alpha > 0$, $p_1^* > 1/2$. Naturally, $p_1^* = p_2^* = 1/2$ if $\alpha = 0$.*

Proof. If $\beta/V > 1/4$, both players do not copy if condition (3.8) holds. The probability that player 1 wins in this baseline case is $x_1^*/(x_1^* + x_2^*) = 1/(2-\alpha)$. It is clear that $p_1^* > 1/2$ if $\alpha > 0$. If condition (3.8) does not hold, player 1 does not copy, and player 2 mixes between copying and not copying in equilibrium. The probability that player 1 wins the contest is then

$$p_1^* = (1 - q_2^*)\frac{x_1^*}{x_1^* + x_2^{n*}} + q_2^* \frac{1}{2}.$$

Since $x_1^*/(x_1^* + x_2^{n*}) = 1 - \sqrt{1/2 - \beta/V} > 1/2$ if $\beta/V > 1/4$, we have that $p_1^* > 1/2$.

If $\beta/V < 1/4$, we know from Figure 1 that there are four cases to consider. If player 1 never copies and player 2 mixes, $p_1^* = \sqrt{\beta/V} < 1/2$. If player 1 never copies and player 2 always copies, we have $p_1^* = 1/(3-\alpha) < 1/2$. If player 1 mixes and player 2 always copies, $p_1^* = (1-q_1^*)\sqrt{(1/2 - \beta/V)/2} + q_1^*/2 < 1/2$. Finally, if both players mix between copying and not copying, the probability that player 1 wins the contest is given by $p_1^* = 1/2 - \alpha(\beta/V)^2/(1/2 - \beta/V)^2$. If $\alpha > 0$, $p_1^* < 1/2$. □

Intuition. In sharp contrast to the baseline contest model without copying, a player with a productivity advantage is less likely to win the contest if the cost of copying is low. If player 2 copies, her chance of winning is at least 1/2 since her effort when she copies is positive, $x_2^c \geq 0$. Copying by player 2 reduces the incentive for player 1 to exert effort. This in turn can be exploited by player 2 when she does not copy, in which case she exerts more effort than player 1, $x_2^n > x_1^n$ if $\beta/V < 1/4$ and $\alpha > 0$, giving her a higher probability of winning than player 1. Despite this, player 1 does not have an incentive to copy more often than she does, since effort is not very costly to her relative to copying due to her productivity advantage.[11] If the cost of copying is high, i.e. $\beta/V > 1/4$, player 2 does not have an incentive to copy often enough to cause the same effect and player 1 is more likely to win. Note that both players have an equal chance of winning in equilibrium, $p_i^* = 1/2$, if they are symmetric, i.e. $\alpha = 0$. The same holds if $\beta/V = 1/4$. Further, from our results we can easily deduct the probability of winning for the potentially weaker player 2 since $p_2 = 1 - p_1$.

[11] To some extent, this mechanism resembles the "paradox of power" discussed by Hirshleifer (1991), which describes the phenomenon in a guns-and-butter model that poorer contenders appropriate a larger share of the pie relative to their endowment than richer contenders. Although in the present model there are no resource constraints, one player is more productive in generating innovative contest effort, which leads the "weaker" player to be more motivated to "appropriate" this effort by copying.

3.5 Comparative statics

Suppose that a government wants to enhance the winning chances or the profit of a domestic firm that competes in an innovation contest by subsidising the firm's innovative effort. Or imagine that a contest designer considers changing the cost of copying to maximise the innovative effort exerted by all participants. In these situations, it is important to know how changes in player 1's productivity advantage α and the cost of copying β affect the equilibrium outcome. In this section, I go through the comparative statics of the model and highlight the most surprising effects that arise when players can copy effort in contests.

3.5.1 Decision variables

Let us first consider the players' decision variables: the probability with which a player copies q_i, the effort she exerts if she does not copy x_i^n, and the effort if she does x_i^c. In general, and unsurprisingly, we have that $dq_1^*/d\alpha \leq 0$ and $dq_2^*/d\alpha \geq 0$, and $dq_1^*/d_\beta \leq 0$ and $dq_2^*/d\beta \leq 0$. The probability with which a player copies decreases in their relative productivity advantage and the cost of copying.

The effort player 1 exerts when she does not copy weakly increases in both her productivity advantage and the cost of copying, $dx_1^{n*}/d\alpha \geq 0$ and $dx_1^{n*}/d\beta \geq 0$. The latter effect is driven by the fact that player 2 copies less often if β increases, making it more attractive for player 1 to exert high effort. Since player 1 never exerts effort if she does copy, we naturally have $dx_1^{c*}/d\alpha = dx_1^{c*}/d\beta = 0$. If player 2 does never copy in equilibrium, $q_2^* = 0$, her effort is decreasing in player 1's productivity advantage. If she copies at least sometimes however, her effort if she does not copy is weakly increasing, $dx_2^{n*}/d\alpha_{|q_2^*>0} \geq 0$. Copying by player 2, which is increasing in α, lowers the incentive for player 1 to exert high effort, which in turn increases the incentive for player 2 to exert high effort herself when she does not copy. If the normalised cost of copying is sufficiently high, $\beta/V > 1/4$, player 2's effort when she does not copy is decreasing in said cost, $dx_2^{n*}/d\beta_{|\beta/V>1/4} < 0$. In contrast, we have $dx_2^{n*}/d\beta_{|\beta/V<1/4} > 0$. If player 1 never copies, player 2's effort if she does copy is weakly decreasing in both α and β, $dx_2^{c*}/d\alpha_{|q_1^*=0} \leq 0$ and $dx_2^{c*}/d\beta_{|q_1^*=0} \leq 0$. If player 1 mixes between copying and not copying, the opposite is the case, $dx_2^{c*}/d\alpha_{|q_1^*>0} > 0$ and $dx_2^{c*}/d\beta_{|q_1^*>0} > 0$. In this latter case, a decrease in player 1's probability to copy incentivises player 2 to exert higher effort.

3.5.2 Win probabilities

In the baseline case without copying, player 1's probability of winning the contest is intuitively increasing in her productivity advantage α. If at least one player copies, this only holds if player 1 never copies and player 2 always copies, $dp_1^*/d\alpha_{|q_i^*=0,q_2^*=1} > 0$. In both cases, both players do not or cannot react to a change in α by adjusting their probability of copying. Otherwise, player 1's win probability is weakly decreasing in α, $dp_1^*/d\alpha \leq 0$ if $q_1^* \in (0,1) \vee q_2^* \in (0,1)$. This surprising effect is closely related to Proposition 1 and follows the same intuition. If player 1 does not copy, her win probability is weakly increasing in the cost of copying, $dp_1^*/d\beta_{|q_1^*=0} \geq 0$. If player 1 mixes between copying and not copying, an increase in the cost of copying leads to a decrease in her chance of winning the contest, $dp_1^*/d\beta_{|q_1^*>0} < 0$.

3.5.3 Utilities

The changes in the players' equilibrium behaviour in reaction to changes in player 1's productivity advantage α and the cost of copying β lead to some counter-intuitive effects on the game's outcomes. Let us consider the players' expected utilities and first discuss the most surprising effect that arises.

Proposition 2 (**Player 1's utility can decrease in $\boldsymbol{\alpha}$**)**.** *If player 2 mixes between copying and not copying while player 1 never copies, and if the cost of copying is sufficiently high, $\beta/V > \sqrt{2}-1$, an increase in player 1's productivity advantage leads to a decrease in her expected utility. Otherwise, we have $du_1^*/d\alpha \geq 0$.*

Proof. In the baseline without copying, the expected utility of player 1 is $u_1^* = V/(2-\alpha)^2$. It is clear that then $du_1^*/d\alpha > 0$. If player 2 mixes between copying and not copying and player 1 never copies, and if additionally $\beta/V > 1/4$, we have that

$$u_1^* = V\left(\frac{1}{2} + (1-\alpha)\frac{1-\sqrt{1/2-\beta/V}}{\sqrt{1/2-\beta/V}}\left(\left(1-\sqrt{1/2-\beta/V}\right)^2 - \frac{1}{2}\right)\right).$$

We see that $du_1^*/d\alpha < 0$ if $(1-\sqrt{1/2-\beta/V})^2 > 1/2$ which is the case if $\beta/V > \sqrt{2}-1$. Otherwise, $du_1^*/d\alpha \geq 0$. If $\beta/V \leq 1/4$ and player 2 mixes between copying and not copying while player 1 never copies, player 1's utility is $u_1^* = \sqrt{\beta V} - (1-\alpha)\beta$. It is easy to see that here $du_1^*/d\alpha > 0$. If payer 2 always copies and player 1 never copies in equilibrium we have that $u_1^* = 2V/(3-\alpha)^2$ and it is clear that $du_1^*/d\alpha > 0$. If player 1 mixes between copying and not copying, she exerts zero effort when she copies. This means that her utility when she copies is $V/2-\beta$. Since she must be indifferent between copying and not copying, this means her expected utility is $u_1^*=V/2-\beta$. It is clear that then $du_1^*/d\alpha=0$. □

Intuition. An increase in her productivity advantage α benefits player 1 in the baseline without copying and in equilibria with copying if the cost of copying is sufficiently low. However, if the cost of copying is high, $\beta/V>\sqrt{2}-1$, and the productivity advantage high enough to incentivise player 2 to copy, i.e. condition (3.8) does not hold, a further increase in α harms player 1. This means for example that a government who subsidises a domestic firm's effort in a contest can end up harming the firm, even without taking into account the additional cost such a subsidy would incur. The effect can be explained by the fact that if the cost of copying is high but not prohibitive, an increase in α eventually leads to a sharp increase in player 2's probability of copying q_2, which more than offsets the benefit for player 1 of the decrease in her effort cost.

More intuitively, player 1's expected equilibrium utility is weakly increasing in the cost of copying β if player 1 never copies, and decreasing in β if player 1 mixes between copying and not copying. We can write $du_1^*/d\beta_{|q_1=0}\geq 0$ and $du_1^*/d\beta_{|q_1>0}<0$. This result is similar for player 2, for whom we have $du_2^*/d\beta_{|q_2=0}=0$, since then no player ever copies, and $du_2^*/d\beta_{|q_2>0}<0$. The effect of an increase in α on player 2's utility is negative if player 1 never copies, $du_2^*/d\alpha_{|q_1=0}\leq 0$, and positive if player 1 mixes, $du_2^*/d\alpha_{|q_1>0}>0$. The latter effect is also somewhat counter-intuitive. It is due to the fact that an increase in α leads player 1 to copy less often, which benefits player 2. If we combine these effects and look at aggregate expected utility in equilibrium, $U^*=\sum_i u_i^*$, we find that an increase in α leads to an increase in U^* and, if at least one player copies at least sometimes, an increase in β leads to a decrease in U^*. For each effect, there is one exception: First, we have $dU^*/d\alpha_{|q_2^*>0\,\wedge\,\beta/V>\sqrt{2}-1}<0$, which is driven by the decrease in player 1's utility discussed in Proposition 2. Second, it is the case that $dU^*/d\beta_{|q_2^*>0\,\wedge\,\alpha<\gamma}>0$, where

$$\gamma=\frac{1/(2(1/2-\beta/V))-3}{1/(2(1/2-\beta/V))-3+2\sqrt{1/2-\beta/V}}.$$

3.5.4 Aggregate effort

A contest designer might not take into account the participants' utilities. Rather, she might aim to maximise aggregate exerted contest effort $X=\sum_i x_i$. This reflects the sum of generated original ideas. Why would a contest designer care about aggregate exerted effort? For instance, original research generated within a specific contest often has major positive externalities beyond this contest. Examples include the race to the moon, which lead to innovations for instance in freeze-drying of foods, fireproof materials, and integrated circuits, and *The Netflix Prize*, a machine learning competition with a \$1 million award announced by the content platform Netflix in 2006, which spurred innovation in recommender systems and machine learning in general.[12] As with players' utilities, when considering aggregate exerted effort, copying of effort in contests can change some of the dynamics compared to the baseline without copying.

Proposition 3 (**Aggregate exerted effort can decrease in $\boldsymbol{\alpha}$**). *If player 2 mixes between copying and not copying while player 1 never copies, expected aggregate exerted effort in equilibrium X^* decreases in α. This is in contrast to the the baseline case without copying and the cases in which player 2 always copies. If both players mix, the sign of $dX^*/d\alpha$ is ambiguous.*

Proof. In the baseline without copying, the aggregate exerted effort in equilibrium is $X^* = V/(2-\alpha)$, and it is clear that then $dX^*/d\alpha > 0$. If player 1 never copies in equilibrium while player 2 mixes between copying and not copying, expected aggregate exerted effort is

$$X^* = x_1^* + (1 - q_2^*)x_2^{n*} + q_2^* x_2^{c*}.$$

Since $dx_1^*/d\alpha = dx_2^{n*}/d\alpha = dx_2^{c*}/d\alpha = 0$, $x_2^{n*} > x_2^{c*}$, and $dq_2^*/d\alpha > 0$, we have that then $dX^*/d\alpha < 0$. If player 2 always copies while player 1 never copies, $X^* = V(2-\alpha)/(3-\alpha)^2$. Then, $dX^*/d\alpha > 0$. If player 2 always copies and player 1 mixes in equilibrium, aggregate effort is $X^* = (1 - q_1^*)x_1^{n*} + x_2^*$. Since in this case $dq_1^*/d\alpha < 0$, $dx_1^{n*}/d\alpha > 0$, and $dx_2^*/d\alpha > 0$, we have that $dX^*/d\alpha > 0$. If both players mix between copying and not copying, we can write

$$\frac{dX^*}{d\alpha} = -\frac{dq_1^*}{d\alpha}x_1^{n*} + \frac{dq_2^*}{d\alpha}(x_2^{c*} - x_2^{n*}) + \frac{dx_2^{n*}}{d\alpha}.$$

This simplifies to $(\beta/V)^2(4\beta/V - 1/2 - \alpha\beta/V)/(1/2 - \alpha\beta/V)^3$. We can show that $dX^*/d\alpha > 0$ if $\alpha < 4 - 1/(2\beta/V)$ and $dX^*/d\alpha < 0$ if $\alpha > 4 - 1/(2\beta/V)$. □

Intuition. A contest designer interested in maximising aggregate exerted effort in a contest might not always benefit from a contest participant becoming more productive. This is due to the fact that an increase in productivity for player 1 incentivises player

[12] Related are arguments made by Terwiesch and Xu (2008) and Bessen and Maskin (2009) for why diversity of ideas might be good from the perspective of a contest designer.

2 to copy more frequently, which in turn lowers the expected effort for player 2 and lowers the incentive for player 1 to exert effort. For intermediary cost of copying, this means that such a contest designer would like to exclude a "strong" player in favour of a "weaker" player whose productivity is more similar to her opponent's.[13]

More intuitively, expected aggregate exerted effort is generally weakly increasing in the cost of copying, $dX^*/d\beta \geq 0$. This implies that an international coalition of governments who would like to increase global research efforts have an incentive to make copying more costly, for example by increasing the punishment of intellectual property law infringements.

3.5.5 Winner's effort

While some contest designers might take into account positive externalities of innovation and thus try to maximise aggregate exerted effort, others might only care about the innovation that wins the contest. For example, a company running a research contest might only implement the winning entry.[14] The natural objective of such a contest designer is the expected winner's effort, $y_w = \sum_i \mathbb{E}[p_i y_i]$. Recall that y_i denotes player i's effective effort and is defined as $y_i = x_i + c_i x_j, i \neq j$. For the contest designer, it is irrelevant whether the innovation is completely original or at least partly copied. While intuitively we might expect copying of effort to be potentially beneficial for a contest designer who wants to maximise the winner's effort, we can formulate our last proposition, which disagrees with this intuition.

Proposition 4 (The expected winner's effort increases in $\boldsymbol{\beta}$). *The expected winner's effort weakly increases in the cost of copying, $dy_w^*/d\beta \geq 0$.*

Proof. In the baseline case without copying, the winner's expected effort in equilibrium is $y_w^* = V(1+(1-\alpha)^2)/(2-\alpha)^3$, and naturally independent of the cost of copying β. If player 2 mixes between copying and not copying and player 1 does not copy in equilibrium, and if additionally $\beta/V > 1/4$, we can write

$$y_w^* = \left((1-\alpha)\left(2\sqrt{1/2-\beta/V}-1\right)+1\right)\left(1-\sqrt{1/2-\beta/V}\right)^2.$$

Making use of condition (3.8), one can show that then $dy_w^*/d\beta > 0$. In the same case, if we have $\beta/V \leq 1/4$, the expected winner's effort is $\beta(2\sqrt{\beta/V}+1/\sqrt{\beta/V}-2)$. Its

[13] Note that a contest designer would still prefer two strong players, however.

[14] See Serena (2017) for a deeper exploration of this idea and more examples of such settings, which he calls "quality contests".

derivative is $dy^*_w/\beta = 3\sqrt{\beta/V} + 1/(2\sqrt{\beta/V}) - 2 \geq 2(\sqrt{3/2} - 1) > 0$. If player 2 always copies and player 1 never copies in equilibrium, we have $y^*_w = V(1 + (2-\alpha)^2)/(3-\alpha)^3$ and thus $dy^*_w/d\beta = 0$. If player 2 always copies while player 1 mixes, the expected winner's effort is

$$y^*_w = (1 - q^*_1)\frac{(x^{n*}_1)^2 + (x^{n*}_1 + x^*_2)^2}{2x^{n*}_1 + x^*_2} + q^*_1 x^*_2.$$

Since $dq^*_1/d\beta < 0$, $dx^{n*}_1/d\beta > 0$, $dx^*_2/d\beta > 0$, and $x^{n*}_1 > x^*_2$, we have that $dy^*_w/d\beta > 0$. Lastly, if both players mix, the expected winner's effort is $y^*_w = (1 - q^*_1)\hat{y} + q^*_1((1 - q^*_2)x^{n*}_2 + q^*_2 x^{c*}_2)$, where $\hat{y} = ((x^{n*}_1)^2 + (x^{n*}_2)^2)/(x^{n*}_1 + x^{n*}_2)$ is the expected winner's effort if no one copies. The derivative is then

$$\frac{dy^*_w}{d\beta} = \frac{dq^*_1}{d\beta}(x^{n*}_2 - q^*_2\beta - \hat{y}) + (1 - q^*_1)\frac{d\hat{y}}{d\beta} + q^*_1\left(-\beta\frac{dq^*_2}{d\beta} + (1 - q^*_2)\frac{dx^{n*}_2}{d\beta} + q^*_2\frac{dx^{c*}_2}{d\beta}\right).$$

Since $dq^*_1/d\beta < 0$, $x^{n*}_2 - q^*_2\beta - \hat{y} < 0$, $d\hat{y}/d\beta > 0$, $dq^*_2/d\beta < 0$, $dx^{n*}_2/d\beta > 0$, and $dx^{c*}_2/d\beta > 0$, we must have that $dy^*_w/d\beta > 0$. □

Intuition. An increase in the cost of copying β makes players less likely to copy and thus less likely to have access to the other player's effort. However, this is offset by the fact that players usually also increase their exerted effort in response to an increase in β. This means that a contest designer who wants to maximise the expected winner's effort would like to increase the cost of copying, for example by punishing players who are caught copying or by making it technically more difficult to copy.[15]

The expected winner's effort in equilibrium y^*_w is also generally weakly increasing in player 1's productivity advantage α, with the exception of the case that both players mix between copying and not copying and the normalised cost β/V being very small. Specifically, we can write $dy^*_w/d\alpha_{|\beta/V>1/4} > 0$, $dy^*_w/d\alpha_{|q^*_1=0, q^*_2\in(0,1)\wedge\beta/V\leq 1/4} = 0$, and $dy^*_w/d\alpha_{|q^*_2=1} > 0$. If both players mix, $q^*_i \in (0,1)\ \forall\ i$, $dy^*_w/d\alpha$ is strictly negative as β/V tends to zero and positive as β/V tends to $1/4$.[16] This implies that a contest designer aiming to maximise the expected winner's effort in most cases benefits from one player gaining a productivity advantage.

[15]Note that an increase in the cost of copying β might not be costless to the contest designer, complicating her optimal policy.

[16]The derivative of the expected winner's effort is given by the rather tedious term $\frac{\beta^2/V(-3/16 + 7/4(\beta/V) + \alpha\beta/(2V) + 4(\beta/V)^2 + \alpha(\beta/V)^2 + (\alpha\beta/V)^2/2 + \alpha^2(\beta/V)^3 - 2(\alpha\beta/V)^3 - 4\alpha^3(\beta/V)^4 + (\alpha\beta/V)^4}{(1/2 - \beta/V)^5}$ in this case.

3.6 Conclusion

In innovation contests, participants may try to enhance their chance of winning by spying on and copying their opponents' effort. I investigate this mechanism in a stylised model and characterise the Nash equilibrium in dependence on the cost of copying and the potential productivity advantage of one of the players. I show that, when copying of effort is possible, the weaker player is more likely to win the contest if copying costs are low, that the more productive player's utility and the aggregate effort may decrease in the productivity advantage of the more productive player, and that the expected winner's effort generally increases in the cost of copying. Hence, a government trying to increase the profit of a domestic firm competing in an innovation contest may not want to subsidise this firm's efforts, even if this subsidy were costless. The designer of a contest who tries to maximise aggregate innovative effort may want to handicap a player with a productivity advantage or substitute her for a weaker one. And a contest designer whose objective is the maximisation of the expected winner's effort generally has an incentive to make copying of effort more costly.

There are multiple avenues for future research, which relate to some of the potential limitations of my model I have discussed. I assume players are successful to copy their opponent's effort with certainty. The model becomes mostly intractable if copying success is uncertain, unfortunately. Specifying a model that is fully tractable also with uncertain copying success may allow us to test the robustness of the stylised model I have presented and may offer additional insights. One may even go further and endogenise the probability of copying success by allowing players to take protective action, such as for example increasing IT security. Another improvement in realism might be to make the cost of copying dependent on the magnitude of the effort that is to be copied. Moreover, I solve the model in its main specification with additive efforts and briefly show that the resulting findings are robust to modelling efforts as redundant. It might still be interesting to analyse an intermediate case, in which some of the ideas the players generate are redundant by chance while the rest are not. Finally, the game I describe is static. Large innovation contests such as the space race often have multiple stages and involve dynamic interaction of the players' strategies. It might be fruitful to see how the integration of such dynamic interactions affect the results I have presented.

Appendices

3.A Appendix – Derivation of Tullock success function

We have that player $i \neq j$ wins the prize if $\theta_i y_i > \theta_j y_j$, with θ_i and θ_j being independent draws from the distribution $F(\theta) = 1 - e^{-\lambda\theta}$, $\lambda > 0$. Fix $\theta_j = \hat{\theta}_j$. Then, $P(\theta_i y_i > \hat{\theta}_j y_j) = P(\theta_i > \hat{\theta}_j y_j / y_i) = 1 - P(\theta_i \leq \hat{\theta}_j y_j / y_i)$, and thus

$$P(\theta_i y_i > \hat{\theta}_j y_j) = e^{-\lambda(\hat{\theta}_j y_j / y_i)}.$$

We can write the unconditional probability as

$$\begin{aligned}
P(\theta_i y_i > \theta_j y_j) &= \int_0^\infty e^{-\lambda(\theta_j y_j / y_i)} f(\theta_j) d\theta_j \\
&= \int_0^\infty e^{-\lambda(\theta_j y_j / y_i)} \lambda e^{-\lambda\theta_j} d\theta_j \\
&= \lambda \int_0^\infty e^{-\lambda\theta_j \frac{y_j + y_i}{y_i}} d\theta_j \\
&= \lambda \left(-\frac{1}{\lambda} \frac{y_i}{y_i + y_j} e^{-\lambda\theta_j \frac{y_j + y_i}{y_i}} \right) \Bigg|_0^\infty ,
\end{aligned}$$

which simplifies to

$$P(\theta_i y_i > \theta_j y_j) = \frac{y_i}{y_i + y_j}.$$

$\square$

3.B Appendix – Proof: player 1 copies less often than player 2

I prove that player 1 cannot copy with a higher probability than player 2 in equilibrium. Throughout Section 3.4 we have that $q_1 \leq q_2$. We must additionally show that it cannot be an equilibrium if player 1 always copies while player 2 mixes or never copies, or if player 1 mixes and player 2 never copies.

Proof that player 1 always copying and player 2 never copying/mixing cannot be an equilibrium

Assume that player 1 always copies in equilibrium. Additionally, assume that player 2 never copies. Solving the first order conditions yields efforts $x_1 = V/(3 - 2\alpha)^2$ and $x_2 = (1 - \alpha)V/(3 - 2\alpha)^2$. If player 2 deviates and copies, she optimally exerts effort

$x_2^c = 0$. For her to not have an incentive to do so it must hold that

$$\frac{\beta}{V} \geq \frac{1}{2} - 2\left(\frac{1-\alpha}{3-2\alpha}\right)^2. \tag{3.14}$$

If player 1 deviates and does not copy, she optimally exerts effort $x_1^n = (2-\alpha)V/(3-2\alpha)^2$, which keeps her probability of winning unchanged. She does not have an incentive to do so if

$$\frac{\beta}{V} \leq \left(\frac{1-\alpha}{3-2\alpha}\right)^2. \tag{3.15}$$

It is straightforward to show that the right-hand side of inequality (3.14) is strictly larger than the right-hand side of (3.15) since $\alpha \in [0,1)$. Hence, both inequalities cannot be true at the same time. It follows that player 1 always copying and player 2 never copying cannot be an equilibrium.

Now assume an equilibrium exists in which player 1 always copies while player 2 mixes, $q_2 \in (0,1)$. We can solve for efforts as $x_1 = (1-q_2)^2V/((1-q_2)+2(1-\alpha))^2$, $x_2^n = (1-\alpha)(1-q_2)V/((1-q_2)+2(1-\alpha))^2$, and, naturally, $x_2^c = 0$. The probability with which player 2 copies is $q_2 = 1-(1-\alpha)(\sqrt{2/(1/2-\beta/V)}-2)$. If player 1 deviates and does not copy, she optimally exerts effort $x_1^n = x_1 + x_2^n$, which keeps her probability of winning unchanged. She does not have an incentive to do so if $\beta \leq (1-\alpha)x_2^n$. Plugging in the closed form solution for x_2^n yields the condition

$$\alpha \leq 1 - \frac{\beta/V}{\sqrt{(1/2-\beta/V)/2} - (1/2-\beta/V))}. \tag{3.16}$$

Since $\sqrt{(1/2-\beta/V)/2} < 1/2$ due to $\beta > 0$, the right hand side of inequality (3.16) is smaller than 0. As we have $\alpha \geq 0$, this condition can never hold. It follows that player 1 always copying and player 2 mixing cannot be an equilibrium. □

Proof that player 1 mixing and player 2 never copying cannot be an equilibrium

Assume that player 1 mixes between copying and not copying, $q_1 \in (0,1)$, and that player 2 never copies in equilibrium. First order conditions yield that we must have $x_1^c = 0$ if $x_1^n < x_2$ and $x_1^c = x_1^n - x_2$ otherwise.

First, assume that $x_1^n < x_2$ and thus $x_1^c = 0$. It follows from the first order conditions that $x_2 = (1-\alpha)(1-q_1)$. We can solve for the probability that player 1 copies as $q_1 = 1-(1/\sqrt{1/2-\beta/V}-1)/(1-\alpha)$. For $x_1^n < x_2$ to hold, we must have that $\beta/V > 1/4$. But this implies that $q_1 < 0$, which is a contradiction.

Now, assume that $x_1^n \geq x_2$ and $x_1^c = x_1^n - x_2$. This implies efforts $x_2 = \beta/(1-\alpha)$, $x_1^n = (\sqrt{\beta V} - \beta)/(1-\alpha)$, and $x_1^c = (\sqrt{\beta V} - 2\beta)/(1-\alpha)$. For $x_1^n \geq x_2$ to hold we must have that $\beta/V \leq 1/4$. We can solve for the probability that player 1 copies as $q_1 = \sqrt{1/(\beta/V)} - (2-\alpha)/(1-\alpha)$. Player 2 has an incentive to deviate and copy while exerting effort $x_2^c = 0$ unless $\sqrt{\beta/V} - \beta/(V(1-\alpha)) \geq 1/2 - \beta/V$. This only holds if $\beta/V = 1/4$ and $\alpha = 0$, which implies $q_1 = 0$, which is a contradiction. □

3.C Appendix – Redundant efforts

If efforts are not additive but redundant, the effective effort of player i is $y_i = \max(x_i, c_i x_j), i \neq j$. Thus, a player's expected utility given the players' decisions is

$$u_i(x_i, x_j, c_i, c_j) = \frac{\max(x_i, c_i x_j)}{\max(x_i, c_i x_j) + \max(c_j x_i, x_j)} V - (1-\alpha_i)x_i - \beta c_i,$$

instead of (3.2). If efforts are redundant, it can never be an equilibrium for a player to copy and exert non-zero effort. Moreover, it cannot be an equilibrium for a player i to always copy. Then, the opponent j's optimal reply would be to not copy and exert no effort. But then, i has an incentive to not copy and exert an infinitesimal effort instead.

With redundant efforts, there are three cases of equilibria: both players never copy, player 2 mixes and player 1 never copies, and both players mix between copying and not copying. Figure 3.C.1 shows these equilibria in dependence of player 1's productivity advantage α and the normalised cost of copying β/V.

If $\beta/V \geq 1/4$ and efforts are additive, player 2 does not exert effort when copying. This means that the players' incentives are exactly the same as with effort redundancy. Hence, the analysis of Section 3.4.1 and Section 3.4.2 goes through. If $\beta/V < 1/4$ and player 2 mixes between copying and not copying while player 1 never copies in equilibrium, we can use the same equilibrium values we have derived in Section 3.4.2 for the case that $x_2^{c*} = 0$, which must hold with redundant efforts by necessity. We can show that player 1 does not have an incentive to deviate and copy if

$$\alpha \geq 1 - \frac{\beta/V}{(1/\sqrt{1/2 - \beta/V} - 1)(1/2 - \sqrt{1/2 - \beta/V})^2}.$$

If both players mix between copying and not copying in equilibrium, we can unfortunately not derive closed-form solutions and thus not evaluate comparative statics ana-

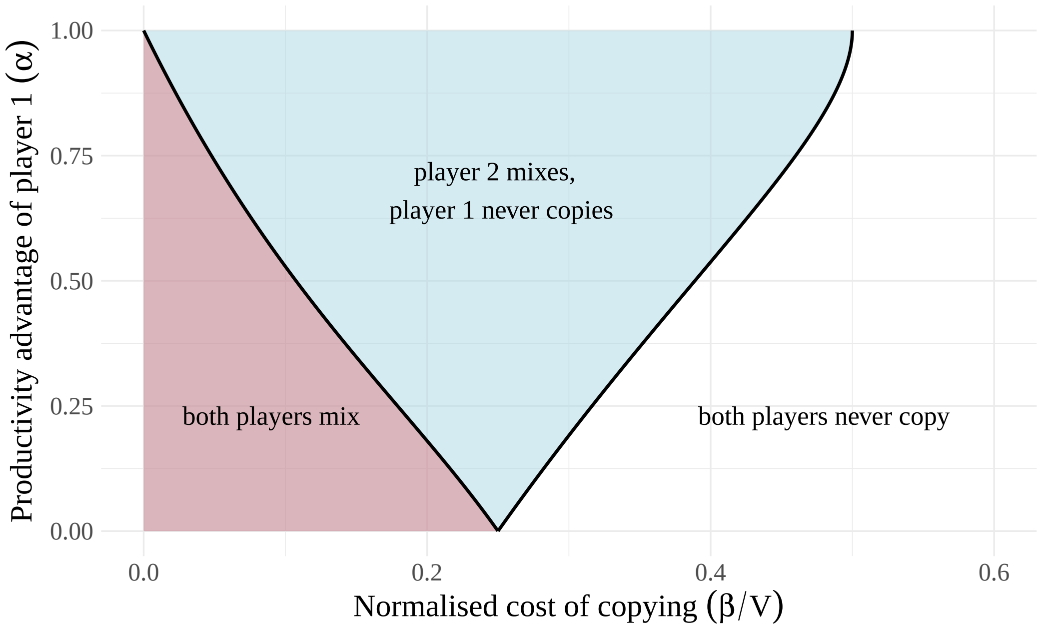

Figure 3.C.1: Nash equilibria in dependence of the normalised cost of copying β/V and player 1's productivity advantage α when efforts are redundant

lytically. However, we can show that equilibrium efforts are given by $x_1^{n*} = (1-q_1^*)/(1+(1-q_1^*)(1-\alpha)/(1-q_2^*))^2$ and $x_1^{n*} = (1-q_1^*)^2(1-\alpha)/((1-q_2^*)(1+(1-q_1^*)(1-\alpha)/(1-q_2^*))^2)$. Using the indifference equations for both players, and defining $\rho = (1-q_1)/(1-q_2)$, we can show that

$$\rho\frac{(\alpha\rho)^2 - (1+\alpha\rho)^2/2}{1-(1-\alpha\rho)^2/2} = 1 \tag{3.17}$$

must hold in equilibrium. Equation (3.17) can be solved numerically, which allows a full numerical characterisation of the equilibrium.

Since the equilibrium with redundant efforts mirrors that with additive efforts if $\beta/V \geq 1/4$, naturally all four propositions also hold. If $\beta/V < 1/4$ and only player 2 mixes in equilibrium, the win probability of player 1 is $p_1^* = (1-q_2^*)(1-\sqrt{1/2-\beta/V})+q_2^*/2 < 1/2$, confirming Proposition 1. In the same case, we have that $du_1^*/d\alpha > 0$, confirming Proposition 2 as well. Since equilibrium exerted efforts are unaffected by a change in α and $dq_2^*/d\alpha > 0$, we have that $dX^*/d\alpha < 0$, confirming Proposition 3. Further, also Proposition 4 holds in this case since we have that $dy_w^*/d\beta > 0$. Interestingly, expected winner's effort y_w^* is decreasing in α in this case, in contrast to the baseline without copying and to the model with additive efforts with copying.

It can be shown numerically that all four propositions are also valid if both players mix. The only qualification is that if efforts are redundant, we have that, if both players mix between copying and not copying in equilibrium, $dX^*/d\alpha > 0$.

References

Abbink, K., Brandts, J., Herrmann, B., and Orzen, H. (2010). Intergroup conflict and intra-group punishment in an experimental contest game. *American Economic Review*, 100(1):420–47.

Adamczyk, S., Bullinger, A. C., and Möslein, K. M. (2012). Innovation contests: A review, classification and outlook. *Creativity and Innovation Management*, 21(4):335–360.

Aghion, P., Harris, C., Howitt, P., and Vickers, J. (2001). Competition, imitation and growth with step-by-step innovation. *The Review of Economic Studies*, 68(3):467–492.

Albano, G. L., Spagnolo, G., and Zanza, M. (2009). Regulating joint bidding in public procurement. *Journal of Competition Law and Economics*, 5(2):335–360.

Alon, N., Emek, Y., Feldman, M., and Tennenholtz, M. (2013). Adversarial leakage in games. *SIAM Journal on Discrete Mathematics*, 27(1):363–385.

Amann, E. and Leininger, W. (1996). Asymmetric all-pay auctions with incomplete information: the two-player case. *Games and Economic Behavior*, 14(1):1–18.

Arbatskaya, M. and Mialon, H. M. (2010). Multi-activity contests. *Economic Theory*, 43(1):23–43.

Baik, K. H. (1993). Effort levels in contests: The public-good prize case. *Economics Letters*, 41(4):363–367.

Baik, K. H. (2008). Contests with group-specific public-good prizes. *Social Choice and Welfare*, 30(1):103–117.

Baik, K. H., Kim, I.-G., and Na, S. (2001). Bidding for a group-specific public-good prize. *Journal of Public Economics*, 82(3):415–429.

Baik, K. H. and Lee, S. (1997). Collective rent seeking with endogenous group sizes. *European Journal of Political Economy*, 13(1):121–130.

Baik, K. H. and Lee, S. (2001). Strategic groups and rent dissipation. *Economic Inquiry*, 39(4):672–684.

Baik, K. H. and Shogren, J. F. (1995a). Competitive-share group formation in rent-seeking contests. *Public Choice*, 83(1-2):113–126.

Baik, K. H. and Shogren, J. F. (1995b). Contests with spying. *European Journal of Political Economy*, 11(3):441–451.

Barrachina, A., Tauman, Y., and Urbano, A. (2014). Entry and espionage with noisy signals. *Games and Economic Behavior*, 83:127–146.

Barrachina, A., Tauman, Y., and Urbano, A. (2021). Entry with two correlated signals: the case of industrial espionage and its positive competitive effects. *International Journal of Game Theory*, 50(1):241–278.

Baumol, W. J. (1992). Innovation and strategic sabotage as a feedback process. *Japan and the World Economy*, 4(4):275–290.

Baye, M. R. and Hoppe, H. C. (2003). The strategic equivalence of rent-seeking, innovation, and patent-race games. *Games and Economic Behavior*, 44(2):217–226.

Baye, M. R., Kovenock, D., and De Vries, C. G. (1993). Rigging the lobbying process: an application of the all-pay auction. *American Economic Review*, 83(1):289–294.

Baye, M. R., Kovenock, D., and De Vries, C. G. (1994). The solution to the tullock rent-seeking game when r >2: Mixed-strategy equilibria and mean dissipation rates. *Public Choice*, 81(3):363–380.

Baye, M. R., Kovenock, D., and De Vries, C. G. (1996). The all-pay auction with complete information. *Economic Theory*, 8(2):291–305.

Baye, M. R., Kovenock, D., and De Vries, C. G. (2012). Contests with rank-order spillovers. *Economic Theory*, 51(2):315–350.

Berentsen, A. (2002). The economics of doping. *European Journal of Political Economy*, 18(1):109–127.

Bergstrom, T., Blume, L., and Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, 29(1):25–49.

Bessen, J. and Maskin, E. (2009). Sequential innovation, patents, and imitation. *The RAND Journal of Economics*, 40(4):611–635.

Billand, P., Bravard, C., Chakrabarti, S., and Sarangi, S. (2010). Spying in multi-market oligopolies.

Blavatskyy, P. R. (2010). Contest success function with the possibility of a draw: Axiomatization. *Journal of Mathematical Economics*, 46(2):267–276.

Bloch, F. (2012). Endogenous formation of alliances in conflicts. *Oxford Handbook of the Economics of Peace and Conflict. Oxford University Press, New York.*

Bloch, F., Sánchez-Pagés, S., and Soubeyran, R. (2006). When does universal peace prevail? secession and group formation in conflict. *Economics of Governance*, 7(1):3–29.

Bolle, F. (1996). Contests with spying: A comment. *European Journal of Political Economy*, 12(4):729–734.

Bozbay, I. and Vesperoni, A. (2018). A contest success function for networks. *Journal of Economic Behavior & Organization*, 150:404–422.

Chaturvedi, A. (2005). Rigging elections with violence. *Public Choice*, 125(1-2):189–202.

Che, Y.-K. and Gale, I. (2003). Optimal design of research contests. *American Economic Review*, 93(3):646–671.

Chen, K.-P. (2003). Sabotage in promotion tournaments. *Journal of Law, Economics, and Organization*, 19(1):119–140.

Chen, P.-L. et al. (2016). Cross-country economic espionage and investment in research and development. *International Journal of Economics and Finance*, 8(4):146–155.

Chen, Z. C. (2019). Spying in contests. *Available at SSRN 2874296.*

Cherry, T. L. and Cotten, S. J. (2011). Sleeping with the enemy: The economic cost of internal environmental conflicts. *Economic Inquiry*, 49(2):530–539.

Choi, J. P., Chowdhury, S. M., and Kim, J. (2016). Group contests with internal conflict and power asymmetry. *The Scandinavian Journal of Economics*, 118(4):816–840.

Chowdhury, S. M. and Gürtler, O. (2015). Sabotage in contests: a survey. *Public Choice*, 164(1):135–155.

Chowdhury, S. M., Jeon, J. Y., and Ramalingam, A. (2016). Identity and group conflict. *European Economic Review*, 90:107–121.

Chowdhury, S. M. and Kovenock, D. (2012). A combinatorial multiple winner contest with package designer preferences. *Paper presented at the Coalition Theory Network Conference.*

Chowdhury, S. M., Lee, D., and Sheremeta, R. M. (2013). Top guns may not fire: Best-shot group contests with group-specific public good prizes. *Journal of Economic Behavior & Organization*, 92:94–103.

Chowdhury, S. M. and Topolyan, I. (2016). The attack-and-defense group contests: Best shot versus weakest link. *Economic Inquiry*, 54(1):548–557.

Clark, D. J. and Konrad, K. A. (2007). Asymmetric conflict: Weakest link against best shot. *Journal of Conflict Resolution*, 51(3):457–469.

Clark, D. J. and Riis, C. (1998a). Competition over more than one prize. *American Economic Review*, 88(1):276–289.

Clark, D. J. and Riis, C. (1998b). Contest success functions: an extension. *Economic Theory*, 11(1):201–204.

Corchón, L. C. and Serena, M. (2018). Contest theory. *Handbook of Game Theory and Industrial Organization*, 2:125–146.

Cornes, R. and Hartley, R. (2005). Asymmetric contests with general technologies. *Economic Theory*, 26(4):923–946.

Cozzi, G. (2001). Inventing or spying? implications for growth. *Journal of Economic Growth*, 6(1):55–77.

Cozzi, G. and Spinesi, L. (2006). Intellectual appropriability, product differentiation, and growth. *Macroeconomic Dynamics*, 10(1):39.

Crane, A. (2005). In the company of spies: When competitive intelligence gathering becomes industrial espionage. *Business Horizons*, 48(3):233–240.

Dasgupta, P. and Stiglitz, J. (1980). Uncertainty, industrial structure, and the speed of r&d. *Bell Journal of Economics*, 11(1):1–28.

d'Aspremont, C. and Jacquemin, A. (1988). Cooperative and noncooperative R & D in duopoly with spillovers. *American Economic Review*, 78(5):1133–1137.

Davis, D. D. and Reilly, R. J. (1998). Do too many cooks always spoil the stew? an experimental analysis of rent-seeking and the role of a strategic buyer. *Public Choice*, 95(1):89–115.

Davis, D. D. and Reilly, R. J. (1999). Rent-seeking with non-identical sharing rules: An equilibrium rescued. *Public Choice*, 100(1):31–38.

Dechenaux, E., Kovenock, D., and Sheremeta, R. M. (2015). A survey of experimental research on contests, all-pay auctions and tournaments. *Experimental Economics*, 18(4):609–669.

Dixit, A. (1987). Strategic behavior in contests. *American Economic Review*, 77(5):891–98.

Eber, N. and Thépot, J. (1999). Doping in sport and competition design. *Recherches Économiques de Louvain/Louvain Economic Review*, pages 435–446.

Epstein, G. S. and Hefeker, C. (2003). Lobbying contests with alternative instruments. *Economics of Governance*, 4(1):81–89.

Epstein, G. S. and Mealem, Y. (2009). Group specific public goods, orchestration of interest groups with free riding. *Public Choice*, 139(3-4):357–369.

Esteban, J. and Ray, D. (2001). Collective action and the group size paradox. *American Political Science Review*, 95(3):663–672.

Esteban, J. and Ray, D. (2008). On the salience of ethnic conflict. *American Economic Review*, 98(5):2185–2202.

Esteban, J. and Sákovics, J. (2003). Olson vs. coase: Coalitional worth in conflict. *Theory and Decision*, 55(4):339–357.

Franke, J. and Öztürk, T. (2015). Conflict networks. *Journal of Public Economics*, 126:104–113.

Fullerton, R. L. and McAfee, R. P. (1999). Auctionin entry into tournaments. *Journal of Political Economy*, 107(3):573–605.

Gallini, N. T. (1992). Patent policy and costly imitation. *The RAND Journal of Economics*, pages 52–63.

Garfinkel, M. R. (2004a). On the stability of group formation: Managing the conflict within. *Conflict Management and Peace Science*, 21(1):43–68.

Garfinkel, M. R. (2004b). Stable alliance formation in distributional conflict. *European Journal of Political Economy*, 20(4):829–852.

Gilpatric, S. M. (2011). Cheating in contests. *Economic Inquiry*, 49(4):1042–1053.

Glazer, A. and Hassin, R. (1988). Optimal contests. *Economic Inquiry*, 26(1):133–143.

Grabiszewski, K. and Minor, D. (2019). Economic espionage. *Defence and Peace Economics*, 30(3):269–277.

Gradstein, M. and Konrad, K. A. (1999). Orchestrating rent seeking contests. *The Economic Journal*, 109(458):536–545.

Groh, C., Moldovanu, B., Sela, A., and Sunde, U. (2012). Optimal seedings in elimination tournaments. *Economic Theory*, 49(1):59–80.

Grossman, G. M. and Helpman, E. (1991). Quality ladders and product cycles. *The Quarterly Journal of Economics*, 106(2):557–586.

Grossman, H. I. (2005). Inventors and pirates: creative activity and intellectual property rights. *European Journal of Political Economy*, 21(2):269–285.

Gürtler, O. (2008). On sabotage in collective tournaments. *Journal of Mathematical Economics*, 44(3-4):383–393.

Gürtler, O. and Münster, J. (2010). Sabotage in dynamic tournaments. *Journal of Mathematical Economics*, 46(2):179–190.

Haugen, K. K. (2004). The performance-enhancing drug game. *Journal of Sports Economics*, 5(1):67–86.

Hausken, K. (2005). Production and conflict models versus rent-seeking models. *Public Choice*, 123(1-2):59–93.

Hausken, K. et al. (2020). Additive multi-effort contests. *Theory and Decision*, 89(2):203–248.

Haynes, J. E. and Klehr, H. (2000). *Venona: decoding Soviet espionage in America*. Yale University Press.

Heijnen, P. and Schoonbeek, L. (2020). Cross-shareholdings and competition in a rent-seeking contest. *International Journal of Industrial Organization*, 71:102625.

Helpman, E. (1993). Innovation, imitation, and intellectual property rights. *Econometrica: Journal of the Econometric Society*, pages 1247–1280.

Herbst, L., Konrad, K. A., and Morath, F. (2015). Endogenous group formation in experimental contests. *European Economic Review*, 74:163–189.

Hillman, A. L. and Riley, J. G. (1989). Politically contestable rents and transfers. *Economics & Politics*, 1(1):17–39.

Hirshleifer, J. (1989). Conflict and rent-seeking success functions: Ratio vs. difference models of relative success. *Public Choice*, 63(2):101–112.

Hirshleifer, J. (1991). The paradox of power. *Economics & Politics*, 3(3):177–200.

Hirshleifer, J. and Riley, J. G. (1992). *The analytics of uncertainty and information*. Cambridge University Press.

Ho, S. (2008). Extracting the information: espionage with double crossing. *Journal of Economics*, 93(1):31–58.

Jia, H., Skaperdas, S., and Vaidya, S. (2013). Contest functions: Theoretical foundations and issues in estimation. *International Journal of Industrial Organization*, 31(3):211–222.

Katz, E., Nitzan, S., and Rosenberg, J. (1990). Rent-seeking for pure public goods. *Public Choice*, 65(1):49–60.

Katz, E. and Tokatlidu, J. (1996). Group competition for rents. *European Journal of Political Economy*, 12(4):599–607.

Katz, M. L. and Shapiro, C. (1987). R and D rivalry with licensing or imitation. *American Economic Review*, pages 402–420.

Ke, C., Konrad, K. A., and Morath, F. (2013). Brothers in arms–an experiment on the alliance puzzle. *Games and Economic Behavior*, 77(1):61–76.

Ke, C., Konrad, K. A., and Morath, F. (2015). Alliances in the shadow of conflict. *Economic Inquiry*, 53(2):854–871.

Kolmar, M. and Rommeswinkel, H. (2013). Contests with group-specific public goods and complementarities in efforts. *Journal of Economic Behavior & Organization*, 89:9–22.

König, M. D., Rohner, D., Thoenig, M., and Zilibotti, F. (2017). Networks in conflict: Theory and evidence from the great war of africa. *Econometrica*, 85(4):1093–1132.

Konrad, K. A. (2000). Sabotage in rent-seeking contests. *Journal of Law, Economics, and Organization*, 16(1):155–165.

Konrad, K. A. (2004). Bidding in hierarchies. *European Economic Review*, 48(6):1301–1308.

Konrad, K. A. (2005). Tournaments and multiple productive inputs: the case of performance enhancing drugs.

Konrad, K. A. (2006). Silent interests and all-pay auctions. *International Journal of Industrial Organization*, 24(4):701–713.

Konrad, K. A. (2009). *Strategy and dynamics in contests*. Oxford University Press.

Konrad, K. A. (2014). Strategic aspects of fighting in alliances. *The Economics of Conflict: Theory and Empirical Evidence. MIT Press, Cambridge, MA*.

Konrad, K. A. and Kovenock, D. (2009a). The alliance formation puzzle and capacity constraints. *Economics Letters*, 103(2):84–86.

Konrad, K. A. and Kovenock, D. (2009b). Multi-battle contests. *Games and Economic Behavior*, 66(1):256–274.

Kovenock, D. and Roberson, B. (2010). Conflicts with multiple battlefields.

Kozlovskaya, M. (2018). Industrial espionage in duopoly games. *Available at SSRN 3190093*.

Kräkel, M. (2005). Helping and sabotaging in tournaments. *International Game Theory Review*, 7(02):211–228.

Kräkel, M. (2007). Doping and cheating in contest-like situations. *European Journal of Political Economy*, 23(4):988–1006.

Krishna, V. and Morgan, J. (1997). An analysis of the war of attrition and the all-pay auction. *Journal of Economic Theory*, 72(2):343–362.

Lazear, E. P. (1989). Pay equality and industrial politics. *Journal of political economy*, 97(3):561–580.

Lee, D. (2012). Weakest-link contests with group-specific public good prizes. *European Journal of Political Economy*, 28(2):238–248.

Lee, D. (2015). Group contests and technologies. *Economics Bulletin*, 35(4):2427–2438.

Lee, T. and Wilde, L. L. (1980). Market structure and innovation: A reformulation. *The Quarterly Journal of Economics*, 94(2):429–436.

Loury, G. C. (1979). Market structure and innovation. *The Quarterly Journal of Economics*, 93(3):395–410.

Marion, J. (2015). Sourcing from the enemy: Horizontal subcontracting in highway procurement. *The Journal of Industrial Economics*, 63(1):100–128.

Marjit, S. and Yang, L. (2015). Does intellectual property right promote innovations when pirates are innovators? *International Review of Economics & Finance*, 37:203–207.

Matsui, A. (1989). Information leakage forces cooperation. *Games and Economic Behavior*, 1(1):94–115.

Millner, E. L. and Pratt, M. D. (1989). An experimental investigation of efficient rent-seeking. *Public Choice*, 62(2):139–151.

Millner, E. L. and Pratt, M. D. (1991). Risk aversion and rent-seeking: An extension and some experimental evidence. *Public Choice*, 69(1):81–92.

Moldovanu, B. and Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review*, 91(3):542–558.

Moldovanu, B. and Sela, A. (2006). Contest architecture. *Journal of Economic Theory*, 126(1):70–96.

Moretti, L. and Valbonesi, P. (2015). Firms' qualifications and subcontracting in public procurement: an empirical investigation. *The Journal of Law, Economics, and Organization*, 31(3):568–598.

Münster, J. (2007a). Selection tournaments, sabotage, and participation. *Journal of Economics & Management Strategy*, 16(4):943–970.

Münster, J. (2007b). Simultaneous inter-and intra-group conflicts. *Economic Theory*, 32(2):333–352.

Münster, J. (2009). Group contest success functions. *Economic Theory*, 41(2):345–357.

Nalbantian, H. R. and Schotter, A. (1997). Productivity under group incentives: An experimental study. *American Economic Review*, 87(3):314–341.

Nitzan, S. (1991a). Collective rent dissipation. *The Economic Journal*, 101(409):1522–1534.

Nitzan, S. (1991b). Rent-seeking with non-identical sharing rules. *Public Choice*, 71(1-2):43–50.

Nitzan, S. and Ueda, K. (2009). Collective contests for commons and club goods. *Journal of Public Economics*, 93(1-2):48–55.

Nitzan, S. and Ueda, K. (2011). Prize sharing in collective contests. *European Economic Review*, 55(5):678–687.

Noh, S. J. (2002). Resource distribution and stable alliances with endogenous sharing rules. *European Journal of Political Economy*, 18(1):129–151.

Nti, K. O. (1998). Effort and performance in group contests. *European Journal of Political Economy*, 14(4):769–781.

Ogburn, W. F. and Thomas, D. (1922). Are inventions inevitable? a note on social evolution. *Political Science Quarterly*, 37(1):83–98.

Olson, M. (1965). The logic of collective action. *Harvard University Press*.

Osório, A. (2018). Conflict and competition over multi-issues. *The BE Journal of Theoretical Economics*, 18(2):1–17.

Pavan, A. and Tirole, J. Expectation conformity in strategic cognition.

Pearson, N. O. (2020). Did a chinese hack kill canada's greatest tech company? https://www.bloomberg.com/news/features/2020-07-01/did-china-steal-canada-s-edge-in-5g-from-nortel. Accessed on September 1, 2021.

Perez-Castrillo, J. D. and Verdier, T. (1992). A general analysis of rent-seeking games. *Public Choice*, 73(3):335–350.

Potters, J., De Vries, C. G., and Van Winden, F. (1998). An experimental examination of rational rent-seeking. *European Journal of Political Economy*, 14(4):783–800.

Rai, B. K. and Sarin, R. (2009). Generalized contest success functions. *Economic Theory*, 40(1):139–149.

Ray, D. and Vohra, R. (2015). Coalition formation. In *Handbook of Game Theory with Economic Applications*, volume 4, pages 239–326. Elsevier.

Reinganum, J. F. (1982). A dynamic game of r and d: Patent protection and competitive behavior. *Econometrica*, 50(3):671–688.

Riaz, K., Shogren, J. F., and Johnson, S. R. (1995). A general model of rent seeking for public goods. *Public Choice*, 82(3-4):243–259.

Roberson, B. (2006). The colonel blotto game. *Economic Theory*, 29(1):1–24.

Robinson, J. A. (2003). Social identity, inequality and conflict. In *Conflict and Governance*, pages 7–21. Springer.

Rosen, S. (1986). Prizes and incentives in elimination tournaments. *American Economic Review*, 76(4):701–715.

Sánchez-Pagés, S. (2007). Endogenous coalition formation in contests. *Review of Economic Design*, 11(2):139.

Scherer, F. M. (1967). Research and development resource allocation under rivalry. *The Quarterly Journal of Economics*, 81(3):359–394.

Segerstrom, P. S. (1991). Innovation, imitation, and economic growth. *Journal of Political Economy*, 99(4):807–827.

Sen, A. (2007). *Identity and violence: The illusion of destiny*. Penguin Books India.

Send, J. (2020). Conflict between non-exclusive groups. *Journal of Economic Behavior & Organization*, 177:858–874.

Serena, M. (2017). Quality contests. *European Journal of Political Economy*, 46:15–25.

Sheremeta, R. M. (2010). Experimental comparison of multi-stage and one-stage contests. *Games and Economic Behavior*, 68(2):731–747.

Sheremeta, R. M. (2018). Behavior in group contests: A review of experimental research. *Journal of Economic Surveys*, 32(3):683–704.

Sheremeta, R. M. and Zhang, J. (2010). Can groups solve the problem of over-bidding in contests? *Social Choice and Welfare*, 35(2):175–197.

Skaperdas, S. (1996). Contest success functions. *Economic Theory*, 7(2):283–290.

Skaperdas, S. (1998). On the formation of alliances in conflict and contests. *Public Choice*, 96(1-2):25–42.

Skaperdas, S. and Grofman, B. (1995). Modeling negative campaigning. *American Political Science Review*, 89(1):49–61.

Solan, E. and Yariv, L. (2004). Games with espionage. *Games and Economic Behavior*, 47(1):172–199.

Solitander, M. and Solitander, N. (2010). The sharing, protection and thievery of intellectual assets: The case of the formula 1 industry. *Management Decision*, 48(1):37–57.

Spence, M. et al. (1984). Cost reduction, competition, and industry performance. *Econometrica*, 52(1):101–121.

Taylor, C. (1995). Digging for golden carrots: An analysis of research tournaments. *American Economic Review*, 85(4):872–90.

Terwiesch, C. and Xu, Y. (2008). Innovation contests, open innovation, and multiagent problem solving. *Management Science*, 54(9):1529–1543.

Tullock, G. (1980). Efficient rent-seeking. In Buchanan, J. M., Tollison, R. D., and Tullock, G., editors, *Towards a theory of the rent-seeking society*. College Station: Texas A&M Press.

Ursprung, H. W. (1990). Public goods, rent dissipation, and candidate competition. *Economics & Politics*, 2(2):115–132.

US Intellectual Property Commission (2017). The theft of American intellectual property: Reassessments of the challenge and united states policy. *The National Bureau of Asian Research*.

Wang, T. (2020). Competitive intelligence and disclosure of cost information in duopoly. *Review of Industrial Organization*, 57(3):665–699.

Wärneryd, K. (1998). Distributional conflict and jurisdictional organization. *Journal of Public Economics*, 69(3):435–450.

Wärneryd, K. (2003). Information in conflicts. *Journal of Economic Theory*, 110(1):121–136.

Wesley, F. (1967). The kidnapping of the lunik. *Studies in Intelligence*, 11(1).

Whitney, M. E. and Gaisford, J. D. (1996). Economic espionage as strategic trade policy. *Canadian Journal of Economics*, 29(s1):627–32.

Whitney, M. E. and Gaisford, J. D. (1999). An inquiry into the rationale for economic espionage. *International Economic Journal*, 13(2):103–123.