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Electronically controlled polarization beat length in Kerr nonlinear media

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ABSTRACT

The polarization beat length of propagating optical fields in nonlinear birefringent Kerr medium is investigated in the presence of an externally applied DC electric field. We show that the critical power, at which the effective polarization beat length becomes infinite, can be controlled through adjusting the externally applied electric field. The principle of operation is based on modifying the polarization instability by electronically adjusting the effective birefringence through an external electrical bias. The presented analytical expressions describe the beat length and the polarization instability as a function of the applied electric field for an arbitrary optical input state.

Introduction

Polarization instability in a medium arises when the nonlinear change of the refractive index is comparable with the linear birefringence. This phenomenon manifests when the nonlinear birefringence cancels completely the linear birefringence and the beat length escalates to infinity. Physically, the beat length (L_B^{eff}) is the length at which the optical power is transferred from one polarization to another. In a nonlinear medium, such as the Kerr medium, the L_B^{eff} length becomes infinite at a critical input power for a propagating light that is polarized along the fast axis [1–3]. It then follows that a substantial change in the output polarization state is observed when the input power (or its polarization state) is slightly differing.

Controlling the polarization dynamics and obtaining non-trivial polarization evolution is vital [4–15] to optimize the operation of several photonic devices [16]. These include the birefringent optical fibers (BOFs), [17–21], the multimode interference (MMI) couplers [22], the Y-branches [23] and also, the integrated photonic circuits [24], specially the electric-field-induced second harmonic generation (EFISHG) could be considered as a practical possibility, in integrated photonics, due to the fact that the nonlinear susceptibility χ^3 in silicon is two order of magnitude larger than in silicon oxide, and that in integrated photonics the non-linear modal area is reduced by a large factor when compared to typical optical fibers [25]. This keeps the electric fields required below the silicon breakdown, although not too far from it. Another favorable condition of integrated devices is that the required field may be produced across a small distance (few microns), thus avoiding the requirement of high voltage components [26].

Interestingly, for propagating optical fields in non-resonant Kerr nonlinear medium, a biasing electric field induces birefringence even if the medium is optically isotropic [27]. In [28], the authors have studied the impact of applying a DC electric field (i.e., E_{ext}), to a third-order nonlinear medium, on the evolution of propagating optical waves. They found that the polarization evolution can be controlled by the applied E_{ext} field. As a matter of fact, the E_{ext} field turns the third-order nonlinearity into a second-order-like as if one deals with an electro-optic-like effect.

While these effects in a nonlinear and birefringent medium have been known for long, and examined in details [28], the polarization instability in nonlinear medium with the presence of externally applied DC electric field has received little attention. Both the L_B, which is the beat length when nonlinear optical effects are neglected, and the L_B^{eff} are important quantities that must be characterized in fibers and waveguides. For instance, reducing these lengths can improve the stability of the optical system and enhance the communication capacity significantly [29]. However, in practice, these lengths are fixed once the geometry, the materials, and the input power are selected. Thus, a limited capability to adaptively designing/monitoring the performance of the pertinent optical systems is experienced.

In this letter, we present a theoretical description for electronically controlled polarization instability [28]. The considered scheme implies adjusting the critical power (at which the polarization instability takes place) through modifying the effective birefringence by applying an external electric field E_{ext}. We have carried out analytical expressions that relate L_B and L_B^{eff} with the DC applied field. The derived expression

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shows that both the critical power and the effective beat length can be arbitrarily shifted by adjusting the applied DC electric field.

The theoretical analysis begins by deriving the governing coupled differential equations of the evolution of the optical field in Kerr nonlinear medium while an external electric field is applied. On assuming a slow-varying-envelope approximation (whereby the second derivatives are neglected), considering harmonic fields, and omitting the transverse variations, a well-known first-order differential equation relating the optical electric field and the polarization is obtained, given by [28]:

$$\frac{\partial \vec{E}}{\partial z} = -i\,\frac{k}{2\,\epsilon}\,\vec{P}_{NL}, \tag{1}$$

where $\vec{E}$ is the optical electric vector field, k is the propagation constant, ϵ is the material permittivity, and $\vec{P}_{NL}$ is the nonlinear polarization vector.

In the following, without losing any aspect of generality, we analyze the x–polarization component while it is coupled to the y–polarization component. It then follows that the nonlinear polarization $P_{x_{NL}}$ is given by:

$$P_{x_{NL}} = \epsilon_0 \chi \left[\frac{A_x}{4}|A_{x,y}|^2 + \frac{A_x^*}{8}A_{x,y}^2 + \frac{3}{8}A_x E_{ext}^2\right], \tag{2}$$

where A_x and A_y are the complex amplitudes of two orthogonal modes (in case of an optical fiber) or TE modes (in case of planar waveguides), ϵ_0 is the vacuum permittivity, and $\chi \equiv \chi^{(3)}$ is the nonlinear susceptibility.

Substituting (2) into (1) yields the spatial evolution of the polarization state, given by:

$$\frac{1}{A_x}\frac{\partial A_x}{\partial z} = i\gamma\left[|A_x|^2 + |A_y|^2 + 4E_{ext}^2 + \frac{1}{3}|A_y|^2\left(\frac{A_x^* A_y}{A_x A_y^*} - 1\right)\right], \tag{3}$$

Here, A and E_{ext} are normalized such that $|A|^2$ and E_{ext}^2 are in power unit (i.e., W). The parameter $\gamma = 3\chi k_0/(8n_L)$ and $n_L = \sqrt{\epsilon_0(1+\chi^{(1)})}$, where $\chi^{(1)}$ is the linear susceptibility.

At this point, we propose to re-write (3) in the following form:

$$\begin{aligned}\frac{\partial A_x}{\partial z} &= i\,\gamma\left[\left(|A_x|^2 + \frac{2}{3}\,|A_y|^2\right)A_x + 4\,E_{ext}^2\,A_y \right.\\ &\quad \left. + \frac{1}{3}\,\frac{A_x^* A_y}{A_x A_y^*}\,|A_y|^2\,A_x\right],\end{aligned} \tag{4}$$

here we have $(A_x^* A_y/A_x A_y^*)|A_y|^2 A_x = A_y^2 A_x^*$, yielding:

$$\begin{aligned}\frac{\partial A_x}{\partial z} &= i\gamma\left[\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x + \frac{1}{3}A_y^2 A_x^* + 4\,E_{ext}^2 A_x\right],\\ \frac{\partial A_y}{\partial z} &= i\gamma\left[\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y + \frac{1}{3}A_x^2 A_y^* + \frac{4}{3}\,E_{ext}^2 A_y\right].\end{aligned} \tag{5}$$

The above two equations can be expressed in the circular polarization bases using the following transformation:

$$\begin{aligned}A_1 &= \frac{A_x + iA_y}{\sqrt{2}}\,\exp\left(-i\frac{8}{3}\kappa_{ext}\,z\right),\\ A_2 &= \frac{A_x - iA_y}{\sqrt{2}}\,\exp\left(-i\frac{8}{3}\kappa_{ext}\,z\right).\end{aligned} \tag{6}$$

where $\kappa_{ext} = \gamma\,E_{ext}^2$. It then follows that the evolution equations are given by:

$$i\,\frac{\partial A_s(z)}{\partial z} = \kappa_{ext}\,A_{3-s} + \frac{2\gamma}{3}\left(|A_s|^2 + \frac{2}{3}|A_{3-s}|^2\right)A_s\,, \tag{7}$$

where $s = 1,2$ pertains to the right 1 and left 2 circular polarization. This equation governs the self-induced polarization rotation- and the polarization instability, resulting from a subtle balance between linear birefringence and self — as well as cross-phase modulation. This is a significant result, which is a generalization of [2,30–32]. Hereby, the polarization dynamics in this regime are controlled by a static electric field. Here, κ_{ext} (which is in m^{-1} unit) is a controlled parameter that is a function of the applied DC electric field. For $E_{ext} = 0$, the equations in (7) are identical to those in [30–32] which governs nondispersive cross-phase modulation (XPM) in birefringent fibers. The solutions in [30–32] are also applicable for short pulses (i.e., 100 ps) given that the fiber length is adequately shorter than the dispersion length and the walk-off length [2].

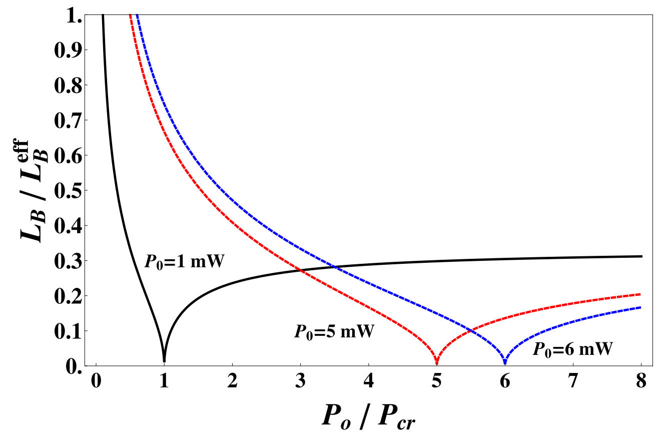

Fig. 1. (Color online) Evolution of the inverse effective beat length versus P_{cr}. Different P_0 are considered.

The polarization state is determined by the complex ratio $\xi = A_1/A_2$. The azimuth of the polarization ellipse is $\theta = (1/2)\arg(\xi)$. We consider an input beam linearly polarized at angle θ_0 with respect to the slow axis. Thus, the slow axis is represented by $\theta_0 = 0^o$ and the fast axis is represented by $\theta_0 = 90^o$.

The solutions of (7) can be sought in the form of $A_s = \left(P_{cr}\,p_s\right)^{1/2}\,\exp(i\,2\theta_0)$ [1,2], where for convenience we have defined the normalization parameter by $P_{cr} = 3\,E_{ext}^2$. Here, p_s are the normalized power in the s mode satisfying $p \equiv P_0/P_{cr} = p_s + p_{(3-s)}$, where P_0 is the total power launched into the medium.

It follows from (7) that when optical nonlinear effects are neglected, the medium shows only linear birefringence. Considering this scenario, the propagating light beams along the principal axes preserve their polarization state and the instability is not taking place. The governing equation in this case can be written as:

$$i\,\frac{\partial A_s(z)}{\partial z} = \gamma\,E_{ext}^2\,A_{3-s}. \tag{8}$$

From (8), one can infer the low-power polarization beat length, given by: $L_B = \pi/(\gamma\,E_{ext}^2)$. It is straight forward to reduce the above expressions in (8) as a system of uncoupled linear ordinary differential equations which behaves like a harmonic oscillator. See [1] for more details. For an isotropic medium, as the external electric field $E_{ext} \to 0$ the beat length also $L_B \to \infty$ [2]. However, for birefringent medium, the right hand side of (8) is given by $(\Delta\beta/2 + \gamma E_{ext}^2)A_{3-s}$, where $\Delta\beta = \beta_{0x} - \beta_{0y}$. Here, β_{0x} and β_{0y} are the propagation constants of slow and fast polarization modes, respectively. Thus, L_B approaches $(2\pi/\Delta\beta)$ as $E_{ext} \to 0$. In this work, we consider the case of isotropic optical medium. We also remark that in a case of pulse propagation, a similar system of Eqs. (7) can be obtained in the quasi-CW regime [33].

On the other hand, for intensive optical input power, the polarization evolution can be described in term of Jacobian elliptic function as detailed in [1,2]. By following the same approach, one can obtain the effective beat length from (7), given by:

$$\begin{aligned}L_B^{eff}(P_0;P_{cr}) &= \frac{2\,\mathrm{K}(m)}{\gamma\,\sqrt{|q|}}\,\frac{1}{E_{ext}^2},\\ &= \frac{2\,\mathrm{K}(m)}{\pi\sqrt{|q|}}\,L_B,\end{aligned} \tag{9}$$

where $\mathrm{K}(m)$ is the quarter-period argument of the Jacobian elliptic function. For completeness, we also present the solution of the power

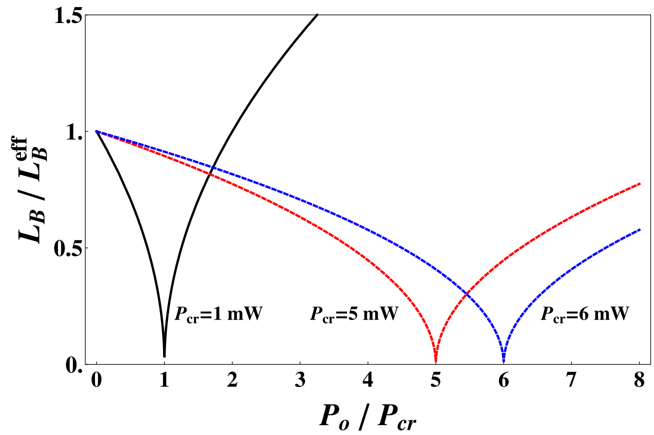

Fig. 2. (Color online) Evolution of the inverse effective beat length versus P_0. Different P_{cr} are considered.

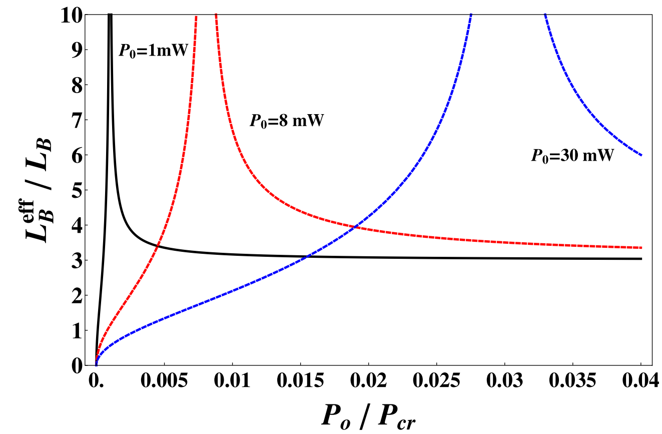

Fig. 3. (Color online) Normalized effective beat length evolution versus P_{cr}. Different P_0 are considered.

evolution p_s, given by:

$$p_s(z) = \frac{P_0}{2\,P_{cr}} - \sqrt{m|q|}\ \mathrm{Cn}\left[\sqrt{|q|}\,2\gamma\,E_{ext}^2\,z + \mathrm{K}(m)\right], \qquad (10)$$

where $\mathrm{Cn}(.|m)$ is the Jacobian elliptic function, $m = \left[1 - Re(q)/|q|\right]/2$, and $q = 1 + p\,e^{i\,2\theta_0}$. In Fig. 1, the normalized inverse effective beat length is calculated against the normalized input power p. Here, the propagating beam is polarized along the fast axes ($\theta = 90^0$). We have also computed 3 examples with different P_0. The first one is the black continuous curve (one on the left) for which we have assumed $P_0 = 1\,mW$, with the instability present at $p = 1$ when P_{cr} reaches the value of P_0. As can be seen, as P_{cr} varies, the critical power shifts as governed by (9). This scenario can be utilized for electronically controlling the optical switching. Also, the instability broadens in terms of the power p while having smaller interval of L_B^{eff} affected by the instability for larger P_{cr} (i.e., larger DC electrical field). While similar what was observed previously in [1], the beat length monotonically decreases for power values increased beyond the critical power.

Fig. 2 is devoted to show the inverse beat length versus P. This is illustrated by computing some examples of very distinctive regions for constant P_{cr} while P_0 is increased. Similar to Fig. 1, the effective beat length becomes infinite as input power becomes identical to the critical power. Further increment in the input power turns the fiber birefringent again but with reversed slow and fast axes. Once the condition for the instability is passed (the input power is increased beyond the critical power), L_B^{eff} decreases monotonically in a similar behavior to the case of slow axis oriented beam.

Finally, we present an illustrative example using real experimental parameters. We consider an optical fiber with 6.6 μm effective mode radius, $n_L = 1.46$ refractive index, and $\gamma = 0.0043\,W^{-1}m^{-1}$ nonlinear coefficient. The corresponding normalized effective beat length for these values is presented in Fig. 3 as function of P_{cr}. As an example, if one considers $P_0 = 1\,mW$ and $E'_{ext} = 80\,V/\mu m$, the beat lengths are $L_B = 3.11\,m$ and $L_B^{eff} = 9.35\,m$. We note that the quantity $E_{ext} = (2\,n_L\,A_{eff}^2\,\epsilon_0\,c)^{1/2}\,E'_{ext}$ in our theoretical description is normalized so that E_{ext}^2 is in $[W]$ unit, while E'_{ext} is the physical applied DC electric field in $[V/m]$ and c is the light speed in free space.

The presented scheme in this work is also applicable for micro-photonic devices including planar waveguides and photonic integrated circuits. Several interesting devices/systems are natural candidates to benefit from electronically controlled polarization instability. These include fiber laser devices that incorporate birefringent cavities [34], supercontinuum photonic crystal fibers that use polarization dynamics of Raman solitons [35], vector cavity solitons in birefringent resonators [36], semiconductor lasers that utilize vertical cavity resonators, and vertical cavity surface-emitting lasers, just to mention few examples. Alternatively, the proposed modality can be devised as a sensitive polarizer (utilizing the polarization instability) that is directly integrable with a specific device (e.g., a semiconductor vertical-cavity surface-emitting laser) to monitor its extreme operation [37].

In conclusion, we have theoretically demonstrated the possibility of controlling the polarization instability of optical fields propagating in Kerr nonlinear medium through applying an external electric field. The proposed scheme implies electronically modifying the effective birefringence of the medium and thus varying the critical power required for the polarization instability.

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CRediT authorship contribution statement

Artorix de la Cruz: Conceptualization, Methodology, Software, Formal analysis, Investigation, Writing - original draft, Writing - review & editing, Visualization. **Montasir Qasymeh:** Investigation, Formal analysis, Writing - review & editing. **Jaromir Pistora:** Supervision. **Michael Cada:** Project administration, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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