Contents lists available at ScienceDirect

# **Engineering Fracture Mechanics**

journal homepage: www.elsevier.com/locate/engfracmech

# Suitability of fatigue crack growth thresholds at negative stress ratios for ferritic steels and aluminum alloys in flaw evaluation procedures

Kunio Hasegawa <sup>a,\*</sup>, David Dvořák<sup>a</sup>, Vratislav Mareš<sup>a</sup>, Bohumir Strnadel<sup>a</sup>, Saburo Usami<sup>b</sup>

<sup>a</sup> Centre for Advanced and Innovative Technologies – VŠB-Technical University of Ostrava, 17. listopadu 2172/15, Ostrava-Poruba, 708 00, Czech Republic

<sup>b</sup> Hitachi Ltd., 1-1-1 Kokubu-cho, Hitachi-shi, Ibraraki-ken, 316-8501, Japan

## ARTICLE INFO

Keywords: Fatigue crack growth threshold Stress ratio Stress intensity factor range Flaw evaluation Ferritic steel Aluminum alloy

#### ABSTRACT

Fatigue crack growth thresholds for ferritic steels and aluminum alloys in flaw evaluation documents are reviewed. The WRC (Welding Research Council) Bulletin, ASME (American Society of Mechanical Engineers) Code Section VIII, IIW (International Institute of Welding) and BS (British Standards) 7910 give constant fatigue crack growth threshold values at negative stress ratios. However, the definitions of the thresholds at negative stress ratios are different between these flaw evaluation documents. From fatigue crack growth tests and collection of fatigue crack growth threshold data, the thresholds are affected by compressive stresses. It can be said that the thresholds at negative stress ratios are not constant. Therefore, the threshold given by the WRC Bulletin, ASME Section VIII and IIW are slightly un-conservative. The threshold given by BS 7910 is considerably conservative. A suitable definition of the threshold at negative stress ratios for application in flaw evaluation procedures is proposed as the variable threshold expressed by full range of stress intensity factors.

## 1. Introduction

It is well known that fatigue crack growth rate da/dN is expressed by stress intensity factor range  $\Delta K_I$ , and the relationship between da/dN and  $\Delta K_I$  is used for reference fatigue crack growth rate curves applied in flaw evaluation procedures such as fitness-for-service codes [1–3].

Fatigue crack growth (FCG) thresholds  $\Delta K_{th}$  are the stress intensity factor range  $\Delta K_I$  values below which fatigue crack growth rates da/dN are negligible. The thresholds of fatigue crack growth rates are important in order to determine whether detected defects propagate or not. FCG thresholds for many materials were published as a data book for practical engineering applications [4]. The thresholds are also employed in several flaw evaluation documents, such as the Welding Research Council (WRC) Bulletin [5], American Society of Mechanical Engineers (ASME) Code Section VIII [6] and Section XI [7], International Institute of Welding (IIW) [8] and British Standards (BS) 7910 [9].

Fatigue crack growth thresholds  $\Delta K_{th}$  are difficult to obtain by experiments, because it takes a long time to determine whether the

\* Corresponding author. *E-mail address:* kunioh@kzh.biglobe.ne.jp (K. Hasegawa).

https://doi.org/10.1016/j.engfracmech.2021.107670

Received 13 November 2020; Received in revised form 28 January 2021; Accepted 7 March 2021

Available online 26 March 2021







<sup>0013-7944/© 2021</sup> The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

Nomenclature

Homene	
da/dN	cyclic crack growth rate
K <sub>max</sub>	maximum stress intensity factor
K <sub>max-th</sub>	maximum stress intensity factor threshold
K <sub>min</sub>	minimum stress intensity factor
$K_{op}$	crack opening stress intensity factor
R	stress ratio ( = $\sigma_{min}/\sigma_{max} = K_{min}/K_{max}$ )
U	crack opening stress ratio $(=(K_{max}-K_{op})/(K_{max}-K_{min}))$
$\Delta K_I$	stress intensity factor range
$\Delta K_{eff}$	effective stress intensity factor range ( = $U \cdot \Delta K_I$ )
$\Delta K_{th}$	fatigue crack growth threshold
$\Delta K_{th0}$	fatigue crack growth threshold at $R = 0$
$\sigma_{max}$	maximum applied stress
$\sigma_{min}$	minimum applied stress

crack is growing or not. Almost all FCG thresholds for metals have been determined under cyclic tensile-tensile loading, which corresponds to positive stress ratios. There are many literatures on the relationship between FCG thresholds at positive stress ratios [10-13]. In the range of positive stress ratios, most of materials show apparently linear increase in FCG thresholds with deceasing stress ratios [14-16]. Compared with the values determined under tensile-tensile loading, there are not sufficient data on FCG thresholds determined under cyclic tensile-compressive loading, equal to negative stress ratios. In the range of negative stress ratios, the relationship between FCG thresholds and stress ratios shows constant values or decrease with decreasing stress ratio [17]. It was not easy to determine thresholds at negative stress ratio without sufficient data. The WRC Bulletin, ASME Section XII provides a constant FCG thresholds for ferritic steels and aluminum alloys at negative stress ratios. ASME Section XI provides a constant FCG threshold for ferritic steels.

Fatigue crack growth tests for many steels and aluminum alloys at positive and negative stress ratios in an air environment have been performed by Usami, one of the authors [18]. FCG thresholds for ferritic steels and aluminum alloys have been recently proposed for wide range of stress ratios [19–21]. However, these flaw evaluation documents on FCG thresholds at negative stress ratios differ with the author's proposal. Besides, the definitions of the thresholds at negative stress ratios are found to be different between these documents.

This paper reviewed the thresholds for ferritic steels and aluminum alloys at negative stress ratios provided by the WRC Bulletin, ASME Section VIII, IIW and BS 7910. From fatigue crack growth tests and collection of FCG thresholds, it is shown that a suitable definition of the threshold at negative stress ratios for application in flaw evaluation procedures is the variable threshold expressed by full range of stress intensity factors.

## 2. Definitions of FCG thresholds at negative stress ratios R

Experimental data on FCG thresholds for ferritic steels and aluminum alloys gathered from the 1970s to the 2000s was employed the WRC Bulletin 194 [5], ASME Code Section VIII [6], IIW [8] and BS 7910 [9]. During that period there were not enough data on FCG thresholds at negative *R*.

According to the 2001 edition of ASTM E 647 [22], fatigue crack growth rate da/dN is expressed by a function of the stress intensity factor range  $\Delta K_I$ , and the growth rate da/dN depends on the applied stress ratio R, where the stress ratio is given by  $R = \sigma_{min}/\sigma_{max} = K_{min}/K_{max}$  are the minimum and maximum cyclic stresses, and  $K_{min}$  and  $K_{max}$  are the minimum and maximum stress intensity factors. The stress intensity factor range  $\Delta K_I$  is expressed by

$$\Delta K_I = K_{max} - K_{min} \quad \text{for } 0 \le R, \text{ and} \\ \Delta K_I = K_{max} \quad \text{for } R < 0$$
(1)

When the stress ratio is  $0 \le R$ , the fatigue crack growth rate da/dN depends on the range of the stress intensity factor  $K_{max} - K_{min}$ . When the stress ratio is negative (R < 0), the stress intensity factor range  $\Delta K_I = K_{max} - 0$ . This is because applied compressive stress does not contribute to the crack growth parameter. ASTM E 647 notes that it is conventional to use only the positive portion of the stress range to calculate the crack growth force. It can be inferred that the definition of the thresholds given in the WRC Bulletin, ASME Section VIII and IIW use Eq. (1).

Note that in the 2015 edition of ASTM E 647, an alternative definition is given for the full stress intensity factor range  $K_{max} - K_{min}$  for all stress ratios R, because it is beneficial to compare fatigue crack growth rates for  $R \le 0$  and R > 0 conditions and to understand the effect of compressive stress on fatigue crack growth rates.

#### 3. FCG thresholds given in flaw evaluation documents

#### 3.1. Thresholds for ferritic steels in flaw evaluation documents

Fatigue crack growth thresholds  $\Delta K_{th}$  given in the WRC Bulletin 194 [5], ASME Code Section VIII [6], IIW [8] and BS 7910 [9] for ferritic steels in an air environment are illustrated in Fig. 1. ASME Code Section XI [7] also gives similar FCG thresholds to the WRC Bulletin. The thresholds of Section XI are currently under discussion at the Working Group of the ASME Section XI Code Committee, and the revised thresholds will be published in the near future. The thresholds given by Section XI are therefore excluded from this paper for reasons of brevity.

The FCG thresholds given in the WRC Bulletin, ASME Code Section VIII, IIW and BS 7910 are expressed by a function of stress ratio R, as shown in Fig. 1. When the stress ratio R is negative,  $\sigma_{min}$  is a compressive stress and  $K_{min}$  becomes a negative value.

The WRC Bulletin 194 gives FCG thresholds for mild, low-alloy and austenitic stainless steels as follows:

$$\Delta K_{th} = 6.4(1 - 0.85R), \text{ksi}\sqrt{\text{in.}} = 7.0(1 - 0.85R), \text{MPa}\sqrt{\text{m}} \quad \text{for } 0.1 < R < 1.0, \text{ and} \\ \Delta K_{th} = 5.5, \text{ksi}\sqrt{\text{in.}} = 6.0, \text{MPa}\sqrt{\text{m}} \quad \text{for } R \le 0.1.$$
(2)

The threshold  $\Delta K_{th}$  increases with decreasing stress ratio *R*. When R < 0.1,  $\Delta K_{th}$  is given by a constant value, as shown in Fig. 1. The ASME Code Section VIII gives thresholds for ferritic steels for entire ranges of stress ratios *R*. For high strength low alloy steels when yield stress 630 MPa is exceeded and for martensitic precipitation hardening steels,  $\Delta K_{th}$  is given as lesser than 7.0(1 –0.85*R*) or 6.0, and not lesser than 2.2 MPa $\sqrt{m}$ . This is the same equation as Eq. (2).

The IIW Commission gives thresholds  $\Delta K_{th}$  in units of MPa $\sqrt{m}$  for ferritic steels at room temperature.  $\Delta K_{th}$  for ferritic steels is given as follows:

$$\Delta K_{th} = 2.0 for 0.5 \le R, \Delta K_{th} = 5.4 - 6.8R for 0 \le R < 0.5, and \Delta K_{th} = 5.4 for R < 0. (3)$$

For elevated temperatures other than room temperature or for metallic materials other than steel,  $\Delta K_{th}$  may be determined by  $\Delta K_{th} = \Delta K_{th,steel} \cdot (E/E_{steel})$ , where  $\Delta K_{th,steel}$  is given by Eq. (3) and *E* is the modulus of elasticity at elevated temperature or for the metallic materials.  $E_{steel}$  is the modulus of elasticity for steels at room temperature. The threshold  $\Delta K_{th}$  for ferritic steels is a constant value for R < 0.

BS 7910 gives recommended fatigue crack growth thresholds for steels (excluding austenitic steels) and aluminum alloys for assessing welded joints and unwelded components. For unwelded steels (excluding austenitic steels) in air and with cathodic protection in marine environments at temperatures up to 20 °C, the thresholds  $\Delta K_{th}$  given by BS 7910 are the same as in Eq. (3).

Table 1 shows the FCG thresholds for ferritic steels given in the WRC Bulletin 194, ASME Code Section VIII, IIW and BS 7910. The values of the FCG thresholds are almost the same in all these flaw evaluation documents.

## 3.2. Thresholds for aluminum alloys in flaw evaluation documents

Fatigue crack growth thresholds  $\Delta K_{th}$  for aluminum alloys given by ASME Section VIII, IIW and BS 7910 are depicted in Fig. 2. The thresholds are expressed for an entire range of stress ratios *R*.

FCG thresholds  $\Delta K_{th}$  for aluminum alloys provided by ASME Section VIII are expressed as "lesser 2.34(1 – NA · *R*) or 2.01 MPa $\sqrt{m}$ , but needed not to be less than 0.73 MPa $\sqrt{m}$ , where NA means not available". This is expressed as



Fig. 1. Fatigue crack growth thresholds for ferritic steels provided by the WRC Bulletin, ASME Section VIII, IIW and BS 7910.

#### Table 1

Fatio	7110	crack	growth	thresholds	for f	erritic	steels	and	aluminum	allows	given	in f	flaw	evaluation	documen	te
raux	sue i	LIACK	growui	unesitoius	101 1	ennuc	SICCIS	anu	aiuiiiiiiuiii	anoys	given	111 1	llavv	evaluation	uocumen	ιs.

Flaw Evaluation Document	$\Delta K_{th}$ , MPa $\sqrt{\mathrm{m}}$	Definition of $\Delta K_{th}$	
	Ferritic steel	Aluminum	
WRC Bulletin 194	7.0(1–0.85 <i>R</i> ) for $0.1 < R < 1.0$ 6.0 for $R \le 0.1$	_	$K_{\max}^{**}$
ASME Code Sec. VIII	Lesser 7.0(1.0–0.85 <i>R</i> ) or 6.0 But not less than 2.2	$min(2.34(1 - NA^* \cdot R, 2.01))$ and $\geq 0.73$	$K_{\max}^{**}$
IIW Commission	2.0 for $0.5 \le R$ 5.4-6.8R for $0 \le R < 0.5$	$0.7 \text{ for } 0.5 \le R$ $1.8-2.3R \text{ for } 0 \le R < 0.5$	K <sub>max</sub> **
BS 7910	5.4 for $R < 0$ 2.0 for 0.5 $\leq R$ 5.4–6.8R for 0 $\leq R < 0.5$ 5.4 for $R < 0$	1.8 for <i>R</i> < 0 0.7 for all <i>R</i>	K <sub>max</sub> - K <sub>min</sub>

Note: \*NA means Not Available, \*\* The authors have predicted the threshold definition because the definition is not stipulated in the documents.



Fig. 2. Fatigue crack growth thresholds for aluminum alloys provided by ASME Section VIII, IIW and BS 7910.

$$\Delta K_{th} = \min(2.34(1 - NA \cdot R), 2.01), \text{ and}$$

$$\Delta K_{th} \ge 0.73 \text{MPa}\sqrt{\text{m}}$$
(4)

Therefore,  $\Delta K_{th}$  for aluminum alloys given by ASME Section VIII is between 0.73 (lower) and 2.01 MPa $\sqrt{m}$  (upper), as shown in Fig. 2.

The threshold  $\Delta K_{th}$  in units of MPa $\sqrt{m}$  for aluminum alloys given by IIW is as follows:

$$\Delta K_{th} = 0.7 for 0.5 \le R, \Delta K_{th} = 1.8 - 2.3R for 0 \le R < 0.5, and \Delta K_{th} = 1.8 for R < 0. (5)$$

The threshold  $\Delta K_{th}$  for aluminum alloys is also a constant value at R < 0. Fig. 2 depicts the threshold for aluminum alloys given by IIW.

For welded joint aluminum alloys, the threshold in air or non-aggressive environments at temperatures up to 20  $^{\circ}$ C is given by BS 7910, as follows:

$$\Delta K_{th} = 0.7 \quad \text{for all } R, \tag{6}$$

where the threshold in units of  $MPa\sqrt{m}$  for aluminum alloys is a constant value at both positive and negative stress ratios.

Fatigue crack growth thresholds for aluminum alloys are tabulated in Table 1. Compared with FCG thresholds for ferritic steels, the FCG thresholds for aluminum alloys differ among these flaw evaluation documents. IIW gives a large FCG threshold and BS 7910 gives a small FCG threshold. ASME Section VIII does not give precise FCG thresholds, but the maximum value of the threshold is close to that given by IIW and the minimum value of the threshold is close to that given by BS 7910.

#### 3.3. Definitions of FCG thresholds in flaw evaluation documents

The FCG threshold given by the WRC Bulletin is well known, and it is important since the three flaw evaluation documents being reviewed. The WRC Bulletin does not define the  $\Delta K_{th}$  for negative stress ratios. It appears that the FCG threshold at negative stress

.

ratios is expressed by  $K_{max}$ , estimated from the original paper [17]. Although ASME Section VIII likewise does not stipulate the definition of  $\Delta K_{th}$  at R < 0,  $\Delta K_{th}$  is deemed to be  $K_{max}$  only. This is because the threshold is the same equation as that in the WRC Bulletin. The IIW Commission document does not give the definition of the FCG threshold at R < 0. The threshold at R < 0 is inferred to be  $K_{max}$  only.

BS 7910 stipulates the definition of the FCG threshold. The requirement stipulated in BS 7910 (in the chapter "Assessment for Fatigue") is to "use full stress range regardless of applied stress ratio (*R*)". The full stress range means  $\sigma_{max} - \sigma_{min}$ , which corresponds to  $K_{max} - K_{min}$ . The definition of the threshold  $\Delta K_{th}$  for R < 0 is  $\Delta K_{th} = K_{max} - K_{min}$ .

Table 1 shows the definitions of  $\Delta K_{th}$  for the WRC Bulletin, ASME Code Section VIII, IIW and BS 7910. The definitions of the thresholds for R < 0 take two forms:  $\Delta K_{th} = K_{max}$  or  $\Delta K_{th} = K_{max} - K_{min}$ . The expression by either  $\Delta K_{th} = K_{max}$  or  $\Delta K_{th} = K_{max} - K_{min}$  is significantly different from the results of actual flaw evaluation analyses.

## 4. FCG threshold tests and thresholds based on data collection

#### 4.1. FCG threshold test at negative stress ratios

In order to clarify the expression of FCG thresholds at R = 0 and R < 0, fatigue crack growth tests for ferritic steels and aluminum alloys performed by Usami have reviewed [18,23]. The test was conducted using a Vibrophore fatigue testing machine (100 kN load capacity) in air at ambient temperature. In accordance with ASTM E-647, the definition of  $\Delta K_{th}$  is given as that which corresponds to a fatigue crack growth rate of  $10^{-7}$ mm/cycle [22]. Fatigue crack growth rates da/dN and fatigue thresholds  $\Delta K_{th}$  were taken by using flat plate tension specimens with through-wall center cracks. Fatigue crack growth rates were acquired by decreasing the load amplitude until crack growth was not observed. In order to avoid the effect of plastic zones at crack tips on FCG thresholds, the ratio of decrease of the load amplitude complied with ASTM E-647. The effect of load history on FCG thresholds is considered to be small because the specimens employed were center cracked flat plate tension specimens. The thresholds were determined after  $2 \times 10^7$  cycles by a microscope at  $50 \times$  magnification with a minimum scale of division of 0.02 mm.

The materials employed were ferritic steels and aluminum alloys. There were four steels: JIS (Japan Industrial Standards) SM 410 (class 410 rolled steel for welded structures), JIS HT 800 (class 800 high tensile strength steel), A533-B (low-alloy steel weldments), and JIS FC 200 (gray cast iron). The mechanical properties of these steels and the geometries of the center cracked flat plate specimens are shown in Table 2. The aluminum alloys used in the tests were JIS A1100-0 (with 99.7% aluminum) and JIS AC4A-T6 cast aluminum. The mechanical properties of the specimens for aluminum alloys are tabulated in Table 3.

Fatigue crack growth rates da/dN versus stress intensity factor range  $\Delta K_I$  for ferritic steels and aluminum alloys were obtained for wide ranges of positive and negative stress ratios. Figs. 3 and 4 show da/dN versus  $\Delta K_I$  at only R = 0 and R < 0, for the purposes of brevity. The definition of the stress intensity factor range is  $\Delta K_I = K_{max} - K_{min}$  for R = 0 and for R < 0. When R = 0, the stress intensity factor range becomes  $\Delta K_I = K_{max}$ , because of  $K_{min} = 0$ . As can be seen in Figs. 3 and 4, fatigue crack growth rates da/dN for ferritic steels and aluminum alloys decrease with decreasing stress ratios R, and the thresholds  $\Delta K_{th}$  increase with decreasing stress ratios R. It is obvious that  $\Delta K_{th}$  are not constant at R < 0. The thresholds  $\Delta K_{th}$  for ferritic steels and aluminum alloys obtained from Figs. 3 and 4 are tabulated in Table 4.

#### 4.2. Fatigue thresholds based on data collection

Hasegawa, et al. had proposed FCG thresholds  $\Delta K_{th}$  for ferritic steels and aluminum alloys at a wide range of stress ratios *R*, based on data collection from a literature survey. Fig. 5 shows the thresholds  $\Delta K_{th}$  as a function of stress ratio *R*.

For ferritic steels [21],

$\Delta K_{th} = 5.5(1 - 0.8R)$	for $R < 0.8$ , and	
$\Delta K_{th} = 2.0$	for $0.8 \le R < 1.0$ .	

For aluminum alloys [20],

 $\Delta K_{th} = 1.7 - R \quad \text{for all } R,$ 

Table 2					
Mechanical	properties and te	st conditions	for	ferritic	steel

Ferritic steel	Mechanical p	roperties			Test specimen and conditions				
	$\sigma_y$ , MPa	$\sigma_u$ , MPa	δ, %	$\phi$ , %	W, mm	t, mm	2a, mm	<i>f</i> , Hz	
SM 410	270	446	41.9	69.8	100	5	25.0	120	
HT 800	804	859	25.9	67.9	70	5	17.6	120	
A533-B weld	484	586	33.1	73.4	70	5	17.6	120	
FC200 cast	180	240	0.6	-	70	5	15.0	120	

Note:  $\sigma_y$  = yield stress,  $\sigma_u$  = ultimate tensile strength,  $\delta$  = elongation,  $\phi$  = reduction of area, W = specimen width, t = specimen wall thickness, 2a = initial total notch length, f = load frequency.

(7)

(8)

#### Table 3

Mechanical properties and test conditions for aluminum alloys.

Aluminum alloy	Mechanical pr	Mechanical properties			Test specimen and conditions			
	$\sigma_y$ , MPa	$\sigma_u$ , MPa	δ, %	W, mm	t, mm	2 <i>a</i> , mm	<i>f</i> , Hz	
A1100-0, 99.7% Al AC4A-T6, Al-Si cast	31 -	113 225	28.0 0.1	100 70	6 5	15.0 17.6	120 120	

Note:  $\sigma_y =$  yield stress,  $\sigma_u =$  ultimate tensile strength,  $\delta =$  elongation, W = specimen width, t = specimen wall thickness, 2a = initial total notch length, f = load frequency.



**Fig. 3.** Fatigue crack growth rates at R = 0 and R < 0 for ferritic steels in air at room temperature.



Fig. 4. Fatigue crack growth rates at R = 0 and R < 0 for aluminum alloys in air at room temperature.

#### Table 4

Test results for fatigue thresholds $\Delta k$	K <sub>th</sub> , K <sub>max-th</sub> an	nd ratio <i>K<sub>max-th</sub>/</i>	$\Delta K_{th0}$ for	ferritic steels and	aluminum alloys
--	--	-------------------------------------	----------------------	---------------------	-----------------

Materials		Stress ratio R	$\Delta K_{th}$ , MPa $\sqrt{m}$	$K_{max-th}$ , MPa $\sqrt{m}$	$K_{max-th}/\Delta K_{th0}$
Ferritic steels	SM 410	0	8.8	8.8	1.0
		-1	15.0	7.5	0.85
	HT 800	0	6.3	6.3	1.0
		-1	12.0	6.0	0.95
	A533-B weld	0	6.4	6.4	1.0
		-1	11.0	5.5	0.86
		-5	27.5	4.6	0.72
	FC 200 cast	0	5.6	5.6	1.0
		-1	11.2	5.6	1.0
Aluminum alloys	A1100-0, 99.7% Al	0	1.9	1.9	1.0
		$^{-1}$	3.2	1.6	0.84
	AC4A-T6, Al-Si cast	0	2.5	2.5	1.0
		-1	4.8	2.4	0.96
		-2	7.0	2.3	0.92



Fig. 5. Fatigue crack growth thresholds for ferritic steels and aluminum alloys based on collection of experimental data [20,21].

where  $\Delta K_{th}$  is expressed by stress intensity factor range  $K_{max} - K_{min}$ . The applicable ferritic steels for Eq. (7) are mild steels, cast steel and high tensile strength steels, except maraging steels. Aluminum alloys for Eq. (8) are high purity aluminum, cast aluminum, 2xxx and 7xxx series. Experimental threshold data for ferritic steels and aluminum alloys shown in Figs. 3 and 4 are plotted in Fig. 5 for references.

As shown in Fig. 5, the thresholds  $\Delta K_{th}$  for ferritic steels and aluminum alloys increase with decreasing stress ratio *R*. The trend of thresholds  $\Delta K_{th}$  at R < 0 differs from the trends given by the flaw evaluation documents.

#### 5. Effect of compressive stress on FCG thresholds

FCG thresholds  $\Delta K_{th}$  can be expressed by maximum stress intensity factors  $K_{max}$  using stress ratios R, where  $K_{max}$  at the threshold is replaced to give  $K_{max-th}$ . The WRC Bulletin, ASME Section VIII and IIW give constant thresholds  $\Delta K_{th}$  at negative stress ratios R, and these definitions at negative stress ratios are  $\Delta K_{th} = K_{max}$ , as mentioned above. Therefore,  $K_{max-th}$  for R < 0 are equal to  $\Delta K_{th}$  at R = 0. Figs. 6 and 7 show  $K_{max-th}$  for ferritic steels and aluminum alloys, respectively. Maximum stress intensity factor thresholds  $K_{max-th}$ given by the WRC Bulletin, ASME Section VIII and IIW are constant at R < 0.

BS 7910 also gives constant thresholds  $\Delta K_{th} = 5.38$  MPa $\sqrt{m}$  for ferritic steels and  $\Delta K_{th} = 0.7$  MPa $\sqrt{m}$  for aluminum alloys at R < 0, as shown in Eqs. (3) and (6). In accordance with BS 7910,  $\Delta K_{th}$  is given by  $K_{max} - K_{min}$ , which is equal to  $K_{max} - RK_{max}$ . Therefore, the thresholds are expressed as  $(1 - R)K_{max} = 5.38$  for ferric steels and  $(1 - R)K_{max} = 0.7$  for aluminum alloys, when R < 0. Then,  $K_{max-th}$  are expressed by

$$K_{max-th} = 5.38/(1-R)$$
 for ferritic steels, and (9)

$$K_{max-th} = 0.7/(1-R)$$
 for aluminum alloys. (10)

Eqs. (9) and (10) are depicted in Figs. 6 and 7. The maximum stress intensity factor thresholds  $K_{max-th}$  significantly decrease with decreasing stress ratios *R*.

Thresholds  $\Delta K_{th}$  determined by the fatigue test data and collection of fatigue thresholds are variable values at negative stress ratios, as shown in Eqs. (7) and (8).  $\Delta K_{th}$  is expressed by  $K_{max} - K_{min}$  at R < 0. Therefore, the maximum stress intensity factor thresholds  $K_{max-th}$ 

(12)



Fig. 6. Maximum stress intensity factor thresholds K<sub>max-th</sub> for ferritic steels given by the WRC Bulletin, ASME Code Section VIII, IIW and BS 7910.



Fig. 7. Maximum stress intensity factor thresholds  $K_{max-th}$  for aluminum alloys given by the ASME Code Section VIII, IIW and BS 7910.

are also given by

$$K_{max-th} = 5.5(1 - 0.8R)/(1 - R) \quad \text{for ferritic steels, and}$$
(11)

 $K_{max-th} = (1.7 - R)/(1 - R)$  for aluminum alloys.

The relationship between  $K_{max-th}$  and R for ferritic steels and aluminum alloys are shown in Figs. 6 and 7, respectively. It is shown that the value of  $K_{max-th}$  at R < 0 differs significantly from the values determined via either  $\Delta K_{th} = K_{max} - K_{min}$  or  $\Delta K_{th} = K_{th} - K$ 



**Fig. 8.** Relationship between  $K_{max-th}/\Delta K_{th0}$  and stress ratio *R*, where  $K_{max-th}$  is the maximum stress intensity factor threshold and  $\Delta K_{th0}$  is the threshold at R = 0.

 $K_{max}$ , and from the values given by either constant or variable  $\Delta K_{th}$ .

From the values of  $K_{max-th}$  and R as shown in Figs. 6 and 7,  $K_{max-th}$  are normalized by  $\Delta K_{th0}$ , where  $\Delta K_{th0}$  is the threshold at R = 0. The normalized  $K_{max-th}/\Delta K_{th0}$  can be seen the effect of compressive stress for ferritic steels and aluminum alloys, together. Fig. 8 illustrates the ratio  $K_{max-th}/\Delta K_{th0}$  as per the WRC Bulletin, ASME Section VIII, IIW, BS 7910 and the proposed thresholds based on test data.

The ratios  $K_{max-th}/\Delta K_{th0}$  as per the WRC Bulletin, ASME Section VIII and IIW are constant at 1.0, irrespective of materials, because  $\Delta K_{th} = K_{max} = \text{constsnt}$  at all negative ratios *R*.  $K_{max-th}$  are not affected by compressive stresses, even when the compressive stresses are large.

The definition of the threshold in BS 7910 is expressed by  $K_{max} - K_{min} = \text{constant}$ , and the  $K_{max}$  is expressed by Eqs. (9) and (10) for ferritic steels and aluminum alloys. As  $\Delta K_{th0} = 5.38 \text{ MPa}\sqrt{\text{m}}$  for ferritic steels and  $\Delta K_{th0} = 0.7 \text{ MPa}\sqrt{\text{m}}$  for aluminum alloys, the ratio  $K_{max-th}/\Delta K_{th0}$  as per BS 7910 is easily obtained by

$$K_{max-th}/\Delta K_{th0} = 1/(1-R)$$
(13)

The ratio  $K_{max-th}/\Delta K_{th0}$  denoted as per BS 7910 is also irrespective of materials. The ratio  $K_{max-th}/\Delta K_{th0}$  significantly decreases with decreasing stress ratio R, as shown in Fig. 8. This means that  $K_{max-th}$  is significantly affected by cyclic compressive stress.

The ratios  $K_{max-th}/\Delta K_{th0}$  as per the fatigue thresholds based on test data are obtained by Eqs. (11) and (12), where  $\Delta K_{th0} = 5.5$  MPa $\sqrt{m}$  for ferritic steels and  $\Delta K_{th0} = 1.7$  MPa $\sqrt{m}$  for aluminum alloys.

$$K_{max-th}/\Delta K_{th0} = (1 - 0.8R)/(1 - R) \quad \text{for ferritic steels, and}$$
(14)

$$K_{max-th}/\Delta K_{th0} = (1.7 - R)/(1.7(1 - R))$$
 for aluminum alloys. (15)

As can be seen in Fig. 8,  $K_{max-th}/\Delta K_{th0}$  decreases slowly with decreasing *R*.  $K_{max-th}$  is gradually affected by compressive stress. Table 4 shows the maximum stress intensity factor thresholds  $K_{max-th}$  and the ratios  $K_{max-th}/\Delta K_{th0}$  for ferritic steels and aluminum alloys obtained by Figs. 3 and 4. The ratios  $K_{max-th}/\Delta K_{th0}$  are plotted in Fig. 8. All data are <1.0, except for FC 200 cast steels. The results for  $K_{max-th}$  suggest that  $K_{max-th}$  are affected by cyclic compressive stresses.

#### 6. Discussions on thresholds in flaw evaluation procedures

Fatigue crack growth rates da/dN are well explained by effective stress intensity factor range  $\Delta K_{eff}$ , where  $\Delta K_{eff}$  is given by  $K_{max} - K_{op}$  and  $K_{op}$  is the crack opening stress intensity factor [24–26].  $\Delta K_{eff}$  represents the stress intensity factor range while the crack tip is opening by applied cyclic loading.  $\Delta K_{eff}$  is given as  $\Delta K_{eff} = U \times \Delta K_{I}$ , where U is the crack opening stress ratio expressed by  $U = (K_{max} - K_{op})/(K_{max} - K_{min})$ . There are several proposals of the crack opening stress ratios [27–32]. Fatigue thresholds were also expressed by the effective stress intensity factor range  $\Delta K_{eff}$  [33].  $\Delta K_{eff}$  at thresholds for ferritic steels were normally obtained by compact tension specimens, center cracked flat plate specimens, single edge crack tension specimens, etc.  $\Delta K_{eff}$  at thresholds are expressed as constant values at positive stress ratios [34].

From a perspective of practical application, the expression of  $\Delta K_{eff}$  at thresholds is not convenient for engineers. This is because actual plant components are not simple geometries and defects for evaluations are surface/subsurface cracks with various aspect ratios. Besides, the crack opening stress ratios *U* depend on materials, applied stress [28], stress ratio *R* [35], etc. It is difficult to measure  $K_{op}$  for complex geometry components with surface/subsurface cracks and to use crack opening stress ratios *U*. Therefore, fatigue thresholds expressed by  $\Delta K_{eff}$  are inconvenient at flaw evaluation, when engineers encounter problems of trouble shooting. They can easily obtain  $K_{max}$  and  $K_{min}$  from stress intensity factor handbooks [36,37], codes and standards [3,7,38] and/or FEM (finite element method) analyses for components with defects. Fatigue thresholds expressed by only  $K_{max}$  and  $K_{min}$  are useful for flaw evaluations, instead of  $\Delta K_{eff}$ .

From fatigue thresholds obtained by the fatigue tests and the collection of thresholds data, it is apparent that the thresholds expressed by  $K_{max}$  are affected by cyclic compressive stresses. That is, the thresholds decrease with decreasing compressive stresses. However, fatigue crack growth thresholds  $\Delta K_{th}$  given by the WRC Bulletin, ASME Section VIII, IIW and BS 7910 are constant, as mentioned above. The definition of  $\Delta K_{th}$  for these flaw evaluation documents is either  $K_{max}$  or  $K_{max} - K_{min}$ . It can be said that the thresholds expressed by  $K_{max}$  for the WRC Bulletin, ASME Section VIII and the IIW are not affected by compressive stresses and they are slightly un-conservative. Thresholds expressed by  $K_{max} - K_{min}$  for BS 7910 are strongly affected by compressive stresses and the thresholds are considerably conservative.

From the perspective of application in flaw evaluation procedures, the definition of thresholds at negative stress ratios can suitably be written using variable threshold values with  $K_{max} - K_{min}$ , because fatigue tests are conducted on testing machines by pre-setting the values  $\sigma_{max}$  and  $\sigma_{min}$ , so the tests proceed within this interval. A suitable definition of the threshold  $\Delta K_{th}$  at negative stress ratio for application in flaw evaluation procedures is a variable threshold expressed by full range of stress intensity factor.

#### 7. Conclusions

Fatigue thresholds  $\Delta K_{th}$  for ferritic steels and aluminum alloys given in the WRC Bulletin, ASME Section VIII, IIW and BS 7910 are constant values at negative stress ratio *R*. However, there are two types of definitions of  $\Delta K_{th}$ :  $K_{max}$  in the WRC Bulletin, ASME Section VIII and IIW, and  $K_{max} - K_{min}$  in BS 7910.

From the review of fatigue tests and collection of threshold data, it was found that the thresholds expressed by  $K_{max} - K_{min}$  increase with decreasing stress ratio R, and the thresholds expressed by  $K_{max}$  decrease with decreasing stress ratio R. Maximum stress intensity factor thresholds  $K_{max-th}$  are affected by cyclic compressive stress.  $\Delta K_{th}$  expressed by either  $K_{max} - K_{min}$  or  $K_{max}$  are not constant at negative stress ratios. Therefore, the thresholds given in the WRC Bulletin, ASME Section VIII and IIW are slightly un-conservative. The thresholds given in BS 7910 are considerably conservative. It is concluded that a suitable way of expressing the thresholds at negative stress ratios for application in flaw evaluation procedures is to use variable thresholds  $\Delta K_{th}$  expressed by  $K_{max} - K_{min}$ .

## **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgement

This paper was elaborated as part of the project No. CZ.02.1.01/0.0/0.0/17\_048/0007373 "Damage Prediction of Structural Materials" within the Research, Development and Education Operational Programme financed by the European Union and from the state budget of the Czech Republic.

### References

- ASME Boiler and Pressure Code Section XI, Rules for inservice inspection of nuclear power plant components construction of nuclear facility components, American Society of Mechanical Engineers, New York, 2019.
- [2] AME Standard Section IV, Unified procedure for lifetime assessment of components and piping in WWER NPPs, Association of Mechanical Engineers, Prague, Czech, 2016.
- [3] JSME S NA1-2011, Codes for Nuclear Power Generation Facilities, Rules on fitness-for-service for nuclear power plants, The Japan Society of Mechanical Engineers, Tokyo, 2018.
- [4] Taylor D. A compendium of fatigue thresholds and growth rates. London: Engineering Materials Advisory Service Ltd.; 1985.
- [5] Barsom JM. Fatigue behavior of pressure-vessel steels, WRC Bulletin 194, The Welding Research Council, 1974.
- [6] ASME Boiler and Pressure Code Section VIII, Rules for construction of pressure vessels, Division 3, Article KD-4, fracture mechanics evaluation, American Society of Mechanical Engineers, New York, 2019.
- [7] ASME Boiler and Pressure Code Section XI, Rules for inservice inspection of nuclear power plant components, Nonmandatory Appendix A, Analytical evaluation of flaws, American Society of Mechanical Engineers, New York, 2019.
- [8] IIW Commissions, Recommendations for fatigue design of welded joints and components, International Institute of Welding, IIW document XIII-2151-07 / XV 1254-07, Paris, 2007.
- [9] BS 7910, Guide to methods for assessing the acceptability of flaw in metallic structures, The British Standard Institution, London, 2019.
- [10] Hudson CM. Effect of stress ratio on fatigue-crack growth in 7075-T6 and 2024-T3 aluminum-alloy specimens, NASA TN-D-5390, 1969.
- [11] Pook LP, Beveridge AA. Threshold for fatigue crack growth in ferritic steels at 300 C, ASTM STP 520, American Society for Testing and Materials, p. 179–91, 1973.
- [12] Davenport RT, Brook R. The threshold stress intensity range in fatigue. Fatigue Eng Mater Struct 1979;1:151-8.
- [13] Benz C. Fatigue crack growth at negative stress ratios: On the uncertainty of using K and R to define the cyclic crack tip load. Eng Fract Mech 2017;189:194–203.
- [14] Kobayashi H, Park KD. Effect of oxidation on fatigue crack growth threshold of alloy and carbon steels at elevated temperature. J High Pressure Inst 1992;30(1): 14–22 (in Japanese).
- [15] Zhang J, He XD, Du SY. A simple engineering approach in the prediction of the effect of stress ratio on fatigue threshold. Int J Fatigue 2003;25:953–1938.
- [16] Samel GK, Sasikala G, Ray SK. On R ratio dependence of the threshold stress intensity factor range for fatigue crack growth in type 316 (N) stainless steel weld. Mater Sci Technol 2011;27(1):371–6.
- [17] Pook LP, Beveridge AA. Threshold for fatigue crack growth in ferritic steels at 300 C, ASTM STP 520, Fatigue at Elevated Temperatures, 1973, p. 179-91.
- [18] Usami S. Development of fracture mechanics evaluation methods for some fatigue problems of machine structures, Dissertation, The Tokyo University (in Japanese), 1982.
- [19] Hasegawa K, Strnadel B. Definition of fatigue crack growth thresholds for ferritic steels in fitness-for-service codes, Proceedings of the ASME pressure vessel and piping conference, PVP 2018-84940, Prague, Czech, 2018.
- [20] Hasegawa K, Usami S, Strnadel B. Fatigue crack growth thresholds under negative stress ratio for aluminum alloys. Key Engng Mater 2019;810:34-9.
- [21] Hasegawa K, Strnadel B, Usami S, Lacroix V. Fatigue crack growth thresholds at negative stress ratio for ferritic steels in ASME Code Section XI. J Pressure Vessel Technol. 2019;141:031101-1 to 6.
- [22] ASTM E 647, Standard test method for measurement of fatigue crack growth rates, American Society of Testing and Materials, Philadelphia, PA, 2001.
- [23] JSMS, Data book on fatigue crack growth rates of metallic materials, The Society of Materials Science, Japan, Vol. 1, 1983.
- [24] Kurihara M, Katoh A, Kawahara M. Analysis on fatigue crack growth rates under a wide range of stress ratio. J Pressure Vessel Technol 1986;108:209–13.
- [25] Tanaka Y, Soya I. Effects of stress ratio and stress intensity factor range on fatigue crack closure in steel plate. J Japan Weld Soc 1987;5(1):119–28 (in Japanese).
   [26] Kang KJ, Song JH, Earmme YY. Fatigue crack growth and closure through a tensile residual stress field under compressive applied loading. Fatigue Frac Engng
- Mater Struct 1989;12(5):363–76.
- [27] Bloom JM, Hechmer JL. High stress crack growth Part II Predictive methodology using a crack closure model, ASME PVP conference, Vol. 350. Fatigue Fract. 1997;1:351–70.
- [28] Newman JC. An assessment of the small crack effect for 2024-T3 aluminum alloy, small Fatigue cracks. Proceedings of the Second Engineering Foundation International Conference/Workshop, Santa Barbara, CA, January 5-10, 1986.
- [29] Makabe C, Kaneshiro H, Nishida S, Yafuso T. Measurement of fatigue crack opening and closing points in thin plate specimen and it's difficulties, Japan Soc Mater Sci. 1992;(41)465:951–6 [in Japanese].
- [30] Eason ED, Gilman JD, Jones DP, Andrew SP. Technical basis for a revised fatigue crack growth rates reference curves for ferritic steels in air. J Pressure Vessel Technol. 1992;(114)Feb.:80–7.
- [31] Bloom J. An approach to account for negative R ratio effects in fatigue crack growth calculations for pressure vessels based on crack closure concepts. J Pressure Vessel Technol 1994;116:30–5.
- [32] Stoychev S. Kujawski, D, Methods for crack opening load and crack tip shielding determination: a review. Fatigue Frac Engng Mater Struct 2003;26:1053–67.
- [33] Pippan R. The effective threshold of fatigue crack propagation in aluminum alloys. Philos Mag A 1998;77(4):861–73.
- [34] Nakai Y, Tanaka K, Kawashima R. Stress-ratio effect on fatigue crack growth threshold in steel. J Soc Mater Sci, Japan 1983;33(371):1045–51 (in Japanese).
- [35] Yamaguchi Y, Hasegawa K, Li Y. Fatigue crack growth for ferritic steel under negative stress ratio. J Pressure Vessel Technol. 2020(142):041507-1 to 6.

- [36] JSMS, Stress intensity factors handbook, The Society of Materials Science, Japan, Vols. 1-5, 2001.[37] Zahoor, A., Ductile fracture handbook, Electric Power Research Institute, NP-6301-D N14-1, Vols. 1-3, 1989.
- [38] AFCEN, Construction and in-service inspection rules for nuclear island components, RSE-M, Appendix 5.4, Analytical methods for calculating stress intensity factors and J integral, French Association for Design, 2010.