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Children's Mathematical Engagement Based on Their Awareness of Different Coding Toys' Design Features

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CHILDREN'S MATHEMATICAL ENGAGEMENT BASED ON THEIR

AWARENESS OF DIFFERENT CODING TOYS' DESIGN FEATURES

by

Joseph S. Kozlowski

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ABSTRACT

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by

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Tangible coding toys have been promulgated as useful learning tools for young children to learn computer science and mathematics concepts and skills. Although research shows coding toys can support mathematics for early childhood-aged children, little is known about specific design features of coding toys that afford mathematical thinking concepts and skills to young children. The purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. The dataset used for this study were collected as part of design-based research NSF project (award #DRL-1842116). I used a multi-phased qualitative analysis with a total of 42 hours of video data of 106, 5- to 6-year-old children engaging in coding toy tasks with four coding toys to answer the three research questions: (a) What design features do kindergarten-aged children perceive and use when interacting with four different coding toys, (b) What mathematics do kindergarten-aged children engage in when they are perceiving design features of four different coding toys, and (c) How do design features of four different coding toys afford kindergarten-aged children’s mathematical engagement?

Results indicated that (a) children used and perceived the grid square and command arrow design features frequently, while other design features were used moderately or rarely; (b) children engaged in a variety of mathematical concepts and skills in five main categories of mathematical topics: spatial reasoning, geometry, comparison, measurement, and number; and (c) the relationship between design features affording mathematics varied depending on the coding toy, and that some design features afforded specific mathematical engagement across all four coding toys, while other design features afforded mathematical engagement only with specific coding toys. This research highlights the importance of specific design features to afford certain mathematical concepts and skills. These findings have important implications as early childhood educators explore ways to implement coding toys to support mathematics and computer science concepts, researchers conduct studies to better understand how coding toys support mathematics and computer science learning, and commercial companies design new coding toys to fill the needs of educators and parents.

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PUBLIC ABSTRACT

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Tangible coding toys have been promulgated as useful learning tools for young children to learn computer science and mathematics concepts and skills. Although research shows coding toys can support mathematics for early childhood aged children, little is known about the specific design features of coding toys that afford mathematical thinking concepts and skills to young children. The purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. The dataset used for this study was collected as part of design-based research NSF project (award #DRL-1842116). I used a multi-phased qualitative analysis with a total of 42 hours of video data of 106, 5- to 6-year-old children engaging in coding toy tasks with four coding to answer the three research questions which were focused on perception of design features, mathematical engagement, and how different design features could afford mathematics.

Results indicated that (a) children used and perceived the grid square and command arrow design features frequently, while other design features were used moderately or rarely; (b) children engaged in a variety of mathematical concepts and skills in five main categories of mathematical topics: spatial reasoning, geometry, comparison, measurement, and number; and (c) the relationship between design features affording mathematics varied depending on the coding toy. This research highlights the importance of specific design features to afford certain mathematical concepts and skills. These findings have important implications as early childhood educators explore ways to implement coding toys to support mathematics and computer science concepts, researchers conduct studies to better understanding how coding toys support mathematics and computer science learning, and commercial companies design new coding toys to fill the needs of educators and parents.

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CHAPTER I

INTRODUCTION

Research suggests that children's engagement with physical and virtual educational manipulatives has benefits for mathematics learning (Carbonneau et al., 2013; Moyer-Packenham & Westenskow, 2013). Importantly, the research shows that specific design features and affordances of educational mathematics manipulatives play an important role in their effectiveness for supporting mathematics learning (e.g., Bullock et al., 2017; Manches & O'Malley, 2012). A new type of educational manipulative (i.e., tangible coding toys) is becoming prevalent in early childhood classrooms. Coding toys were not specifically designed to support mathematics instruction like virtual and physical mathematics manipulatives. However, coding toys do share certain characteristics with these manipulatives such as tangible parts and digital programming of dynamic movements. Emerging research shows that young children who play with coding toys engage with mathematical concepts (Miller, 2019; Palmér, 2017; Shumway et al., 2021) and can even improve their mathematics knowledge (Nam et al., 2019). However, it is not yet known what design features and related affordances of the coding toys support this mathematical development. This study focused on children's awareness of the design features in coding toys and how those design features afford children's engagement with mathematics through the lens of affordance theory (Gibson,1979b) and theories of embodied cognition (Lakoff & Nuñez, 2000).

Background of the Problem

New opportunities for young children to engage with mathematics burgeon as new technology emerges. Young children began playing with coding type interfaces some time ago (Clements & Battista, 1989; Papert, 1980), but the current landscape of technological opportunities for young children is vast. Yu and Roque (2019) described the different types of computer science computational tools that young children can use in educational settings, and highlighted how some are completely digital (e.g., ScratchJr., a totally online computer-based environment), some are completely tangible (e.g., Code-a-pillar, a coding toy that moves around on the floor and is programmed with tangible pieces), and some are hybrid (e.g., LEGO Mindstorms, a coding toy that moves around on the floor but is programmed from a computer).

Considering the wide variety of current computational and coding tools, many educators and researchers have advocated for the use of tangible coding toys with preschool- and kindergarten-aged children. One main reason is because tangible coding toys have been shown to benefit early childhood learning (e.g., Angeli & Valenides, 2019; Bers et al., 2014, 2019; Murcia & Tang, 2019; Shumway et al., 2021). For example, Murcia and Tang demonstrated how young children playing with a coding toy (i.e., Cubetto) engaged in the mathematical number concept of one-to-one correspondence as they matched codes, movements, and grid squares. Another reason educators and researchers find tangible coding toys promising is because health experts recommend that young children should see a reduction in daily screen time (American Academy of Pediatrics [AAP], 2016; Bers et al., 2019); tangible coding toys offer children the opportunity to experience computer science and mathematical experiences without a screen.

Curriculum standards are emerging which require educators to teach computer science concepts to children at younger and younger ages (e.g., Computer Science Teachers Association [CSTA], 2017; International Society for Technology in Education [ISTE], 2007). For example, CSTA's set of standards for kindergarten to second grade suggests children should learn to perform basic computing concepts such as "Decompose (break down) the steps needed to solve a problem into a precise sequence of instructions" (1A-AP-11, K-2). Due to limited instructional time and many existing curriculum standards, teachers struggle to incorporate these computing standards into their general instructional time (Hunsaker & West, 2020). However, educators of young children are seeing the possibilities of integrating mathematics standards and computing standards in a single lesson. For example, Israel and Lash (2020) demonstrate the promising opportunity to blend computer science and mathematics topics into a single lesson, however most educator's forefront one skill over the other. One context for integrating computer science and mathematics is coding toys, which various practitioners have found as a promising environment where mathematics and computer science concepts can be experienced (Shumway et al., 2019; Winters et al., 2020).

Researchers are beginning to identify different physical design features of such coding toys such as external manipulative coding blocks or tactile buttons for control (Hamilton et al., 2020). However, very specific design features of coding toys (e.g., flashing lights, buttons on coding toy body) have not yet been investigated in terms of their affordances for mathematics learning. In research on other mathematics tools such as virtual manipulatives and physical manipulatives, some researchers employ affordance theory (Gibson, 1979b) to show that design features and affordances of the mathematics tools either support or hinder learning (e.g., Boyer-Thurgood, 2017). The existing knowledge on how design features and affordances of these other mathematics tools (i.e., physical and virtual manipulatives) holds special insights on how design features of coding toys—which share some characteristics with these other mathematics tools—could relate to mathematics.

Statement of the Problem

Research shows that young children who play with coding toys use certain mathematics such as number, spatial, and measurement concepts (Miller, 2019; Nam et al., 2019; Palmér, 2017; Shumway et al., 2021). Additionally, scholars are beginning to evaluate the design features of coding toys (Hamilton et al., 2020; Yu & Roque, 2019). Therefore, the field has a general sense of the mathematical benefits of coding toys, as well as an emerging understanding of the different design features of such coding toys. However, how design features of coding toys afford young children's engagement with mathematics is still unknown. It is important to understand this relationship so teachers, researchers, and designers of coding toys can most effectively take advantage of this new learning tool, and possibly design new coding systems as well as instructional lessons that leverage the coding toy's possible beneficial affordances.

Significance of the Study

How coding toy design features afford children's engagement with mathematics is important because it has implications for classroom teachers, researchers, and designers. Understanding the link between design features and mathematics would benefit classroom teachers interested in implementing technological tools by offering them tasks and instructional strategies to benefit the child's mathematical experiences. For example, if findings indicated that a grid space floor mat supported discrete counting skills, teachers may include these types of supplementary materials when teaching with coding toys.

Understanding the link between design features and mathematics would also benefit researchers because it could begin to explain a contextual variable that would be important to account for in future research. For example, if findings indicated that children demonstrated more one-to-one correspondence with coding toys that had buttons on the coding toy's body rather than on a separate programming interface, it could explain why children may not have benefitted from an intervention with a coding toy with a certain set of design features.

Finally, research on this topic would have important implications for coding toy designers. Commercial companies commonly claim that their products will engage young children in problem solving, critical thinking, computing, and mathematics. For example, Primo Toys—the designer of Cubetto the coding toy—claim on their website: "Coding, STEM, numeracy and creativity in a single product." However, little empirical research exists which supports such claims, and the research that does exist does not analyze the specific design features of the platforms that might aid in children's learning of mathematics topics. Understanding the link between design features and mathematics would benefit designers of coding toys by providing a research-based design framework which could suggest optimal design features to support mathematics.

Purpose of the Study and Research Questions

Because little is known about coding toy design features and early childhood mathematics, the purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. Three research questions guide this study:

1. What design features do kindergarten-aged children perceive and use when interacting with four different coding toys?
2. What mathematics do kindergarten-aged children engage in when they are perceiving design features of four different coding toys?
3. How do design features of four different coding toys afford kindergarten-aged children's mathematical engagement?

Summary of Research Study Design

The dataset I used in this study was collected as part of a large National Science Foundation (NSF)-funded (Award #: DRL-1842116) design-based research (DBR) project. The research design for this original project was DRB (Cobb et al., 2003), which allows for initial theory development through iterative testing and implementing of interventions and resources. During the original project, coding toy activities were iteratively designed and updated based on what data demonstrated children were able to do and whether or not their engagement with the coding toy elicited intended purposes of the lesson. Data in the project were collected from 106 participants who engaged in 84 lessons in small groups of three to five children with four different coding toys (i.e., Cubetto by Primo Toys, Code-a-pillar by Fisher Price, Botley by Learning Resource, Bee-Bot by Terrapin). A total of 42 hours of video data are a part of this dataset.

I used a multi-phased qualitative research design when investigating the existing DBR dataset. Overall, qualitative methods were appropriate because the dataset from the large DBR project was collected using video recordings in children's natural environments, and therefore methods were needed to understand the phenomena considering the rich context. The use of qualitative methods allowed me to explore new concepts in complex learning environments and helped me to develop initial theory around such topics. Because no empirical work has been conducted which investigates design features and mathematics with early childhood coding toys, qualitative methods were most appropriate for this inquiry. Specifically, a multi-phased design is appropriate for a phased analysis of video data because different phases of the analysis build off of one another, and elucidating knowledge in early phases of the investigation is imperative for understanding knowledge in the later phases. Five phases of analysis were conducted in this multi-phased qualitative research design.

Scope of Study

The scope of this study was constrained by several factors, of which three are notably important: (a) only four tangible and screen-free coding toys were examined, (b) only the introductory lessons were analyzed (i.e., the first two lessons with the coding toy), and (c) only 5- and 6-year-old children comprised the participant population.

Although a variety of other coding toys that are commercially available, this study was designed to investigate four specific coding toys. Commercially available coding toys have a vast variety of design features. Therefore, it could be inappropriate to generalize the findings of this study to coding toys which were not examined. Although there may be localized generalizability of findings to other specific design features of other coding toys, to generalize the findings to all coding toys would be inappropriate.

Additionally, for the purposes of the current study, the dataset was limited to the first two lessons each child participated in with each coding toy (i.e., introductory lessons). This means that results from the study should be considered only in terms of early stages of learning to use coding toys. It may be that once children become experienced with these coding toys, the design features have different relations to children's mathematical understandings. Considering this, the implications for the current study are focused on in-classroom learning episodes when young children initially engage with coding toys.

Finally, the research participants in this study are 5- to 6-year-olds. Due to rapid cognitive development in these early years of life, additional future research will be needed to understand the relationship between design features and mathematics with older children and younger ones. It should not be assumed that the findings of this study would be able to transfer to children of different ages.

Definition of Terms

Physical manipulatives—"Concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p. 11).

Virtual manipulative—"An interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (Moyer-Packenham & Bolyard, 2016, p. 13).

Affordances—"cues of the potential uses of an artefact by an agent in a given environment"; refer to possibilities that the agent has for action (Burlamaqui & Dong, 2014, p. 13).

Design features—In digital games, design features have been defined as game attributes that can determine learning potential (Bedwell et al., 2012); elements that are programmed into the game to determine how it functions (Boyer-Thurgood, 2017). A definition for design features specifically for tangible coding toys was not located. However, this study uses the term design features to mean the components of the coding toy that can be visually perceived or can be physically manipulated during child-coding toy interaction (Hamilton et al., 2020).

Mathematical engagement—Appleton et al. (2008) analyzed 19 definitions of engagement in psychological literature. Findings indicated that "All definitions describe engagement as *participation* in some activity, either *behavioral* or c*ognitive* participation" (Middleton et al., 2017, p. 668). Therefore, this study adopts an operational definition of mathematical engagement as "participating in mathematics behaviorally."

Coding toys—The term coding toys (i.e., tangible coding toys) are physical objects that allow child-artifact interaction that is not entirely mediated by a screen. An example of a tangible coding toy is Fisher Price's Code-a-pillar (Figure 1). This tangible coding toy allows children to physically append and remove body segments. Each body segment represents a different code which dictates the movements of the tangible coding toy. For a comprehensive list of current tangible coding toys, screen-based coding environments, and computer science computational thinking kits, refer to Hamilton et al. (2020) and Yu and Roque (2019). However, this study uses the term coding toy to refer explicitly to tangible coding toys.

Figure 1

A Coding Toy: Code-a-Pillar

CHAPTER II

LITERATURE REVIEW

Because little is known about coding toy design features and affordances that relate to early childhood mathematics, the purpose of the current study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. Three distinct areas of research provide a conceptual grounding for the current study which are: (a) early childhood mathematics with coding toys—specifically number concepts, measurement concepts, and spatial concepts; (b) design features of mathematics tools, and (c) engagement in mathematics through perceptions, affordances, and embodiment.

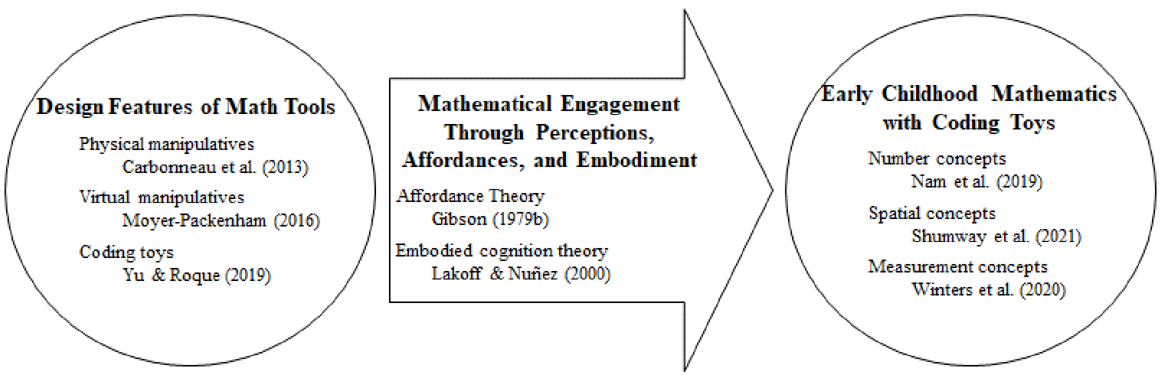
This chapter begins with an overview of the conceptual framework which relates the three main constructs. The conceptual framework highlights the theorized relationship between design features of mathematical tools and young children's engagement in mathematics with coding toys while implementing embodied cognition theory and affordance theory to understand the relationship. Next, the chapter will focus on early childhood mathematics with coding toys, in terms of three main mathematical concepts: number concepts, spatial concepts, and measurement concepts. Then, the chapter contains a summary of the corpus of research around design features of physical manipulatives, virtual manipulatives, and coding toys. The final section of the chapter contains a discussion on affordance theory and embodied cognition theories and how they were appropriate interpretive lens' for the current study.

Conceptual Framework

The conceptual framework depicted in Figure 2 highlights the theory of design features of mathematical tools (left oval), and how children's mathematical engagement with the design features of mathematical tools (through the lens of perceptions, affordances, and embodied cognition; connecting arrow) related to early childhood mathematics concepts (right oval).

Figure 2

Conceptual Framework



The right oval in the framework represents the first main topic, which is early childhood mathematics with coding toys. More specifically, the right oval represents the body of knowledge known about early childhood mathematics with coding toys in the three mathematics domains that are the focus of the current study: number, spatial, and measurement concepts. Importantly, the research on these three topics shows that certain number, spatial, and measurement skills have certain developmental progressions for certain ages and coding contexts can support these mathematics concepts. The left oval of the framework represents the second main topic, which is design features of mathematics tools. The left oval represents an empirically based corpus of literature that demonstrates that design features of certain mathematics tools (i.e., physical manipulatives, virtual manipulatives, coding toys) have the potential to support children's mathematics. Finally, the middle connecting arrow represents the theorized relationship between design features of mathematics tools and young children's mathematical engagement in number, spatial, and measurement concepts. This theorized relationship indicates that children's mathematical engagement can be observed through their perceptions and embodiments of design features while using the coding toys. As children in this study engaged with design features of coding toys, they perceived the beneficial affordances of the toys, which supported their engagement with mathematics. I was able to observe the children's mathematical engagement through their awareness of design features and their physical embodiments in the coding space, for example, when a child rotated a coding toy in the air and said it needed to "look that way," I observed mathematical engagement with spatial concepts through the embodiment of the child physically moving the toy in their physical space. Directly related to my conceptual framework, I looked at the way design features afforded mathematics which allowed me to understand relational links between the two that may be somewhat generalizable. For example, certain design features (e.g., flashing lights) afforded specific mathematical concepts or skills (e.g., coordination), which allowed me to evaluate the relational link between the two (e.g., simultaneous linking) that may be generalizable to possible design of future features. I theorize that affordances of design features of coding toys have the potential to support early childhood mathematics in three key domains: number concepts, spatial concepts, and measurement concepts through awareness of the design features of the coding toys. Affordance theory (Gibson, 1979b) was used as a theory to understand the mathematical affordances of the coding toys and embodied cognition theory (Lakoff & Nunéz, 2000) was used to understand how the children's embodied experience in the coding toy activities relates to their mathematical engagement.

Early Childhood Mathematical Thinking with Coding Toys

The first main area of research that was pertinent to the current study is early childhood mathematics with coding toys, specifically number concepts, spatial concepts, and measurement concepts. The current study pertains to how design features of coding toys afford early childhood mathematical engagement in these three specific domains and therefore, it is important to understand the literature surrounding early childhood mathematics in general, but also more specifically within these three domains of early childhood mathematics with coding toys. This section is organized in four main subsections. In the first section, a broad view of early childhood mathematics is described. The second section focuses on early childhood number concepts. The third section focuses on early childhood spatial concepts. The final section focuses on early childhood measurement concepts.

Early Childhood Mathematics Education

This section highlights a broad view of early childhood mathematics education and is important to the current study because it provides a foundation for an in-depth understanding of the three mathematical concepts investigated in the study (i.e., number, spatial, measurement). This section specifically highlights early childhood mathematical learning trajectories (LTs) and young children's informal mathematics knowledge (IMK).

Learning Trajectories as a Model to Understand Early Childhood Mathematics

An important topic that aids in understanding early childhood mathematics is learning trajectories (LTs), which have been a predominant model for understanding young children's mathematical thinking. Simon (1995) originally described a hypothetical LT as "the learning goal, the learning activities, and the thinking and learning in which students might engage" (p. 133). Later, Clements and Sarama (2004) reconceptualized a LT as:

> Descriptions of children's thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domains. (p. 83)

Ultimately, what makes LTs unique and powerful is their ability to account for the developmental aspect of learning as well as for the instructional task aspect of learning. The Bill and Melinda Gates Foundation, Heising-Simons Foundation, and the Institute of Education Sciences have recently funded entire curricula, websites, and research into the learning and teaching with LTs. One such funded project is called *Learning and Teaching with Learning Trajectories* (LT^2; Clements & Sarama, 2021), which is run by two leading experts in the field—Julie Sarama and Douglas Clements. This website, full of activities and resources directed at birth-to-third-grade children, builds on emerging evidence which demonstrates that teaching young children using a LT approach is more beneficial in helping them learn mathematics than teaching to standard outcomes (Clements et al., 2020; Frye et al., 2013). Clements et al. investigated 291 kindergarten children's learning in order to determine if teaching mathematics was more beneficial if the instruction was LT-oriented or teach-to-target oriented. Children in the LT-oriented group received mathematics instruction at one LT level above their thinking. Children in the teach-to-target group received instruction geared at the goal, which in some cases was many levels above the child's current level of thinking. The results indicated that children receiving LT-oriented instruction made significantly larger gains than the other children, while gender and amount of intervention did not have significant interaction effects.

LTs are a validated way to understand children's mathematical understanding. Through a series of papers and research, Clements andSarama (2004, 2020, 2021), Sarama and Clements (2003, 2009) and Clements et al. (2013,2019, 2020) have designed and empirically validated LTs for young children. These LTs originated through the NSF-funded project called *Building Blocks* (Sarama & Clements, 2003, which focused on creating curriculum, assessment, and learning trajectories that aligned with empirical research and theoretical foundations of early childhood development of mathematics. Sarama and Clements first focused on creating and validating early numeracy LTs, then with colleagues (Szilágyi et al., 2013) validated the measurement LTs, and recently went on to validate shape composition LTs (Clements et al., 2019). Understanding learning trajectories is important to the current study because it offers a nuanced view on early childhood mathematics education and provides a detailed perspective on the skills and activities appropriate for young children's mathematics learning.

What Children Know Before School: Informal Mathematics Knowledge

It is important to understand early childhood mathematics in terms of (IMK), which is the mathematics knowledge that is experienced and acquired at home, in the individuals' environments, and in informal settings, as opposed to school-taught mathematics. A variety of factors have been empirically shown to lead to individual differences in IMK at early ages such as family income (Starkey et al., 2004), number talk at home (Levine et al., 2010, 2011), home mathematics activities (Anders et al., 2012), and parental affect about mathematics (Maloney et al., 2015). A challenge exists in early childhood mathematics education which is when school mathematics instruction does not match children's IMK, it leads to struggles in early numeracy development which is a strong predictor of future mathematics success and success in other domains (Aunio & Niemivirta, 2010; Claessens & Engel, 2013; Jordan et al., 2007; Koponen et al., 2013). For this reason, it is important for instruction to align with, and support, young children's IMK. Baroody and Purpura (2017) offer three effective teaching strategies that can do this: (a) addressing informal barriers, such as previously learned misconceptions; (b) teaching to big ideas, overarching concepts and strategies; and (c) using LTs to guide instruction. Ultimately, it is important to consider IMK in the current study because the participants of the study are kindergarten children and have incredibly diverse IMK which may affect the way they engage in mathematics in the coding toy activities.

These broad topics of early childhood mathematics education are important to the current study because they provide a broad research perspective which helps frame the understanding of early childhood mathematical thinking. They are also important to the current investigation because the discussion section of this study will relate some findings back to LTs and IMKs, therefore an understanding of these foundational early childhood mathematics topics is essential. This broad perspective also provides a foundation for the next three sections, in which three specific early childhood mathematical concepts that children learn in kindergarten are described.

Early Childhood Number Concepts

This section focuses on the specific mathematical domain of early childhood number concepts. Early childhood number sense and number learning is a rich body of research. Of importance, research in the last 100 years has made remarkable progress from Thorndike's (1922) drill theory of number learning to number meaning theory (Brownell, 1935), to number sense theories (Baroody & Purpura, 2017; Sarama & Clements, 2009), which account for early childhood developmental characteristics. This section has two main purposes which are to: (a) highlight early childhood number concept research, and (b) elucidate research that demonstrates how young children demonstrate number concepts in a coding toy context.

Early Childhood Research on Number Concepts

Learning trajectories on early childhood number sense are currently a leading model used to indicate what number concepts young children can understand (e.g., Baroody & Purpura, 2017; Sarama & Clements, 2009). These LTs emerged from decades of research aimed at understanding what number concepts young children are capable of learning and demonstrating (e.g., Baroody & Lai, 2007; Carpenter & Moser, 1982; Fuson & Secada, 1986). For example, Baroody and Purpura (2017) provide an in-depth number sense learning trajectory which begins with verbal-based subitizing of numbers one to six and ends with automatic retrieval of number from a memory system. These number sense trajectories have also helped inform national standards such as Common Core State Standards for Mathematics (CCSS-M; Common Core State Standards Initiative [CCSSI], 2010). For example, the counting and cardinality domain of the CCSS-M (CCSSI, 2010) indicates kindergarten children should be able to understand the relationship between numbers and quantities and compare the number of items in a small group.

Children in the current study closely align to Baroody and Purpura's (2017) LT level *Meaningful Object Counting*, which focuses on number concepts such as stable order of number words, one-to-one correspondence, and cardinality (also a focus in Sarama & Clements, 2009). These specific number concepts describe whether a child has a consistent and sequential order of number words (stable order of number words), whether the child matches a specific unit of value to a specific number word (one-to-one correspondence), and whether the last number word represents the total quantity (cardinality). Children in the current study are kindergarten-aged children and should be developing and becoming proficient with these types of number concepts.

Early Childhood Research on Number Concepts with Coding Toys

Research suggests that young children use number concepts as they engage in coding toy tasks (Fessakis et al., 2013; Moore et al., 2020; Nam et al., 2019; Shumway et al., 2021; Sung et al., 2017). Many times, these number concepts emerge in the form of counting movements and matching those with corresponding amounts of coding tiles or arrows (coordination), referencing the total quantity of different physical aspects of the coding toy environment (cardinality), or adding or subtracting certain codes to accomplish programming goals (operations). Important to this study, Shumway et al. defined *coordinating* as "Coordinates the totals of two quantities and/or matches 1-to-1 counting with movements or codes" and observed students engaging in coordination when they did things like associate number of movements on a grid space with number of coding tiles/arrows. This same operational definition is used in the current study.

Nam et al. (2019) suggests that engaging with coding toys can improve the use of numbers in mathematical problem-solving. In conducting a quasi-experimental design with 53 Korean kindergarten children, half the children participated in 12 typical classroom instructional sessions (control) and the other half participated in 12 coding robot activities with TurtleBot (experimental). The Turtlebot activities involved children doing a range of tasks including mastering basic function, directing the robot to go here and there, and then creating a dance with the TurtleBot. Results of a pre- to post-test mathematics number assessment indicated that the experimental group who interacted with the TurtleBot activities significantly outperformed their peers on the number assessment. The current study demonstrated how mathematics, and even the use of numbers, can be improved with participation in coding toy tasks. It is important to understand how children engage broadly in number concepts, but also what the current literature says about how they engage in number concepts in coding toys tasks.

Early Childhood Spatial Concepts

This section focuses on the specific mathematical domain of early childhood spatial concepts. Spatial concepts as a mathematical domain are often overlooked in the U.S. Although it is implicit in resources such as the CCSS-M, innovative LTs (Clements & Sarama, 2021) have more explicitly outlined its structure and importance as a mathematical concept for young children. This section has two main purposes which are to: (a) highlight research on early childhood spatial research, and (b) review research that demonstrates how young children demonstrate spatial concepts in a coding toy context.

Early Childhood Research on Spatial Concepts

Sarama and Clements (2009) presented a learning trajectory (LT) for young children's spatial thinking, which is broken into two categories—spatial visualization and spatial orientation. The spatial visualization levels on the LT range from the *Intuitive Mover level*, where children explore characteristics of objects through movements and use trial-and-error to fit objects into spaces, to the *Mental Mover level*, where children begin to mentally predict end-phase of moved and rotated objects. The spatial orientation levels on Clements and Sarama's LT range from the *Foundations level*, where children use themselves as a reference system to get around the world, to the *Framework User level,* where children can follow and use maps even when the reference frame of the object has changed. There is still little empirical research regarding the validity of these new spatial LTs and whether or not they appropriately align to children's development.

Early Childhood Research on Spatial Concepts with Coding Toys

One place to start when thinking about early childhood spatial concepts in a coding toy context is with Papert's (1972) LOGO and Turtle Geometry. A cybernetic turtle in Turtle Geometry represented a virtual point that could be moved around and programmed using directional movement and rotational commands. Papert (1980) described how spatial mathematics through engagement with Turtle Geometry can be an important mathematical experience for young children:

> A Turtle is at some place–it, too, has a position–but it also faces some direction–its heading. In this, the turtle is like a person–I am *here* and I am facing north—or an animal or a boat. And from these similarities comes the Turtle's special ability to serve as a first representative of formal mathematics for a child. Children can *identify* with the Turtle and are thus able to bring their knowledge about their bodies and how they move into the work of learning formal geometry. (pp. 55-56)

Papert was interested in the ways that young children could perceive these different spatial organizations of the Turtle, and how that spatial referencing and development was linked to mathematics. Later, researchers began to build off Papert's work and further investigate spatial mathematics in similar environments (Clements & Battista, 1989; Clements et al., 1996; Cittá et al., 2019; Cuneo, 1985). For example, Cuneo conducted a factorial designed study with 32, 4- and 5-year-old and 32, 6- and 7-year-old children using LOGO's Turtle Geometry. In the study, children saw two turtles appear next to one another on the screen. Then one of the turtles would do one of the four commands (i.e., rotate 90 degrees right, rotate 90 degrees left, move forward one movement, move backward one movement). Children were asked to identify the corresponding code (i.e., rotate 90 degrees right, rotate 90 degrees left, move forward one movement, move backward one movement) that would make the second turtle match the one that had changed position. Children completed two replications of the complete design and the starting orientation of the turtles would change throughout each replication. Results indicated that across both ages, children consistently struggled to indicate the correct turn (4 to 5-year-olds, 27%; 6 to 7-year-olds, 40%) and when answering about movements, children referred to the end-state of the turtle in relation to their own bodies and the screen rather than in relation to the turtle. The implications to the current study are that the geometric application of movements and rotations in this coding context heavily relied on the child's spatial concepts and whether or not they could spatially orient movements and rotations.

Recent research has also demonstrated how spatial mathematical concepts emerge as young children play with coding toys (Moore et al., 2020; Palmér, 2017; Shumway et al., 2019, 2021). Palmér studied eight, 3- to 5-year-old children by giving them a pretest on basic programming, providing three to four weeks of a 'body coding' intervention, and then providing another basic programming posttest. During the intervention phase, children would program the researcher around the room by saying words to subsequently move the person. These intervention activities would progress until the children was putting paper arrows on a grid to program a robot to move around. Results pertaining to spatial thinking indicated that children mentally compared the grid map to the real life-size map, as well as associate movements with symbols. Additionally, the children had to mentally envision paths, movements, and changes in orientation which are all aspects of spatial thinking and predictors of later mathematics success (Cross et al., 2009).

Research also shows that spatial reference frames play an important role in coding contexts for older children (Smith et al., 2014) and for younger children (Clarke-Midura et al., 2021). Klatzky (1998) distinguished the two commonly defined categories of reference frames as egocentric—when the heading of the perceiver and the object is shared, and allocentric—when the heading of the perceiver differs from that of the object. Clarke-Midura et al. (2021) conducted an in-depth qualitative investigation on 16, 5- and 6-year-old children who participated in lessons with Fisher Price's Code-a-pillar coding toy. Their results indicated that additional reference frame categories were needed to describe the young children's imprecise spatial and measurement coordination, which were called ProtoEgocentric and ProtoAllocentric. Results also indicated the children most regularly shifted from ProtoAllocentric to ProtoEgocentric frame of reference. In practice, this means children naturally assume an imprecise egocentric spatial association to the coding toy (shared a heading), but when trying to figure out problems they shift into an imprecise allocentric spatial association to the coding toy (did not share a heading). These results support other cognitive science research that suggests young children lack mathematical precision when working in a numeric system (Dehaene, 2011).

This section highlighted the emerging work on early childhood spatial thinking in mathematics and in coding toy contexts. Although research on the importance of spatial thinking for young children's mathematics learning is still emerging, coding toys may be a valuable context in which to support such spatial concepts. Specifically, the dynamic nature of coding toys—their movement in a 3-D space—may offer unique spatial opportunities for children.

Early Childhood Measurement Concepts

This section focuses on the specific mathematical domain of early childhood measurement concepts. Szilagyi et al. (2013) highlighted the importance of measurement learning in the early years because it serves as a sort of bridge between numeracy development and geometrical development. However, U.S. children have not demonstrated strong measurement understanding on international comparisons (Ginsburg et al., 2005). These realities have precipitated an interest in developing assessment, curriculum, strategies, and professional development opportunities to support the teaching and learning of early childhood measurement concepts (Maloney et al., 2014). This section has two main purposes which are to: (a) highlight research on early childhood measurement research, and (b) review research that demonstrates how young children demonstrate measurement concepts in a coding toy context.

Early Childhood Research on Measurement Concepts

Early childhood measurement concepts have been broken into two general types: (a) length measurement, and (b) area, volume, and angle measurement (Sarama & Clements, 2009). Sarama and Clements describe length measurement as the process of subdividing an object by a unit and then placing that unit end to end next to an object to compare its length. Certain skills have been shown to support early children's ability to measure length such as equal partitioning (MacDonald, 2011), units and unit iteration (Steffe, 1991), accumulation of distance (Piaget et al., 1960), and conservation of length

(Inhelder et al., 1974). In terms of angle measurement, research demonstrates a variety of misconceptions young children have about measuring angle such as perceiving angles as figures or turns (Lehrer et al., 1989) and being confused about what angles actually measure (Clements & Battista, 1989, 1992).

Children in the current study likely align with Sarama and Clements (2009) linear measurement LT levels called *Serial Orderer* and *End-to-End Length Measurer*, and the angle measurement LT levels called *Implicit Angle User* and *Angle Matcher*, due to the fact the participant ages for the current study were 5- to 6-years old and these LT levels have been constructed for children at these ages. The linear measurement levels describe children's ability to compare more than two physical objects and order them according to their magnitude (serial orderer) and lay units end-to-end to understand the resulting length has holistic comparison power (end-to-end length measurer). This is similar to the CCSS-M (CCSSI, 2010) standard which describes kindergarten children being able to directly compare two objects. In the current study, there are opportunities with the coding toys for children to compare and measure the linear lengths of different paths, as well as to use specific units of linear movements. The angle measurement levels describe how the young children begin to apply notions of angles outside of physical touch, as well as uses an approximate visual image of an angle. In the current study, there are opportunities for children to act upon the coding toys in ways that align to these angle measurement levels, such as conserving the amount of turn that occurs with a code and conserving this turn to use in other coding situations.

Early Childhood Measurement in Coding Toy Contexts

I only found two empirical studies that suggested children engage in early measurement concept as they engage in coding toy contexts (Murcia & Tang, 2019; Shumway et al., 2021). However, two practitioner pieces were published in *Teaching Children Mathematics* and *Mathematics Teacher: Learning & Teaching* that provide a vision for how measurement is ever present as young children engage in coding robot activities (Shumway et al., 2019; Winters et al., 2020). Specifically, Winters et al. created a progression of instructional activities with her Kindergarten to second-grade children where they would have experiences (a) observing and exploring, (b) interpreting, (3) developing and writing, and (d) critiquing and refining, as they played with two coding toys named Ozobot and Bee-Bot. Winters et al. closely examined the children's play with the coding toys and maintained a perspective on mathematical engagement. Teacher observations indicated that children engaged in with measurement concepts. The children made length estimations as they tried to figure out the distance Bee-Bot would move. Additionally, the children employed the use of metersticks to use units to standardize movements and use units of measurement. Although not empirical in nature, this article sheds light on the potential for young children to engage in measurement concepts as they play with coding toys.

In this section, literature on early childhood measurement and measurement concepts in a coding toy context was reviewed. Important to the current study, literature suggests that young children are developing angle and linear measurement concepts that may align with certain aspects of coding toy contexts (e.g., angle rotations of the coding toy, coding toy's linear movement along a path). A coding toy context may support young children's engagement in measurement concepts.

Summary of Early Childhood Mathematics with Coding Toys

This section provided a general review of early childhood mathematics education research, and then focused on three specific mathematic domains (i.e., number, spatial, measurement) and how they relate to coding toys. This research is important to the current study because it provides a perspective on early childhood mathematics and demonstrates that the mathematical domains of investigation for the current study (i.e., number, spatial, measurement) are appropriate. Additionally, this review highlighted emerging research which demonstrates that coding toy environments are promising contexts to support these specific mathematical domains for early-childhood aged children. These three mathematics domains were the focus for the current investigation and therefore, it is critical to understand what is already known about them in terms of coding toys and young children.

Design Features of Mathematical Tools

The second main area of research that was pertinent to the current study is research on design features of mathematics tools, specifically physical manipulatives, virtual manipulatives, and coding toys. There is well documented research that demonstrates the mathematical benefits of using physical mathematics manipulatives (e.g., Carbonneau et al., 2013) and virtual mathematics manipulatives (e.g., Moyer-

Packenham & Westenskow, 2013). Additionally, research on these topics has evaluated the design features and affordances that aid in mathematics learning (e.g., Bullock et al., 2017; Manches & O'Malley, 2012).

New educational manipulatives (i.e., coding toys) are becoming quite pervasive in educational spaces and, unlike physical and virtual manipulatives which have been specifically designed for mathematics learning, coding toys were designed for computer science learning and coding learning. A coding toy shares blended features with physical and virtual mathematics manipulatives, such as tangible parts (physical) and digital programming (virtual). For this reason, the research on physical and virtual mathematics manipulatives can inform the current study on the design features of coding toys. The first two parts of this section highlight the research on physical and virtual mathematics manipulatives. The final part of this section highlights the research on coding toys.

Physical Manipulatives: Benefits, Design Features and Affordances

Physical mathematics manipulatives and coding toys share specific characteristics (e.g., tangible pieces, movable objects). Therefore, understanding the research on physical manipulatives can inform how coding toys may relate to engagement in mathematics. Physical manipulatives (also called concrete or tangible manipulatives) refer to physical objects that children can move, manipulate, and physically engage with while learning. Physical manipulative materials have been defined as "devices or tools that engage the senses of sight and touch by handling or using them. These concrete objects, when handled by the children, are thought to enhance their opportunity for understanding mathematical concepts" (Moyer, 1997, p. 15). Some of the early forms of physical manipulatives were simple wooden blocks and colored rods used by educational innovators, such as Maria Montessori or John Dewey (Dewey, 1938; Montessori, 1912). Research demonstrates the positive mathematical effects of physical manipulative use (e.g., Carbonneau et al., 2013, Manches & O'Malley, 2016), and some research investigates the specific affordances of design features of manipulatives (e.g., Antle, 2007; Mix, 2009).

Mathematical Benefits of Physical Manipulative Use

The corpus of literature demonstrates that children who use physical manipulatives show increased mathematics learning compared to children who do not (e.g., Anderson, 1957; Carbonneau et al., 2013; Driscoll, 1983; Guarino et al., 2013; Lesh & Johnson, 1976; Manches & O'Malley, 2016; Sowell, 1989). Although research exists that provides counterevidence to this relationship (e.g., Fennema, 1972); Carbonneau et al.'s meta-analysis of 55 empirical studies comparing mathematics learning across two conditions (i.e., use of physical manipulatives and non-use) found a significant positive effect size for physical manipulative use, and notably, a large effect size for mathematical retention.

Clements (1999) and Sarama and Clements (2016) share an important nuance to learning mathematics with manipulatives, which is that sole implementation of manipulatives in instruction is not enough to support mathematical learning. In order for manipulatives to support mathematics, children need to create a connection between the concrete representation and the abstract mathematical target concept. In order to do this, they proffer that a mental conversion from one representation (physical manipulative) to another (mental mathematical symbol) has to be made. Pouw et al. (2014) suggested that learning is afforded by manipulatives based in embodied cognition. According to this view, individuals internalize sensorimotor actions on manipulatives which have subsequent implications on cognition around the mathematical concepts. This means that the physical actions children take on manipulatives may lead to mathematics learning. This is important to the current study because children in the study will physically act upon coding toys in many of the same ways that they act upon physical manipulatives. Therefore, mathematics supported by physical manipulatives may have similarities with the mathematics supported by coding toys.

Specific Design Features and Affordances of Physical Manipulatives

Researchers have attempted to parse out the specific positive affordances of the design features of physical manipulatives (e.g., Antle, 2007; Manches & O'Malley, 2012; Mix, 2009). For example, Mix offers four categories of affordances of physical manipulatives: *offloading intelligence, focusing attention, representing conceptual metaphors,* and *generating action.* Similarly, Antle's Child Tangible Interaction (CTI) framework provides five broad affordances of physical manipulatives which have corresponding design implications: *spaces for action, perceptual mappings, behavioral mappings, semantic mappings,* and *space for friends.* Although physical manipulative affordances have been proffered as specificities of the beneficial impact of such learning objects, there exists surprisingly little empirical research on the specific affordances of physical manipulatives and how specific affordances support mathematical learning.

Research on physical manipulatives is important to the current study because coding toys share certain tangible characteristics with physical manipulatives. This means that what is known about physical manipulatives and mathematics may also inform what can be known about coding toys and engagement in mathematics.

Virtual Manipulatives: Benefits, Design Features and Affordances

Coding toys share specific characteristics with virtual mathematics manipulatives (e.g., digitalized movements, simultaneous linking). Therefore, it is important to the current study to understand the research on virtual manipulatives in order to inform the understanding of how coding toys related to engagement in mathematics. Virtual manipulatives began to emerge in the 1990s when computers and technology were becoming increasingly prevalent in educational settings (e.g., Dorward & Heal, 1999; Resnick et al., 1998). The National Library of Virtual Manipulatives (http://nlcm.usu.edu/) was created in 1999 and served as an original database for a wide variety of virtual manipulatives for research and education. Moyer-Packenham and Bolyard (2016) defined virtual manipulatives as

> …an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge. (p. 13)

Virtual manipulative researchers have investigated the similarities and differences between virtual manipulatives and physical manipulatives (e.g., Bouck et al., 2014; Burns

& Hamm, 2011; Gecu-Parmaksiz & Delialioglu, 2019; Ha & Fang, 2018), investigated the design features and specific affordances of virtual manipulatives (Karakirik, 2016; Ladel & Kortenkamp, 2016; Suh & Moyer-Packenham, 2016), and investigated the beneficial mathematics learning that virtual manipulatives provide (e.g., Desoete et al., 2016; Moyer-Packenham & Westenskow, 2013; Moyer-Packenham & Suh, 2012). The following sections will focus on the beneficial mathematics learning afforded by virtual manipulatives and the specific affordances of design features of virtual manipulatives.

Mathematical Benefits of Virtual Manipulative Use

A rich body of evidence indicates that virtual manipulatives support mathematical learning (e.g., Desoete et al., 2016; Moyer-Packenham & Westenskow, 2013; Moyer-Packenham & Suh, 2012; Paek, 2012). Of notable importance to this review, Moyer-Packenham and Westenskow (2013, 2016) conducted two meta-analyses on the effects of virtual manipulatives on children achievement and learning. They found that five categories of virtual manipulative affordances consistently supported mathematics learning: “focused constraint, creative variation, simultaneous linking, efficient precision, and motivation” (p. 2013, p. 35).

The mathematical benefits of virtual manipulatives are an important topic for the current study because coding toys share certain characteristics with virtual manipulatives (e.g., simultaneous linking, digitalized control) and may support the same types of mathematical benefits as virtual manipulatives.

Specific Design Features and Affordances of Virtual Manipulatives

Research on design features and affordances of virtual manipulatives has been conducted in the context of digital games. Virtual manipulatives are sometimes embedded in dynamic digital math games (Moyer-Packenham et al., 2019) and therefore, research on digital math games offers insight on the specific affordances of virtual manipulatives. A strong line of research has been dedicated to understanding the design features and affordances of digital games and apps (Boyer-Thurgood, 2017; Larkin & Milford, 2018) and how the design features and their affordances support learning (Bullock et al., 2017, 2021; Falloon, 2013; Moyer-Packenham et al., 2019, 2020). Moyer-Packenham et al. (2020) examined the digital game play of 193 elementary children and found that three specific design features (i.e., *providing information*, *manipulable math objects*, and *focused constraint*) had unique benefits when they were perceived by the children. Bullock et al. also reported that, in order to take advantage of the potentially beneficial mathematics affordances of design features, children must be aware of them. Taken together, research on digital games and embedded virtual manipulatives demonstrates that not all design features are high quality (Larkin & Milford, 2018), and children miss the potentially beneficial affordances of high-quality design features if they are unaware of them.

The research on virtual manipulatives and their design features is important to the current study because coding toys share some digital characteristics with virtual manipulatives. If research on virtual manipulatives aligns with coding toys, then coding toys could support children's mathematics and the design features of coding toys could play an important role in how they support children's engagement in mathematics.

Coding Toys: Benefits, Design Features, and Affordances

The current study is about children's mathematics while they engage in coding toy tasks and therefore it is important to understand the literature surrounding coding toys in mathematics education. Coding toys are designed to support children's computer science learning, but commercial companies often claim that they also support mathematics. Figure 3 highlights four commercially available coding toys. For example, Bee-Bot is a coding toy that is programmed by pushing the directional buttons on top of

Figure 3

The Four Coding Toys Used in this Study

the toy, and then it moves around on wheels according to the sequence of depressed buttons. As can be seen in Figure 3, coding toys come in many shapes and sizes. Some have separate programming boards, and some have integrated programming buttons. Coding toys are considered a new educational manipulative that share features with physical and virtual manipulatives. For example, children can physically move them around and interact with them like physical mathematic manipulatives; they can also receive feedback from them (e.g., sounds, flashing lights) like virtual manipulatives.

Across the early childhood literature, research on coding toys typically falls within three categories: understanding computer science content (e.g., Bers et al., 2014, 2019; Chalmers, 2018; Muñoz-Repiso & Caballero-González, 2019), understanding the coding toy context (e.g., Angeli & Valenides, 2019; Clarke-Midura et al., 2021; Gomes et al., 2018) or understanding cross-curriculum connections between coding toys and mathematics (e.g., Moore et al., 2020; Murcia & Tang, 2019; Nam et al., 2019; Shumway et al., 2021; Sung et al., 2017). Computer science content studies use these systems to explore computer science skills that can be acquired, like how Bers (2014, 2019) investigated preschool through kindergarten-aged children's ability to learn sequencing, conditionals, and action-instruction correspondence with a coding toy named KIBO. Context studies focus on the specific learning environments that foster and support learning. For example, Angeli and Valenides were interested in different scaffolding techniques in a coding toy environment and if these were more or less beneficial to the learning of girls or boys. Finally, cross-curricular studies typically focus on how mathematics can be integrated into coding toy experiences. These types of studies are of primary importance to this research study because they analyze children's engagement with coding toys and evaluate the mathematics that is involved in the process. The following sections focus on how coding toys support mathematics and then on specific design features of coding toys.

Mathematical Benefits of Coding Toys

The unique blend of physical and digital characteristics in coding toys have been shown to support the mathematical thinking of young children (Fessakis et al., 2013; Moore et al., 2020; Murcia & Tang, 2019; Nam et al., 2019; Palmér, 2017; Shumway et al. 2019, 2021; Sung et al., 2017). Much of this emerging research on young children's mathematics and coding toys justifies the use of these systems because, in part, the tangible aspects of the systems match appropriate developmental considerations for the age group, while exposing children to the computer science skills that are concomitantly afforded by the semi-digital systems (Moore et al., 2020; Nam et al., 2019). Generally, researchers interested in understanding computer science learning—which encompasses mathematical thinking skills such as decomposition and sequencing—end up developing theory and research findings which indicate an inextricable connection between mathematical processes and computer science processes (e.g., Israel & Lash, 2020; Peréz, 2020; Shute et al., 2017; Weintrop et al., 2016). This research is important to the current study because, overall, it illustrates that coding toy contexts are promising spaces to support the development of young children's mathematics concepts.

Specific Design Features and Affordances of Coding Toys

Although research suggests that coding toys support young children's mathematics concept development, there is limited research regarding how the design features of such tools specifically afford the mathematics. The only literature located that lends insight into the design features of coding toys was conducted by Clarke-Midura et al. (2019), Hamilton et al. (2020) and Yu and Roque (2019). These researchers focused on identifying design features of the coding toys, but not on understanding how the design features afforded mathematics. Yu and Roque conducted an examination of current early childhood computer science computational kits (which includes coding toys) both in academic research and in commercial venues. They found that design features varied across the kits, sometimes there were differences and sometimes there were similarities. Sometimes there were differences in design features across platforms such as one coding toy being programmed directly from the body and another coding toy being programmed from a separate interface. Sometimes there were similarities in design features across platforms such as all coding toys incorporating some sort of flashing light or sound. This research is important to the current study because it is beginning to identify the different design features of coding toys. The current study aims at understanding such design features in relation to mathematics, so identifying the design features of the coding toys themselves is an important step in the process.

Summary of Design Features of Mathematical Tools

The research on design features of physical manipulatives, virtual manipulatives, and coding toys converges on three critical topics. The first is that these tools have demonstrated their potential to support children's mathematics. Next, design features of manipulatives provide specific mathematical affordances. Finally, research suggests that children need to perceive and demonstrate awareness of the design features of the manipulatives in order to take advantage of the beneficial affordances. Although little research exists in which the design features and affordances of coding toys are investigated, research on physical and virtual manipulatives aids in understanding the possible mathematical benefits of design features and affordances of coding toys.

Young Children's Perception and Embodiment

The final section of this review focuses on affordance theory and embodied cognition theory as a theoretical lens to understand how young children engaged with mathematics through their perceptions and use of design features as they play with coding toys. The review begins with a discussion of Gibson's (1977, 1979b) Affordance Theory, which is a commonly used theory when investigating children's perceptions of design features. The second portion of the section will review theories of embodied (or grounded) cognition (Barsalou, 2008; Gibbs, 2005; Glenberg, 2008; Lakoff & Johnson, 1999; Lakoff & Nuñez, 2000), which are commonly used when investigating knowledge and learning. These sections are important to the current study because they provide a theoretical frame for interpreting how children engaged with mathematics through their perceptions and use of design features as they play with coding toys.

Young Children's Perceptions from an Affordance Theory Perspective

Children's perceptions of design features are an important part of the current study because it helps the researcher understand how the children engaged in mathematics concepts. To frame the understanding of children's perceptions of design features, Gibson's Affordance Theory (1977, 1979a, 1979b) is adopted as a theoretical perspective. Affordance Theory is based on the "complementarity of the animal and the environment" (Gibson, 1979b, p. 56). By this, Gibson means that environmental objects have an inherent influence on perceptions and actions of an animal, or in the current case, an individual. Affordances have also been defined by Burlamaqui and Dong (2014) as "cues of the potential uses of an artefact by an agent in a given environment" (p. 13) and refer to possibilities that the agent has for action. The main way that affordance theory diverges from orthodox psychology is that it relates our classification and understanding of objects not principally to their qualities of properties, but by the affordances they offer the individual. According to affordance theory for example, a coffee mug would not principally be understood in terms of its handle, density, round edges, or size, but by its ability to offer containment and hold-ability. Greeno (1994) describes a nuance to Gibson's Affordance Theory which indicates the distinction between the agent and the environment, and that the abilities of the agent are co-defining alongside the affordance of the environment. In the previous example with the coffee mug, the agent (person) has specific abilities (desire to use with hands). These abilities directly relate to the object's (mug) affordances (containment, hold-ability).

It is important to note that affordances of the objects can be beneficial or injurious

(Gibson, 1979b). An example given to us by Gibson is that of a cliff. One side of the cliff—the side you may find yourself walking on—affords certain beneficial affordances such as locomotion, support, or stability. However, the other side of the cliff—the side you would be wise to avoid—affords certain injurious affordances such as pain, injury, or falling. It is important to know that any environmental object likely offers both beneficial and injurious affordances depending on the interaction the agent's abilities takes on the object. An inherent and underlying process behind affordance theory is perception.

Adopting affordance theory as a theoretical perspective in the current study helps to explain how children might perceive design features of coding toys, which could support or indicate the child is engaging with mathematics. For example, it helps explain what is afforded by a specific design feature (e.g., flashing light, moving wheels) on the coding toys and whether or not that feature supported engagement with mathematics.

Young Children's Engagement with Coding Toys from an Embodied Cognition Perspective

Due to the dynamic nature of coding toys, children generally employ their bodies in a physical way during coding toy activities. Children often demonstrate engagement in mathematics and awareness of design features through physical interaction with the task or with the coding toy. For this reason, embodied cognition theory was also adopted in the current study to situate understanding of mathematical engagement through children's physical interactions with the coding toy environment (e.g., gestures, body turning, body movement)

Two terms have been used, nearly synonymously, to describe the theoretical positioning that cognition stems from bodily interactions with the world. These terms are embodied cognition (Gibbs, 2005; Glenberg, 2008; Lakoff & Johnson, 1999; Lakoff & Nuñez, 2000) and grounded cognition (Barsalou, 2008). Barsalou suggests a slight difference between the two theories, that embodied cognition theorists hypothesize only bodily states or sensory-motor processes precede cognition (Ionescu & Vasc, 2014), while grounded cognition theorists hypothesize that bodily states, as well as other forms of grounding (e.g., situated action, simulations) precede cognition. Despite this distinction, these theories differ from more traditional cognitivist theories in that cognition is not the antecedent of learning and thinking, rather it is the consequence of sensory-motor interactions with the environment that precede learning and thinking. Ionescu and Vasc provide two defining statements that help set embodied cognition theories apart from other theories: (a) cognition is not abstract or amodal and is grounded in multimodality of representations in sense, brain, and action; and (b) cognition is more than just about thinking; it heavily relies upon perception, action, and bodily states.

There is an inextricable connection between theories of embodied cognition and body syntonicity as described by Papert (1980). Body syntonicity, or syntonic learning, focuses on instruction where children connect to the activities with their bodies, allowing them to form an association between sensory-motor perception (i.e., sensory engagement) and cognition. In his work with Logo the programming Turtle, Papert describes the activities as syntonic, and because of that, "encourages the conscious, deliberate use of problem-solving and mathematic strategies" (pp. 63-64). Paek (2012) also found that more mathematics emerged as children increased their embodied interaction with the virtual manipulative, *Puzzle Blocks*. This suggests that embodiment during virtual manipulative interaction may be an important characteristic of learning environments that influences mathematics.

Theories of embodied cognition are important to this study because children are constantly using their bodies when they play with coding toys. Sometimes they move around after the toy, turn their heads, make gestures in the air, move the robot with their hands, and perform a variety of other actions that demonstrate physical interaction. These types of actions help indicate whether the child is engaging in mathematics as well as help the children make connections amongst activity and the design features of the coding toys.

Summary of Young Children's Perceptions, Affordances, and Embodiment

The theoretical underpinnings of perception, affordances, and embodiment are critical for this study because they help the researcher know if the child is engaging in mathematics. Children's perceptions of design features and what they afford are grounded in Gibson's (1979b) affordance theory. For, according to Gibson, objects contain affordances, and the ways that children perceive such affordances influence their learning. Additionally, it can be challenging to understand children's perception and use of affordances without adopting some of the embodied cognition theories on sensory-motor experiences and their relation to learning. Sometimes, children demonstrate use and perception of design feature affordances through embodiment. For this reason, these two theories (i.e., affordance theory, embodied cognition theory) were used as theoretical perspective to understand how children engage in mathematical concepts.

Summary

In this literature review, I presented a conceptual framework for the current study that helps to explain the phenomenon of how design features of coding toys relate to early childhood mathematics with coding toys, and I interpreted the children's engagement with the mathematics through affordance theory and embodied cognition theory. Research demonstrates that coding toy contexts have the potential to support mathematics for young children, specifically in three mathematical domains (i.e., number, spatial, measurement). Research on virtual and physical mathematics manipulatives suggests that specific affordances and design features support mathematics learning, and that some of these affordances and design features may also relate to coding toy manipulatives. Finally, theories on perception and embodiment provide a lens for understanding how the children engage in mathematics through perception and use of design features. Taken together, this study aimed to understand children's awareness of the design features in coding toys and to understand how those design features afforded children's engagement with mathematics. In the next chapter, the method employed for the study is outlined.

CHAPTER III

METHOD

The purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. In this chapter, I start by describing how qualitative methods and a multi-phased qualitative research design were most appropriate to answer the research questions, and I discuss the participants and school sites. Then, I describe the coding toys and tasks which were the main context of the study. Afterwards, I outline the data sources, how I obtained the existing dataset, the procedures for data collection, the data analyses, and a description of the steps taken to ensure trustworthiness. Finally, I briefly report on pilot work that I conducted which informed design of the current investigation.

Research Design of Original NSF Project and Current Research Project

I will first describe the research design that the dataset was collected in, which is design-based research (DBR). Then, I will explain used a multi-phased qualitative research design to investigate the dataset.

Research Design of the Original Coding in Kindergarten Project

The dataset included 42 hours of video recordings that were collected as part of a large DBR project, known as Coding in Kindergarten (CiK; DRL#-1842116). The research design for the CiK project was DBR (Cobb et al., 2003; diSessa & Cobb, 2004). Typical DBR is characterized by interventions that establish long lasting relationships with participants, iteratively implement and revise intervention designs, and carefully examine data gathered from multiple cycles to document changes in effectiveness, changes in understanding of learning, and changes in understanding of theory

The CiK project used a DBR design to iteratively design teaching and learning situations in order to create theory on young children's development of computational thinking. The pragmatic side of the CiK project was to develop and test activities to support young children's computational thinking within a classroom setting. Based on the theoretical and pragmatic goals of the CiK project (a) the coding toy activities were iteratively revised throughout data collection to achieve specific learning outcomes targeted toward children's development of computational thinking through learning to program, and (b) the design and revision of the coding toy activities were informed by ongoing analysis of children's reasoning and the learning environment.

Research Methods and Design

I utilized a multi-phased qualitative research design for this investigation. Overall, qualitative methods for this investigation were appropriate because they aided the researcher in understanding the "why" and "how" of human interaction (Agee, 2009). The dataset used in this study included video recordings, which are a natural fit for an in-depth qualitative analysis because they can be viewed repeatedly allowing analysis from different perspectives. Additionally, this dataset was collected in a naturalistic setting (Armstrong, 2010), which means the focus was on maintaining a naturalistic learning environment rather than in a controlled environment.

Qualitative methods are appropriate for attempting to understand complex learning in naturalistic environments. Qualitative methods were appropriate for this study because they have the ability to develop initial theory around a concept (i.e., coding toy design features and mathematics), which has received very little previous exploration. Specifically, a multi-phased qualitative research design analysis was appropriate because questions and data were analyzed in five different phases, one building off the next. For example, the first part of the qualitative analysis allowed understanding of design features that existed within each coding toy. Only after this first phase could the researcher begin the next phase and analyze how the children perceived or used those design features.

The multi-phased qualitative analysis was distinctly unique from the original CiK DBR project. For example, one CiK research question focused on understanding the mathematics children used as they coded with two coding toys (Shumway et al., 2021). However, my research question focused on children's awareness of the design features of four coding toys and how those design features afforded children's engagement with mathematics. Therefore, I aimed to understand the learning episodes in which the children engaged in mathematics as they perceived and used design features, which is a uniquely different inquiry than the one conducted by Shumway et al. in the original CiK DBR project, which focused on mathematical engagement holistically as children code.

Participants and School Sites

The video data for this study included 106 participants (47 females, 59 males).

The participants were 5- and 6-year-old kindergarten children from six different school sites (83 full-day public kindergarten, 15 full-day private kindergarten, and 8 after-school programs) in the Rocky Mountain Region. Broadly, the demographic composition of the public schools (Sites 3-6) is: 1% identified as Asian, 19% identified as Hispanic, 76% identified as White, and 4% identified as another race. Table 1 indicates the school type, number of participants, and gender for each site.

Table 1

School Type, Number of Participants, and Participant Gender

School site	School type	Number of participants	Females	Males
Site 1	Private	15	10	5
Site 2	Private after-school	8	4	4
Site 3	Public	35	14	21
Site 4	Public	15	6	9
Site 5	Public	17	5	12
Site 6	Public	16	8	8
Total		106	47	59

Across the six school sites, there were aspects of time, space, and structure that were relatively consistent in terms of implementing the coding toy tasks, though certainly each class was unique. Four things were relatively stable across sites and contexts: activities occurred on the ground, they lasted approximately 30-minutes, the groups of children working on the activity ranged from three to five children, and there was space for a teacher-researcher and a videographer-researcher.

Research Context: The Coding Toys and Tasks

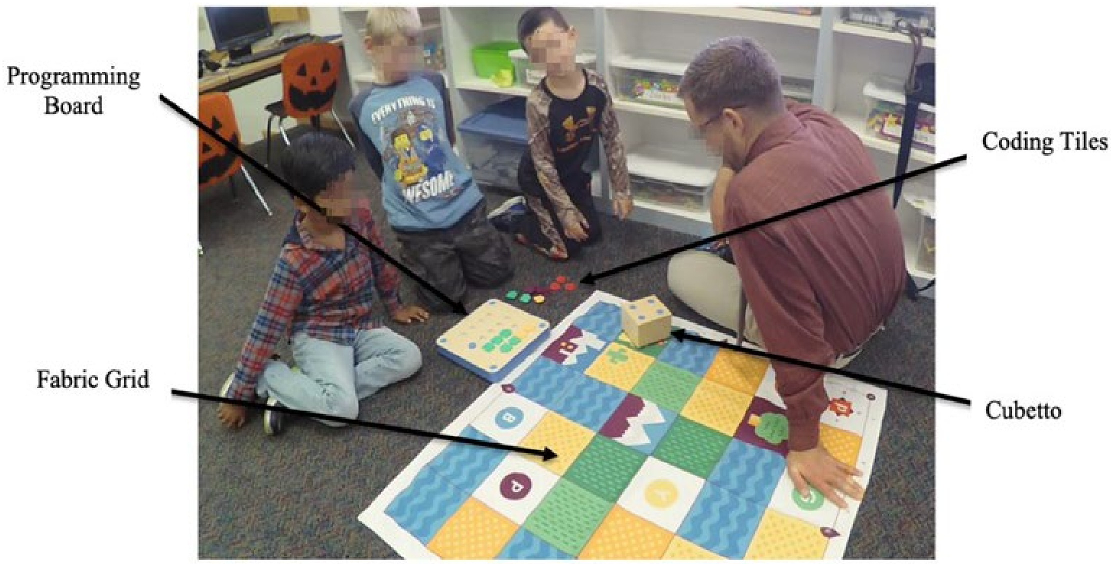
The video dataset shows the 5- and 6-year-old participants (n = 106) using four specific coding toys: (a) Cubetto by Primo Toys, (b) Code-a-pillar by Fisher Price, (c) Botley by Learning Resource, and (d) Bee-Bot by Terrapin. Each of these coding toy systems is commercially available, designed for young children, have a blend of physical and digital characteristics, and are advertised to support young children's problem-solving and coding skills. For example, when one navigates to the Primo Toys page to inquire about the product Cubetto, large font pops up across the page that reads "*Coding, STEM numeracy and creativity in a single product.*" This advertising pitch is similar across products and demonstrates the intended purpose for the products. The following four sections describe each coding toy and its supplementary materials. Some of the supplementary materials are provided with the product and some of them were created by the DBR project research team.

Cubetto by Primo Toys

Cubetto, by Primo Toys, includes a programming board, coding tiles, and a fabric grid. Figure 4 illustrates the materials that accompany Cubetto. Cubetto is controlled to move around the fabric grid by the coding tiles that are placed on the programming board. Each coding tile is laid sequentially in the programming board, and when the blue button is pressed on the programming board, Cubetto enacts the sequence by moving around with the small wheels on the underside of the toy. The green coding tile indicates one grid space forward, the purple indicates one grid space backwards. The red coding tile indicates one 90-degree right rotation, the yellow indicates one 90-degree left rotation.

Figure 4

Cubetto and Supplementary Materials



Code-a-Pillar by Fisher Price

The second coding toy system is Code-a-pillar by Fisher Price. This system includes a moveable Code-a-pillar body that has wheels, body segments that are the codes, and a researcher-created large mat with grid spaces. Figure 5 illustrates the materials that accompany Code-a-pillar.

The moveable body consists of the Code-a-pillar head with little antennas and wheels on the bottom. In order to move Code-a-pillar, the connectable segments must be "plugged in" to the rest of the body. Each body segment represents a different action (e.g., move one grid square forward, rotate left). As these are appended to the Code-a-pillar body in a long line, they form a sequence of codes which then is enacted as an entire program. When the program is enacted and the Code-a-pillar starts moving, the eyes blink blue and it makes singing sounds. The large researcher-created grid map was

Figure 5

Code-a-Pillar and Supplementary Materials

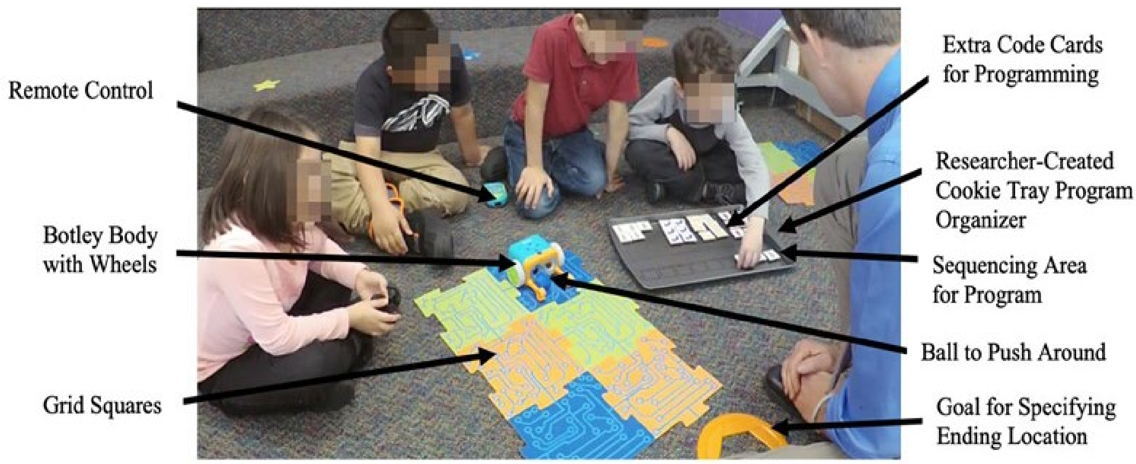
created so the children could track the distance the Code-a-pillar traveled with each movement. The size of the grid squares aligns with the movements of Code-a-pillar, meaning each forward command moves it from the center of one grid square to the center of another.

Botley by Learning Resources

The third coding toy system is Botley by Learning Resources. This system includes a moveable Botley body that has wheels, a remote that programs Botley, grid squares that provide a space for Botley to move around on, supplementary materials like a goal and a ball, as well as a researcher-created cookie tray program organizer with coding cards to help the children keep track of the codes they programmed into Botley. Figure 6 illustrates the materials that accompany Botley.

Figure 6

Botley and Supplementary Materials



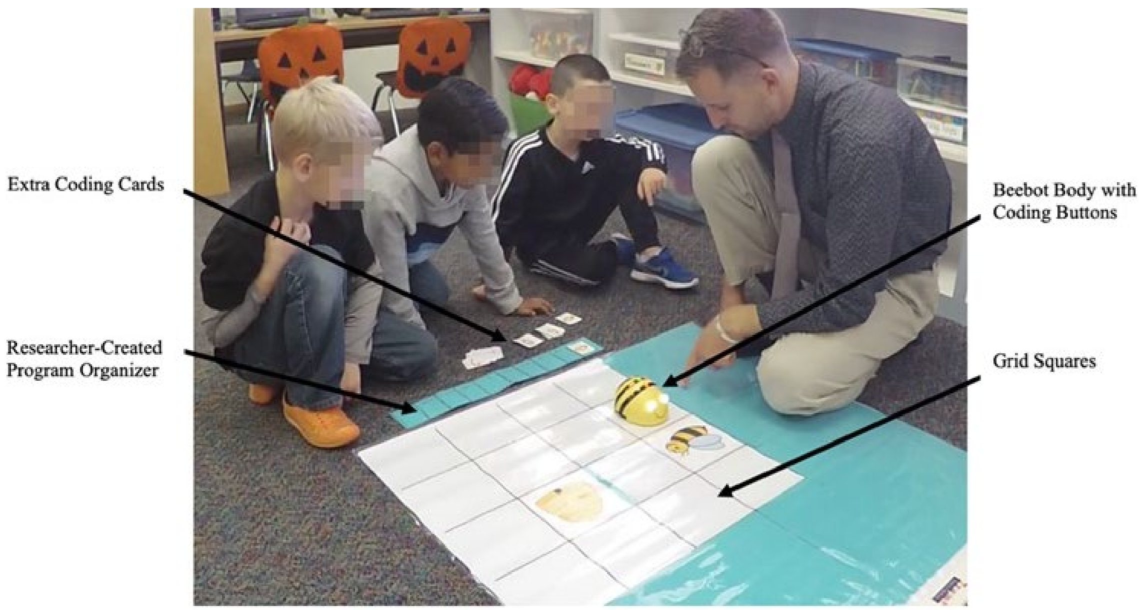
Botley's body is controlled entirely by using the remote control. The remote control keeps track of the different action button depresses (e.g., move forward, move forward, rotate left, move backward) and then when the 'enact' button is depressed, the Botley body enacts the sequence of codes stored in the remote control. The grid squares align with Botley movements so that one movement translates the Botley from the center of one grid square to another. Sometimes children forget the different codes that they have pressed on the remote control. In order to help provide structure and organization for the children's programs, the researchers created a cookie tray program organizer with coding cards. This organizer makes it so the children can plan the codes they want to press in the remote, as well as remember the codes already pressed into the remote.

Bee-Bot by Terrapin

The final coding toy system is Bee-Bot by Terrapin. This system includes a moveable Bee-Bot body that has wheels, grid squares that provide a space for Bee-Bot to move around on, and a researcher-created program organizer with coding cards to help children keep track of their programs. Figure 7 illustrates the materials that accompany Bee-Bot.

Figure 7

Bee-Bot and Supplementary Materials



Bee-Bot is programmed using the buttons that are directly on top of its head. The four directional arrows on the top of Bee-Bot's head can be pressed in any order and the computer inside the Bee-Bot keeps track of the arrow pushes. Then, when a green center button is pushed that says *go*, the Bee-Bot will enact the codes following the sequence they were entered. To write new programs, there is a blue button that says *clear* on the Bee-Bot that clears out all the previous codes that were stored in its memory. The researcher-created program organizer allows the children to keep track of the codes they have entered into Bee-Bot's computer. They use the extra coding cards and place them sequentially on the program organizer as they plan and write programs.

An Example Activity: Introducing Cubetto and Building a Sequence

This section briefly describes one specific coding toy activity (i.e., *Introduction to Cubetto and Building a* Sequence). At the beginning of the lesson, a researcher read an introductory story to the children which introduced them to Cubetto, the coding tiles, the programming board, and the fabric grid. Next, the researcher posed a coding challenge for the children to get Cubetto to, first, travel three spaces forward, and then travel backwards the same three spaces. After this initial task, which was meant to help the children become familiar with the materials, the participants were challenged to get Cubetto to a castle grid square, which added the complexity of a rotational code. This challenge prompted the introduction of the rotation coding tiles and allowed the children to problem solve by creating a sequence that completed the challenge. Following the completion of this challenge, the participants were invited to try a slightly more complex challenge where they tried to direct Cubetto across the fabric grid to land on a boat. As the children worked on these tasks, they collaborated, discussed, and took turns making decisions and putting the coding tiles on the programming board. The main role of the researcher during this task was to help guide and prompt children's thinking, as well as help manage turn-taking and group work logistics.

Data Source

The main source of data for this study was a video dataset of 84 lessons using the four coding toys (a total of 42 hours of data). The dataset included 16 lessons with Code-a-pillar (8 hours), 30 lessons with Botley (15 hours), 30 lessons with Cubetto (15 hours), and 8 lessons with Bee-Bot (4 hours). Table 2 presents a detailed breakdown of the video dataset. The rows indicate the amount of video data per coding toy, the columns indicate the amount of video data per school site, and the colored cells indicate the amount of data per year of data collection. The final column and row in Table 2 highlight the total amount of coding toy lessons as well as the total hours of video data. There are more hours of video data for some coding toys than others because of the DBR tradition in which data was collected. The Bee-Bot coding toy was used early in the CiK project, but after use in one school, it was no longer included at other sites. I obtained every

Table 2

Number of Introductory Lessons in Dataset for Each Coding Toy by Site and Year

	Number of lessons						Total	
Coding Toy	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6	Lessons	Hours
Code-a-pillar	6 2	8					16	8
Botley	2		8	8	8	4	30	15
Cubetto	2		8	8	8	4	30	15
Bee-Bot				8			8	4
Total (hrs.)	12 (6)	8 (4)	16 (8)	24 (12)	16 (8)	8 (4)	84	42

Note. Each lesson lasts approximately 30 minutes. The total hours of data equaled the total lessons multiplied by .5 hours. For example, Site 1 Code-a-pillar lessons from 2018 (in orange) totaled 3 hours of video data: 6 lessons x .5 hours = 3 hours of video data.

[a] Year of data collection colored

2018	2019	2020

introductory lesson that was conducted with each coding toy which meant more or less hours of video data for each coding toy, which was acceptable due to the qualitative nature of this investigation. Pertaining to Bee-Bot, four hours of in-depth qualitative analysis was sufficient to gain advanced understanding of ways children interacted with the design features.

The video dataset was collected with a video camera that was placed on a large tripod during the coding toy lessons and was angled to capture their gestures and facial expresses at all possible moments. The tripod and camera were managed by a researcher-videographer who ensured that it captured children's verbal and physical interactions with the robot. The video data were stored on an encrypted storage system (i.e., BOX) and on an external hard drive in a locked research laboratory.

Procedures for Acquiring the Dataset

The first part of this section describes my role in the dataset and how I acquired the video dataset for analysis. The second part of this section describes how the researchers in the CiK project collected the data.

How the Researcher Acquired the Video Dataset

This section describes my role in the dataset used for the study, and how I acquired the dataset from the CiK project. The dataset used for this study was collected from 2018-2020 by the CiK project. Across three years of the CiK project, I was a graduate research assistant with CiK and some of my roles on the project were helping write the institutional review board application, coordinating with participating school sites, designing coding toy tasks, implementing tasks in schools, video recording the tasks, and storing and managing the video data. The CiK principal investigators (i.e., Drs. Jody Clarke-Midura, Jessica Shumway, Victor Lee) main tasks in this project were to operationalize young children's computational thinking, design tasks using coding toys to study children's computational thinking and mathematical problem solving and design a computational thinking assessment. One subresearch question that has not yet been examined by the research team is about the design features of the coding toys and in what ways they afford computational thinking and engagement in mathematics. This study aimed to meet this need and to study the coding toys' design features and ways they afforded children's engagement in mathematics. So far, the CiK team has used some of this data to investigate mathematics using child-centered theories on learning. However, this study applied affordance theory as a new theoretical lens to help better understand the relationship between specific design features and affordances of the coding toys and mathematical engagement.

I received permission to use existing project video data for my dissertation from all three principal investigators, with the condition that I work closely with Dr. Shumway to ensure the dissertation project would (a) not interfere with current CiK investigations, (b) benefit the CiK project's vision and direction, and (c) establish a purpose that aligned to the project data. Following this approval from CiK, Dr. Shumway and I met with Utah State University's institutional review board director (Nicole Vouvalis; 2020_11_12) to ask about the process of allowing NSF-funded project data to be used in a dissertation project. The response was positive, and that following successful defense of the dissertation proposal, amendments to the existing CiK institutional review board certifications (#8928, #9569) were made that allowed me to officially use the data for my dissertation.

Procedures Used by CiK Researchers to Collect the Dataset

This section describes how the CiK research team collected the dataset. In general, the CiK team coordinated with schools and teachers, prepared and implemented the activities, and then transported and stored the data at the university. The CiK research team consisted of principal investigators (PIs), graduate research assistants (GRAs), and undergraduate research assistants (URAs).

Coordinating with Schools and Teachers

The CiK research team coordinated with local schools and teachers to obtain sites and participants for data collection. Informed consent was administered and obtained before data collection began. The CiK researchers coordinated with classroom teachers about an appropriate time, space, and structure in which to implement the coding toy activities.

Preparing and Implementing Coding Toy Activities

Two members of the CiK research team—typically a PI paired with a GRA or URA—were assigned to each group of children who worked on a coding toy activity. These two researchers worked as a pair in planning, implementing, and refining the coding toy activities for the children. Prior to the lesson, each researcher within a pair was assigned a different role (i.e., teacher-researcher, videographer-researcher). The main roles of the teacher-researcher were to present the task, guide and prompt children's thinking, and provide collaboration scaffolding (e.g., turn-taking, group work logistics). The teacher-researcher urged problem-solving by asking questions such as "Why do you think that will work?" "Do you all agree with this strategy?" or "What is another strategy you think is worth trying?" The main roles of the researcher-videographer were to make sure the video camera captured the participants' verbal and physical interactions and to conduct detailed notes on a design memo about critical events of the teaching episode. The researcher-videographer moved around the activity space in order to capture the interactions with the children.

The coding toy activities implemented by the researchers varied by year, school site, and coding toy, but had some general similarities. The similarities important to the current study are: (a) they were either the first or second time the children had participated in a coding toy activity with the researchers, and are considered *introductory lessons*; (b) they involved the children learning how to use basic codes to program the coding toys; (c) they were designed so children were actively engaged in testing and trying, rather than listening and absorbing; and finally (d) each lesson lasted approximately 30-minutes.

Transporting and Storing Data

The CiK researchers transported and stored data immediately after data collection. After the researchers debriefed about the coding toy activities, they immediately drove back to the university where one of the researchers took the data directly to a locked research laboratory. Upon arriving in the laboratory, the researchers downloaded the data from the video camera memory cards onto an external hard drive. Then, the videos were also uploaded onto a secure, double-authenticated, cloud storage system (i.e., BOX). The video data were deleted from the original video camera memory cards and the external hard drive remained in the locked research laboratory.

Data Analysis

I conducted a multi-phased qualitative video analysis (DeCuir-Gunby et al., 2012; Erickson, 2006). Some advantages of video data in mathematics education research are its permanence, flexible features (e.g., slow motion), and the ability to be reviewed multiple times and the experiences interpreted from many perspectives (Powell et al., 2003). A variety of educational researchers offer beneficial techniques when describing how to rigorously analyze video data. Some suggested techniques are identifying critical events, coding, and composing a narrative (Powell et al., 2003); focusing on subject matter content, and focusing on verbal and nonverbal activity (Erickson, 2006); and rewatching video data from multiple perspectives for triangulation purposes (DeCuir-Gunby et al., 2012).

The perspective I took in analyzing the data was an observational one, one geared at watching the children's actions, words, and observable interactions in order to understand the data. Thereby, the unit of analysis was the specific interactions of individual children working in a small group. I implemented an interpretivist lens on the video data analysis, meaning that truth statements are context-bound and consider the group interactions as important aspects when developing theory around the topics of interest (Alharhsheh & Pius, 2020).

All of the qualitative analysis was conducted using MAXQDA software (VERBI, 2020). This software housed all of the video data and allowed me to attach specific codes to video segments. The software has built-in analysis features for counting frequencies of codes, displaying codes in multiple ways, and sorting, changing, and categorizing codes. One of the primary benefits of this software is that it allowed me to attach various codes to segments of video, and then go back and reanalyze those segments more closely for later phases of analysis. For example, video segments were assigned codes during first-cycle coding (phase 1), then all of the segments that were coded in first-cycle coding were reanalyzed and assigned new codes during second-cycle coding (phase 2) and later coding phases.

The video dataset was analyzed in direct alignment to the three research questions. The analysis processes I used to answer each research question are described in the sections below. Table 3 shows the alignment among the research questions, the data source, and the data analysis techniques.

RQ#1 Analysis

The first research question focused on the design features children perceive and use when interacting with the four coding toys. To answer this question, I used descriptive coding strategies (Miles et al., 2020; Saldaña, 2021) to create a comprehensive set of design features of the coding toys. Saldaña (2021) described first-cycle descriptive coding, also called topic coding, as an initial labeling of small portions

Table 3

Alignment of Research Questions, Data Sources, and Data Analysis Techniques

Research questions	Data source for all three questions	Data analysis
What design features do kindergarten-aged children perceive and use when interacting with four different coding toys?	42 hours of video data from NSF grant # DRL-1842116 Code-a-pillar (8 hours) Botley (15 hours) Cubetto (15 hours) Bee-Bot (4 hours)	Descriptive Coding (Saldaña, 2021) Process Coding (Charmaz, 2002; Corbin & Strauss, 2015)
What mathematics do kindergarten-aged children engage in when they are perceiving design features of four different coding toys?		A priori Coding and Open Coding (Saldaña, 2021)
How do design features of four different coding toys afford kindergarten-aged children's mathematical engagement?		Causation Coding (Saldaña, 2021) Variable-Oriented Strategy (Miles et al., 2020)

of topic with a word or short phase. The topic is design features, and during this phase of coding I assigned a short phrase to each design feature, across coding toys, that aligned with that topic.

During the second phase of analysis, I conducted second cycle coding (Miles et al., 2020) to record the incidences that children perceived, or engaged, with the design features. I used process coding (Corbin & Strauss, 2015; Saldaña, 2021), which uses gerunds ("-ing" words) to depict action in the data (Charmaz, 2002). Coding for these positive incidences in the data—when children perceived design features through action—also informed negative incidences in the data—when children did not perceive the design features. Understanding both how children perceived, and did not perceive, the design features was important to this study. Overall, this phase of coding allowed a nuanced view of how children perceived or used the design features of the coding toys or did not. Hypothetical examples of codes developed in the first and second cycle of coding are depicted in Table 4.

Table 4

Example First- and Second-Cycle Codes for RQ#1 Analysis

First-cycle codes for design features	Description	Second-cycle codes for children's perception or engagement with design features	Description
Blinking Light (Board)	A small blue light flashes on the board for each code that is enacted. This flashing light is located on the separate programming board.	Watching □ Blinking Light (Board)	This second cycle code captures how the child is watching (perceiving) the blinking light on the board (design features).
Blinking Light (Body)	A small blue light flashes on the coding toy for each code that is enacted. This flashing light is located on the coding toy itself.	Discussing □ Blinking Light (Body)	This second cycle code captures how the child is discussing (perceiving) the blinking light on the body(design features).
Grid Squares	The array-based environment which the coding toy moves around on. Each grid square in the array is measured to precisely correspond to one movement of code with the coding toy.	Gesturing □ Grid Squares	This second cycle code captures how the child is gesturing (using) the grid squares on the mat (design features)
Symbol Shape	This code indicates the specific shape of the coding tile, image, or card. The tangible tiles have different points on them regarding their purpose, just like the buttons have different images depending on the purpose.	Rotating □ Symbol Shape	This second cycle code captures how the child is rotating (using) the symbolic code shapes (design features).
Face	This code indicates the face features that is on the side of the different coding toys.	Referencing □ Face	This second cycle code captures how the child is referencing (perceiving) the face feature on the side of a coding toy (design features).

RQ#2 Analysis

The second research question focused on the mathematics that the kindergarten-aged children engage in when perceiving and using design features of the four coding toys. Mathematical engagement was operationalized as children demonstrating behavioral (e.g., verbal, gestural, physical) participation in the mathematics concept. For this analysis, I used a priori and open coding (Saldaña, 2021). The three a priori coding schemes I used were created by Shumway et al. (2021) during the NSF CiK study and set within Site 3 only of the naturalistic investigation where young children coded two robot coding toys: Botley and Cubetto (see Appendices B, C, and D). These coding schemes indicate how young children engage in number, spatial, and measurement concepts and skills as they code Botley and Cubetto. Shumway et al's. research question was not focused on the specific design features of the coding toys, but on the mathematics that emerged, overall, as the children actively programmed Botley and Cubetto. Therefore, when I reanalyzed some of the Site 3 data, I identified different learning episodes to answer the research question for this analysis. Specifically, I identified the learning episodes in which the children engaged in mathematics as they perceived and used design features, which is a unique question from the one answered by Shumway et al. Boyatzis (1998) cautions that adopting a priori codes demands an adoption of the perspectives and assumptions made by the previous researchers who developed the codes. Considering this, I chose to adopt these codes for the following two reasons: (a) the Shumway et al. study was conducted with the Site 3 participants, and hence, the codes were developed within similar contexts to those that were used in this analysis; and (b) I was part of the Shumway et al. research team, and therefore, I understood and acknowledged the perspectives and assumptions of the codes. During this phase of coding, I reanalyzed each previously coded learning episode when a child perceived or used a design feature to see if the child also engaged in mathematics during that learning episode.

The set of a priori codes developed by Shumway et al. (2021) has three overarching mathematical topics: number, spatial, and measurement. Each of these overarching mathematical topics is comprised of specific concepts and skills as follows: number (i.e., counting, counting on, coordinating counts, operations), measurement (i.e., units of measure, distance measurement), and spatial (i.e., spatial orientation, spatial visualization, spatial language, spatial knowledge in codes). Certain behavioral indications help researchers know when one of these mathematics codes is appropriate over another. For example, when a child verbally counted each grid square, it was coded as *counting*; when a child counted grid squares and then counted codes to match, it was coded as *coordinating counts.* Sometimes, mathematics codes overlapped and there was more than one code in a certain learning episode. For example, if a child traced a linear movement length on the grid map and counted, it was coded as *units of measure* and *counting*.

As I conducted this phase of a priori coding, I also conducted open coding for any other mathematical concepts and skills that emerged as the children perceived or used the design features. This allowed me to account for mathematical concepts and skills that children engaged with in this context that were not have been represented in Shumway et al.'s (2021) a priori scheme.

RQ#3 Analysis

The third research question focused on how design features of the coding toys afford mathematical engagement to the kindergarten-aged children. Analysis for this question occurred in two phases. During the first phase, I used causation coding (Munton et al., 1999; Saldaña, 2021), which helped to determine the relational force (link) between perception/use of design features (cause) and children's engagement with the mathematics (outcome). For example, I looked for a special relationship between a distinct design feature and a mathematical concept and skill. Munton et al. (1999) described causation coding as understanding the *why* behind a connected cause and effect. There are three necessary components to understanding causation: the cause, the outcome, and the link between the two. This phase of the analysis resulted in understanding what specific design features of each individual coding toy afforded mathematical engagement for the kindergarten children.

During the second phase of analysis, I conducted a variable-oriented cross-case comparison across toys (Miles et al., 2020) to understand the similarities and differences between the toys. This phase brought forward the patterns and trends in the previously conducted causation coding. Miles et al. described two specific strategies for orienting a cross-case comparison: case-oriented and variable-oriented. Each orientation has its own benefits and weaknesses and should be selected based on the data and the purpose of the research questions. A variable-oriented approach was most appropriate for this study because the focus is on the specific relationship between design features of coding toys and mathematics. Additionally, there were various cases (i.e., different children groups working with different robots) that needed to be examined in conjunction with one another to find the commonalities across cases. The case-oriented approach focuses on explaining and depicting the similarities and differences of a few select cases of individuals, rather than on a specific topic-related theme across more cases. The variable-oriented approach was more appropriate than the case-oriented approach for this study because there are clear variables (i.e., design features and mathematics) that are targeted throughout the analysis across the various cases.

Affordance and embodied cognition theory were explicitly connected to this analysis process through children's verbalizations, gestures, movements, and perceptions of the design features and the mathematics. For example, embodied cognition theory is about the relationship between children's sensory motor interactions and thinking. Therefore, children's physical actions related to design features were analyzed and coded while answering research question 1 (e.g., moving body forward, turning body left, rotating hand).

Trustworthiness and Validity of Data Collection and Analysis

Lincoln and Guba (1985) described four characteristics of a rigorous investigation which help determine the trustworthiness and validity of a qualitative study. These four characteristics are *credibility, transferability, dependability,* and *confirmability.* Credibility is similar to internal validity and represents how well the data sources (often participants) think the analysis and interpretations match reality. Transferability describes how the findings and interpretations have at least some power of transferability to others within an extremely similar context. Dependability can also be thought of as stability with procedures and conscious methodological decisions throughout the study. Finally, confirmability is related to credibility and describes whether processes were taken to ensure matching of study design and goals with the topic. Some of these factors were accounted for throughout original data collection by the CiK project team by constantly debriefing with classroom teachers, maintaining long periods of engagement with classrooms and teachers, and engaging in researcher reflexivity. During analysis, I helped ensure rigorous content validity by consulting a content expert and meeting with one knowledgeable professional in the field of coding toys, mathematics, and early childhood to ensure that codes, themes, and interpretations were grounded in reality. This occurred in phase one of RQ#1 to help generate a set of design features were captured in the coding process. It also occurred throughout phase two and three to help understand the mathematics and link them to design features.

Examining Design Features of a Coding Toy: Pilot Work

Pilot work was conducted to evaluate how the specific design features of one coding toy in a single task related to mathematics (Kozlowski et al., 2021). The purpose of this pilot work was to understand how the specific design features could afford mathematical engagement with the Cubetto coding robot toy. The researcher analyzed four, 30-minute small group lessons as children completed a task with Cubetto and conducted qualitative analysis for (a) design features, (b) mathematics, (c) and how the design features afforded mathematical engagement. Results indicated important findings that helped to inform this dissertation study. One was that children's lack of perception of certain design features (i.e., blinking lights) impeded the beneficial mathematical affordance of such design features. Another important finding was that different design features afforded different mathematics to the children. This pilot work highlighted initial design features code groups (e.g., anthropomorphized features, simultaneous linking) that were used in this study. Also, it revealed important methodological considerations in the current dissertation design such as first-cycle coding for children's perceptions of and engagement with design features, rather than just on the design features themselves.

Summary

In this chapter, I discussed the methods I used to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afford children's engagement with mathematics. Ultimately, the video dataset for this study was analyzed in phases, each building on the last. First, the children's perceptions and use of design features was analyzed. Second, those learning episodes when children perceived or used design features was reanalyzed to understand how the children engaged in mathematics during those instances. Then, (a) the learning episodes that children engaged in mathematics while perceiving or using design features were reanalyzed to understand the relationship between the perception and use of the design features and mathematics, and (b) those relationships between design features and mathematics were compared across the four coding toys.

CHAPTER IV

RESULTS

The purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afforded children's engagement with mathematics. The three research questions that guided this study were as follows.

1. What design features do kindergarten-aged children perceive and use when interacting with four different coding toys?
2. What mathematics do kindergarten-aged children engage in when they are perceiving design features of four different coding toys?
3. How do design features of four different coding toys afford kindergarten-aged children's mathematical engagement?

The sections below are organized around the three research questions. The first section presents the results for Research Question #1, the second section present results for Research Question #2, and the third section presents results for Research Question #3.

Research Question #1: Design Features Perception and Use

Research Question #1 focused on the design features that kindergarten-aged children perceived and used when interacting with four different coding toys. I structured this subsection to (a) present the results of the analysis demonstrating the design features that were perceived and used across all four coding toys, and then to (b) depict three design features that showed similar patterns of perception and use across the four coding toys (i.e., grid spaces, command arrows, lights, and sounds).

Design Features that were Perceived and Used

The main design features that children perceived across the four coding toys were the grid square features and command arrows/tiles. Other design features were perceived at lesser degrees of frequencies. These results are presented in Table 5. Each coding toy has its own highlighted set of rows. The columns list the different design features categorized by system (i.e., body, controller, environment, program organizer). Sometimes, a coding toy did not have a system—like Bee-Bot not having a remote control because the coding arrows are on the top of the physical body. Other times, a coding toy had all the systems—like Botley having a body, remote control, environmental features, and a program organizer. Table 5 presents the frequency of child perception and use for each design feature by coding toy. The frequencies are percentages calculated using the number of times a design feature of a coding toy was used or perceived in relation to all occurrences of use of design features for that coding toy. For example, there were a total of 810 cases of design feature perception and use with Botley, and of those 810 cases, only 10 were of the children perceiving and/or using the flashing body lights. To indicate the percentage of use for each design feature, either an R was placed to represent *Rare Use* (<5% of cases), an M was placed to represent *Moderate Use* (5% to 25% of cases), or an F was placed to represent *Frequent Use* (>25% of cases). These percentages of frequencies were based on the distribution of the data. In the case of the flashing body lights, an R was inserted into the frequency column across from "Light for Code" because 10:810 represents <5% of the cases.

As can be observed in Table 5, there was a notable range of use and perception of

Table 5

Children's Perception and Use of Design Features of the Four Coding Toys

Coding toy (# of cases)	Body	Frequency	Separate controller	Frequency	Environment	Frequency	Program organizer	Frequency
Code-a-pillar (389)	Coding Arrows	Frequent	N/A		3X3 Grid	Frequent		
	Cont. Moving	Moderate			Grid Pictures	Rare		
	Light for Code	Rare						
	Wall Hit Light	Rare						
	Face on Body	Rare					N/A	
	Ending Song	Rare						
	Sing w/ Motion	Rare						
	Colored Codes	Rare						
	Codes on Body	Rare						
Cubetto (555)	Stops w/ Codes	Moderate	Cod. Arrow/Tiles	Frequent	6X6 Grid	Frequent		
	Separate Body	Moderate	Col. Arrow/Tiles	Moderate	Human Loc.	Rare		
	Beep w/ Codes	Rare	Prog. Board	Rare	Comp. Rose	Rare		
	Beeps at End	Rare	Line Con. Holes	Rare			N/A	
	Face on Body	Rare	Back and Forth	Rare				
	Slow Motion	Rare	Flash w/ Code	Rare				
Bee-Bot (208)	Stops w/ Codes	Moderate	N/A		6X6 Grid	Frequent	Code Cards	Frequent
	Codes on Body	Moderate			Map Pictures	Rare	Seq. Spaces	Moderate
	Eyes Flash	Moderate					Bottom Line	Rare
	Light for Code	Rare						
	Lights at End	Rare						
	Face on Body	Rare						
	X Button	Rare						
Botley (810)	Cont. Moving	Moderate	Colored Codes	Frequent	Adj. Grid	Moderate	Magnet Cod.	Frequent
	Light for Code	Rare	Trash Can	Rare	Ball	Rare	Seq. Spaces	Moderate
	Beep for Code	Rare	Flash for Button	Rare	Goal	Rare	Preset Prog.	Rare
	Face on Body	Rare			Flags	Rare		
	On/Off Voice	Rare			Barriers	Rare		
	Pause Whistle	Rare						
	Say's "WEEE"	Rare						

Note. Rare = Design features perceived or used rarely (i.e., <5% of cases); Moderate = Design features perceived or used moderately (i.e., 5% - 25% of cases); Frequent = Design features perceived or used frequently (i.e., >25% of cases).

The percentages of frequencies were based on the distribution of the data.

design features across the four coding toys, indicated by the variety of R, M, and F frequency markings, and the most frequently perceived design features across all four coding toys were the grid squares and command arrows. All four coding toys had certain design features that were used frequently, moderately, and rarely. When analyzed closely, Table 5 reveals that there were three important patterns of perception and use across all four coding toys. Two design features were perceived and used frequently for all four coding toys: grid squares and command arrows; one design feature was rarely perceived and used: light and sound features. These three patterns are presented in more detail in the next section. The grid squares and command arrows are presented in more detail in these results because they were the most frequently perceived. The lights and sounds features are presented in more detail because they were very explicitly and intentionally designed by the creators of these coding toys as features to aid children in use of the toy.

Design Features with Similar Patterns of Perception and Use Across Four Coding Toys

Children Frequently Perceived and Used Grid Squares

One common pattern was that children perceived and used the grid square design features frequently across all the coding toys. Grid square design features are the specific environmental grid spaces on which each coding toy is designed to travel. Each of the specific grid squares are precisely designed so the distance from the center of one grid square to the center of the next grid square is one movement of the coding toy. Figure 8 shows the four types of grid squares of the four coding toys. Each of the grid spaces represents one unit of code, or movement, for the corresponding coding toy. There are a

Figure 8

The Four Coding Toy Grid Square Design Features

few things to notice regarding the differences in the grid square designs across the four coding toys. One is that the Botley grid squares are adjustable in terms of arrangement. They can be laid out in an array or in a path. The other is that the Code-a-pillar grid square is large, with approximately three feet of space from the center of one grid square to the center of the next. Due the large size, the entire grid space is a 3x3 array of grid squares and it was created by the CiK research team because no grid existed for Code-a-pillar. Cubetto and Bee-Bot have relatively similar grid square design features except the

Bee-Bot grid squares are on a solid mat with a laminate cover, and the Cubetto grid squares are on a fabric mat; both are arranged in a 6x6 array.

The video data showed children's awareness and use of the grid spaces through certain child observables (e.g., verbalizing, gesturing, full body movement, tracing, gaze). For example, Figure 9 shows a child who gestured on the Botley grid to indicate a path that he thought the coding toy should follow. The child in the green and blue shirt answered a question from the teacher about where he thought the coding toy was going to travel. The child leaned forward and made a sweeping motion on the grid spaces with his hand to indicate that it would follow a specific path. This gesture showed the child used the grid squares design feature as he completed the coding toy activity.

Figure 9

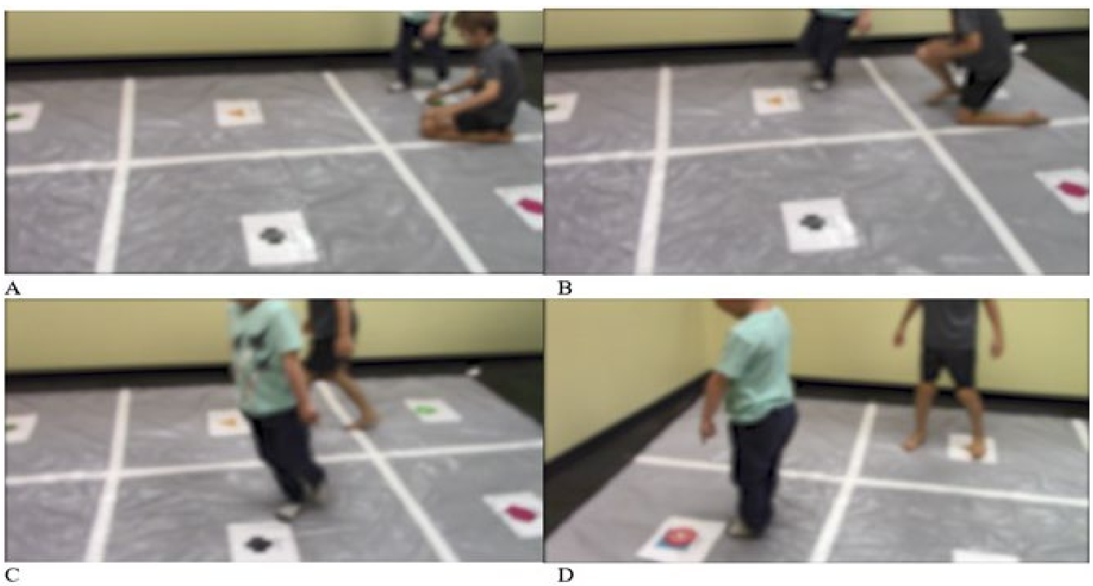
A Child Gestured to Indicate Use of Botley's Grid Square Design Feature

Another example of children using the grid squares design feature was when they used their full body to indicate use of the Code-a-pillar grid squares. Figure 10 shows a child who used his body to step grid square by grid square when planning a path that he and his partner thought Code-a-pillar would travel. You can see his feet in the top right corner of Pane A when he started on the beginning location of the Code-a-pillar. Then in

Pane B, he walked to the center of the next grid square in the intended path. The child continued by turning and moving up to the next grid square in the intended path (Pane C) until he arrived in the center of the final grid square that he and his partner believed the Code-a-pillar should stop in (Pane D). This full body use shows how the child was using the design feature of the grid spaces while engaging in the Code-a-pillar activities.

Figure 10

A Child Used his Full Body to Indicate Use Code-a-pillar's Grid Square Design Feature

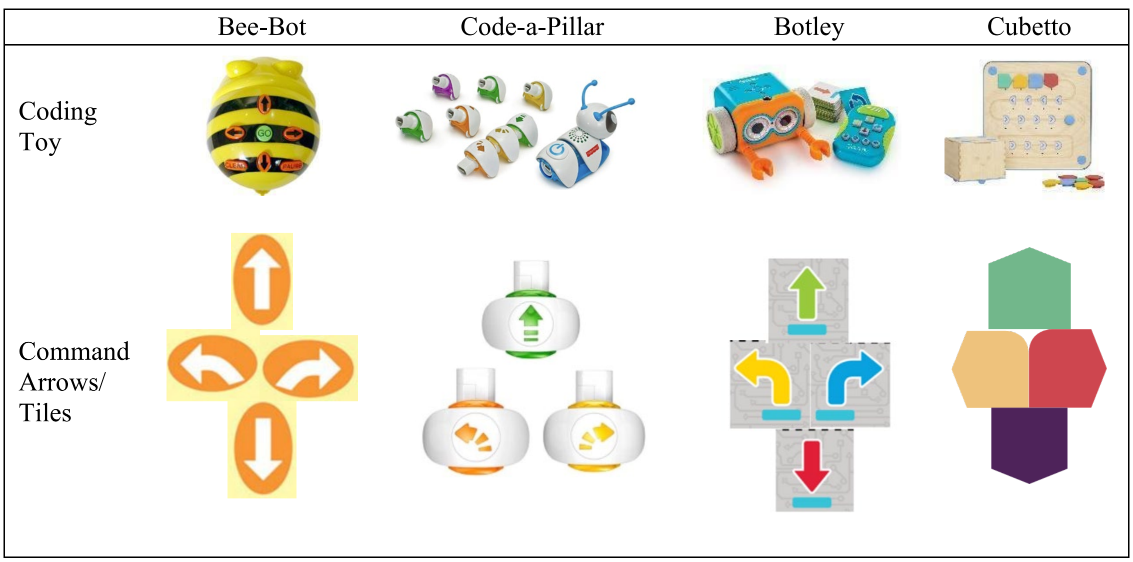


Children Frequently Perceived and Used the Command Arrows

Another common pattern was that children frequently used the command arrows/tiles across the four coding toys. Figure 11 indicates the command arrows that correspond to each of the coding toys. Differences exist between the ways that the arrows relate to and operate each coding toy. While Bee-Bot and Botley have arrow cards for planning a program, Code-a-pillar and Cubetto do not have planning cards. Instead, Code-a-pillar's arrows are attached directly to the body and Cubetto's command tiles are placed directly on the programming board. Cubetto's colored tiles have subtle points on them and can only be inserted in the programming board in one specific way. Ultimately, Bee-Bot and Botley have two sets of arrows (i.e., arrows for commanding, arrows for planning) and Code-a-pillar and Cubetto have one set of arrows/tiles (i.e., arrows/tiles for commanding).

Figure 11

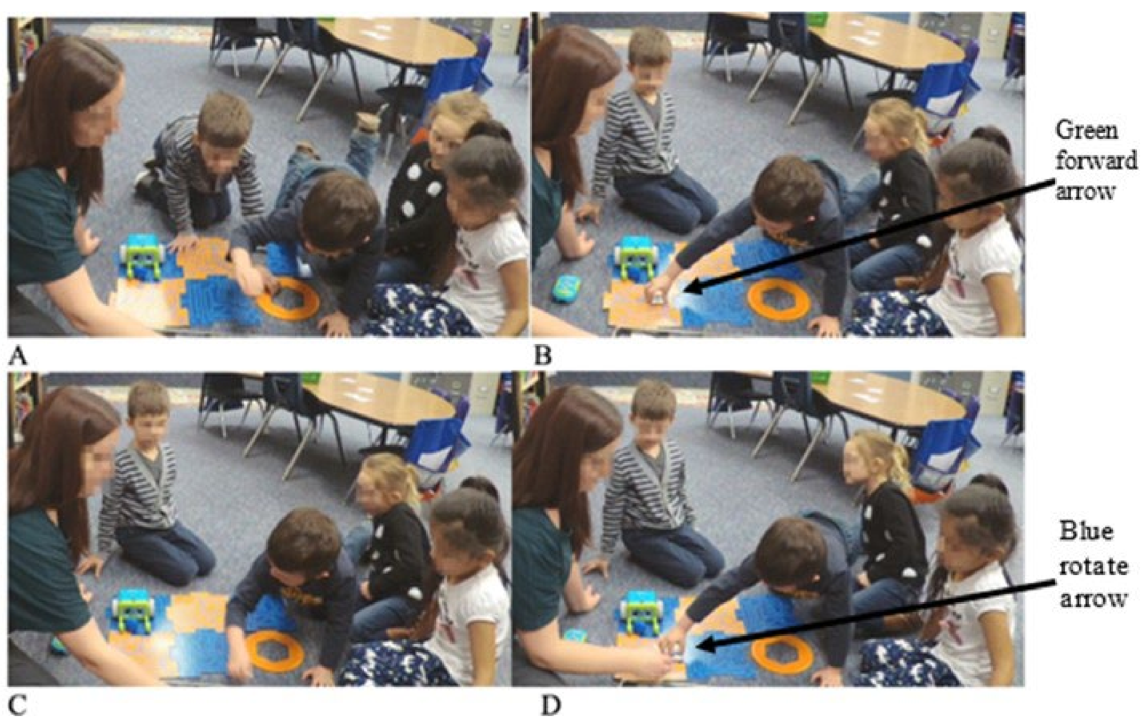
Coding Toys with Their Respective Command Arrows



Children used and perceived the command arrows/tiles in a variety of ways throughout their participation in the coding activities. Some of these different ways included matching command arrows with planning arrows, holding arrows up to see which way they were facing, counting arrows, and describing the directional shape of the arrows. Figure 12 highlights one child who placed the command arrows on the grid to help figure out which arrows to push on the remote control. In Pane A, the child reached onto the program organizer to grab a green forward arrow. He placed that green forward arrow on the orange grid square (Pane B) which is one space forward for Botley to move. Then, he put that green forward arrow back on the program organizer (Pane C) and grabbed a blue rotate arrow and placed it on the halfway line between the grid squares (Pane D). In this case, the child used the blue rotate arrow to indicate that after the initial forward movement, the blue rotate arrow would help the coding toy turn the direction of the orange goal. This example shows how the child used the design feature of the separate arrows when trying to plan a path and write a program by placing the separate arrows on the grid space to determine which ones to put in the remote control.

Figure 12

A Child Used the Arrows to Help Determine Correct Code



Another way children perceived the arrow design features was by glancing back and forth to the arrows as they entered the codes into the coding toy. Figure 13 shows one child who iteratively went back and forth between glancing at the arrows on the program organizer and pushing the arrows on Bee-Bot's body. Although subtle, Figure 13 (Pane A) shows the child with both hands on the mat and her head turned sideways looking at the arrows on the program organizer. Then, in Pane B she brought her head back down and picked one hand off the mat to press the corresponding arrow on Bee-Bot's body. Not pictured in Figure 13 is the fact that this child iteratively repeated these steps for all of the arrows on the program organizer. She turned her head sideways, glanced at the next arrow on the program organizer, and then brought her head back over the Bee-Bot's body to press the corresponding arrow. In this example, the child perceived the arrows

Figure 13

A Child Iteratively Perceived the Arrows While Pressing Buttons

A B

through her glancing and gaze of the arrows which allowed her to press the matching buttons on the Bee-Bot's body.

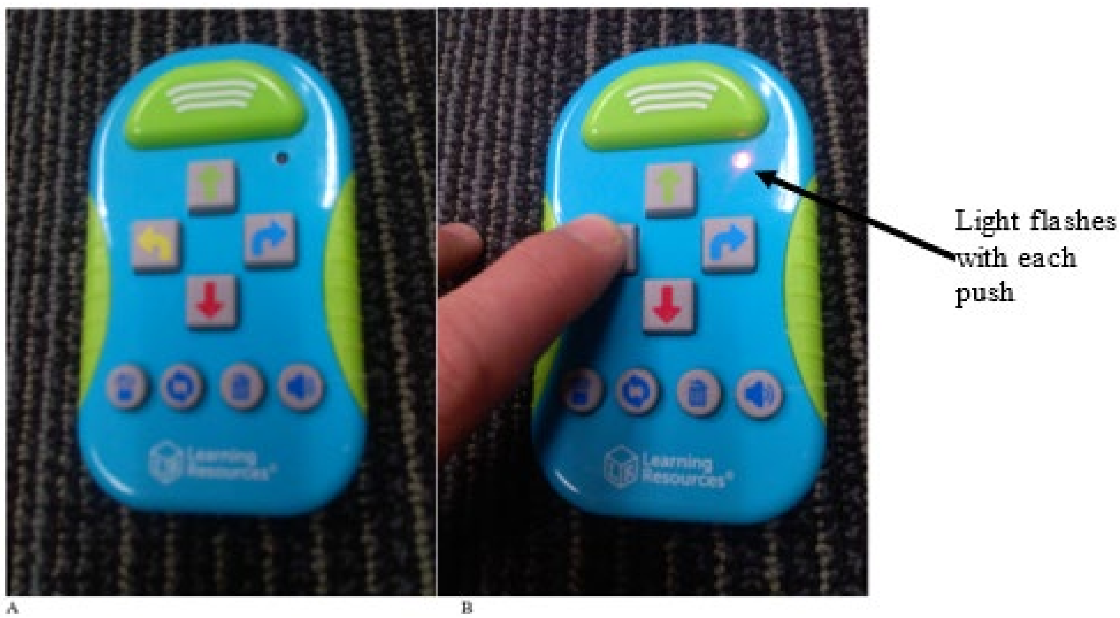
Children Rarely Perceived and Used Lights and Sounds

A third common pattern was that the children rarely perceived and used the lights and sounds across all four coding toys. There were a variety of different light and sound design features that were intentionally built into these products by designers to aid child interaction with the coding toys. Many of the lights and sounds corresponded to an action (e.g., button push, code enactment) in order to help children make a connection between two things. However, the children who participated in this study rarely perceived these simultaneous linking light and sound features.

One example of when children did not perceive or use the lights and sounds had to do with the Botley remote control. Figure 14 shows the Botley remote control when an arrow button is not being pushed (Pane A) and when it is being pushed (Pane B). The only difference between the button being pushed and not being pushed is the small red light that flashes when the button is correctly pushed. This light is an important design feature because it indicates whether the button is correctly pushed. Often, children knew what the correct program was, but made an error when entering it into the remote. Sometimes they didn't push a button hard enough and other times they accidentally pushed the button multiple times, both of which led to incorrect programs. Then, upon watching Botley enact the erroneous program, the children would get confused because they did not realize they input the program incorrectly. In these instances, the children missed the flashing red light on the remote that was meant to verify whether or not codes were being input into the remote.

Figure 14

Light on Remote to Verify Button Pushes



With Cubetto, children also rarely perceived or used the lights and sounds. One of the flashing light features with Cubetto was the small blue light that is directly underneath each one of the coding tile insert holes on the programming board (Figure 15, Pane A). When a coding tile was correctly inserted into the hole, the light maintained a steady blue glow—indicating it was properly in the queue. Then, when the program was enacted and the Cubetto moved in correspondence to each coding tile, the small blue light underneath one tile flashed as that specific tile was enacted. This light was meant to help the children see a correspondence between the flashing light of a specific code, and the

Figure 15

Children's Eyes on Body Instead of Flashing Lights on Programming Board

specific movement of that code. However, because the programming board and the Cubetto body are two different interfaces, the children's eyes primarily watched the body move around the mat instead of looking at the programming board and noticing the flashing blue light that corresponded to the codes (Figure 15, Pane B). This example demonstrates how the children rarely perceived the lights design feature of Cubetto because their eyes were gazing in a different direction than where the lights were.

Conclusion

The results for Research Question #1 regarding children's perception and use of design features revealed that children did perceive and use a variety of the design features with varying levels of frequency across the coding toys (Table 5). Notably, the grid square and the command arrows design features were frequently perceived and used by children across all four coding toys. Conversely, the lights and sounds design features were rarely perceived and used across all four coding toys.

Research Question #2: Children's Mathematical Engagement

Research Question #2 focused on the *mathematics that the children engaged in when interacting with the four coding toys.* I structured this subsection to (a) present the results of the mathematical concepts and skills that children engaged in, and (b) describe three synthesizing patterns of mathematical engagement. These three patterns were that across the four coding toys, children (1) engaged in spatial reasoning concepts and skills, (2) engaged in coordination, linear/discrete units, and counting, and (c) engaged in interesting mathematical concepts and skills that were not a set of the a-priori codes (e.g., multiplicative reasoning, patterning, subitizing/cardinality).

Mathematical Concepts Children Engaged in While Interacting with Coding Toys

Results indicated that children engaged in a variety of mathematical concepts and skills as they interacted with these coding toys. The five mathematical topics children engaged in were spatial reasoning, geometry, comparison, measurement, and number. These results are presented in Table 6. Each coding toy is represented in a column and the mathematical concepts and skills are represented in a row. Throughout Table 6, the words "observed" or "not observed" indicate when a mathematical concept and skill was present as a child perceived or used design features for that toy. For example, the concept and skill of describing location (i.e., using mathematical language such as next to, passing, besides) was observed as children perceived and used design features of Bee-Bot, Code-a-pillar, and Cubetto; however, describing location was not observed when children perceived and used design features of Botley.

Table 6

Mathematical Concepts and Skills Children Engaged in During Perception and Use of Design Features

Math concepts and skills	Bee-Bot	Botley	Code-a-pillar	Cubetto
Spatial Reasoning				
Spatial orientation	Observed	Observed	Observed	Observed
Estimation	Observed	Observed	Observed	Observed
Matching symbols	Observed	Observed	Observed	Observed
Visualization: URF[a]	Observed	Observed	Observed	Observed
Geometry				
Describing location	Observed	Not observed	Observed	Observed
Describing shapes	Not observed	Observed	Observed	Observed
Comparison				
Matching movements	Observed	Observed	Observed	Observed
More/less/same	Observed	Observed	Observed	Observed
Coordination	Observed	Observed	Observed	Observed
Patterning	Not observed	Observed	Observed	Observed
Measurement				
Angle	Not observed	Not observed	Observed	Not observed
Linear/discrete unit	Observed	Observed	Observed	Observed
Velocity	Not observed	Observed	Not observed	Observed
Number				
Multipl. reasoning	Not observed	Observed	Not observed	Not observed
Decomposition	Not observed	Not observed	Observed	Observed
Counting on	Observed	Observed	Not observed	Observed
Subitizing/cardinality	Observed	Observed	Observed	Observed
Counting	Observed	Observed	Observed	Observed
Subtraction	Observed	Observed	Observed	Observed
Addition	Observed	Observed	Observed	Observed
Sequencing	Observed	Observed	Observed	Observed

Note. Green is used to highlight mathematical concepts and skills observed with each coding toy.
[a] Updating Reference Frame

The mathematical concepts and skills in Table 6 are a mixture of a-priori codes (Appendices B, C, and D) and open codes (Appendix E). A few notable patterns in Table 6 are that all of the spatial reasoning concepts and skills are highlighted, meaning that they were observed with all four coding toys. Table 6 also shows that coordination, counting, and linear/discrete units were observed across all four coding toys; these three mathematics concepts and skills are presented in this section because they were often coded together. Finally, several mathematical concepts and skills that were not in the set of a-priori codes emerged during open coding, and included multiplicative reasoning, patterning, and subitizing/cardinality.

Three Synthesizing Data Patterns of Mathematical Engagement

Data analysis revealed three synthesizing patterns in the data. The first pattern is that children engaged in each of the spatial reasoning mathematics concepts and skills with all four of the coding toys. That is, children engaged in each of the specific spatial concepts and skills nested under the larger mathematical topic of spatial reasoning (i.e., spatial orientation, estimation, matching symbols, spatial visualization: updating reference frame) with every single coding toy. The second pattern is that three specific mathematical concepts and skills typically co-occurred in the data and were also present with all four of the coding toys (i.e., counting, linear/discrete unit, coordination). The third pattern is that children engaged in a few interesting mathematical concepts and skills that were not in the a-priori codes (i.e., multiplicative reasoning, patterning, subitizing/cardinality).

Children Engaged in all Spatial Reasoning Concepts and Skills with all Coding Toys

One important result pertaining to mathematical engagement was that children engaged in each of the spatial reasoning concepts and skills while perceiving and using the design features of the four coding toys (see Appendices B and E). One example of a spatial reasoning concept and skill children engaged in was *spatial visualization: updating reference frame (URF)*. This mathematical skill was observed when children made hand gestures in the air to show they were changing orientation or planning a path using mental images. It also occurred when children held the arrow codes or planning cards to determine whether or not the coding card helped them reorient or move the coding toy in the intended direction. The two parts of this code (i.e., spatial visualization + updating reference frame) were coded in this way because updating reference frame is an inherent element, or outcome, of spatial visualization. As children spatially visualized paths and movements, they recreated mental maps—imaginal updating—based on the face that was on the side of Cubetto; they had to visualize a new path based on the new reference frame that the Cubetto's face was using.

Children demonstrated the spatial skill, called "Visualization: Updating Reference Frame (URF)" with all four coding toys (see Figure 16). For each coding toy depicted in Figure 16, we see that children often gestured to indicate a visualized change in orientation of the coding toy. Accompanying many of the children's gestures were comments that also indicated this visualization. For example, the child using Botley (pseudonym: Kylee) in the final row of Figure 16, made a gesture to the right and seemed to be trying to visualize an intended rotation to get the Botley to face the small ladybug that is laying off the grid between her knees and the grid squares. The teacher-researcher prompted the whole group, "Botley wants to look at the ladybug. Can you get Botley to

Figure 16

Spatial Visualization with Gestures and Language

Coding toy	Spatial visualization	Description
Bee-Bot		This child pointed his finger to the right when prompted what the Bee-Bot needed to do next. Then, the other student simulated the Bee-Bot to test out what effect his point might have on the actual Bee-Bot. This finger point indicated he is visualizing a change in orientation, or possibly a change in intended directional path.
Cubetto		The group was trying to code Cubetto to go from the mountains to the desert. When trying to figure out how to get the Cubetto to turn right and get to the desert, the child pictured says "it needs to go down" and put her thumb in the downward position. This indicated her visualization of a change in position of Cubetto.
Code-a-pillar		This child was trying to plan a path for Code-a-pillar and as she looked around the large grid mat, she gestured and used language to indicate possible movements. These gestures and language indicated engagement in spatial visualization.
Botley		Initially, this child used the language "turn" to indicate what needed to happen to get Botley to rotate. Then upon another child putting a backward code on the board, she said "no, it needs to do this (gesturing), like turn around!" This gesture and language indicated engagement in spatial visualization.

turn and look at the ladybug?" Kylee made a turn gesture with her hand and arm and declared "It needs a turn!" Her partner then programed a backwards code, and upon seeing that this motion was not what she wanted, Kylee proceeded to repeat her previous turn motion and declared, "No, it needs to do this [gesturing] like turn around!"

In this example, Kylee visualized an action that she intended for Botley to do. When Botley did something different, Kylee repeated her hand gestures and included language to describe a visualized path. The design feature that Kylee perceived in this example was the face on the Botley body. The language used in the example (e.g., turn, turn around, look at) indicated that part of the physical body needed to be oriented in a certain direction, which is the face on the side of the body. Therefore, she was using the face on the side of the Botley body to visualize intended reorientations and changes in position.

Children Engaged in Counting, Linear/Discrete Unit, and Coordination with all Coding Toys

A second important result regarding mathematical engagement was that children engaged in counting, linear/discrete unit, and coordination while using all four coding toys. Descriptions of how children engaged in these three mathematical concepts and skills are presented below. While they are presented separately, notice that these concepts and skills tended to co-occur within each example.

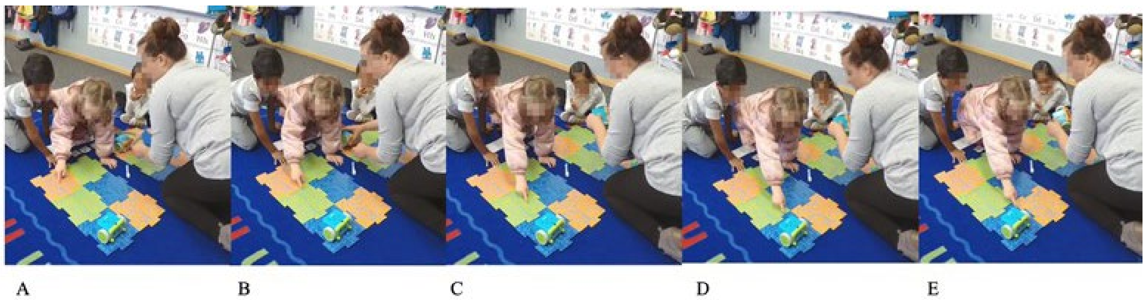
Counting. Counting occurred most often when the children counted the command arrows or the grid spaces. Counting the command arrows of all four coding toys generally resembled the children first laying the command arrows in a sequential nature, either on the program organizer (i.e., Botley, Bee-Bot), programming board (Cubetto), or on the body (Code-a-pillar), and then counting them in the same sequential manner, typically touching each command arrow as they counted. It also was a frequent occurrence for children to count as they were gathering up a set of tiles (Cubetto), body segments (Code-a-pillar), magnetic coding cards (Botley), or coding cards (Bee-Bot). The other common design feature that was counted was the grid squares. Sometimes when the children planned paths, they counted the grid squares by tapping each square and verbally chanting "1, 2, 3. . ." Other times, the children were prompted by the teacher-researcher to answer questions about how far the coding toy travelled, which prompted explicit counting of the grid squares. Similarly, although rarely, children counted the movements of the coding toy itself, however this was almost always when the teacher-researcher directed the children's attention to the movement and elicited their verbal counting of the movements. These examples of counting the command arrows, as well as the grid squares, were common across the four coding toys.

Linear/discrete units. Children also engaged in linear/discrete unit concepts and skills as they perceived and used design features of all four coding toys. This was observed when the children verbally, or with gestures, indicated a type of unit. Sometimes, children made motions or verbalizations that depicted their use of discrete units, such as Cubetto's command arrow tiles. Children gathered up tiles in their hands, in a pile, or laid them out on the program organizer and then they counted the discrete objects. In this case, there were two mathematics codes, *counting* as well as *discrete unit* because the children were clearly attending to the discrete nature of the tiles in order to form a counting sequence. The children demonstrated this counting of discrete units with all arrow/tile types, including forward, backwards, and rotations. In other words, the tiles as physical objects, represented a unit of one. However, in other cases, children engaged in *linear unit* concepts when they dealt with the movement of the coding toy on the grid space with the command arrows/tiles. Ultimately, children assigned a linear movement to a command arrow/tile. This engagement in linear units occurred with all the different arrow/tile types, sometimes correctly and sometimes incorrectly. For example, sometimes children assigned a linear movement to a forward or backward arrow/tile, which is accurate. Other times, children would assign a linear movement to a rotational arrow/tile which is inaccurate. An example of this engagement in linear units is when children tried to program Botley to move from the center of one grid square to the center of another. Figure 17 illustrates an instance where one child explicitly demonstrated engagement with linear units as she communicated her intended program for Botley. In Figure 17, Pane A, the child started her finger on the center of the orange square, and progressively dragged ger finger (Pane B) to the center of the next square (Pane C). As she dragged her finger, she verbally counted the number "one." Then, she continued to follow this same progression as she slowly dragged her finger across the grids (Pane D) to the center of the next square (Pane E) and counted the number "two."

This example demonstrates how the child engaged with linear units, because she was using the linear movement between two points to assign a unit of one to the green forward arrow. These two examples highlight how children engaged in two types of unit concepts, discrete and linear.

Figure 17

A Child Demonstrated Linear Units Through Counting and Gestures with Botley



Coordination. Last, children engaged with coordination as they perceived and used design features of all four coding toys. In this study, *coordinating* was defined as "Coordinates the totals of two quantities and/or matches 1-to-1 counting with movements or codes" (Shumway et al., 2021). Importantly, this operational definition included data incidences where children coordinated within a numerical system (e.g., coordinated numbers of squares to number of tiles) as well as data incidences where children coordinated pre-numerical actions (e.g., coordinated gestures to numbers of tiles). In terms of the four coding toys, children coordinated number words, movements, grid squares, button pushes, and command arrows. Coordinating was almost always double coded with another mathematics code, such as counting. A few examples of children coordinating include counting grid squares and then counting that same number of forward/backward command arrows; moving their bodies on the large Code-a-pillar mat, and saying the specific body code segment (e.g., forward, backward, right rotation) that matched that movement; counting movements of the coding toy and then acquiring the same number or command arrows/tiles. One clear example of coordination is represented in Figure 18, when a group of children coded Cubetto. The teacher-researcher prompted the children to get the Cubetto to go four spaces backwards to land on the tree. As depicted in Figure 18 (Pane A), the children touched the squares and verbally count "1, 2, 3, 4." Then, they sorted through the pile of codes, pulled out four purple backwards codes, and placed them on the programming board (Pane B). Finally, after they enacted the program to see if it worked, the child pointed to each tile as she counted, and the child is holding up counting fingers to match with each counting number (Pane C). In this example, there were various instantiations of coordinating. The children coordinated their counting of the grid squares with the number of purple backward arrows they chose to place on the programming board, the child coordinated the tiles with movements, and the young child coordinated his finger counts with verbal counts.

Figure 18

Children Coordinate Counting of Grid Spaces with Command Arrows with Cubetto

Children Engaged in Interesting Mathematics While Using Design Features

A third important result was that children engaged in interesting mathematical concepts and skills that were not part of the a-priori codes (Appendix E). Three mathematical concepts and skills highlighted in this section that were important are (a) multiplicative reasoning, (b) patterning, and (c) subitizing/cardinality.

Multiplicative reasoning with Botley. Multiplicative reasoning emerged in one case with Botley and was not part of the a-prior set of codes. Multiplicative reasoning is different than additive reasoning because it is thinking in terms of iterating a unit some number of times to form a result. Although it was only observed one time with a single child, evidence in the video data demonstrated a young child using multiplicative reasoning. The following transcript occurred in one of the rare instances when a child perceived the flashing lights on the top of the Botley head. The teacher-researcher in this incident prompted the children to decide how many forward arrows the Botley needed to move two grid squares forward. One child (Ben) eagerly responded to her prompts in the following manner:

Researcher: Let me show you again. Who has some ideas about what I did? Watch this, watch one more time [Enacts a program of two forwards for Botley while group of children watch].

Ben: Twice!! Forward twice. It's like a green forward, twice!

Kylie: Three!

Researcher: Oh, you noticed that on Botley? Okay, so some of us aren't sure whether it is two or three. Okay Ben, what made you think twice instead of three times [enacts the program again]?

Ben: [Reaches forward and points at the Botley while it is rolling] Because the light blinked twice!

[Video Bot_Hill_5_1, 7:06-7:20]

In this excerpt, we see Ben using language that indicated his multiplicative reasoning through the iteration of a unit, the green forward arrow. Specifically, when he said, "It's like a green forward, twice!" we see there is acknowledgement of the unit (i.e., green forward) and then an iteration of that unit (i.e., twice). This type of explicit multiplicative reasoning was only observed one time, and there were very few cases of children perceiving the flashing light design features, which made this a unique case.

Patterning. Another interesting mathematics concept that arose during analysis was patterning. Patterning was observed with Botley, Code-a-pillar, Cubetto, but not with Bee-Bot. When children engaged in patterning with these three coding toys, sometimes it had to do with the color of the command arrows. For example, with Cubetto, there was one red, two greens, and another red placed on the programming board and one child said, “It’s a Christmas pattern!” In this case, the child perceived the colors of the coding tiles and then engaged in patterning concepts and skills. Similarly, different children programmed Code-a-pillar with the codes: green forward, orange left turn, yellow right turn. Then, one child said, “Let’s finish the pattern” and they proceeded to add on three more codes saying the color of the codes (i.e., green, orange, yellow). The teacher-researcher asked the children what they were trying to do, and one child responded, “We’re trying to make a pattern!” In this example, the children created the first pattern module, module A (i.e., green, orange, yellow). Then, they iterated module A, a second time to form module B (i.e., green, orange, yellow, green, orange, yellow) by attending to the colors of the Code-a-pillar body segments. These examples show that children attended to the colors rather than the directional arrows, based on their use of the color words instead of directional words when discussing the codes they were patterning.

Subitizing/cardinality. Subitizing/cardinality is another mathematics concept that was not in the a-priori codes. First, it is important to note why this mathematical concept is labeled the way it is. Sometimes, it was hard to know whether a child was truly subitizing (e.g., rapidly perceiving the value of a small number of items), or if they had somehow previously counted the set of values and were just demonstrating cardinality by expressing the total. Therefore, when children expressed a number for a group of values, it was coded as subitizing/cardinality.

Similar to the mathematics concept counting, subitizing/cardinality was observed often when it dealt with the command arrows and the grid spaces. Also, subitizing/ cardinality was typically observed with numbers 4 or less. One example of subitizing was when children were first learning about Cubetto's colored command tiles that represented codes or movement. The teacher-researcher had one green forward command tile on the programming board and enacted it to show the children where it would make Cubetto go. Then, she put another green forward command tile on the board. Importantly, the children did not verbally or physically count each tile, but when the teacher-researcher asked the children "How many do we have?" they all yelled out "two!" and some held up two fingers. While it is possible the children were tracking the amount by counting the individual green forward command tiles, subitizing seemed more likely because when asked about the quantity of the group, children were able to quickly communicate an innate sense of the value and amount of two.

In regard to cardinality, sometimes it could be seen clearly when children counted the command tiles and then verbalized, they knew the full amount of all the command tiles together. For example, with Cubetto, a group of children had coded four green forward command tiles on the programming board. The teacher asked the children how many green forward command tiles were in the program. One child leaned forward and whispered the counting numbers “1, 2, 3, 4” and then in a loud voice he called out “There’s four!” This example shows how he counted the discrete units within a set and then demonstrated cardinality by aggregating the units in the set to form a value of the whole group.

Conclusion

In conclusion, children engaged in a variety of mathematical concepts and skills as they perceived and used design features of the four coding toys. Specifically, across all four coding toys, children engaged in (a) all spatial reasoning concepts and skills, (b) three co-occurring mathematical concepts and skills (i.e., counting, linear/discrete units, coordination), and (c) interesting mathematical concepts and skills that were not in the a-priori codes.

Research Question #3: Design Features Affording Mathematical Engagement

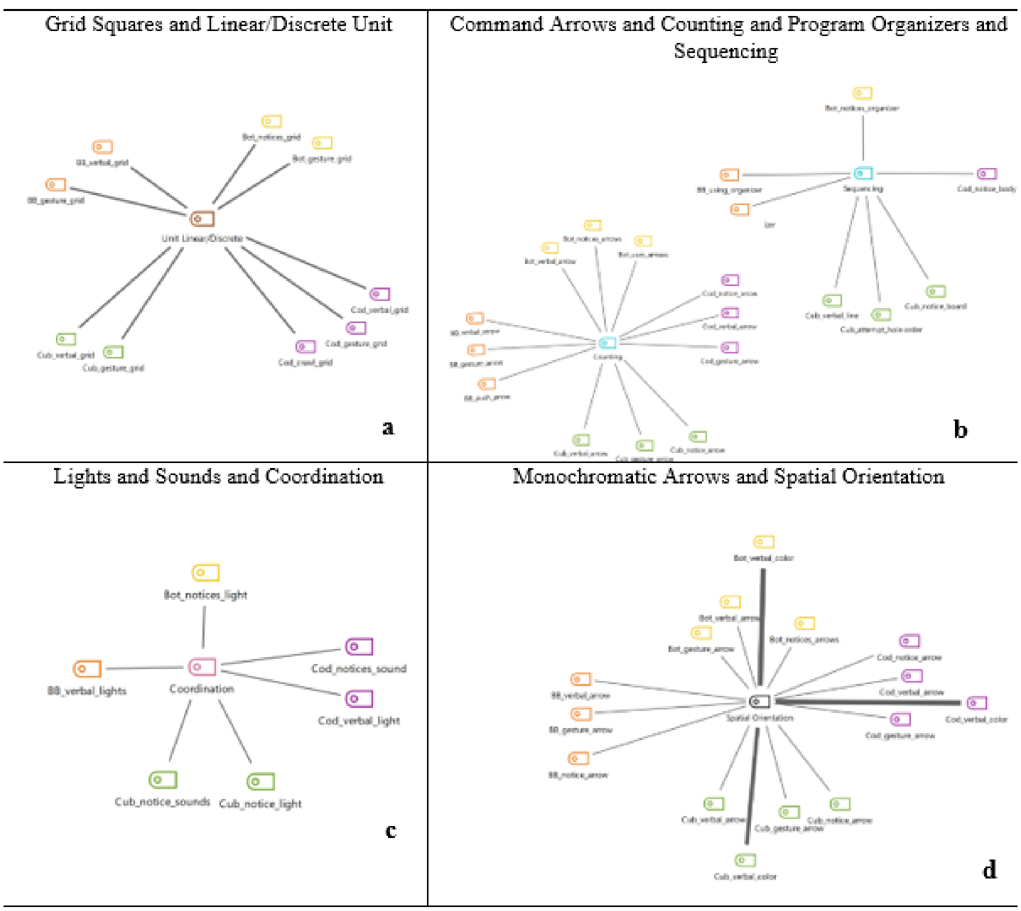
Research Question #3 focused on how design features of the four coding toys afforded children’s mathematical engagement and was investigated by analyzing the data incidents that had overlapping mathematics and design feature codes. The first part of this section presents similarities in ways the coding toy design features afforded engagement in mathematics, and the second part presents differences in ways the coding toy design features afforded engagement in mathematics.

Similarities in ways Coding Toy Design Features Afforded Mathematical Engagement

Across the four toys, there were four similarities in the ways design features afforded mathematical engagement. These four similarities included: (a) grid squares afforded linear/discrete unit construction, (b) program organizers and command cards afforded sequencing and counting, (c) certain lights and sounds afforded coordination when perceived, and (d) monochromatic arrows afforded spatial orientation concepts and skills. These relationships in the data can be viewed in Figure 19(a)-19(d). Figure 19

Figure 19

Similarities in Affordances Across Coding Toys Construction



shows data visualizations from the MAXQDA software which highlight how a specific design feature of each coding toy commonly afforded a specific mathematical concept or skill. For example, Figure 19(a) shows a math concept or skill in the middle (i.e., linear/discrete unit) and design features of the different coding toys that afforded it around the outside (e.g., all four coding toys; Grid Squares). In the sections that follow, I refer to Figure 19 to describe similarities.

Grid Squares Afforded Linear/Discrete Unit Construction

There was an important relationship between the grid square design features of all four coding toys and children's engagement in construction of linear and discrete units (Figure 19a). Throughout their interactions with the coding toys, children engaged in construction of units, either linear units (i.e., the linear distance from the center of one square to the center of another) and/or discrete units (i.e., one square as one unit). Figure 20 highlights these two types of units that children constructed from the grid squares while using the four coding toys.

An interesting example of both discrete and linear unit construction occurred as a child tried to program Botley to go two squares forward (Figure 21). Initially, the child touched each square, including the square that Botley started on and counted "1, 2, 3, it needs three." Pane A shows his physical touch of a grid square and then him verbalizing a counting number that corresponded to the touch of the discrete grid square (i.e., touch-count). This touch-count demonstrated his use of the grid squares as discrete units, each grid square represented a unit of one. Then, after programming the three forwards and

Figure 20

Grid Squares for Discrete and Linear Units Construction

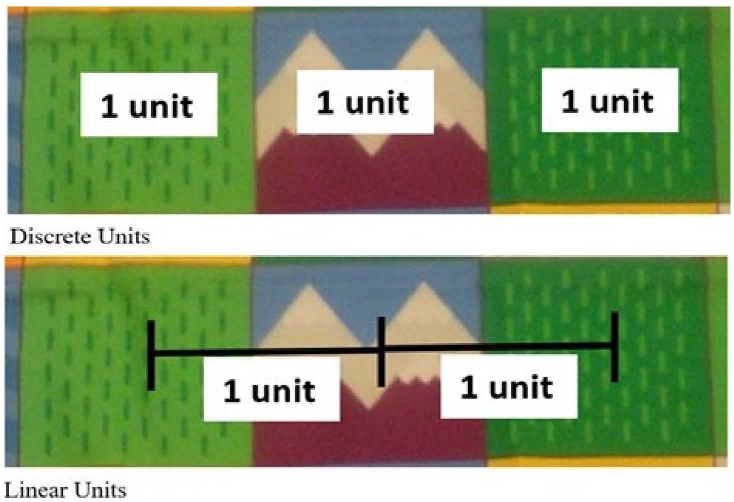
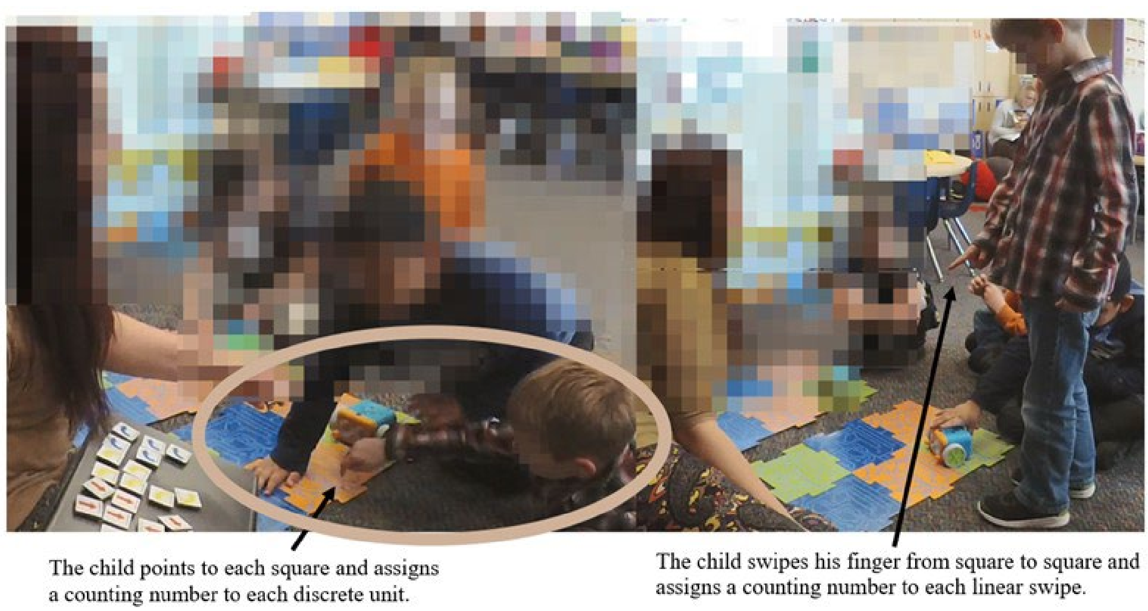


Figure 21

A Child Shifted from Counting Discrete Units to Counting Linear Units



watching it go too far over the intended landing path, the child shifted to using linear units. The child pointed his finger to the original starting spot of Botley, and instead of counting the point of his finger like in his initial attempt, he slid his finger in the air from the start position to the center of the next square and counted “1” (Pane B) and then he slid his finger from the center of that square to the center of the final square and counted “2, it needs 2.” This sliding of the finger and counting the slide (e.g., slide-count) indicated that the child shifted from counting the discrete squares (i.e., touch-count) to counting the linear movement from the center of one square to the center of another (i.e., slide-count); he constructed a linear unit.

This example highlights how a child engaged in both linear and discrete unit construction from his awareness and use of the grid square design features of the coding toys. His touch-count demonstrated his initial engagement with a discrete unit and his slide-count demonstrated his engagement with a linear unit with the grid supporting his tracking of each of these units.

Program Organizers and Command Cards Afforded Sequencing and Counting

A relationship existed between the program organizers and sequencing as well as the command cards and counting (Figure 19b). The two design features (i.e., program organizers, command cards) were often coded together because the inherent design of these systems was to place the command cards on the program organizer. Although the program organizers looked different across the four coding toys, all had the same basic principle which was an organizational system to aid in sequencing and organizing the codes (i.e., Code-a-pillar, connectable sequenced body segments; Cubetto, sequenced insert holes for command tiles; Botley and Bee-Bot, sequenced boxes for command cards). Ultimately, children engaged frequently in sequential counting as they used the command arrows that were placed on the program organizers. Children's interactions with the coding toys showed that command arrows related to counting, and program organizers related to the sequential nature of counting.

Often, children counted the command arrows on the paper program organizer of Botley and Bee-Bot. Both of these coding toy program organizers look the same and were research-created materials. They are thin strips of paper that have squares on them in a single row that are fitted for the command cards to be placed in. It was common for children to point their fingers to the first individual command card on the program organizer and count "1," progress their finger to the next command card and count "2," and continue in this manner until they reached the final command card. Figure 22 highlights this relationship with the Bee-Bot program organizer and command cards. The child in Figure 22 counted one by one, from left to right, the command cards and labeled them with the appropriate counting numbers as she progressed. After being prompted by the teacher-researcher, the child pointed to the first arrow in the program organizer's sequence (Pane A) and said "1." Then, she moved her finger to the next arrow on the program organizer (Pane B) and said "2." She continued this progression until she placed her finger on the final command arrow in the program organizer and counted "9."

This example shows the two direct relationships that emerged in data analysis. The first is the explicit counting of the discrete command cards, and the second is the

Figure 22

A Child Touched and Counted each Command Cards One by One from Left-to-Right Sequence

sequential nature that she counted the command arrows afforded by the structure of the program organizer.

Another example of this is when two children attempted to program Code-a-pillar to move it two spaces forward. The two children programmed five green forward arrows by appending them sequentially to the body of Code-a-pillar and then put the whole coding toy on the starting location. The teacher-researcher prompted the children to explain why they thought the program would work to get the Code-a-pillar to end in the correct spot. The child put his finger towards the five green body segments and pointed one by one down the body. After he sequentially pointed to each of the body segments, his partner called out “five, I just did math problems, I’m good at math.” In this example, the child demonstrated an awareness of the sequential nature of the program organizing system (i.e., linear connectable body segment structure) by his gestures one by one down the body, and the child demonstrated a counting of each of the individual command codes (i.e., discrete body segments).

Certain Lights and Sounds Afforded Coordination when Perceived

There were a variety of light and sound design features across the four coding toys, though children perceived and used them rarely. However, during the rare instances where children did perceive and use the lights and sounds, they engaged in mathematics through coordination of the quantities of lights and sounds of the coding toy and the codes they used to create the program (Figure 19c). For example, one teacher-researcher prompted a child to look at the programming board while Cubetto was enacting a program. The child took her finger and pointed to the tile on the programming board as it flashed, turned her head quickly to look at Cubetto, and said "It's moving and it's blinking every time it does it!" In the subsequent lessons with this same child, she made various references to the flashing blue light and the movements of Cubetto. One instance was when the Cubetto enacted a program and she called out "The light's blinking, backwards!" Another example is when the Cubetto enacted a longer program of 10 codes, she reached forward in the middle of Cubetto's enactment of the program and started pointing at the code on the programming board that was being enacted. She used the blinking light as a reference to know which code was currently in use. These instances demonstrate how children could make an explicit coordination between the light design features, movements, and command arrows.

Monochromatic Command Arrows Afforded Spatial Orientation

Another relationship between design features and mathematical engagement that was common across all four coding toys was between the coloration of the command arrows/tiles and spatial orientation concepts and skills (Figure 19d). With three of the four coding toys (i.e., Cubetto, Botley, Code-a-pillar), each of the command arrows/tiles was a specific color. This coloration could be used to directly identify each code. For example, the forward code arrows/tile for Cubetto, Botley, and Code-a-pillar were all green; the rotate right arrow for Botley is blue. In contrast, the arrows on Bee-Bot were all white (i.e., monochromatic), without a coloration of each arrow.

The analysis illustrated that when children needed to plan paths, program the coding toys, and discuss their programs with Botley, Code-a-pillar, and Cubetto (colored-arrow toys), they took advantage of color terms to communicate their reasoning. However, when children did these same activities with Bee-Bot (monochromatic arrow toy), they tended to use spatial orientation language and gestures, spatial visualization, and symbols matching. For example, the short transcript below highlights an incident where a child (Tom) used exclusively color words to describe the coding tiles he thought were needed to program Cubetto (colored-arrow toy) to match a specific path. In the excerpt, Tom watched a pre-programmed Cubetto move on the grid area. As it moved, he pointed to the Cubetto and called out colors. Then after it stopped, he started grabbing the colored tiles and programming his own Cubetto to match the program.

Tom: [Presses the go program and watches the pre-programmed Cubetto rotate to the right] Red [watches the Cubetto move forward] Green [then stops talking as the Cubetto finishes by rotating left and moving forward]

Researcher: Do you want to watch again or do you want to try?

Tom: I'll try.

Researcher: Okay, you can code it when you're ready. Actually, let's watch it one more time. [Presses go on the pre-programmed Botley]

Tom: [As the pre-programmed Cubetto is moving, Tom is programming his other Botley, and chants] Red [codes a red] Green [codes a green, and then codes a yellow and green without saying other words]

[Video Cub_C_3_1, 21:55-22:32]

In this excerpt, Tom used color language to communicate and reason with the command tiles and the movements of Cubetto. Both the codes that he verbalized (i.e., green, red) have corresponding directional actions (i.e., more forward, rotate right), however Tom used color language rather than directional language when reasoning with these codes. In contrast, the following excerpt demonstrates when a child used spatial language when planning a path with Bee-Bot (monochromatic arrow toy). Kyle looked at a program that another child had constructed (i.e., forward, forward, forward) to try and get Bee-Bot forward, forward, forward, backward, backward, backward.

Researcher: [Points to Kyle] What do you think, thumbs up or thumbs down? Do you think this program here [gesturing to the forward, forward, forward program on organizer] is going to get Bee-Bot up to the beehive and back? [sliding finger on grid three forward to beehive and three backwards to starting location].

Kyle: [Puts thumb down]

Researcher: Thumb down, why?

Kyle: [Leans forward and points to the program organizer spaces right after the three forwards] Because there's back, back, one, two, three.

[Video BB_Nib_1_1, 22:02-22:28]

In this excerpt, we see that the child discussed the coding arrows and the intended movement of Bee-Bot using spatial orientation language to communicate spatial reasoning. Specifically, when contrasted to the previous excerpt, we see the child using spatial orientation language such as "back, back" instead of the color language in the previous excerpt such as "red, green." The important relationship to mathematical engagement highlighted by these two excerpts is that one child used spatial orientation language to communicate reasoning about coding (i.e., back, back) and one child used color language to communicate reasoning about coding (i.e., red, green). Throughout their interactions with the colored-arrow coding toys, children sometimes used color language when available. On the other hand, the monochromatic arrow coding toy (i.e., Bee-Bot) necessitated children's engagement in spatial orientation concepts and skills because they could not rely on color associations of the codes. Ultimately, the monochromatic design of the Bee-Bot arrows forced children to find other, spatial-based ways to communicate and reason with the command arrows because children could not use color terms.

Differences in Ways Coding Toy Design Features Afforded Mathematical Engagement

Across the four toys, there were differences in the ways that design features afforded mathematical engagement. The following five differences are presented because of their significance in terms of possible design implications. The first three all relate to coordination and the second two relate to spatial concepts: (a) Stopping motions between movements of the Bee-Bot and Cubetto coding toys afforded unit coordination, (b) tangible body designs of the Bee-Bot, Cubetto, and Botley coding toys afforded unit coordination, (c) Botley's remote control afforded inaccurate coordination, (d) Arrows on body of the Bee-Bot and Code-a-pillar coding toys afforded spatial orientation, and (e) Code-a-pillar's physically appended codes and large grid squares afforded spatial orientation, visualization, and estimation. These relationships in the data can be viewed in Figure 23(a)-(b) which shows how certain design features of specific coding toys afforded mathematics concepts and skills. Again, the mathematics concept or skill is the code in the middle of the MAXQDA visualization, and the nodes connected to the skill are the different coding toy design features.

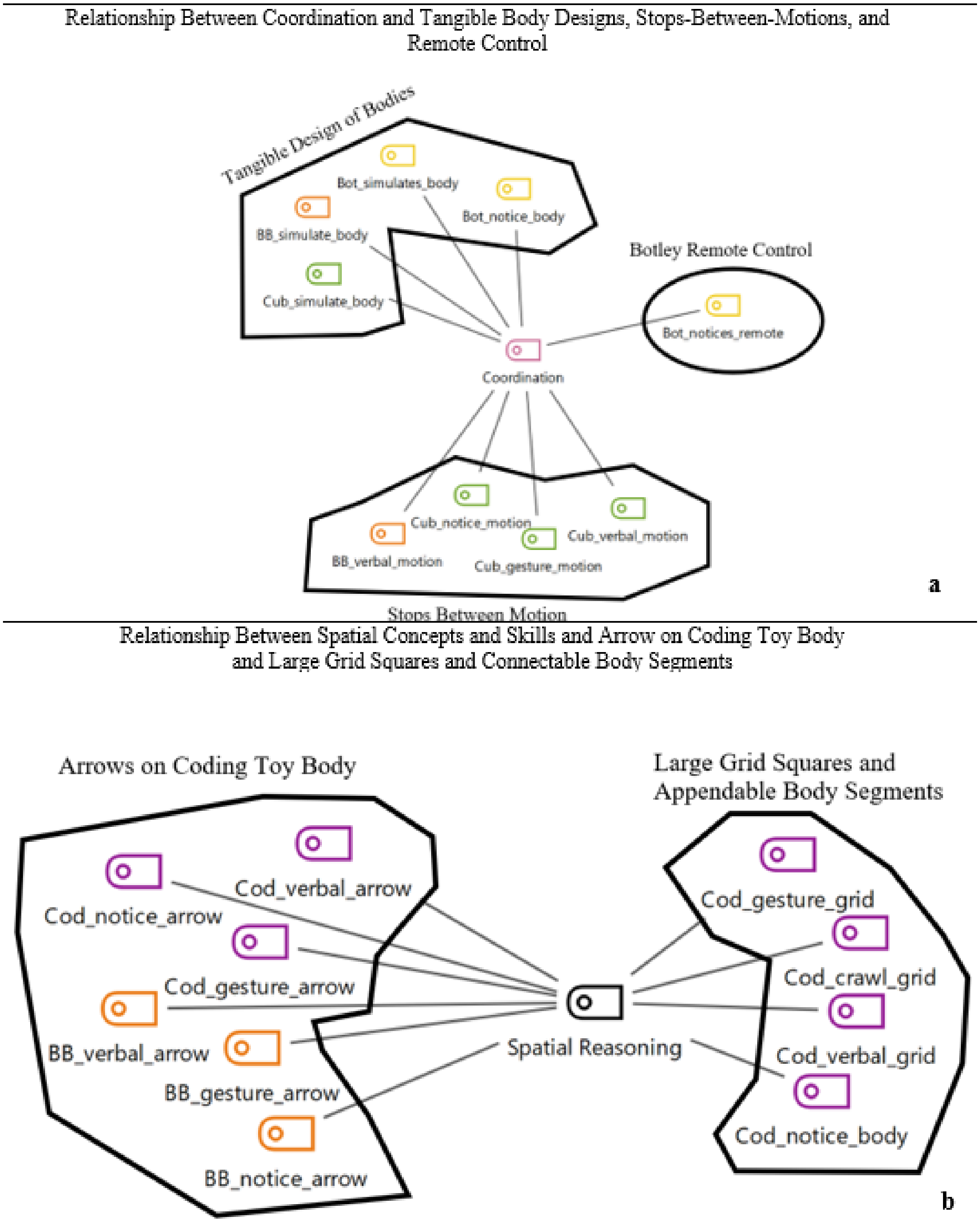
Bee-Bot and Cubetto: Stops Between Movements Afford Units Coordination

One relationship between design features and mathematical engagement unique to Bee-Bot and Cubetto was that the stops between movements afforded units coordination (Figure 23a). These two coding toys were designed to stop between enactment of each code (i.e., Bee-Bot, Cubetto), while Botley and Code-a-pillar were designed to continue moving without any hesitation between enactment of codes. Children tended to coordinate units with specific movements of Bee-Bot and Cubetto, such as coordinating counting words with individual movements, coordinating codes with individual movements, and even coordinating distinct hand gestures with individual movements.

Unit coordination was often observed when children coordinated a counting word with a unit of linear movement. Children counted number words that were associated to the unit of one forward movement of these coding toys in real time, moving to the next

Figure 23

Differences in Affordances Across Coding Toys Construction



counting word after the toy stopped and then initiated its next movement. For example, as Bee-Bot moved from one square to the next, the children would call out “one” and then when it moved on to the next square they would call out “two.” Additionally, children often dragged out or pronounced the syllables of counting words to match the movement (e.g., oonnee, twwoo) so that the duration of the counting word aligned to the entire unit of movement of the toy. Figure 24 demonstrates children performing this coordination between counting words and units of linear movement with Cubetto’s stops between each code enactment. The children watched as Cubetto progressed from one square to the next as it enacted its program (i.e., four forwards). As it moved, the children chanted counting words that aligned to each of the movements. The children said each counting word so that the word lasted the entirety of the move, and they terminated the word once it stopped in the middle of the next square. Pane A of Figure 24 is the first stop in the four-forward program, and it is when the children ended the counting word “oonnee.” Then, upon Cubetto’s starting to move again, they began the counting word “twwooo,” and ended the counting word once it stopped in the next square (Pane B). The children continued this coordination between counting word and linear movement until the Cubetto stopped in its ending location.

This example highlights how the children coordinated the counting word with the unit of a linear movement, and the specific stop in between motion of each code enactment is what allowed them to accurately coordinate the number word and the movement. By drawing out the counting words, children attempted to match the counting word with the movement of the coding toy.

Figure 24

Children Coordinate Counting Words with Movements

A B

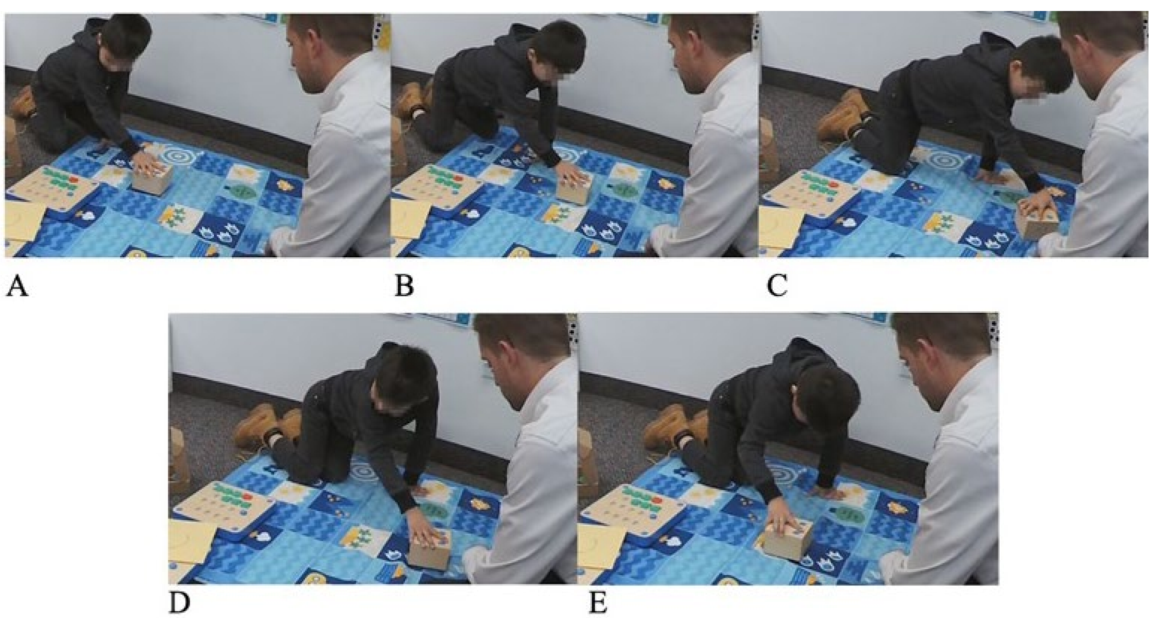
Bee-Bot, Cubetto, and Botley: Tangible Body Design Affords Units Coordination

There was also a relationship between the toys' small tangible body design and units coordination (Figure 23a). Although children in this study simulated all four coding toys while referencing coding arrows and units (i.e., grabbed the coding toys with their hands and moved them around), it occurred most prominently with Bee-Bot, Cubetto, and Botley. During these simulations, children typically referenced the codes that were on the program organizer, and then coordinated each code on the program organizer with one simulated movement of the coding toy. In these cases, there was an explicit coordination between the simulated movements of the coding toy body and the command arrows.

Figure 25 demonstrates a data incident where one child explicitly coordinated the command arrows on the Cubetto's programming board with the simulated movements of the Cubetto body. The child was creating a program when the teacher prompted him to explain where the program was going to take Cubetto. He started out by looking directly at the command arrows on the programming board to ensure that he was simulating the correct actions (Pane A). Then, he went on to coordinate those arrows with simulated Cubetto body movements (Pane B-C). The child stopped and turned his head back to look at the arrows (Pane D) to verify that the next simulations of the Cubetto body were actually coordinated with the command arrows on the programming board. Finally, the child turned his head back toward the Cubetto body and simulated the remainder of the program (Pane E).

Figure 25

A Child Explicitly Coordinated Simulations of the Cubetto Body and the Command Arrow



This example demonstrates how the simulated movements of the small and tangible Cubetto body afforded engagement in coordination. Important in this example is the way the child turned his head to make explicit coordination between the simulated Cubetto body movements and the command arrows. Also, the shape and size of the Cubetto body allowed the child to fit his hand on the body and make physical changes to its position.

Botley: Remote Afforded Inaccurate Coordination

There was a relationship between the Botley remote control and engagement in inaccurate coordination (Figure 23a). Throughout the Botley lessons, the children accidentally pushed buttons, or did not push buttons and made inaccurate coordination between what they thought was stored in the memory bank and the actual movements of the coding toy. Although design features were in place to help the child know when the codes they programmed were received in the system—such as the red light that flashed for each button push and a sound that activated with each button push—the children often pushed buttons they are unaware of. Similarly, they often thought they pushed a button when they actually did not. In these cases, children inaccurately coordinated movements with codes and pushes because there was receiving error they were unaware of. Finally, they often did not use the trash button and added more codes to the end of a program when really they thought it was going to start over.

The first way the remote control afforded inaccurate coordination was when children did not press a button hard enough. In these cases, they thought a certain code would be enacted and when it was not, they were confused about what the codes actually represented. For example, Figure 26 highlights a time when a child tried to enter the codes forward, left rotation, backwards. Pane A-C highlights his fingers pressing the three buttons as the teacher is pointing to each of the codes that he should enter. However, his fingers only pressed the forward arrow and the backward arrow hard enough, and the left rotation was never received into the remote. He then pressed the GO button and the Botley only went forward and backward. In this example, the remote afforded an inaccurate coordination because the children all had three codes in their minds (i.e., forward, left rotation, right rotation) and they assumed that whatever Botley actions ensued was a result of these three codes. The teacher in this example, and most other similar examples, quickly corrected the problem for the children and made sure they reentered the codes and that the remote received the intended program.

Figure 26

A Child Believed He Entered the Correct Codes, but did not, Leading to Inaccurate Coordination

Another common way that the remote control afforded inaccurate coordination was when children pushed extra buttons they were unaware of and then enacted the program and observed an enactment that was different than what they had planned. For example, Figure 27 shows a child who tried to enter a program (e.g., right rotation, forward, forward) to get Botley to the orange goal. The child attempted to push the remote-control buttons that matched the magnetic coding card on the program organizer however, when he entered the codes, he accidentally pushed an extra right rotation arrow (Pane A). Therefore, instead of programming right rotation, forward, forward, he accidentally programmed right rotation, right rotation, forward, forward. Then he enacted the program and it rotated too far and travelled in the wrong direction (Pane B). In this case, the remote afforded inaccurate coordination because the child thought he programmed one thing (i.e., right rotation, forward, forward) when he actually programmed a different thing (i.e., right rotation, right rotation, forward, forward). Because the child was unaware of the incorrect pushes, he could have inaccurately coordinated the code he thought he pushed—one right rotation—with a 180-degree rotation.

Figure 27

A Child Entered an Extra Code which Led to Inaccurate Coordination

A: The program the child thought he pressed to get Botley to the goal: right rotation, forward, forward.

B: Where Botley went with the child's actual presses: right rotation, right rotation, forward, forward.

The final way the remote control afforded inaccurate coordination was when children did not understand, or use appropriately, the trash can button, which is meant to reset (or clear out) the codes stored in the memory of Botley. When this happened, programs the children thought started Botley from the beginning, actually continued adding on to a previous program. Children often wanted to change their program entirely so they put the Botley back on the starting place, not select the trash can, enter their newly desired program, and then, upon enactment, were confused about what was happening because Botley enacted the original program plus the newly entered program. These instances with Botley happened pervasively and were handled in a variety of ways by the teacher-researcher. Regardless of the quick handling by the teacher-researcher, there was obvious confusion by children because an inaccurate coordination between entered program and what actually happened with Botley.

The three ways described in this section were all examples of ways the remote control afforded inaccurate coordination. In essence, the remote control and corresponding design features, such as a flashing light for each button push and sound for each button push, did not help children accurately coordinate one button push with precisely one physical instantiation of the coding toy.

Bee-Bot and Code-a-pillar: Arrows on Body Afforded Spatial Orientation

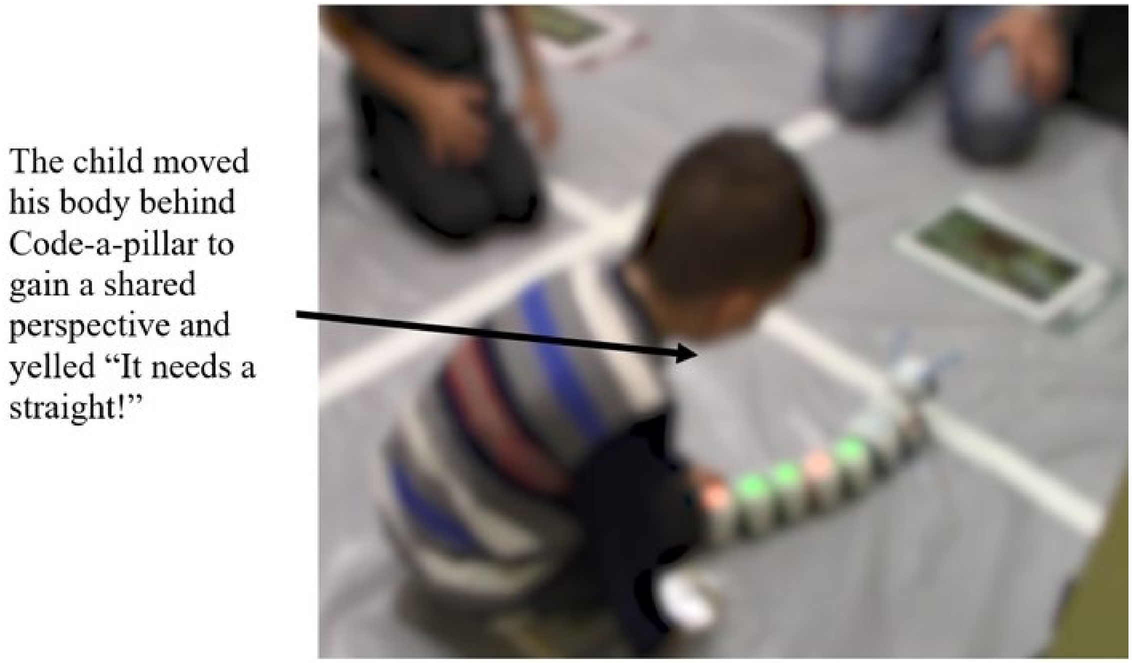
Bee-Bot and Code-a-pillar were distinct from Cubetto and Botley because the coding arrows were directly on the body of the toy when the program was being enacted. This means that the arrows were always aligned with the coding toy's orientation, so children did not need to re-coordinate the position and meaning of the code with the orientation of the toy every time it moved. This design feature of the coding arrows on the body more easily afforded engagement with accurate spatial orientation (Figure 23b).

With Code-a-pillar, children often crawled behind the coding toy or moved to where the coding toy was to share a perspective. When they did this, their own body and perspective was shared with Code-a-pillar's orientation and perspective, thereby eliminating the need to visualize or imagine Code-a-pillar's perspective and the toy's changing orientation of forward movements or rotations. Being on the body of Code-a-pillar, its arrows changed position and realigned to Code-a-pillar's orientation, unlike arrows on a remote (e.g., Botley) or program organizer or board (e.g., Cubetto). Figure 28 highlights how the children repositioned their bodies to take advantage of the new orientation of the arrows. In Figure 28, the child tried to get Code-a-pillar to land on the grid square with the green tree in it (top right of Figure 28). The Code-a-pillar stopped in the position indicated in Figure 28, and the child moved his body from another location on the grid to be positioned directly behind the Code-a-pillar in the crouching position. He looked down the body of Code-a-pillar with a shared perspective and called out, "It needs a straight!" In this example, the arrows already attached to Code-a-pillar were positioned in alignment with the Code-a-pillar's head. Then, when the child also positioned his own body to be in alignment with the Code-a-pillar's head, he was able to accurately call out the code because the arrows were already facing directly at the green tree and all he had to do was select the arrow that was facing the same way.

With Bee-Bot, the children also took advantage of the arrows on the body to

Figure 28

A Child Used the Repositioned Arrows Through Body Movements



navigate spatial orientation situations, but it was mostly observed through reaching and touching the arrows directly. Figure 29 highlights one child who demonstrated this relationship between the arrows on the body and spatial orientation. The child tried to get Bee-Bot to land on the grid square with the beehive, but the Bee-Bot had stopped one space in front of it. Earlier in the lesson, she consistently used incorrect codes when trying to rotate or move Bee-Bot, however in this instance, she reached forward, slightly turned her head and body, and touched the one forward arrow that was facing the hive. Potentially the aiding factor in this situation was that the Bee-Bot had enacted part of the program and now the arrows on the Bee-Bot body were facing the intended ending location. She used the directions of the arrows on the Bee-Bot body to select the appropriate next arrow in the sequence. In this example, we see the child use the arrows on the body to help assist her in understanding Bee-Bot's movements according to its orientation.

Figure 29

A Child Reached for and Touched the Repositioned Arrows

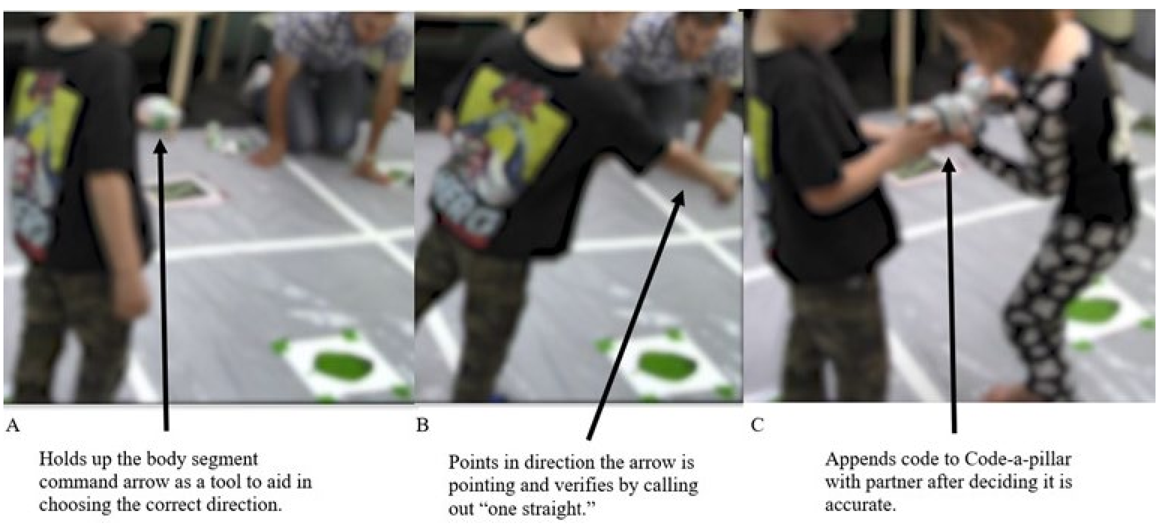
Code-a-Pillar: Physically Appended Codes and Large Grid Afford Spatial Orientation, Spatial Visualizations, and Spatial Estimation

Two design features of Code-a-pillar (i.e., physically appended codes, large grid squares) related to spatial orientation, spatial visualization, and spatial estimation. The uniqueness of the command arrows of Code-a-pillar was that they could be discrete command arrows when unattached from the Code-a-pillar body, but when they were attached, they essentially became part of the coding toy. Because of this, children used the command arrows in two different ways, depending on whether or not they were attached to the body. When the command arrows were not attached to the body, the children used the movable command arrow body segments to help them solve spatial orientation problems, and when they were attached to the body, they elicited the child's spatial visualization and spatial estimation (Figure 23b).

When the separate command codes were not attached to the body, children partnered their bodies with the separate codes to aid in solving spatial orientation problems. This was often seen as children called out codes that their partner needed to program when they walked their bodies around on the large grid spaces. When children reached a point on the grid where they were unsure of the code that should be called out to their partner, they grabbed a physical code and took it to the last space on the grid where they were calling out codes from. They held the code up in front of them to see if the arrow on the code was facing the correct way or not. If it was, they put that code on the Code-a-pillar. If it was not, they switched to the other rotation arrow. In these instances, the children used the detachable arrows as tools to help them solve spatial orientation problems that had to do with the new direction that the Code-a-pillar needed to rotate. Figure 30 provides an example of a child who called out codes and used a detached body segment to help solve a spatial orientation problem. The child travelled around the large grid with his body and called out codes for his partner to add to the Code-a-pillar. He reached one instance where he aligned his body in the grid square and was deciding how to get to the next square forward. Pane A shows how he grabbed a green forward arrow and used it as a tool to help him identify which code was needed. He aligned the code to the grid squares, and then pointed forward with his hand and stated, "one straight" (Pane B). After this use of the discrete coding body segment to solve the spatial orientation problem, his partner brought the Code-a-pillar body over and appended the code to the body (Pane C).

Figure 30

A Child Used the Separated Command Arrow to Solve a Spatial Orientation Problem



In this example, the child demonstrated the relationship between the two design features (a) large grid squares, and (b) separate discrete body segment code, with mathematical engagement in spatial orientation. The large grid squares allowed the child to physically engage his body in the coding process when faced with a spatial orientation challenge. This was seen when he moved his body to a square that was important for the intended path of Code-a-pillar. Then, we saw how the discrete coding arrows allowed the child to physically align the arrow with the direction that he wanted the Code-a-pillar to move. The command arrow in this sense took on the role of a spatial orientation tool which helped him verify that it was the correct code that got Code-a-pillar to travel in the intended direction.

The other way children demonstrated this relationship between the connectable body segments and engagement in spatial mathematics concepts and skills was when they visualized a path and made spatial estimations based on the overall length and size of the Code-a-pillar body when codes were attached. With Code-a-pillar, the length of the body increased with every additional coding body segment that was added to the program. This change-in-size of the Code-a-pillar body with increased body segments afforded engagement in spatial estimation of linear movement. When there were lots of body segments attached on the Code-a-pillar, children made linear estimations of a 'far' visualized path, or an ending spot that is a 'long' way away. One example of this was when two children were using the Code-a-pillar and trying to get it to the end of the mat—only two spaces forward.

Jill: How about we just do straights?

Megan: Yeah, cause then it goes *ALLLLL* the way to the bee [the intended ending spot. The children proceed by putting on three forwards].

Researcher: So you're thinking three of them?

Megan: No, we're not. Five! [The children proceed to put on two more forwards. Then, Jill starts putting on all the rest of the available codes, which were two left rotations and two right rotations]. No Jill, those won't. . .

Jill: It's a *long* caterpillar!

[Video Cod_DDE_3_1, 8:10-8:53]

In this excerpt, we see a relationship between the design feature of the connectable coding segments making the Code-a-pillar longer and the children's spatial linear estimations of a long-visualized path. Specifically in this excerpt, the children reference a long path, or a 'far off' destination by the way they enunciated the phrase "ALLLL the way to the bee." Then, when developing the program, the children explicitly indicated that a "*long*" caterpillar is needed to match the visualized path. In this case we see the children linking the long body of Code-a-pillar with spatial linear estimations of long visualized lengths and far off distances.

In conclusion, results for Research Question #3 indicated that certain design features afforded engagement in mathematics. Some design features afforded engagement in mathematics across all four coding toys (i.e., grid squares afforded linear/discrete unit construction; program organizers and command cards afforded sequencing and counting; lights and sounds afforded coordination when perceived; monochromatic arrows afforded spatial orientation concepts and skills), while other design features afforded engagement in mathematics in unique ways with specific coding toys (i.e., Bee-Bot and Cubetto's stops between movements afforded unit coordination; Bee-Bot and Code-a-pillar's arrows on body afforded spatial orientation; Bee-Bot, Cubetto, and Botley's tangible body design afforded unit coordination; Botley's remote control afforded inaccurate coordination; Code-a-pillar's physically appended codes and large grid squares afforded spatial orientation, spatial visualization, and spatial estimation).

Summary

The results presented in this chapter highlight the findings on the design features children used and perceived, the mathematics that children engaged in while using design features, and the design features that afforded specific mathematical engagement. The results revealed that children used and perceived the grid spaces and command arrows design features frequently, while others were used moderately or rarely.

The results related to children's mathematical engagement revealed that children engaged in a variety of mathematical concepts and skills in five main categories of mathematical topics: spatial reasoning, geometry, comparison, measurement, and number concepts and skills. Children engaged in certain mathematical concepts and skills with all four coding toys (e.g., spatial concepts and skills) and sometimes, mathematical concepts and skills common across all four coding toys tended to co-occur, such as coordination, units, and counting. Additionally, children engaged in mathematical concepts and skills such as multiplicative reasoning, patterning, and subitizing/cardinality that were added to the a priori list of mathematics codes.

The results also indicated that the relationship between design features affording mathematics varied depending on the coding toy, and that some design features afforded specific mathematical engagement across all four coding toys (e.g., grid squares afforded unit construction), while other design features afforded specific mathematical engagement only with specific coding toys (e.g., Cubetto and Bee-Bot's stop in-between movement afforded unit coordination).

CHAPTER V

DISCUSSION

The purpose of this study was to examine kindergarten-aged children's awareness of the design features in coding toys and to understand how those design features afforded children's engagement with mathematics. Results indicated that there were specific design features that children used and perceived with different levels of frequencies, children engaged with a variety of mathematics concepts and skills as they used and perceived design features, and there were connections among specific affordances of design features and mathematical engagement.

Based on the results of this study, three important themes emerged. The discussion section is organized to address the following themes: (a) children perceive and use coding toy design features with different frequencies and engage in a variety of mathematics concepts and skills when they do so, (b) design features afford mathematical engagement in similar and different ways across coding toys, and (c) design features engage children in meaningful coordination. Accompanying the discussion of each theme are recommendations and suggestions for future research. This discussion section closes with limitations and a conclusion.

Design Feature Use Leads to Mathematical Engagement

The first theme in the results related to the way children's perception and use of coding toy design features led to engagement in mathematics. To discuss this theme, two sections are presented. The first discusses the way that children perceived and use design features with different frequencies, and the second discusses the variety of mathematics that children engaged in when perceiving the design features.

Children Perceived and Used Design Features with Different Levels of Frequency

One important result of this study was that, across the four coding toys, children perceived different design features of toys with different levels of frequencies. Sometimes children observed certain design features frequently, sometimes moderately, and sometimes rarely. This means that design features had different opportunities to afford mathematics. For example, children frequently perceived the command arrow and grid square design features, and therefore were able to take advantage of their beneficial affordances. Other times, design features were perceived rarely (e.g., flashing lights) and, therefore, may have had less opportunity to afford mathematics to children. Research with virtual manipulatives suggests that children must first perceive the design feature to take advantage of its possible beneficial affordances (Bullock et al., 2017; Moyer-Packenham et al., 2020). The current findings support this literature and indicate that, although design features may be specifically incorporated by the designers to support mathematical engagement (e.g., flashing lights to support coordination between the number of movements and codes), children may not take advantage of the beneficial affordances because they did not perceive the design feature frequently. This result may be important to designers of coding toys to better inform their designs. If designers were more explicit about the types of mathematical concepts and skills they hope children will engage in with these coding toys, they could design more noticeable features of the toys—to increase perception—that could elicit those concepts and skills. Therefore, designers must find this balance of creating design features that are explicitly available and accessible to children while clearly eliciting mathematical engagement. When using coding toys in classrooms, one recommendation for teachers is that they spend considerable time explicitly teaching the design features and helping children to perceive and use them. Although not a specific question pertaining to this study, it seemed that teacher-researcher prompting influenced children's awareness of design features, and then this awareness led to mathematical engagement. Research suggests that connections across representations is useful for mathematics learning (Cramer, 2003), and that teachers can support these connections (Clement, 2004). However, research is needed to know exactly what the role of teacher prompting plays regarding supporting children's awareness of design features and the subsequent implications on mathematical engagement.

Children Engaged in a Variety of Mathematical Concepts and Skills While Perceiving and Using Design Features

A second important result related to the way children perceived and used coding toy design features. In specific, children engaged with a variety of mathematical concepts and skills within the topics of spatial reasoning, geometry, comparison, measurement, and number. Literature suggests that young children engage with spatial, number, and measurement concepts and skills when playing with coding toys holistically (Nam et al., 2019; Palmér, 2017; Shumway et al., 2021; Winters et al., 2020). This finding in the current study supports previous findings and found that children engaged with these same mathematics concepts and skills. Findings in the current study add to the literature in two ways. The first way the current study adds to the literature is by providing a nuanced lens as to what is precipitating mathematical engagement. Previous studies examined children's engagement in mathematics holistically as they played with coding toys. The current study examined what aspects of the design features led to the mathematical engagement. Therefore, this finding adds a nuanced theoretical lens to suggest that it is not a *coding toy* that affords engagement in mathematics, but it is the *specific design features of the coding toy* that afford engagement in mathematics. From an embodied cognition perspective, these coding toy design features offer an embodied experience due to their tangibility and movement in space. These design features, when perceived by children, elicit embodied mathematics and affords engagement in embodied mathematics

Additionally, the current study adds a few mathematical concepts and skills that were not identified by other studies, namely patterning, subitizing/cardinality, and multiplicative reasoning. Previous studies have not made explicit documentation of children engaging in these mathematics concepts and skills, and thereby, these new ways that children demonstrate mathematical engagement are potentially important contributions. On a broader picture, early childhood research on mathematics education suggests that children at this age are just beginning to recognize AB patterns and some can begin to identify the smallest core unit (Sarama & Clements, 2021); their cardinality/subitizing activity is changing from perceptual to conceptual (MacDonald & Wilkins, 2019); and they may be too young to begin reasoning multiplicatively, as this skill emerges in nationally adopted standards in the third grade (CCSSI, 2010).

Therefore, the fact that children in the current study demonstrated engagement with these concepts and skills is important. For example, the colors and sequential nature of the coding cards and symbols provided opportunities for children to engage in early patterning skills. Additionally, the program organizers, coding cards, and grid squares may have supported perceptual cardinality/subitizing as the children demonstrated quick apprehension of the numerosity of small sets of these features. It is important to point out that there is a current and ongoing discussion in the literature regarding the qualitative difference between subitizing and cardinality (Baroody & Purpura, 2017; Carey, 2009; Simon et al., 2021). Because of this ongoing discussion on the distinction between the two constructs, I made the decision in analysis to code them together during my analysis. An interesting idea for future research may be to analyze these incidences with a specific theoretical framework designed to observe and understand the qualitative difference between specific cardinality and subitizing incidences. Lastly, command cards and coding toy movements afforded mathematical engagement in multiplicative reasoning. All of these design features allowed children some sort of sensory-motor engagement. Interpreting these findings through embodied cognition theory, they support the Paek (2012) study which found that increased bodily engagement led to more emergence of mathematics. Ultimately, these learning tools and their associated design features may have afforded unique mathematical experiences for young children because they engaged the children in sensory-motor activity (Papert, 1980). Coding toy curriculum materials are being developed that have been shown to support first- and second-grade students' computational thinking skills (Bers et al., 2019; Relkin et al., 2021; Wang et al., 2021).

Similarly, non-coding toy curriculum materials are being developed that support early childhood integrated computational and mathematical thinking (Lavigne et al., 2019). However, early childhood curriculum which uses coding toys to directly target mathematical learning is not as available. Although early work is beginning to crop up that specifically targets mathematics lessons using coding toys as learning tools (e.g., Winters et al., 2020), there is still insufficient work done to offer educators a comprehensive set of materials that would allow them to incorporate these coding toys into mathematical instruction. Further research—specifically design-based research—could be conducted to generate materials that aid teachers in incorporating coding toys into their mathematics instruction.

Design Features Afford Mathematics in Similar and Different Ways Across Coding Toys

The second theme in the findings was the mixed results regarding how the specific design features afforded mathematics. Certain design features afforded mathematical engagement, while others did not. Some design features afforded the same mathematics across all four coding toys, and some design features afforded mathematics in specific ways with specific coding toys. There may be reasons as to why design features afforded mathematical engagement in similar or different ways across the coding toys. In the following sections, I theorize about important links between design feature-affordance-mathematics and make recommendations and suggestions for future research.

Grid Squares may have Afforded Linear/Discrete Unit Construction through Depicted Spaces

One significant result of this study was that the grid square design feature afforded linear and discrete unit construction across all four coding toys. Children counted the squares to construct discrete units and they counted the movements from square to square to construct linear units. This means that children engaged in construction of two different types of units (i.e., linear, discrete) when perceiving and using the grid squares. Often this was observed by children touching the grid squares and verbalizing a unit, or children tracing a linear movement from one grid square to another and verbalizing a unit. Literature is mixed on young children's developmental readiness to work with continuous linear units versus discrete units (Boyer & Levine, 2015; Friso-van de Bos et al., 2018; Welch et al., 2021). Basically, some research suggests that continuous and linear units are developmentally appropriate and beneficial for kindergarten-aged children to reason and work with; other research suggests they are not, and that children at this age are only ready to work with discrete units. This finding in the current study is important because it shows that these coding toys, and specifically the grid square design features, offered children an opportunity to reason with and construct multiple unit types, including a linear unit. This new type of dynamic learning tool may have offered the children in this study an environment where a linear unit was understandable and appropriate. Moreno-Armella et al. (2008) documented a progression of mathematical tools from static-to-dynamic. Moreno-Armella et al. state that *continuous dynamic* tools allow in-the-moment re-orientation of the tool and body. Through an embodied cognition lens, there is promising evidence in the current study's data to suggest that kindergarten-aged children constructed linear units in an appropriate manner with these special tools (coding toys) potentially because of their dynamic nature and the way the children could engage their bodies and re-orient their perspectives in-the-moment. Although the grid squares afforded construction of both units, they tended to afford discrete unit construction more than linear movements. I suggest that future designers make the grid squares with an explicit marking to show the movements, such as a line from the center of one to the next. This may afford the children more opportunities to engage in linear unit construction because they could visually see the start and stop between squares. Another design suggestion is to omit the grid squares entirely and create an environment for the coding toys to move on that has lines with ticks—like rulers—and the coding toy moves from one tick to the next tick. This design idea would more explicitly highlight the linear unit of movement from one number line tick to the next number line tick.

Program Organizers and Command Cards May have Afforded Sequencing and Counting through Scaffolding

Another important result from this study was that the program organizers afforded sequencing and the command cards afforded counting. This means that when children counted the command cards that were laid on the program organizer, they usually followed a specific sequenced counting order such as "one, two, three..." and followed along with their fingers on the program organizer. A recent literature review on robot-mediated activities with preschool-aged children identified three categories of scaffolding for these activities (i.e., narrative, auxiliary objects, embodied examples; Bakala et al., 2021). The program organizers and command cards in the current study would fall into the category of auxiliary objects, which are objects associated with the coding toy environment to help support learning. The current finding is important because it demonstrates how auxiliary objects—as a form of scaffolding—may afford specific mathematical engagement. I suggest that explicit design features could be added to the program organizer to afford sequencing even more. For example, the program organizer could have numbered slots, so the children know which position is first, second, third, etc. This would help the child not reverse sequence as occasionally occurred in the current unnumbered design of the program organizer. Additionally, the design feature of the *connecting line* for Cubetto may have hindered sequencing because it was ordered in a way that was not intuitive to the children.

Angeli and Valenides (2019) conducted a study with 5-year-olds and found that program organizers, as a form of scaffolding, were more helpful for young boys when coding, and discursive forms of scaffolding were more helpful for young girls. Unfortunately, the current study is unable to provide further information on whether or not the program organizer—as a form of scaffolding—more beneficially afforded sequencing to boys or girls. This is a promising area of future research because there may be gender differences regarding ways design features afford mathematical engagement.

Monochromatic Arrows May Have Afforded Spatial Reasoning through Restrictive Descriptive Terms

Another important result of this study was that monochromatic arrows afforded spatial orientation language. This means that due to the white nature of the arrows on

Bee-Bot, children used terms like "*turn around, go straight, turn to the right, back up,"* whereas with the toys that had multi-colored arrows (i.e., green for forward, yellow for backward, etc.) children took advantage of the colors and used terms like "*green, red, blue*." Because research has demonstrated that technological tools with programmed directionality support the development of spatial reasoning (Cittá et al., 2019; Terroba et al., 2021), it is important to understand how the design features of these coding toys supported different directional and spatial terms and discriminations. Literature on discriminating spatial language, including left and right, varies (Benton, 1959; Harris, 1972; Piaget, 1968). However, big ideas that remain relatively constant are that developing directional discrimination is happening from 4- to 8-years old, and that discriminating left versus right is more challenging for young children than discriminating up versus down or front versus back. The results of the current study are important because they indicate that the coloration of the codes could act as a differentiating feature that could allow young children—still unable to discriminate between some of the more technical spatial language (e.g., left, right)—to simplify the cognitive process by allowing them to refer to a color for communication. On the other hand, for children more advanced in their development of technical spatial terms, monochromatic arrows may afford more engagement in spatial concepts and skills due to the restrictions on descriptive terms. Future research could focus in on the age where a child should begin to regularly work with these more specific spatial terms, target a specific audience of child with the design of the coding toy, and then design the coding toy accordingly. For example, if research shows that Kindergarten children should begin to use more technical spatial terms, such as right rotation or left rotation, then the design of the toys could omit the colors so they will be more likely to engage in such spatial concepts and skills. That means that coding toys designed for early age groups (e.g., 3-year-olds) could include the colored codes to help children be productive and successful with the coding toy. Another option is to design a toy with the option of adding the colors to the lights or not, or having colored coding cards/tiles, or not. This would allow the design of one toy to meet the needs of both groups of children.

Arrows on Body May Have Afforded Spatial Orientation through a Shared Perspective

Another important result was that the arrows directly on the Bee-Bot and Code-a-pillar body afforded spatial orientation. This means that the way children accessed the arrows allowed them to engage in spatial concepts and skills. I hypothesize this is due to the way the arrows on the body always maintained a shared spatial perspective with the direction the coding toy was facing. Research on early childhood coding indicates that young children struggle to understand rotation arrows (Cuneo, 1985), but mental rotational thinking begins developing as early as 3 years old (Krüger et al., 2014). Results from the current study support previous literature because children struggled with rotation codes but did show the ability to use mental rotations. The current findings highlight how the arrows on the coding toy body may aid children in use of rotation codes and mental rotations. The coding arrows being directly on the body afforded a shared spatial perspective with the child and the way the coding toy was facing. This allowed children to match the coding cards with the desired movement without having to mentally recreate a spatial map through imaginal updating (Klatzky, 1998). The future design of coding toys for this age group could have the codable arrows on top of the body. This could aid in the children's ability to match the correct movements with the correct spatial rotation codes. Additionally, teachers who are considering using coding toys could consider the benefit of the arrows on the body. If the coding toy does not have arrows on the body, the teacher may need to prompt children to do more body movements to orient themselves to the coding toy because the arrows will not be sharing a perspective with the coding toy body.

From an embodied cognition perspective, research shows that young children do adopt different reference frames and they align their physical bodies to share an orientation with coding toys when learning how to code (Clarke-Midura et al., 2021). Future research is needed to fully understand the learning benefits of this shared spatial orientation, both shared orientation of the arrows on the coding toy and shared orientation of the child's physical body with the coding toy. This would help us know if shared orientation of the child's physical body with the coding toy provides similar or different benefits as when the arrows share an orientation with the body of the coding toy.

Code-a-pillar Features Afforded Spatial Skills Through Embodied Engagement and Visual-Spatial Correspondence

Another important finding of the current study was that the physically appended codes and large grid squares of Code-a-pillar afforded spatial orientation, visualization, and estimation. This result may have occurred because of the way the large grid squares allowed the children to engage their bodies. This finding supports extant early childhood computational thinking and mathematics education research which has shown that children's embodied engagement increases success (Paek, 2012; Sung et al., 2017), ultimately supporting the theoretical position that active bodily engagement in experience strengthens and supports learning connections (Lakoff & Johnson, 1999; Papert, 1980). Furthermore, research has shown that spatial structuring occurs in second and third grade (Battista, 1999; Battista et al., 1998). Therefore, embodied cognition theory would suggest that exposing kindergarten-aged children to activities that physically engage them in spatial orientation, visualization, and estimation could support them in their readiness to structure their spatial thoughts more abstractly and formally in the later grades. This finding is important because Code-a-pillar offers an environment where children engage in spatial visualization and updating of reference frames by using their body as a tool to code. This embodied engagement was essential to success with actualizing spatial visualizations and determining correct codes based on updating reference frames. The suggestion here is that coding toys for this young age group should consider increasing length of movements—similar to Code-a-pillar—so that children can engage their full body when reasoning with spatial concepts and skills. It is important to consider the practical drawbacks of this however, because practitioners have limited space to use materials. The grid space for Code-a-pillar took up about 36 square feet, a considerable space for a classroom. If these coding toys are to be used for school instruction, creativity in use of classroom space will be needed to make using a space-needy coding toy a reality.

Another important finding was about Code-a-pillar's physically appended codes affording spatial orientation, visualization, and estimation, through visual-spatial correspondence. Code-a-pillar's physically appended codes afforded these spatial concepts in the way the children (a) saw a physical body of a certain length, (b) saw an intended path of a certain length, and (c) determined a body-path match or mismatch. This body-path matching has been called *visual-spatial correspondence,* making a connection between two visual-spatial entities. If there is a mismatch between these two design features (e.g., long Code-a-pillar body, short, intended path on the grid squares) the child sometimes removed codes from Code-a-pillar to shorten the body, or moved the Code-a-pillar farther away in order to lengthen the intended path. Children intuitively looked to match the physicality of the body and the path. Children encountered many problems with this visual-spatial correspondence with Botley and Bee-Bot because any number of codes programmed into those systems yielded nonvisual feedback of amount of codes, essentially, there was no way to see the physical *length* of the set of codes programmed into the toys. Because of this, children using these two coding toys created programs that were either way too long or way too short because there was no way for them to determine a match or a mismatch between length of program and length of intended path. Although the researcher-created program organizers were designed to ameliorate this in some way, they were not always effective seemingly due to the nature of the child trying to organizationally manage the various materials. This finding supports recent research that demonstrated young children sometimes tried to use one code to make Cubetto move continuously until stopped by an outside force (Welch et al., 2021). Because robust research on early childhood and mathematics suggests that visual-spatial ability is linked to mathematics development (Barnes et al., 2011; Gunderson et al., 2012; Zhang, 2016), it is important to incorporate design features that ensure visual-spatial correspondence. Informed by embodied cognition theory and the tenant of bridging physical-perceived experiences with mental constructions, I suggest that future design features support visual-spatial correspondence by providing a way for the children to see the length of the program they have entered into the coding toys directly on the coding toy body. This could include a small screen on the body of the coding toys that depicts all the codes that are entered into the remote. Another idea is that coding toys could include other tangible code designs that could be appended to the bodies of the coding toy similar to that of Code-a-pillar. By incorporating these features, designers would allow children to experience knowledge (codes) more concretely in a physical or visualized way by ensuring the program codes were instantiated in a perceivable reality.

Importance of Design Features to Engage Children in Meaningful Coordination

The third theme in the results related to design features engaging children in meaningful coordination. Toy designers created features such as flashing lights or stops between movements to help children see how the codes are coordinated to the movements of the toy. Some of these design features were perceived by children and afforded coordination between codes and movements (e.g., stops between movements), and some design features were not perceived and did not afford coordination (e.g., light on Botley remote). Literature on coding and mathematics suggests that coordination—

sometimes called one-to-one correspondence and/or action-instruction correspondence—is an important concept young children can learn while playing with coding toys (Bers et al., 2014, 2019; Munoz-Repiso & Caballero-Gonzalez 2019; Murcia & Tang, 2019). However, it is still unknown exactly what design features best afford this coordination. The results of this study highlight a few specific design features that afforded coordination for the children. For example, the coding toys that had the stop-between-movement design feature afforded coordination between code-movement. A possible special link between the stop-between-movement design feature and the engagement in coordination could be an idea called *specification of dynamic movement*. Children at this age mostly work with concrete, static, and discrete units and, therefore, their ability to recognize a unit of dynamic movement may not yet be well developed. Making sure to have a design feature which explicitly specifies each dynamic movement of the coding toy, like Bee-Bot and Cubetto's stop-between-movement feature, may be essential in helping young children coordinate between each dynamic movement and each code.

Additionally, the tangible design of certain coding toys (i.e., Bee-Bot, Botley, Cubetto) led to simulation of the toy, which afforded coordination. When children simulated the coding toys—meaning they picked up the coding toy and moved it around on the grid—they often tried to coordinate the simulated movements with the codes. This was doable with certain coding toys because they were small, child-hand sized, and could be moved without being damaged. The small size and tangible aspect of these coding toys may have supported simulation, and simulation may have afforded coordination between movements and codes. This is important because it may be a first step in documenting the power of the hand simulation of the coding toys, which may support future design of toys to be of certain sizes, shapes, and configurations to allow for these hands-on-coding-toy actions.

Lastly, a significant result pertaining to design features and coordination was that the lights and sounds afforded coordination when perceived. This means that when the children used and perceived design features like the flashing lights that mirrored the code enactments, and the sounds that aligned with code enactments, they engaged in coordination of codes and movements. Literature suggests that design features that include simultaneous linking of features and representations are beneficial to mathematics learning; however, children must first be aware of the features in order to take advantage of the potentially beneficial affordances (Bullock et al., 2017; Moyer-Packenham et al., 2020). The current study demonstrated that the children were rarely aware of the simultaneous linking design features and, therefore, may not have been able to take full advantage of their beneficial affordances. In the few times that children observed the simultaneous linking features, they afforded coordination between movements, codes, and numbers. Therefore, one could hypothesize that, if children became more aware of the design features, they may have engaged in more coordination. Because of this, I suggest that future design of coding toys incorporate more explicit simultaneous linking features, such as sounds that say and count the spatial movements out loud as the toy is enacting codes—*count code calling.* This design feature would look like the coding toy moving around and making auditory noises, "*one movement forward, one movement forward, 90-degree right rotation, 90-degree right rotation*." Another suggestion is that the lights on the coding toy could be in the exact shape and color as the command card/code button, which they are currently not. For example, there could be lights on the top of Cubetto that flash the same shape and color as the command tiles for each of the movements. This may help the children be more aware of the simultaneous linking design features. Additionally, the simultaneous linking features should be directly on the moveable body, so the children gaze is on the design feature and the light at the same instance, making this connection between movement and design feature more visible to the children. Overall, this final theme highlights how future design of toys should incorporate coordination-affording features that either currently exist (e.g., stop-between-motion) or have not yet been developed (e.g., auditory count code calling).

Limitations

Two primary limitations of the current study are: (a) mathematical engagement was primarily measured by behavioral indicators rather than cognitive ones, and (b) the research questions were posed after the original study design and data collection.

The first limitation of the current study is that it primarily relied on behavioral participation (engagement) in certain mathematical concepts and skills and did not primarily capture cognitive engagement. Appleton et al. (2008) examined 19 definitions of engagement in the psychological literature and found that all definitions of engagement included two similarities: *participation* either *behaviorally* or *cognitively*. The current study used videos as the main data source, which lends itself to analysis of behavioral indicators (e.g., gestures, movements, verbalizations, gaze). Although children sometimes verbalized potential cognitive processes which can be examined, the analysis was primarily limited to the children's' behavioral engagement. This is a limitation because it is not possible to report children's cognitive engagement with the mathematics based on the video data or the analysis.

The second limitation of the current study is that the research questions were posed after the original project design and data collection. Ideally, design of a research study and data collection should be driven by a research question. There were two primary reasons that I posed these research questions after the original project design and data collection. The first is that one of the original NSF-Funded CiK Project questions focused on design features of coding toys and computational thinking. Therefore, the research questions in the current study were designed to be aligned to that focus on the design features, which had similarities to the questions posed in the original CiK Project. Therefore, the data were a good fit for answering my posed research questions. The second influence on the timing of the research questions is that this study was designed during the height of the worldwide COVID-19 pandemic. Research guidelines at Utah State University limited in-person data collection during this time period, especially with young children in K-12 classroom settings. This required a change to my original plan for data collection and caused me to seek out the use of an existing data set to answer my research questions.

Conclusion

This study on young children and coding toys examined kindergarten-aged children's awareness of the design features in coding toys and sought to understand how those design features afforded children's engagement with mathematics. Results showed that (a) children used and perceived the grid square and command arrow design features frequently, while others were used moderately or rarely; (b) children engaged in a variety of mathematical concepts and skills in five main categories of mathematical topics: spatial reasoning, geometry, comparison, measurement, and number; and, (c) the relationship between design features affording mathematics varied depending on the coding toy, and that some design features afforded specific mathematical engagement across all four coding toys (e.g., grid squares afforded unit construction), while other design features afforded mathematical engagement only with specific coding toys (e.g., Cubetto and Bee-Bot's stop in-between movement afforded unit coordination). These findings have implications on future designs of coding toys for young children and can also support teacher implementation of coding toys in mathematics instruction by providing an empirical link between features of the toys and specific mathematical concepts and skills.

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APPENDICES

Appendix A

IRB Approval

IRB Approval

Utah State University Institutional Review Board (IRB) Proposal #8928 and #9569: Approved

Human Subject Approval or Completion Date: 2018-05-07

Responsible Conduct Approval or Completion Date: 2021-05-10

Appendix B

Spatial Concepts A Priori Coding Scheme

Table B-1

Spatial Concepts and Skills Students Used or Developed

Concepts and Skills	Description	Sample of Indicators
Spatial orientation	Understands and operates on relationships between different positions in space (e.g., the robot's movement in relation to a location on the grid or the robot's position in relation to themselves)	• Manually rotates the robot in relation to the endpoint to help plan one's program • Watches one robot move across the grid in a linear movement and determines how to move another robot in the same manner • Traces paths on the grid with hands from a start point to an end point • Considers robot's orientation in relation to the direction it will move
Spatial visualization	Understands and performs imagined transformations of objects (e.g., mental images of movement in the space such as a 90-degree rotation)	• Mentally rotates the robot to know which rotation code to add to one's program • Adjusts head or body when thinking of path • Describes imagined path or ending destination in a way to indicate they have visualized the robot's movements prior to the movements occurring
Spatial language	Describes movement of robot using spatial language correctly (e.g., forward, backward, rotate left) or intuitively (e.g., straight, down, turn) or position of the robot relative to locations or objects (e.g., next to)	• Describes robot's linear movements with terms such as forward, backward, up, down, straight, go • Describes robot's rotational movements with terms such as rotate, turn, left, right, that way, over • Describes position of robot relative to locations on the grid such as next to, almost there, in front of
Spatial knowledge in codes	Connects spatial orientation, spatial movement, and spatial language to a representational system (e.g., codes for the program represented as arrows or tiles)	• Uses a code for a program to symbolize the movement they described • Interprets a code for a program as an instruction for movement on the grid • Describes a movement and names the code that is needed for that movement (e.g., it needs to go forward so we need a green tile; it needs to turn so use the arrow that looks like this)

Appendix C

Measurement Concepts A Priori Coding Scheme

Table C-1

Measurement Concepts and Skills Students Used or Developed

Concepts and Skills	Description	Sample of Indicators
Units of measure	Understands and operates with a unit of measure, usually one linear forward movement	• Gestures individual linear movements on the grid, pausing between each iterated unit of distance • Describes the number of units needed for a program or makes sounds indicating a unit of measure (e.g., deet, deet, deet) • Describes or simulates a rotation as a unit of distance from one orientation to another (90 degrees for these robots)
Distance measurement	Understands that distance can be measured by units of linear movement either by counting the units of measure or describing or showing a distance from one point to another	• Describes the measurable attributes of distance or the robot's movements • Gestures a distance, for instance, between two hands representing the distance between a start point and end point • Uses words to describe the distance between a start and end point or the robot's position and a desired position on the grid, such as far or long or too short • Compares distances by describing a distance as equal, longer, or shorter using words such as more, less, farther, shorter • Gestures or describes how much more distance the robot needs to cover compared to current position

Appendix D

Number Concepts A Priori Coding Scheme

Table D-1

Number Concepts and Skills Students Used or Developed

Concepts and Skills	Description	Sample of Indicators
Counting	Counts movements (a distance quantity) or codes (objects)	• Uses number words to count either movements or objects • Names the correct number of movements or codes when asked how many • Gestures on the board or grid or with fingers a specific number of squares or movements while using number words
Counting on	Counts on from a given space on the grid (i.e., robot's starting point) which involves understanding that we are counting the movements of the robot, not the squares on the grid	• Touches grid as a sliding motion or jumping motion from the starting point to show counting on from a given point in space (e.g., like jumps on a number line) • Explains verbally that counting starts after the initial square because that is the starting point
Coordinating counts	Coordinates the totals of two quantities and/or matches 1-to-1 counting with movements or codes	• Associates number of movements along a path with the number of commands or codes a program needs to complete the path • Connects the number of movements and number of codes (blocks on programming board, arrows on remote, or arrows on program organizer) through a verbal explanation or gestures • Uses number words (e.g., three forwards) instead of spatial language (e.g., forward, forward, forward) for counting movements and coordinated with another quantity (tiles or arrows)
Operations	Uses addition or subtraction to operate on quantities	• Physically adds or subtracts from a quantity of codes, most often +/-1 block or arrow from the programming board or organizer • Adds or subtracts from a quantity of movements most often +/-1, such as "we need one more forward"

Appendix E

New Mathematical Codes that Emerged from Analysis

Table E-1

New Mathematical Codes that Emerged from Analysis

Concepts and Skills		Description	Sample of Indicators
Geometry	Location Description	Child verbalizes relativity of the coding toy to another thing using mathematical language such as next to, besides, on top of, underneath, around.	● Child uses mathematical language such as above, besides, around, next to, passing
	Shape Description	Child describes a geometrical shape while engaging in the coding activity	● Child says, "Look, it makes a circle!" ● Child says, "The board is in the shape of an L!"
Comparison	Matching Movements	Child attempts to imitate, mimic, or match individual movements or a path	● Child simulates the coding toys with their hand to show an existing path. ● Child traces a path with their fingers that is supposed to match another existing path.
	Comparing Quantity	Child compares two or more things using mathematical language such as more than, less than, same as	● Child uses mathematical terms such as: more than, less than, same as.
	Patterning	Child acknowledges some sort of pattern; repeating or singular module.	● Child verbally mentions a pattern. ● Child describes a pattern (e.g., forward, right, right! Forward, right, right!)
Measurement	Angle	Child reasons with various angles other than 90 degrees.	● Child says, "it needs to turn this much!" and gestures an angle in the air. ● Child places the coding toy on the mat in a way indicating a non 90-degree angle.
	Velocity	Child perceives some aspect of speed	● Child says, "This toy is fast!" ● Child says, "This toy is slow!"
Number	Multiplicative Reasoning	Child uses numbers and quantity using multiplicative reasoning rather than additive.	● Child says, "I need to use one green forward, three times!"
	Subitizing & Cardinality	Child says the amount of a quantity without explicitly counting, by looking at accumulated sets of objects	● Child says, "it needs four!" without counting. ● Child previously counted an amount, and then verbalizes the whole set.
	Sequencing	Child attends to the order of things, such as something being first, next, or last.	● Child uses mathematical language such as first, next, or last. ● Child sequences command arrows in environment.
	Decomposing	Child explicitly breaks numbers or paths apart.	● Child says "Bee-Bot did this part first, then did this part second!"
Spatial	Visual Estimation	Child estimates ending location of a coding toy without enacting codes or individual movements	● Child points to a grid square when asked where the coding toy will end. ● Child verbally approves or disapproves of whether a program will get the coding toy to a destination.

CURRICULUM VITAE

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EDUCATION

Ph.D. Utah State University; Spring 2022
Ph.D. in Education
Specialization: Curriculum and Instruction
Concentration: Mathematics Education and Leadership

M.S. University of Wyoming; June 2018
Master of Science
Concentration: Middle Level Mathematics Education

B.S. University of Wyoming; May 2014
Bachelor of Elementary Education
Minor: Early Childhood Education
Wyoming State Endorsement PreK-6
Wyoming State Endorsement Early Childhood Education

EMPLOYMENT HISTORY
Utah State University—4 Years

Elementary Teacher (2021-present)
Edith Bowen Laboratory School
Three main roles of this position include providing high quality instruction to elementary students, mentor and train future teachers who are enrolled in Utah State Universities Teacher Education and Leadership program, and engage in educational research.

Research Assistant (2019-present).
NSF-Funded Position, Coding in Kindergarten Project. Award# DRL-1842116. PIs: Jody Clarke-Midura and Jessica Shumway, Utah State University; Victor Lee, Stanford University.
College of Education and Human Services, Utah State University.
Research responsibilities include working with an interdisciplinary team and assisting with research tasks that include, but are not limited to: develop and teach curriculum, collect data (administer assessments, conduct clinical interviews with students, and/or videotape teaching episodes), analyze data, write research papers, assist with professional development for teachers, prepare materials, and present at research conferences. Leadership roles on the research team include: serve as a mentor for undergraduate research assistants, manage the literature database, lead monthly team discussions about current literature, and coordinate teaching and observation schedules.

Adjunct Faculty (Spring 2019)
School of Teacher Education and Leadership.
College of Education and Human Services, Utah State University.
Teaching responsibilities include but are not limited to: plan, prepare, present, and assess lessons; organize practicum experiences; supervise practicum experiences. Instruct a mathematical methods course to develop pre-service teacher skills in mathematics education.

Research and Teaching Assistant (Fall 2018).
School of Teacher Education and Leadership.
College of Education and Human Services, Utah State University.
Teaching responsibilities include but are not limited to: plan, prepare, present, and assess lessons; organize practicum experiences; supervise practicum experiences. Instruct a mathematical methods course to develop pre-service teacher skills in mathematics education.
Research responsibilities include: assist supervising professor with any research-based tasks that include, but are not limited to: collect, organize, and code data; write reports; run statistical analysis; analyze emergent themes; interview research participants; and present at conferences.

Public School Teaching Experience - 4 Years

Elementary School Teacher, Grade 3, All subjects (2014-2018).
Sheridan County School District #1, Ranchester, Wyoming.
Teaching responsibilities include but are not limited to: plan lessons, present lessons, administer assessments, participate in Professional Learning Communities, conduct parent/teacher conferences, organize school activities, develop district curriculum (i.e., create common assessments, validate assessments, develop outcomes/components, construct proficiency scales).

Early Childhood Teaching Experience - 3 Years

Classroom Assistant Teacher, Ages 1-6, All subjects (2011-2014).
Basic Beginnings Early Childhood Education Center, Laramie, Wyoming
Teaching responsibilities include but are not limited to: plan lessons, present lessons, administer assessments. Further responsibilities include: communicate with parents, organize student portfolios, and assist with parent/teacher conference.

AWARDS & PROFESSIONAL RECOGNITION

- **Outstanding Doctoral Researcher Award** (2020). Department of Teacher Education and Leadership
- **Outstanding Graduate Student Reviewer Award** (2020). AERA SIG-RME
- **NSF Grant Funded Assistantship** (2019-Present). Utah State University
- **Department Level Graduate Research and Teaching Assistantship** (2018). Utah State University
- **Cowboy State Highly Effective Literacy Teacher** (2018). University of Wyoming
- **Anderson, Rex, and Florence Scholarship** (2016-2018). Science and Math Teaching Center, University of Wyoming
- **Sigrid See Scholarship** (2016-2018). Science and Math Teaching Center, University of Wyoming
- **Teacher of the Month** (2016). Sheridan County School District #1, Ranchester, WY

RESEARCH

Research Interests:

- Mathematical creativity
- Early childhood mathematical thinking in coding contexts
- Early childhood computational thinking

Research Projects:

Coding in Kindergarten: An Exploratory Study of Coding Toys in Kindergarten Classrooms—NSF Grant-Funded Project. (2019-present). Utah State University (with Lead PI—Jody Clarke-Midura, Utah State University; and Co-PIs—Jessica Shumway, Utah State University; Victor Lee, Stanford University). My role: develop and teach curriculum, collect data (administer assessments, conduct clinical interviews with students, and/or videotape teaching episodes), analyze data, write research papers, assist with professional development for teachers, prepare materials, and present at research conferences. Leadership roles on the research team include: serve as a mentor for an undergraduate research assistant, manage the literature database, lead monthly team discussions about current literature, and coordinate teaching and observation schedules.

Affordances of Virtual Manipulatives Grades 3-6. (2018-2021). Utah State University (with PI Dr. Patricia Moyer-Packenham and the Virtual Manipulatives Research Group). My role: conduct iPad-based interviews with participants; collect and code data; analyze data; author and co-author collaborative presentations and publications.

PUBLICATIONS

Journal Articles (Refereed)

1. Shumway, J. F., Welch, L. E., **Kozlowski, J. S.,** Clarke-Midura, J., & Lee, V. R. (2021). Kindergarten students' mathematics knowledge at work: the mathematics knowledge for programming robot toys. *Mathematics Thinking and Learning*. http://doi.org/1080/10986065.2021.1982666.

2. Moyer-Packenham, P. S., Roxburgh, A. L., Litster, K., & **Kozlowski, J. S.** (2021). Relationships between semiotic representations and mathematical outcomes in digital games. *Technology, Knowledge, and Learning.* https://doi.org/10.1007/s10758-021-09506-5

3. Clarke-Midura, J., Silvis, D., Shumway, J. S., Lee, V. R., & **Kozlowski J. S.** (2021). Integrated computational thinking and mathematical assessment for young children with coding robots. *Computer Science Education*; *Special Issue, Assessment of Computational Thinking,* 1-24. doi: 10.1080/08993408.2021.1877988

4. Clarke-Midura, J., **Kozlowski, J. S.**, Shumway, J. F., Lee, V. R., Silvis, D., & Welch, L. (2021). How young children engage in and shift between reference frames when playing with coding toys. *International Journal of Child-Computer Interaction, 28*(100250), 1-12. https://doi.org/10.1016/j.ijcci.2021.100250

5. **Kozlowski, J. S.,** Jenkins, J., & Baggaley, S. (2020). A math RTI structure to support equity and learning for all students. *Utah Mathematics Teacher: Utah Council of Teachers of Mathematics, 13,* 33-49. Available at https://drive.google.com/file/d/1TSaQhgnbdjCDRnX8A1GzW80O1KmfAes_/view

6. Moyer-Packenham, P. S., Ashby, M. J., Litster, K., Roxburgh, A. L., & **Kozlowski, J. S.** (2020). Examining how design features promote children's awareness of affordances in digital math

games. *Journal of Computers in Mathematics and Science Teaching, 39*(2), 169-180.

7. **Kozlowski, J. S.,** & Chamberlin, S. A., (2019). Raising the bar for gifted students in mathematics through a creativity-based teaching approach. *Gifted and Talented International, 33*(1-2), doi: 10.1080/15332276.2019.1690954

8. **Kozlowski, J. S.**, & Si, Shouqing. (2019). Mathematical creativity: A vehicle to foster equity. *Thinking Skills and Creativity, 33*, 1-8, doi: 10.1016/j.tsc.2019.100579

9. **Kozlowski, J. S.**, Chamberlin, S. A., & Mann, E. L. (2019). Factors that influence mathematical creativity. *The Mathematics Enthusiast, 16*(26), 505-540. https://scholarworks.umt.edu/tme/vol16/iss1/26

Book Chapters (Refereed)

1. **Kozlowski, J. S.**, & Chamberlin, S. A. (2020). Creativity-based mathematical instruction: Fostering a holistically creative mathematical environment. In. S. A. Chamberlin & E. L. Mann (Eds.), *Creativity and affect in mathematics* (pp. 97-119). Prufrock Press

2. Moyer-Packenham, P. S., Litster, K., Roxburgh, A. L., **Kozlowski, J. S**., & Ashby, M. J. (2019). Relationships between mathematical language, representation connections, and learning outcomes in digital games. In D. C. Gibson & M. N. Ochoa (Eds.), *Research highlights in technology and teacher education 2019* (pp. 55-64). Association for the Advancement of Computing in Education (AACE).

Conference Proceedings (Refereed)

1. Silvis, D., Lee, V. R., Clarke-Midura, J., Shumway, J., & **Kozlowski, J. S,** (2020, April). Blending Everyday Movement and Representational Infrastructure: An Interaction Analysis of Kindergarteners Coding Robot Routes. In Gresalfi, M. & Horn I. S. (Eds.), *The Interdisciplinarity of the Learning Sciences, International Conference for the Learning Sciences* (ICLS) 2020, Volume 1 (pp.98-105). Nashville, TN, United States: International Society of the Learning Sciences.

2. Litster, K., Moyer-Packenham, P. S., Ashby, M. J., Roxburgh, A. L., & **Kozlowski, J. S**. (2019). Digital math games: Importance of strategy and perseverance on elementary children's learning opportunities. In K. Graziano (Ed.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 2157-2162). Las Vegas, NV, United States: Association for the Advancement of Computing in Education (AACE).

3. Moyer-Packenham, P. S., Ashby, M. J., Litster, K., Roxburgh, A. L., & **Kozlowski, J. S.** (2019). How design features promote children's awareness of affordances in digital math games. In K. Graziano (Ed.), *Proceedings of the Society for Information Technology and Teacher Education (SITE) International Conference* (pp. 2192-2200). Las Vegas, NV, United States: Association for the Advancement of Computing in Education (AACE).

Unpublished Manuscripts

1. Si, Shouqing., & **Kozlowski, J. S.** (in development). Culturally responsive practices improve cultural responsiveness: A guide for preservice teachers. Projected outlet, *Journal of Teacher Education.*

2. **Kozlowski, J. S.,** & Chamberlin, S. A. (in press). Mathematical creativity in the elementary grades. In M. Savic, P. Liljedahl, & S. Chamberlin (Eds.), *Mathematical creativity research in grades K-16.* In Series J. Cai, & J. A. Middleton (Series Eds.), *Research in Mathematics Education*. Springer.

GRANTS, CONTRACTS, & PRIVATE FUNDING

Utah Department of Education (1,000). Engaging students in after-school social experiences through disc golf and ultimate frisbee. State funded allocations to get the startup equipment needed to start an after-school club.

Research Assistant (1.6 million). *Coding in Kindergarten: An Exploratory Study of Coding Toys in Kindergarten Classrooms*. (2019-2022). National Science Foundation. Project goal: Design and validate a computational thinking assessment for early childhood-aged students. To develop understanding of how early-childhood aged students learn mathematics and computational thinking skills through interaction with coding robots (with Lead PI—Jody Clarke-Midura, Utah State University; and Co-PIs—Jessica Shumway, Utah State University; Victor Lee, Stanford University).

$300. *Travel Grant.* (2021). Utah State University. Research and Graduate Studies. Presentation at Psychology of Mathematics Education—North America's annual meeting (PME-NA). Philadelphia, PA

$300. *Travel Grant.* (2021). Utah State University. School of Teacher Education and Leadership (TEAL). Presentation at Psychology of Mathematics Education—North America's annual meeting (PME-NA). Philadelphia, PA

$300. *Travel Grant.* (2020). Utah State University. Research and Graduate Studies. Presentations at American Educational Research Associations annual meeting (AERA). San Francisco, CA.

$300. *Travel Grant.* (2020). Utah State University. School of Teacher Education and Leadership (TEAL). Presentations at American Educational Research Associations annual meeting (AERA). San Francisco, CA.

$300. *Travel Grant.* (2018). Utah State University. Research and Graduate Studies. Presentation at National Association of Gifted Children (NAGC). Minneapolis, MN.

$300. *Travel Grant.* (2018). Utah State University. School of Teacher Education and Leadership (TEAL). Presentation at National Association of Gifted Children (NAGC). Minneapolis, MN.

$1,000. *Newcomb Grant.* (2015). University of Wyoming. College of Education. Project goal: Further development of educator knowledge and pedagogic skill.

UNIVERSITY TEACHING

Utah State University, Logan, Utah—2 semesters; (2018-2019).
College of Education and Human Services

Courses Taught—Utah State University

ELED 4062—Teaching Elementary School Mathematics II: Number, Operations, and Algebraic Reasoning
Undergraduate course. Development of pedagogical content knowledge in number, operations, and algebraic reasoning for teaching grades preschool to grade 6. Methods for designing and implementing mathematics instruction, assessment, remediation, and intervention will be applied in a field-based placement. Face-to-Face Course combined with Practicum In-school Supervision.

- Course Evaluation Scores: 2018 (63, top 20%); 2019 (65, top 10%).

Guest Lecturer

Guest Lecture on Elementary Data Use, ELED 4062 Mathematics Methods (2021, April). For Dr. Jessica Shumway, Utah State University

Guest Lecture on Elementary Data Use, ELED 4062 Mathematics Methods (2021, April). For Dr. Beth MacDonald, Utah State University

Guest Lecture on Measurement and Data, ELED 4062 Mathematics Methods (2020, December). For Dr. Jessica Shumway, Utah State University

Guest Lecture on Place Value and Place Value Manipulatives, ELED 4062 Mathematics Methods (2020, January). For Dr. Jessica Shumway, Utah State University

Guest Lecture on Coding Robots for Integrating Mathematics and Computational Thinking, ELED 4062 Mathematics Methods (2019, November). For Dr. Jessica Shumway, Utah State University

Guest Lecture on Coding Robots for Integrating Mathematics and Computational Thinking, ELED 4062 Mathematics Methods (2019, October). For Rachel Reeder, Utah State University

PRESENTATIONS

National Presentations—Scholarship

Psychology of Mathematics Education—North America (PME-NA)
Kozlowski, J. S., Shumway, J. F., Clarke-Midura, J., & Lee, V. (2021, October 15-17). *Eliciting kindergarten students' mathematics with a coding toy: A pilot study on design features* [Poster presentation]. 43rd Annual Conference for Psychology of Mathematics Education—North America (PME-NA), Philadelphia, PA.

American Education Research Association (AERA)
Clarke-Midura, J. E., **Kozlowski, J. S.,** Shumway, J. G., Evans, H., Lee, V. R., & Welch, L. E. (2020, April 17-21) *Perspectives and Shifts of Young Children Playing with Coding Toys* [Paper Session]. AERA Annual Meeting San Francisco, CA. http://tinyurl.com/rkhbka (Conference Canceled)

Lee, V. R., Clarke-Midura, J. E., Shumway, J. F., **Kozlowski, J.,** Welch, L. E. & Evans, H. (2020, Apr 17 - 21) *Capturing Kindergarteners' Computational Thinking Through Commercial Toy-Centered Task and Assessment Development* [Symposium]. AERA Annual Meeting San Francisco, CA. http://tinyurl.com/yx2wzh53 (Conference Canceled)

Shumway, J. F., Clarke-Midura, J. E., Lee, V. R., Welch, L. E., **Kozlowski,** J. & Evans, H. (2020, Apr 17 - 21) *Identifying the Mathematics in Kindergarteners' Play with Coding Toys* [Paper Session]. AERA Annual Meeting San Francisco, CA. http://tinyurl.com/shh4hle (Conference Canceled)

Moyer-Packenham, P. S., Roxburgh, A. L., Litster, K. & **Kozlowski, J.** (2020, Apr 17 - 21) Students Connections Among Semiotic Representations in Digital Games and Their Impact on Mathematics Learning [Paper Session]. AERA Annual Meeting San Francisco, CA. http://tinyurl.com/t6bxst7 (Conference Canceled)

School Science and Mathematics Association (SSMA)
Moyer-Packenham, P., Roxburgh, A. L., & **Kozlowski, J. S.** (November 2019). *Students' Uses of Mathematical Representations and Their Learning Outcomes in Digital Games.* Research Presentation, School Science and Mathematics Association National Convention (SSMA), Salt Lake City, Utah.

Shumway, J. F., Clarke-Midura, J., **Kozlowski, J. S.**, Welch, L. E., & Evans, H. (November 2019). *Coding and Math: Playing with Screen-Free Robots to Develop Spatial and Measurement Reasoning.* Workshop Presentation, School Science and Mathematics Association National Convention (SSMA), Salt Lake City, Utah.

Clarke-Midura, J., Shumway, J. F., Welch, L. E., **Kozlowski, J. S.**, & Evans, H. (November 2019). *Integrated STEM: Using Coding Toys in Kindergarten Mathematics Lessons.* Presentation, School Science and Mathematics Association National Convention (SSMA), Salt Lake City, Utah.

National Association for Gifted Children (NAGC)
Chamberlin, S. A., & **Kozlowski, J. S**., Payne, A. (2021, November). *The relationship between creativity and affect in mathematics.* Research Presentation, 68th Annual Convention of the National Association for Gifted Children (NAGC), Aurora, CO.

Chamberlin, S. A., & **Kozlowski, J. S**. (2018, November). *What can be done to facilitate creativity in elementary mathematics classrooms?* Research Presentation, 65th Annual Convention of the National Association for Gifted Children (NAGC), Minneapolis, MN.

Society for Information Technology and Teacher Education (SITE)
Litster, K., Moyer-Packenham, P. S., Ashby, M. J., Roxburgh, A. L., & **Kozlowski, J. S**. (2019, March). *Digital Math Games: Importance of Strategy and Perseverance on Elementary Children's Learning Opportunities.* Research Paper Presentation, Society for Information Technology and Teacher Education (SITE), Las Vegas, NV.

Moyer-Packenham, P. S., Ashby, M. J., Litster, K.., Roxburgh, A. L., & **Kozlowski, J. S**. (2019, March). *How Design Features Promote Children's awareness of Affordances in Digital Math Games.* Research Paper Presentation, Society for Information Technology and Teacher Education (SITE), Las Vegas, NV.

Moyer-Packenham, P. S., Litster, K.., Roxburgh, A. L., **Kozlowski, J. S**., & Ashby, M. J. (2019, March). *Relationships between Mathematical Language, Representation Connections, and Learning Outcomes in Digital Games.* Research Paper Presentation, Society for Information Technology and Teacher Education (SITE), Las Vegas, NV.

NSF, STEM+C PI Assembly
Clarke-Midura, J., Shumway, F. J., Lee, R. V., Silvis, D., **Kozlowski, J. S.,** Welch, L., Evans, H., & Peterson, R. (2019, October). *Coding in kindergarten: Research on the development of an assessment to measure kindergarten children's abilities to reason computationally with mathematical problem-solving skills.* Poster Presentation, National Science Foundation, STEM+C PI Assembly, Washington, DC.

State & Regional Presentations

NCTM Regional

Kozlowski, J. S., Shumway, J. F., & Roxburgh, A. L. (2019, October). *How Do I Make My Textbook Lessons More Inquiry-Oriented? Some Simple Adaptations.* Burst Presentation, National Council for Teachers for Mathematics (NCTM), Salt Lake City, UT.

Roxburgh, A. L., & **Kozlowski, J. S**. (2019, October). *Fostering Mathematical Discourse Through Inquiry-Based Tasks.* Burst Presentation, National Council for Teachers for Mathematics (NCTM), Salt Lake City, UT.

Wyoming Innovations in Learning
Kozlowski, J. S., & Ross, S. (2020, November). *Creative Mathematical Journaling for Student Success*. Teaching Method Presentation. 4th Annual Wyoming Innovations in Learning. Wyoming State Office of Education. Online Conference.

Kozlowski, J. S., & Hayward, M. C. (2018, November). *Innovating Methods to Teach Conceptual Mathematics*. Teaching Method Presentation. 2nd Annual Wyoming Innovations in Learning. Wyoming State Office of Education. Evanston, WY.

Utah State University Research Symposium
Kozlowski, J. S., Welch, L. E., & Evans, H. Mentors: Shumway, J. F., Clarke-Midura, J., & Lee, V. R. (2019, April). *An Exploration of Kindergarten Students' Use of Perspective and Computational Thinking*. Oral Presentation, Utah State Research Symposium, Logan, UT.

Welch, L. E., **Kozlowski, J. S.,** & Evans, H. Mentors: Shumway, J. F., Clarke-Midura, J., & Lee, V. R (2019, April). *Coding to Develop Early Mathematical and Computational Thinking in Kindergarten: A Case Study*. Oral Presentation, Utah State Research Symposium, Logan, UT.

University of Wyoming College of Education Research Symposium
Kozlowski, J. S., & Chamberlin, S. A. (2018, March). *Factors that Influence Creative Output in Mathematical Problem-Solving Episodes*. Research Paper Presentation. College of Education Research Symposium. Laramie, WY.

Proposals Submitted

Welch, L. E., **Kozlowski, J. S.,** Silvis, D., Clarke-Midura, J., D., Shumway, J. F., & Lee, V. R. (accepted, 2022). *Identifying Kindergarten Students' Strategies as they Solve Computational Thinking Performance Assessment Tasks.* [Paper Session]. Annual Meeting of the American Educational Research Association (AERA), San Diego, CA.

Welch, L. E., Silvis, D., Clarke-Midura, J., D., Shumway, J. F., **Kozlowski, J. S.,** & Lee, V. R. (under review for 2022). *Assessment Designs that Elicit Multimodal Strategies: What We Can Learn about Early Childhood CT by Design.* [Paper Session]. The International Society for the Learning Sciences (ISLS), Hiroshima, Japan.

PROFESSIONAL SERVICE—INSTITUTIONAL

Institutional Service—University Level
Utah State University

Founder/President, Utah State Students for Life, USUSA Club (2019-2021)
Founded a USUSA sponsored club and acted as the first president. Founding the club involved recruiting members; drafting a mission and constitution; coordinating with the university-, state-, and national-level stakeholders for support; fundraising; organizing events; and leading and guiding the club with a shared mission.

Student Mock Job interviewer, Aggies Elevated (2020)
Served as a mock job interviewer for students in the USU Aggies Elevated program. This helped the students get feedback on their interviewing skills, their application materials, and their communication skills.

Volunteer, USU Food Recovery Program (2019-2020)
Supported this program through various roles including picking up leftover food from various on-campus dining services and sorting and organizing the food to make available to individuals who needed it.

Institutional Service—College Level
College of Education, University of Wyoming

Mentor Teacher (2017-Present)
Served as a full-time mentor teacher to provide guidance, instruction, and feedback on pedagogic skills to a student teacher.

Alyssa Brown—2018
Kylie Wilcox—2021
Lauren Nixon—2022

Institutional Service—National Level

Journal Reviewer Mentoring Program

Journal for Research in Mathematics Education (JRME; 2020-Present)
Selected from a large applicant pool of doctoral candidates to work with a veteran reviewer for JRME to co-review manuscripts in order to support the development of high-quality journal reviewers for mathematics education research.

Conference Submissions Reviewer

American Education Research Association—Awarded Outstanding Graduate Student Reviewer (AERA; 2020)
Served as a presentation submission reviewer. Synthesize, critique, and offer recommendations for revisions for proposal submissions.

Peer Reviewed Journals

Journal for Research in Mathematics Education (JRME; 2020-2021)—2 manuscripts
Served as an article reviewer. Synthesize, critique, and offer recommendations for revisions of this peer reviewed journal.

Creativity Research Journal (CRJ; 2021)—1 manuscript
Served as an article reviewer. Synthesize, critique, and offer recommendations for revisions of this peer reviewed journal.

Gifted and Talented International (GTI; 2021)—1 manuscript
Served as an article reviewer. Synthesize, critique, and offer recommendations for revisions of this peer reviewed journal.

Journal of Empowering Teaching Excellence (JETE; 2019-2020)—2 manuscripts
Served as an article reviewer. Synthesize, critique, and offer recommendations for revisions of this peer reviewed journal.

International Journal of Evaluation and Research in Education (IJERE; 2019)—1 manuscript
Served as an article reviewer. Synthesize, critique, and offer recommendations for revisions of this peer reviewed journal.

Journal on Mathematics Education (JME; 2019)—1 manuscript
Served as an article reviewer. Synthesize, critique, and offer recommendations for revisions of this peer reviewed journal.

Institutional Service—Public School Systems
Tongue River Elementary, Sheridan County School District #1

Mathematics Professional Development (2015-2016)
Provided professional development to colleagues, centered on hands-on mathematics instruction. The 2-hour district day sessions involved embedding manipulatives into mathematics instruction for various ages of students.

John Oliver's ABC's of Life Staff Development (2018)
Introduced new staff members to the Self-Control module of the John Oliver's classroom management program. New staff who were learning how to implement this session as well as veteran teachers reviewing the protocol attended.

Edith Bowen Laboratory School, Utah State University

Mathematics RTI Program Evaluation (2019-2020)
Collaborated with 6th grade teachers, instructional specialists, special education specialists, and administration team to develop new mathematics pedagogy structure to meet academic goals of all students and of school. The 6th-grade team and myself published an article based on this work together.

Mathematics RTI Instruction Volunteer (2020-2021)
Volunteered as a 6th-grade RTI group instructor for one hour, two times a week throughout the year. In addition, I was happy to volunteer my service to the 6th-grade team in any other way they saw fit which sometimes included teaching a whole class lesson, collaborating on assessment creation, collaborating on lesson creation, or accompanying the class on an experiential learning trip!

PROFESSIONAL AFFILIATIONS & LEADERSHIP ROLES

State of Utah

Leadership
Utah Council of Teacher of Mathematics Board Member, Leadership Committee. (2020)

Sheridan County School District #1

Adventure Club Coordinator (2017-2018)
Coordinated with stakeholders to fundraise for, plan, recruit students and teachers, and implement single-day and multi-day experiential learning trips with the 3rd-5th-grade elementary school students. These trips included extensive planning to ensure trips related students' experiences to learning. Some trips included: Buffalo Bill Center of the West Museum and river rafting on the Shoshone River in Cody, WY; History at Devil's Tower and Mt. Rushmore, Black Hills, SD; and Wyoming History and Dog Sled Races, Casper, WY.

Adventure Club Co-Coordinator (2016-2017)
Assisted the principal (Deb Hofmeier) in all activities listed under *Adventure Club Coordinator.*

Head Wrestling Coach (2016-2018)
Recruited athletes, planned and facilitated practices, organized trips, and managed team finances. Throughout my tenure as head coach, the program increased from 7 to18 athletes. We were a new team in the state, and we became affiliated with a district. We greatly improved our team standing in district rankings during my coaching tenure. Most importantly, our student athletes found individual success both in their academics and in their competitions.

Assistant Wrestling Coach (2015-2016)
Assisted the head wrestling coach with all activities listed under *Head Wrestling Coach.*

Memberships

American Education Research Association (AERA)
School Science and Mathematics Association (SSMA)
National Association for Gifted Children (NAGC)
Utah Council of Teachers of Mathematics (UCTM)

SPECIAL TRAININGS & CERTIFICATIONS

Research on Early Mathematics Assessment - REMA (2019)
Trained and certified with the extensive REMA. Underwent a two-day training and passed an administration-based examination to certify as an assessor.

Successful Grant Writing Workshop—AtKisson Training Group (2019)
The one-day extensive workshop detailed successful grant writing techniques with a focus on large scale grants (e.g., NFS, IES, NIH)