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Locus filters

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Abstract: In this paper, directly following from Gage [J. Opt. Soc. Am. **23**, 46 (1993)], we study the design of a particular theoretical filter for photography, that we call the locus filter. It is built in such a way that a Wien-Planckian light (of any temperature) is spectrally mapped to another Wien-Planckian light. We provide a physical basis for designing such a filter based on the Wien approximation of Planck's law, and we prove that there exists a unique set of filters that have the desired property. While locus filtered Wien-Planckian lights are on the locus, the amount they shift depends both on the locus filter used and on the color temperature of the light. In experiments, we analyze the nature of temperature change when applying different locus filters and we show that real lights shift more or less as if they were Planckians in terms of the changes in their correlated color temperatures. We also study the quality of the filtered light in terms of distance from the Planckian locus and color rendering index.

1. Introduction

Colored filters have been historically used in photography to modulate the color of the prevailing illuminant. A filter is usually a piece of colored glass (or another substrate) that is placed in front of the lens of the camera leading to a change in the colors of the scene in the acquired photo. One of the main goals of using these filters is to change the way the camera sees the light which depends on the spectral properties of the filter used. As an example, we might use a 'warm' filter when taking a picture under a cool light to render the effective illuminant color neutral.

In Fig. 1(a), we show the spectrum of a locus filter, a theoretical filter which we will specify and explore its properties in this paper (the max transmittance is normalised to 100%). This filter is designed to, exactly, spectrally, map a Wien-Planckian light (Wien-Planckians are very similar to Planckian lights but are described by a simpler equation, which we exploit in this paper) with a color temperature 10000 Kelvin, or 10000K (henceforth, 'K' to denote Kelvin), to 4300K. Notice the filter is very smooth. In Fig. 1(b), we plot CIE 10000K Daylight illuminant (also referred to as D100) and also this light multiplied by the locus filter (where both lights are normalized to their maximum intensity). As a Daylight does not lie on the locus, its color temperature is actually a correlated color temperature (CCT), which informally equates to the actual temperature of a close Planckian. In this example we find that - although not explicitly designed for this purpose - our locus filter correctly maps D100 to D43. In Fig. 1(c) we plot, in the CIE uv chromaticity space, the Planckian locus and the coordinates of D100 and D43. The reason behind using CIE uv chromaticity space in our plots is to keep coherence as uv coordinates are the ones used to calculate the CCTs of real lights. In Fig. 1(d) we 'zoom in' and plot the same data. In the more detailed view, we see that D100 and D43 do not actually fall on the Planckian locus (they lie on the Daylight locus which runs parallel to the Planckian)

Of course there will be many shapes of filters which will map a light with a given CCT to a target CCT (since there are many metamers that integrate to the same color tristimulus). And, there are many lights that have different chromaticities which map to the same CCT. In the context of this paper, we propose that a good filter should have four properties. First, it should

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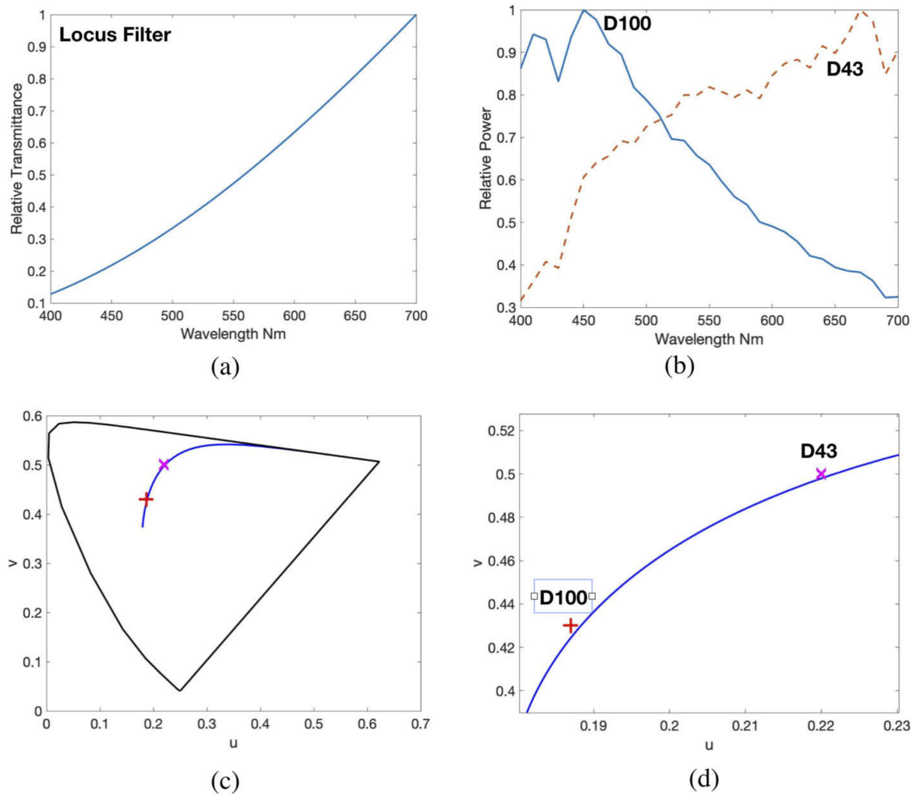

Fig. 1. An example of a locus filter applied on a daylight. (a) The spectral transmittance of a locus filter. (b) Two Daylight illuminants, D100 and D43, in solid blue and dashed red lines respectively (the filter in (a) multiplying D100 results in D43). (c) Their uv coordinates in the uv chromaticity diagram, where a red '+' and a magenta 'x' denote D100 and D43 and the Planckian locus is drawn in blue. (d) A zoomed-in view of our data

map a typical light to another typical light. In our example, one daylight is mapped to another, meeting this criterion. Second, that the distance in uv chromaticity space, from the Planckian locus should be maintained after filtering, to mimic a change in temperature along the locus. Third, the filter should be smooth, palpably true for our illustration in Fig. 1. When a filter is spiky, it is less likely it will work for all lights. Fourth, and importantly, the interaction of light and filter should be predictable. Ideally, for a known filter and a light with a known CCT we should a priori be able to predict how the CCT will change when the filter is applied.

In this paper, we start with the idea that it would be a useful property if a colored filter is guaranteed to map all lights from one position (color temperature) on the Planckian locus to another. That is, we wish to avoid the circumstance where a colored filter might for example, make a Planckian light (yellows, whites and blues) slightly greenish. Following directly from Gage [1], we show that it is indeed possible to design a filter that guarantees that the resulting light stays on the Wien-Planckian locus. A Wien-Planckian is defined by a slightly different formula than a Planckian that is useful for the derivations we set forth in this paper. We call such a filter a 'locus filter', and present a physical basis for the design of such filters. Significantly, the locus filters are the only ones that have the property that they map Wien-Planckian lights [2] to other lights that are also on the Wien-Planckian locus.

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As the reader will have noticed, in this paper, we will sometimes need to speak of Planckian lights and Wien-Planckians. Though they are similar and can often be thought of interchangeably they are not the same. Thus, to avoid confusion we will say a light is Planckian if it is described by Planck's blackbody radiator equation, and Wien-Planckian if we are using Wien's approximation. However, the reader - in their mind's eye - might wish to read Planckian and Wien-Planckian as *meaning* the same thing, at least on the first read through, since the difference in the spectra the two equations produce ranges from negligible to small.

A Planckian light (and a Wien Planckian), by definition, is dependent on a single number, the color temperature (measured in Kelvin). As we move from 3000 to 5000 to 10000 Kelvin the corresponding Planckian (and Wien-Planckian) light 'looks' respectively yellowish, whitish and bluish. The higher the temperature the cooler the color of the light. Our locus filter maps a given input color temperature to another (the formulas are presented later) and this mapping is non-linear in temperature. Not only does the locus filter map color temperatures accurately, one Wien-Planckian is exactly spectrally mapped to another. Applying the locus filter on a Wien-Planckian light results in shifting its temperature making the light more yellowish or more bluish. Significantly, given a locus filter and the color temperature of a given Wien-Planckian light we can explicitly, in closed form, calculate the color temperature of the filtered Wien-Planckian.

In this work, we are also interested in studying the behavior of locus filters when applied to real lights. The analogous correlated color temperature (CCT) [3] is often used as a measure of the color temperature of real lights. For a real light, we calculate the CCT before and after application of the locus filter. Experiments demonstrate that the CCTs of real lights shift very similarly to the corresponding Planckian (note CCTs are calculated according to the actual Planckian locus and not the Wien-Planckian counterpart). Other metrics are also analyzed in our experiments to assess how a locus filter filters real lights including the distance, before and after filtering, to the Planckian locus (Duv) in the uv chromaticity diagram [4], and the color rendering index (CRI) [5] of the filtered light.

In section 2 of this paper we present background material. Then, in section 3, we give the physical and mathematical basis for the design of the locus filter. Here, we will provide a proof of uniqueness: only a filter formulated as a locus filter has the property that it will *always* map a Wien-Planckian to another Wien-Planckian. We will then, in section 4, empirically study how locus filters affect non-Planckian lights. Lastly, we discus how some properties of real lights affect the behaviour of locus filters when applied on those lights in section 5, then conclude in section 6.

2. Background

In a Lambertian scene, the sensor responses (of a camera or the XYZ Color matching functions (CMFs)) ρ_k can be written as a function of the surface spectral reflectance $S(\lambda)$, the spectral power distribution of the illuminant $E(\lambda)$, and the spectral sensitivities $Q_k(\lambda)$ (where $k = R, G, B$):

$$\rho_k = \int_\omega Q_k(\lambda)S(\lambda)E(\lambda)d\lambda, \tag{1}$$

where ω is the range of visible wavelengths. Throughout this paper, we consider the 1931 color matching functions in our calculation.

Ignoring complexities such as interreflections, when a colored filter is put in front of the lens of the vision system, the filtered sensor responses ρ_k^F can be written as:

$$\rho_k^F = \int_\omega Q_k(\lambda)S(\lambda)F(\lambda)E(\lambda)d\lambda, \tag{2}$$

$F(\lambda)$ defines the filter spectral transmittance.

A Planckian blackbody illuminant E^P is a function of color temperature T and wavelength λ, and is written as:

$$E_P(\lambda, T) = kc_1\lambda^{-5}(e^{\frac{c_2}{T\lambda}} - 1)^{-1}, \tag{3}$$

where c_1 and c_2 are constants equal to $3.74183 \times 10^{-16} Wm^2$ and $1.4388 \times 10^{-2} mK$, respectively. The scalar k modulates the intensity of the Planckian light.

Much research in the literature has focused on finding a particular filter that changes a specific illuminant in the scene to another chosen one. Gage [1] presented a study of the properties of a filter that brings a Wien Planckian illuminant (see next section for a definition) with one color temperature to an another specific one. As we present in the next section, it turns out that a Gage-filter also maps any Wien-Planckian to another Wien-Planckian (our own analysis starts with this result). For known film-types, given lights, and a set of approximate, physically realizable, Gage-type filters, McCamy [6] considered how these filters could be practically used to optimise the exposure of color films. It has also been observed by Henry Hemmendinger - and reported in [7] - that the the yellowing of the lens of the eye can be thought of as approximately following a Gage-type equation. Weaver [8] also considered the design of a filter that converts lights from one color temperature to another. In complementary work, Estey [9] studied the properties of color temperature change for some commercially available filters.

Interestingly, the question of how the filters modulate arbitrary light has not, to our knowledge, been considered quantitatively. If a filter maps (say) 6500K to 4500K how will it modulate a 10000K illuminant and will the filtered light still be on the Planckian locus? This predictability question is a key concern of this paper.

Of course, there are other ways to measure how color temperature shifts when a light is filtered. The Mired temperature [10] (micro-reciprocal-degree), also called is sometimes used when we wish to measure how similar one light color is to another. Reciprocal color temperature, T^{rec}, measured in Mired units, is calculated from a color temperature T as follows:

$$T^{Mired} = \frac{10^6}{T} \tag{4}$$

It has been shown empirically that the just noticeable difference (JND) is 5.5 mired units [10,11].

As discussed in the introduction we are also interested in how a filter changes the correlated color temperature (CCT) of a real light. The CCT is the temperature of the Planckian illuminant giving the closest perceived color to the real illuminant. Interestingly, how to calculate the CCT is still an active area of research [12–14]. In this paper we will adopt the method [4] to calculate CCTs.

According to this method, we first calculate the uv chromaticity coordinates for each Planckian light at a resolution of color temperatures of 1 Kelvin, in the temperature interval [1000K,100000K]. We remember that the CIE uv chromaticity diagram (part of the LUV color space) is useful because the distance between chromaticity points correlates with perceived difference in color (see Fig. 1(a) to see an example plot in the uv diagram, there the Planckian locus is also shown). Then, the uv coordinates, (u, v) of an actual real light are calculated. The closest precomputed Planckian uv coordinate to (u, v) determines the closest Planckian light. The temperature of the closest Planckian Light is the correlated color temperature, the CCT.

The color rendering index (CRI) will also be used in our experimental evaluation. We are interested in how the CRI changes (or does not change) when a real light is filtered. The CRI [5] is a quantitative measure of how close a light source to a natural or ideal illuminant in terms of the way they render colors. CRI is a score in the range [0,100] with 100 meaning perfect color rendition. A CRI of 90+ can be a taken as an indication of good color indexing (though the perceptual meaning/significance of CRI is a source of debate). We will adopt CIE 1995 method for calculating the CRI, which is based on the color difference between standard samples

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rendered with the light source and the same samples rendered using an ideal light source with the same color temperature as the first one. More details about the calculation algorithm that we used can be found here [15].

Regarding color filter design in general, more recent research has focused on the design of filters for other tasks in image processing and digital photography. Some approaches in the state of the art have focused on filter design and use in the objective of making camera sensors more colorimetric [16–18]. In computational color constancy, specific color filters have also been shown to be useful in the estimation of the color of the light and the determination of where lights change in an image in scenes with multiple illumination conditions [19–21]. Moreover, filters are also used in multispectral imaging [22,23].

3. Locus filter model

In the range of typical lights (2000K to 20000K), a simple approximate form of Planck's equation - called Wien's approximation - can be used to describe black body illumination [2]. These Wien-Planckian lights are written as:

$$E(\lambda, T) = kc_1\lambda^{-5}e^{-\frac{c_2}{T\lambda}} \tag{5}$$

Here, k modulates intensity and the constants c_1 and c_2 are the same (defined after Eq. (3)). For most practical purposes (and the ones we consider in this paper) we can use Planckian and Wien Planckian lights interchangeably. However, there are small differences (when T>4000K). Thus, throughout this paper we always say Wien-Planckian when we are, specifically, using Eq. (5) and Planckian when we are using Eq. (3).

Now, let us focus on the problem of designing a filter that maps one light to another. Suppose we have a Planckian illuminant with a temperature T_1, and that after applying the filter we would like to obtain another Planckian illuminant with a temperature T_2. By dividing the spectra of these two illuminants provided by Eq. (5) we obtain a filter transfer function $Tr(\lambda, T_f)$ that has the desired property:

$$Tr(\lambda, T_f) = e^{-\frac{c_2}{T_f\lambda}} \tag{6}$$

We note that Gage [1] actually derived the same equation by finding a filter for a single pair of Wien-Planckian lights. But, he did not consider what happens when the filter is applied to other Planckian (and non-Planckian, see our experiments section) lights.

We call the T_f parameter **the locus filter-temperature (LFT)**, and it is equal to:

$$T_f = \frac{1}{\frac{1}{T_2} - \frac{1}{T_1}} \tag{7}$$

We prove in the following section that the form of the filter equation is unique. That is, there is no other filter which always maps all Wien-Planckian lights to corresponding lights that themselves are Wien-Planckians.

Clearly, from Eq. (7), the LFT, T_f, is not uniquely defined by the pair of Wien-Planckian lights with temperatures T_1 and T_2. That is there are many pairs of temperatures that can lead to the same LFT. Also following from Eq. (7), the LFT can be negative.

Let us now calculate the product of the filter $Tr(\lambda, T_f)$ with a third light that has a temperature T_3 to make a new light spectrum $E'(\lambda)$:

$$\begin{aligned} E'(\lambda) &= E(\lambda, T_3)Tr(\lambda, T_f) \\ &= Ic_1\lambda^{-5}e^{-\frac{c_2}{T_3\lambda}}e^{-\frac{c_2}{T_f\lambda}} \\ &= Ic_1\lambda^{-5}e^{-\frac{c_2}{T'\lambda}} \end{aligned} \tag{8}$$

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Clearly, $E'(\lambda)$ lies on the Wien-Planckian locus with temperature T' calculated as:

$$T' = \frac{1}{\frac{1}{T_f} + \frac{1}{T_3}} \tag{9}$$

Curiously, Eq. (9) teaches that a filtered light can have a negative color temperature as LFT can go negative (see Eq. (7)) but, of course, this is not physically possible. However, for the purposes of our work we will allow negative color temperatures and, here, we briefly consider the import of this choice.

Admitting negative temperatures, it turns out, leads to even bluer lights than occur when the color temperature tends to infinity. Indeed, in Fig. (2(a)), we show on the uv chromaticity diagram the Wien-Planckian locus in solid blue and the additional colors available in an extended locus (that admits -ve colors) is shown in a red line. For reference a uniform white light has a uv chromaticity coordinate of (0.21,0.47). Lights with a coordinate u<0.21 are cooler in color than white and when u>.21 the color of lights become progressively warmer. To elaborate this point on the Wien-Planckian locus (blue line) from left to right the temperature monotonically decreases from $\lim_{T\to\infty}$ to $\lim_{T\to 0^+}$. Conversely, in the extended locus from right to left the temperature monotonically increases from $\lim_{T\to-\infty}$ to $\lim_{T\to 0^-}$.

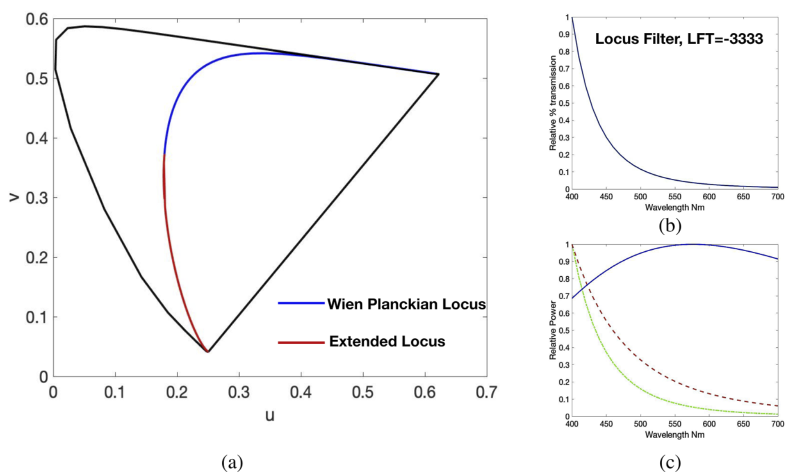

Fig. 2. Negative locus filter temperature example. (a) The Wien- Planckian locus (blue for +ve Temperatures) and the extended locus (red for -ve Temperatures in the uv chromaticity diagram).(b) Spectral transmittance of a locus filter with an LFT of −3333K. (c) the red dashed line shows the spectral power distribution for a light with a color temperature tending to infinity. Respectively, the solid blue line and the dash-dotted green line show a 5000K Wien-Planckian and its filtered counterpart (filtered by the filter shown in (b)

Intuitively, the extended locus is quite a natural extension. As temperatures decrease, in the limit a Wien-Planckian light has almost all its power concentrated in the very end of the long-wave part of the visible spectrum (effectively, it becomes almost monochromatic). As such, the reddest red light intersects with the spectral locus. In contrast, the bluest blue +ve temperature Wien-Planckian (as the temperature tends to infinity) is actually not so blue. See dashed red line in panel (2(c)). Admitting negative temperatures effectively allows a transit of light colors that becomes progressively bluer until they too reach the spectral locus.

In (2(b)) we show a locus filter with a negative LFT of −3333. Now, we plot a 5000K Wien-Planckian light (blue, solid) in (2(c)). Applying the locus filter to the light results in a filtered light (green dash-dotted line), which has color temperature of −9997K. Compared to the

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spectrum for an infinite +ve color temperature (red dotted line), it is clear the filtered spectrum is bluer than the bluest physically possible Planckian. All spectra in (2(b)) and (2(c)) are normalised to have a maximum value of 1.

Finally, we note that the definition of locus filter has another degree of freedom. Specifically we can scale the locus filter to transmit more or less light. In Eq. (10) we add the scalar k to model this variable transmittance. Clearly, if one locus filter is a scalar, say <1, from another then it will map Wien-Planckian lights to the same counterparts but they will be darker.

$$Tr(\lambda, T_f, k) = ke^{-\frac{c_2}{T_f \lambda}} \tag{10}$$

3.1. Proof of uniqueness

Often colored filters are designed to mimic a change in illuminant color. A bluish light is filtered by a yellowish filter results in a warmer colored light. Here, we prove that a Wien-Planckian illuminant filtered by a locus filter results in a second Wien-Planckian illuminant (for all Wien-Planckians) and, across all choices of filters, and only the locus filter has this property.

Theorem: Only locus filters - with the form defined by Eq. (10) - will map all Wien-Planckians to other Wien-Planckians (where negative as well as positive temperatures are allowed).

Proof: We will present our argument in the log-domain (since, because the logarithm function is bijective, if the filter maps the logarithm of all Wien-Planckians to log domain counterparts then uniqueness is also proved in the non-log domain). Let us define $\alpha = \log k$, $v(\lambda) = -\frac{c_2}{\lambda}$, $\beta = T^{-1}$ and $w(\lambda) = \log(c_1 \lambda^{-5})$. Adopting this notation, we write the logarithm of a Wien-Planckian as

$$e(\lambda, \alpha, \beta) = \alpha + \beta v(\lambda) + w(\lambda), \tag{11}$$

where, by construction there exists a k and T such that $exp(e(\lambda, \alpha, \beta)) = kE(T, \lambda)$.

Remember, we are allowing T to be positive and negative. As a consequence $\beta \in \mathbb{R}$. Similarly, $\alpha \in \mathbb{R}$ as the log of the scalar $k \in (0, \infty)$. It follows that there exists a log Wien-type illuminant for all real scalars α and β.

A filter $F(\lambda)$ (which has a multiplicative role in the non log domain) is an additive offset in the log domain. Denoting $f(\lambda) = \log(F(\lambda))$, for our proof, we need to show that, if $F(\lambda)$ is a Locus Filter, $\forall \alpha, \beta, \exists \alpha', \beta'$:

$$e(\lambda, \alpha, \beta) + f(\lambda) = e(\lambda, \alpha', \beta') \tag{12}$$

Substituting, Eq. (11) in Eq. (12) we are interested in log filters where:

$$\alpha + \beta v(\lambda) + w(\lambda) + f(\lambda) = \alpha' + \beta' v(\lambda) + w(\lambda) \tag{13}$$

or, equivalently

$$\alpha + \beta v(\lambda) + f(\lambda) = \alpha' + \beta' v(\lambda) \tag{14}$$

Let us define a filter as;

$$f(\lambda) = \alpha_f + \beta_f v(\lambda) \tag{15}$$

Then if we set $\alpha' = \alpha - \alpha_f$ and $\beta' = \beta - \beta_f$ in Eq. (14) we see that $f(\lambda)$ maps all log-Wien-Planckians to counterpart spectra that are also log-Wien-Planckians. And, of course, by construction, $F(\lambda) = exp(f(\lambda))$ is a locus filter.

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Now, suppose we cannot write $f(\lambda)$ using Eq. (15) (it is not a locus filter):

$$f(\lambda) = \alpha_f + \beta_f v(\lambda) + \delta(\lambda) \tag{16}$$

There do not exist scalars α_δ and β_δ such that $\alpha_\delta + \beta_\delta v(\lambda) = \delta(\lambda)$:

$$\forall \alpha_\delta, \beta_\delta \ , \ \alpha_\delta + \beta_\delta v(\lambda) \neq \delta(\lambda) \tag{17}$$

Returning to Eq. (13), and substituting $f(\lambda)$ from Eq. (16) our theorem is not true iff:

$$\alpha + \beta v(\lambda) + \alpha_f + \beta_f v(\lambda) + \delta(\lambda) = \alpha' + \beta' v(\lambda) \tag{18}$$

Let $\alpha_c = \alpha + \alpha_f - \alpha'$ and $\beta_c = \beta + \beta_f - \beta'$ our theorem would be contradicted if

$$-\alpha_c - \beta_v v(\lambda) = \delta(\lambda) \tag{19}$$

but, from Eq. (17), this is impossible ($\delta(\lambda)$ cannot be written as a sum of α and $\beta v(\lambda)$). Only filters of the form in Eq. (15) (i.e. only locus filters) always map Wien-Planckian lights to Wien-Planckian lights. This completes our proof of uniqueness.

3.2. Planckian vs the Wien-Planckian approximation

At this point we should also be clear in stating that the Wien-Planckian locus is not identical to the Planckian locus. However, they are surprisingly similar. Indeed, if we plot them on the same uv chromaticity diagram they lie more or less on top of each other. Including and below 4000K the spectra that are produced by Wien's or Planck's equation are nearly identical. Though, there is some divergence for higher temperatures. In the normal temperature range for daylights and artificial lights (say 2000K to 20000K) the largest spectral difference is found for a 20000K Wien approximation compared with the true Planckian light. We show these spectra - normalised to their maximum intensity (it is set to 1), in Fig. 3.

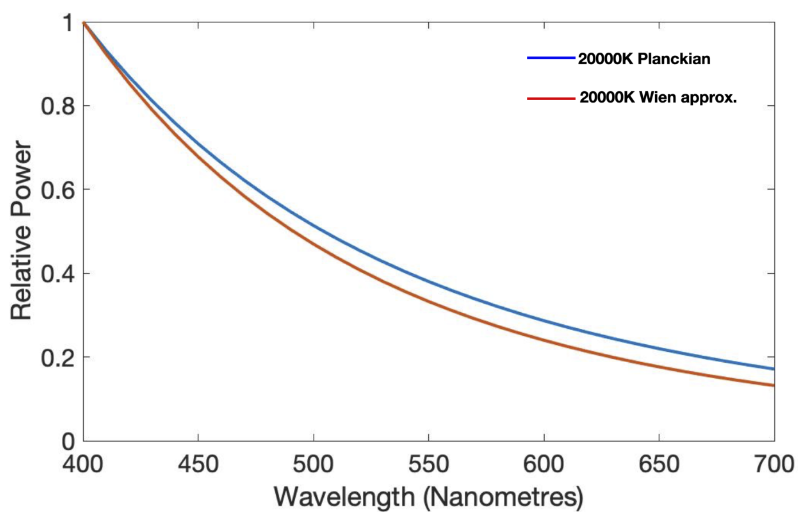

Fig. 3. A 20000K Planckian light (blue) compared with a 20000K Wien-Planckian approximation (red).

Respectively, the Wien-Planckian and Planckian uv chromaticities for these lights are - to three decimal places - (0.183, 0.402) and (0.185, 0.412), respectively. The Euclidean distance between these points is 0.011. To understand the visual significance of these numbers let us multiply the

Duv by 650 (i.e. we use the CIE LUV formulae to visualize the chromaticities when lightness $L^* = 50$, see [24]) then we can say something about the visual perceptibility of the difference. For reference, the color difference Delta E here is $650 * 0.011 = 7$ which is visually noticeable but, the difference is small. Indeed, studies [25] have shown that in complex imagery, unless a Delta E is larger than 5 then color differences are not noticeable at all.

Having acknowledged the difference between Planck's and Wien's approximations because the differences - from a perceptibility viewpoint are likely small - we will use the same temperature when discussing each equation (despite the small spectral shift at higher temperatures). Returning to Fig. 1(d), here a real D100 light (which is referenced to the real Planckian locus) has a locus filter that maps a Wien-Planckian light of 10000K to 4300K. Here, the filtered Daylight also maps to 4300K meaning that, at least for this example, a Daylight is shifting as predicted by our Wien-Planckian locus analysis.

Later, we will return to this issue when we discuss how the correlated color temperature of real lights shifts when we apply our locus filters. Specifically, we compare how Correlated Color Temperature (CCT, calculated according the the Planckian-locus) shifts when we filter with a locus filter (even though a locus filter is designed to shift color temperature along the Wien-Planckian locus).

4. Experiments

4.1. Filtering Wien-Planckian lights

We demonstrated earlier that when applying a locus filter on Wien-Planckian lights, filtered lights also lie on the Wien-Planckian locus. However, the change of temperature after applying the filter depends on both the initial temperature of the Wien-Planckian light and LFT (locus filter temperature, see Eq. (7) for its definition) of the applied filter (see Eq. (9)). In order to better observe the nature of this change, we apply some locus filters with different LFTs with positive and negative values to several Wien-Planckian lights with different color temperatures. The first three columns of Table 1 shows the change of temperature when applying filters with a locus filter with a positive LFT of 5000, 7500 and 10000 to different Wien-Planckian illuminants. The last two columns are for the negative LFTs, −12000 and −20000.

Table 1. Color temperature shifts after applying locus filters on Wien-Planckian lights. Column one records the color temperature in Kelvin for 4 input lights, ranging from 4000K to 10000K. The columns show the LFT T_f ranging from −12000 to 10000. In position (i, j) we see the output colour temperature for the ith input light filtered by a locus filter with the jth LFT.

Input Light Temperature	Output colour temperature for LFT ($\mathbf{T_f}$),				
	(5000)	**(7500)**	**(10000)**	**(−12000)**	**(−20000)**
4000K	2222K	2609K	2857K	6000K	5000K
6000K	2727K	3333K	3750K	12000K	8571K
8000K	3077K	3871K	4444K	24000K	13333K
10000K	3333K	4286K	5000K	60000K	20000K

From Table 1, we see that a $4000K$ light filtered by a locus filter with $LFT = 5000$ shifts to $2222K$ (i.e. a color temperature change of $1778K$). However, the same light shifts to $6000K$ when filtered with a locus filter with $LFT = -12000$. More generally, notice that filtering a Wien-Planckian light with a filter with +ve LFT always makes the light warmer and conversely a negative LFT will result in a filter that will always make lights cooler in appearance.

In Fig. 4, we show in the u, v chromaticity diagram, how a Wien-Planckian light with a color temperature of $5000K$ shifts differently when being filtered with two different locus filters. In the first case, the light is filtered with a locus filter with a positive temperature ($LFT = 5000$) making it warmer. In the second case, the same light is filtered with a locus filter with a negative temperature ($LFT = -12000$) which makes it cooler. In both cases, the new lights are, as they must be, on the Wien-Planckian locus.

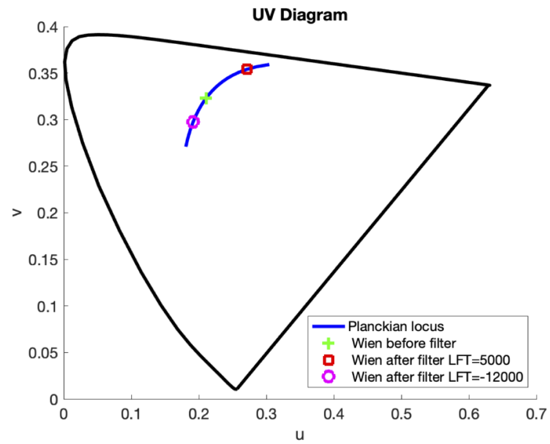

Fig. 4. Color temperature shifts on the Wien-Planckian Locus in the u, v chromaticity diagram after applying two different locus filters one with a positive LFT and the other with a negative one.

4.2. Filtering real lights using a locus filter

After seeing how the locus filter behaves when applied to Wien-Planckian lights, let us now investigate its behavior with respect to real lights. To do that, we will quantify the change of correlated color temperature (CCT) using a locus filter.

In our experimental investigation, we use two sets of lights: the 102 illuminants compiled by Barnard *et al.* [26] which includes many artificial lights, and the set of 99 measured daylights in Granada, Spain [27]. We will refer to these sets as Barnard set and Granada set respectively.

One hypothesis, to be validated in our study, is that the change of CCT after applying a locus Filter is consistent with the change of color temperature when the same filter is applied to a Wien-Planckian illuminant whose temperature is equal to that of the real light. In order to confirm this hypothesis, for each real light, we first obtain an *Equivalent Planckian* light: the Planckian light whose color temperature is equal to the CCT of that light using Eq. (5). Then, a locus filter is applied to these two lights - real and Planckian counterpart - resulting in a filtered real light and a filtered Planckian one. Lastly, the color temperature of the filtered Planckian illuminant is compared to the CCT of the filtered real one to obtain a temperature error.

Here, it is prudent to refer the reader to our earlier discussion in section 3.2. There we considered the fact that color temperature for Planckian lights and Wien-Planckians are different. While they induce almost the same spectra when color temperature $T < 4000K$, there is a small divergence for larger color temperatures. However, this difference is quite small even for very high color temperatures, see Fig. 3. In our experiments we are designing a locus filter that shifts Wien-Planckian Lights by a calculable temperature. The corresponding shift for real Planckians and for real lights is almost the same and in our experiments almost always within 1% of that predicted for Wien-Planckians. Thus, in interpreting the figures in this section and the next three sections the reader should be aware of this inaccuracy. This said, it will be clear that the predicted shifts for a locus filter operating on a Wien-type Planckian broadly hold for the CCTs of real lights too. Even though the CCT is calculated according to the Planckian locus as opposed to the Wien-type Planckian locus.

In detail, suppose a real light, $L(\lambda)$ has a correlated color temperature T_1^c - we use the superscript ^{c} to denote a correlated color temperature - and its Wien-Planckian equivalent is denoted, $E(\lambda, T)$ (see Eq. (5)), where $T = T^c$. Now for a given LFT T_f, the corresponding locus filter is denoted $Tr(\lambda, T_f, k)$, see Eq. (6). By filtering the real light with the locus filter, $L(\lambda)Tr(\lambda, T_f, k)$, we obtain a second light and we calculate its CCT, T_2^c Similarly, we can filter our Wien-Planckian with the same filter, $E(\lambda, T_1)Tr(\lambda, T_f, k)$, and we can use Eq. (9) to calculate the actual temperature T_2 of the filtered Wien-Planckian. Ideally, we would like the change in CCT to be close to the change in corresponding temperature of the Wien-Planckian and its filtered counterpart i.e. $T_2 \approx T_2^c$, but they will be different and we are interested in how much they differ,

We use two metrics to measure difference in temperatures. First, the absolute temperature error, is defined as $|T_2^c - T_2|$, which reflects the absolute difference in color temperature of filtered real lights compared to filtered Wien-Planckian counterparts; and the relative temperature error, equal to $\frac{|T_2^c - T_2|}{T_2}$, indicating the color temperature difference relatively to the original color temperature.

As a case study, we use a locus filter with an LFT of 5000 . In the first two rows of Table 2 we show the average shift of temperature between filtered and unfiltered lights, and the mean absolute temperature error between the real light case and the Wien-Planckian ones for the Barnard and the Granada sets. One can observe that the mean absolute error is small compared to the actual mean temperature shift. For both sets of lights, the average CCT shift is more than 3000K, while the absolute CCT error is 66K for Granada set and 135K for the Barnard set.

Table 2. The absolute and relative temperature errors for filtered real lights compared to filtered Wien-Planckian counterparts (both filtered with a locus filter with LFT=5000).

	Barnard et al [26]	Hernandez-Andres et al. [27]
Mean temperature shift	3270K	3560K
Mean absolute temperature error	135K	66K
Mean relative temperature error	0.056	0.023
Median relative temperature error	0.035	0.008
Max relative temperature error	0.237	0.192
95th percentile relative temperature error	0.204	0.126

Table 2 also reports the mean of the relative temperature error for both sets in the third row. This mean % error is small (respectively 5.6% and 2.3% for the Barnard and Granada Data sets. The median value is also reported in the forth row, showing smaller values than the mean one (respectively 3.5% and 0.8% for the Barnard and Granada Data sets). Two more relative temperature metrics are added to have more statistical understanding of the temperature error: the max and the 95th percentile relative temperature errors.

Perhaps unsurprisingly, the Daylight spectral set has smaller temperature errors (as Daylights themselves are often expressed by their color temperature and the Daylight locus is close to and runs parallel to the Wien-Planckian locus). While the temperature errors are larger for the Barnard set they are also in general fairly low, even at the 95% percentile level the error is only 20%. Broadly, It can be concluded that the change in CCT of real lights mirrors the change in Wien-Planckian temperature.

Finally, we remark that although our case study considers a locus filter with an LFT of 5000, changing the LFT does not change the trend of our results. As per our case study, we have found that, the amount the CCTs of a light (Daylight or artificial) shifts when filtered (by a locus filter) is well predicted by the corresponding shift of the corresponding equivalent Wien-Planckian.

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4.3. Mired temperatures

We repeat our experiment for Mired units and report results in Table 3 (see section for a discussion). In the first row we report the average temperature shift between filtered and unfiltered lights (averaged over our two sets of lights). In Mired units the average shift is 179 and 208 for the Barnard and Granada sets respectively (visually, significant, divide by 5.5). The Table also records the mean, median, max and 95th percentile Mired absolute errors (we compare the temperatures as for the first experiment (but now compute the absolute difference between the CCT of the filtered light and the actual Wien-Planckian shift). It is noticeable that Granada set has significantly better Mired errors compared to Barnard set. Indeed, the mean and the median are sufficiently low as to be, on average, almost visually insignificant.

Table 3. The reciprocal temperature errors for filtered real lights compared to filtered Wien-Planckian counterparts (both filtered with a locus filter with *LFT* = 5000).

	Barnard et al [26]	Hernandez-Andres et al. [27]
Mean mired shift	179	208
Mean mired error	21.5	8.9
Median mired error	11.9	3.1
Max mired error	87.4	79.6
95th % mired error	81.5	46.1

4.4. Delta uv error

Here, we wish to investigate the effect of the locus filter in more detail. As well as predicting a CCT shift, intuitively, we'd like lights to shift along loci. In order to verify that filtered real light are at least as close to the Planckian locus as real lights, we calculate the Duv distance, which is the Euclidean distance between the u, v coordinates of the light and the u, v coordinates of the closest point on the Planckian locus (i.e. the Planckian light that has a color temperature equivalent to the CCT of the light).

Table 4 shows the average Duv of real lights of both light sets compared to the average Duv of these same lights but after being filtered with a locus filter whose LFT is equal to 5000. We can observe that the filtered lights are slightly closer to the Planckian locus. Of course these numbers are hard to interpret (we are looking as raw chromaticity distances in the uv chromaticity space). To understand the visual significance of these numbers suppose we multiply the Duv by 650 (i.e. we use the CIE LUV formulae to visualize the chromaticities when lightness $L^* = 50$, see [24]) then we can say something about the visual perceptibilty of the difference. For reference $650 * 0.002 = 1.3$ which means - according to our Delta E visualisation, that the filtered and unfiltered lights are, on average, the same distance from the Planckian Locus.

Table 4. The average Duv of real lights and filtered real lights after applying a locus filter with LFT=5000.

	Barnard et al [26]	Hernandez-Andres et al. [27]
Mean Duv	0.005	0.002
Mean filtered Duv	0.004	0.001

Figure 5, shows an example of the temperature shift in the u, v chromaticity diagram after applying a locus filter of a filter temperature of 5000 on a light with a CCT of 5450K and on its equivalent Wien-Planckian light.

One can observe that the shifts are consistent between real and Wien approximated lights and that the filtered real stays close to the Planckian locus.

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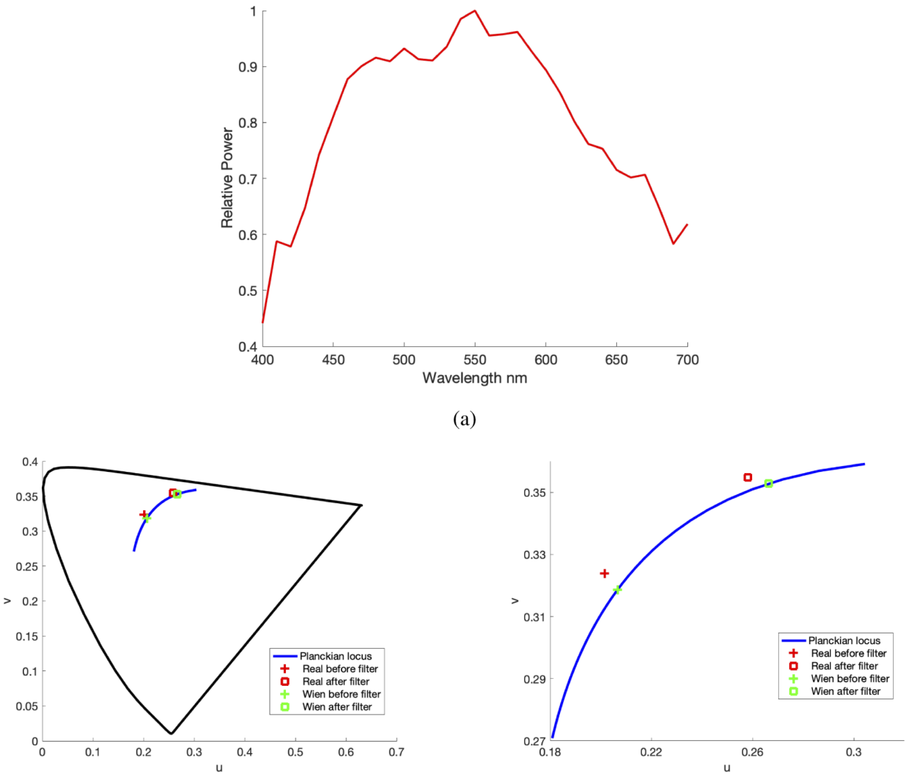

(a)

(b) (c)

Fig. 5. The change in color temperature between a real light whose temperature is 5450K, and its Wien-Planckian equivalent applying a locus filter with an LFT of 5000. (a) Spectral power distribution of that real light. (b) Change of temperature in u,v chromaticity diagram. (c) Zooming in.

4.5. CRI error

Table 5, records the average Color Rendering Indices of the CRIs - see the Background section for a discussion - of both Barnard and Granada lights being unfiltered or filtered with locus filters with different LFTs. One can observe that CRIs of filtered lights are approximately equal to the ones of unfiltered lights, which allows us to conclude that locus filters do not negatively (or positively) impact the color rendering indices of real lights. The filtered lights, on average, have the same CRI as their unfiltered counterparts.

Table 5. Comparison of average CRIs between unfiltered and filtered real lights using different locus filters .

		locus filter temperature			
Light set	unfiltered	5000	7000	9000	11000
Barnard et al [26]	89.96	89.50	89.52	89.61	89.67
Hernandez-Andres et al. [27]	95.70	95.08	95.204	95.29	95.37

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5. Discussion

Returning to Table 3, we reported that the mean Mired error between the temperature shift predicted by a locus filter and the actual observed CCT shift is larger than 5.5 Mired units, so perceptually significant. In addition, we note that the error was higher for Barnard set which includes artificial lights as opposed to the Granada set of Daylights. In this section, we test the hypothesis that the larger the difference between a light and its Wien-Planckian equivalent (the one whose color temperature is equal to the real light CCT) then the observed color shift when a locus filter is applied will have a higher Mired error (between the actual CCT for the filtered light) and the predicted Wien-Planckian temperature shift).

In order to validate this hypothesis, we take the 10 lights from the Barnard set that have 5 highest and 5 lowest Mired errors (according to our Experiment presented in section 4.3). We call these sets LOW and HIGH. For each of these lights we calculate the corresponding Wien-Planckian (i.e. the Wien-Planckian with the same CCT). Given a pair of a real and Wien-Planckian equivalent we calculate the angular error between the spectra and the CRI for the real Light. Given a real light $L(\lambda)$ with a CCT of T^c then the matching equivalent Wien-Planckian is denoted $E(\lambda, T^c)$. In reality, both lights are represented by vectors of measurements at a discrete number of sample points across the visible spectrum, which we denote $\underline{L}$ and $\underline{E}$. With respect to this discrete representation, the angular error is written as

$$\text{AngularError}(\underline{L}, \underline{E}) = acos(\frac{\underline{L}.\underline{E})}{||\underline{L}|| * ||\underline{E}||}) \tag{20}$$

where '.' denotes the vector dot-product, $||.||$ is the vector magnitude and *acos* is the inverse cosine. The angular error, by construction, is independent of the magnitude of the vectors and compares the shape of the underlying spectra.

In Fig. 6, the left and right columns show results for the HIGH and LOW sets (those whose CCT shifts are not well predicted by the locus filter filtering a Wien-Planckian light and those that are well described). In top left, (6(a)), we plot the 5 highest errors against the spectral angular error. Here we see a good correlation between high Mired error and a large angular error. Conversely when the Mired Error is low (6(b)) the angular error is low. Or, in summary, the more the spectral shape of the light corresponds to a Wien-Planckian, the more (and this makes intuitive sense) it behaves Like a Wien-Planckian when filtered by a locus filter. In panels (6(c)) and (6(d)) we respectively see that the HIGH and LOW mired sets have, respectively, low and high CRIs.

Even for this experiment we can see that the relationship between the Mired error and either the angular error or the CI is not, respectively, monotonically increasing or decreasing. Regarding panel (6(a)), we see that the recorded Mired error of 82.9 results in an angular error of about 35 degrees. In contrast the light with the smaller Mired error of 82.6 has a higher angular error. Regarding the CRI, we see in panel (6(c)) a similar trend. Our hypothesis predicts that as the mired error decreases so the CRI should increase. But we do not see a monotonically decreasing curve.

However, over all the lights, the trend of the data adheres to our hypothesis. A large mired error correlates with a large angular error and a smaller CRI. Conversely, for small Mired errors we find the angular error becomes small and the CRI increases.

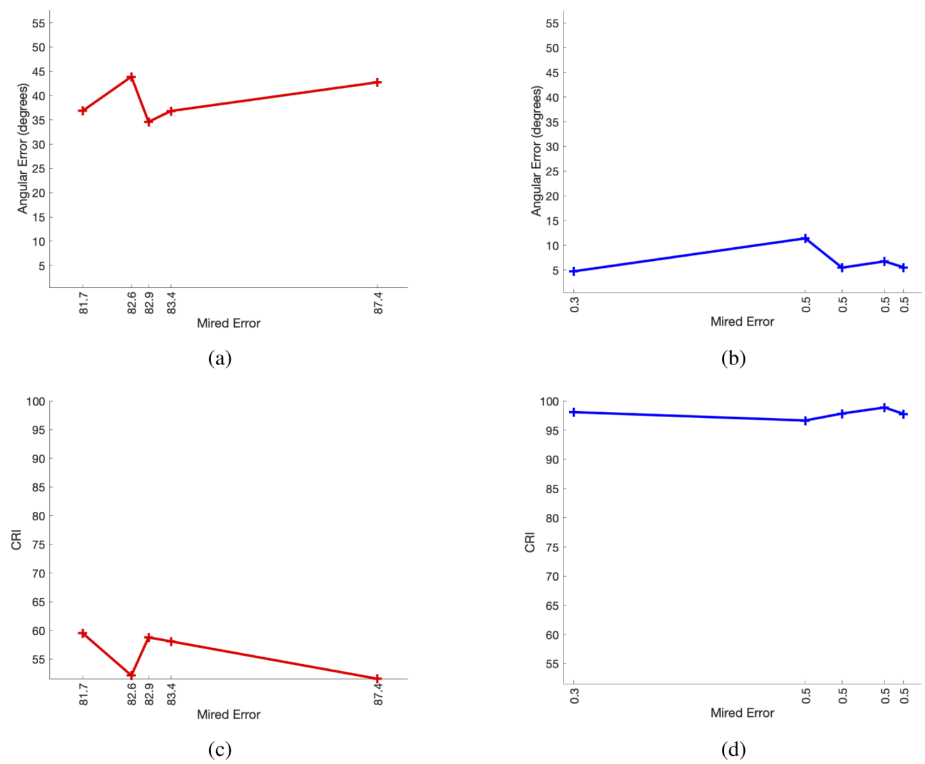

Fig. 6. The relation between Mired error calculated between real and Wien-Planckian case and the properties of the real light in terms of its CRI and the angular error with its Planckian counterpart. (a) Angular errors of 5 the lights with the highest Mired errors. (b) Angular errors of 5 the lights with the lowest Mired errors. (c) CRIs of the the 5 lights with the highest Mired errors. (d) CRIs of the the 5 lights with the lowest Mired errors.

6. Conclusion

The locus filters, designed in this paper, have the - unique over all and any other formulations of transmissive filter - property that they map all Wien-Planckian lights (Planckian lights described using Wien's approximation [2]) from one temperature to another. Each locus filter is defined by a single parameter that we call the locus filter-temperature, or LFT. Experiments demonstrated that the locus filter shifts illuminant toward a warmer or a cooler color but the magnitude of this shift is related to the illuminant color temperature and the filter LFT.

On average, the correlated color temperature, CCT, of real lights were also shown to shift analogously to the corresponding Wien-Planckian lights. Though, there are some lights that shift less like Wien-Planckians (and so how a locus filter filters these lights is harder to predict). However, when the predicted CCTs have higher errors, these lights, empirically, are much less smooth than Wien-Planckian counterparts and also have a low color rendering index, CRI (they score poorly in terms of how colors are rendered). Conversely, real lights that filter like Wien-Planckians tend to have high CRIs and are spectrally similar in shape to Wien-Planckian lights.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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References

1. H. Gage, "Color filters for altering color temperature. pyrometer absorption and daylite glasses," J. Opt. Soc. Am. **23**(2), 46–54 (1933).
2. W. Wien, "Xxx. on the division of energy in the emission-spectrum of a black body," The London, Edinburgh, Dublin Philos. Mag. J. Sci. **43**(262), 214–220 (1897).
3. R. Davis, "A correlated color temperature for illuminants," Bur. Stand. J. Res. **7**(4), 659 (1931).
4. Y. Ohno, "Practical use and calculation of cct and duv," Leukos **10**(1), 47–55 (2014).
5. D. Nickerson, "Light sources and color rendering," J. Opt. Soc. Am. **50**(1), 57–69 (1960).
6. C. McCamy, "A nomograph for selecting light balancing filters," Photogr. Sci. Eng. **3**, 302–304 (1959).
7. M. Brill, "How our eye's lens yellows with age," Inter-Society Color Council News **390**, 11–12 (2001).
8. K. Weaver, "The calculation of filters for color temperature conversion," J. Opt. Soc. Am. **26**(9), 339–342 (1936).
9. R. S. Estey, "The selection of color temperature altering filters," J. Opt. Soc. Am. **26**(7), 293–297 (1936).
10. D. B. Judd, "Sensibility to color-temperature change as a function of temperature," J. Opt. Soc. Am. **23**(1), 7–14 (1933).
11. S. Tominaga and B. A. Wandell, "Natural scene-illuminant estimation using the sensor correlation," Proc. IEEE **90**(1), 42–56 (2002).
12. C. S. McCamy, "Correlated color temperature as an explicit function of chromaticity coordinates," Color Res. Appl. **17**(2), 142–144 (1992).
13. J. Hernandez-Andres, R. L. Lee, and J. Romero, "Calculating correlated color temperatures across the entire gamut of daylight and skylight chromaticities," Appl. Opt. **38**(27), 5703–5709 (1999).
14. C. Li, G. Cui, M. Melgosa, X. Ruan, Y. Zhang, L. Ma, K. Xiao, and M. R. Luo, "Accurate method for computing correlated color temperature," Opt. Express **24**(13), 14066–14078 (2016).
15. C. I. de l'Eclairage, "Method of measuring and specifying colour rendering properties of light sources: Technical report: Cie 13.3-1995," (CIE, 1995).
16. G. D. Finlayson, Y. Zhu, and H. Gong, "Using a simple colour pre-filter to make cameras more colorimetric," in *Color and Imaging Conference*, vol. 2018 (Society for Imaging Science and Technology, 2018), pp. 182–186.
17. G. D. Finlayson and Y. Zhu, "Finding a colour filter to make a camera colorimetric by optimisation," in *International Workshop on Computational Color Imaging*, (Springer, 2019), pp. 53–62.
18. G. D. Finlayson and Y. Zhu, "An improved optimization method for finding a color filter to make a camera more colorimetric," Electron. Imaging **2020**(15), 163 (2020).
19. G. D. Finlayson, S. D. Hordley, and P. Morovic, "Colour constancy using the chromagenic constraint," in *2005 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR'05)*, vol. 1 (IEEE, 2005), pp. 1079–1086.
20. G. D. Finlayson, S. D. Hordley, and P. Morovic, "Chromagenic colour constancy," in *10th Congress of the International Colour Association*, vol. 2 (2005), p. 3.
21. C. Fredembach and G. Finlayson, "Bright chromagenic algorithm for illuminant estimation," J. Imaging Sci. Technol. **52**(4), 40906 (2008).
22. J. Y. Hardeberg, F. J. Schmitt, and H. Brettel, "Multispectral image capture using a tunable filter," in *Color Imaging: Device-Independent Color, Color Hardcopy, and Graphic Arts V*, vol. 3963 (International Society for Optics and Photonics, 1999), pp. 77–88.
23. J. Y. Hardeberg, "Filter selection for multispectral color image acquisition," J. Imaging Sci. Technol. **48**, 105–110 (2004).
24. G. Wyszecki and W. S. Stiles, *Color science* (John Wiley & Sons, Inc., 1982), 2nd ed.
25. B. J. Lindbloom, "Accurate color reproduction for computer graphics applications," in *Proceedings of the 16th Annual Conference on Computer Graphics and Interactive Techniques*, (Association for Computing Machinery, New York, NY, USA, 1989), SIGGRAPH '89, p. 117–126.
26. K. Barnard, V. Cardei, and B. Funt, "A comparison of computational color constancy algorithms. i: Methodology and experiments with synthesized data," IEEE Trans. on Image Process. **11**(9), 972–984 (2002).
27. J. Hernández-Andrés, J. Romero, J. L. Nieves, and R. L. Lee, "Color and spectral analysis of daylight in southern europe," J. Opt. Soc. Am. A **18**(6), 1325–1335 (2001).