Imperial-TP-AT-2021-05

On type 0 string theory in solvable RR backgrounds

Torben Skrzypek[a,1] and Arkady A. Tseytlin[a,2]

[a] *Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*

Abstract

Motivated by a possibility of solving non-supersymmetric type 0 string theory in $AdS_5 \times S^5$ background using integrability, we revisit the construction of type 0 string spectrum in some solvable examples of backgrounds with RR fluxes that are common to type IIB and type 0B theories. The presence of RR fluxes requires the use of a Green-Schwarz description for type 0 string theory. Like in flat space, the spectrum of type 0 theory can be derived from the type II theory spectrum by a $(-1)^F$ orbifolding, i.e. combining the untwisted sector where GS fermions are periodic with the twisted sector where GS fermions are antiperiodic (and projecting out all spacetime fermionic states). This construction of the type 0 spectrum may also be implemented using a Melvin background that allows to continuously interpolate between the type II and type 0 theories. As an illustration, we discuss the type 0B spectrum in the pp-wave background which is the Penrose limit of $AdS_5 \times S^5$ with RR 5-form flux and also in the pp-wave background which is the Penrose limit of $AdS_3 \times S^3 \times \mathrm{T}^4$ supported by mixed RR and NSNS 3-form fluxes. We show that increasing the strength of the RR flux increases the value of the effective normal ordering constant (which determines the mass of the type 0 tachyon in flat space) and thus effectively decreases the momentum-space domain of instability of the ground state. We also comment on the semiclassical sector of states of type 0B theory in $AdS_5 \times S^5$.

[1] t.skrzypek20@imperial.ac.uk

[2] Also at the Institute of Theoretical and Mathematical Physics (ITMP) of Moscow U. and Lebedev Institute. tseytlin@imperial.ac.uk

Contents

1 **Introduction** 2

2 **From type II to type 0 theory via Melvin twist** 3

3 **Type 0B string in pp-wave background with RR 5-form flux** 6

4 **Type 0B string in pp-wave background with mixed NSNS and RR 3-form fluxes** 10

5 **Remarks on the $AdS_5 \times S^5$ case** 14

A **Type 0 string in flat space** 16

B **Melvin twist on pp-wave background** 18

C **Type 0B effective action expanded near pp-wave background** 20

1 Introduction

It was suggested some time ago [1, 2, 3] that type 0 string theory [4, 5, 6] may provide an example of planar AdS/CFT duality in a non-supersymmetric setup. The closed type 0B theory admits the same classical $AdS_5 \times S^5$ solution with RR 5-form flux as the type IIB theory but a major problem is the presence of the tachyon T (with flat-space mass $m_0^2 = -\frac{2}{\alpha'}$). The coupling of T to the RR field strength [2] implies that the mass of the tachyon may be shifted by the presence of RR flux and thus the tachyon may disappear for some small enough critical value of the effective string tension $\sqrt{\lambda} \equiv \frac{R^2_{\rm AdS}}{\alpha'}$ [3, 7].[1]

Testing this conjecture requires an understanding of the spectrum of type 0 string theory in $AdS_5 \times S^5$ background beyond the large tension $\sqrt{\lambda} \gg 1$ limit. In the case of type IIB string theory in $AdS_5 \times S^5$ at genus zero (dual to planar $\mathcal{N} = 4$ SYM) the spectrum of string states is at least in principle computable using integrability (see, e.g., [16, 17, 18, 19]). Since the bosonic part of the $AdS_5 \times S^5$ string worldsheet action (shared by type 0 and type II theories) is integrable, one may conjecture that integrability may also apply to the type 0 case. Let us note that the spectral problem for integrable superstrings on (non-supersymmetric) orbifolds and TsT transformations of $AdS_5 \times S^5$ was addressed in [20, 17, 21, 22] but the case of type 0 theory remains to be treated explicitly.

As $AdS_5 \times S^5$ is supported by the RR flux, this requires formulating the type 0 string action in the Green-Schwarz (GS) approach. In flat space the type 0 string has a straightforward description in the NSR formalism (involving a diagonal GSO projection which excludes spacetime

[1] From the weak-coupling gauge theory side the presence of the closed string tachyon may be seen as the appearance of an imaginary part of the anomalous dimension of the dual operator at some large enough critical value of the 't Hooft coupling λ [7]. The suggestion of duality [3] between type 0 string theory in $AdS_5 \times S^5$ and non-supersymmetric gauge theory on self-dual type 0 D3 branes [8] (which is a $(-1)^F$-type orbifold of $\mathcal{N} = 4$ SYM [3, 9]) has of course several known caveats (even in the strict planar limit). One may view this type 0 example as being analogous to non-supersymmetric $AdS_5 \times S^5/\Gamma$ orbifolds of type IIB theory [10]. In all of these cases there is also a problem of generating new double-trace terms in the action with couplings having complex values at zeroes of the corresponding beta-functions [11, 12, 13, 14, 15]. These may be interpreted in terms of tachyons appearing at small enough gauge coupling or small enough effective string tension. Here we shall focus only on the type 0 closed string tachyon present at large string tension and the dependence of its mass on the strength of the RR background.

fermions) [4, 5, 23]. Using light-cone gauge it can also be described in the GS formalism as a $(-1)^F$ orbifold of type II string theory, i.e. as a combination of the two sectors – with periodic and antiperiodic GS fermions [4]. How to generalise this construction to the case of non-trivial RR backgrounds like $AdS_5 \times S^5$ without applying any compactification and limits may first appear to be non-trivial.

Motivated by the example of the Melvin background that allows to interpolate between the type II and type 0 theories [24, 25, 26] we shall argue that starting with the type II superstring in a consistent RR background one can find the spectrum of the type 0 string in this background by the same orbifolding prescription as in flat space – as a combination of the untwisted sector (described by the type II GS string action with periodic worldsheet fermions but all spacetime fermions projected out from the spectrum) and the twisted sector (described by the type II GS action with antiperiodic worldsheet fermions).

Our aim below will be to discuss a few simple examples of solving type 0 string theory in pp-wave backgrounds with RR fluxes using the GS formulation where the connection to the corresponding solution of type II theory is straightforward. We shall demonstrate how the orbifolding construction works and draw some lessons that may be useful in the $AdS_5 \times S^5$ case. From the exact expression of the type 0 string spectrum available in these cases we shall see that the presence of the RR fluxes indeed increases the value of the effective tachyon mass-squared compared to its negative flat-space value.

We shall start in section 2 with reviewing the solution of type II string theory in Melvin background and explain the relation to the corresponding spectrum of type 0 string theory.

In section 3 we shall use the known solution [27, 28, 29] of type IIB string theory in the pp-wave background which is the Penrose limit of $AdS_5 \times S^5$ and the above orbifolding prescription to construct the corresponding type 0 string spectrum [30] and the modular-invariant partition function [31]. We shall find the explicit dependence of the tachyon mass on the strength of the RR field and show that it approaches zero in the limit of infinite flux.

In section 4 we shall perform a similar analysis in the case of the pp-wave background which is the Penrose limit of $AdS_3 \times S^3 \times \mathrm{T}^4$ supported by a combination of RR and NSNS 3-form fluxes (with the solution of the corresponding type IIB theory found in [28, 32]). Here we will see again that the type 0 tachyon mass-squared is shifted up by the RR flux.

Some comments on 1-loop corrections to the energy of semiclassical states in the type 0B spectrum in $AdS_5 \times S^5$ will be made in section 5.

In Appendix A we shall review the construction of the type 0 string spectrum in flat space. In Appendix B we shall consider type II string theory in pp-wave background with an extra Melvin twist in a plane and recover from its solution the pp-wave type 0 string spectrum by taking a special limit.[2] In Appendix C we shall demonstrate how to reproduce the lower-level part of the type 0 spectrum in pp-wave background discussed in section 3 by expanding the type 0B low-energy effective action to quadratic order in fluctuations.

2 From type II to type 0 theory via Melvin twist

The construction of type 0 string theory in flat space as a $(-1)^F$ orbifold of type II theory in GS description [4] reviewed in Appendix A suggests that the same recipe may apply also in general curved backgrounds that are common to type II and type 0 theories. The full type 0 spectrum will be a sum of untwisted and twisted sectors. The untwisted sector is obtained from the corresponding type II spectrum by projecting out all spacetime fermion states. The

[2] One may also consider starting directly with type II theory in $AdS_5 \times S^5$ (rather than its pp-wave limit) and add a Melvin twist (cf. [33, 34] for related constructions) in order to interpolate to type 0 theory. However, solving such an interpolating string theory (even if it may still be integrable) will be non-trivial.

twisted sector should be found by starting again with the type II GS action and now taking fermions to be antiperiodic on the cylinder.[3]

To further clarify this relation between the type II and type 0 theories, here we shall discuss the type II solution in Melvin background that allows to continuously interpolate between the type II and type 0 spectra [24, 26].[4] The flux tube or KK Melvin background is represented by the locally flat 10d metric[5]

$$\mathrm{d}s^2 = \mathrm{d}s^2_{1,6} + \mathrm{d}y^2 + \mathrm{d}\rho^2 + \rho^2(\mathrm{d}\varphi + q\,\mathrm{d}y)^2 \ , \tag{2.1}$$

where y is compactified on a circle of radius R and q is an arbitrary parameter. Moving around the y-circle induces a rotation of $2\pi Rq$ in the (ρ, φ) plane. When

$$\xi = qR \tag{2.2}$$

is integer valued, this rotation does not affect bosonic fields but changes the sign of the fermions and thus supersymmetry is broken unless ξ is even; in the latter case this background is topologically trivial and the resulting superstring spectrum is the same as in flat space.

Fixing the light-cone gauge, the corresponding GS action may be written as [24]

$$\mathcal{S} = \frac{1}{\pi\alpha'}\int \mathrm{d}^2\sigma \ \ (G_{\mu\nu}\partial_+ x^\mu \partial_- x^\nu + iS_R\mathcal{D}_+ S_R + iS_L\mathcal{D}_- S_L) \ , \tag{2.3}$$

where $\mathcal{D}_\pm = \partial_\pm + \frac{1}{4}\omega_\mu^{mn}\Gamma_{mn}\partial_\pm x^\mu$ with $\omega^{12} = -\omega^{21} = -q\mathrm{d}y$. Using $x = x_1 + ix_2 = \rho e^{i\varphi}$ and setting

$$x = e^{-iqy}X \ , \qquad\qquad S_{R,L} = e^{\mp\frac{iq}{2}y}\Lambda_{R,L} \tag{2.4}$$

one finds that X and $\Lambda_{R,L}$ satisfy free equations of motion. Since y is compact, the string can wind w times around it and then the prefactors in (2.4) pick up a phase $\exp(-2\pi i\gamma)$ or $\exp(-\pi i\gamma)$ under a shift $\sigma \to \sigma + 2\pi$, where

$$\gamma = wqR = w\xi \, . \tag{2.5}$$

Since the fields x and $S_{R,L}$ should be periodic in σ, these phases are to be compensated by twisting periodicities of X and $\Lambda_{R,L}$. This results in frequencies $(n \pm \gamma)$ for bosonic modes and $(n \pm \frac{1}{2}\gamma)$ for the fermionic ones. Our notation follow [43, 24], i.e. we label the creation and annihilation operators $\alpha^\dagger_{n\pm}$, $\alpha_{n\pm}$ for bosons and $\beta^\dagger_{n\pm}$, $\beta_{n\pm}$ for fermions (with canonical normalisation $[\alpha_{n-}, \alpha^\dagger_{n-}] = \delta_{nm}$, etc.). The mass operator may be written as [24]

$$\alpha' M^2 = \frac{\alpha'}{R^2}\hat{\mathrm{m}}^2 + \frac{R^2}{\alpha'}w^2 + 2\big(\hat{N}_R + \hat{N}_L + A\big) \ , \qquad\qquad \hat{\mathrm{m}} \equiv \mathrm{m} - \xi(\hat{J}_b + \tfrac{1}{2}\hat{J}_f) \ , \tag{2.6}$$

[3]As is well known, the GS action in generic type II background is κ-symmetric if the background solves the corresponding string low-energy (or supergravity) equations. The same should be true also in the type 0 case provided the GS string is coupled only to the massless fields that are common to type II and type 0 theories (i.e. the background values of the type 0 tachyon field and the second copy of the RR fields should be set to zero).

[4]Here we will consider only perturbative 10d string theory setting. Relations between type II and type 0 theories were also discussed from 11d M-theory perspective in [35, 36, 37]. In particular, ref. [37] discussed the spectrum and the tachyon mass in the interpolating background.

[5]The Kaluza-Klein generalization of the Melvin solution was introduced in [38, 39] and its interpretation as flat 5d space with a specific identification was given in [40, 41]. The embedding of the Melvin solution into string theory was done in [42, 43, 24].

where m is the integer momentum in y direction, which is shifted in $\hat{\mathrm{m}}$ by the angular momenta of bosonic $\hat{J}_b$ and fermionic modes $\hat{J}_f$. A is the normal ordering constant. Explicitly, we have

$$
\begin{aligned}
\hat{J}_b &= \tilde{\alpha}_0^\dagger\tilde{\alpha}_0 - \alpha_0^\dagger\alpha_0 + \sum_{n=1}^{\infty}\left(\alpha_{n+}^\dagger\alpha_{n+} - \alpha_{n-}^\dagger\alpha_{n-} + \tilde{\alpha}_{n+}^\dagger\tilde{\alpha}_{n+} - \tilde{\alpha}_{n-}^\dagger\tilde{\alpha}_{n-}\right) , \\
\hat{J}_f &= \tilde{\beta}_0^\dagger\tilde{\beta}_0 - \beta_0^\dagger\beta_0 + \sum_{n=1}^{\infty}\left(\beta_{n+}^\dagger\beta_{n+} - \beta_{n-}^\dagger\beta_{n-} + \tilde{\beta}_{n+}^\dagger\tilde{\beta}_{n+} - \tilde{\beta}_{n-}^\dagger\tilde{\beta}_{n-}\right) , \qquad (2.7)\\
\hat{N}_R &= \sum_{n=1}^{\infty}(n-\gamma)\alpha_{n+}^\dagger\alpha_{n+} + \sum_{n=0}^{\infty}(n+\gamma)\alpha_{n-}^\dagger\alpha_{n-} + \sum_{n=1}^{\infty}\left(n-\tfrac{\gamma}{2}\right)\beta_{n+}^\dagger\beta_{n+} + \sum_{n=0}^{\infty}\left(n+\tfrac{\gamma}{2}\right)\beta_{n-}^\dagger\beta_{n-} , \\
\hat{N}_L &= \sum_{n=1}^{\infty}(n-\gamma)\tilde{\alpha}_{n-}^\dagger\tilde{\alpha}_{n-} + \sum_{n=0}^{\infty}(n+\gamma)\tilde{\alpha}_{n+}^\dagger\tilde{\alpha}_{n+} + \sum_{n=1}^{\infty}\left(n-\tfrac{\gamma}{2}\right)\tilde{\beta}_{n-}^\dagger\tilde{\beta}_{n-} + \sum_{n=0}^{\infty}\left(n+\tfrac{\gamma}{2}\right)\tilde{\beta}_{n+}^\dagger\tilde{\beta}_{n+} .
\end{aligned}
$$

Here the tilde denotes left-moving modes and the spinor indices as well as the contributions of 6 flat bosonic dimensions have been suppressed. For $0 \leq \gamma \leq 1$, the normal ordering constant is given in terms of the Hurwitz zeta-function $\zeta(s,a) = \sum_{n=0}^{\infty}(n+a)^{-s}$ as

$$
A(\gamma) = 2\,\zeta(-1,\gamma) - 8\,\zeta(-1,\tfrac{1}{2}\gamma) + 6\,\zeta(-1,1) = -\gamma , \qquad \gamma \in [0,1] . \qquad (2.8)
$$

When $1 \leq \gamma < 2$ the energy contribution of the bosonic mode of frequency $(1-\gamma)$ becomes negative and has to be reordered; this results in $A(\gamma) = -2+\gamma$. At $\gamma = 2$ both bosonic and fermionic modes are to be reshuffled and we can absorb the γ-shift in renaming of all modes, getting back to the flat-space spectrum. This is exactly what happens in the globally trivial case of $\xi = 2$ for any value of w. Extended to all values of γ, $A(\gamma)$ looks like a triangle wave, oscillating between values 0 and -1. Thus for non-trivial cases of $\gamma \notin 2\mathbb{Z}$, we find tachyonic ground states in the winding sectors and broken supersymmetry [25].

Let us now consider the limits $R \to \infty$ and $R \to 0$ while keeping ξ fixed. For $R \to \infty$ the KK momentum $p^y = \mathrm{m}/R$ becomes continuous while the winding modes become infinitely heavy. Thus we end up with the $w = 0$ sector and recover the type II string theory on Minkowski space.

Considering $R \to 0$ and specifying to the $\xi = 1$ case, the periodicity in $\gamma = w\xi$ implies that we have to split the spectrum in two sectors, an "untwisted" one with even γ and a "twisted" one with odd γ. Then taking the limit $R \to 0$, the winding number w becomes effectively continuous (i.e. is replaced by the momentum of the T-dual coordinate $p^{\tilde{y}} = w/\tilde{R}$, $\tilde{R} = \alpha'/R$) and we get just two distinct sectors of states.

Furthermore, in the limit $R \to 0$ the first "KK-momentum" term in (2.6) blows up unless $\hat{\mathrm{m}} = \mathrm{m} - \xi(\hat{J}_b + \frac{1}{2}\hat{J}_f)$ vanishes. As m is integer, for $\xi = 1$ we can find a solution of the condition $\hat{\mathrm{m}} = 0$ only if $\hat{J}_f$ in a given state is even. This projects out all fermionic states from the spectrum, i.e. the only states that have finite masses in the $R \to 0$ limit are the bosonic ones. The untwisted sector thus consists of the bosonic spectrum of the T-dual type II theory. The twisted sector has bosonic states with a tachyonic ground state ($\alpha' m_0^2 = 2A = -2$) and fermionic states corresponding to odd numbers of excitations of the half-integer frequency modes; imposing the level-matching condition the latter are projected out and we end up with the bosonic states only (even before imposing the $\hat{\mathrm{m}} = 0$ constraint).

The resulting spectrum is exactly the same as that of type 0 string theory in flat space. Thus we can obtain the type 0 spectrum from the type II one via Melvin twist and T-duality.[6] This construction can be used to find the type 0 spectrum from the type II one on non-trivial backgrounds that are solutions to both type II and type 0 theories (assuming these backgrounds

[6] Note that this relation between type II and type 0 theories is not "mutual": we cannot obtain the type II spectrum from the type 0 one by reversing this procedure.

admit a translational as well as rotational isometries so that one can compactify one dimension and apply the Melvin twist in a 2-plane). Then the above transformation at $\xi = 1$ will result in the type 0 string theory on the T-dual background. Thus at least for the backgrounds allowing the introduction of a Melvin twist, the type 0 spectrum can be obtained from the type II one by the same orbifolding procedure as in flat space [4].

One immediate application of the above discussion is to derive the type 0 spectrum on the Melvin background. The untwisted sector will be given by the bosonic part of the type II spectrum. In the twisted sector the mass operator is the same as in (2.6) but since we have to take the GS fermions to be antiperiodic the corresponding fermionic number operators here are

$$\hat{N}_{R,f} = \sum_{n=0}^{\infty} \big(n + \tfrac{1}{2} - \tfrac{\gamma}{2}\big)\beta^\dagger_{n+}\beta_{n+} + \sum_{n=0}^{\infty} \big(n + \tfrac{1}{2} + \tfrac{\gamma}{2}\big)\beta^\dagger_{n-}\beta_{n-} \ ,$$
$$\hat{N}_{L,f} = \sum_{n=0}^{\infty} \big(n + \tfrac{1}{2} - \tfrac{\gamma}{2}\big)\tilde{\beta}^\dagger_{n-}\tilde{\beta}_{n-} + \sum_{n=0}^{\infty} \big(n + \tfrac{1}{2} + \tfrac{\gamma}{2}\big)\tilde{\beta}^\dagger_{n+}\tilde{\beta}_{n+} \ , \tag{2.9}$$

and the twisted-sector normal ordering constant turns out to be ($\gamma \in [0,1]$)

$$A(\gamma) = 2\,\zeta(-1,\gamma) - 8\,\zeta(-1, \tfrac{1}{2} + \tfrac{1}{2}\gamma) + 6\,\zeta(-1,1) = -1 + \gamma \ . \tag{2.10}$$

If we take $R \to \infty$ we end up with the type 0 spectrum in flat Minkowski space. $R \to 0$ at $\xi = 0$ leads to the T-dual type 0 theory in flat space. In the limit of $R \to 0$ at $\xi = 1$ (discussed above in the type II case) for each value of the winding number we find one sector of integer frequency fermionic modes ($A = 0$) and one sector of half-integer frequency fermionic modes ($A = -1$), which originate alternatingly from untwisted and twisted sector. The condition that $\hat{\mathrm{m}}$ should vanish to get a finite-mass state from (2.6) can always be satisfied and we again get the flat-space spectrum of the T-dual type 0 theory.

Finally, let us comment on the corresponding torus partition functions. In [43, 26] the partition function of type IIB theory on Melvin background was given as

$$Z_{\rm IIB} = \frac{V_7 R}{(2\pi)^{10}\alpha'^4}\int_{\mathcal{F}_0}\frac{\mathrm{d}^2\tau}{\tau_2^5}\sum_{w,w'\in\mathbb{Z}}\frac{\big|\vartheta_{11}(\frac{1}{2}\xi\chi;\tau)\big|^8}{|\eta(\tau)|^{18}\,|\vartheta_{11}(\xi\chi;\tau)|^2}\exp\Big(-\frac{\pi R^2}{\alpha'\tau_2}\chi\bar{\chi}\Big) \ , \tag{2.11}$$
$$\chi \equiv w' - \tau w \ , \qquad \bar{\chi} \equiv w' - \bar{\tau} w \ . \tag{2.12}$$

$Z_{\rm IIB}$ vanishes at $\xi = 0$ when supersymmetry is restored. The functions $\vartheta_{ab}(z;\tau)$ (see (A.5)) are quasi-periodic with respect to shifts $z \to z+1$ and $z \to z+\tau$ but shifting z by $\frac{1}{2}$ or $\frac{1}{2}\tau$ relates the functions $\{\vartheta_{00}, \vartheta_{01}, \vartheta_{10}, \vartheta_{11}\}$ to each other. As a result, for $\xi = 1$ the numerator in (2.11) splits into four separate contributions while the denominator becomes proportional to $\vartheta_{11}(0;\tau)$. The denominator vanishes, indicating the appearance of new continuous zero modes. Finally, in the $R \to 0$ limit the exponential factor becomes 1 and we recover the flat-space type 0A partition function given in (A.8).

In the case of type 0B string theory in Melvin background we should find the following analog of both (A.8) and (2.11)

$$Z_{\rm 0B} = \frac{V_7 R}{(2\pi)^{10}\alpha'^4}\int_{\mathcal{F}_0}\frac{\mathrm{d}^2\tau}{\tau_2^5}\sum_{w,w'\in\mathbb{Z}}\frac{\mathcal{I}_{0B}}{|\eta(\tau)|^{18}\,|\vartheta_{11}(\xi\chi;\tau)|^2}\exp\Big(-\frac{\pi R^2}{\alpha'\tau_2}\chi\bar{\chi}\Big) \ , \tag{2.13}$$

$$\mathcal{I}_{\rm 0B} = \big|\vartheta_{00}(\tfrac{1}{2}\xi\chi;\tau)\big|^8 + \big|\vartheta_{01}(\tfrac{1}{2}\xi\chi;\tau)\big|^8 + \big|\vartheta_{10}(\tfrac{1}{2}\xi\chi;\tau)\big|^8 + \big|\vartheta_{11}(\tfrac{1}{2}\xi\chi;\tau)\big|^8. \tag{2.14}$$

3 Type 0B string in pp-wave background with RR 5-form flux

A non-trivial example of type 0 string theory in curved space where the use of GS formulation is crucial is the $D = 10$ pp-wave supported by RR 5-form flux. This is the same background as in type IIB theory, i.e. the Penrose limit of $AdS_5 \times S^5$ with 5-form flux [44] ($i = 1, ..., 8$)

$$ds^2 = 2\mathrm{d}u\mathrm{d}v - f^2 x_i^2 \mathrm{d}u^2 + \mathrm{d}x_i^2 \ , \qquad F_{u1234} = F_{u5678} = 4f \ . \tag{3.1}$$

The corresponding type IIB string action and its spectrum were found in [27, 28, 29]. After fixing the light-cone gauge on u, the type IIB GS action is as in flat space (A.9) but now 8 bosons and 8 fermions acquire mass μ proportional to the flux or the curvature scale f [7]

$$u = u_0 + \alpha' p^u \tau \ , \qquad \mu = \alpha' p^u f \ . \tag{3.2}$$

The resulting light-cone Hamiltonian H can be written as [29][8]

$$H = -p^v = \frac{1}{\alpha' p^u} \sum_{n=-\infty}^{\infty} \sqrt{n^2 + \mu^2} \left(\alpha_n^\dagger \alpha_n + \tilde{\alpha}_n^\dagger \tilde{\alpha}_n + \beta_n^\dagger \beta_n + \tilde{\beta}_n^\dagger \tilde{\beta}_n \right) \ , \tag{3.3}$$

where the zero-mode part has harmonic-oscillator type spectrum. The effective mass operator is then given by ($p_v = p^u, \ p^u = \frac{p^y + p^t}{\sqrt{2}}, \ p^v = \frac{p^y - p^t}{\sqrt{2}}, \ E = p^t$)

$$M^2 = E^2 - (p^y)^2 = -2p^u p^v = 2p^u H \ , \tag{3.4}$$

reducing to the one in flat space in the limit of $\mu = 0$. Note that since the eigenvalues of H are expressed in terms of μ which depends on $p^u = \frac{1}{\sqrt{2}}(E + p^y)$ one gets here a non-trivial dispersion relation for the energy E as a function of p^y (see also below).

One way to find the type 0B string spectrum in this background is to compactify $y = \frac{u+v}{\sqrt{2}}$ on a circle, introduce a Melvin twist in some 2-plane of the transverse x-space, find the resulting type IIB spectrum and then apply the same limit ($\xi \equiv qR = 1, \ R \to 0$) as in section 2. This is discussed in Appendix B. A short-cut is to apply the $(-1)^F$ orbifolding procedure, i.e. to combine the bosonic part of the pp-wave spectrum of type IIB theory (untwisted sector) with a similar spectrum found by imposing antiperiodic boundary conditions on the GS fermions (twisted sector). This procedure was used in [30, 31].

To cover both the case of the untwisted and of the twisted sector we may write the corresponding light-cone Hamiltonian as

$$H = \frac{1}{\alpha' p^u} \left(\hat{N}_b + \hat{N}_f + A \right) \ , \tag{3.5}$$

$$\hat{N}_b = \sum_{n=0}^{\infty} \omega_n (\alpha_n^\dagger \alpha_n + \tilde{\alpha}_n^\dagger \tilde{\alpha}_n) + \sum_{n=1}^{\infty} \omega_n (\alpha_{-n}^\dagger \alpha_{-n} + \tilde{\alpha}_{-n}^\dagger \tilde{\alpha}_{-n}) \ , \qquad \omega_n = \sqrt{n^2 + \mu^2}, \tag{3.6}$$

$$\hat{N}_f = \sum_{n=a}^{\infty} \omega_n (\beta_n^\dagger \beta_n + \tilde{\beta}_n^\dagger \tilde{\beta}_n) + \sum_{n=1-a}^{\infty} \omega_n (\beta_{-n}^\dagger \beta_{-n} + \tilde{\beta}_{-n}^\dagger \tilde{\beta}_{-n}) \ , \qquad a = 0, \ \tfrac{1}{2} \ . \tag{3.7}$$

The untwisted sector corresponds to the choice of the lower summation limit $a = 0$ in (3.7) and then the normal ordering constant A vanishes. We are to project out all fermionic states, i.e. to consider only states created by an even number of worldsheet fermion operators.

[7]Here and below $p^u = p_v$ is the component of the string momentum defined with the standard factor of string tension $\frac{1}{2\pi\alpha'}$.

[8]Here we suppress indices on the complex mode operators (8 real modes are described by 4 complex ones and the sum is over both positive and negative n). In [29] the fermionic modes appear multiplying certain combinations of gamma matrices. Here we redefined β_n and $\beta_n^\dagger$ to absorb these. We have also fixed the ambiguity of choosing a Clifford vacuum for the fermionic zero modes.

The twisted sector corresponds to $a = \frac{1}{2}$ and here the normal ordering constant is a non-trivial function of μ [9]

$$A \equiv A_8(\mu) = 8 \times \tfrac{1}{2}\Big(\sum_{n=0,\pm1,\dots} \sqrt{n^2+\mu^2} - \sum_{r=\pm\frac{1}{2},\dots} \sqrt{r^2+\mu^2} \Big) . \tag{3.8}$$

It determines the mass of the ground state in the twisted sector. In the flat-space limit ($\mu = 0$) we get[10]

$$A_8(0) = 8\left[\zeta(-1,0) - \zeta(-1,\tfrac{1}{2})\right] = -1 , \tag{3.9}$$

determining the flat-space value of the type 0 tachyon mass ($m_0^2 = \frac{2}{\alpha'} A(0) = -\frac{2}{\alpha'}$).

Separating the flat-space value and the bosonic $n = 0$ term we can represent (3.8) as

$$A_8 = -1 + 4|\mu| + 8\mathcal{A}(\mu) , \tag{3.10}$$

$$\mathcal{A} \equiv \sum_{n=1}^{\infty} \Big[\sqrt{n^2+\mu^2} - n - \sqrt{\big(n-\tfrac{1}{2}\big)^2 + \mu^2} + \big(n - \tfrac{1}{2}\big) \Big] . \tag{3.11}$$

Expanding the square roots in (3.11) in powers of μ we can write $\mathcal{A}$ as[11]

$$\mathcal{A} = \sum_{n=1}^{\infty}\sum_{k=1}^{\infty} \tfrac{\mu^{2k}(-1)^k\Gamma(2k+1)}{2(1-2k)(\Gamma(k+1))^2} \left[\tfrac{1}{(2n)^{2k-1}} - \tfrac{1}{(2n-1)^{2k-1}}\right] = \sum_{k=1}^{\infty} \tfrac{\mu^{2k}\,(-1)^k\,\Gamma(2k-1)}{\Gamma(k)\Gamma(k+1)}\ \eta_D(2k-1) , \tag{3.12}$$

where we expressed the sum over n in terms of the Dirichlet η-function

$$\eta_D(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = (1 - 2^{1-s})\zeta(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \mathrm{d}x\ \frac{x^{s-1}}{e^x + 1} . \tag{3.13}$$

The sum in (3.12) converges for $\mu < \frac{1}{2}$. Analytic continuation to all μ can be achieved by using the integral representation of η_D thus getting

$$A_8 = -1 + 4|\mu| - 8|\mu| \int_0^{\infty} \mathrm{d}x\ \frac{J_1(2|\mu|x)}{x\,(e^x+1)} , \tag{3.14}$$

where J_1 is the Bessel function of first kind.[12] The resulting function A_8 in (3.10) has the following asymptotics

$$A_8|_{\mu\to 0} = -1 + 4|\mu| - 8\ln(2)\,\mu^2 + 6\,\zeta(3)\,\mu^4 - 15\,\zeta(5)\,\mu^6 + \mathcal{O}\big(\mu^8\big) , \tag{3.15}$$

$$A_8|_{\mu\to\infty} = 0 + \mathcal{O}\big(e^{-|\mu|}\big) , \tag{3.16}$$

and its plot is given in Figure 1. Note that the coefficient $\ln 2$ of μ^2 in (3.15) is the value of $\eta_D(1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$. Also, $A_8|_{\mu\to\infty} \to 0$ has to do with the fact that the difference between the integer and half-integer mode contributions to (3.1) disappears in the large μ limit.

Thus A_8 is asymptotically approaching zero from below, i.e. the ground state is "tachyonic" for all finite values of μ, becoming massless only at $\mu = \infty$.[13] An important conclusion is that increasing the value of the RR flux makes the twisted-sector ground state less tachyonic.

[9] We use the label 8 to indicate that this model has 8 copies of massive excitations corresponding to 8-dimensional transverse space in (3.1).

[10] The sum in (3.8) for $\mu = 0$, i.e. $\frac{1}{2}\sum_{n=0,\pm1,\dots}|n| - \frac{1}{2}\sum_{r=\pm\frac{1}{2},\dots}|r|$ can be computed explicitly as: $\sum_{n=1}^{\infty} n - \sum_{n=1}^{\infty}(n-\frac{1}{2}) \to \sum_{n=1}^{\infty}\left[n\,e^{-\epsilon n} - (n-\frac{1}{2})\,e^{-\epsilon(n-1/2)}\right] = -\frac{1}{2}e^{-\epsilon/2}(1+e^{-\epsilon/2})^{-2} = -\frac{1}{8} + \mathcal{O}(\epsilon^2)$.

[11] In [30] (3.8) was expressed in terms of the Epstein function. Here we will obtain a more convenient integral representation.

[12] We used the following representation for the Bessel function: $J_\alpha(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(k+1)\Gamma(k+1+\alpha)} \left(\frac{x}{2}\right)^{2k+\alpha}$.

[13] In the limit of $\mu \to \infty$ A_8 receives only exponential corrections (as was already mentioned in [30]). The structure of the exponential corrections in the integral in (3.14) is similar to the one discussed in [45] and in [46].

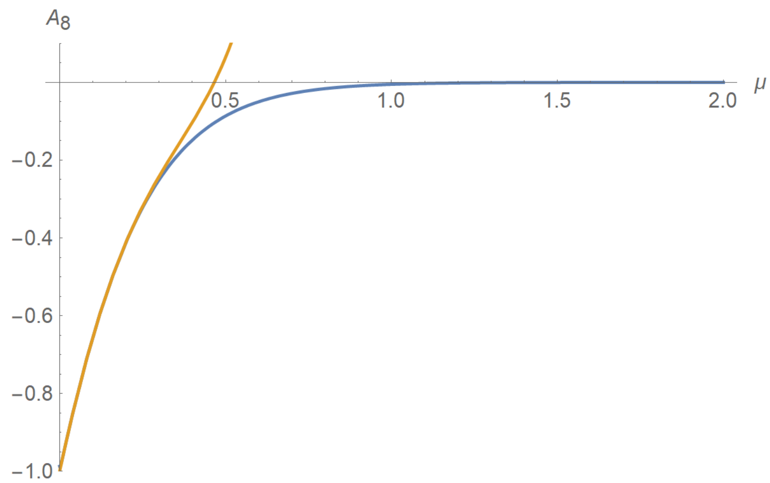

Figure 1: Dependence of the twisted-sector normal ordering constant A_8 on the parameter $\mu = \alpha' p^u f$. The curve approaching zero at large μ is obtained by numerical integration of (3.14), while the upper curve represents the small μ approximation (3.15).

To summarise, the untwisted sector of type 0 theory consists of the bosonic states of the type IIB spectrum. For example, the lowest energy states are given by even excitations of β_0 which are 128 bosonic states with masses 0, 4μ, 8μ, 12μ and 16μ. The twisted sector has a unique tachyonic ground state with the effective mass $m^2 = \frac{2}{\alpha'} A_8(\mu)$. Both sectors contain also towers of massive states which are all bosonic.

Let us now comment on the meaning of the negative mass-squared of a field in the pp-wave background. Since the eigenvalues of H in (3.4) or (3.5) are expressed in terms of μ which depends on $p^u = \frac{1}{\sqrt{2}}(p^y + E)$ one gets a non-trivial "dispersion relation" for the energy $E = E(p^y)$ and one may wonder if the condition for stability may change compared to that in flat space. To see this let us start with a simple massive scalar equation in the pp-wave metric (3.1)

$$(\nabla^2 - m_0^2)T = (2\partial_u\partial_v + f^2 x_i^2 \partial_v^2 + \partial_i^2 - m_0^2)T = 0 \ . \tag{3.17}$$

After the Fourier transformation in u, v this becomes the equation for the wave function of the quantum harmonic oscillator with frequency $2fp_v$. Thus we get the following relation for $E = p^t$ as a function of p^y $\ (p^v = \frac{p^y - p^t}{\sqrt{2}},\ \ p^u = \frac{p^y + p^t}{\sqrt{2}})$

$$-2p^u p^v = m_0^2 + 2fp^u N_0 \ , \qquad\qquad N_0 = \sum_{i=1}^{8}(n_i + \tfrac{1}{2}) \ , \tag{3.18}$$

$$(p^t)^2 - (p^y)^2 = m_0^2 + \sqrt{2}f(p^t + p^y)N_0 \ , \qquad\qquad E = p^t = \tfrac{fN_0}{\sqrt{2}} \pm \sqrt{(p^y + \tfrac{fN_0}{\sqrt{2}})^2 + m_0^2} \ . \tag{3.19}$$

This implies that if m_0^2 is negative there is always a region of real momentum $p^y \sim -\frac{fN_0}{\sqrt{2}}$ for which the energy becomes imaginary, signalling an instability, i.e. the existence of solutions growing indefinitely with time (see also a discussion in [47]).

In the case of the string-theory tachyon in a non-trivial background, its equation receives α' corrections that in the pp-wave case translate into its mass m being itself a function of $\mu = \alpha' p^u f$. For the type 0 spectrum discussed above this implies that

$$m_0^2 = -\tfrac{2}{\alpha'} \quad \to \quad m^2 = \tfrac{2}{\alpha'} A_8(\alpha' p^u f) = m_0^2 + 8p^u f - 16\ln(2)\ \alpha'(p^u f)^2 + \mathcal{O}(\alpha'^2) \ , \tag{3.20}$$

with A_8 given in (3.10),(3.15). From (3.4) and (3.5) we then learn that for the lowest excitation mode we get

$$E^2 - (p^y)^2 = \tfrac{2}{\alpha'} A_8\Big(\tfrac{\alpha'}{\sqrt{2}}(p^y + E)f\Big) \ . \tag{3.21}$$

Note that the α'-independent term $8p^u f$ for (3.20) corresponds to the $2fp^uN_0$ term in (3.18) in the oscillator ground state case of $n_i = 0$, i.e. with $N_0 = 4$.

Eq. (3.21) for $E(p^y)$ should be defined at any point of the real axis of p^y. Since $A_8 \le 0$ (cf. (3.16) and Figure 1) we conclude that for small enough $(p^y)^2$ the value of E is always imaginary, implying an instability. Note that the region of p^y that leads to instability effectively shrinks with increasing f as then A_8 grows towards zero.

The torus partition function corresponding to type 0B string theory in the above pp wave background was found in [31] and can be written as[14]

$$Z_{0\mathrm{B}}(f) = \int \frac{\mathrm{d}^2\tau}{2\tau_2} \int \mathrm{d}p^u \mathrm{d}p^v \; e^{-2\pi\alpha'\tau_2\, p^u p^v} \; \mathcal{I}(\tau; \alpha' p^u f) \; , \tag{3.22}$$

$$\mathcal{I}(\tau;\mu) = \frac{(\mathcal{Z}_{0,0})^4 + (\mathcal{Z}_{1/2,0})^4 + (\mathcal{Z}_{0,1/2})^4 + (\mathcal{Z}_{1/2,1/2})^4}{(\mathcal{Z}_{0,0})^4} \; , \tag{3.23}$$

$$\mathcal{Z}_{a,b}(\tau;\mu) = e^{4\pi\tau_2\Delta_b(\mu)} \prod_{n\in\mathbb{Z}} \left(1 - e^{-2\pi\tau_2|\omega_{n+b}| + 2\pi i\tau_1(n+b) + 2\pi i a}\right) \left(1 - e^{-2\pi\tau_2|\omega_{n-b}| + 2\pi i \tau_1(n-b) - 2\pi i a}\right) ,$$

$$\Delta_b(\mu) = \sum_{k=1}^{\infty} c_k(\mu)\, \cos(2\pi b\, k) \; , \qquad c_k = -\tfrac{1}{2\pi^2} \int_0^\infty \mathrm{d}s \; e^{-k^2 s - \frac{\pi^2\mu^2}{s}} \; . \tag{3.24}$$

Here ω_n was defined in (3.6) and c_k can be expressed in terms of the K_1 Bessel function. don't know this integral rep. – check mathematica $\mathcal{Z}^4_{0,0} + \mathcal{Z}^4_{1/2,0}$ in (3.23) correspond to the contribution of the untwisted sector and $\mathcal{Z}^4_{0,1/2} + \mathcal{Z}^4_{1/2,1/2}$ to that of the twisted sector.

The structure of (3.22) is very similar to that of the flat space partition function (A.8), with $\vartheta_{ab}\bar\vartheta_{ab}$ replaced by $\mathcal{Z}_{\frac{1-b}{2},\frac{1-a}{2}}$ (which indeed reduces to $\vartheta_{ab}\bar\vartheta_{ab}$ in the flat space limit $\mu \to 0$). Note that in the type IIB case the pp-wave partition function has a similar structure but with the integrand being proportional to $\mathcal{Z}^4_{0,0}/\mathcal{Z}^4_{0,0} = 1$, which mirrors the flat space expression (A.10).[15]

Since the partition function (3.22) encodes the type 0 spectrum, one is able to recover from it the value of the effective mass of the ground state. Taking the limit $\tau_2 \to \infty$ the ground state contribution to $\mathcal{Z}_{a,b}$ comes from the exponential $e^{4\pi\tau_2\Delta_b(\mu)}$ and thus the twisted sector contribution gives[16]

$$\frac{\mathcal{Z}^4_{0,1/2} + \mathcal{Z}^4_{1/2,1/2}}{\mathcal{Z}^4_{0,0}} \;\to\; e^{16\pi\tau_2[\Delta_{1/2}(\mu) - \Delta_0(\mu)]} = e^{-2\pi\tau_2 A_8} \; , \tag{3.25}$$

where A_8 is the same as in (3.10).

4 Type 0B string in pp-wave background with mixed NSNS and RR 3-form fluxes

Let us now consider type 0B theory in the pp-wave background which is a Penrose limit of $AdS_3 \times S^3 \times \mathrm{T}^4$ supported by a mixture of NSNS and RR 3-form fluxes. This background is a solution of both type IIB and type 0B theories and thus the type 0B spectrum can be found

[14] Ref. [31] used different notation: the parameter f was called μ. Similar partition functions for type IIB theory in pp-wave background at finite temperature were discussed in [48, 49, 47].

[15] The type IIB pp-wave partition function does not automatically vanish, which is due to the zero modes lifting the flat-space degeneracy of the ground state so that instead of a full supermultiplet only one state remains massless [31]. In the limit $\mu \to 0$ special attention has to be paid to these zero modes and one gets the vanishing flat space result [50]. At the same time, one can also argue that the type IIB pp-wave partition function actually vanishes provided one uses an analytic continuation in the momentum integral in (3.22): integrating over the imaginary p^v sets p^u and thus μ to zero and one recovers the flat-space result $Z = 0$.

[16] In the untwisted sector similar exponentials cancel corresponding to a massless lowest-energy state.

again using the GS description and applying the $(-1)^F$ orbifolding procedure. The explicit form of the metric and fluxes is ($r = 1,2,3,4;\ s = 5,6,7,8$)

$$
\begin{aligned}
\mathrm{d}s^2 &= 2\mathrm{d}u\mathrm{d}v - f^2 x_r^2 \mathrm{d}u^2 + \mathrm{d}x_r^2 + \mathrm{dx}_s^2 \ , \\
H_{u12} &= H_{u34} = -2qf, \qquad\qquad F_{u12} = F_{u34} = -2\sqrt{1-q^2} f \ ,
\end{aligned}
\tag{4.1}
$$

where x_s are coordinates of the 4-torus T^4. The parameter q (that we shall assume to take values in the interval $[0,1]$) interpolates between the pure RR case ($q = 0$) and the pure NSNS case ($q = 1$). The type IIB string in this background and its spectrum was discussed in [28, 32, 51] (see also [52, 53]).

The light-cone Hamiltonian is given again by (3.5), i.e.

$$
H = \frac{1}{\alpha' p^u} \left(\hat{N}_b + \hat{N}_f + \hat{N}_T + A \right) \ , \tag{4.2}
$$

where $\hat{N}_b$ and $\hat{N}_f$ have the same form as in (3.6)(3.7) but now with half of the massive bosonic and fermionic oscillator modes and the frequencies given by

$$
\omega_n = \sqrt{(n + q\,\mu)^2 + (1-q^2)\,\mu^2} \ , \qquad\qquad \mu = \alpha' p^u f \ . \tag{4.3}
$$

$\hat{N}_{\mathrm{T}}$ stands for the contribution of the 4 massless torus bosons and 4 massless decoupled fermions (there is also the standard T^4 momentum and winding mode contribution that we suppress). The NSNS flux acts to shift the mode number n (introducing certain periodicity, see below) while RR flux produces the effective mass $\mu' = \sqrt{1-q^2}\mu$.

The normal ordering constant A vanishes in the type IIB case and thus in the untwisted sector of the type 0B spectrum, while in the twisted sector where GS fermions are taken to be antiperiodic we get (cf. (3.8),(3.9))

$$
A \equiv A_4(\mu, q) = A_{\mathrm{T}} + \bar{A}_4(\mu, q) \ , \qquad\qquad A_{\mathrm{T}} = \bar{A}_4(0, q) = 4\left[\zeta(-1,0) - \zeta(-1,\tfrac{1}{2})\right] = -\tfrac{1}{2} \ , \tag{4.4}
$$

$$
\bar{A}_4(\mu, q) = 4 \times \tfrac{1}{2}\Big[\sum_{n=0,\pm1,\ldots} \sqrt{(n + q\,\mu)^2 + (1-q^2)\mu^2} - \sum_{r=\pm\frac{1}{2},\ldots} \sqrt{(r + q\,\mu)^2 + (1-q^2)\mu^2} \Big] . \tag{4.5}
$$

A_{T} in (4.4) is the contribution of massless oscillators in $\hat{N}_{\mathrm{T}}$. In the limit of $\mu = 0$ we recover the flat space value $A_4 = A_{\mathrm{T}} + \bar{A}_4(0,q) = -1$ as in (3.9).

Separating the flat-space part and the $n = 0$ bosonic contribution in the sum in (4.5) as in (3.10),(3.11), the function A_4 can be represented as

$$
A_4(\mu, q) = -\tfrac{1}{2} + \bar{A}_4(\mu, q) = -1 + 2|\mu| + 2\sum_{n=1}^{\infty} X_n \ , \tag{4.6}
$$

$$
\begin{aligned}
X_n = &\sqrt{(n + q\,\mu)^2 + (1-q^2)\mu^2} + \sqrt{(n - q\,\mu)^2 + (1-q^2)\mu^2} \\
&- \sqrt{(n - \tfrac{1}{2} + q\,\mu)^2 + (1-q^2)\mu^2} - \sqrt{(n - \tfrac{1}{2} - q\,\mu)^2 + (1-q^2)\mu^2} \ - 1 \ .
\end{aligned}
\tag{4.7}
$$

For $n \gg 1$ one has $X_n \to -\frac{\mu^2(1-q^2)}{2n^2} + \mathcal{O}(\frac{1}{n^3})$, i.e. the sum in (4.6) is convergent.

In the pure RR flux case of $q = 0$ we get (cf. (3.8) and (4.5))

$$
A_4(\mu, 0) = -\tfrac{1}{2} + \tfrac{1}{2} A_8(\mu) \ , \tag{4.8}
$$

i.e. $A_4(\mu, 0)$ is always negative, growing to $-\frac{1}{2}$ at $\mu \to \infty$.

In the pure NSNS flux case of $q = 1$ (4.6) simplifies to

$$A_4(\mu, 1) = -1 + 2|\mu| + 2\sum_{n=1}^{\infty}\Big(|n+\mu| + |n-\mu| - |n - \tfrac{1}{2} + \mu| - |n - \tfrac{1}{2} - \mu| \; - 1\Big) \; . \tag{4.9}$$

For $0 \le \mu \le \frac{1}{2}$ we find that $\bar{A}_4(\mu, 1) = 2\mu - 1$ and for general μ this function oscillates between -1 at $|\mu| = k$ and 0 at $|\mu| = k + \frac{1}{2}$, $k = 0, 1, 2, ...$ with period 1 (see Figure 2).[17] Thus the normal ordering constant vanishes for special values of μ. To analyse stability we need to fix the background parameter f; since p^u may vary, we will still find an instability at other values of $\mu = \alpha' p^u f$.

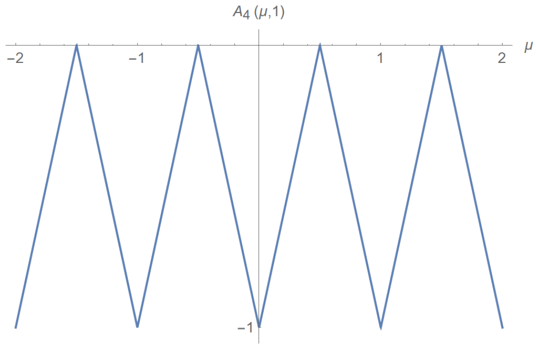

Figure 2: Plot of A_4 as a function of μ in the $q = 1$ case (pure NSNS flux).

For $q \in (0, 1)$ the asymptotics of $A_4 = -\frac{1}{2} + \bar{A}_4$ are given by (cf. (3.15),(3.16))

$$A_4|_{\mu\to 0} = -1 + 2|\mu| - 4\ln(2)(1-q^2)\,\mu^2 + 3\zeta(3)(1-q^2)(1-5q^2)\,\mu^4 + \mathcal{O}\big(\mu^6\big) \; , \tag{4.10}$$
$$A_4|_{\mu\to\infty} = -\tfrac{1}{2} + \mathcal{O}\big(e^{-|\mu|}\big) \; . \tag{4.11}$$

For general q one can show that $\bar{A}_4$ vanishes at special values of μ where $4q\mu$=odd integer, i.e.

$$A_4 = -\tfrac{1}{2} + \bar{A}_4 = -\tfrac{1}{2} \qquad \text{if} \quad q\,\mu = \tfrac{1}{4}(2k+1) \; , \qquad k \in \mathbb{Z} \; . \tag{4.12}$$

This follows from rearranging terms in the convergent sum in $\bar{A}_4$ in (4.6). For example, for $q\mu = \frac{1}{4}$ the second and the third terms in (4.7) cancel and then including the zero-mode term $2|\mu| = \sqrt{(q\mu)^2 + (1-q^2)\mu^2}$ in the sum of the first term, in (4.7) and renaming n in the sum of the fourth term, one concludes that $\bar{A}_4\big|_{q\mu=\frac{1}{4}} = 0$.

One can find an integral representation for A_4 similar to the one for A_8 in (3.14) by considering the intervals between the points $q\mu = \frac{1}{4}(2k+1)$ separately. For example, for $q\mu \in [-\frac{1}{4}, \frac{1}{4}]$ we get

$$A_4 = -\tfrac{1}{2} + 2\sqrt{(q\mu)^2 + \mu'^2} - 4|\mu'| \int_0^{\infty} \mathrm{d}x \, \frac{J_1(2|\mu'|\,x)}{x\,(e^x+1)} \, \cosh(2q\mu\,x) \; , \qquad \mu' \equiv \sqrt{1-q^2}\,\mu \; . \tag{4.13}$$

To get the analog of (4.13) for $q\mu \in [\frac{1}{4}, \frac{3}{4}]$ we need to introduce an overall minus sign for the $\bar{A}_4$ contribution and shift μ by $\frac{1}{2q}$ while leaving μ' unaltered. A_4 is continuous at the glueing points $q\mu = \frac{1}{4}(2k+1)$ where it takes the value $A_4 = -\frac{1}{2}$. The plots of A_4 as a function of μ for some values of q are shown in Figure 3.

[17] This is similar to the periodicity in the normal ordering constant (2.8) in the Melvin background discussed in section 2, where a renaming of mode numbers allowed to absorb integer shifts of the parameter γ in (2.5).

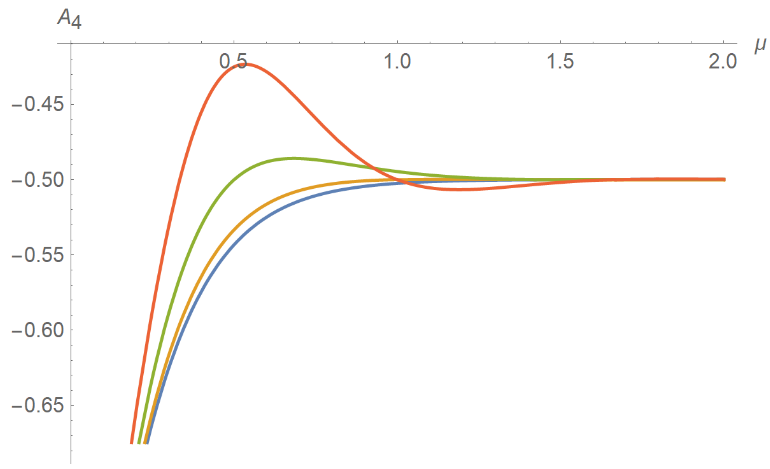

Figure 3: Dependence of the normal ordering constant A_4 on the parameter μ for different values of q: $\frac{3}{4}$ (upper red curve), $\frac{1}{2}$, $\frac{1}{4}$, and 0 (lower blue curve). Note that $A_4 = -\frac{1}{2}$ for $q\mu = \frac{1}{2}k + \frac{1}{4}$, $k \in \mathbb{Z}$.

Let us now consider the corresponding type 0 partition function, i.e. the analog of (3.22). We will need to include the torus T^4 contribution, i.e. factors of the massless oscillator partition function in general defined as

$$\begin{aligned}\mathcal{Z}^{\mathrm{T}}_{a,b}(\tau) =& e^{4\pi\tau_2\left[\frac{1}{24}-\frac{1}{8}(2b-1)^2\right]} \prod_{n\in\mathbb{Z}} \left(1 - e^{-2\pi\tau_2|n+b|+2\pi i\tau_1(n+b)+2\pi i a}\right) \\ & \times \left(1 - e^{-2\pi\tau_2|n-b|+2\pi i\tau_1(n-b)-2\pi i a}\right) = |\eta(\tau)|^{-2} \left|\theta\left[\begin{smallmatrix}\frac{1}{2}-b\\ \frac{1}{2}-a\end{smallmatrix}\right](0,\tau)\right|^2 , \end{aligned} \tag{4.14}$$

where $b \in [0,1]$ and the θ-function is defined in (A.5). The bosonic T^4 momentum and winding modes contribute

$$\mathcal{Z}_0(\tau) = \mathrm{Tr}\, e^{i\pi(\tau p_R^2 - \bar{\tau} p_L^2)} = \sum_{p_L, p_R} e^{i\pi\tau_1(p_R^2-p_L^2)} e^{-\pi\tau_2(p_R^2+p_L^2)} , \tag{4.15}$$

where for the simplest rectangular torus with $R = \sqrt{\alpha'}$ one has $(p_{L,R})_s = \mathrm{m}_s \pm w_s$. This factor replaces the contribution $\tau_2^{-4/2}$ of 4 continuous bosonic zero modes. Let us also introduce

$$\widetilde{\mathcal{Z}}_{a,b}(\tau;\mu,q) = \mathcal{Z}_{a-q\mu\tau,\, b+q\mu}(\tau; \sqrt{1-q^2}\,\mu) , \tag{4.16}$$

where $Z_{a,b}(\tau;\mu)$ was defined in (3.23) Then the total partition function may be represented as (cf. (3.22))

$$Z_{0\mathrm{B}}(f,q) = \int \frac{\mathrm{d}^2\tau}{2\tau_2} \int \mathrm{d}p^u \mathrm{d}p^v\, e^{-2\pi\alpha'\tau_2 p^u p^v}\, \mathcal{Z}_0(\tau)\, \mathcal{I}(\tau;\alpha' p^u f,\, q) , \tag{4.17}$$

$$\mathcal{I} = \frac{(\widetilde{\mathcal{Z}}_{0,0})^2(\mathcal{Z}^{\mathrm{T}}_{0,0})^2 + (\widetilde{\mathcal{Z}}_{1/2,0})^2(\mathcal{Z}^{\mathrm{T}}_{1/2,0})^2 + (\widetilde{\mathcal{Z}}_{0,1/2})^2(\mathcal{Z}^{\mathrm{T}}_{0,1/2})^2 + (\widetilde{\mathcal{Z}}_{1/2,1/2})^2(\mathcal{Z}^{\mathrm{T}}_{1/2,1/2})^2}{(\widetilde{\mathcal{Z}}_{0,0})^2\, |\eta(\tau)|^8} , \tag{4.18}$$

where in (4.18) we suppressed the arguments of the $\mathcal{Z}$-functions. Modular invariance can be checked along the same lines as for (3.22) (see [31]). A few useful relations are

$$\widetilde{\mathcal{Z}}_{a,b}(\tau+1;\mu,q) = \widetilde{\mathcal{Z}}_{a+b,b}(\tau;\mu,q), \qquad \widetilde{\mathcal{Z}}_{a+1,b}(\tau;\mu,q) = \widetilde{\mathcal{Z}}_{a,b+1}(\tau;\mu,q) = \widetilde{\mathcal{Z}}_{a,b}(\tau;\mu,q), \tag{4.19}$$

$$\mathcal{Z}^{\mathrm{T}}_{a,b}(\tau+1) = \mathcal{Z}^{T}_{a+b,b}(\tau), \qquad \mathcal{Z}^{T}_{a+1,b}(\tau) = \mathcal{Z}^{\mathrm{T}}_{a,b+1}(\tau) = \mathcal{Z}^{\mathrm{T}}_{a,b}(\tau) \qquad \mathcal{Z}^{\mathrm{T}}_{a,b}\left(-\tfrac{1}{\tau}\right) = \mathcal{Z}^{\mathrm{T}}_{-b,a}(\tau). \tag{4.20}$$

Also, we may use that [31]

$$\mathcal{Z}_{a,b}\left(-\tfrac{1}{\tau}; |\tau|\mu\right) = \mathcal{Z}_{-b,a}(\tau;\mu) . \tag{4.21}$$

In the present case the periodicities transform with μ as a modular parameter rather than a positive constant, so we have to be careful about where the absolute value should be taken. Following a similar discussion in [54], one can show that

$$\widetilde{\mathcal{Z}}_{a,b}\left(-\tfrac{1}{\tau};\tau\mu,q\right)=\mathcal{Z}_{a+q\mu,b+q\mu\tau}\big(-\tfrac{1}{\tau};\tau\sqrt{1-q^2}\mu\big)=\mathcal{Z}_{-b-q\mu\tau,a+q\mu}(\tau;\sqrt{1-q^2}\mu)=\widetilde{\mathcal{Z}}_{-b,a}(\tau;\mu,q). \tag{4.22}$$

As a result, one can check that the integrand in (4.17) is invariant under $\tau\to-\frac{1}{\tau}$ provided we also redefine the integration variables $p^u\to\tau p^u$, $p^v\to\bar{\tau}p^v$.

Taking the large τ_2 limit one finds as in (3.25) that it is controlled by the mass of the ground state in the twisted sector, i.e. by the normal ordering constant A_4 in (4.4). Namely, we get

$$\exp\Big(2\pi\tau_2\Big[2\Delta_{\frac{1}{2}+q\mu}(\sqrt{1-q^2}\mu)-2\Delta_{q\mu}(\sqrt{1-q^2}\mu)+\tfrac{1}{2}\Big]\Big)\ , \tag{4.23}$$

where $\Delta_b(\mu)$ was defined in (3.24). As follows from (3.24), this expression exactly matches the normal ordering constant (4.13). Since A_4 is negative we find as in (3.21) that there is always a region of instability.

5 Remarks on the $AdS_5\times S^5$ case

As we discussed above, in flat space and in the light cone gauge the NSR description of type 0 string theory and its GS description based on the orbifolding procedure are equivalent [4, 55]. This generalises to solvable examples of curved NS-NS models like pp-wave and Melvin-type background. The case of RR backgrounds is non-trivial and to argue that the same orbifolding procedure applies we studied the simplest example of pp-wave supported by RR flux. The corresponding spectrum of type 0 string theory in pp-wave background supported by F_5 flux was already found using the orbifolding procedure [30, 31], while we demonstrated (in Appendix B) that like in flat space the reasoning for this can be understood by starting with the type IIB case, adding a Melvin twist and taking the special limit that leads to the type 0 string.

Let us now comment on the example of our main interest – $AdS_5\times S^5$ background supported by self-dual RR 5-form flux which is a solution of both type IIB and type 0B theories [2]. The spectrum of the type 0B string in this background should again be given by a combination of the untwisted and twisted sectors described by the $AdS_5\times S^5$ GS action with periodic and antiperiodic fermions respectively. Indeed, the above Melvin-twist procedure leading to an orbifolding construction of the spectrum in an RR-flux supported pp-wave background should apply also to the $AdS_5\times S^5$ case. However, in this case the type IIB spectrum (with Melvin twist added) is not known in an explicit form and thus one may not be able to directly implement the limiting procedure leading to the type 0 spectrum. Thus this orbifolding prescription may still be viewed as a conjecture. We believe it is a natural one, given that the corresponding dual gauge theory may be interpreted as an orbifold of $\mathcal{N}=4$ SYM theory [3, 9].

The classical type IIB GS string in $AdS_5\times S^5$ background [56] is known to be integrable [57] and by now its spectrum is effectively known, at least implicitly [16, 18]. The integrability property should be expected to determine the spectrum in the twisted sector as well since it should also apply to the case of antiperiodic GS fermions.

To get an idea of the effect the choice of fermion periodicity has, let us consider some semiclassical string states with large quantum numbers in the limit of large effective string tension $\frac{\sqrt{\lambda}}{2\pi}=\frac{R^2_{\rm AdS}}{2\pi\alpha'}$. In the semiclassical limit [58, 59, 60, 61] one fixes the parameters of the classical string solution or the ratios of spins to string tension while taking $\sqrt{\lambda}\gg1$, e.g.,

$$\frac{S}{\sqrt{\lambda}}\,,\ \frac{J}{\sqrt{\lambda}}=\text{fixed}\ ,\qquad \sqrt{\lambda}\gg1\ , \tag{5.1}$$

where S is a spin in AdS_5 and J is an angular momentum in S^5. One important example is the long folded spinning string in AdS_5 with the energy [58]

$$E = S + f(\lambda)\ln S + ... \ , \qquad \sqrt{\lambda} \gg 1, \quad \frac{S}{\sqrt{\lambda}} \gg 1 \ , \tag{5.2}$$

$$f(\lambda) = \tfrac{1}{\pi}\big[a_0\sqrt{\lambda} + a_1 + \tfrac{1}{\sqrt{\lambda}}a_2 + ...\big] \ , \qquad a_0 = 1 \ , \quad a_1 = -3\ln 2 \ , \quad a_2 = -\mathrm{K} \ , ... \ . \tag{5.3}$$

The function $f(\lambda)$ is the cusp anomalous dimension which is known exactly from the integrability [62, 63]. In the semiclassical string expansion, a_0 is determined by the classical bosonic string solution [58], a_1 – by the 1-loop correction due to bosonic plus fermionic string fluctuations [59], a_2 (proportional to Catalan's constant K) – by the 2-loop GS string correction [64], etc. . Explicitly, a_1 is given by the sum of contributions of the 8 bosonic and 8 fermionic fluctuation modes ($\mathrm{s} = \frac{1}{\pi}\ln\frac{S}{\sqrt{\lambda}} \gg 1$) [59]

$$\begin{aligned} a_1 &= \lim_{\mathrm{s}\to\infty}\frac{1}{\mathrm{s}^2}\sum_{n=1}^{\infty}\big(\sqrt{n^2+4\mathrm{s}^2} + 2\sqrt{n^2+2\mathrm{s}^2} + 5\sqrt{n^2} - 8\sqrt{n^2+\mathrm{s}^2}\ \big) \\ &= \int_0^\infty dp\ \big(\sqrt{p^2+4} + 2\sqrt{p^2+2} + 5p - 8\sqrt{p^2+1}\ \big) = -3\ln 2 \ . \end{aligned} \tag{5.4}$$

A similar semiclassical state should exist in the twisted sector of type 0B theory where the GS fermions are antiperiodic. Because of the limit $\frac{S}{\sqrt{\lambda}} \gg 1$ the folded string is infinitely long, i.e. the world sheet circle becomes effectively decompactified and the sign of periodicity of the fermions becomes irrelevant. Thus we should find the same value of the function $f(\lambda)$. Indeed, in the antiperiodic case the sum in (5.4) is replaced by (cf. (3.8))

$$\sum_{n=1}^{\infty}\Big[\sqrt{n^2+4\mathrm{s}^2} + 2\sqrt{n^2+2\mathrm{s}^2} + 5\sqrt{n^2} - 8\sqrt{(n-\tfrac{1}{2})^2+\mathrm{s}^2}\ \Big] \ . \tag{5.5}$$

The trivial s-independent divergent part in this sum drops out from a_1 for $\mathrm{s} \to \infty$[18] and then setting $n \to p\,\mathrm{s}$ we conclude that the $-\frac{1}{2}$ shift in the summation index in the fermionic contribution does not play a role in the large spin limit. Thus we arrive at the conclusion that the energy of the corresponding fast spinning string state in the twisted sector should be the same as in the untwisted sector.

To get a non-trivial effect from the change of periodicity of the GS fermions one is to consider solutions where the size of the closed string remains fixed in units of $\sqrt{\lambda}$. For example, in the limit where the S^5 angular momentum parameter $\mathcal{J} = \frac{J}{\sqrt{\lambda}}$ is taken to be larger than other semiclassical string parameters one should recover the pp-wave spectrum with the same twisted sector ground state energy as in (3.8). Another simple example is provided by the rigid circular string rotating in two orthogonal planes in the S^3 part of S^5 with the two equal angular momenta $J_1 = J_2 = \frac{1}{2}J$ [66, 67]. Its classical AdS_5 energy is given by

$$E_0 = \sqrt{\lambda}\,\kappa = \sqrt{J^2 + \lambda \mathrm{w}^2} = J + \tfrac{\lambda \mathrm{w}^2}{2J} + ... \ , \tag{5.6}$$

where w is the winding number of the string and in the last equality we expanded in large $\mathcal{J} = \frac{J}{\sqrt{\lambda}}$.[19]

[18] An alternative is to isolate the divergent $\mathrm{s} = 0$ part explicitly as in (3.10),(3.11) which will "symmetrise" the fermionic contribution (cf. [65]): $8\sqrt{(n-\frac{1}{2})^2+\mathrm{s}^2} \to 4\sqrt{(n-\frac{1}{2})^2+\mathrm{s}^2} + 4\sqrt{(n+\frac{1}{2})^2+\mathrm{s}^2}$.

[19] The large $\mathcal{J}$ limit is similar to the expansion near the pp-wave background (cf. (3.2),(3.5)) with $p^u \to J$ and $\mu = \alpha' p^u f \to \mathcal{J} = \frac{J}{\sqrt{\lambda}} = \alpha' J R_{\rm AdS}^{-2}$.

Starting with the form of this solution in [67], one finds for the 1-loop correction to the energy $E = E_0 + E_1 + ...$ [68, 69][20]

$$E_1 = E_{1,0} + E_1' \ , \qquad E_1' = \sum_{n=1}^{\infty} S_n \ , \tag{5.7}$$

$$E_{1,0} = \big[2 + \sqrt{1 - \tfrac{2\mathrm{w}^2}{\kappa^2}} + 5\sqrt{1 - \tfrac{\mathrm{w}^2}{\kappa^2}}\big] - 8\sqrt{1 - \tfrac{\mathrm{w}^2}{\kappa^2}} \ , \qquad \kappa^2 = \mathcal{J}^2 + \mathrm{w}^2 \ , \tag{5.8}$$

$$S_n = 2\sqrt{1 + \tfrac{(n+\sqrt{n^2-4\mathrm{w}^2})^2}{4\kappa^2}} + 2\sqrt{1 + \tfrac{n^2-2\mathrm{w}^2}{\kappa^2}} + 4\sqrt{1 + \tfrac{n^2}{\kappa^2}} - 8\ \sqrt{1 + \tfrac{n^2-\mathrm{w}^2}{\kappa^2}} \ . \tag{5.9}$$

Here $E_{1,0}$ is the contribution of the zero modes and the negative contributions come from 8 periodic fermionic fluctuations. For the corresponding state in the twisted sector of type 0B theory one needs to take the fermions to be antiperiodic, i.e. drop the fermionic zero mode term in $E_{1,0}$ and sum the negative term in (5.9) with $n \to r = n - \frac{1}{2}$. Then the difference between the energies of the untwisted and twisted sector states is found to be

$$\begin{aligned}\Delta E_1 = E_1 - \tilde{E}_1 &= -\tfrac{1}{\kappa}\, 8 \times \tfrac{1}{2}\Big[\sum_{n=0,\pm1,...}^{\infty} \sqrt{n^2 + \mathcal{J}^2} - \sum_{r=\pm\frac{1}{2},...}^{\infty} \sqrt{r^2 + \mathcal{J}^2}\Big] \\ &= -\frac{1}{\sqrt{\mathcal{J}^2 + \mathrm{w}^2}} A_8(\mathcal{J}) \ . \end{aligned} \tag{5.10}$$

Here $A_8(\mathcal{J})$ is exactly the same function (3.8),(3.14) as in the ground state energy of the twisted sector of the type 0B theory in pp-wave background. A similar discussion can be repeated for another simple case of the circular (S, J) solution [72].

One may also use a semiclassical approach to study quantum corrections to energies of short strings [73] but addressing the question about the energy of the type 0B tachyon as a function of the string tension $\sqrt{\lambda}$ requires an exact solution for the twisted sector of the $AdS_5 \times S^5$ spectrum. Hopefully, this can be achieved using the integrability of the $AdS_5 \times S^5$ GS string theory that should hold regardless the choice of periodicity for the GS fermions.

Acknowledgements

AAT is grateful to M. Beccaria, F. Levkovich-Maslyuk, J. Russo and S. van Tongeren for related discussions. We also thank R. Metsaev for comments on the draft. TS acknowledges funding by the President's PhD Scholarships of Imperial College London. The work of AAT was supported by the STFC grant ST/T000791/1.

A Type 0 string in flat space

Here we shall recall the standard description of type 0 string in flat space first in the NSR and then in the light-cone GS approach.

We shall start with NSR closed string theory on a cylinder (τ, σ) and use the notation $\sigma^{\pm} = \tau \pm \sigma$, $\partial_{\pm} = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$. The light-cone action is $(i = 1, ..., 8)$

$$\mathcal{S} = \frac{1}{\pi\alpha'}\int d^2\sigma \ \left(\partial_+ x^i \partial_- x^i + i\bar{\psi}^i_+ \partial_- \psi^i_+ + i\bar{\psi}^i_- \partial_+ \psi^i_-\right) \ . \tag{A.1}$$

[20] The fluctuation frequencies were found in [70] starting with the form of the solution in [66] related to the one in [67] by an $SO(4)$ transformation. In [65] it was assumed that after a rotation needed to diagonalise the fermionic fluctuation Lagrangian the GS fermions become effectively antiperiodic. In fact, this rotation is not needed if one starts with the form of the solution in [67], i.e. the type IIB fermions remain periodic. This issue was further clarified in [71].

Depending on periodicity of left- and right-moving fermions we get 4 sectors. In each left- or right-moving sector the mass operator is given by

$$\alpha' M^2 = \sum_{n=1}^{\infty} n\alpha^i_{-n}\alpha^i_n + \sum_{r=b}^{\infty} r\beta^i_{-r}\beta^i_r + A \ , \tag{A.2}$$

where $\alpha^i_{-n}(\alpha^i_n)$, $\beta^i_{-n}(\beta^i_n)$ are creation (annihilation) operators of bosonic and fermionic modes respectively. The values of the parameter b in the fermionic sum and the normal ordering constant A depend on the sector: in the NS (antiperiodic) sector $b = \frac{1}{2}$ and $A = -\frac{1}{2}$ while in the R (periodic) sector $b = 1$ and $A = 0$. The ground state $|0\rangle_{NS}$ is a scalar tachyon while $|0\rangle_R$ is an $SO(8)$ spinor. Combining the left- and right-moving sectors and imposing the level matching condition $M_L^2 = M_R^2$ one then generates the closed string spectrum. In particular, one gets[21]

Sector	Lowest mass state	Rep of SO(8)	Statistics
$(\mathrm{NS}_-, \mathrm{NS}_-)$	$\|0\rangle_{NS} \otimes \|0\rangle_{NS}$	$\mathbb{1} \otimes \mathbb{1}$	Bosonic
$(\mathrm{NS}_+, \mathrm{NS}_+)$	$\xi_{ij}\beta^i_{-1/2}\|0\rangle_{NS} \otimes \beta^j_{-1/2}\|0\rangle_{NS}$	$8_v \otimes 8_v$	Bosonic
$(\mathrm{R}_+, \mathrm{R}_+)$	$\|+\rangle_R \otimes \|+\rangle_R$	$8_c \otimes 8_c$	Bosonic
$(\mathrm{R}_+, \mathrm{R}_-)$	$\|+\rangle_R \otimes \|-\rangle_R$	$8_c \otimes 8_s$	Bosonic
$(\mathrm{R}_-, \mathrm{R}_-)$	$\|-\rangle_R \otimes \|-\rangle_R$	$8_s \otimes 8_s$	Bosonic
$(\mathrm{NS}_+, \mathrm{R}_+)$	$\xi_i\beta^i_{-1/2}\|0\rangle_{NS} \otimes \|+\rangle_R$	$8_v \otimes 8_c$	Fermionic
$(\mathrm{NS}_+, \mathrm{R}_-)$	$\xi_i\beta^i_{-1/2}\|0\rangle_{NS} \otimes \|-\rangle_R$	$8_v \otimes 8_s$	Fermionic

Imposing modular invariance leads to the following consistent theories

$$\begin{array}{lllll} \text{type IIA:} & (\mathrm{NS}_+, \mathrm{NS}_+) & (\mathrm{R}_+, \mathrm{R}_-) & (\mathrm{NS}_+, \mathrm{R}_-) & (\mathrm{R}_+, \mathrm{NS}_+) \\ \text{type IIB:} & (\mathrm{NS}_+, \mathrm{NS}_+) & (\mathrm{R}_+, \mathrm{R}_+) & (\mathrm{NS}_+, \mathrm{R}_+) & (\mathrm{R}_+, \mathrm{NS}_+) \\ \text{type 0A:} & (\mathrm{NS}_-, \mathrm{NS}_-) & (\mathrm{NS}_+, \mathrm{NS}_+) & (\mathrm{R}_+, \mathrm{R}_-) & (\mathrm{R}_-, \mathrm{R}_+) \\ \text{type 0B:} & (\mathrm{NS}_-, \mathrm{NS}_-) & (\mathrm{NS}_+, \mathrm{NS}_+) & (\mathrm{R}_+, \mathrm{R}_+) & (\mathrm{R}_-, \mathrm{R}_-) \end{array} .$$

In particular, type 0 theory contains a tachyon (with $m_0^2 = -\frac{2}{\alpha'}$), the same massless NSNS states, double the number of massless RR states (as compared to type II theory) and no spacetime fermions.

The corresponding torus partition function is given by ($\tau = \tau_1 + i\tau_2$)

$$Z = \int_{\mathcal{F}_0} \frac{\mathrm{d}\tau_1 \mathrm{d}\tau_2}{\tau_2} \mathrm{Tr}\left(e^{-2\pi\tau_2 H + 2\pi i \tau_1 P}\right) = \int_{\mathcal{F}_0} \frac{\mathrm{d}^2\tau}{4\tau_2}\left(\mathrm{q}^{L_0}\bar{\mathrm{q}}^{\bar{L}_0}\right) \ , \qquad \mathrm{q} = \exp(2\pi i\tau) \ . \tag{A.3}$$

Separating the bosonic zero mode factor $(2\pi\sqrt{\alpha'})^{-8}\tau_2^{-5}$, the contribution of 8 bosonic oscillator modes is expressed in terms of the Dedekind η-function $\eta(\tau) = \mathrm{q}^{1/24}\prod_{n=1}^{\infty}(1-\mathrm{q}^n)$. Then

$$Z = \frac{1}{4(2\pi)^8\alpha'^4}\int_{F_0} \frac{\mathrm{d}^2\tau}{4\tau_2^6}\left|\eta(\tau)\right|^{-16}\mathcal{I} \ , \qquad \mathcal{I} = \mathrm{Tr}_{\text{ferm.}}\left(\mathrm{q}^{L_0}\bar{\mathrm{q}}^{\bar{L}_0}\right) \ . \tag{A.4}$$

The fermionic contributions lead to factors of $\prod_{n=1}^{\infty}(1 \pm q^{n+a})$ (where $a = -\frac{1}{2}$ for NS and 0 for R sectors) which can be expressed in terms of the Jacobi theta functions $\vartheta_{ab}(0;\tau)$ defined as

$$\vartheta_{ab}(z;\tau) = \theta\begin{bmatrix} a/2 \\ b/2 \end{bmatrix}(z;\tau) = \sum_{n\in\mathbb{Z}} e^{i\pi(n+a/2)^2\tau} e^{2\pi i(n+a/2)(z+b/2)}. \tag{A.5}$$

[21] One introduces a $\mathbb{Z}_2$ grading by defining "G-parity" as $(-1)^{\mathrm{F}+1}$ for the NS-sector and $(-1)^{\mathrm{F}}\Gamma_9$ for the R sector (here F counts the number of worldsheet fermionic excitations).

One finds that the only modular invariant combinations of these factors are

$$\mathcal{I}_{\rm IIA,B} = \frac{(\vartheta_{00}^4 - \vartheta_{01}^4 - \vartheta_{10}^4 + \vartheta_{11}^4)(\bar\vartheta_{00}^4 - \bar\vartheta_{01}^4 - \bar\vartheta_{10}^4 \mp \bar\vartheta_{11}^4)}{|\eta|^8} \ , \tag{A.6}$$

$$\mathcal{I}_{\rm 0A,B} = \frac{\vartheta_{00}^4\bar\vartheta_{00}^4 + \vartheta_{01}^4\bar\vartheta_{01}^4 + \vartheta_{10}^4\bar\vartheta_{10}^4 \mp \vartheta_{11}^4\bar\vartheta_{11}^4}{|\eta|^8} \ , \qquad \vartheta_{ab} \equiv \vartheta_{ab}(0;\tau) \ , \tag{A.7}$$

corresponding to the above two type II and two type 0 theories. In the case of type II theories the Jacobi identity $\vartheta_{00}^4 = \vartheta_{01}^4 + \vartheta_{10}^4$ and $\vartheta_{11}(0;\tau) = 0$ implies that $Z_{\rm IIA,B} = 0$. In the type 0 case we cannot further simplify

$$Z_{\rm 0A,B} = \frac{1}{4(2\pi)^8\alpha'^4}\int_{\mathcal{F}_0} \frac{{\rm d}^2\tau}{4\tau_2^6}\ \frac{\vartheta_{00}^4\bar\vartheta_{00}^4 + \vartheta_{01}^4\bar\vartheta_{01}^4 + \vartheta_{10}^4\bar\vartheta_{10}^4 \mp \vartheta_{11}^4\bar\vartheta_{11}^4}{|\eta(\tau)|^{24}} \ . \tag{A.8}$$

Here there is an IR divergence at $\tau_2 \to \infty$ indicating the presence of the ground-state tachyon.

Let us now discuss the equivalent light-cone GS description. In the IIB case one starts with

$$\mathcal{S} = \frac{1}{\pi\alpha'}\int {\rm d}^2\sigma\ \left(\partial_+ x^i \partial_- x^i + i S_R^\alpha \partial_+ S_R^\alpha + i S_L^\alpha \partial_- S_L^\alpha\right) \ , \tag{A.9}$$

while in the type IIA case one has $S_L^\alpha \to S_L^{\dot\alpha}$ ($\alpha = 1,...,8$). Here $S_{L,R}$ are worldsheet bosons and spacetime fermions that satisfy periodic boundary conditions. The fermionic zero modes imply degeneracy of the ground-state corresponding to $8_c \oplus 8_v$ of $SO(8)$. Combining the left and right movers, the massless modes are $(8_{s/c}\oplus 8_v)\otimes(8_c\oplus 8_v)$ which is the same as in the NSR case after the GSO projection and all higher order excitations arise from this single sector. The partition function vanishes since it involves ϑ_{11}

$$Z_{\rm II} = \frac{1}{4(2\pi)^8\alpha'^4}\int_{\mathcal{F}_0} \frac{{\rm d}^2\tau}{4\tau_2^6}\ \frac{\vartheta_{11}^4\bar\vartheta_{11}^4}{|\eta(\tau)|^{24}} = 0. \tag{A.10}$$

The way to describe flat-space type 0 string theory starting from type II theory in the GS framework [4] is to orbifold by a 2π rotation in a plane or equivalently by $(-1)^F$ where F is the spacetime fermion number.[22] Namely, one gets the "untwisted" sector built out of type II states keeping only those with $(-1)^F = 1$, i.e. projecting out all spacetime fermions. To preserve modular invariance one is to then add a "twisted" sector where the string closes only up to a transformation by $(-1)^F$, i.e. one is to take the GS fermions S_L and S_R to be antiperiodic on the cylinder. Like in the NSR formulation that gives a non-zero normal ordering constant and thus a tachyonic ground state.

In the partition function, the projection onto bosonic modes introduces a term $\vartheta_{10}^4\bar\vartheta_{10}^4$ while the twisted sector adds $\vartheta_{00}^4\bar\vartheta_{00}^4 + \vartheta_{01}^4\bar\vartheta_{01}$ and we end up with the same partition function (A.8) as in the NSR description.

B Melvin twist on pp-wave background

Starting with type IIB pp-wave background (3.1) we may compactify the coordinate $y = \frac{u+v}{\sqrt{2}}$ on a circle of radius R and add a Melvin twist (2.1) in the (x_1, x_2) plane as

$$-dt^2 + dy^2 + {\rm d}x_1^2 + {\rm d}x_2^2 \ \ \to\ \ -dt^2 + dy^2 + ({\rm d}x + iqx\,{\rm d}y)({\rm d}\bar x - iq\bar x\,{\rm d}y) \ , \qquad x = x_1 + ix_2 \ . \tag{B.1}$$

[22] A similar light-cone GS description for a Z_2 orbifold of type 0 string theory viewed as Z_4 orbifold of type II string theory was given in [55]. Related constructions with antiperiodic spacetime fermion sectors appeared in [74, 75] and also [76, 25, 26].

For compact y we may fix light-cone gauge as

$$u = \alpha' p^u \tau + wR\sigma \ . \tag{B.2}$$

The transverse string fluctuations now acquire mass (cf. (3.2))

$$\mu^2 = f^2\big[\alpha'^2(p^u)^2 - (wR)^2\big] \, . \tag{B.3}$$

The derivation of the spectrum is similar to the case of a Melvin twist in flat space in section (2). The zero-mode momentum in y direction is now $p^y = \frac{m}{R}$ and thus we get (below $\xi = qR$ and $\gamma = w\xi$ as in (2.2),(2.5), cf. (2.6))

$$\hat{p}^u = \tfrac{\hat{\mathrm{m}}}{R} + \sqrt{\left(\tfrac{\hat{\mathrm{m}}}{R}\right)^2 + \left(\tfrac{wR}{\alpha'}\right)^2 + \tfrac{2}{\alpha'}\hat{N}} \ , \qquad\qquad \hat{\mathrm{m}} \equiv \mathrm{m} - \xi(\hat{J}_b + \tfrac{1}{2}\hat{J}_f) \ , \tag{B.4}$$

$$\hat{H} = -\hat{p}^v = -\tfrac{1}{2}\tfrac{\mathrm{m}}{R} + \tfrac{1}{2}\sqrt{\left(\tfrac{\hat{\mathrm{m}}}{R}\right)^2 + \left(\tfrac{wR}{\alpha'}\right)^2 + \tfrac{2}{\alpha'}\hat{N}} \ , \tag{B.5}$$

$$\hat{E}^2 = (\hat{p}^t)^2 = \left(\tfrac{\hat{\mathrm{m}}}{R}\right)^2 + \left(\tfrac{wR}{\alpha'}\right)^2 + \tfrac{2}{\alpha'}\hat{N} \ . \tag{B.6}$$

Here the expressions for $\hat{N}$ (before normal ordering) and the bosonic contribution to the angular momentum operator $\hat{J}_b$ are given by

$$\hat{N} = \hat{N}_x + \hat{N}_\perp + \hat{N}_f \ , \tag{B.7}$$

$$\hat{N}_x = \sum_{n=1}^{\infty}\big(\omega_{n-\gamma}\alpha^\dagger_{n+}\alpha_{n+} + \omega_{n+\gamma}\alpha_{n-}\alpha^\dagger_{n-} + \omega_{n-\gamma}\tilde{\alpha}_{n-}\tilde{\alpha}^\dagger_{n-} + \omega_{n+\gamma}\tilde{\alpha}^\dagger_{n+}\tilde{\alpha}_{n+}\big) + \omega_\gamma(\alpha_{0-}\alpha^\dagger_{0-} + \tilde{\alpha}^\dagger_{0+}\tilde{\alpha}_{0+})$$

$$\hat{N}_\perp = \tfrac{1}{2}\sum_{i=3}^{8}\Big[\sum_{n=1}^{\infty}\omega_n(\alpha^{i\dagger}_n\alpha^i_n + \alpha^i_n\alpha^{i\dagger}_n + \tilde{\alpha}^{i\dagger}_n\tilde{\alpha}^i_n + \tilde{\alpha}^i_n\tilde{\alpha}^{i\dagger}_n) + \tfrac{1}{2}\mu(\alpha^{i\dagger}_0\alpha^i_0 + \alpha^i_0\alpha^{i\dagger}_0 + \tilde{\alpha}^{i\dagger}_0\tilde{\alpha}^i_0 + \tilde{\alpha}^i_0\tilde{\alpha}^{i\dagger}_0)\Big]$$

$$\hat{J}_b = \sum_{n=1}^{\infty}\big(\alpha^\dagger_{n+}\alpha_{n+} - \alpha^\dagger_{n-}\alpha_{n-} + \tilde{\alpha}^\dagger_{n+}\tilde{\alpha}_{n+} - \tilde{\alpha}^\dagger_{n-}\tilde{\alpha}_{n-}\big) + \tilde{\alpha}^\dagger_0\tilde{\alpha}_0 - \alpha^\dagger_0\alpha_0 \ , \tag{B.8}$$

where frequencies are given by $\omega_n = \sqrt{n^2+\mu^2}$ as in (3.6) and the modes are again labelled by their sign in $\hat{J}_b$. The fermionic part of the GS Lagrangian in the light-cone gauge takes the form (we follow [29] and use the same spinor notation, i.e. $\Pi = \gamma^1\bar{\gamma}^2\gamma^3\bar{\gamma}^4$, etc.)

$$\mathcal{L} = i\theta^1\bar{\gamma}^v(\partial_+ - \tfrac{1}{2}q\gamma^{12}\partial_+ y)\theta^1 + i\theta^2\bar{\gamma}^v(\partial_- - \tfrac{1}{2}q\gamma^{12}\partial_- y)\theta^2 - 2\mu\theta^1\bar{\gamma}^v\Pi\theta^2 + \dots . \tag{B.9}$$

Introducing the commuting projectors $P^\pm = \frac{1\mp i\gamma^{12}}{2}$, $\mathrm{P}^\pm = \frac{1\pm\Pi}{2}$ we may define

$$\Lambda^{1,2} = (e^{\frac{i}{2}qy}P^+ + e^{-\frac{i}{2}qy}P^-)\theta^{1,2} \ . \tag{B.10}$$

and thus rewrite (B.9) as

$$\mathcal{L} = i\Lambda^1\bar{\gamma}^v\partial_+\Lambda^1 + i\Lambda^2\bar{\gamma}^v\partial_-\Lambda^2 - 2\mu\Lambda^1\bar{\gamma}^v\Pi\Lambda^2 + \dots \ . \tag{B.11}$$

Then we can use the expressions in [29] accounting for the shifted periodicities arising from the phase factors. The fermionic equations of motion split under $\mathrm{P}^\pm$ into

$$\partial_+\Lambda^1 \mp \mu\Lambda^2 = 0, \qquad\qquad \partial_-\Lambda^2 \pm \mu\Lambda^1 = 0 \ , \tag{B.12}$$

and finally the fermionic contributions to the oscillator number operator (B.7) and the angular momentum in the (1,2)-plane which appears in (B.4) are found to be

$$\hat{N}_f = \sum_{n=1}^{\infty}\big(\omega_{n-\frac{\gamma}{2}}\beta^\dagger_{n+}\beta_{n+} + \omega_{n+\frac{\gamma}{2}}\beta^\dagger_{n-}\beta_{n-} + \omega_{n+\frac{\gamma}{2}}\tilde{\beta}^\dagger_{n+}\tilde{\beta}_{n+} + \omega_{n-\frac{\gamma}{2}}\tilde{\beta}^\dagger_{n-}\tilde{\beta}_{n-}\big) + \omega_{\frac{\gamma}{2}}(\beta^\dagger_0\beta_0 + \tilde{\beta}^\dagger_0\tilde{\beta}_0)$$

$$\hat{J}_f = \sum_{n=1}^{\infty}(\beta^\dagger_{n+}\beta_{n+} - \beta^\dagger_{n-}\beta_{n-} + \tilde{\beta}^\dagger_{n+}\tilde{\beta}_{n+} - \tilde{\beta}^\dagger_{n-}\tilde{\beta}_{n-}) + \tilde{\beta}^\dagger_0\tilde{\beta}_0 - \beta^\dagger_0\beta_0 \ , \tag{B.13}$$

where the tilde distinguishes between left- and right-moving modes and the subscript $\pm$ corresponds to the eigenvalues of γ^{12}. As the fermions couple to y with "charge" $\frac{1}{2}q$, as in (2.6), there is an extra $\frac{1}{2}$ coefficient between $\hat{J}_b$ and $\hat{J}_f$ in $\hat{\mathrm{m}}$ (B.4). Setting $\xi = 1$ and taking the limit $R \to 0$ like in the case of the flat space Melvin twist in section 2, and then applying T-duality[23] we end up with the spectrum of type 0B theory in the pp-wave background (3.1), i.e. with the sum of the untwisted and twisted sectors discussed in section 3.

C Type 0B effective action expanded near pp-wave background

In the case of type IIB theory [29] one can reproduce the low-energy part of the string spectrum in pp-wave background (3.1) by expanding the string theory effective action or the corresponding equations for the target space fields. Here we shall do the same in the type 0B case.

The effective action of type 0B string theory can be reconstructed from the string scattering amplitudes [2] [24]

$$S = \tfrac{1}{2\kappa^2}\Big\{ \int \mathrm{d}^{10}x\sqrt{-G}\Big(e^{-2\Phi}\Big[R + 4\partial_\mu\Phi\partial^\mu\Phi - \tfrac{1}{4}(\partial_\mu T\partial^\mu T + m_0^2 T^2) - \tfrac{1}{2}|H_3|^2\Big] \tag{C.1}$$
$$- \tfrac{1}{4}e^{h(T)}\big(\big|F_1^+\big|^2 + |\tilde{F}_3^+|^2 + |\tilde{F}_5|^2\big) - \tfrac{1}{4}e^{-h(T)}\big(\big|F_1^-\big|^2 + |\tilde{F}_3^-|^2\big)\Big) + \tfrac{1}{2}\int F_5 \wedge F_3^- \wedge B_2\Big\} + \ldots \, .$$

Compared to type IIB theory where we have a graviton g, a dilaton Φ, a Kalb-Ramond field B with field strength H and RR potentials C_0, C_2 and C_4 with field strengths F_1, F_3 and F_5 (the later restricted by a self-duality constraint), here we have an additional tachyon scalar T with $m_0^2 = -\frac{2}{\alpha'}$ and a doubled set $F_1^\pm$, $F_3^\pm$, F_5 of the massless RR fields[25] which appear in the combinations

$$\tilde{F}_n = F_n - H_3 \wedge C_{n-3}\ , \qquad n = 3, 5\ . \tag{C.2}$$

The coupling of the tachyon to the RR fields is given by an odd function

$$h(T) = T + \mathcal{O}\big(T^3\big)\ . \tag{C.3}$$

One should note that the effective action including the tachyon is not, strictly speaking, well defined as a derivative expansion. In particular, the tachyon effective action reconstructed from on-shell amplitudes and expanded in powers of derivatives is ambiguous [80, 81]: one can use field redefinitions and the leading-order equation $\alpha'\Box T = -2T$, and thus cannot distinguish between $(-\frac{\alpha'}{2}\Box)^n T$ and T factors in the action.

Our aim will be to compute the spectrum of small fluctuations of the tachyon and massless fields in (C.1) near the pp-wave background (3.1). Since the latter has only metric and (self-dual) F_5 being non-trivial, the cubic interaction terms in (C.1) involving the RR fields which are relevant for the study of quadratic fluctuations are

$$S_3 = \tfrac{1}{4\kappa^2}\int \Big[C_2^+ \wedge H_3 \wedge \star F_5 + F_5 \wedge F_3^- \wedge B_2 - T F_5 \wedge \star F_5\Big]\ . \tag{C.4}$$

[23] Without applying T-duality we get the spectrum of type 0A theory in the background T-dual to the pp-wave (3.1), i.e. in a generalisation of the fundamental string (or F-model [77]) type background supported also by a combination of RR fluxes.

[24] Additional information about the structure of the action can be inferred indirectly by imposing consistency with T-duality [78]. Here we use the sign conventions of [79] and the normalisation of [2]. In [2] the action is defined with an additional minus-sign in front of the action and $\kappa = \frac{1}{2}$. To connect to [78] we have to introduce an overall minus sign in front of the metric and another one in front of the Ricci scalar, set $\kappa = \frac{1}{\sqrt{2}}$ and rescale $T \to \sqrt{2}T$ and $F_n \to \sqrt{2}F_n$.

[25] Here there is one copy of F_5 but it is not self-dual. We use the notation $|F_p|^2 = \frac{1}{p!}F_{\mu_1\ldots\mu_p}F^{\mu_1\ldots\mu_p}$.

The relevant terms in the resulting (linearised) equations of motion may be written as

$$R_{\mu\nu} = \tfrac{1}{4\cdot 4!}(1+T)\Big(F_{\mu\alpha\beta\gamma\delta}F_{\nu}^{\alpha\beta\gamma\delta} - \tfrac{1}{10}G_{\mu\nu}F_{\alpha\beta\gamma\delta\epsilon}F^{\alpha\beta\gamma\delta\epsilon}\Big) \ , \tag{C.5}$$

$$\nabla^2\Phi = 0 \ , \qquad (\nabla^2 - m_0^2)T = 4f(F_{v1234} + F_{v5678}) \ , \qquad \nabla^2 C^{\pm} = 0 \ , \tag{C.6}$$

$$\nabla^\mu F^{\pm}_{\mu\nu\rho} = \mp 4fH^{\alpha\beta\gamma}(\delta^{u1234}_{\nu\rho\alpha\beta\gamma} + \delta^{u5678}_{\nu\rho\alpha\beta\gamma}) \ , \tag{C.7}$$

$$\nabla^\mu H_{\mu\nu\rho} = 2fF^{\alpha\beta\gamma +}(\delta^{u1234}_{\nu\rho\alpha\beta\gamma} + \delta^{u5678}_{\nu\rho\alpha\beta\gamma}) - 2fF^{\alpha\beta\gamma -}(\delta^{u1234}_{\nu\rho\alpha\beta\gamma} + \delta^{u5678}_{\nu\rho\alpha\beta\gamma}) \ , \tag{C.8}$$

$$\nabla^\mu F_{\mu\nu\rho\sigma\eta} = -4f\partial^\alpha T(\delta^{u1234}_{\nu\rho\sigma\eta\alpha} + \delta^{u5678}_{\nu\rho\sigma\eta\alpha}) \ . \tag{C.9}$$

Here $\delta^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} = \delta^{\mu_1}_{[\nu_1}\ldots\delta^{\mu_n}_{\nu_n]}$. Introducing $F_3 = \frac{1}{\sqrt{2}}(F_3^+ - F_3^-)$ and $\bar{F}_3 = \frac{1}{\sqrt{2}}(F_3^+ + F_3^-)$ we can write the equations for $F_3^{\pm}, H$ as:

$$\nabla^\mu \bar{F}_{\mu\nu\rho} = 0 \ , \qquad \nabla^\mu F_{\mu\nu\rho} = -2^{5/2}fH^{\alpha\beta\gamma}(\delta^{u1234}_{\nu\rho\alpha\beta\gamma} + \delta^{u5678}_{\nu\rho\alpha\beta\gamma}) \ , \tag{C.10}$$

$$\nabla^\mu H_{\mu\nu\rho} = 2^{3/2}fF^{\alpha\beta\gamma}(\delta^{u1234}_{\nu\rho\alpha\beta\gamma} + \delta^{u5678}_{\nu\rho\alpha\beta\gamma}) \ . \tag{C.11}$$

For the dilaton Φ and the RR scalars $C^{\pm}$, we get the massless wave equation (cf. (3.17))

$$\nabla^2\Phi = \frac{1}{\sqrt{-G}}\partial_\mu(\sqrt{-G}G^{\mu\nu}\partial_\nu\Phi) = \left(2\partial_u\partial_v + f^2x_i^2\partial_v^2 + \partial_i^2\right)\Phi = 0 \ , \tag{C.12}$$

which is solved as in [29] or in (3.17)–(3.19) with the effective light-cone Hamiltonian $H = -p_v = f(\sum_{i=1}^8 a_i^\dagger a_i + 4)$ with the lowest energy state having $H_0 = f\mathcal{E}_0$, $\mathcal{E}_0 = 4$. This matches what one finds in the type 0B spectrum: the contribution 4 arises from either 4 fermionic excitations of the untwisted sector (for Φ, C^+ as in type IIB case [29]) or from the first fermionic excitation in the twisted sector once combined with the normal ordering constant (3.15) expanded in α'.

Following the notation in [29] for a field satisfying a similar wave equation with an extra linear ∂_v term one can read off the lowest light-cone energy value H_0 as

$$(\nabla^2 + 2if\,c\,\partial_v)\phi = 0 \qquad \to \qquad H_0 = f\mathcal{E}_0 \ , \qquad \mathcal{E}_0 = 4 + c \ . \tag{C.13}$$

Fixing the gauge $\bar{C}_{v\mu} = 0$ for the field $\bar{F}_3$ one finds as in [29] that $\nabla^2\bar{C}_{ij} = 0$ and thus 28 massless modes with $\mathcal{E}_0 = 4$. In the following we need to distinguish two sets of indices $i = 1,2,3,4$ and $i' = 5,6,7,8$ as these couple to the 5-form flux independently. For (F_3, H_3) one finds that $\nabla^2 C_{ii'} = \nabla^2 B_{ii'} = 0$ while modes with indices from the same set as e.g. $A_{12} = C_{12} + \sqrt{2}iB_{34}$ and $\bar{A}_{12} = C_{12} - \sqrt{2}iB_{34}$ satisfy

$$(\nabla^2 - 4if\partial_v)A_{12} = 0, \quad (\nabla^2 + 4if\partial_v)\bar{A}_{12} = 0 \quad \to \quad \mathcal{E}_0 = 4 \pm 2 \ . \tag{C.14}$$

The most complicated sector is that of the fluctuations of the metric, the 5-form and the tachyon. Since T is coupled to F_5 in (C.1), the non-zero F_5-background leads to mixing of the fluctuations of F_5 with the tachyon. Expanding to linear order in fluctuations

$$G_{\mu\nu} \to G_{\mu\nu} + h_{\mu\nu}, \quad C_{\mu\nu\rho\sigma} \to C_{\mu\nu\rho\sigma} + c_{\mu\nu\rho\sigma} \ , \quad R_{\mu\nu} \to R_{\mu\nu} + r_{\mu\nu}, \quad F_{\mu\nu\rho\sigma\eta} \to F_{\mu\nu\rho\sigma\eta} + a_{\mu\nu\rho\sigma\eta},$$

and choosing the light-cone gauge $h_{v\mu} = 0$, $c_{v\mu\nu\rho} = 0$, the linearised Einstein equation takes the form

$$\begin{aligned} r_{\mu\nu} =& f a_\mu^{\alpha\beta\gamma\delta}\delta^{u1234}_{\nu\alpha\beta\gamma\delta} + f a_\nu^{\alpha\beta\gamma\delta}\delta^{u1234}_{\mu\alpha\beta\gamma\delta} - f^2 h^{\alpha\alpha'}G^{\beta\beta'}G^{\gamma\gamma'}G^{\delta\delta'}\delta^{u1234}_{\mu\alpha\beta\gamma\delta}\delta^{u1234}_{\nu\alpha'\beta'\gamma'\delta'} + (1234 \to 5678) \\ &+ 8f^2T\delta^{uu}_{\mu\nu} - G_{\mu\nu}f(a_{v1234} + a_{v5678}) \ , \qquad (\text{C.15}) \\ r_{\mu\nu} =& \tfrac{1}{2}\left(-\nabla^2 h_{\mu\nu} + \nabla_\mu\nabla^\rho h_{\rho\nu} + \nabla_\nu\nabla^\rho h_{\rho\mu} - \nabla_\mu\nabla_\nu h^\rho_\rho + 2R_{\mu\rho\sigma\nu}h^{\rho\sigma} + R_{\mu\rho}h^\rho_\nu + R_{\nu\rho}h^\rho_\mu\right) , \end{aligned}$$

with the non-zero components being (background curvature is $R_{iuui} = -f^2, \quad R_{uu} = 8f^2$)

$$
\begin{aligned}
r_{uu} &= 2f(a_{u1234} + a_{u5678}) - 2f^2 \textstyle\sum_{i=1}^{8} h^{ii} + 8f^2 T + f^2 x_I^2 (a_{v1234} + a_{v5678}) \\
r_{ij} &= f(a_{v1234} - a_{v5678})\delta_{ij}, \quad r_{i'j'} = f(a_{v5678} - a_{v1234})\delta_{i'j'}, \quad r_{ui} = f(\epsilon_{ijkl} a_{vujkl} + a_{i5678}), \\
r_{ui'} &= f(\epsilon_{i'j'k'l'} a_{vuj'k'l'} + a_{i'1234}), \quad r_{ii'} = f(\epsilon_{ijkl} a_{vi'jkl} + \epsilon_{i'j'k'l'} a_{vij'k'l'}) \ .
\end{aligned} \tag{C.16}
$$

where $\epsilon_{1234} = \epsilon_{5678} = 1$ and repeated indices i or i' are summed over. All other components vanish. As in [29] the (vv) component of the Einstein equation vanishes and thus gives the zero-trace condition for the transverse modes $h^i_i + h^{i'}_{i'} = 0$.[26] The (vi) components vanish as well, which determines the non-dynamical modes h_{ui} via $\partial^u h_{ui} + \partial^j h_{ij} + \partial^{j'} h_{ij'} = 0$. The (vu) component implicitly fixes h_{uu} via $\partial^u h_{uu} + \partial^i h_{iu} + \partial^{i'} h_{i'u} = 0$. The off-diagonal (ij) and $(i'j')$ components give the free field equations for h_{ij}, $h_{i'j'}$

$$
\nabla^2 h_{ij} = 0, \qquad \nabla^2 h_{i'j'} = 0 \qquad \to \qquad \mathcal{E}_0 = 4 \ . \tag{C.17}
$$

The remaining metric fluctuations are mixed with C_4 potential fluctuations via

$$
\begin{aligned}
\nabla^2 h_{ij} &= -2f\partial_v (c_{1234} - c_{5678})\delta_{ij} \ , \qquad \nabla^2 h_{i'j'} = -2f\partial_v (c_{5678} - c_{1234})\delta_{i'j'} \ , \\
\nabla^2 h_{ii'} &= -2f\partial_v (\epsilon_{ijkl} c_{i'jkl} + \epsilon_{i'j'k'l'} c_{ij'k'l'}) \ .
\end{aligned} \tag{C.18}
$$

The equation for F_5 expanded to first order is

$$
\begin{aligned}
\nabla^\mu a_{\mu\nu\rho\sigma\eta} - G^{\alpha\beta}\nabla^\mu h_{\nu\alpha} F_{\mu\beta\rho\sigma\eta} - G^{\alpha\beta}\nabla^\mu h_{\rho\alpha} F_{\mu\nu\beta\sigma\eta} - G^{\alpha\beta}\nabla^\mu h_{\sigma\alpha} F_{\mu\nu\rho\beta\eta} - G^{\alpha\beta}\nabla^\mu h_{\eta\alpha} F_{\mu\nu\rho\sigma\beta} \\
= -\, 2f\partial^\alpha T (\delta^{u1234}_{\nu\rho\sigma\eta\alpha} + \delta^{u5678}_{\nu\rho\sigma\eta\alpha}) \ .
\end{aligned} \tag{C.19}
$$

Here we can gauge fix $c_{v\mu\nu\rho} = 0$ and then eliminate $c_{u\mu\nu\rho}$. Then $\nabla^2 c_{iji'j'} = 0 \to \mathcal{E}_0 = 4$ while the remaining physical fluctuations satisfy

$$
\begin{aligned}
\nabla^2 c_{1234} &= 4f\partial_v (h_{11} + h_{22} + h_{33} + h_{44}) - 4f\partial_v T \ , \\
\nabla^2 c_{5678} &= 4f\partial_v (h_{55} + h_{66} + h_{77} + h_{88}) - 4f\partial_v T \ , \\
\nabla^2 c_{i'jkl} &= 4f\epsilon_{ijkl}\partial_v h_{ii'} \ , \qquad \nabla^2 c_{ij'k'l'} = 4f\epsilon_{i'j'k'l'}\partial_v h_{ii'} \ .
\end{aligned} \tag{C.20}
$$

To simplify these equations let us introduce the following combinations of fields (using $h^i_i = 0$)

$$
\begin{aligned}
h &= h_{11} + h_{22} + h_{33} + h_{44} \ , & h' &= h_{55} + h_{66} + h_{77} + h_{88} = -h \ , \\
g_a &= h_{11} + h_{aa} - h_{bb} - h_{cc} \ , & (a,b,c) &= \sigma(2,3,4) \ , \\
g_{a'} &= h_{55} + h_{a'a'} - h_{b'b'} - h_{c'c'} \ , & (a',b',c') &= \sigma(6,7,8) \ , \\
c_{ii'} &= \epsilon_{ijkl} c_{i'jkl} + \epsilon_{i'j'k'l'} c_{ij'k'l'} \ , & \bar{c}_{ii'} &= \epsilon_{ijkl} c_{i'jkl} - \epsilon_{i'j'k'l'} c_{ij'k'l'} \ .
\end{aligned} \tag{C.21}
$$

Then $\bar{c}_{ii'}$, g_a and $g_{a'}$ satisfy free massless equations as in (C.17) and thus yield 22 modes with $\mathcal{E}_0 = 4$. The field $c_{ii'}$ combines with $h_{ii'}$ and yields 16 modes of both $\mathcal{E}_0 = 2$ and $\mathcal{E}_0 = 6$. It remains to solve the equations for the 4 interdependent fluctuations $c = c_{1234}$, $c' = c_{5678}$, h and T. They satisfy the equations

$$
\begin{aligned}
\nabla^2 c &= 4f\partial_v h - 4f\partial_v T \ , & \nabla^2 c' &= -4f\partial_v h - 4f\partial_v T \ , \\
\nabla^2 h &= -8f\partial_v (c - c') \ , & (\nabla^2 - m_0^2) T &= 4f\partial_v (c + c') \ .
\end{aligned} \tag{C.22}
$$

These equations can be obtained also by directly expanding the action (C.1) to quadratic order in fluctuations

$$
\begin{aligned}
S[h,h',c,c',T] = \tfrac{1}{2\kappa^2} \int \mathrm{d}^{10}x \sqrt{-G} \Big[& \tfrac{1}{16} h\nabla^2 h + \tfrac{1}{16} h'\nabla^2 h' + \tfrac{1}{4} T(\nabla^2 - m_0^2) T + \tfrac{1}{4} c\nabla^2 c + \tfrac{1}{4} c'\nabla^2 c' \\
& - 2fT\partial_v (c + c') + f(h - h')\partial_v (c - c') \Big] \ ,
\end{aligned} \tag{C.23}
$$

[26] This also follows directly from the 10d Weyl invariance of the action for F_5 and thus tracelessness of its stress tensor in (C.5).

and imposing the zero-trace condition $h = -h'$ after the variation. Introducing

$$c_- = c - c' \ , \qquad\qquad c_+ = c + c' \ , \tag{C.24}$$

we find that c_- and h satisfy

$$\nabla^2 c_- = 8f\partial_v h \ , \qquad \nabla^2 h = -8f\partial_v c_- \ , \qquad\qquad \nabla^2(h \pm ic_-) = \pm 8if\partial_v(h \pm ic_-) \ , \tag{C.25}$$

and thus using (C.13) $h \pm ic_-$ have $\mathcal{E}_0 = 0$ and 8. From (C.22) the remaining fields c_+ and T satisfy a coupled system

$$\nabla^2 c_+ = -8f\partial_v T \ , \qquad (\nabla^2 - m_0^2)T = 4f\partial_v c_+ \ , \tag{C.26}$$

implying $(\nabla^2 - m_0^2)\nabla^2 T = -32f^2\partial_v^2 T$. It can be diagonalised by introducing the new fields

$$K^{\pm} = 4f\partial_v T + \hat{\mathcal{M}}_{\mp}\, c_+ \ , \qquad \hat{\mathcal{M}}_{\pm} \equiv \tfrac{1}{2}m_0^2 \pm \sqrt{\tfrac{1}{4}m_0^4 - 32f^2\partial_v^2} \ , \tag{C.27}$$

$$(\nabla^2 - \hat{\mathcal{M}}_{\pm})K^{\pm} = 0 \qquad \rightarrow \qquad \mathcal{E}_0 = 4 + \frac{m_0^2}{4fp^u} \pm \sqrt{\frac{m_0^4}{16(fp^u)^2} + 8} \ , \tag{C.28}$$

where in (C.28) we used the momentum representation and $p^u = p_v$.

The resulting spectrum may be summarised as follows:

Fields	Components	$\mathcal{E}_0$	#
Φ	Φ	4	1
F_1^+, F_1^-	C^+, C^-	4	2
F_3^+, F_3^-, H_3	$\bar{C}_{ij}$,$\bar{C}_{i'j'}$,$\bar{C}_{ii'}$	4	28
	$C_{ii'}$, $B_{ii'}$	4	32
	A_{ij}, $A_{i'j'}$	2	12
	$\bar{A}_{ij}$, $\bar{A}_{i'j'}$	6	12
G, F_5, T	h_{ij}, $h_{i'j'}$, g_a, $g_{a'}$	4	18
	$c_{iji'j'}$, $\bar{c}_{ii'}$	4	52
	$h_{ii'} + ic_{ii'}$	2	16
	$h_{ii'} - ic_{ii'}$	6	16
	$h + ic_-$	0	1
	$h - ic_-$	8	1
	K^+	$4 + \frac{m_0^2}{4fp^u} + \sqrt{\frac{m_0^4}{16(fp^u)^2} + 8}$	1
	K^-	$4 + \frac{m_0^2}{4fp^u} - \sqrt{\frac{m_0^4}{16(fp^u)^2} + 8}$	1

Here we gave the values of the rescaled light-cone energy $\mathcal{E}_0$, with the eigenvalue of the light-cone Hamiltonian being $H = -p^v = f\mathcal{E}_0$. The effective mass or dispersion relation for the corresponding state may be written as in (3.4) or (3.18),(3.21), i.e. ($E = p^t$, $p^u = p_v = \frac{p^y+E}{\sqrt{2}}$)

$$m^2 = E^2 - (p^y)^2 = 2fp^u\mathcal{E}_0 \ . \tag{C.29}$$

The spectrum of the subset of fields (dilaton Φ, metric G, H_3, one copy of F_1 and F_3 and the self-dual part of F_5) present also in type IIB theory, i.e. of the untwisted sector of type 0B theory, is of course the same as in [29].

The remaining states – the 63 extra "massless" ($\mathcal{E}_0 = 4$) modes and "massive" mixtures $K^\pm$ of the tachyon and the RR 5-form should belong to the twisted sector of type 0 states. For the "lower" or ground-state mode K^+ we get from (C.29)

$$\begin{aligned} m^2 =& 8fp^u + \tfrac{1}{2}m_0^2 + \tfrac{1}{2}m_0^2\sqrt{1+128m_0^{-4}(fp^u)^2} \\ =& -\tfrac{2}{\alpha'} + 8fp^u - 16\alpha'\,(fp^u)^2 + 128\alpha'^3(fp^u)^4 + \dots \,. \end{aligned} \tag{C.30}$$

We used that $m_0^2 = -\frac{2}{\alpha'}$ and expanded in α' or in powers of $\frac{fp^u}{m_0^2} = -\frac{1}{2}\mu$ where $\mu = \alpha' p^u f$ is the parameter used in the exact string spectrum (cf. (3.6),(3.11)). Comparing (C.30) to $m^2 = -m_0^2 A_8(\mu)$ in (3.20) we observe that the first two terms match while the coefficient of the leading order correction $\alpha'(fp^u)^2$ is - 16 in (C.30) and $-16\ln 2$ in (3.20). This mismatch is not too surprising as here the starting point was the low-energy effective action (C.1) which (as was mentioned below (C.3)) contains an ambiguity when expanded in derivatives of the tachyon field[27] and also does not include higher α' corrections.

Similarly, for K^- we get

$$m^2 = 8fp^u + \tfrac{1}{2}m_0^2 - \tfrac{1}{2}m_0^2\sqrt{1+128m_0^{-4}(fp^u)^2} = 8fp^u + 16\alpha'\,(fp^u)^2 - 128\alpha'^3(fp^u)^4 + \dots \,. \tag{C.31}$$

Thus to leading order K^- is effectively massless and should combine with the other 63 modes of $\mathcal{E}_0 = 4$ in the above table to complete the first excited level of the twisted sector. Indeed, the states on the first excited level in the twisted sector are created by applying a pair of fermionic creation operators in (3.5),(3.7) to the vacuum and thus the corresponding analog of the ground-state mass relation (3.20) is

$$m^2 = \tfrac{2}{\alpha'}\left[A_8(\mu) + \sqrt{1+\mu^2}\,\right] = 8fp^u + \mathcal{O}\big(\alpha'(fp^u)^2\big)\ , \tag{C.32}$$

i.e. has the same leading term as in (C.31). Here we expanded in powers of $\mu = \alpha' p^u f$ and the flat-space tachyon part -1 in A_8 cancelled against the leading excited state contribution. The mismatch of subleading order α' terms is again attributed to the fact that the action (C.1) does not contain α' corrections. One may, in fact, turn this around and try to use the information about the exact pp-wave spectrum to fix the structure of higher α' terms in the effective action.

References

[1] A.M. Polyakov, *The Wall of the cave*, *Int. J. Mod. Phys. A* **14** (1999) 645 [hep-th/9809057].

[2] I.R. Klebanov and A.A. Tseytlin, *D-branes and dual gauge theories in type 0 strings*, *Nucl. Phys. B* **546** (1999) 155 [hep-th/9811035].

[3] I.R. Klebanov and A.A. Tseytlin, *A Nonsupersymmetric large N CFT from type 0 string theory*, *JHEP* **03** (1999) 015 [hep-th/9901101].

[4] L.J. Dixon and J.A. Harvey, *String Theories in Ten-Dimensions Without Space-Time Supersymmetry*, *Nucl. Phys. B* **274** (1986) 93.

[5] N. Seiberg and E. Witten, *Spin Structures in String Theory*, *Nucl. Phys. B* **276** (1986) 272.

[6] C. Thorn, *unpublished*, .

[7] I.R. Klebanov, *Tachyon stabilization in the AdS / CFT correspondence*, *Phys. Lett. B* **466** (1999) 166 [hep-th/9906220].

[27] For example, the TF_5F_5 coupling can be replaced by $-\frac{1}{2}\alpha'\partial^2 TF_5F_5$, etc.

[8] O. Bergman and M.R. Gaberdiel, *A Nonsupersymmetric open string theory and S duality*, *Nucl. Phys. B* **499** (1997) 183 [hep-th/9701137].

[9] N. Nekrasov and S.L. Shatashvili, *On nonsupersymmetric CFT in four-dimensions*, *Phys. Rept.* **320** (1999) 127 [hep-th/9902110].

[10] S. Kachru and E. Silverstein, *4-D conformal theories and strings on orbifolds*, *Phys. Rev. Lett.* **80** (1998) 4855 [hep-th/9802183].

[11] A.A. Tseytlin and K. Zarembo, *Effective potential in nonsupersymmetric SU(N) $\times$ SU(N) gauge theory and interactions of type 0 D3-branes*, *Phys. Lett. B* **457** (1999) 77 [hep-th/9902095].

[12] A. Adams and E. Silverstein, *Closed string tachyons, AdS / CFT, and large N QCD*, *Phys. Rev. D* **64** (2001) 086001 [hep-th/0103220].

[13] A. Armoni, E. Lopez and A.M. Uranga, *Closed strings tachyons and noncommutative instabilities*, *JHEP* **02** (2003) 020 [hep-th/0301099].

[14] A. Dymarsky, I.R. Klebanov and R. Roiban, *Perturbative gauge theory and closed string tachyons*, *JHEP* **11** (2005) 038 [hep-th/0509132].

[15] E. Pomoni and L. Rastelli, *Large N Field Theory and AdS Tachyons*, *JHEP* **04** (2009) 020 [0805.2261].

[16] N. Beisert et al., *Review of AdS/CFT Integrability: An Overview*, *Lett. Math. Phys.* **99** (2012) 3 [1012.3982].

[17] S.J. van Tongeren, *Integrability of the* $\mathrm{AdS}_5 \times \mathrm{S}^5$ *superstring and its deformations*, *J. Phys. A* **47** (2014) 433001 [1310.4854].

[18] N. Gromov, V. Kazakov, S. Leurent and D. Volin, *Quantum spectral curve for arbitrary state/operator in AdS_5/CFT_4*, *JHEP* **09** (2015) 187 [1405.4857].

[19] N. Gromov, *Introduction to the Spectrum of $N = 4$ SYM and the Quantum Spectral Curve*, 1708.03648.

[20] M. de Leeuw and S.J. van Tongeren, *The spectral problem for strings on twisted $AdS_5 \times S^5$*, *Nucl. Phys. B* **860** (2012) 339 [1201.1451].

[21] V. Kazakov, S. Leurent and D. Volin, *T-system on T-hook: Grassmannian Solution and Twisted Quantum Spectral Curve*, *JHEP* **12** (2016) 044 [1510.02100].

[22] F. Levkovich-Maslyuk and M. Preti, *Exploring the ground state spectrum of γ-deformed $N = 4$ SYM*, 2003.05811.

[23] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2007).

[24] J.G. Russo and A.A. Tseytlin, *Magnetic flux tube models in superstring theory*, *Nucl. Phys. B* **461** (1996) 131 [hep-th/9508068].

[25] A.A. Tseytlin, *Magnetic backgrounds and tachyonic instabilities in closed string theory*, *AIP Conf. Proc.* **607** (2002) 269 [hep-th/0108140].

[26] T. Takayanagi and T. Uesugi, *Orbifolds as Melvin geometry*, *JHEP* **12** (2001) 004 [hep-th/0110099].

[27] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys. B* **625** (2002) 70 [hep-th/0112044].

[28] D.E. Berenstein, J.M. Maldacena and H.S. Nastase, *Strings in flat space and pp waves from N=4 superYang-Mills*, *JHEP* **04** (2002) 013 [hep-th/0202021].

[29] R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in Ramond-Ramond plane wave background*, *Phys. Rev. D* **65** (2002) 126004 [hep-th/0202109].

[30] F. Bigazzi, A.L. Cotrone, L. Girardello and A. Zaffaroni, *PP wave and nonsupersymmetric gauge theory*, *JHEP* **10** (2002) 030 [hep-th/0205296].

[31] T. Takayanagi, *Modular invariance of strings on PP waves with RR flux*, *JHEP* **12** (2002) 022 [hep-th/0206010].

[32] J.G. Russo and A.A. Tseytlin, *On solvable models of type 2B superstring in NS NS and RR plane wave backgrounds*, *JHEP* **04** (2002) 021 [hep-th/0202179].

[33] O.J. Ganor, A. Hashimoto, S. Jue, B.S. Kim and A. Ndirango, *Aspects of Puff Field Theory*, *JHEP* **08** (2007) 035 [hep-th/0702030].

[34] D. Dhokarh, S.S. Haque and A. Hashimoto, *Melvin Twists of global AdS(5) x S(5) and their Non-Commutative Field Theory Dual*, *JHEP* **08** (2008) 084 [0801.3812].

[35] O. Bergman and M.R. Gaberdiel, *Dualities of type 0 strings*, *JHEP* **07** (1999) 022 [hep-th/9906055].

[36] M.S. Costa and M. Gutperle, *The Kaluza-Klein Melvin solution in M theory*, *JHEP* **03** (2001) 027 [hep-th/0012072].

[37] J.G. Russo and A.A. Tseytlin, *Magnetic backgrounds and tachyonic instabilities in closed superstring theory and M theory*, *Nucl. Phys. B* **611** (2001) 93 [hep-th/0104238].

[38] G.W. Gibbons, *Quantized flux tubes in Einstein-Maxwell theory and noncompact internal spaces*, in *22nd Winter School of Theoretical Physics: Fields and Geometry*, pp. 597–615, 5, 1986.

[39] G.W. Gibbons and K.-i. Maeda, *Black Holes and Membranes in Higher Dimensional Theories with Dilaton Fields*, *Nucl. Phys. B* **298** (1988) 741.

[40] F. Dowker, J.P. Gauntlett, D.A. Kastor and J.H. Traschen, *Pair creation of dilaton black holes*, *Phys. Rev. D* **49** (1994) 2909 [hep-th/9309075].

[41] F. Dowker, J.P. Gauntlett, S.B. Giddings and G.T. Horowitz, *On pair creation of extremal black holes and Kaluza-Klein monopoles*, *Phys. Rev. D* **50** (1994) 2662 [hep-th/9312172].

[42] A.A. Tseytlin, *Melvin solution in string theory*, *Phys. Lett. B* **346** (1995) 55 [hep-th/9411198].

[43] J.G. Russo and A.A. Tseytlin, *Exactly solvable string models of curved space-time backgrounds*, *Nucl. Phys. B* **449** (1995) 91 [hep-th/9502038].

[44] M. Blau, J.M. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A New maximally supersymmetric background of IIB superstring theory*, *JHEP* **01** (2002) 047 [hep-th/0110242].

[45] S. Schafer-Nameki and M. Zamaklar, *Stringy sums and corrections to the quantum string Bethe ansatz*, *JHEP* **10** (2005) 044 [hep-th/0509096].

[46] M. Beccaria, G.V. Dunne and A.A. Tseytlin, *Strong coupling expansion of free energy and BPS Wilson loop in $\mathcal{N} = 2$ superconformal models with fundamental hypermultiplets*, *JHEP* **08** (2021) 102 [2105.14729].

[47] F. Bigazzi and A.L. Cotrone, *On zero point energy, stability and Hagedorn behavior of type IIB strings on pp waves*, *JHEP* **08** (2003) 052 [hep-th/0306102].

[48] L.A. Pando Zayas and D. Vaman, *Strings in RR plane wave background at finite temperature*, *Phys. Rev. D* **67** (2003) 106006 [hep-th/0208066].

[49] B.R. Greene, K. Schalm and G. Shiu, *On the Hagedorn behaviour of PP wave strings and N=4 SYM theory at finite R charge density*, *Nucl. Phys. B* **652** (2003) 105 [hep-th/0208163].

[50] A.B. Hammou, *One loop partition function in plane waves RR background*, *JHEP* **11** (2002) 028 [hep-th/0209265].

[51] G. Grignani, M. Orselli, G.W. Semenoff and D. Trancanelli, *The Superstring Hagedorn temperature in a pp wave background*, *JHEP* **06** (2003) 006 [hep-th/0301186].

[52] A. Cagnazzo and K. Zarembo, *B-field in AdS(3)/CFT(2) Correspondence and Integrability*, *JHEP* **11** (2012) 133 [1209.4049].

[53] B. Hoare and A.A. Tseytlin, *On string theory on $AdS_3 \times S^3 \times T^4$ with mixed 3-form flux: tree-level S-matrix*, *Nucl. Phys. B* **873** (2013) 682 [1303.1037].

[54] Y. Sugawara, *Thermal amplitudes in DLCQ superstrings on PP waves*, *Nucl. Phys. B* **650** (2003) 75 [hep-th/0209145].

[55] I.R. Klebanov, N.A. Nekrasov and S.L. Shatashvili, *An Orbifold of type 0B strings and nonsupersymmetric gauge theories*, *Nucl. Phys. B* **591** (2000) 26 [hep-th/9909109].

[56] R.R. Metsaev and A.A. Tseytlin, *Type IIB superstring action in $AdS_5 \times S^5$ background*, *Nucl. Phys. B* **533** (1998) 109 [hep-th/9805028].

[57] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5 \times S^5$ superstring*, *Phys. Rev. D* **69** (2004) 046002 [hep-th/0305116].

[58] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A Semiclassical limit of the gauge / string correspondence*, *Nucl. Phys. B* **636** (2002) 99 [hep-th/0204051].

[59] S. Frolov and A.A. Tseytlin, *Semiclassical quantization of rotating superstring in AdS(5) × S5*, *JHEP* **06** (2002) 007 [hep-th/0204226].

[60] A.A. Tseytlin, *Semiclassical strings and AdS/CFT*, in *NATO Advanced Study Institute and EC Summer School on String Theory: From Gauge Interactions to Cosmology*, pp. 265–290, 9, 2004 [hep-th/0409296].

[61] A.A. Tseytlin, *Review of AdS/CFT Integrability, Chapter II.1: Classical AdS5xS5 string solutions*, *Lett. Math. Phys.* **99** (2012) 103 [1012.3986].

[62] N. Beisert, B. Eden and M. Staudacher, *Transcendentality and Crossing*, *J. Stat. Mech.* **0701** (2007) P01021 [hep-th/0610251].

[63] B. Basso, G.P. Korchemsky and J. Kotanski, *Cusp anomalous dimension in maximally supersymmetric Yang-Mills theory at strong coupling*, *Phys. Rev. Lett.* **100** (2008) 091601 [0708.3933].

[64] R. Roiban, A. Tirziu and A.A. Tseytlin, *Two-loop world-sheet corrections in $AdS_5 \times S^5$ superstring*, *JHEP* **07** (2007) 056 [0704.3638].

[65] S.A. Frolov, I.Y. Park and A.A. Tseytlin, *On one-loop correction to energy of spinning strings in S5*, *Phys. Rev. D* **71** (2005) 026006 [hep-th/0408187].

[66] S. Frolov and A.A. Tseytlin, *Multispin string solutions in $AdS_5 \times S^5$*, *Nucl. Phys. B* **668** (2003) 77 [hep-th/0304255].

[67] G. Arutyunov, J. Russo and A.A. Tseytlin, *Spinning strings in $AdS_5 \times S^5$: New integrable system relations*, *Phys. Rev. D* **69** (2004) 086009 [hep-th/0311004].

[68] N. Beisert, A.A. Tseytlin and K. Zarembo, *Matching quantum strings to quantum spins: One-loop versus finite-size corrections*, *Nucl. Phys. B* **715** (2005) 190 [hep-th/0502173].

[69] J.A. Minahan, A. Tirziu and A.A. Tseytlin, *$1/J^2$ corrections to BMN energies from the quantum long range Landau-Lifshitz model*, *JHEP* **11** (2005) 031 [hep-th/0510080].

[70] S. Frolov and A.A. Tseytlin, *Quantizing three spin string solution in $AdS_5 \times S^5$* , *JHEP* **07** (2003) 016 [hep-th/0306130].

[71] V. Mikhaylov, *On the Fermionic Frequencies of Circular Strings*, *J. Phys. A* **43** (2010) 335401 [1002.1831].

[72] I.Y. Park, A. Tirziu and A.A. Tseytlin, *Spinning strings in $AdS_5 \times S^5$: One-loop correction to energy in SL(2) sector*, *JHEP* **03** (2005) 013 [hep-th/0501203].

[73] M. Beccaria, S. Giombi, G. Macorini, R. Roiban and A.A. Tseytlin, *'Short' spinning strings and structure of quantum $AdS_5 \times S^5$ spectrum*, *Phys. Rev. D* **86** (2012) 066006 [1203.5710].

[74] R. Rohm, *Spontaneous Supersymmetry Breaking in Supersymmetric String Theories*, *Nucl. Phys. B* **237** (1984) 553.

[75] J.J. Atick and E. Witten, *The Hagedorn Transition and the Number of Degrees of Freedom of String Theory*, *Nucl. Phys. B* **310** (1988) 291.

[76] J.D. Blum and K.R. Dienes, *Strong / weak coupling duality relations for nonsupersymmetric string theories*, *Nucl. Phys. B* **516** (1998) 83 [hep-th/9707160].

[77] G.T. Horowitz and A.A. Tseytlin, *On exact solutions and singularities in string theory*, *Phys. Rev. D* **50** (1994) 5204 [hep-th/9406067].

[78] P. Meessen and T. Ortin, *Type 0 T duality and the tachyon coupling*, *Phys. Rev. D* **64** (2001) 126005 [hep-th/0103244].

[79] K. Becker, M. Becker and J.H. Schwarz, *String theory and M-theory: A modern introduction*, Cambridge University Press (2006).

[80] T. Banks, *The Tachyon potential in string theory*, *Nucl. Phys. B* **361** (1991) 166.

[81] A.A. Tseytlin, *On the tachyonic terms in the string effective action*, *Phys. Lett. B* **264** (1991) 311.