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The approximate number system and mathematics achievement: it's complicated. A thorough investigation of different ANS measures and executive functions in mathematics achievement in children

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ABSTRACT

The ability to represent approximate numerical magnitudes is often referred to as the approximate number system (ANS) and has regularly been proposed as foundational to mathematics achievement. However, some argue that the relation between ANS acuity and mathematics achievement ceases to exist when controlling for domain-general cognitive abilities. The current debate in the literature on whether ANS acuity remains a predictor to mathematics after strict control is applied leads to the need to simultaneously investigate domain-specific and domain-general foundational abilities in different ages. 174 Children took part in two computerised ANS tasks, two executive function tasks, a verbal skills task, two intelligence subscales, and a mathematics achievement task (i.e. global, formal, and informal mathematics achievement). Results demonstrated that, when controlling for intelligence and visuospatial memory, the relation between ANS acuity and mathematics achievement ceased to exist, and that ANS acuity might only play a predictive role in early informal mathematics.

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Introduction

Humans appear to possess the ability to represent and process non-symbolic quantities (Dehaene, 2011; Feigenson et al., 2013). This ability is often called the Approximate Number System (ANS) and is present in all humans as young as newborns (Dehaene, 2011; Izard et al., 2009; Xu & Spelke, 2000; Xu et al., 2005). In addition to their acquired approximate sense of quantities, humankind has also developed a symbolic number code and mathematics that can only be learned through educational instructions. Supported by correlational studies, it has been suggested that ANS acuity and mathematics achievement are positively associated (Hyde et al., 2016; Szkudlarek & Brannon, 2017). Further investigation through longitudinal studies proposed that individual differences in ANS acuity is predictive of mathematics achievement (Hyde et al., 2016; Libertus et al., 2011; Mazzocco et al., 2011). However, not all studies have managed to replicate results finding a link between ANS acuity and mathematics achievement (Castronovo & Göbel, 2012; Holloway & Ansari, 2009; Iuculano et al., 2008; Kolkman et al., 2013; Lyons et al., 2014; Sasanguie et al., 2014; Soltész et al., 2010; Vanbinst et al., 2012). When controlling for cognitive capacities (such as general intelligence, verbal skills and executive functions), a link between ANS acuity and mathematics achievement is more difficult to find (Fuhs & McNeil, 2013; Passolunghi et al., 2014; Simms et al., 2016). In addition, many studies reporting a link between ANS acuity and mathematics achievement had little or no control of non-numerical cognitive skills, and it is possible that such ANS acuity tasks rely, at least in part, on these skills (Hyde et al., 2016; Libertus et al., 2011; Starr et al., 2013; Wang et al., 2016).

In particular, other research in numerical cognition has focused on executive functions (EFs) and how they relate to mathematics abilities (Bull &

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Lee, 2014; Bull & Scerif, 2001; Bull et al., 2008; D'Amico & Guarnera, 2005). Similar to the results found on ANS acuity and mathematics achievement, correlations have been found between EFs and mathematics achievement, with longitudinal studies also suggesting that individual differences in EFs are predictive of mathematics abilities in children (Cragg & Gilmore, 2014; Holmes et al., 2008; Hornung et al., 2014). Furthermore, some studies investigating the link between ANS acuity and mathematics achievement found the association between ANS acuity and mathematics achievement to decrease, or even to disappear, when controlling for EFs, such as inhibition (Gilmore et al., 2013; Purpura et al., 2017; Schmitt et al., 2017), shifting skills (Purpura et al., 2017; Schmitt et al., 2017) or working memory (Purpura et al., 2017; Schmitt et al., 2017; Simms et al., 2016).

In light of the literature, inconsistent results concerning ANS, EFs and mathematics abilities have been found in different studies using different research paradigms. The current study attempts to further and thoroughly investigate the predictive roles of the ANS in mathematics achievement in primary school children (aged 4–7 years) with the use of the main different ANS tasks and measures, interchangeably used in the literature while also controlling for different EFs measures, as well as verbal skills, intelligence, age and gender. Such thorough investigation might help unravel the level of prediction of different cognitive factors in mathematical development. In addition, this should lead to a better understanding of which key skills (i.e. ANS, EFs, general cognitive skills) play a foundational role in the development and acquisition of mathematics achievement in children. Such thorough investigation on the key predictors of mathematics achievement in children could be key in the long run in developing enhanced mathematics interventions and teaching strategies.

Approximate number system and mathematics achievement

So far, a large number of studies have reported a link between individual differences in ANS acuity and symbolic mathematics achievement throughout the life span (Feigenson et al., 2013; Hyde et al., 2016; Piazza et al., 2010; Szkudlarek & Brannon, 2017). Correlational studies have abundantly observed a relation between ANS acuity and mathematics from early childhood (Chu et al., 2015; Halberda & Feigenson, 2008; Keller & Libertus, 2015; Libertus et al., 2013a) to adulthood (Agrillo et al., 2013; Dewind & Brannon, 2012; Libertus et al., 2012; Lindskog et al., 2014). Three meta-analyses of a large number of cross-sectional studies further support the existence of a link between ANS acuity and mathematics achievement (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2016).

Furthermore, longitudinal data have shown that individual differences in ANS acuity can predict mathematics achievement (Szkudlarek & Brannon, 2017). Some studies found that ANS acuity measured in children in preschool predicts their mathematics performance in primary school (Jordan et al., 2007; Libertus et al., 2011; Mazzocco et al., 2011). Similar predictive results have been found across longitudinal studies in different age groups and with different time spans: from 6 months apart (Libertus et al., 2013a; Mussolin et al., 2014) to 2–3 years between testing periods (Landerl, 2013; Libertus et al., 2013b; Mazzocco et al., 2011; Starr et al., 2013). Chen and Li (2014) also included longitudinal studies investigating the link between ANS acuity and mathematics achievement in their meta-analysis and further support that ANS acuity predicts later mathematics performance.

Further evidence for a relation between ANS acuity and mathematics achievement has been suggested in ANS training studies, where training the ANS seems to be associated with better and faster performance on symbolic arithmetic problems in both adults (Park & Brannon, 2013, 2014; Park et al., 2016; but see Lindskog & Winman, 2016; Merkley et al., 2017; Szucs & Myers, 2017) and children (Hyde et al., 2014; Khanum et al., 2016). For instance, in one training experiment by Park and Brannon (2013), adults were trained on approximate addition and subtraction tasks, resulting in an improvement in arithmetic performance. In an additional training study, Hyde et al. (2014) assigned 6- to 7-year-old-children to one of four training conditions: a non-symbolic approximate addition task, a non-symbolic approximate comparison task, a line-length addition task or a brightness comparison task. Results revealed that children who received a training session on the non-symbolic approximate addition task and the non-symbolic approximate comparison task solved symbolic addition problems faster than the children trained on length addition and brightness comparison.

The literature on the existence of an association between ANS and mathematics achievement, as well as the ANS training studies led to the suggestion that ANS might have an important role to play in the acquisition of symbolic mathematics and might even be foundational to formal mathematics (Dehaene, 2011; Szkudlarek & Brannon, 2017; but see Carey & Barner, 2019; Szucs & Myers, 2017). Starr and Brannon (2015) proposed several possible explanations as to how the ANS might relate to mathematics achievement. One hypothesis was that because humans are able to mentally manipulate non-symbolic numerical representation in approximate arithmetic operations (Gilmore et al., 2010; Hyde et al., 2014), the ANS could play a foundational role in the development of symbolic arithmetic operations. Another hypothesis was that ANS acuity could be particularly critical when learning symbolic numbers and therefore be critical in acquiring symbolic number skills rather than maintaining and progressing in mathematics skills (Fazio et al., 2014; Starr & Brannon, 2015; Szkudlarek & Brannon, 2017). On the other hand, Carey and Barner (2019) suggested that approximate number representations might not be foundational in how children learn number. The authors hypothesised that children learn to understand whole numbers in two phases. First, children associate small exact number words with small exact sets. Second, children learn larger number words by mastering the logic of exact counting algorithms (e.g. by one-to-one-correspondence). In other words, children are provided with a counting list, which is a seemingly meaningless list and they gradually learn how this list matches with numbers.

Current challenges in the relation between ANS acuity and mathematics achievement

Despite the abundance of evidence suggesting a link between ANS acuity and mathematics achievement, not all studies have reported a correlation between them (Castronovo & Göbel, 2012; Holloway & Ansari, 2009; Iuculano et al., 2008; Kolkman et al., 2013; Lyons et al., 2014; Sasanguie et al., 2014; Soltész et al., 2010; Vanbinst et al., 2012). Three main hypotheses have been suggested to account for these conflicting results. First, age could play a role in whether the link between ANS acuity and mathematics achievement is found or not. Second, various dependent and independent measures have been used in the ANS and mathematics achievement literature so far, including in the literature described in this introduction. Third, domain-general skills, such as general intelligence, verbal skills and EFs (such as inhibition and visuospatial skills) might be a contributing factor to the link between ANS acuity and mathematics achievement. These three hypotheses are now discussed in turn.

Age and the relation between ANS acuity and mathematics achievement

Age has been suggested as a likely contributing factor as to when a link between ANS acuity and mathematics achievement is found (Castronovo & Göbel, 2012). Indeed, it has been hypothesised that the link between individual differences in ANS acuity and mathematics skills appears to be largely found in studies in children up to early adulthood rather than studies in adults (Fazio et al., 2014). Moreover, Gimbert et al. (2019) demonstrated that ANS acuity was predictive of mathematics achievement in 5-year-old-children, but no longer in 7-year-old-children. It has been suggested that the ANS plays a particularly important foundational role in the acquisition of symbolic numerical knowledge and mathematics mainly at key stages in human development, such as at the start of formal mathematics learning (Shusterman et al., 2016). Due to the suggested foundational role of ANS, in particular at the start of formal mathematics schooling (e.g. Bonny & Lourenco, 2013; Lindskog et al., 2014; Nys et al., 2013), one aim of this study will be to explore whether age-related differences can be found in the link between ANS and mathematics achievement in children at different stages of formal mathematics schooling: (a) prior to formal schooling (4 years); (b) at the start of formal schooling (5–6 years); and (c) after at least one year of formal schooling in mathematics (7 years).

Various dependent and independent measures

It has also been suggested that different findings on the association between ANS acuity and mathematics achievement result from the use of different tasks, and measures, to address ANS acuity and mathematics achievement (Dietrich et al., 2015, 2016; Szkudlarek & Brannon, 2017). For example, measures for mathematics skills can range from more informal mathematics skills (e.g. counting skills first acquired in children outside of the context of formal schooling, such as through spontaneous interactions with their environment or through imitations of adults and siblings), to complex formal mathematics skills developed through education (e.g. calculations; arithmetic problem solving) (Gebuis & van der Smagt, 2011; Libertus et al., 2013b). Gebuis and van der Smagt (2011) found in their research that performance on a non-symbolic magnitude comparison task could only be significantly predicted by symbolic additions, but not subtractions, multiplications or divisions. A second example is when Libertus et al. (2013b) found a correlation between ANS acuity and informal mathematics questions (e.g. "count up as high as you can"), as defined by the Test of Early Mathematics-3 (TEMA-3; Ginsburg & Baroody, 2003), but not with the formal TEMA-3 questions (e.g. "write the number 7"). In addition, support has been found for a stronger link between ANS acuity and mathematics prior to formal instructions (Fazio et al., 2014) and it has been suggested that the ANS becomes less involved in mathematics the more children advance in formal mathematics (Geary, 2013). Therefore, the current study will aim at exploring the link between ANS and mathematics achievement in general, but it will also aim at further exploring the link between ANS acuity and both formal and informal mathematics achievement via the use of formal and informal mathematics scores from the TEMA-3 standardised maths test, as used in the literature (e.g. Libertus et al., 2013b, 2016). Such differentiation between formal and informal mathematics achievement should help to better understand the nature of the link between ANS acuity and different types of mathematics skills.

As previously mentioned, different tasks and measures have also been used in the literature to assess ANS acuity (Clayton et al., 2015; Gilmore et al., 2010; Gimbert et al., 2016; Sella et al., 2016; Xenidou-Dervou et al., 2017). Often used ANS acuity tasks are approximate magnitude comparison tasks (Halberda & Feigenson, 2008; Inglis et al., 2011; Libertus et al., 2013b) or more complex approximate arithmetic tasks (Gilmore et al., 2010; Hyde et al., 2014). In non-symbolic approximate comparison tasks, such as the Panamath task by Halberda et al. (2008), participants have to judge which of two quantities is more numerous, whilst perceptual variables (i.e. dot size, density and total area) are controlled for. ANS acuity in non-symbolic comparison tasks is indexed by using reaction times, mean accuracy or Weber fraction (w). Approximate non-symbolic arithmetic tasks on the other hand require participants to mentally manipulate two non-symbolic quantities and decide whether a third quantity is less or more numerous than the addition or subtraction of the first two quantities (Gilmore et al., 2010; Hyde et al., 2014). For instance, in an Approximate Addition task, two separate arrays of dots would disappear sequentially inside a box displayed on a screen. A third dot array would then be presented next to the box. The participant's task would be to decide whether the dot array next to the box has more or less dots than the two sets of dots inside the box. Mean accuracy and reaction times can then be recorded as measures of ANS acuity.

It has been suggested that different ANS tasks entail different cognitive demands, which might account for the inconsistency of results found in the literature and might also preclude direct comparison of different results found when different ANS tasks have been used (Dietrich et al., 2016, 2015; Gilmore et al., 2011, 2014). For example, previous literature proposed that while inhibition is required in non-symbolic comparison tasks (e.g. Gilmore et al., 2013), non-symbolic Approximate Addition tasks on the other hand would rely more on visuo-spatial memory instead (Xenidou-Dervou et al., 2014). Moreover, research also suggests that even similar non-symbolic comparison tasks can have a low inter-reliability, when using different ANS acuity measures as index of ANS acuity (e.g. accuracy vs. Weber fraction; Dietrich et al., 2016; Inglis & Gilmore, 2014; Norris & Castronovo, 2016) or controlling for different continuous perceptual variables (e.g. cumulative surface area of the dot array vs. convex hull; Clayton et al., 2015; Gilmore et al., 2016). Finally, it has been shown that the immediate test-retest reliability of the approximate comparison task when using Panamath appears to be low (Clayton et al., 2015). In the current study, the two different ANS tasks, commonly used in the literature, will be pitted against each other, so that the differences in predictive value of ANS acuity (as measured by mean accuracy and the Weber fraction of the magnitude comparison task and mean accuracy of the Approximate Addition task) to mathematics achievement can be compared.

Domain-general cognitive abilities and mathematics achievement

Following the suggestion that different types of ANS task involve different cognitive functions, domain-general cognitive competences and their involvement in solving ANS tasks gathered further attention. It has even been suggested that general cognitive competences (e.g. intelligence) play a role in the observed association between ANS acuity and mathematics achievement (Dietrich et al., 2015). Indeed, learning mathematics requires a spectrum of both domain-general (e.g. IQ; verbal skills; EFs) and domain-specific (e.g. ANS acuity) cognitive abilities (Passolunghi et al., 2014; Vanbinst & De Smedt, 2016). A number of studies in children found support for domain-general abilities as predictors of mathematics skills, such as intelligence (Passolunghi et al., 2014, 2015), memory (Cirino et al., 2016; Passolunghi et al., 2014) and phonologic skills (Cirino et al., 2016; Passolunghi et al., 2015; Vanbinst et al., 2014). Moreover, some studies found a reduction or suppression of the link between ANS acuity and mathematics achievement when controlling for these domain-general cognitive abilities (Passolunghi et al., 2014; Vanbinst & De Smedt, 2016). However, other research, including a meta-analysis by Chen and Li (2014) found that the link between ANS acuity and mathematics achievement remained significant when controlling for domain-general abilities (Halberda et al., 2008; Hornung et al., 2014; Xenidou-Dervou et al., 2017). Therefore, similarly to the previous issues raised regarding the literature on ANS, control of general cognitive competences such as intelligence and verbal skills has often been lacking and when control of general cognitive competences was applied, inconsistent results were found. Although general intelligence and verbal skills are not primary variables of interest within this study, they have been found to underlie general academic achievement and will thus be included as covariates. Accordingly, the current study will also aim at applying a thorough control of general intelligence and verbal skill measures when investigating the link between ANS acuity and mathematics achievement in children.

More recently, EFs in particular have been suggested to play a critical role in the association found between ANS acuity and mathematics achievement (Purpura et al., 2017; Schmitt et al., 2017). Such hypothesis emerged from the existing debate as to whether responses on non-symbolic comparison tasks truly reflect numerical judgements rather than judgements based on non-numerical sensory cues, such as confounding continuous perceptual variables (Clayton et al., 2015; DeWind et al., 2015; Gilmore et al., 2016). Indeed, control of perceptual variables in non-symbolic comparison tasks involve stimuli (i.e. sets of dots to be compared) to either be congruent (i.e. the most numerous, the largest surface area and average dot size) or incongruent (i.e. the more numerous, the smaller the dots on average or the smaller the total surface area). Gilmore et al. (2013) argued that inhibitory control plays a significant role in approximate comparison tasks, since in congruent trials (i.e. when continuous perceptual and numerical variables are congruent—the visually larger set corresponds to the more numerous set), the visual cues play a facilitator role for the participants to make their numerical judgement; whilst in incongruent trials (i.e. when continuous perceptual and numerical variables are incongruent—the visually larger set corresponds to the less numerous set), there is interference between the visual and numerical cues, so that participants have to inhibit perceptual information to process numerical information. Moreover, when controlling for inhibition skills, Gilmore et al. (2013) failed to find a link between ANS acuity and mathematics achievement, therefore leading the authors to assume that inhibition rather than non-symbolic numerical skills is significantly associated with mathematics achievement in approximate comparison tasks.

On the other hand, visual short-term memory and working memory appear to play a key role in a non-symbolic Approximate Addition task. This is because in that particular task, the dot arrays used as addends in a trial need to be maintained in the visual short-term memory and manipulated in visual working memory to be added together and compared to the third dot array, before a numerical judgement can be made (Xenidou-Dervou et al., 2014).

Furthermore, it is widely agreed that EFs, such as short-term and working memory (Brankaer et al., 2014; Bull et al., 2008; Cirino et al., 2016; Purpura et al., 2017; Vanbinst et al., 2014), inhibition (Bull & Scerif, 2001; Gilmore et al., 2013; Purpura et al., 2017) and shifting skills (Bull & Scerif, 2001; Yeniad et al., 2013) constitute significant predictors of mathematics achievement in children. However, once more, some contradictory findings have been found, since in some studies a relation between ANS acuity and mathematics skills could be preserved even after controlling for EFs (Cirino et al., 2016; Keller & Libertus, 2015; Xenidou-Dervou et al., 2013). For instance, Keller and Libertus (2015) found that the association between an approximate magnitude comparison task and mathematics performance persisted in children aged 3–6 years, even when controlling for inhibition skills. Similar findings were reported by Hornung et al.

(2014), demonstrating that performance on a magnitude comparison task was predictive of early number competences (verbal counting, dot counting and Arabic number comparison) in kindergarten even after controlling for short-term and working memory.

The all-encompassing aim of this study is to examine whether ANS measures are still predictive of mathematics achievement when a strict control of domain-general skills, such as intelligence, verbal skills and EFs known to be involved in ANS tasks or mathematics achievement is applied. In doing so, the design of this study further allows us to explore the involvement of various EF (such as inhibition, shifting and visuospatial memory) on ANS tasks, mathematics achievement and the link between ANS acuity and mathematics achievement.

The present study

This cross-sectional study is part of a wider longitudinal project (where the same children are followed-up after one year). The results reported here focus on the current debate of domain-general versus domain-specific foundations to mathematics achievement and their importance at one time point. The current study aimed at further investigating and clarifying the nature of the association between ANS acuity and mathematics achievement in 4- to 7-year-old-children, with the use of a thorough research design, including formal and informal measures of mathematics achievement and different measures of ANS acuity; whilst controlling for different measures of EFs, verbal skills and intelligence. Mathematics achievement was measured with the Test of Early Mathematics Ability (TEMA-3), which includes questions that can be classified as either assessing formal or informal mathematics skills (Ginsburg & Baroody, 2003). The ANS was measured using both an approximate comparison task (i.e. Panamath) (Halberda & Feigenson, 2008; Halberda et al., 2008) and an approximate arithmetic task (i.e. Approximate Addition task) (Hyde et al., 2014). Executive functions such as inhibitory control and shifting skills were measured using the Eyes task (Burns et al., 2012), whilst visuospatial short-term memory was tested using the Pathspan (created by Hume, 2012, available at https://hume.ca/ix/pathspan/). Verbal skills and intelligence were measured using the British Picture Vocabulary Scale and the Block Design and Information subtests of the Wechsler Preschool and Primary Scale of Intelligence. The use of such a thorough research design in young children should allow us to gain a clearer understanding of what skills constitute significant factors in mathematics achievement, whilst addressing most of the issues raised previously which could account for the discrepancy of results found in the literature on the foundational role of ANS acuity and its association with mathematics achievement.

Method

Participants

174 children between the ages of 4 and 7 years participated in this study (87 females, $M_{age} = 70.34$ months, SD = 9.76 months). 62 children attended Foundation stage 2 (reception; ages 4–5 years) (35 females, $M_{age} = 59.7$ months, SD = 2.9 months). 57 children were in Year 1 (ages 5–6 years) (28 females, $M_{age} = 71.4$ months, SD = 4.8 months) and 55 in Year 2 (ages 6–7 years) (24 females, $M_{age} =$ 81.7 months, SD = 4.2 months). Foundation stage 2 or reception is the first year of compulsory education in the UK and corresponds to Year 0 of primary school. Teaching is informal and happens through games and play. Year 1 and Year 2 correspond to the first and second year of formal primary education. The sample size was determined by several restrictions, in particular the number of schools, parents and children agreeing to take part in this study within the time limit of one academic year. Children were recruited from 3 different schools in East Yorkshire. Each school was invited by e-mail to participate in the study and information sheets for school and parents were provided. Once approval was gained from a school, parents were sent information on the study and had the option to request their child or children be removed from the study. At the start of each experimental session, children were verbally asked whether they were willing to participate or not. Teachers were asked to point out any children with severe learning difficulties or limited English speech ability to be excluded from the study. The study received ethical approval from the Ethics Committee of the University of Hull.

The anonymised data that support the findings of this study as well as the output of the analyses have been made available on OSF and can be accessed at https://osf.io/g3ah7/?view_only=3c0103d449f643d2849b84dfda7b4e24.

Material

Approximate number system

Approximate numerical comparison task. The first task used to measure ANS acuity was a version of the Panamath Software (Halberda et al., 2008). This task was chosen as it is often used as an ANS task, which would then allow us to compare our results to those found so far in the literature. Reliability of the Panamath measures are inconsistent in the literature varying from poor to good reliability, with test-retest reliability of the Weber fraction ranging from $r = .41$ to $r = .60$ (Inglis & Gilmore, 2014) and split-half reliabilities being reported between $r = .47$ (Price et al., 2012) and $r = .78$ (Halberda et al., 2012). Similarly for mean accuracy, the literature previously reported test-retest reliability between $r = .47$ and $r = .79$ (Inglis & Gilmore, 2014) and split-half reliability around $r = .69$ (Libertus et al., 2016). The perceptual controls chosen are the default settings, as this is often used in research using the Panamath in children. In this task, two arrays of spatially separated yellow and blue dots were simultaneously presented on a 15.6 inch laptop screen. Children had to choose whether there were more yellow or blue dots displayed on a screen. The two arrays were presented simultaneously, the blue dots in a blue frame on the right side of the screen and the yellow dots in a yellow frame on the left side. The frames were accompanied by child friendly characters in matching colours to the frames, namely the yellow Big Bird and the blue Cookie Monster from the TV show Sesame Street. The dots were presented for 2,000 ms, as in previous studies with children (Halberda & Feigenson, 2008; Halberda et al., 2008) followed by the empty frames until the child gave a response by pressing a yellow-dotted key (the letter "A" on the left side of a Qwerty keyboard) if they thought there were more yellow dots and a blue-dotted key (the letter "L" on the right side of a Qwerty keyboard) if they thought that there were more blue dots. The experimenter initiated each trial by pressing the space bar when the child paid attention to the screen. The number of dots ranged from 4 to 15 and the ratios between the two quantities to be compared varied from 1.2 to 2 (1:2, 2:3, 3:4, 6:7) based on the parameters used in Halberda and Feigenson (2008). The Panamath task included 8 practice trials and 60 testing trials. In 30 of the trials the cumulative surface area was congruent with the number of dots (number of dots and cumulative area were correlated) and the other 30 trials the individual dot size was on average the same size on both sides. Within each frame the dot sizes varied randomly with an average of 36 pixels and variations up to 20%. The correct answer was on the left 50% of the time and on the right 50% of the time. ANS acuity was obtained by calculating the Weber fraction and mean accuracy of all responses for the Panamath task.

Approximate addition task. As a recent debate in the literature suggested that the link between ANS acuity, measured with the Panamath task, and mathematics abilities might be driven by inhibitory demands rather than by the ANS itself, an Approximate Addition task was also introduced to measure ANS acuity in children. This task was chosen as it was previously reported to successfully increase mathematics achievement after training. Its reliability has also been found to be good in the literature ($r = .87$ and $r = .90$; Gilmore et al., 2011, 2014 respectively).

This computerised non-symbolic Approximate Addition task was adapted[1] from Hyde et al. (2014), where two dot arrays appeared sequentially on a screen and the child had to decide whether a third dot array was more or less numerous than the sum of the first two arrays presented. The task started with a rectangle in the middle of the screen presented to the child as a box, followed by the first array of dots appearing after 500 ms on the left of the rectangle. After one second the array disappeared behind the rectangle, while the child was told that the dots went inside the box. A second array was then displayed on the right of the rectangle for the same period of time before moving behind the rectangle as well. The child was informed that the second group of dots joined the first group of dots inside the box and was then instructed to keep in mind the sum of both groups of dots, which were then out of sight "inside the box". After 1,250 ms the rectangle disappeared and revealed a

[1]The adaptations consisted of changes made to the introductory instructions and shortening the task. In this study, children were told that dots moved into a box to limit testing time, while a child friendly introduction was used in Hyde et al. (2014). Experimental trials in this study were identical to the ones used in Hyde et al. (2014) with a ratio of 0.57.

third array that was displayed until the child pressed one of two keys. The letter "J" on a Qwerty keyboard held a sticker with the written word "more" and had to be pressed when the child judged the third set of dots to be larger than the sum of the first two arrays; and the letter "F", with the written word "less" had to be pressed when the third group of dots was judged as being less numerous than the sum of the first two sets of dots. After each trial, the children were verbally reminded to press either the button "more" (with the experimenter pointing to the button with the word "more" written on it) or the button "less" (with the experimenter pointing to the button with the word "less" written on it) if they thought that the third set of dots presented was more or less numerous than the first two sets of dots combined. The dots were black on a white background and the rectangle was light grey. The addends were between 7 and 34 dots, while the outcome of the sum was between 16 and 56 dots. The ratio between the displayed outcome and the actual outcome was 0.57 (7:4), which corresponds to the easier trials used in Hyde et al. (2014). Only the easiest ratio was chosen in order to adapt the task to the younger age range tested in this study and to shorten the duration of the task. The addends used per trials were identical to the ones used in Hyde et al. (2014). For every numerical value of the dot arrays (addends and outcomes), two sets of images of dot arrays were created (each set containing five different images produced for that numerical value). One set of dot arrays was controlled on dot size and inter-dot spacing and varied on total area and luminance, while the second set was controlled for total area and total luminance, but varied on dot size and spacing. The programme randomly selected one of the two sets and consecutively randomly selected one of the five images within that set to be displayed for every trial. The task consisted of 30 trials with 4 practice trials. The mean accuracy of all trials was used as the ANS acuity measure for this task. Due to a large variability in the reaction times of young children on this task, reaction times were not used in the analyses (Bucsuházy & Semela, 2017).

Mathematics achievement

Children's mathematics achievement was measured using form A of the Test of Early Mathematics Ability-third edition (TEMA-3A) (Ginsburg & Baroody, 2003). TEMA-3 assesses a child's informal and formal mathematical knowledge and provides overall raw and standardised scores of mathematics achievement in children aged 3–8 years. This mathematics test has often been used in research on numerical cognition with children as a composite score of mathematics (Bonny & Lourenco, 2013; Chu et al., 2015; Mazzocco et al., 2011; Wang et al., 2016). This test has also been used in previous research to explore the distinction between informal and formal mathematics (Libertus et al., 2013b, 2016). Informal mathematics is defined by TEMA-3 as the skills that children acquire outside of the context of formal schooling, such as through spontaneous interactions with their environment (e.g. observations that adding an object to a collection of objects results in more objects or learning through children's television shows) and through imitations of adults and siblings (e.g. learning birthdays and counting the age of siblings). Formal mathematics as defined by TEMA-3 are the knowledge and understanding about numbers that children get taught in school, such as writing symbols, understanding additions and subtractions or grouping items by five or ten (Ginsburg & Baroody, 2003). Informal mathematics in the TEMA-3 task covers areas such as numbering skills (e.g. verbally counting quantities), number comparison facility (e.g. "which of two spoken number words is larger?"), informal calculations (e.g. solving word problems aided by tokens or fingers) and informal number concepts (e.g. cardinality principle: the knowledge that the last number in a counting sequence when counting a set of objects is also the number of objects in the set). Formal mathematics covers areas in numeral literacy (e.g. reading and writing Arabic numerals), mastery of number facts (e.g. retrieving addition, subtraction and multiplication facts), calculation skills (e.g. solving mental and written addition and subtraction problems) and understanding number concepts (e.g. "how many tens are in a hundred"). The total test consists of 72 questions with the test ending after 5 consecutive incorrect responses. 40 questions are informal mathematics questions and 32 are formal mathematics questions. The TEMA-3 has been reported as having good reliability (between $r = .82$ and $r = .93$) (Ginsburg & Baroody, 2003).

Control measures

Verbal skills. The second edition of the British Picture Vocabulary Scale (BPVS-2) was used to assess children's verbal skills as a control measure of basic verbal intelligence. This test is a normed test of hearing vocabulary for Standard English, comprised of 14 sets of 12 test items gradually increasing in difficulty. The test can provide a standardised score and a raw score. For each trial, a page with four pictures was presented to the participant. The examiner said a word and the participant had to point to the picture that best illustrates the meaning of the word. The test ended if a participant had four or fewer correct items in one set. BPVS-2 split-half reliability has also been reported to be good in previous literature at $r = .86$ (Gilmore et al., 2014).

Intelligence. Two subtests of the Wechsler Preschool and Primary Scale of Intelligence—Third Edition (WPPSI-III) were used as a measure for general abilities. The WPPSI-III subtests "block design" and "information" assess intellectual functioning in performance and verbal cognition respectively. In the block design subtest, participants had to recreate block designs that were shown either as a constructed model or a picture by using one-colour blocks or two-colour blocks within a specific time limit, from 30 seconds up to 120 seconds for more complex block designs. The test ended after 3 consecutive wrong answers. The second subtest was information where participants had to verbally answer a variety of 34 questions addressing general knowledge, ranging from easy questions (e.g. "what do people write with") to more difficult questions (such as "what is the biggest ocean in the world"). The test terminated after 5 consecutive incorrect answers or when the final question was reached. The mean percentage of correct answers of block design and information were taken together to form a WPPSI-III score. The average split-half reliability for WPPSI-3 subtests range between $r = .83$ and $r = .95$, and test-retest reliability for subtests are between $r = .70$ and $r = .90$.

Executive functions

EFs have been suggested to be crucial in mathematics achievement such as, working memory, inhibition, shifting skills and visuospatial skills (Gilmore et al., 2013; Passolunghi et al., 2014). Two EF tasks were chosen accordingly: (1) the Eyes Task measuring inhibition and shifting skills; and (2) the Pathspan, measuring visuospatial short-term memory. Note that the forwards Pathspan task is not a pure measure of executive functions as it measures visuospatial short-term memory rather than visuospatial working memory which can be measured with a backwards Pathspan task. Nevertheless, due to the young age of the children, a backwards visuospatial span task proved too difficult and would not have allowed a follow-up of the same measure over all ages. Thus, throughout the paper the term EF will be used to represent both the Eyes Task and the forwards Pathspan task, reflecting respectively inhibition and shifting skills, and visuospatial short-term memory.

Inhibitory control and shifting skills. The Eyes Task from Burns et al. (2012) was used to examine inhibitory control as well as shifting skills. Due to the multitude of tasks already in this study, a short task combining inhibition and shifting skills was used to limit testing time for children. A face appeared on the left or the right side of a screen that would either present a gaze straight down or down and across an angle of 45°. The instructions were to press either the right or left button depending on where the eyes were looking. Participants had to press the button on the same side as the location of the face when the eye gaze was straight down (congruent trial) and press the button on the opposite side of the location when the eye gaze was across (incongruent trial). Consequently, participants had to inhibit the tendency to press the button on the same side as the location of the face in incongruent trials, as a traditional Simon task (i.e. suppressing the irrelevant location of a stimulus appearing either left or right, while responding with a left or right located button depending on the colour of the stimulus) (Simon & Berbaum, 1990). The task consisted of 20 randomised trials and 4 practice trials. The child needed to constantly switch between the rule of the eyes gazing down or across, which makes this task a measure for switching ability as well as inhibitory control (Burns et al., 2012; Davidson et al., 2006). Mean accuracy of all trials was used in this study. Reaction time measures were recorded but not used due to a large variability because of the young age range of the children (Bucsuházy & Semela, 2017). Reliability information about this specific test is unknown and the task conducted in this study did not consist of enough trials to calculate a split-half reliability. Note that Burns et al. (2012) demonstrated that the trials for which switching was required were responded to more slowly and with less accuracy than the non-switch trials, showing the validity of this task as shifting task as well as inhibition task.

Visuospatial short-term memory. To assess visuospatial short-term memory, the Pathspan, an adaptation of the Corsi block tapping task on Ipad, was used (developed by Hume, 2012). Nine buttons appeared on screen in a pseudorandom pattern. The buttons flashed for 1,000 ms per button in a certain pattern until a tone was heard. The child then had to remember the sequence by touching the buttons in the same order as the flashing. The experiment started with a series length of two up to nine. Every length was presented three times with a different order to the sequence and the experiment ended with three incorrect responses on the same length. The score for this task was the highest sequence length with at least one correct response. Internal consistency of the Pathspan was previously reported with a Cronbach's alpha of .70 and test-retest correlations between Pathspan performances one year apart was $r = .44$ and between Pathspan performances two years apart it was reported to be $r = .56$ (LeFevre et al., 2010).

Procedure

Children were tested individually in a quiet room in their school. The complete assessment time was approximately 1 hour and 10 minutes split into 3 sessions with minimum 24 hours and maximum 1 week between sessions. After every session, children received a small reward in the form of a sticker. The tasks were conducted in the following order: session 1—TEMA-3A; session 2—Panamath, Eyes task, WPPSI-3 (block design and information); session 3—Approximate Addition task, BPVS-2 and the Pathspan. The order of testing was the same for all participants to ensure the same testing experience for all children. Instructions were presented orally by the examiner.

Results

Across all statistical analyses for each age group (foundation, year 1 and year 2) all scores above or below three standard deviations from every task's mean were deleted (1.38% of all data points). Children with either the dependent variable missing or three or more scores missing from the 8 tasks were excluded from the data (16 children; total sample size of 174 was reduced to 158 for analyses (78 females)). The remaining missing scores (3.4%) were deleted pairwise for the correlations, but were imputed with the mean[2] for the regression analyses in order to be able to retain a large enough sample size. Although, there is no established cut-off regarding an acceptable percentage of missing data, a missing rate of 5–10% or lower is considered as inconsequential (Dong & Peng, 2013; Schafer, 1999). Nevertheless, the causes of our missing data were explored. The most frequent causes for missing data were problems with the task or computer used, or interruption of the task by a third party (e.g. teacher, classmate). However, excluded children were mostly from foundation stage as some of the youngest children did not possess some necessary skills (e.g. being able to sit still during the larger part of a task, being able to concentrate on the task, being able to verbally answer mathematics questions) to complete all tasks. Six children from foundation stage scored 0 on formal mathematics achievement of which 2 children scoring 0 were excluded from the data as they failed to reach item 14 (following 5 consecutive incorrect answers), which corresponds to the 1st item testing formal mathematics in the TEMA-3. The other 4 children scoring 0 on the formal TEMA-3 task, of which 3 children were asked 2 formal mathematics questions and 1 child was asked 3 formal mathematics questions, were included in the data. As a data check prior to the main analyses, the performance on the Approximate Addition task was explored for a size effect. A size effect was found in our data on the Approximate Addition task with better performance on smaller magnitudes, $F\ (1,3148) = 27.97$, $p < .001$. The size effect is a characteristic of the ANS, indicating that the Approximate Addition task tests the same underlying ANS concept as the magnitude comparison task, as repeatedly suggested in the literature (Gilmore et al., 2010; Hyde et al., 2014; Khanum et al., 2016; Park & Brannon, 2013, 2014). Raw correlational analyses (see Table A1) and descriptive statistics per task (see Table A2) used for these analyses are displayed in the Appendix. The analyses were conducted on SPSS and will be reported in two

[2]Due to some debate in the current literature on replacing missing values by the mean, the same regression analyses were run when missing values were estimated with multiple imputations on SPSS. The results converge completely regardless of the imputation strategy, likely due to the small number of missing values.

parts. The alpha value or *p*-value was set to 0.05 for all analyses conducted. In the first part, a series of partial correlations were conducted depending on the control measures (1. controlling for demographics (i.e. gender, age and school); 2. Controlling for demographics, verbal skills and intelligence; 3. Controlling for demographics, verbal skills, intelligence and the individual EF tasks), as this provides a clear overview to understand the data. Note that gender was controlled for in all analyses due to controversial findings in the literature on potential gender differences in mathematics achievement (for a review see Spelke, 2005). No specific hypotheses were made on the role of gender in this study, but gender was added as covariate in the statistical analyses due to the debated potential gender differences in mathematics achievement. Furthermore, the school was controlled for to account for possible differences in mathematics achievement between schools and to account for the different deprivation decile of the areas where the schools were based (of which parents come from the area around the school; deprivation decile of 1 in two schools and 10 in another school). In a second part, hierarchical regression analyses were conducted, which we deemed to be the best analyses to answer the current research question on whether ANS acuity has any added predictive value to mathematics achievement after controlling for EFs. All analyses output has been made available on OSF and can be accessed at https://osf.io/g3ah7/?view_only=3c0103d449f643d2849b84dfda7b4e24. Note that supplementary Bayesian regressions have also been carried out and are equally available on OSF.

Correlations

Correlational analyses between ANS measures and TEMA-3A scores were conducted in four steps according to the measures controlled for. Since most tasks used in this study were not standardised, age in months was always controlled for. Therefore, the raw scores of the TEMA-3A task were used in the analyses instead of the standardised scores so that age would not be controlled for twice. A first correlation analysis between TEMA-3A and the three ANS measures controlled for demographic variables, such as gender, age and school. Results showed that TEMA-3A scores correlated with accuracy, $r(134) = .261$, $p < .01$, and Weber fraction in the Panamath task, $r(134) = -.332$, $p < .001$ and with accuracy in the Approximate Addition task, $r(134) = .255$, $p < .01$. All three ANS measures also correlated with each other ($p_s < .05$).

In addition to gender, age and school as control measures, a second partial correlation was conducted to control for verbal skills and intelligence (BPVS-2 and WPPSI-3 scores[3]). Panamath measures still significantly correlated to TEMA-3A scores (Panamath accuracy, $r(130) = .254$, $p < .01$; Weber fraction, $r(130) = -.224$, $p < .01$), however accuracy on the Approximate Addition task only marginally correlated with TEMA-3A scores, $r(130) = .162$, $p = .060$. Moreover, when controlling for verbal and cognitive skills, measures of ANS acuity from different tasks (i.e. Weber fraction in Panamath vs. Accuracy scores in Approximate Addition task) no longer correlated with each other, $r(130) = -.120$, $p = .169$. Nevertheless, the accuracy of Panamath and the accuracy of the Approximate Addition task were still positively correlated with each other at this stage, $r(130) = .178$, $p < .05$.

In the third step, the added control measure was the Eyes task. TEMA-3A was still significantly correlated with ANS acuity measures in Panamath (accuracy, $r(123) = .248$, $p < .01$; Weber fraction, $r(123) = -.213$, $p < .05$), but only marginally correlated with ANS acuity measured in the Approximate Addition task, $r(123) = .171$, $p = .057$. Moreover, measures of ANS acuity in both ANS tasks were no longer correlated (Panamath Weber fraction vs. accuracy Approximate Addition task, $r(123) = -.096$, $p = .288$; Panamath accuracy vs. accuracy Approximate Addition task, $r(123) = .165$, $p = .066$). Finally, Table 1 displays the correlations between the three ANS measures and TEMA-3A, when controlling for age, gender, school, BPVS-2, WPPSI-3, Eyes Task and Pathspan. When all these control measures were controlled for, then the only ANS acuity measure which still presented a significant correlation with mathematics achievement (TEMA-3A) was Panamath accuracy.

In addition to these correlations, a partial correlation was also carried out with TEMA-3A scores separated between formal mathematics and informal mathematics scores. This enabled us to investigate whether there is a difference between the link that ANS acuity has with formal versus informal mathematics. Table 1 also illustrates the partial correlations between all three ANS acuity measures and

[3]WPPSI-3 scores are the mean percentage of both subtests, block design and information.

Table 1. Partial correlations with overall, formal versus informal TEMA-3A scores, controlled for gender, age, school, BPVS-2, WPPSI-3, Eyes task and Pathspan ($N = 131$).

	TEMA-3A	Informal	Formal	Panamath acc	Panamath w
TEMA-3A	___				
Informal TEMA-3A	.803***	___			
Formal TEMA-3A	.738***	.341***	___		
Panamath acc	.191*	.235**	.009	___	
Panamath w	−.157	−.289**	.080	−.790***	___
Addition acc	.134	.149	.149	.131	−.061

Note: *$p < .05$, **$p < .01$, ***$p < .001$.

formal as well as informal TEMA-3A scores, controlled for age, gender, school, BPVS-2, WPPSI-3, Eyes Task and Pathspan. Results demonstrate that the Panamath ANS acuity measures positively correlated with informal TEMA-3A scores, whilst they did not correlate with formal mathematics scores. Furthermore, the accuracy of the Approximate Addition task did not correlate with either formal or informal TEMA-3A scores.

Finally, as exploratory analyses, partial correlation analyses were conducted to investigate the relation between EFs and formal/informal mathematics achievement scores, as well as ANS acuity measures. When controlling for age, gender, school, BPVS-2 and WPSSI-3, both the Eyes task and Pathspan significantly correlated with Panamath accuracy and Weber fraction, see Table 2. However, EF measures did not correlate with accuracy in the Approximate Addition task. Pathspan measures correlated with both informal and formal TEMA-3A scores, but the Eyes task only correlated with formal TEMA-3A scores.

Hierarchical regressions

In order to examine whether the three different ANS measures might have a predictive role, and not just a correlational role in mathematics achievement, three separate 4-stage hierarchical multiple regression analyses were conducted with TEMA-3A raw scores as the dependent variable. Prior to conducting the regression analyses, the necessary assumptions were examined. The sample size used for the multiple regressions was 158, which was enough for this analysis (Stevens, 2009; Tabachnick & Fidell, 2001). According to the collinearity statistics all Tolerance (between .398 and .995) and VIF (between 1.005 and 2.518) were within the limits indicating that multicollinearity was no concern in these analyses. The assumptions of independent errors was met in our data with the Durbin-Watson value close to 2 in all 3 models (*Durbin-Watson* $value_1 = 2.009$*; Durbin-Watson* $value_2 = 1.984$*; Durbin-Watson* $value_3 = 1.974$). The histogram and P–P plot of standardised residuals indicated that the assumption of normality was met and the scatterplot showed that the assumption of homoscedasticity and linearity were satisfied.

Table 2. Partial correlations with formal versus informal TEMA-3A scores and EFs, controlled for gender, age, school, BPVS-2, WPPSI-3 ($N = 131$).

	Eyes task	Pathspan
TEMA-3A	.159	.295***
Informal TEMA-3A	.133	.280**
Formal TEMA-3A	.184*	.212*
Panamath acc	.299**	.274**
Panamath W	−.329***	−.261**
Addition task	.101	.167

Note: *$p < .05$, **$p < .01$, ***$p < .001$.

In order to be able to observe possible age related interaction effects, z-scores were used for all variables. Table 3 describes the obtained results from the three hierarchical regression models in an intent to compare the different ANS acuity measures as predictors of TEMA-3A raw scores. All three models had the same first three steps. In step 1 the demographic components gender and age were entered as control variables. Given that the different schools were not significant predictors in this model, they were left out of the final models. Note that this was done to simplify the final models. Deletion of non-significant predictors was only done for covariates that were of no interest to address the research question. Non-significant predictors that enable us to answer our research question and provide the strict control that was hypothesised to impact the link between ANS acuity and mathematics achievement were kept in the final models as planned. In order to control for general abilities, raw scores for verbal skills in the BPVS-2 and the mean percentage of the two WPPSI-3 subscales (block design and information) used as measures of verbal skills and intelligence were entered as step 2. Step 3 consisted of the EF

Table 3. Summary of Hierarchical regression analysis for variables predicting TEMA-3A ($N = 158$).

Predictor	Panamath acc.		Weber fraction		Addition acc.	
	Model 1[14]		Model 2[25]		Model 3[36]	
	β	ΔR^2	β	ΔR^2	β	ΔR^2
Step 1: demographics		.612***		.612***		.612***
Gender	.097*		.095*		.092*	
Age	.494***		.501***		.502***	
Step 2: control		.105***		.105***		.105***
WPPSI-3	.286***		.274***		.286***	
BPVS	.070		.069		.053	
Step 3: EFs		.027***		.027***		.027***
Eyes task	.043		.041		.025	
Pathspan	.141**		.148**		.146**	
Step 4: ANS acuity		.006		.004		.001
Acc. Panamath	.088					
Weber fraction			−.079			
Addition task					.041	

*$p < .05$. **$p < .01$. ***$p < .001$.[47]

measures Pathspan (visuospatial short-term memory) and the Eyes task (inhibition and shifting skills). In step 4 the different ANS acuity measures were entered. All interactions were entered in step 5, however, since no interactions were significant, these were left out of the final model. Results showed that demographic variables accounted for 61.2% of the variance of raw TEMA-3A scores, $F(2,155) = 122.13$, $p < .001$. An additional 10.5% of the variance was explained by intelligence and verbal skills, $F(2, 153) = 28.25$, $p < .001$, and a further 2.7% of the variance was explained by the EFs, $F(2,151) = 8.02$, $p < .001$. None of the ANS acuity measures were significant predictors of raw TEMA-3A scores after controlling for demographics, verbal skills, intelligence and EFs (Panamath accuracy, $F(1,150) = 3.36$, $p = .069$; Weber fraction, $F(1,150) = 2.48$, $p = .117$; Approximate Addition task accuracy, $F(1,150) = 0.73$, $p = .396$).

The same regression analyses were also conducted with informal TEMA-3A scores as displayed in Table 4 and formal TEMA-3A scores as displayed in Table 5 as dependent variables. Because formal TEMA-3A scores were judged to violate the assumptions of normality, linearity and homoscedasticity, a square root transformation was carried out before entering formal TEMA-3A scores as the dependent variable in the regression analyses. ANS acuity measures in the Panamath task were significant predictors of informal TEMA-3A scores after controlling for demographics, verbal skills, intelligence and EFs (Panamath Accuracy: $F(1,150) = 5.91$, $p < .05$; Panamath Weber fraction: $F(1,150) = 7.01$, $p < .01$). However, Weber fraction was not significantly predictive of formal TEMA-3A scores, $F(1,150) = 2.63$, $p = .107$, while Panamath accuracy was a marginally significant predictor of formal TEMA-3A, $F(1,150) = 3.17$, $p = .077$. The Approximate Addition task, on the other hand, was not a significant predictor of either formal or informal TEMA-3A scores after controlling for demographics, verbal skills, intelligence and EFs (Informal, $F(1,150) = 1.00$, $p = .321$; formal,

Table 4. Summary of Hierarchical regression analysis for variables predicting informal TEMA-3A ($N = 158$).

Predictor	Panamath acc.		Weber fraction		Addition acc.	
	Model 4[18]		Model 5[29]		Model 6[310]	
	β	ΔR^2	β	ΔR^2	β	ΔR^2
Step 1: demographics		.610***		.610***		.610***
Gender	.069		.071		.049	
Age	.487***		.492***		.497***	
Step 2: control		.107***		.107***		.107***
WPPSI-3	.295***		.273***		.298***	
BPVS	.063		.063		.052	
Step 3: EFs		.029***		.029***		.029***
Eyes task	.041		.032		.062	
Pathspan	.137**		.141**		.157***	
Step 4: ANS acuity		.010*		.011**		.002
Acc. Panamath	.115*					
Weber fraction			−.130**			
Addition task					.048	

*$p < .05$. **$p < .01$. ***$p < .001$.[411]

Table 5. Summary of Hierarchical regression analysis for variables predicting formal TEMA-3A ($N = 158$).

Predictor	Panamath acc.		Weber fraction		Addition acc.	
	Model 7[112]		Model 8[213]		Model 9[314]	
	β	ΔR^2	β	ΔR^2	β	ΔR^2
Step 1: demographics		.538***		.538***		.538***
Gender	.118*		.116*		.105*	
Age	.433***		.440***		.430***	
Step 2: control		.119***		.119***		.119***
WPPSI-3	.277***		.264***		.279***	
BPVS	.095		.095		.081	
Step 3: EFs		.040**		.040**		.040**
Eyes task	.131**		.128**		.145**	
Pathspan	.112*		.118*		.120*	
Step 4: ANS acuity		.006		.005		.005
Acc. Panamath	.093					
Weber fraction			−.088			
Addition task					.079	

*$p < .05$. **$p < .01$. ***$p < .001$.[415]

F (1,150) = 2.30, $p = .132$). The degree to which the individual predictor variables predict TEMA-3A scores are represented by the standardised coefficients (β). The significance for each predictor variable in the model is tested through t-tests. Regarding EFs—Pathspan was a significant predictor of informal TEMA-3A (Model 4, $t(150) = 2.97$, $p < .01$; Model 5, t (150) = 3.10, $p < .01$; Model 6, $t(150) = 3.39$, $p < .01$) and formal TEMA-3A (Model 7, t (150) = 2.19, $p < .05$; Model 8, t (150) = 2.34, $p < .05$; Model 9, t (150) = 2.38, $p < .05$) in all models. On the other hand, the Eyes task was not a significant predictor of informal TEMA-3A scores (Model 4, t (150) = 0.96, $p = .340$; Model 5, t (150) = 0.73, $p = .464$; Model 6, t (150) = 1.44, $p = .153$), whilst it did significantly predict formal TEMA-3A scores in all models (Model 7, t (150) = 2.75, $p < .01$; Model 8, t (150) = 2.63, $p < .01$; Model 9, t (150) = 3.10, $p < .01$).

Discussion

This study investigated whether ANS acuity, measured through different tasks, is a good predictor of mathematics achievement in children aged 4–7 years, when other possible predictive factors such as EFs, verbal skills and intelligence are controlled for. The results demonstrate that none of the ANS acuity tasks and their respective measures appear to be good predictors of mathematics achievement when strict control measures are applied. A secondary aim was to further understand the extent to which intelligence and EFs influence mathematics achievement, as well as the possible relation between ANS acuity and mathematics achievement. Our findings indicate that intelligence and EFs, such as visuospatial memory, are good predictors of mathematics achievement. Finally, this study also investigated the role of ANS acuity, verbal skills, intelligence and EFs in both informal and formal mathematics achievement. The results demonstrate the existence of an association between ANS acuity measured in a non-symbolic comparison task and informal mathematics achievement, whilst no association was found with formal mathematics achievement. When ANS acuity was measured in a non-symbolic arithmetic task, no association was found with either form of mathematics achievement.

In this study, we were able to address several challenges faced by researchers exploring the link between ANS acuity and mathematics achievement, which are believed to have produced inconsistent results in the literature. In a first instance, this study explored the link between ANS acuity and mathematics achievement in the ages 4–7 years in order to explore whether this link differs in children prior to formal schooling and after as some of the past literature's findings suggested. Our results

[9]Model 5 is the hierarchical regression model with the Weber fraction of Panamath as ANS predictor to informal TEMA-3A scores.
[10]Model 6 is the hierarchical regression model with the accuracy of the approximate addition task as ANS predictor to informal TEMA-A3 scores.
[11]All standardised β coefficients are from the final step in the analysis.

[9]Model 5 is the hierarchical regression model with the Weber fraction of Panamath as ANS predictor to informal TEMA-3A scores.
[10]Model 6 is the hierarchical regression model with the accuracy of the approximate addition task as ANS predictor to informal TEMA-A3 scores.
[11]All standardised β coefficients are from the final step in the analysis.

show that none of the age-related interactions included in the model were significant, suggesting that there were no age-related differences in the link between ANS acuity and mathematics achievement for children attending Foundation stage prior to formal schooling and children attending either Year 1 or Year 2 after having started formal schooling. Nevertheless, longitudinal data would be better suited to answer this question and would provide more insight into possible age-related differences in the link between ANS acuity and mathematics achievement. The second challenge found in the literature resulted from the fact that various dependent and independent measures have been used interchangeably to measure ANS acuity and mathematics achievement, leading to inconsistent results and conclusions. By using most main measures for ANS and mathematics achievement in the same study, our findings show that the link between ANS acuity and mathematics achievement seems to be highly dependent on the chosen measure to assess ANS acuity and mathematics achievement, which would explain the inconsistent findings in the literature. This was demonstrated in this study by the existence of a link between ANS acuity and mathematics achievement, only when using a non-symbolic comparison task, but not an Approximate Addition task to measure ANS acuity. Similarly, for mathematics achievement measures, the link between mathematics achievement and ANS acuity appears to only be found when using informal mathematics achievement measure rather than the overall or formal mathematics achievement measures in the TEMA-3. The third challenge arising from the literature has to do with the need to control for domain-general cognitive abilities such as verbal skills, intelligence and EFs when exploring the link between ANS acuity and mathematics achievement. The current findings confirm that domain-general skills, such as general intelligence, can affect the link between ANS acuity and mathematics achievement, in particular when assessing ANS acuity with the Approximate Addition task. Finally, there has been a debate in the literature regarding the possible role of EFs on the link between ANS acuity and mathematics achievement. Our findings indicate that the link between ANS acuity and mathematics achievement could still be found when controlling for inhibition and shifting skills. However, it appeared that visuospatial short-term memory seemed to largely account for the link between ANS acuity and mathematics achievement. These findings will be discussed in more detail.

Approximate number system and mathematics achievement

A common issue in the literature concerning ANS research is the frequent use of different ANS tasks, different ANS acuity measures as well as different mathematics ability measures. In order to take a thorough approach, this study assessed ANS acuity with two commonly used ANS tasks and different ANS acuity measures, while examining the role for these ANS measures in the overall mathematics ability scores as well as formal and informal mathematics ability scores. The main finding concerning ANS acuity as predictor of mathematics achievement is that, when domain-general cognitive abilities and EFs are controlled for, no ANS measure seems to predict overall and formal mathematics achievement scores, whilst ANS acuity seems to predict informal mathematics achievement scores only when measured using the non-symbolic comparison task and not the Approximate Addition Task.

In previous research, correlational analyses have often been used to demonstrate the existence of a link between ANS acuity and mathematics achievement abilities (Hyde et al., 2016; Szkudlarek & Brannon, 2017). The correlations found in this study, when initially not controlling for verbal skills, intelligence and EFs, showed a significant relation between all ANS measures and mathematics achievement in children. This is in accordance with a number of previous studies reporting a link between ANS acuity and mathematics achievement abilities (Hyde et al., 2016; Szkudlarek & Brannon, 2017). However, when controlling for intelligence and EFs, the ANS acuity measure from the Approximate Addition task, reported as being effective in ANS training studies (Hyde et al., 2014), no longer correlated with any of the three mathematics achievement scores (overall, formal and informal). This finding, for the Approximate Addition task, suggests that its high cognitive demands are

[13]Model 8 is the hierarchical regression model with the Weber fraction of Panamath as ANS predictor to formal TEMA-3A scores.
[14]Model 9 is the hierarchical regression model with the accuracy of the approximate addition task as ANS predictor to formal TEMA-A3 scores.
[15]All standardised β coefficients are from the final step in the analysis.

central to the relation found between ANS acuity and mathematics achievement. Indeed, the Approximate Addition task requires high cognitive resources, such as retention and manipulation of quantities in short-term and working memory, similar to the demands required in mathematics calculation exercises with symbolic numbers. Our results therefore suggest that such high cognitive demands may well account for the previous reported link between ANS acuity (in the Approximate Addition task) and mathematics achievement, which does not hold when controlling for general intelligence.

In accordance with other studies comparing non-symbolic comparison tasks with non-symbolic arithmetic tasks, different results were found concerning their association with mathematics achievement (Gilmore et al., 2011). Indeed, both ANS measures (i.e. Weber fraction, mean accuracy) in the non-symbolic comparison task were still significantly correlated to mathematics achievement, above and beyond verbal skills, intelligence and EFs, such as inhibition and shifting skills—but this was not true for the Approximate Addition task. However, when adding visuospatial memory as a control measure, only accuracy in the non-symbolic comparison task remained significantly correlated with mathematics achievement. A possible reason for the remaining correlation of the mean accuracy, but not the Weber fraction, can be related back to prior research indicating that accuracy in the Panamath task has a higher test-retest reliability than Weber fraction and might therefore constitute a more valid measure of ANS acuity (Inglis & Gilmore, 2014).

Furthermore, measures of ANS acuity in the Approximate Addition and non-symbolic comparison tasks did not correlate with each other after controlling for verbal skills, intelligence and EFs. This finding supports previous literature suggesting that different tasks of ANS acuity might target different cognitive processes (Gilmore et al., 2011, 2014; Xenidou-Dervou et al., 2014). This could provide some explanation as to why discrepant results are found when ANS acuity is measured in non-symbolic comparison tasks or in non-symbolic Approximate Addition tasks. Indeed, discrepancies have been found between studies in adults and studies in children (Gilmore et al., 2011, 2014). While the performance of adults on non-symbolic comparison and Approximate Addition tasks does not correlate with each other (Gilmore et al., 2011), performance on non-symbolic comparison and Approximate Addition tasks does correlate in children (Gilmore et al., 2014). Gilmore et al. (2014) proposed that both ANS acuity tasks could reflect domain-general demands (beyond verbal skills) more than ANS acuity and that the extent to which these tasks recruit either the ANS or domain-general abilities might differ between children and adults. The results from the current study provide further support for the idea that different ANS tasks are likely to entail different cognitive and EF demands, which might directly impact on the association found between ANS acuity and mathematics achievement (Dietrich et al., 2015, 2016; Gilmore et al., 2011, 2014; Purpura et al., 2017; Schmitt et al., 2017).

Nevertheless, the significant correlation between the accuracy in the non-symbolic comparison task and mathematics ability after controlling for domain-general skills indicates that a role for the ANS in relation to mathematics is still possible. However, little is known concerning the link between ANS acuity with either formal or informal mathematics achievement, when domain-general cognitive abilities are controlled for. The current study however, demonstrates that ANS acuity measured through a non-symbolic comparison task remains correlated to informal mathematics but not to formal mathematics, when controlling for verbal skills, intelligence and EFs. This suggests that the ANS could still have a role to play in informal mathematics, but only when measured through a non-symbolic comparison task. Moreover, this further supports and expands the findings of Hornung et al. (2014) that ANS acuity as measured in non-symbolic comparison tasks might have a significant contribution to early, informal number competence beyond the control of visuospatial memory. Hornung et al. (2014) found a significant correlation between a non-symbolic comparison task performance and early number competence, such as verbal counting, dot counting and Arabic number comparison in 5- to 6-year-olds, when controlling for working and short-term memory. The current study expands these findings with a correlational contribution of the non-symbolic comparison task performance to informal mathematics achievement after controlling for verbal skills, intelligence, visuospatial short-term memory as well as inhibition and shifting skills in children aged 4–7 years.

Further regression analyses conducted in the current study demonstrated that no ANS acuity measures were significantly predictive of overall scores of mathematics achievement in children when controlling for cognitive skills and EFs. Nevertheless, once more, the non-symbolic comparison task remained a significant predictor of informal mathematics above and beyond verbal skills, intelligence and EFs.

Together, these findings suggest that domain-general skills might contribute more to overall and formal scores of mathematics achievement than ANS acuity, although a link between the non-symbolic comparison task and informal scores of mathematics achievement remains above and beyond domain-general cognitive control.

Domain-general cognitive abilities and mathematics

As a secondary aim, this study also addressed the extent to which domain-general abilities influence mathematics achievement, as well as the association between ANS acuity and mathematics achievement. Overall, the results demonstrate that intelligence scales and visuospatial short-term memory might account for the relation found between ANS acuity and mathematics achievement, while inhibition and shifting skills might only relate to formal mathematics scores.

The results concerning visuospatial short-term memory seem to contradict previous research reporting a joint contribution for both short-term memory and non-symbolic comparison skills to early number competence (Hornung et al., 2014). Indeed, our results show that the contribution of non-symbolic comparison skills to mathematics achievement disappears when controlling for visuospatial short-term memory. Therefore, the current study further supports a rather opposing suggestion—that EFs, such as visuospatial short-term memory constitute an important predictor in mathematics achievement (Bull & Scerif, 2001; Hornung et al., 2014; Xenidou-Dervou et al., 2013), as well as accounting for the link between ANS and mathematics achievement (Xenidou-Dervou et al., 2014). Moreover, visuospatial short-term memory seemed to be a good predictor for all scores of mathematics (overall, formal and informal).

Aside from (visuospatial) short-term memory, intelligence was a good predictor of all scores of mathematics achievement (overall, formal and informal). This finding adds to previous literature, suggesting a role for intelligence in mathematics achievement (Passolunghi et al., 2014, 2015).

On the other hand, not all domain-general cognitive abilities were equally critical in mathematics achievement. Contradictory to previous studies reporting an important role for verbal skills (Passolunghi et al., 2015; Vanbinst & De Smedt, 2016) and inhibition (Fuhs & McNeil, 2013; Gilmore et al., 2013) in the overall score of mathematics achievement, such findings were not fully replicated in the current study. A possible reason why inhibition was not predictive of the overall score of mathematics, might be due to our use of a less conventional EFs task. In the Eyes task used in our study, both inhibition and shifting skills were measured together and combined in one accuracy measure. The introduction of shifting skills, in combination with inhibition, might have reduced the impact of inhibition as a predictive factors of mathematics achievement. Nevertheless, Price and Wilkey (2018) found a mediating role for a similar combined shifting-inhibition task (i.e. hearts and flower task) on the link between a non-symbolic comparison task and mathematics achievement albeit in an older group of 11- to 13-year-old-children. It should be noted that to our knowledge no data regarding the reliability of the Eyes task used in this study is currently available and due to the relatively low number of trials included in the Eyes task in our study, it was not possible to calculate its reliability. Therefore, caution should be taken in ruling out inhibition as a potential predictor of general mathematics achievement. Nevertheless, the Eyes task and similar tasks, have previously successfully been used as a measure of inhibition and shifting skills with the same number of trials (Burns et al., 2012; Davidson et al., 2006).

Our results do however illustrate that even though inhibition and shifting skills seem not to be related to the overall score and the informal score of mathematics achievement, there is a correlational as well as a predictive role for inhibition and shifting skills on the formal score of mathematics. In a similar way, Price and Wilkey (2018) found their inhibition-shifting task to relate to numeration, algebra and geometry in 11- to 13-year-old-children, which involved formal mathematics competence rather than informal mathematics competence as defined by the TEMA-3 task used in our study. Furthermore, Gilmore et al. (2013) also found a role for inhibitory control on the link between ANS acuity and mathematics achievement in children's formal arithmetic calculations. Hence, consistent with our results, the current literature seems to mainly find a role for inhibition skills on the association between ANS acuity and formal mathematics achievement. Therefore, we suggest that inhibition and shifting might have a role to play in formal mathematics, but not necessarily in informal mathematics.

The findings of the current study offer insight on the two hypotheses suggested by Starr and Brannon (2015) concerning the specific role of ANS acuity in mathematics achievement. The hypothesis that the ANS could be foundational in symbolic arithmetic operations (Starr & Brannon, 2015) is not supported, since the current findings demonstrate that intelligence and visuospatial short-term memory appear to largely impact the link between performance on a non-symbolic Approximate Addition task and mathematics achievement in children, rather than ANS acuity per se. However, our results support the second hypothesis by Starr and Brannon (2015) stating the role that ANS acuity plays during the early process of learning symbolic numbers. This is not in accordance with the hypothesis of Carey and Barner (2019) suggesting that approximate number representations do not play a foundational role when learning whole number meanings, nevertheless, the current findings do not provide empirical support for or against this hypothesis.

Conclusion

In summary, this study provides insight into how ANS acuity relates to mathematics achievement and the extent to which verbal skills, intelligence and EFs are involved in the association between ANS acuity and mathematics achievement, by addressing three main challenges. The main findings suggest that intelligence and EFs contribute more to mathematics achievement than any ANS measure. Nevertheless, the non-symbolic approximate comparison task remains predictive of informal mathematics achievement scores above and beyond intelligence and EFs. This provides further support to the proposition that ANS acuity, measured through non-symbolic comparison tasks, might be critical in learning symbolic numbers and in the acquisition of early symbolic number skills (Hornung et al., 2014; Starr & Brannon, 2015). Therefore ANS acuity might be more critical in acquiring symbolic number skills during the first years of primary education rather than maintaining and progressing in mathematics skills (Fazio et al., 2014; Starr & Brannon, 2015; Szkudlarek & Brannon, 2017). Finally, the results concerning domain-general cognitive abilities suggest that intelligence and visuospatial short-term memory have a significant predictive role in all mathematics achievement in children aged 4–7 years, while inhibitory control and shifting skills might only have a significant role when more complex formal mathematics education takes place.

By examining the association between ANS acuity and mathematics achievement thoroughly, the current study has highlighted the importance of domain-general cognitive control measures in the association found between ANS acuity and mathematics, since formal mathematics scores seems to be predominantly predicted by intelligence and EFs rather than ANS acuity. Nevertheless, an important role for ANS acuity measured through a non-symbolic comparison task was still found, but only in informal mathematics achievement.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendix

Table A1. Correlation matrix ($N_{max.} = 158$).

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1. Age (months)	–	–	–	–	–	–	–	–	–	–
2. TEMA raw	.78***	–	–	–	–	–	–	–	–	–
3. Informal TEMA	.78***	.95***	–	–	–	–	–	–	–	–
4. Formal TEMA	.70***	.90***	.79***	–	–	–	–	–	–	–
5. Panamath acc.	.32***	.40***	.43***	.31***	–	–	–	–	–	–
6. Panamath W	−.35***	−.45***	−.50***	−.32***	−.84***	–	–	–	–	–
7. Addition %	.46***	.49***	.50***	.46***	.35***	−.33***	–	–	–	–
8. Pathspan	.36***	.50***	.50***	.44***	.37***	−.36***	.37***	–	–	–
9. Eyes task %	.14	.24**	.25**	.27***	.32***	−.39***	.20*	.19*	–	–
10. WPPSI %	.58***	.73***	.75***	.67***	.33***	−.44***	.42***	.40***	.24**	–
11. BPVS	.50***	.61***	.60***	.59***	.23**	−.31***	.39***	.34***	.22**	.74***

Note: *$p < .05$, **$p < .01$, ***$p < .001$.

Table A2. Descriptive statistics for all tasks.

Task	*N*	*M*	SD	Range
Age (months)	158	70.77	9.77	36
TEMA-3A (raw)	158	32.38	12.86	64
Informal TEMA-3A	158	24.36	7.67	35
Formal TEMA-3A	158	6.54	4.82	23
Panamath acc. %	153	80.74	11.23	48.33
Weber fraction	149	0.41	0.41	2.51
Addition task %	147	74.56	11.30	56
Eyes task acc %	151	88.77	12.14	55
Pathspan	152	3.89	1.20	5
WPPSI mean %	154	66.98	9.61	53.38
BPVS raw	152	56.38	14.60	70