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Full Length Research Paper

On the behavior of the solutions of the system of rational difference equations $x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}-1},\ y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}-1},\ z_{n+1}=\frac{x_nz_{n-1}}{y_n}$

Abdullah Selcuk Kurbanli[1]*, Ibrahim Yalcinkaya[1] and Ali Gelisken[2]

[1]Department of Mathematics, Faculty of Education, Necmettin Erbakan University, Konya, Turkey.
[2]Department of Mathematics, Faculty of Science, Karamanoglu Mehmetbey University, Karaman, Turkey.

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There has been a great interest in studying difference equations and systems. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, geometry, economics, probability theory, genetics, physics etc. In this paper, we investigate the solutions of the system of rational difference equations $x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}-1},\ y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}-1},\ z_{n+1}=\frac{x_nz_{n-1}}{y_n}$ where the initial values x_{-1},x_0,z_{-1},z_0 are real numbers and the initial values y_{-1},y_0 are non-zero real numbers such that x_0y_{-1} and y_0x_{-1} are not equal to 1. We give general solutions of the system. Also, we obtain necessary conditions for every solution of the system to be limited or unlimited.

Key words: Difference equation, system of difference equations, solutions.

INTRODUCTION

In this study, we investigate the behavior of the solutions of the difference equation system

$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}-1},\ y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}-1},\ z_{n+1}=\frac{x_nz_{n-1}}{y_n} \tag{1}$$

Where $x_{-1},\ x_0,\ y_{-1},\ y_0,\ z_{-1},\ z_0$ are real numbers such that $y_0x_{-1}\neq 1,\ x_0y_{-1}\neq 1,\ y_{-1}\neq 0$ and $y_0\neq 0$.

Similar nonlinear systems of rational difference equations were investigated, for examples, Kurbanlı et al. (2011a) studied the behavior of the positive solutions of the system:

$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}+1},\ y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}+1}.$$

Cinar (2004) studied the solutions of the system:

$$x_{n+1}=\frac{1}{y_n},\ y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}}.$$

Kurbanli (2011b) studied the behavior of the solutions of the system:

$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}-1},\ y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}-1},\ z_{n+1}=\frac{z_{n-1}}{y_nz_{n-1}-1}.$$

Clark and Kulenović (2002) and Clark et al. (2003)

*Corresponding author. E-mail: akurbanli@yahoo.com.

investigated the global asymptotic stability of the system:

$$x_{n+1} = \frac{x_n}{a + cy_n},\ y_{n+1} = \frac{y_n}{b + dx_n}.$$

Kulenović and Nurkanović (2005) studied the global asymptotic behavior of the solutions of the system:

$$x_{n+1} = \frac{a + x_n}{b + y_n},\ y_{n+1} = \frac{c + y_n}{d + z_n},\ z_{n+1} = \frac{e + z_n}{f + x_n}.$$

Zhang et al. (2006) investigated the behavior of the positive solutions of the system of difference equations:

$$x_{n+1} = A + \frac{1}{y_{n-p}},\ y_{n+1} = A + \frac{y_{n-1}}{x_{n-r}y_{n-s}}.$$

Zhang et al. (2007) studied the boundedness, the persistence and global asymptotic stability of the positive solutions of the system:

$$x_{n+1} = A + \frac{y_{n-m}}{x_n},\ y_{n+1} = A + \frac{x_{n-m}}{y_n}.$$

Yalcinkaya (2008) studied the global asmptotic stability of the system:

$$z_{n+1} = \frac{t_n z_{n-1} + a}{t_n + z_{n-1}},\ t_{n+1} = \frac{z_n t_{n-1} + a}{z_n + t_{n-1}}.$$

MAIN RESULTS

Theorem 1

Let $y_0 = a,\ y_{-1} = b,\ x_0 = c,\ x_{-1} = d,\ z_0 = e,\ z_{-1} = f$ be real numbers such that $ad \neq 1,\ cb \neq 1,\ a \neq 0,\ b \neq 0$ and let (x_n, y_n, z_n) be a solution of the system of Equation 1. Then all solutions of of Equation 1 are:

$$x_n = \begin{cases} \dfrac{d}{(ad-1)^{\frac{n+1}{2}}}, & n-odd \\ c(cb-1)^{\frac{n}{2}}, & n-even \end{cases} \qquad (2)$$

$$y_n = \begin{cases} \dfrac{b}{(cb-1)^{\frac{n+1}{2}}}, & n-odd \\ a(ad-1)^{\frac{n}{2}}, & n-even \end{cases} \qquad (3)$$

$$z_n = \begin{cases} \dfrac{c^n f (cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}}, & n-odd \\ \dfrac{d^n e (cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=0}^{n} i}}, & n-even \end{cases} \qquad (4)$$

Proof

From Equation 1, we have

$$x_1 = \frac{x_{-1}}{y_0 x_{-1} - 1} = \frac{d}{ad-1},$$

$$y_1 = \frac{y_{-1}}{x_0 y_{-1} - 1} = \frac{b}{cb-1},$$

$$z_1 = \frac{x_0 z_{-1}}{y_0} = \frac{cf}{a},$$

$$x_2 = \frac{x_{01}}{y_1 x_0 - 1} = \frac{c}{\frac{b}{cb-1}c - 1} = c(cb-1),$$

$$y_2 = \frac{y_0}{x_1 y_0 - 1} = \frac{a}{\frac{d}{ad-1}a - 1} = a(ad-1),$$

$$z_2 = \frac{x_1 z_0}{y_1} = \frac{\frac{d}{ad-1}e}{\frac{b}{cb-1}} = \frac{de(cb-1)}{b(ad-1)},$$

$$x_3 = \frac{x_1}{y_2 x_1 - 1} = \frac{\frac{d}{ad-1}}{a(ad-1)\frac{d}{ad-1} - 1} = \frac{d}{(ad-1)^2},$$

$$y_3 = \frac{y_1}{x_2 y_1 - 1} = \frac{\frac{b}{cb-1}}{c(cb-1)\frac{b}{cb-1} - 1} = \frac{b}{(cb-1)^2},$$

$$z_3 = \frac{x_2 z_1}{y_2} = \frac{c(cb-1)\frac{cf}{a}}{a(ad-1)} = \frac{c^2 f (cb-1)}{a^2 (ad-1)}.$$

So Equations 2, 3 and 4 are true for $n = 1, 2, 3.$ Assume that Equations 2, 3 and 4 are true for $n = 4, 5, ..., k$. Then

$$x_{2k-1} = \frac{x_{2k-3}}{y_{2k-2}x_{2k-3}-1} = \frac{d}{(ad-1)^k},$$
$$x_{2k} = \frac{x_{2k-2}}{y_{2k-1}x_{2k-2}-1} = c(cb-1)^k,$$

$$y_{2k-1} = \frac{y_{2k-3}}{x_{2k-2}y_{2k-3}-1} = \frac{b}{(cb-1)^k},$$
$$y_{2k} = \frac{y_{2k-2}}{x_{2k-1}y_{2k-2}-1} = a(ad-1)^k$$

and

$$z_{2k-1} = \frac{x_{2k-2}z_{2k-3}}{y_{2k-2}} = \frac{c^k f(cb-1)^{\sum_{i=0}^{k-1} i}}{a^k (ad-1)^{\sum_{i=0}^{k-1} i}}, \quad z_{2k} = \frac{x_{2k-1}z_{2k-2}}{y_{2k-1}} = \frac{d^k e(cb-1)^{\sum_{i=1}^{k} i}}{b^k (ad-1)^{\sum_{i=1}^{k} i}}.$$

Now, we must show that Equations 2, 3 and 4 are true for $n = k+1$. From Equation 1, we have

$$x_{2k+1} = \frac{x_{2k-1}}{y_{2k}x_{2k-1}-1} = \frac{\frac{d}{(ad-1)^k}}{a(ad-1)^k \frac{d}{(ad-1)^k}-1} = \frac{d}{(ad-1)^{k+1}},$$

$$y_{2k+1} = \frac{y_{2k-1}}{x_{2k}y_{2k-1}-1} = \frac{\frac{b}{(cb-1)^k}}{c(cb-1)^k \frac{b}{(cb-1)^k}-1} = \frac{b}{(cb-1)^{k+1}}$$

and

$$z_{2k+1} = \frac{x_{2k}z_{2k-1}}{y_{2k}} = \frac{c(cb-1)^k \frac{c^k f(cb-1)^{\sum_{i=0}^{k-1} i}}{a^k (ad-1)^{\sum_{i=0}^{k-1} i}}}{a(ad-1)^k} = \frac{c^{k+1} f(cb-1)^{k+\sum_{i=0}^{k-1} i}}{a^{k+1}(ad-1)^{k+\sum_{i=0}^{k-1} i}} = \frac{c^{k+1} f(cb-1)^{\sum_{i=0}^{k} i}}{a^{k+1}(ad-1)^{\sum_{i=0}^{k} i}}.$$

Also, we have

$$x_{2k+2} = \frac{x_{2k}}{y_{2k+1}x_{2k-1}} = \frac{c(cb-1)^k}{\frac{b}{(cb-1)^{k+1}}c(cb-1)^k-1} = \frac{c(cb-1)^k}{\frac{b}{cb-1}c-1} = c(cb-1)^{k+1}$$

$$y_{2k+2} = \frac{y_{2k}}{x_{2k+1}y_{2k}-1} = \frac{a(ad-1)^k}{\frac{d}{(ad-1)^{k+1}}a(ad-1)^k-1} = \frac{a(ad-1)^k}{\frac{d}{ad-1}a-1} = a(ad-1)^{k+1}$$

and

$$z_{2k+2} = \frac{x_{2k+1}z_{2k}}{y_{2k+1}} = \frac{\frac{d}{(ad-1)^{k+1}} \frac{d^k e(cb-1)^{\sum_{i=1}^{k} i}}{b^k (ad-1)^{\sum_{i=1}^{k} i}}}{\frac{b}{(cb-1)^{k+1}}} = \frac{d^{k+1}e(cb-1)^{(k+1)+\sum_{i=1}^{k} i}}{b^{k+1}(ad-1)^{(k+1)+\sum_{i=1}^{k} i}} = \frac{d^{k+1}e(cb-1)^{\sum_{i=1}^{k+1} i}}{b^{k+1}(ad-1)^{\sum_{i=1}^{k+1} i}}.$$

Therefore, the proof is completed by induction.

Corollary 1

Let (x_n, y_n, z_n) be a solution of the system of Equation 1 and let a,b,c,d be real numbers such that $ad \neq 1, cb \neq 1, a \neq 0$ and $b \neq 0$, then the following results hold:

(1) If $0 < a,b,c,d,e,f < 1$, then $\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = \infty$ and $\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = 0$,

(2) If $0 < a,b,c,d,e,f < 1$, $c < a$ and $cb = ad$ or $0 < a,b,c,d,e,f < 1$, $c = a$ and $b < d$, then $\lim_{n\to\infty} z_{2n-1} = 0$,

(3) If $0 < a,b,c,d,e,f < 1$, $c > a$ and $cb = ad$ or $0 < a,b,c,d,e,f < 1$, $c = a$ and $d > b$, then $\lim_{n\to\infty} z_{2n-1} = \infty$,

(4) If $0 < a,b,c,d,e,f < 1$, $c = a$ and $b = d$, then $\lim_{n\to\infty} z_{2n-1} = f$,

(5) If $0 < a,b,c,d,e,f < 1$, $d < b$ and $cb > ad$ or $0 < a,b,c,d,e,f < 1$, $d < b$ and $cb = ad$ or $0 < a,b,c,d,e,f < 1$, $f < 1$, $b = d$ and $c > a$, then $\lim_{n\to\infty} z_{2n} = 0$,

(6) If $0 < a,b,c,d,e,f < 1$, $d > b$ and $cb = ad$ or $0 < a,b,c,d,e,f < 1$, $d = b$ and $ad < cb$, then $\lim_{n\to\infty} z_{2n} = \infty$,

(7) If $0 < a,b,c,d,e,f < 1$, $d = b$ and $c = a$, then $\lim_{n\to\infty} z_{2n} = e.$

Proof

(1) From $0 < a,b,c,d < 1$, we have $-1 < ad-1 < 0$ and $-1 < cb-1 < 0$. Hence, we obtain

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = d \lim_{n\to\infty} \frac{1}{(ad-1)^n} = \begin{cases} -\infty, & n-odd \\ +\infty, & n-even \end{cases},$$

$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(cb-1)^n} = b \lim_{n\to\infty} \frac{1}{(cb-1)^n} = \begin{cases} -\infty, & n-odd \\ +\infty, & n-even \end{cases}$$

and

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(cb-1)^n = c \lim_{n\to\infty} (cb-1)^n = 0$$

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} c(ad-1)^n = a\lim_{n\to\infty}(ad-1)^n = 0.$$

(2) From $0<a,b,c,d<1$, $c<a$ and $cb=ad$, we have $\frac{cb-1}{ad-1}=1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = 0.$$

Similarly, from $0<a,b,c,d<1$, $c=a$ and $b<d$, we have $\frac{cb-1}{ad-1}<1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = 0.$$

(3) From $0<a,b,c,d<1$, $c>a$ and $cb=ad$, we have $\frac{cb-1}{ad-1}=1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = \infty.$$

Similarly, from $0<a,b,c,d<1$, $c=a$ and $d<b$, we have $\frac{cb-1}{ad-1}>1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = \infty.$$

(4) From $0<a,b,c,d<1$, $c=a$ and $b=d$, we have $\frac{cb-1}{ad-1}=1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = f.$$

(5) From $0<a,b,c,d<1$, $d<b$ and $bc>da$, we have $\frac{cb-1}{ad-1}<1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{d^n e(cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=1}^{n} i}} = \lim_{n\to\infty} e\left(\frac{d}{b}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=1}^{n} i} = 0.$$

Similarly, from $0<a,b,c,d<1$, $d<b$ and $bc=da$, we have $\frac{cb-1}{ad-1}=1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{d^n e(cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=1}^{n} i}} = \lim_{n\to\infty} e\left(\frac{d}{b}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=1}^{n} i} = 0.$$

Similarly, from $0<a,b,c,d<1$, $d=b$ and $c>a$, we have $\frac{cb-1}{ad-1}<1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{d^n e(cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=1}^{n} i}} = \lim_{n\to\infty} e\left(\frac{d}{b}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=1}^{n} i} = 0.$$

(6) From $0<a,b,c,d<1$, $d>b$ and $bc=da$, we have $\frac{cb-1}{ad-1}=1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{d^n e(cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=1}^{n} i}} = \lim_{n\to\infty} e\left(\frac{d}{b}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=1}^{n} i} = \infty.$$

Similarly, from $0<a,b,c,d<1$, $d=b$ and $bc>da$, we have $\frac{cb-1}{ad-1}>1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{d^n e(cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=1}^{n} i}} = \lim_{n\to\infty} e\left(\frac{d}{b}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=1}^{n} i} = \infty.$$

(7) From $0<a,b,c,d<1$, $d=b$ and $c=a$, we have $\frac{cb-1}{ad-1}=1$. Hence, we obtain

$$\lim_{n\to\infty} z_{2n} = \lim_{n\to\infty} \frac{d^n e(cb-1)^{\sum_{i=1}^{n} i}}{b^n (ad-1)^{\sum_{i=1}^{n} i}} = \lim_{n\to\infty} e\left(\frac{d}{b}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=1}^{n} i} = e.$$

Corollary 2

Let (x_n, y_n, z_n) be a solution of the system of Equation 1 and let $a,b,c,d \in (1,+\infty)$ and $a > d > c > b.$ If $ad-1,\ cb-1 \in (1,+\infty)$, then

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} z_{2n-1} = 0,$$

and

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = \infty.$$

Proof

From $a,b,c,d \in (1,+\infty)$, $a > d > c > b$ and $ad-1,\ cb-1 \in (1,+\infty)$, we have $\frac{cb-1}{ad-1} < 1$. Hence, we have

$\lim_{n\to\infty} (cb-1)^n = +\infty$ and $\lim_{n\to\infty} (ad-1)^n = +\infty$. Also, we have

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = d.\lim_{n\to\infty} \frac{1}{(ad-1)^n} = 0,$$

$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(cb-1)^n} = b.\lim_{n\to\infty} \frac{1}{(cb-1)^n} = 0$$

and

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = 0.$$

Similarly, we have

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(cb-1)^n = c \lim_{n\to\infty} (cb-1)^n = +\infty$$

and

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = a \lim_{n\to\infty} (ad-1)^n = +\infty$$ ▪

Corollary 3

Let (x_n, y_n, z_n) be a solution of the system of Equation 1 and let $a,b,c,d \in (1,+\infty)$ and $a = c,\ b > d$. If $ad-1,\ cb-1 \in (0,\ 1)$, then

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} z_{2n-1} = \infty,$$

and

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = 0.$$

Proof

From $a,b,c,d \in (1,+\infty)$, $a = c,\ b > d$ and $ad-1,\ cb-1 \in (0,1)$, we have $\frac{cb-1}{ad-1} > 1$. Hence, we have $\lim_{n\to\infty} (cb-1)^n = 0$ and $\lim_{n\to\infty} (ad-1)^n = 0$. Also, we have

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = d.\lim_{n\to\infty} \frac{1}{(ad-1)^n} = +\infty,$$

$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(cb-1)^n} = b.\lim_{n\to\infty} \frac{1}{(cb-1)^n} = +\infty$$

and

$$\lim_{n\to\infty} z_{2n-1} = \lim_{n\to\infty} \frac{c^n f(cb-1)^{\sum_{i=0}^{n-1} i}}{a^n (ad-1)^{\sum_{i=0}^{n-1} i}} = \lim_{n\to\infty} f\left(\frac{c}{a}\right)^n \left(\frac{cb-1}{ad-1}\right)^{\sum_{i=0}^{n-1} i} = +\infty$$

Similarly, we have

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(cb-1)^n = c.0 = 0$$

and

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = a.0 = 0.$$

REFERENCES

Cinar C (2004). On the positive solutions of the difference equation system $x_{n+1} = \frac{1}{y_n}, y_{n+1} = \frac{y_n}{x_{n-1} y_{n-1}}$. Appl. Math. Comput. 158:303-305.

Clark D, Kulenović MRS (2002). A coupled system of rational difference equations. Comput. Math. Appl. 43:849-867.

Clark D, Kulenović MRS, Selgrade JF (2003). Global asymptotic behavior of a two-dimensional difference equation modelling competition. Nonlinear Anal. Teor. 52:1765-1776.

Kulenović MRS, Nurkanović Z (2005). Global behavior of a three-dimensional linear fractional system of difference equations. J. Math. Anal. Appl. 310:673-689.

Kurbanli AS (2011b). On the behavior of solutions of the system of rational difference equations $x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1}$. Discrete Dyn. Nat. Soc. 2011:1-12.

Kurbanlı AS, Cinar C and Yalcinkaya I (2011a). On the behavior of positive solutions of the system of rational difference equations $x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + 1}, y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + 1}$. Math. Comput. Model. 53:1261-1267.

Yalcinkaya I (2008). On the global asymptotic stability of a second-order system of difference equations. Discrete Dyn. Nat. Soc. 2008:1-12.

Zhang Y, Yang X, Evans DJ, Zhu C (2007). On the nonlinear difference equation system $x_{n+1} = A + \frac{y_{n-m}}{x_n}, y_{n+1} = A + \frac{x_{n-m}}{y_n}$. Comput. Math. Appl. 53:1561-1566.

Zhang Y, Yang X, Megson GM, Evans DJ (2006). On the system of rational difference equations $x_n = A + \frac{1}{y_{n-p}}, y_n = A + \frac{y_{n-1}}{x_{n-r} y_{n-s}}$. Appl. Math. Comput. 176:403-408.