Plasma screening effects on the energies of hydrogen atom under the influence of velocity-dependent potential

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In order to examine the plasma screening and velocity-dependent potential effects on the hydrogen atom, the Schrödinger equation including a more general exponential cosine screened Coulomb and velocity-dependent potential is solved numerically in the framework asymptotic iteration method. The more general exponential cosine screened Coulomb potential is used to model Debye and quantum plasma for the specific values of the parameters in its structure. However, in order to examine effects of velocity-dependent potential on energy values of hydrogen atom in Debye and quantum plasma, the isotropic form factor of velocity-dependent potential is given as harmonic oscillator type, $\rho(r) = \rho_o r^2$. Then, the energies of s and p states are calculated numerically without any approximation. In order to investigate thoroughly plasma screening effects and contribution of velocity-dependent potential on energy values of hydrogen atom, the corresponding calculations are carried out by using different values of parameters of more general exponential cosine screened Coulomb potential and isotropic dependence, results of which are discussed. © *2014 AIP Publishing LLC*.
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I. INTRODUCTION

Since the investigation of the two nucleon system in nuclear and particle physics is crucial, the velocity-dependent potential that is given for modeling interaction among nucleons is often used in the studies of the nuclear and particle physics. Why the velocity-dependent potential for such interactions is appropriate relates to that strong repulsion at short distances is a typical feature of the two nucleon system. So, using the Schrödinger equation with velocity-dependent potential may provide models as good as the static (velocity non-dependent) ones for operating the interactions among particles in the nonrelativistic limit. The examination of the properties of the varied Schrödinger equation by means of velocity-dependent terms thus appears to us to be of importance. The properties of the Schrödinger equation including velocity-dependent potential and the singularities of the corresponding differential equation were studied in order to probe properties of velocity-dependent potential and it was found that there is a simple agreement between the velocity-dependent problem and a static one.[1] The velocity-dependent potential was first considered in order to investigate the scattering of mesons from the complex nuclei.[2] Also, the velocity-dependent potential was used to model nucleon-nucleon interaction and reproduced the ^1S, ^1D, ^1G singlet-even phase shift.[3] The Schrödinger equation with velocity-dependent potential was also considered by using the Morse function as an "effective" potential. Then, approximate analytical solutions and an eigenvalue formula were obtained in order to investigate bound states of O^{17} and Ca^{41}.[4] However, the velocity-dependent interaction can be also used for modeling interactions of particles with position dependent mass. In other words, the corresponding equations are in principle the position-dependent mass Schrödinger equation. The solution of the Schrödinger equation with position dependent mass has been found to be very useful in studying the physical and electronic properties of semiconductors, quantum wells and quantum dots, quantum liquids, ^{3}He clusters, graded alloys and semiconductor heterostructures.[5–10] The position dependent mass concept is not only important for non-relativistic systems but also for relativistic quantum systems.[11] Apart from all these, there are many studies on velocity dependent potential in the field of atomic physics. In this manner, the variational calculation of the binding energy of helium was introduced using a two-body central velocity-dependent potential for ^1S and ^3S states by considering the two parameter Irving wave function.[12] Morever, in order to describe of the bound and scattering states of electrons interacting with atomic oxygen and electrons interacting with neon, the velocity-dependent potential was used together with Yukawa form factor.[13] As previously mentioned, since the investigation of non-relativistic systems with a velocity-dependent potential is vitally important in many different fields such as nuclear, particle and atomic physics, there are ongoing studies for solutions of the corresponding Schrödinger equation. But, to obtain exact analytical solutions of Schrödinger equation with a velocity-dependent potential is extremely difficult. So, in order to solve corresponding equation, rather than exact analytical solutions, some approximate mathematical methods as perturbation theory are preferred.[14–18] In Ref. 18, the Schrödinger equation with isotropic velocity-dependent potential was solved using perturbation theory and in order to calculate effect of perturbing potential on energy and wave function of a state, corresponding wave functions and energies in the s-wave state were obtained using unperturbed wave function. In Ref. 14, for investigation of the amendments in the scattering phase shifts and wave functions, the corresponding Schrödinger equation was studied using the developed analytical formulas, and results of corresponding study were tested for nucleon-nucleon scattering. Then, it was seen

that the results of the mentioned perturbation formalism with those of the exactly solvable other models have a very good agreement. In addition, in Ref. 19, the formal scattering theory was used to obtain scattering and bound states in the corresponding system with velocity-dependent Kisslinger potential. On the other hand, the asymptotic iteration method that developed by Çiftçi *et al*. is an alternative method for solution of the Schrödinger equation with velocity-dependent potential. In order to examine the effect of the isotropic velocity-dependent potentials on the bound state energy values of the Morse potential, the corresponding Schrödinger equation was solved numerically in the framework of AIM.[20] In Ref. 20, the AIM was used to solve this problem in two ways: first, when considering non-velocity dependent case, the Schrödinger equation can be solved analytically by using the Pekeris approximation. Second, if the form factor of velocity-dependent total potential is evaluated as harmonic oscillator type, since obtaining analytical solutions is extremely difficult, the numerical calculations were made by using the AIM. The contribution of the velocity-dependent potential on system was shown for different potential parameters. Furthermore, the perturbative-AIM was also used to examine the effect of velocity-dependent potential on the bound states of Coulomb and harmonic oscillator potentials for any quantum numbers.[21] Given all these studies, the importance and necessity of studies to be able to performed on investigation of velocity-dependent potential effects in applications of plasma physics are clear. So, in this study, the effects of the velocity-dependent potential and plasma screening parameters on energy states of hydrogen atom have been probed. It should be pointed out that there are important studies performed by some researchers on examination of plasma screening effects.[22–28] While the screened Coulomb (SC) potential which also called Debye model is used to investigate screening effects of Debye plasma for hydrogen atom,[29–34] the exponential cosine screened Coulomb (ECSC) potential is used in order to model quantum plasma.[35] Further, as alternatively the SC and ECSC potentials, the more general exponential cosine screened Coulomb (MGECSC) potential has been proposed for the first time by Soylu in order to investigate screening effects on energies of hydrogen atom in Debye and quantum plasmas.[36] It has been shown in Ref. 36 that the MGECSC potential has stronger screening effects than that of ECSC and SC potentials and this potential can be reduced the PC (Pure Coulomb), SC and ECSC potentials by means of special values of parameters in its structure. In the view of above informations, an important point to be noted is that plasma screening effects on the energies of Hydrogen atom under the influence of velocity-dependent potential have been investigated for the first time in the present study.

This paper is organized as follows. In Sec. II, the velocity–dependent potential model in consideration of hydrogen atom in plasma modeled by MGECSC potential has been summarized. In Sec. III, the AIM has briefly been explained. In Sec. IV, it has been outlined that how to solve numerically using AIM a second-order homogeneous differential equation and the corresponding differential equation has been solved. Then, velocity-dependent potential effects on the energies of hydrogen atom in Debye and quantum plasma have been examined in detail. However, the obtained results are given and interpreted. Finally, summary and conclusion are given.

II. MODEL

The following potential that consists of the local ($V_{eff}(r)$) and the isotropic velocity-dependent local ($V(r,p)$) potentials is the velocity-dependent total potential[2–4,14–21,37]

$$V_{total}(r,p) = V_{eff}(r) + \frac{\hbar^2}{2m}\left[F(r)\nabla^2 + \vec{\nabla}F(r).\vec{\nabla}\right], \quad (1)$$

where F(r) is the isotropic function (form factor) of the radial variable r. Then, the corresponding radial Schrödinger equation for a particle with m mass and E energy is

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{total}(r,p) - E\right]\psi(r) = 0, \quad (2)$$

where $\psi(r)$ is radial wave function. Setting $F(r) = \gamma\rho(r)$ for form factor and using $\psi(r) = r^{-1}u(r)$ for radial wave function in Eq. (2), being a constant γ and in units of $2m = \hbar = 1$, Eq. (2) reduces to

$$u''(r) - \left[u'(r) - \frac{u(r)}{r}\right]\frac{\gamma\rho'(r)}{1-\gamma\rho(r)} + \frac{E - V_{eff}(r)}{1-\gamma\rho(r)}u(r) = 0, \quad (3)$$

where the effective potential is

$$V_{eff}(r) = \frac{\ell(\ell+1)}{r^2} + V_{MGECSC}(r), \quad (4)$$

where ℓ is the angular momentum quantum number, $V_{MGECSC}(r)$ is the more general exponential cosine screened Coulomb potential. The MGECSC potential which is considered in order to study plasma screening effects on energy levels of hydrogen atom in the presence of velocity-dependent potential in detail is given by[36]

$$V_{MGECSC}(r) = -\frac{Ze^2}{r}(1+br)e^{-r/\lambda}\cos(cr/\lambda), \quad (5)$$

where b, c and λ are potential parameters as called screening parameters. λ potential parameter is known as Debye screening parameter in Debye plasma. Changing of these screening parameters leads to shifting energy levels of hydrogen atom in plasma by significantly affecting profile of total interaction potential of system.

III. THE AIM

In this section, we review some of the formalism of the AIM that will be necessary for solution of our problem. The AIM can be used in order to solve second order differential equations in the following form:

$$P'' = \lambda_0(y)P' + s_0(y)P. \quad (6)$$

The differential equation (6) has a general solution as follows:[38]

$$P_n(y) = \exp\left(-\int^{y} \alpha d\ell\right)\left[C_2' + C_1' \int^{y} \exp\left(\int^{\ell} [\lambda_0(\upsilon) + 2\alpha(\upsilon)] d\upsilon\right) d\ell\right], \tag{7}$$

where C_1' and C_2' are integration constants, if $k > 0$, in finite value of k, for some $k = 1, 2, \ldots$

$$\frac{s_k}{\lambda_k} = \frac{s_{k-1}}{\lambda_{k-1}} \equiv \alpha, \tag{8}$$

which is termination condition and here λ and s are associated by

$$\lambda_k(y) = \lambda'_{k-1}(y) + s_{k-1}(y) + \lambda_0(y)\lambda_{k-1}(y), \tag{9a}$$

$$s_k(y) = s'_{k-1}(y) + s_0(y)\lambda_{k-1}(y),\ k = 1, 2, \ldots, n, \tag{9b}$$

which are known as recurrence relations.

For a given potential, if possible, the related equation is converted to the form of Eq. (6), $\lambda_0(y)$ and $s_0(y)$ are determined. Then, using together both of Eqs. (9) and termination condition of the method, the following expression used to calculate the energy values is obtained:

$$\delta(y) = \lambda_{k-1}(y)s_k(y) - \lambda_k(y)s_{k-1}(y) = 0. \tag{10}$$

The first part of Eq. (7) has the polynomial solutions that are convergent and physical, whereas the second part of it has nonphysical. So, in order to find the square integrable, the coefficient (C_1') of second part of Eq. (7) is taken as zero. Then, the exact eigenfunctions can be derived from the following wave function generator:

$$P_n(y) = C_2' \exp\left(-\int^{y} \alpha d\ell\right). \tag{11}$$

However, the more detailed information can be found in Refs. 38–40.

IV. APPLICATION

In all calculations, the energy values have been calculated for $Z = e = 1$ as well as units of $2m = \hbar = 1$ and Rydberg units. For the total effective potential in Eq. (3) and form factor, respectively, if the MGECSC potential and $\rho(r) = \rho_0 r^2$ similar to harmonic oscillator use, it is obtained

$$u''(r) - \left[\frac{2\gamma\rho_0 r}{1-\gamma\rho_0 r^2}\right]u'(r) + \left[\frac{2\gamma\rho_0 + E}{1-\gamma\rho_0 r^2} - \frac{\ell(\ell+1)}{r^2(1-\gamma\rho_0 r^2)} + \frac{Ze^2(1+br)e^{-r/\lambda}\cos(cr/\lambda)}{r(1-\gamma\rho_0 r^2)}\right]u(r) = 0. \tag{12}$$

Since it is not possible to solve Eq. (12) analytically, it can be preferred to seek a numerical solution. Note that the AIM can be applied to second-order differential equations similar to Eq. (12), but which is possible numerically.[33] The advantage of AIM is that this method has applicability to other problems and allows us to obtain the energy eigenvalues of corresponding systems in a very systematic way. In order to solve numerically Eq. (12) in the framework of AIM, the asymptotic behavior of this equation for $r \to 0$ and $r \to \infty$ should be considered. Therefore, the following physical wave function is proposed:

$$u(r) = r^{\ell+1}e^{-\beta r}f(r), \tag{13}$$

which has to fulfill the boundary conditions. Inserting this suggested wave function into Eq. (12) allows to implementation of the AIM in a more easy way, which is already for physical solutions. If this wave function is inserted into Eq. (12), the second-order differential equation in the following form is gotten:

$$f''(r) = -\left(\frac{2\ell}{r} + \frac{2(\beta r - 1)}{r(-1+\rho_0\gamma r^2)} + \frac{2\left(2\rho_0\gamma r - \rho_0\gamma\beta r^2\right)}{(-1+\rho_0\gamma r^2)}\right)f'(r) + \left(\begin{array}{l}\dfrac{E - \ell(3+\ell)\rho_0\gamma + \beta^2}{(-1+\rho_0\gamma r^2)} + \dfrac{r(4\beta\rho_0\gamma + 2\beta\rho_0\gamma\ell)}{(-1+\rho_0\gamma r^2)} \\ -\dfrac{\beta^2\rho_0\gamma r^2}{(-1+\rho_0\gamma r^2)} + \dfrac{2\beta(-1-\ell)}{r(-1+\rho_0\gamma r^2)} + \dfrac{(1+br)\cos(cr/\lambda)}{e^{r/\lambda}r(-1+\rho_0\gamma r^2)}\end{array}\right)f(r). \tag{14}$$

If Eq. (14) compare with Eq. (6), $\lambda_0(r)$ and $s_0(r)$ are found as follows:

$$\lambda_0(r) = -\left(\frac{2\ell}{r} + \frac{2(\beta r - 1)}{r(-1+\rho_0\gamma r^2)} + \frac{2\left(2\rho_0\gamma r - \rho_0\gamma\beta r^2\right)}{(-1+\rho_0\gamma r^2)}\right), \tag{15a}$$

$$s_0(r) = \left(\begin{array}{c} \dfrac{E - \ell(3+\ell)\rho_0\gamma + \beta^2}{(-1+\rho_0\gamma r^2)} + \dfrac{r(4\beta\rho_0\gamma + 2\beta\rho_0\gamma\ell)}{(-1+\rho_0\gamma r^2)} - \\ \dfrac{\beta^2\rho_0\gamma r^2}{(-1+\rho_0\gamma r^2)} + \dfrac{2\beta(-1-\ell)}{r(-1+\rho_0\gamma r^2)} + \dfrac{(1+br)\cos(cr/\lambda)}{e^{r/\lambda}r(-1+\rho_0\gamma r^2)} \end{array} \right). \tag{15b}$$

$\lambda_k(r)$ and $s_k(r)$ are obtained by using Eqs. (9). Then, in order to calculate the energy values of the system, the quantization condition given by Eq. (10) will be used. For different iteration number k, the quantization condition depends on two variables E and r. However, considering the quantization condition, it is clear that energy values of hydrogen atom should not depend on the choice of r. The choice of r for iterative solution is very crucial, namely, a special value of r enables to convergence of energy values more quickly. With this, there are two options on how to choice of r: One choice of $r = r_0$ minimizes the V(r) potential or maximizes the wave function (Eq. (13)). Then, the special value of r can be taken as $r_0 = (\ell+1)/\beta$ in order to make maximum the wave function. β is an arbitrary parameter and note that it provides the stability of process, as well as affecting speed of convergence of the iterations. On the one hand, the various values of β parameter can be used in the course of investigations in different values of potential parameters and in order to compute the energies of different quantum states. By doing so, the speed of convergence of the iteration increases and the results are achieved more quickly. Now, in the presence of velocity-dependent potential, plasma screening effects on the energies of hydrogen atom will be probed for each of cases of ($c = 0$, $b \neq 0$), ($b = 0$, $c \neq 0$), and ($c \neq b \neq 0$).

A. Case of c = 0, b ≠ 0

Firstly, the case of $c = 0$, $b \neq 0$ for the MGECSC potential is considered, which holds for Debye plasma, and λ parameter in the transforming potential is considered as Debye screening parameter. For case of $c = 0$, $b \neq 0$, Eqs. (14) and (15) change naturally and applying AIM to the transformed version of Eq. (14) allows us to determine effects of velocity-dependent potential on energies of hydrogen atom in Debye plasma. So, in order to show the effects of b parameter on energies of hydrogen in interaction case with velocity-dependent potential, the corresponding energies have been computed using $c = 0$, $\lambda_D = 100$, $\gamma = 1$, $\rho_o = 1/5$ for 1s, 2s, 3s, 2p, 3p, and 4p quantum states in units of $2m = \hbar = 1$. As shown Table I, the energy values of hydrogen atom have been found for $b = 0, 0.2, 0.4, 0.6, 0.8, 1, 1.5$ and 3 using $\beta = 5/4$. Here, it should be pointed out that the energy values converge after maximum 15 iterations for all quantum states. Having found the energy spectrum of hydrogen atom in Debye plasma, let us try to interpret the results of velocity-dependent potential effect. For increasing values of b screening parameter, as can be shown in Table I, energy values of hydrogen atom in influence of velocity-dependent potential decrease monotonically.

On the one hand, if the behavior of MGECSC potential versus changing of b screening parameter is investigated, it is clear that the corresponding potential becomes more attractive due to the increase of b screening parameter.[36] A result of this case relates to localizations of energies levels of quantum states. Under the same conditions, for more attractive potential, the energy levels localize in the lower levels, in other words, the energy values of system decrease. With this, it is expected that when b screening parameter is increased, the energy values decrease. However, in this system, hydrogen atom in Debye plasma is under effect of velocity-dependent potential and there are important results of this effect on the energy levels of hydrogen atom. In Fig. 1, for case of $c = 0$, $b \neq 0$, the effect of b screening parameter on energy values of hydrogen atom has been examined for $\lambda_D = 200$ value of Debye screening parameter using $\gamma = 1$, $\rho_o = 1/5$, $\beta = 5/4$ in the presence and absence of velocity-dependent potential. As shown in Fig. 1, the energy values of 1s, 2s, and 3s quantum states have been calculated for 0.2, 0.4, and 0.6 values of b screening parameter. According to these results, in both cases energy values decrease when b screening parameter is increased, but here it should be pointed out that while including velocity-dependent potential in the system leads to decrease energy

TABLE I. In the presence of velocity-dependent potential, for different values of b screening parameter, the energy values of hydrogen atom in Debye plasma for 1s, 2s, 3s, 2p, 3p, 4p quantum states in units of $2m = \hbar = 1$, where $\lambda_D = 100$, $\gamma = 1$, $\rho_o = 1/5$.

	c = 0 $\lambda_D = 100, \gamma = 1, \rho_o = 1/5$							
State	b = 0	b = 0.2	b = 0.4	b = 0.6	b = 0.8	b = 1	b = 1.5	b = 3
1s	−0.712882	−0.909859	−1.106836	−1.303815	−1.500795	−1.697775	−2.190231	−3.667638
2s	1.067397	0.870346	0.673295	0.476245	0.279196	0.082147	−0.410472	−1.888312
3s	4.548863	4.351753	4.154644	3.957535	3.760426	3.563318	3.070547	1.592239
2p	0.244167	0.047795	−0.148576	−0.344948	−0.541321	−0.737694	−1.228627	−2.701441
3p	2.889325	2.692459	2.495593	2.298727	2.101861	1.904995	1.412831	−0.063661
4p	7.190604	6.993604	6.796604	6.599604	6.402604	6.205604	5.713105	4.235609

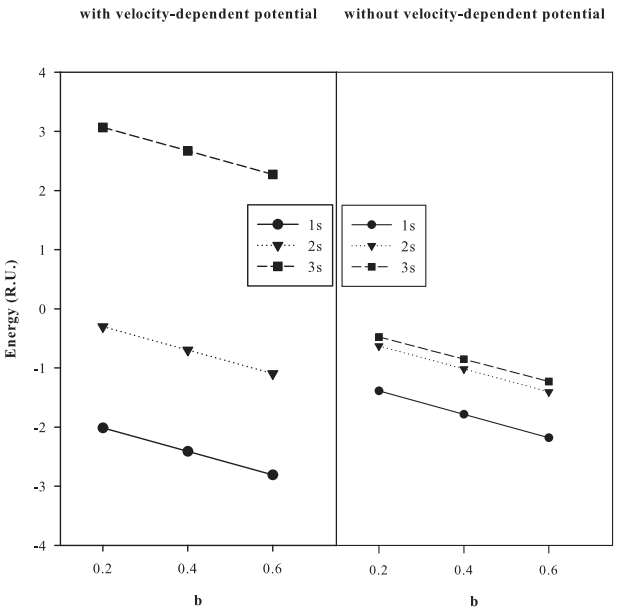

FIG. 1. For c = 0, b ≠ 0, in Rydberg units, comparison of the energy values of hydrogen atom in Debye plasma in the presence of velocity-dependent potential with $\lambda = 200$, $\gamma = 1$, $\rho_o = 1/5$ with the results of Ref. 36.

values of 1s state, it increases energy values of 2s and 3s states for same values of b screening parameter. Then, it can be read that the contribution of velocity-dependent potential was given rise to an asymmetrication on total interaction potential profile of the system. However, it should be mentioned that while energy value of 2s state for b = 0.2 equals to –0.628468 in the absence of velocity dependent potential, for b = 0.6 it is −1.096120 in the presence of velocity-dependent potential, the results in which have been obtained in Rydberg units. As shown, the energy values decrease when going b screening parameter from 0.2 to 0.6 as well as including velocity-dependent potential in the system. The increase of b screening parameter changes significantly total interaction potential profile and hence energy values of bound states. Note that b screening parameter for this system under influence of velocity-dependent potential can be used to model external and screening effects which decrease the energy values of hydrogen in Debye plasma.

In the presence of velocity-dependent potential, how to change bound states of hydrogen atom in Debye plasma versus change of λ_D Debye screening parameter and the

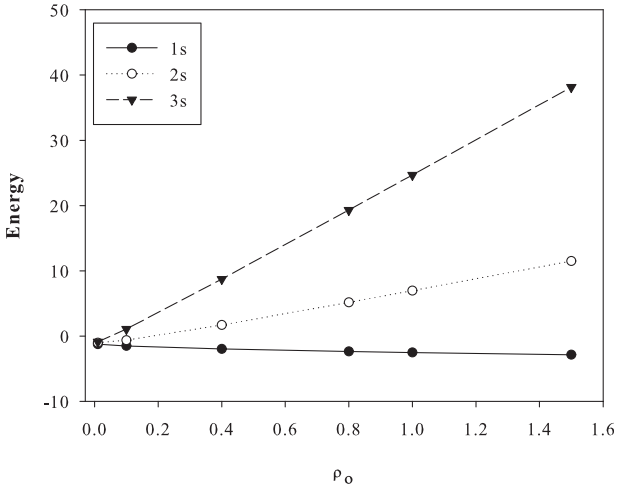

FIG. 2. The behavior of energy values of 1s, 2s, and 3s quantum states versus isotropic dependence of velocity-dependent potential in units of $2m = \hbar = 1$, where $\lambda = 150$, c = 0, b = 1, $\gamma = 1$.

corresponding energy values have been given for case of parameter values c = 0, b = 2, $\gamma = 1$ and $p_0 = 1/5$ in Table II. The best convergence condition is obtained for $\beta = 5/4$.

As shown in Table II, as a result of the increase of λ_D screening parameter from 10 to 500, energy values of all quantum states decrease. Therefore, the increase of λ_D screening parameter in the presence of velocity-dependent potential increases attractiveness of total interaction potential of the system and changes localizations of quantum levels.

The investigation of the effects of velocity-dependent potential parameters is also crucial as well as investigation of the effects of screening parameters of the MGECSC potential on energy values of hydrogen atom in Debye plasma. For $\lambda = 150$, $\gamma = 1$, c = 0, and b = 1, the energy values of 1s, 2s, 3s, 2p, 3p, and 4p quantum states versus isotropic dependence of velocity-dependent potential have been obtained in units of $2m = \hbar = 1$. As can be seen in Fig. 2, while energy values of 1s states decrease slowly with increasing of the isotropic dependence, the energy values of other states increase.

B. Case of b = 0, c ≠ 0

For case of b = 0, c ≠ 0, the MGECSC potential reduces the ECSC potential which is used to model quantum plasma.

TABLE II. For c = 0, b = 2, in the presence of velocity-dependent potential with $\gamma = 1$, $\rho_o = 1/5$, the energy values of 1s, 2s, 3s, 2p, 3p, 4p quantum states of hydrogen atom in Debye plasma for different λ values in units of $2m = \hbar = 1$.

	c = 0, b = 2 $\gamma = 1$, $\rho_o = 1/5$							
State	$\lambda = 10$	$\lambda = 50$	$\lambda = 100$	$\lambda = 150$	$\lambda = 200$	$\lambda = 300$	$\lambda = 400$	$\lambda = 500$
1s	−2.353033	−2.643330	−2.682694	−2.695981	−2.702655	−2.709351	−2.712707	−2.714723
2s	−0.575165	−0.864270	−0.903088	−0.916166	−0.922730	−0.929312	−0.932610	−0.934590
3s	2.900042	2.615983	2.577776	2.564901	2.558436	2.551953	2.548706	2.546755
2p	−1.341741	−1.674275	−1.719563	−1.734856	−1.742540	−1.750248	−1.754112	−1.756433
3p	1.262118	0.961232	0.920666	0.906990	0.900123	0.893236	0.889785	0.887712
4p	5.551519	5.259870	5.220606	5.207371	5.200726	5.194062	5.190724	5.188718

TABLE III. In the presence of velocity-dependent potential, for different eight values of c screening parameter and b = 0, the energy values of hydrogen atom in quantum plasma for 1s, 2s, 3s, 2p, 3p, 4p quantum states in units of $2m = \hbar = 1$, where $\lambda = 100$, $\gamma = 1$, $\rho_o = 1/5$.

b = 0 $\lambda = 100, \gamma = 1, \rho_o = 1/5, \beta = 7/6$								
State	c = 1	c = 1.2	c = 1.4	c = 1.6	c = 1.8	c = 2	c = 2.5	c = 5
1s	−0.712807	−0.712774	−0.712735	−0.712690	−0.712639	−0.712582	−0.712414	−0.711010
2s	1.067470	1.067502	1.067540	1.067584	1.067633	1.067689	1.067853	1.069222
3s	4.548934	4.548966	4.549003	4.549046	4.549095	4.549149	4.549310	4.550652
2p	0.244257	0.244297	0.244343	0.244397	0.244458	0.244527	0.244729	0.246412
3p	2.889403	2.889437	2.889477	2.889524	2.889577	2.889636	2.889810	2.891265
4p	7.190678	7.190711	7.190750	7.190794	7.190845	7.190901	7.191069	7.192462

For b = 0, c ≠ 0 and $\lambda = 100$, $\gamma = 1$, $\rho_o = 1/5$, $\beta = 5/4$, the energy values of 1s, 2s, 3s, 2p, 3p, and 4p quantum states have been obtained in units of $2m = \hbar = 1$. In the presence of the velocity-dependent potential, energy values of hydrogen atom in quantum plasma have been calculated for different eight values from 1 to 5 of c screening parameter. The correct and stable energy values have been found approximately after 15th iteration. As shown in Table III, the energy values of hydrogen atom incerase with the increasing values of c screening parameter. Note, however, that energies of only 1s quantum state are negative. It should be pointed out that influence on energy values of changing of c screening parameter in quantum plasma is less in contrast with influence on energy values of changing of b screening parameter in Debye plasma. So, it can be considered like that this effect is a perturbation. On the other hand, in the presence of the velocity-dependent potential, the increase of c screening parameter in quantum plasma decreases attractiveness of total effective potential of the system.

In order to examine effects of velocity-dependent potential on bound states of hydrogen atom in quantum plasma, the energy values in the presence of velocity-dependent potential should be compared with the energy values in the absence of velocity-dependent potential. Using $\lambda = 200$, $\gamma = 1$, $\rho_o = 1/5$, $\beta = 7/6$, the corresponding calculations have been made in order to probe influence of c screening parameter on energies of hydrogen atom in quantum plasma in the presence and absence of velocity-dependent potential. According to Fig. 3, the increase of c screening parameter increases slowly energy values in both cases. The effect of velocity-dependent potential on energy values of hydrogen atom in quantum plasma modeled by using ECSC potential has been investigated by considering case without velocity-dependent potential and the obtained results have been shown in Fig. 3. The contribution of velocity-dependent potential affect total interaction potential profile as described in Sec. IV A. The including velocity-dependent potential into the system leads to decrease energies of 1s state, increase 2s and 3s energies. However, since the increase of c screening parameter in case with velocity-dependent potential increases energy values, it should be pointed out that velocity-dependent potential effect on ECSC potential is used to model for increasing an interaction on the energy values of hydrogen atom in quantum plasma, in other words, the increase of c screening parameter in case with velocity-dependent potential decreases attractiveness of total interaction potential.

In the presence of velocity-dependent potential, the effect of λ screening parameter on energy values of hydrogen atom in modeled interaction by using ECSC potential, which relates to hydrogen atom in quantum plasma, has been examined. For c = 1.5, b = 0, the energies of 1s, 2s, 3s, 2p, 3p, and 4p quantum states have been calculated for different values from 10 to 500 of λ screening parameter as can be seen in Table IV.

Note that the energies of only 1s quantum state are negative. As a result of increasing λ screening parameter, the energy values of hydrogen atom decrease. When values of λ screening parameter are increased, the attractiveness of total interaction potential of system increases and corresponding bound states localize in the deeper region of potential well.

It is seen in Fig. 4 that how to change energy values versus isotropic form factor of velocity-dependent potential.

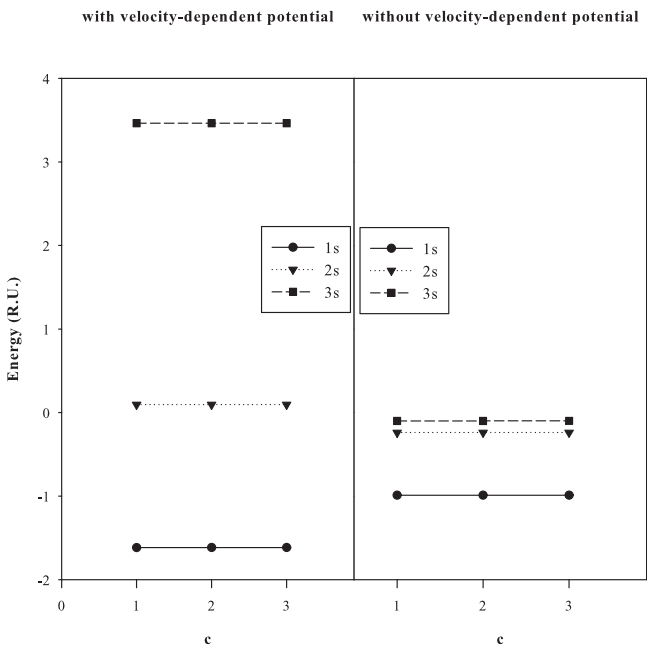

FIG. 3. For b = 0, c ≠ 0, in Rydberg units, comparison of the energy values of hydrogen atom in quantum plasma in the presence of velocity-dependent potential with $\lambda = 200$, $\gamma = 1$, $\rho_o = 1/5$ with the results of Ref. 36.

TABLE IV. For c = 1.5, b = 0, in the presence of velocity-dependent potential with $\gamma = 1$, $\rho_o = 1/5$, the energy values of 1s, 2s, 3s, 2p, 3p, 4p quantum states of hydrogen atom in quantum plasma for different λ values in units of $\hbar = 2m = 1$.

	c = 1.5, b = 0 $\gamma = 1, \rho_o = 1/5, \beta = 5/4$							
State	$\lambda = 10$	$\lambda = 50$	$\lambda = 100$	$\lambda = 150$	$\lambda = 200$	$\lambda = 300$	$\lambda = 400$	$\lambda = 500$
1s	−0.615593	−0.702445	−0.712714	−0.716098	−0.717783	−0.719463	−0.720301	−0.720803
2s	1.164377	1.077822	1.067561	1.064178	1.062494	1.060814	1.059976	1.059474
3s	4.645733	4.559280	4.549024	4.545641	4.543957	4.542279	4.541441	4.540939
2p	0.342594	0.254690	0.244369	0.240975	0.239286	0.237604	0.236765	0.236263
3p	2.986791	2.899778	2.889500	2.886113	2.884428	2.882748	2.881910	2.881408
4p	7.287765	7.201038	7.190771	7.187387	7.185703	7.184023	7.183185	7.182683

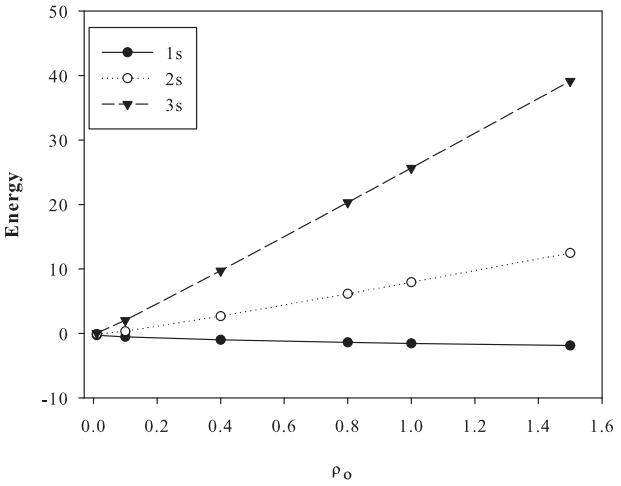

FIG. 4. The behavior of energy values of 1s, 2s, and 3s quantum states versus isotropic dependence of velocity-dependent potential in units of $\hbar = 2m = 1$, where $\lambda = 150$, c = 1, b = 0, $\gamma = 1$.

When strength of isotropic form factor of velocity-dependent potential is increased, as in Debye plasma, while this case gives rise to increase energy values of 2s and 3s quantum states, the same case decreases energies of 1s quantum state.

C. Case of b ≠ 0, c ≠ 0

The corresponding potential in the case of b ≠ 0, c ≠ 0 is the MGECSC potential in Eq. (5) and this potential can be also used to model interaction of hydrogen atom in quantum plasma. For b ≠ 0, c ≠ 0, the energy values of 1s, 2s, 3s, 2p, 3p, 4p states of hydrogen atom have been found using interaction potential parameters $\lambda = 100$, $\gamma = 1$, $\rho_o = 1/5$ in units of $\hbar = 2m = 1$. The iterations in calculations are stable after 20th iteration for $\beta = 5/4$. When $\beta = 1$, the energy values of 1s, 2s, 3s, 2p, 3p, 4p states of hydrogen atom have been computed for different eight values from 0.1 to 5 of c screening parameter as can be seen in Table V. If the obtained results are taken into account, it is realized that when values of c screening parameter are increased, the energy values of hydrogen atom also increase as shown in Table V. Here, it is clear that if values of c screening parameter increase, attractiveness of total interaction potential of system decreases, which has occurred in the presence of velocity-dependent potential.

As well as both of b and c screening parameter of MGECSC potential, individually change of these screening parameter can be worked on differently corresponding system. Using $\lambda = 100$, $\gamma = 1$, $\rho_0 = 1/5$ and taking as 1 c screening parameter of potential, the energy values of 1s, 2s, 3s, 2p, 3p, 4p, states of hydrogen atom have been computed for different eight values from 0.1 to 5 of b screening parameter and results of which have been introduced in Table VI. Unlike results in Table V, energy values of hydrogen atom in quantum plasma decrease with incerasing of b screening parameter as can be seen in Table VI. As a result of increasing b screening parameter, unlike the behavior of the system to variation of c screening parameter, the system exhibits the following behavior that the attractiveness of total interaction

TABLE V. In the presence of velocity-dependent potential, for different values of c screening parameter and b = 1, the energy values of hydrogen atom in quantum plasma for 1s, 2s, 3s, 2p, 3p, 4p quantum states in units of $\hbar = 2m = 1$, where $\lambda = 100$, $\gamma = 1$, $\rho_o = 1/5$.

	b = 1 $\lambda = 100, \gamma = 1, \rho_o = 1/5, \beta = 5/4$							
State	c = 0.1	c = 0.5	c = 1	c = 1.5	c = 2	c = 2.5	c = 3	c = 5
1s	−1.697773	−1.697725	−1.697575	−1.697324	−1.696974	−1.696523	−1.695972	−1.692770
2s	0.082149	0.082198	0.082351	0.082606	0.082964	0.083423	0.083984	0.087248
3s	3.563320	3.563367	3.563516	3.563764	3.564110	3.564556	3.565101	3.568269
2p	−0.737691	−0.737629	−0.737434	−0.737109	−0.736655	−0.736071	−0.735357	−0.731208
3p	1.904997	1.905049	1.905209	1.905476	1.905850	1.906331	1.906918	1.910334
4p	6.205606	6.205655	6.205808	6.206063	6.206419	6.206878	6.207438	6.210695

TABLE VI. In the presence of velocity-dependent potential, for different values of b screening parameter and c = 1, the energy values of hydrogen atom in quantum plasma for 1s, 2s, 3s, 2p, 3p, 4p quantum states in units of $\hbar = 2m = 1$, where $\lambda = 100$, $\gamma = 1$, $\rho_o = 1/5$.

	c = 1 $\lambda = 100, \gamma = 1, \rho_o = 1/5, \beta = 5/4$							
State	b = 0.1	b = 0.5	b = 1	b = 1.5	b = 2	b = 2.5	b = 3	b = 5
1s	−0.811283	−1.205188	−1.697575	−2.189968	−2.682369	−3.174775	−3.667188	−5.636907
2s	0.968957	0.574909	0.082351	−0.410202	−0.902753	−1.395300	−1.887845	−3.857989
3s	4.450392	4.056225	3.563516	3.070808	2.578101	2.085396	1.592691	0.378116
2p	0.146088	−0.246587	−0.737434	−1.228283	−1.719134	−2.209987	−2.700843	−4.664285
3p	2.790983	2.397306	1.905209	1.413112	0.921016	0.428920	−0.063175	−2.031557
4p	7.092192	6.698243	6.205808	5.713373	5.220939	4.728505	4.236071	2.266339

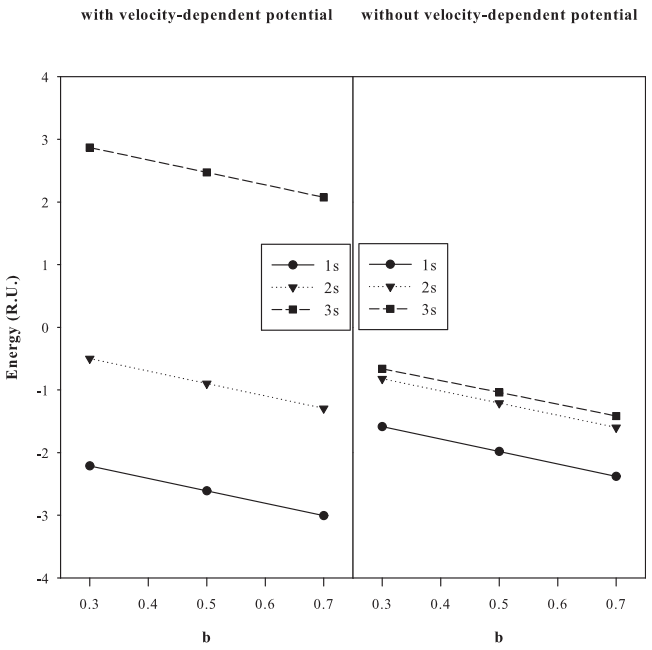

FIG. 5. For c = 1, b ≠ 0, in Rydberg units, comparison of the energy values of hydrogen atom in quantum plasma in the presence of velocity-dependent potential with $\lambda = 200$, $\gamma = 1$, $\rho_o = 1/5$ with the results of Ref. 36.

potential increases, the localizations of quantum levels change and consequently energy values decrease, which has also occurred in the presence of velocity-dependent potential.

For b ≠ 0, c ≠ 0, in order to realize effects of velocity-dependent potential on bound states of hydrogen atom, it is enough that comparison of present results with the results of Ref. 36. Using $\lambda = 200$, $\gamma = 1$, $\rho_0 = 1/5$ and taking as 1 c screening parameter of potential, the energy values of 1s, 2s, 3s states of hydrogen atom have been computed for different three values from 0.3 to 0.7 of b screening parameter as can be seen in Fig. 5. It is clear that the energy values decrease in both cases with and without velocity-dependent potential due to increasing b screening parameter. However, the following is also important to note that, similar to case in Secs. IV A and IV B, when considering same values of b parameter, by means of velocity-dependent potential, 1s energies decrease and the energies of other states increase. But, when going from case without velocity-dependent potential for b = 0.3 to case with velocity-dependent potential for b = 0.7, the energy of 2s state decreases from −0.822287 to −1.294446. Because, as previosly mentioned, b screening parameter have an important influence on total interaction potential.

On the other hand, it should be pointed out that since the increase of b screening parameter in case with velocity-dependent potential leads to decrease energy values, this type of interaction potential can be used to model decreasing some effects for energy values of hydrogen atom in quantum plasma.

In the presence of velocity-dependent potential, the energy values of hydrogen atom in quantum plasma modeled by using MGECSC potential have been computed for different eight values from 10 to 500 of λ screening parameter. The obtained results have been introduced in Table VII. The increase of λ screening parameter causes to decrease energy values as can be seen in Table VII.

TABLE VII. For c = 2.5, b = 3, in the presence of velocity-dependent potential with $\gamma = 1$, $\rho_o = 1/5$, the energy values of 1s, 2s, 3s, 2p, 3p, 4p quantum states of hydrogen atom in quantum plasma for different λ values in units of $\hbar = 2m = 1$.

	c = 2.5, b = 3 $\gamma = 1, \rho_o = 1/5, \beta = 5/4$							
State	$\lambda = 10$	$\lambda = 50$	$\lambda = 100$	$\lambda = 150$	$\lambda = 200$	$\lambda = 300$	$\lambda = 400$	$\lambda = 500$
1s	−3.002283	−3.602662	−3.664831	−3.684643	−3.694369	−3.703973	−3.708728	−3.711567
2s	−1.184505	−1.823412	−1.885391	−1.904988	−1.914581	−1.924037	−1.928712	−1.931501
3s	2.271261	1.655761	1.595062	1.575831	1.566409	1.557118	1.552522	1.549781
2p	1.875785	−2.623783	−2.697702	−2.721058	−2.732486	−2.743743	−2.749308	−2.752626
3p	0.661298	0.004230	−0.060625	−0.081173	−0.091240	−0.101168	−0.106078	−0.109007
4p	4.932086	4.300976	4.238501	4.218695	4.208989	4.199417	4.194682	4.191857

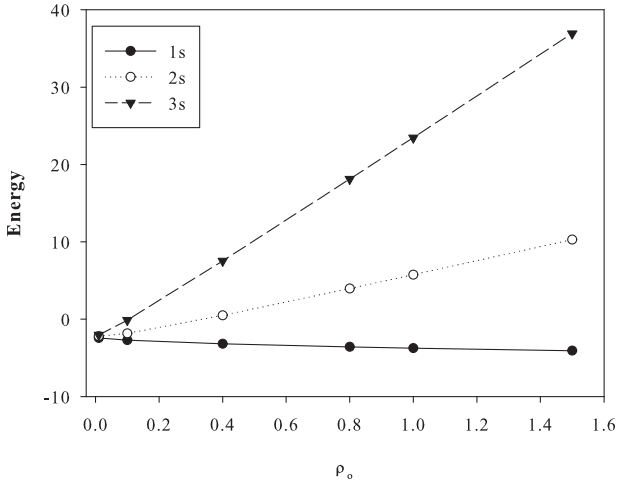

FIG 6. The behavior of energy values of 1s, 2s and 3s quantum states versus isotropic dependent of velocity-dependence potential in units of $\hbar = 2m = 1$, where $\lambda = 150$, c = 1.5, b = 2.2, $\gamma = 1$.

It is shown in Fig. 6 that in the presence of MGECSC potential how to affect isotropic form factor of velocity-dependent potential on energy values. Here, while increasing strength of isotropic form factor leads to decrease energy values of 1s states, it increases energy values of other states.

V. CONCLUSION

In this study, for the first time, screening effects on energy values of hydrogen atom in plasmas have been investigated in the presence of velocity-dependent potential. In addition to this, screening effects on energy values of velocity-dependent potential parameters have also been investigated. As previously mentioned, for the first time the MGECSC potential suggested by Soylu is used in order to model screening effects on bound states of hydrogen in plasma.[36] Because the advantage of the MGECSC potential is that stronger a screening effect is exhibited according to screening effect of case without these terms due to cosine term and b parameter in this potential. The influence of velocity-dependent potential and parameters of corresponding effective potential (V_{eff}) have been probed by considering different cases of b, c, and λ parameters. Namely, the obtained potential form by taking $c = 0$, $b \neq 0$ is used to Debye potential; creating form by taking $b = 0$, $c \neq 0$ as called ECSC potential is used to model quantum plasma and once again creating form by taking $b \neq 0$, $c \neq 0$ is MGECSC potential and this form can also used to model quantum plasma. The made work in this study is fact that investigation of screening effects on bound states of velocity-dependent potential for mentioned these three different cases. So, Eq. (14) has been solved by using AIM for mentioned these three different cases. The AIM is a very practical method and it can be applied to second-order differential equations similar to type of Eq. (14). Also, when results of the AIM is compared with obtained numerical results by using other methods, it is realized that the AIM has a very good agreement. In a nutshell, the AIM is quite an assertive method in this style of plasma physics problems and in order to solve many other quantum mechanical problems. Being parametres b, c, and λ of effective potential (V_{eff}) and ρ_o is parameter that stands for strength of isotropic dependence of velocity-dependent potential, there are four important parameters in this system. In fact, the main points to mean in this study are following that is how to affect parameters of total effective potential bound states of corresponding system and for this creating change what role of velocity-dependent potential is as shown in Tables I and VI, while the increase of b parameter decreases energy values, the increase of c parameter leads to increase of energy values as can be seen Tables III and V. Morever, the increase of λ parameter causes to decrease of energy values for each of case in Secs. IV A–IV C. When it takes into account that above mentioned results have been obtained in the presence of velocity-dependent potential and if Fig. 1, 3, and 5 are examined, it is understood that inclusion of velocity-dependent potential into system do not change characteristic of screening behaviors of b, c, and λ parameters for Debye and quantum plasma in the absence of velocity-dependent. In other words, increasing or decreasing effects on energy values of b, c, and λ parameters are same in the presence and absence of velocity-dependent potential. However, for same conditions while the including velocity-dependent potential into the system decrease energy values of 1s states, it increases energy values of other states (2s, 3s, and p states) as can be seen in Figs. 1, 3, and 5. The effect on energy values of the velocity-dependent potential gets predominantly functionality with strength of isotropic dependence. When strength of isotropic dependence increases for different cases of b and c, while energy values of 1s states decrease, energy values of other states increase, which shows that profile of total interaction potential of system becomes to be an asymmetric situation, in other words velocity-dependent potential does not have same effect on each point of profile of total interaction potential. The influence on bound states of isotropic dependence is seen clearly in Figs. 2, 4, and 6 and being in support above mentioned results of inclusion into system of velocity-dependent potential. The influence of velocity-dependent potential on profile of total interaction potential can be considered as an asymmetrize. Such an effect can be created by using another an effect as applying electric field. Thereby, for hydrogen atom in Debye and quantum plasma, some external influences that lead to constitute mentioned effects in this study on total interaction potential or bound states can be modeled by using velocity-

TABLE VIII. For $c = 1$, $b \neq 0$, in atomic units, comparison of the energy values of 2s state of hydrogen atom in quantum plasma with the results of Refs. 36, 41, and 42.

	2s b = 0, c = 1			
λ	Present Result	Ref. 36	Ref. 41	Ref. 42
10	−0.034941	−0.034941	−0.033500	−0.034967
20	−0.076449	−0.076449	−0.076406	−0.076449
50	−0.105103	−0.105103	−0.105103	−0.105103
100	−0.115013	−0.115013	−0.115013	−0.115013

dependent potential. It should be note that a very good agreement has been obtained in comparison between obtained present results by using AIM with the results obtained in the previous calculations with different methods as can be seen in Table VIII. In order to compare results in Table VIII with obtained results in this study, isotropic dependence of velocity-dependent potential (ρ_o) has been taken as zero. Also, in order to compare corresponding results, since atomic units are used $m = \hbar = 1$ in Refs. 36, 41, and 42, units of $2m = \hbar = 1$ in this study are setted as $m = \hbar = 1$.

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[1]E. M. Ferreıra, N. Guıllen, and J. Sesma, J. Math. Phys. **8**, 2243 (1967).
[2]L. S. Kisslinger, Phys. Rev. **98**, 761 (1955).
[3]M. Razavy, G. Field, and J. S. Levinger, Phys. Rev. **125**, 269 (1962).
[4]A. E. S. Green, G. Darewvch, and R. Berezdıvın, Phys. Rev. **157**, 929 (1967).
[5]L. I. Serra and E. Lipparini, Europhys. Lett. **40**, 667 (1997).
[6]M. Barranco, M. Pi, S. M. Gatica, E. S. Hernández, and J. Navarro, Phys. Rev. B **56**, 8997 (1997).
[7]M. R. Geller and W. Kohn, Phys. Rev. Lett. **70**, 3103 (1993).
[8]J. Yu and S. H. Dong, Phys. Lett. A **325**(3–4), 194–198 (2004).
[9]J. Yu, S. H. Dong, and G. H. Sun, Phys. Lett. A **322**(5–6), 290–297 (2004).
[10]S. H. Dong and M. Lozado-Cassou, Phys. Lett. A **337**(4–6), 313 (2005).
[11]A. D. Alhaidari, Phys. Lett. A **322**(1), 72 (2004).
[12]S. C. Jain and B. K. Srivastava, Phys. Rev. **169**, 833 (1968).
[13]A. E. S. Green, D. E. Rio, and T. Ueda, Phys. Rev. A **24**, 3010 (1981).
[14]M. I. Jaghoub, Phys. Rev. A **74**, 032702 (2006).
[15]M. I. Jaghoub, Eur. Phys. J. A **15**, 443 (2002).
[16]M. I. Jaghoub, Eur. Phys. J. A **13**, 349 (2002).
[17]M. I. Jaghoub, Eur. Phys. J. A **28**, 253 (2006).
[18]M. I. Jaghoub, Eur. Phys. J. A **27**, 99 (2006).
[19]M. I. Jaghoub, Eur. Phys. J. A **11**, 175 (2001).
[20]A. Soylu, O. Bayrak, and İ. Boztosun, Cent. Eur. J. Phys. **10**(4), 953 (2012).
[21]O. Bayrak, A. Soylu, and İ. Boztosun, Chin. Phys. Lett. **28**(4), 040304 (2011).
[22]A. C. H. Yu and Y. K. Ho, Phys. Plasmas **12**, 043302 (2005).
[23]S. Sahoo and Y. K. Ho, Phys. Plasmas **13**, 063301 (2006).
[24]S. Kar and Y. K. Ho, Phys. Plasmas **15**, 013301 (2008).
[25]S. Paul and Y. K. Ho, Phys. Plasmas **16**, 063302 (2009).
[26]S. Paul and Y. K. Ho, Phys. Plasmas **15**, 073301 (2008).
[27]S. Paul and Y. K. Ho, Phys. Rev. A **78**, 042711 (2008).
[28]S. Paul and Y. K. Ho, Phys. Plasmas **17**, 082704 (2010).
[29]G. Ecker and W. Weizel, Ann. Phys. (Leipzi) **452**, 126 (1956).
[30]H. Margenau and M. Lewis, Rev. Mod. Phys. **31**, 569 (1959).
[31]P. Winkler, Phys. Rev. E **53**, 5517 (1996).
[32]L. Zhang and P. Winkler, Int. J. Quantum Chem. **60**(7), 1643 (1996).
[33]D. Bielinska-Waz, J. Karwowski, B. Saha, and P. K. Mukherjee, Phys. Rev. E **69**, 016404 (2004).
[34]S. Dong, G.-H. Sun, and S. H. Dong, Int. J. Mod. Phys. E **22**(6), 1350036 (2013).
[35]A. Ghoshal and Y. K. Ho, J. Phys. B **42**, 075002 (2009).
[36]A. Soylu, Phys. Plasmas **19**, 072701 (2012).
[37]M. A. K. Lodhi, Phys. Rev. C **1**, 365 (1970).
[38]H. Ciftci, R. L. Hall, and N. Saad, J. Phys. A **36**, 11807 (2003).
[39]N. Saad, R. L. Hall, and H. Ciftci, J. Phys. A **39**, 13445 (2006).
[40]H. Ciftci, R. L. Hall, and N. Saad, J. Phys. A **38**, 1147 (2005).
[41]R. Sever and C. Tezcan, Phys. Rev. A **35**, 2725 (1987).
[42]S. Paul and Y. K. Ho, Comput. Phys. Commun. **182**, 130 (2011).