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# Chapter 10

# Representational practices in VMT

Richard Medina, Dan Suthers & Ravi Vatrapu

Abstract

This chapter analyzes the interaction of three students working on mathematics problems over several days in a virtual math team. Our analysis traces out how successful collaboration in a later session was contingent upon the work of prior sessions, and shows how representational practices are important aspects of these participants' mathematical problem solving. We trace the formation, transformation and refinement of one problem-solving practice—*problem decomposition*—and three representational practices—*inscribe first solve second*, *modulate perspective* and *visualize decomposition*. The analysis is of theoretical interest because it suggests that "situated cognition" is contingent upon not only the immediate situation but also the chronologically prior resources and associated practices; shows how *inscriptions become representations* for the group through an interactive process of interpretation; and sheds light on "group cognition" as an interactional process that is not identical to individual cognition yet that draws upon a dynamic interplay of individual contributions.

Prior work in our laboratory at the University of Hawai'i and elsewhere has examined the importance of representational resources to collaborative learning, including experimental studies testing hypotheses concerning how given notations or environments can influence learning processes (Suthers & Hundhausen, 2003; Suthers *et al.*, 2008) and ideographic analyses of how participants make use of representational affordances (Dwyer & Suthers, 2006; Medina & Suthers, 2008). In order to broaden our understanding to a greater diversity of representations and situations, we have begun to analyze data from other sources. Sharing of data and analyses across laboratories is an important strategy for advancing our field, as exemplified by this volume. At Gerry Stahl's recommendation, we selected for examination Team B's work in the VMT Spring Fest 2006, previously analyzed in Stahl (2007b) (now expanded in Chapter 26). In this sequence of four hour-long sessions, students address a breakdown in their understanding of how they solved a problem, making indexical references to inscriptions in a whiteboard as problem representations. The first major concern of this paper is to understand the role of representations in these students' problem solving.

There is a convincing body of work showing that learning, problem solving and other group accomplishments are contingent upon the situation (e.g., Garfinkel, 1967; Goodwin, 2000; Greeno, 2006; Koschmann, Stahl, & Zemel, 2007; Lave & Wenger, 1991). The second major concern of this paper is the claim that this situated contingency is not restricted to the immediate situation or bounded at some temporal threshold, but reaches into the past at successively larger granularities. As Blumer tells us, "any instance of joint action, whether newly formed or long established, has necessarily arisen out of a background of previous actions of the participants" (Blumer, 1969, p. 20). In computer-supported collaborative learning, contingencies may extend back in time with the aid of persistent resources such as inscriptions in a workspace (Latour, 1990). Therefore, to understand the breakdown and repair of the selected VMT episode, we needed to examine participants' prior work together, and we needed to attend particularly to inscriptions in the graphical whiteboard in addition to messages in the chat board.

In the process of examining the data, we chose to focus on earlier sessions than the one analyzed by Stahl (2007b). The episode analyzed by Stahl was the fourth session (each of the four session taking place on a different day, but in the same chat room). Looking back through the data, we found in the third session a remarkable event. It begins when Aznx (a self-selected pseudonym) says, "*I have an interesting way to look at this problem*," and proceeds to describe an innovative representation of the problem that enables its decomposition into mathematically simpler expressions. Aznx's partners seem to quickly understand what he is trying to do, and indeed another participant, Bwang, supplies the actual visualization of the problem representation, using color to distinguish the components of the decomposition. Is this an instance of a brilliant insight arising whole cloth from the mind of an individual? If so, how were the others able to appropriate it so quickly? Or is the insight a product of group cognition (Stahl, 2006)? If so, how did the group build on Aznx's comment without much apparent deliberation, quickly applying methods of problem representation and decomposition? In either case, if they understood each other so well, why was there a breakdown in the later session analyzed by Stahl (2007b)?

To begin to answer these questions, we looked further back at prior sessions to identify how the insight expressed by Aznx and the group's handling of this insight were contingent upon prior interactions. We found that participants' actions in session 3 were the continuation of prior *practices.* These practices were *joint* practices developed in the interaction of group members and shared by those members. We identified abstract problem-solving practices that were largely enacted as *representational practices*: methods for generating, manipulating and interpreting inscriptions that the group developed for handling a class of problems (Enyedy, 2005; Kozma & Russell, 2005; Roth, 2003). This chapter reports on the representational practices we identified, and the manner in which they were developed by participants and applied to generate the insights of session 3. It then returns to some of the theoretical issues raised above.

# Background

### Data

Data for this analysis was drawn from the VMT Spring Fest 2006 project. Groups of three students and a moderator, all at different locations, convened in four separate sessions over four days to derive solutions for different algebraic geometry problems (see Chapters 7 and 8 for excerpts from Team C). Participants interacted using the VMT environment (Figure 10-1), a software environment consisting of a shared whiteboard and a chat tool with the capability of referencing the whiteboard (see Chapter 15). They also used a wiki to post their solutions during and after each session. These wiki pages, the software log files and a re-playable instance of the interaction environment for all sessions served as our data sources. Our analysis focuses on the work of team B.

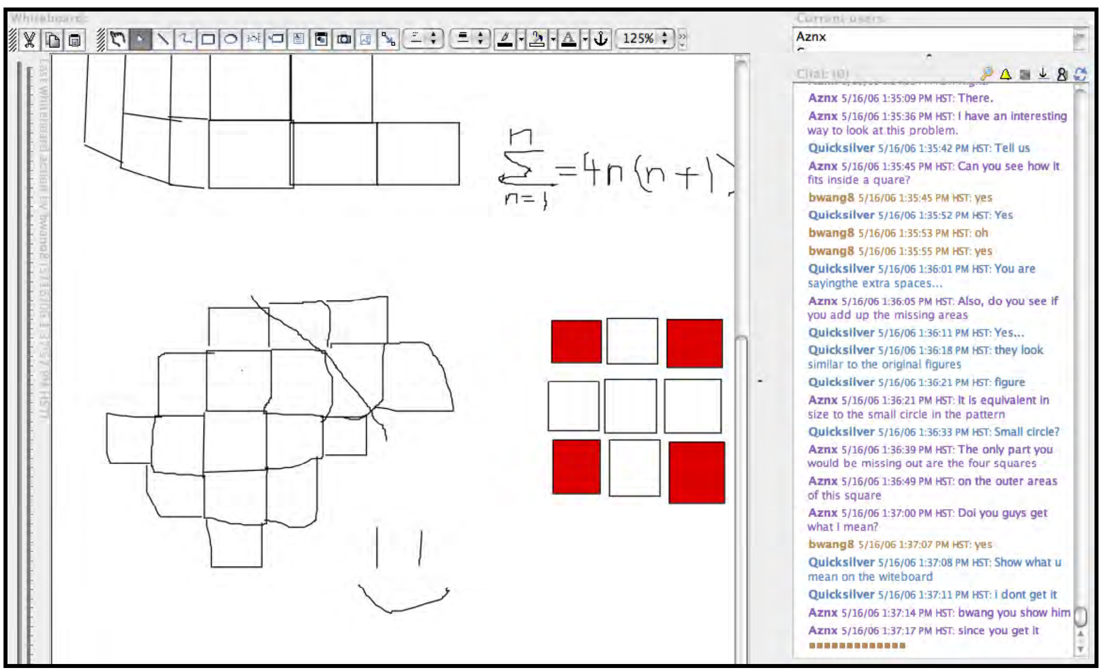



Figure 10-1. Team B in the VMT software environment.

We utilized the software logs and the re-playable version of the VMT sessions, moving from one format to the other as needed. For example, the log maintains the ordering of discrete events and their related information (act, actor, timestamp, media, etc). This is useful for recording annotations and reading participant conversations. The VMT Replayer provided a richer contextual view useful for understanding the participants' inscriptional work and its concurrent development with the interaction in the chat tool. (Screen images in this paper are from the Replayer.)

## Method

The analysis began with identification of an episode of interest, and then worked both backwards and forwards at two granularities (termed *global* and *local* for convenience of reference) to construct accounts of the participants' interaction and accomplishments.

We began with the episode from session 4 analyzed in Stahl (2007a). In this episode, participants reference certain inscriptions available in the whiteboard, construing them as representational resources for resolving the question at hand. At the global granularity of analysis, we searched backwards to find chronologically prior episodes in which these inscriptions or related inscriptions were constructed, in order to understand how they previously functioned as representations for the participants. (Our conception of "related" expanded as the analysis progressed.) We bounded episodes by first identifying where the development of an inscription had been completed, and then worked back to where the construction and discussion of the inscription began as well as forward to the completion of discussion about the inscriptions. Episodes were first identified at the point where inscriptions were completed because this is where the inscriptions had reached the form in which they were available in future episodes. Chat interaction was as important as inscriptional activity in identifying and bounding relevant episodes, since participants' chat referenced, labeled and interpreted inscriptions in the whiteboard. This process of searching backwards for relevant prior episodes was repeated until we had identified a chain back to the first session.

Then the local granularity of analysis worked forwards within each episode to construct an account of the interaction within the episode. (Local analysis was not applied to the episode already analyzed by Stahl, 2007a). This granularity was undertaken in a manner similar to conversation analysis (Heritage, 1995; Sacks, 1992) as it is applied in CSCL (e.g., Koschmann *et al.*, 2005; as well as Stahl, 2007b), but attended to inscriptional acts as well as conversations in the chat tool. Discussions in the chat are often interwoven with inscriptional work in the whiteboard in a manner that distributes conversation across the two media (Suthers, 2006) (see Chapter 7). A trace of the contributions made in each of these media at the level of speech and inscriptional acts provides a resource for understanding contingent interaction. On examination, certain events within each segment were annotated with observational notes to document relationships between individual acts. For example, we may see the reuse or introduction of inscriptional practices or linguistic references that demonstrate contingent relationships from one act to the next. During local analysis, the segment under consideration was sometimes expanded to encompass the episode of meaning making relevant to the question at hand. Issues identified locally also facilitated further global analysis of relationships between episodic frames.

In summary, we worked backwards "globally" to identify prior episodes on which a given episode's accomplishments may have been contingent; and worked forwards "locally" within each episode to identify participants' methods of meaning making with the resources available. The result is a *trace of contingencies* at two granularities that enables us to recognize patterns in the data and better understand collaborative interaction and its accomplishment in shared environments (Medina & Suthers, 2008; Suthers *et al.*, 2007). Traces can be represented as graphs or organized as a sequentially ordered set of events. In the present analysis we relied on the latter to document interactional traces in the data.

# Analysis

In this analysis we initially found a particular episode towards the end of the group project (Session 3 of 4) in which the participants co-constructed a problem representation in the whiteboard. This artifact played an indexical role in the group's interpretation of their solution discussed in Stahl (2007b). Taking the construction of this artifact as a starting point, we began to document the contingent relationships between it and the interaction history.

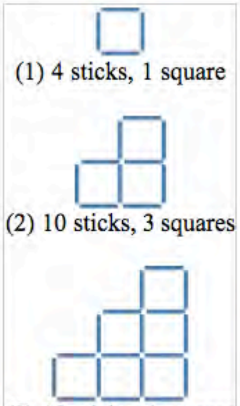
The analyses presented in the following sections reveal that the formation, transformation and refinement of representational practices are important aspects of these participants' mathematical problem solving. The participants demonstrate four practices that are introduced, applied and adapted in ongoing group problem solving that spans four meetings. These practices are reified as inscriptions in the whiteboard, but are also enacted in linguistic interaction in the chat tool. Our analysis shows the emergence and sustaining of one problem-solving practice—*problem decomposition*—and three representational practices—*inscribe first solve second*, *modulate perspective* and *visualize decomposition*. The practices are interdependent and compositional. Each particular enactment of a practice either introduces a previously unutilized practice into the joint work or builds on a previous instantiation.

We begin with a description of an episode in Session 1 of Team B, in which the practices of *problem decomposition* and *inscribe first, solve second* are introduced into the group's work by one of the participants. We then describe a series of subsequent episodes in the next two sessions ending with our analytical entry point in Session 3 briefly discussed above. This ordering is presented to provide evidence for the historical development of representational practices in joint interaction.

### Session 1: Initial Appearance of Practices

In this session participants are meeting for the first time in the collaborative environment. They settle in, ask questions about the software and begin working on their first problem. The problem description and instructions are provided on a wiki page (Figure 10-2). The participants are instructed to derive a growth pattern, and then employ it to complete a table of incremental stages of growth in terms of number of sticks (lines) and number of squares.

Here are the first few examples of a particular pattern or sequence, which is made using sticks to form connected squares:




| N | Sticks | Squares |
|---|---|---|
| 1 | 4 | 1 |
| 2 | 10 | 3 |
| 3 | 18 | 6 |
| 4 | ? | ? |
| 5 | ? | ? |
| 6 | ? | ? |
| ... | ... | ... |
| N | ? | ? |

*Scroll down to see instructions for each Session.*

**Session I**

1. Draw the pattern for N=4, N=5, and N=6 in the whiteboard. Discuss as a group: How does the graphic pattern grow?
2. Fill in the cells of the table for sticks and squares in rows N=4, N=5, and N=6. Once you agree on these results, post them on the VMT Wiki
3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the VMT Wiki.

Figure 10-2. Instructions for session 1.

For the remainder of the discussion we will refer to the three participants using their chat handles *Aznx*, *Bwang* and *Quicksilver*. Transcripts are based on the VMT log file, which includes all actions in the software, including whiteboard edits. To preserve space, we have chosen to include only chat contributions in our transcript presentations and to provide figures as needed to display the inscriptions to be analyzed. Annotations in the right hand column of the transcript relate these inscriptions to the chat. Therefore, line numbers in the log excerpts will at times be nonconsecutive (e.g., lines 183-185 in Log 10-1 are omitted nonlinguistic actions that led to the completion of an inscription shown in Figure 10-3). Shaded fields help the reader pick out lines referenced in our text.

*Chronological Summary of Episode*

Bwang initiates the problem solving at transcript index [182] (refer to Log 10-1 and Figure 10-3 during this discussion) by posting, "*you can divide the thing into two parts*." He immediately begins to draw two sets of lines. One set is horizontal and the other vertical; each set corresponds to the pattern drawn in the instruction information (Figure 10-2), but the horizontal and vertical lines are drawn apart from each other. After completing this inscription (Figure 10-3, top left), Bwang proceeds to explain in the chat window how the arrangement of horizontal and vertical lines can be used to derive a formula [219]. The other two participants orient to both the inscription and the problem at [214] and [237].

Log 10-1.

| | | | | |
|---|---|---|---|---|
| 182 | 18:32:05 | Bwang | you can divide the thing into two parts | |
| 186 | 18:32:10 | Aznx | Let's start this thing. | |
| 206 | 18:32:38 | Quicksilver | my computer was lagging...What are we doing? | |
| 210 | 18:32:49 | Aznx | http://home.old.mathforum.org/SFest.html | |
| 214 | 18:32:58 | Quicksilver | what are the lines for? | Bwang has completed the inscription in the whiteboard (Figure 10-3) |
| 216 | 18:33:01 | Aznx | go to view topic | |
| 219 | 18:33:05 | Bwang | so you can see we only need to figur one out to get the total stick | |
| 222 | 18:33:09 | Aznx | read the problem | |
| 224 | 18:33:32 | Bwang | 1+2+3+........+N+N | |
| 230 | 18:33:38 | Bwang | times that by 2 | |
| 232 | 18:33:40 | Quicksilver | Never mind I figured it out.. | |
| 237 | 18:34:01 | Aznx | Can we collaborate this answer even more? | |
| 240 | 18:34:05 | Aznx | To make it even simpler? | |
| 244 | 18:34:15 | Bwang | ok | |
| 246 | 18:34:16 | Aznx | Because I think we can. | |
| 250 | 18:34:50 | Bwang | ((1+N)*N/2+N)*2 | |
| 253 | 18:34:58 | Bwang | that's the formula, right? | |
| 258 | 18:35:15 | Aznx | How did you come up with it? | |
| 260 | 18:35:16 | Bwang | for total sticks | |
| 270 | 18:35:34 | Bwang | Is a common formual | |
| 274 | 18:35:40 | Bwang | formula | |
| 278 | 18:35:46 | Aznx | Yeah, I know. | |
| 280 | 18:35:59 | Bwang | and just slightly modify it to get this | |
| 292 | 18:36:31 | Aznx | Aditya, you get this right? | |
| 315 | 18:37:45 | Quicksilver | What does the n represent? | |
| 319 | 18:37:57 | Bwang | the given | |
| 322 | 18:37:58 | Bwang | N | |
| 326 | 18:38:02 | Aznx | Yeah. | |
| 330 | 18:38:05 | Aznx | In the problem. | |
| 335 | 18:38:13 | Gerry | Gerry created a textbox | |
| 336 | 18:38:17 | Gerry | Gerry resized some objects | |
| 337 | 18:38:20 | Quicksilver | Quicksilver resized some objects | |
| 341 | 18:38:37 | Quicksilver | Oh | |
| 343 | 18:38:38 | Bwang | The number of squares is just (1+N)*N/2 | |
| 348 | 18:38:50 | Quicksilver | We need that as well. | |
| 351 | 18:38:52 | Gerry | I put Bwang's formula on the whiteboard | |

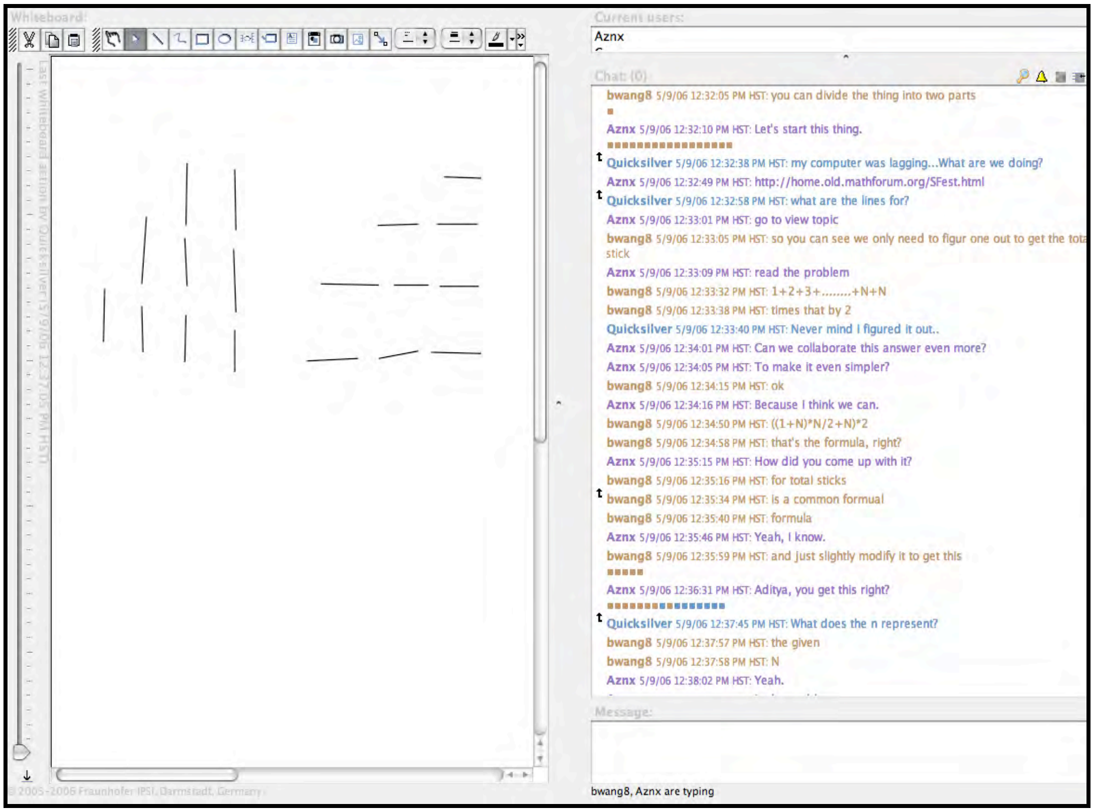



Figure 10-3. Initiating the practice of visualizing problem decomposition.

Following the construction of the inscription, the group begins to develop a formula for the growth pattern. Chat postings [224] through [292] show an exchange in which Bwang and Aznx are discussing the solution and propose two formulas. Aznx then confers with Quicksilver to determine his understanding. With the assistance of the moderator [351], the formulas initially posted in the chat tool by Bwang are inserted into the whiteboard adjacent to Bwang's inscription. After the transcript ends, the formulas are applied by the participants to complete the table, as required by the problem instructions (Figure 10-2).

*Practices Displayed*

Several practices that are taken up in latter sessions make their initial appearance in this episode. Bwang has brought forward two related problem-solving strategies. The first, which we call *decompose problem,* is exemplified by his recognition that the vertical and horizontal lines ("sticks") composing the geometric figure can be separated into two equal sets, so that only one set needs to be counted [182, 219]. The second, which we call *inscribe first, solve second,* is exemplified by his construction of an inscription before deriving the formulas for the number of "sticks" (lines) [250] and squares [343]. This strategy is implied by the steps of the session instructions (Figure 10-2), but is actualized by participants' actions. Bwang has also introduced a representational strategy, which we call *visualize decomposition.* His inscriptions visually decompose the structure of the geometric figure presented in the problem statement (Figure 10-2), spatially separating horizontal and vertical lines in a manner that reflects a problem decomposition that can then be mapped to a formulaic solution. By inscribing a decomposed representation in the white board, Bwang has not only made a specific inscription available to the group, but has also made a strategy for visualizing the problem-decomposition strategy available. In subsequent sessions we see how the persistence of the whiteboard medium preserves and carries these resources forward to the future.

The three strategies are highly integrated in this episode: visualizing the decomposition in an inscription makes it easier to derive the formula from the decomposition. We will justify our identification of these strategies as *practices* by showing that the strategies are taken up in later sessions. We will justify our identification of these three practices as *distinct* practices by showing that they are sometimes enacted in different ways and combinations. For example, in this session *problem decomposition* is distinguished from *visualize decomposition* because the former is first expressed in language.

## Session 2: The Practices Reappear in Different Forms

Moving now to the team's second session, we find that the participants have decided to work on a problem of their own choosing. The previous day's inscriptions remain in the whiteboard.

### *Summary of Episode*

In this episode, Quicksilver takes the initiative and suggests working on generating a pattern for a pyramid at [1379] (refer to Log 10-2). The others agree on the idea and Quicksilver then inscribes a pyramid-shaped figure in the whiteboard (see Figure 10-4, middle and outlined with the referencing tool). On completing the pyramid he then references the figure from the chat posting [1415], explaining that it is a "side view" perspective. In the ensuing discussion, the participants attempt to work out how the inscription can relate to the problem from the previous day [1419]. Participants all attempt to show how the inscription can be decomposed [1440-1445; 1462-1464].

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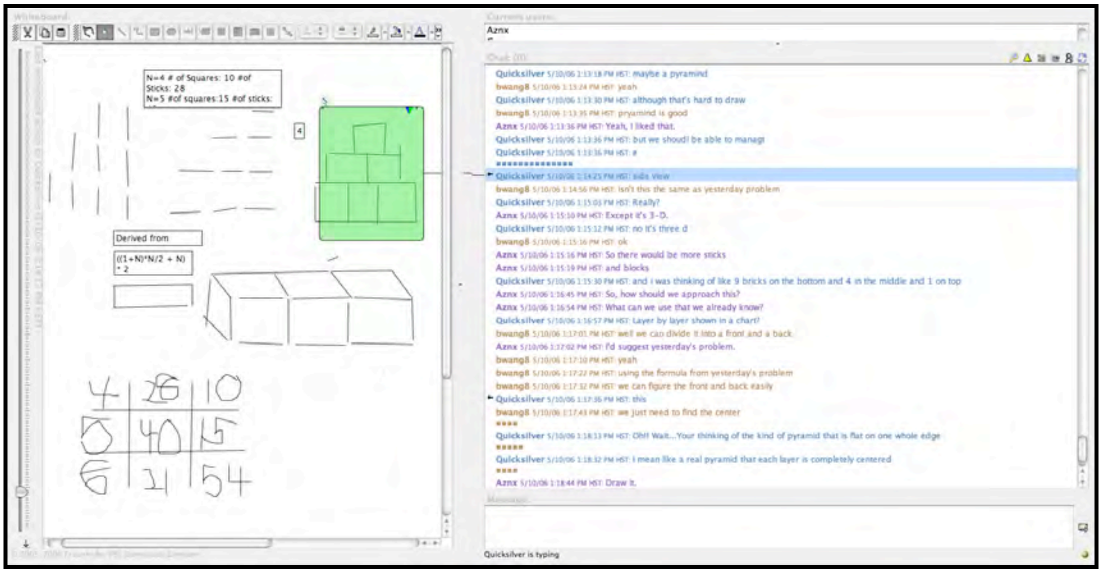


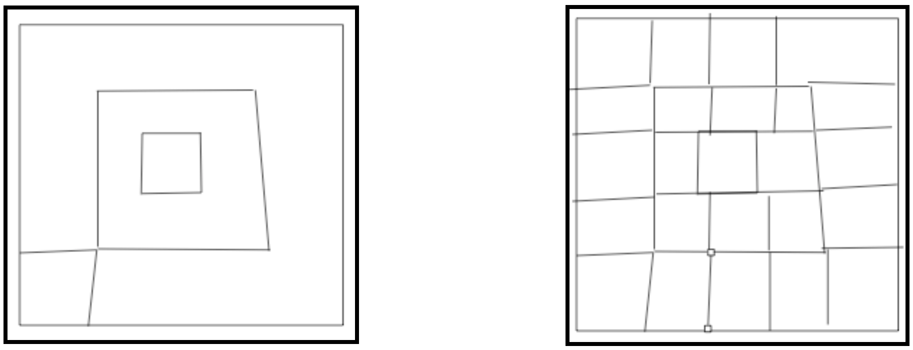

Figure 10-4. Side view of pyramid.

The exchange in the chat window concerning Quicksilver's inscription exposes an instance of the group's practice of *inscribe first, solve second*. At [1466] (Log 10-2) Aznx proposes that the group draw on the approach taken in the previous session. Bwang concurs [1469] and further proposes aligning the current problem with specific formulas from Session 1 [1473]. Following this exchange, Quicksilver indicates that the approach the others are discussing is not compatible with his "side view." At [1493] he articulates that the others are misinterpreting the inscription. He follows this up by restating his objective [1502] and, on Aznx's prompting [1509], proceeds to draw a second inscription. He refers to this inscription (Figure 10-5a) as a "top view" [1543] because it shows a pyramid as viewed from above. Aznx assists by adding additional lines to the drawing to complete the decomposition visualization (Figure 10-5b). With the new inscription drawn from a different perspective, the participants begin a second round of discussion concerning the problem solution.

Log 10-2.

| | | | | |
|---|---|---|---|---|
| 1379 | 19:13:18 | Quicksilver | maybe a pyramind | |
| 1383 | 19:13:24 | Bwang | yeah | |
| 1387 | 19:13:30 | Quicksilver | although that's hard to draw | |
| 1391 | 19:13:35 | Bwang | pryamind is good | |
| 1393 | 19:13:36 | Aznx | Yeah, I liked that. | |
| 1395 | 19:13:36 | Quicksilver | but we shoudl be able to managt | |
| 1398 | 19:13:36 | Quicksilver | e | |
| 1415 | 19:14:25 | Quicksilver | side view | Inscription complete (Figure 10-4) |
| 1419 | 19:14:56 | Bwang | isn't this the same as yesterday problem | |
| 1423 | 19:15:03 | Quicksilver | Really? | |
| 1430 | 19:15:10 | Aznx | Except it's 3-D. | |
| 1433 | 19:15:12 | Quicksilver | no it's three d | |
| 1438 | 19:15:16 | Bwang | ok | |
| 1440 | 19:15:16 | Aznx | So there would be more sticks | |
| 1443 | 19:15:19 | Aznx | and blocks | |
| 1445 | 19:15:30 | Quicksilver | and i was thinking of like 9 bricks on the bottom and 4 in the middle and 1 on top | |
| 1450 | 19:16:45 | Aznx | So, how should we approach this? | |
| 1459 | 19:16:54 | Aznx | What can we use that we already know? | |
| 1462 | 19:16:57 | Quicksilver | Layer by layer shown in a chart? | |
| 1464 | 19:17:01 | Bwang | well we can divide it into a front and a back | |
| 1466 | 19:17:02 | Aznx | I'd suggest yesterday's problem. | |
| 1469 | 19:17:10 | Bwang | yeah | |
| 1473 | 19:17:22 | Bwang | using the formula from yesterday's problem | |
| 1476 | 19:17:32 | Bwang | we can figure the front and back easily | |
| 1479 | 19:17:36 | Quicksilver | this | |
| 1483 | 19:17:43 | Bwang | we just need to find the center | |
| 1493 | 19:18:13 | Quicksilver | Oh!! Wait...Your thinking of the kind of pyramid that is flat on one whole edge | |
| 1502 | 19:18:32 | Quicksilver | I mean like a real pyramid that each layer is completely centered | |
| 1509 | 19:18:44 | Aznx | Draw it. | |
| 1513 | 19:18:57 | Quicksilver | i'll try | |
| 1522 | 19:19:24 | Bwang | use the rectangle tool, it's easier | |
| 1528 | 19:19:32 | Aznx | Yeah. | |
| 1531 | 19:19:33 | Quicksilver | k | |
| 1539 | 19:19:44 | Bwang | o ic | |
| 1543 | 19:19:49 | Quicksilver | top view | |
| | 19:21:36 | Quicksilver | | Inscription complete (Figure 10-5a) |

The "top view" perspective is a resource in a further exchange between Quicksilver and Aznx [1659-1760, Log 10-3]) as they attempt to work out a decomposition pattern based on the new, "top view" inscription. The discussion results in both participants having a slightly different explanation for how the problem should be deconstructed. At issue is whether or not the top-view perspective is a three- or two-dimensional representation [1747-1760]. Aznx's question, "*You want to do 3-D?*" [1760], reveals that the two participants had a different understanding of the role of the inscription in the problem solving. Parallel to this discussion, Quicksilver inscribes a third perspective using blue and red to distinguish different levels of the pyramid (see Figure 10-6). Quicksilver's response to Aznx's question is directed at Bwang at line [1765] in Log 10-3. Quicksilver asks Bwang for assistance in clarifying the group's activity. Bwang responds with a proposal to divide the layers of the pyramid into "levels" [1777].



(a) Top view constructed by Quicksilver

(b) Inscription in (a) extended by Aznx

Figure 10-5. Top view of pyramid

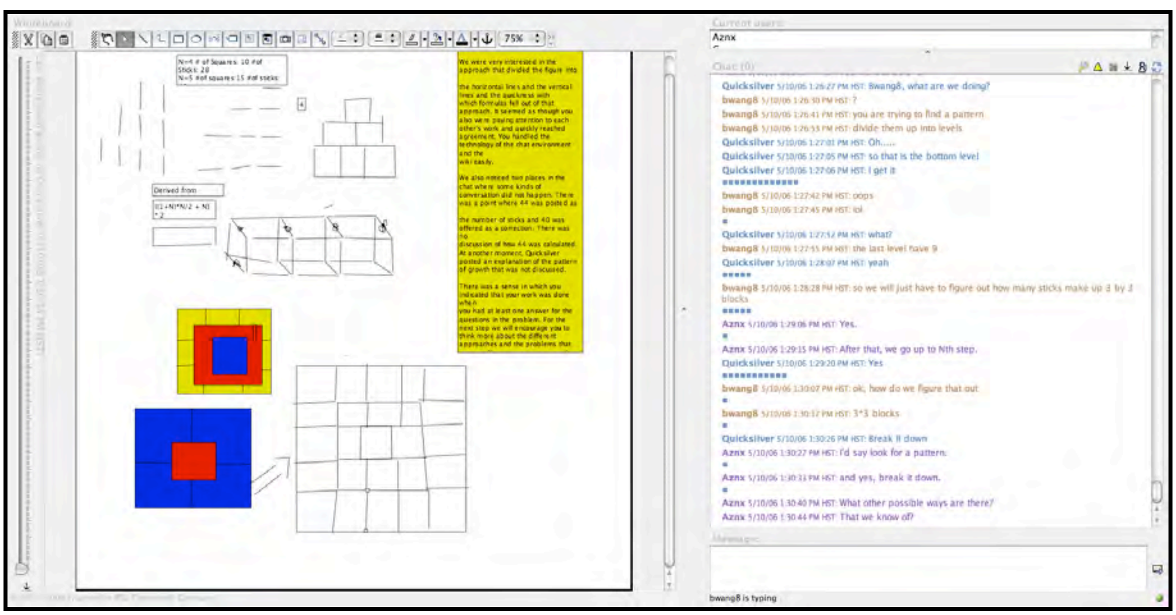



Figure 10-6. Color used to show layers of pyramid

Log 10-3.

| | | | | |
|---|---|---|---|---|
| 1659 | 19:23:42 | Aznx | Instead of a triangular format of the sticks, we do the one you jsut made: the board format? | Reference to top view inscription in figures 10-5a and 10-5b. |
| 1698 | 19:24:18 | Quicksilver | what do u mean? | |
| 1708 | 19:24:42 | Aznx | Look at my arrow. | |
| 1711 | 19:24:42 | Quicksilver | ok | |
| 1715 | 19:24:49 | Aznx | So you start off with one block. | |
| 1718 | 19:24:52 | Quicksilver | And that's a top view right | |
| 1722 | 19:25:00 | Aznx | Yes. | |
| 1725 | 19:25:04 | Quicksilver | Well there's a problem | Quicksilver begins to redraw inscription using color (Figure 10-6, bottom left) |
| 1731 | 19:25:34 | Aznx | So, the first one has 1 block. | Quicksilver completes blue and red, top view pyramid (Figure 10-6, bottom left) |
| 1735 | 19:25:41 | Aznx | and four sticks | |
| 1739 | 19:25:48 | Quicksilver | first block | |
| 1741 | 19:25:51 | Aznx | The second one has 5 blocks. | |
| 1745 | 19:25:59 | Aznx | Wait | |
| 1747 | 19:26:00 | Quicksilver | no it is 3 | |
| 1751 | 19:26:02 | Quicksilver | d | |
| 1753 | 19:26:03 | Aznx | You're doing it wrong. | |
| 1756 | 19:26:04 | Quicksilver | 3d | |
| 1760 | 19:26:12 | Aznx | You want to do 3-D? | |
| 1765 | 19:26:27 | Quicksilver | Bwang8, what are we doing? | |
| 1767 | 19:26:30 | bwang8 | ? | |
| 1771 | 19:26:41 | bwang8 | you are trying to find a pattern | |
| 1777 | 19:26:53 | bwang8 | divide them up into levels | |
| 1781 | 19:27:01 | Quicksilver | Oh..... | |
| 1784 | 19:27:05 | Quicksilver | so that is the bottom level | |
| 1787 | 19:27:06 | Quicksilver | I get it | |
| 1809 | 19:27:42 | bwang8 | oops | |
| 1812 | 19:27:45 | bwang8 | lol | |
| 1818 | 19:27:52 | Quicksilver | what? | |
| 1820 | 19:27:55 | bwang8 | the last level have 9 | |
| 1824 | 19:28:07 | Quicksilver | yeah | Quicksilver begins drawing yellow, red, blue inscription (Figure 6) |
| 1831 | 19:28:28 | bwang8 | so we will just have to figure out how many sticks make up 3 by 3 blocks | |
| 1839 | 19:29:06 | Aznx | Yes. | |
| 1843 | 19:29:15 | Aznx | After that, we go up to Nth step. | |
| 1848 | 19:29:20 | Quicksilver | Yes | |
| 1867 | 19:30:07 | bwang8 | ok, how do we figure that out | |
| 1871 | 19:30:17 | bwang8 | 3*3 blocks | |
| 1876 | 19:30:26 | Quicksilver | Break it down | |
| 1878 | 19:30:27 | Aznx | I'd say look for a pattern. | |
| 1882 | 19:30:33 | Aznx | and yes, break it down. | Quicksilver completes yellow, red, blue inscription (Figure 6) |
| 1886 | 19:30:40 | Aznx | What other possible ways are there? | |
| 1889 | 19:30:44 | Aznx | That we know of? | |
| 1892 | 19:30:52 | bwang8 | top, middle and bottom | |

| | | | |
|---|---|---|---|
| 1905 | 19:31:29 | bwang8 | top and bottom are 3 by 3 squares |
| 1907 | 19:31:33 | Quicksilver | whoops i drew it wrong |
| 1910 | 19:31:36 | Quicksilver | but yes |

*Further Development of Shared Practices*

During the accomplishment of problem solving, the practices of *problem decomposition*, *inscribe first solve second*, and *visualize decomposition* are sustained in this session. These practices are enacted in multiple cycles as the participants attempt to build on their previous work. The references to the prior sessions' work in [1419], [1459] and [1473] indicate that deployment of prior accomplishments is a participants' concern. Our analytic approach of identifying uptake of prior practices is aligned with this concern. This episode is significant because the group has established a "way of doing things" consisting of a recurring set of practices, to be affirmed in the next session.

As the participants worked out the pyramid problem they drew on their *problem decomposition* strategy from Session 1 by deconstructing the pattern into components. Quicksilver enacted the strategy *visualize decomposition* using color rather than spatial separation to visualize the layers of the pyramid. Furthermore, the *inscribe first solve second* practice recurs in this session as several inscriptions are attempted, which brings us to a new practice.

Quicksilver introduced a new practice to indicate dimensionality. He introduced a side view, and then inscribed three successive top-view perspectives of a pyramid. This *modulation of perspective* appears to enable the participants to make progress toward a solution. The side view inscription is almost identical to the original figure provided in the instructional materials (see Figure 10-2). The difference, however, is that in the current context, the figure is a representation of a three-dimensional pyramid, not a two-dimensional triangular form. This distinction is indicated by Quicksilver at lines [1493] and [1502] in which he attempts to clarify what he sees as a misinterpretation on the part of the others. The construction of the top-view pyramid is subsequently initiated to address these different interpretations. The distinction between 2-D and 3-D nature of the inscription remains a point of concern in the ensuing discussion surrounding the top-view representation [1747-1760]. Quicksilver then begins to use color to articulate the three dimensional properties of a pyramid from a top-view perspective (Figure 10-6).

Much of the group's work in this session seeks to coordinate the decomposition problem-solving practice with the group practice of translating the inscribed reifications into algebraic formulas (Alterman, 2007). Aligning these practices is a joint accomplishment that allows the group to progress towards a solution. An inscription can support the decomposition practice only if participants recognize that inscription as meaningful in that way. In dialogue that exposes the utility of inscriptions for problem-solving practices, we are seeing *inscriptions becoming representations*. Quicksilver introduced color into the joint work to amplify both perspective and decomposition of the pyramid [1882] (Log 10-3 and Figure 10-6). Bwang proposes a decomposition strategy at line [1777] (Log 10-3) that is then reified as an inscription by Quicksilver in the whiteboard [1882]. The nested yellow, red, and blue squares in Figure 10-6 correlate to the top, middle, and bottom [1892] of the pyramid. Color is appropriated as a resource for problem-decomposition practice and as a representational tool to highlight the figure as a three-dimensional pyramid viewed from above.

### Session 3: The Practices are Applied to a New Problem

The third session represents a crucial point in the group's collaborative interaction, in which they carry forward elements of their representational practices established in their prior work, applying them to a new problem. In the segment of work described next, Aznx initiates the *inscribe first, solve second* practice—producing an inscription that is then refined by Bwang, who appropriates color and perspective to display structural decomposition. These practices provide a resource for the participants as they proceed to develop the solution for the new problem. This episode shows three of the prior practices being brought to bear, in some cases applied by different individuals or using different inscriptional devices.

*Summary of Session*

Following a suggestion by the moderator to take up another team's solution in a different way, the participants begin working on deriving the equation for growing a diamond pattern. Team C posted this pattern and its equation on a wiki (shared by the several teams that participated in the VMT Spring Fest 2006). Figure 10-7 shows the figure and formulas posted by Team C. The Team B participants view the wiki, and begin to work out their own explanation of the pattern.

At time 19:30:38, Aznx began to inscribe Team C's figure into the whiteboard (Figure 10-8a). On finishing the inscription, he begins reasoning about the pattern [3911] with Quicksilver. Of concern at this early point is how the diamond pattern grows.

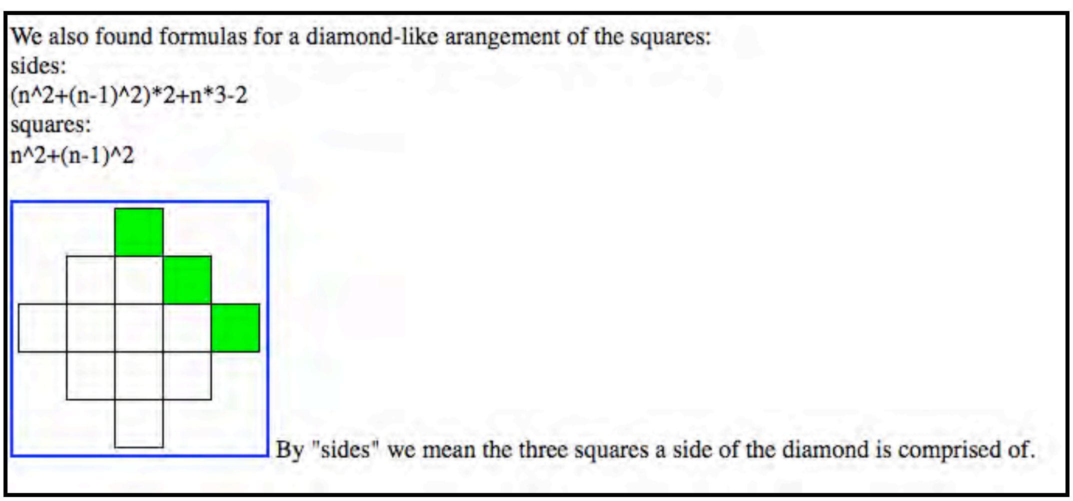
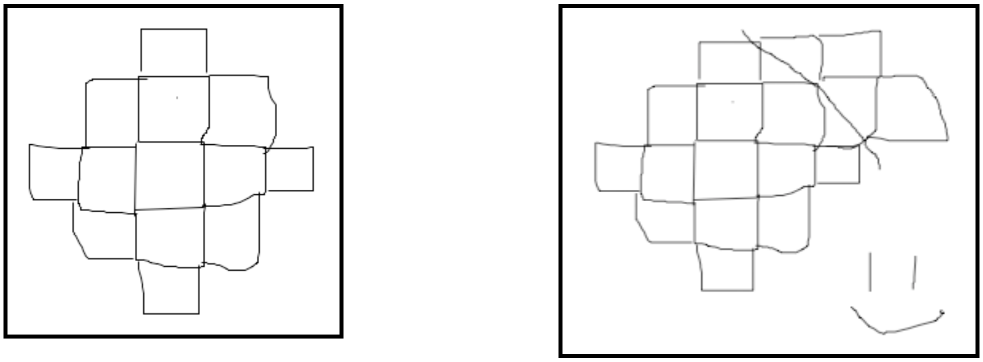



Figure 10-7. Team C's solution in the wiki.

Medina, R., Suthers, D. D., & Vatrapu, R. (2009). Representational practices in VMT. In G. Stahl (Ed.), Studying Virtual Math Teams (pp. 185-205). Cambridge, MA: MIT Press.



(a) as originally drawn

(b) extended to show growth

Figure 10-8. Growth of a diamond pattern

In the exchange presented in Log 10-4, Aznx is arguing that the pattern grows like a tessellation. Quicksilver requests explanation, and Aznx begins drawing additional squares on the top right corner of the diamond inscription (Figure 10-8b). Building on the joint practice of using color to distinguish elements of the representation Quicksilver [3950] suggests using color to bring out the "portion." This portion references the component of the diamond that grows. However, Aznx does not use color but indicates the portion with a line (Figure 10-8b). That this alternative visualization is taken as an appropriate way to meet the request evidences the group's orientation toward *visualize decomposition* as a practice independent of the particular means of visualization.

| | | | | |
|---|---|---|---|---|
| | | | | Aznx draws diamond pattern, figure 10-8a |
| 3898 | 19:30:44 | Aznx | lol, it looks horrible | |
| 3902 | 19:30:48 | Bwang | lol | |
| 3908 | 19:31:01 | Aznx | Ok | |
| 3911 | 19:31:23 | Aznx | How would you grow this pattern? | |
| 3914 | 19:31:32 | Aznx | Like a tesselation? | |
| 3917 | 19:31:40 | Quicksilver | No | |
| 3920 | 19:31:45 | Quicksilver | It doesn't tesselate | |
| 3927 | 19:31:55 | Aznx | Actually it does | |
| 3932 | 19:31:58 | Quicksilver | How? | |
| 3936 | 19:32:03 | Aznx | Hold on | |
| 3950 | 19:32:11 | Quicksilver | color the portion | Aznx draws diagonal line, figure 10-8b. |
| 3959 | 19:32:48 | Quicksilver | Besides, It grows in all directions | |
| 3962 | 19:32:56 | Aznx | But it fits | |
| 3965 | 19:33:05 | Aznx | You can do it on your own scratch piece of paper =P | |
| 3968 | 19:33:06 | Bwang | ok | |

Log 10-4.

As the interaction unfolds, Bwang initiates a transition to developing an equation for generating the growth of the diamond pattern [3971] (Log 10-5). Bwang copies Team C's equations into the chat window [3987 & 3991] and Aznx attempts to make sense of the formulas as Quicksilver attempts to translate this reasoning to the inscription [3996]. At this moment, Aznx provides an opener into an extended explanation of how the pattern can be derived by stating, "*I have an interesting way to look at this problem*" [4009].

Log 10-5.

| | | | | |
|---|---|---|---|---|
| 3971 | 19:33:16 | Bwang | lets think about the equatin | |
| 3974 | 19:33:22 | Bwang | equation | |
| 3977 | 19:33:23 | Quicksilver | yes | |
| 3980 | 19:33:30 | Bwang | how did they derive it | |
| 3984 | 19:33:50 | Aznx | There's the formula | |
| 3987 | 19:33:57 | Bwang | (n^2+(n-1)^2)*2+n*3-2 | |
| 3991 | 19:34:08 | Bwang | n^2+(n-1)^2 | |
| 3994 | 19:34:18 | Aznx | The 3n has to do with the growing outer layer of the pattern I think. | |
| 3996 | 19:34:23 | Quicksilver | the sides and squares | |
| 4000 | 19:34:55 | Aznx | Right. | |
| 4005 | 19:35:09 | Aznx | There. | |
| 4009 | 19:35:36 | Aznx | I have an interesting way to look at this problem. | |
| 4013 | 19:35:42 | Quicksilver | Tell us | |
| 4016 | 19:35:45 | Aznx | Can you see how it fits inside a quare? | |
| 4018 | 19:35:45 | Bwang | yes | |
| 4023 | 19:35:52 | Quicksilver | Yes | |
| 4026 | 19:35:53 | Bwang | oh | |
| 4030 | 19:35:55 | Bwang | yes | |
| 4033 | 19:36:01 | Quicksilver | You are sayingthe extra spaces... | |
| 4035 | 19:36:05 | Aznx | Also, do you see if you add up the missing areas | |
| 4039 | 19:36:11 | Quicksilver | Yes... | |
| 4043 | 19:36:18 | Quicksilver | they look similar to the original figures | |
| 4046 | 19:36:21 | Quicksilver | figure | |
| 4048 | 19:36:21 | Aznx | It is equivalent in size to the small circle in the pattern | |
| 4055 | 19:36:33 | Quicksilver | Small circle? | |
| 4057 | 19:36:39 | Aznx | The only part you would be missing out are the four squares | |
| 4060 | 19:36:49 | Aznx | on the outer areas of this square | |
| 4064 | 19:37:00 | Aznx | Doi you guys get what I mean? | |
| 4067 | 19:37:07 | Bwang | yes | |
| 4069 | 19:37:08 | Quicksilver | Show what u mean on the witeboard | |
| 4072 | 19:37:11 | Quicksilver | i dont get it | |
| 4075 | 19:37:14 | Aznx | Bwang you show him | |
| 4078 | 19:37:17 | Aznx | since you get it | |
| 4096 | 19:38:18 | Bwang | we just have to find the whole square and minus the four corners | Bwang has completed the inscription in Figure 10-9 (bottom right). |

At [4016] Aznx elaborates on the potential solution, noting that the diamond pattern is structurally decomposed from a square. In the ensuing exchange—[4018] through [4060]—Bwang and Quicksilver also engage with the explanation. Bwang indicates that he understands [4067], however Quicksilver is not as convinced. At [4075] Aznx directs Bwang to explain the idea to Quicksilver, presumably using an inscription. Bwang composes a new inscription (see Figure 10-9, bottom right), using color to show the corners of the square that are excluded from the diamond. It is a reification of the description Aznx contributed in the previous exchange, but it also draws on previously shared representational practices of using color to show how the problem can be structurally decomposed. On completing the inscription, Bwang states the solution in simple terms [4096].

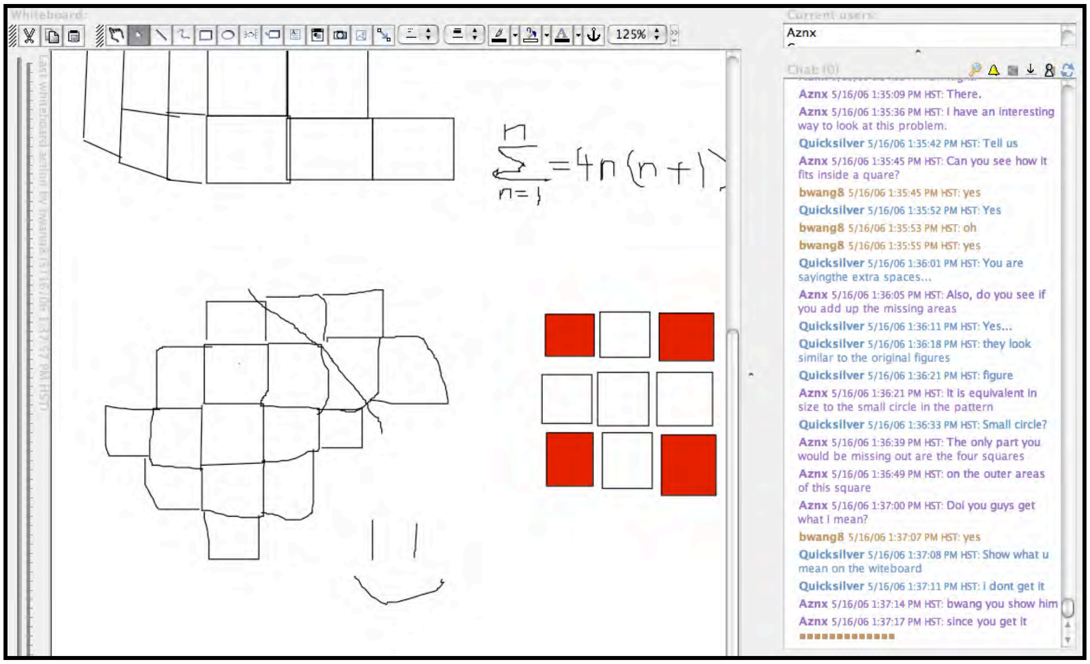



Figure 10-9. Whiteboard at line 4096 in Log 10-5.

*Summary of Practices*

The session discussed above reveals a productive group interaction. Ideas are exchanged and practices are enacted that build upon the prior interaction history of the participants. Across all the episodes we discussed, the participants applied their problem-solving and representational practices as resources in addressing different problems. For example, the practice of inscribing and then discussing a problem solution is a recurring pattern of interaction throughout the throughout the group's work. Further, for each of the above sessions, a different participant initiates the interaction by first producing an inscription that the other two subsequently orient to through the chat discourse (Bwang in Session 1, Quicksilver in Session 2 and Aznx in Session 3): the practice is shared and has been taken up by all participants. In Sessions 2 and 3 we see that the practice of *inscribe first solve second* is iteratively enacted and composed with two additional practices—*modulate perspective* and *visualize decomposition*. In Session 2, Quicksilver's use of color and perspective emerges in the joint work in support of both representational and problem-solving practices. In Session 3, Bwang appropriates color to draw out the particular decomposition previously articulated by Aznx. This is an example of the subtle ways in which the participants draw on prior work and artifacts to facilitate their current meaning-making practices.

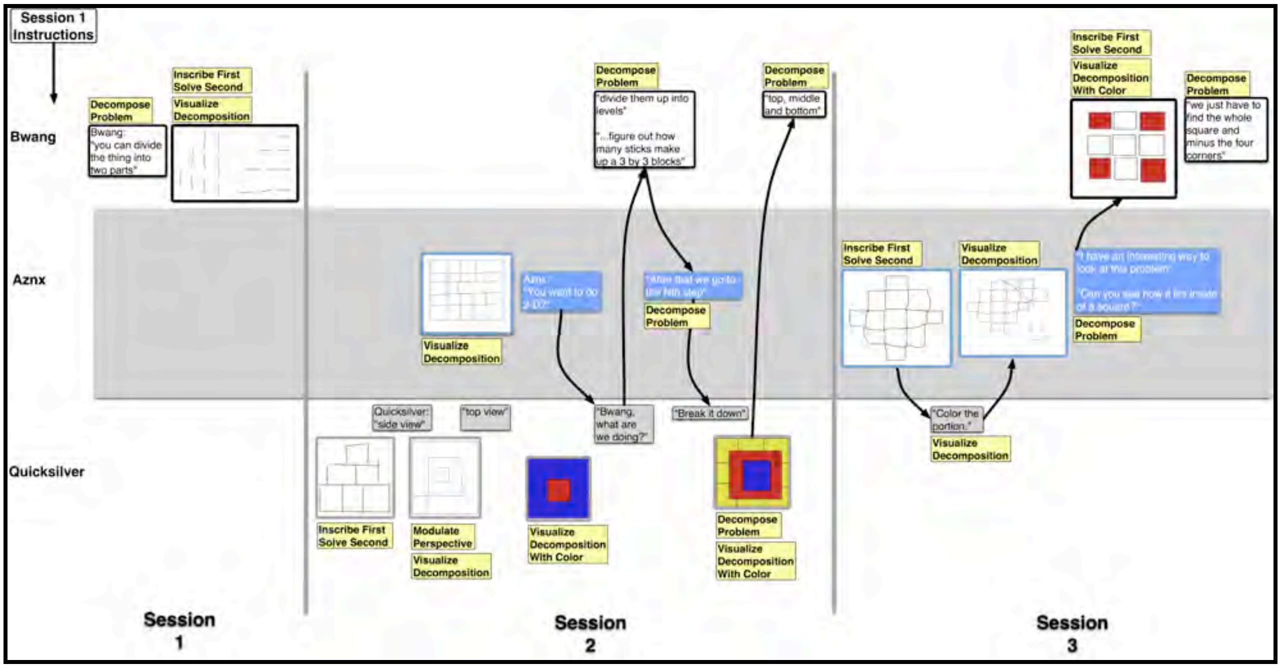



Figure 10-10. Representational practices across people and artifacts.

Figure 10-10 illustrates how these interactions can be related across the sessions, participants and artifacts discussed in this analysis. The figure is composed of three layers. The top shows inscriptions and chat contributions from Bwang, the middle represents the inscriptional work and discussion contributed by Aznx, and the bottom layer shows the work of Quicksilver. Our analysis of each session is organized from left to right in the figure and suggests that practices can be formed, transformed and refined in progressive cycles of group interaction. Three practices are taken up by the participants consistently across the three sessions—inscribe first solve second, visualize decomposition, and decompose problem. A second form of visualize decomposition using color is introduced in Session 2 and reapplied in Session 3. Modulate perspective is also introduced in Session 2 and is intertwined with discussion in the chat. It is noteworthy that each of the three participants initiated a different problem-solving episode. This has provided key evidence for identifying uptake relations (Suthers, 2006) between participants. For example, Bwang's use of color to show a diamond decomposed from a square (right side of Figure 10-10), draws on (1) a problem decomposition strategy that he originally introduced but that was given new manifestations by his partners, (2) Quicksilver's practice of using color to visualize decomposition and (3) the prior practice of using drawings to reason about and structure algebraic formulas.

# Conclusions

Stahl (2007b) provides the following definition of group cognition:

> Here, the term "group cognition" does not refer to some kind of mental content, but to the ability of groups to engage in linguistic processes that can produce results that would be termed "cognitive" if achieved by an individual, but that in principle cannot be reduced to mental representations of an individual or of a sum of individuals.

This definition might be improved to rely less on judgments of what processes are "termed 'cognitive'"—a matter we won't pursue further here. The definition can be generalized to allow for other interactive processes in addition to linguistic ones, an improvement that we must assume here to include our account of representational practices. Also, rather than the "ability" of groups to engaged in such processes, it seems more consistent with Stahl's other writings to take group cognition as the processes themselves. Under this reading, the group must interact each time it "cogitates" about a given problem. One contribution of this paper is to show that they don't do so in a vacuum, and so are not doomed to work out their methods anew each time. They can draw on their prior interactions and on the products of their interactions as resources for progressive group cognition.

Our analysis showed how uptake of prior resources enabled the development and reapplication of practices in the work of one group. It showed how the contingent nature of group accomplishments is temporally extended and is mediated by persistent inscriptions. "Immutable mobiles" (Latour, 1990) are powerful because they bring one moment's resources for interaction into another moment. The ability to re-establish mental representations can also serve this role, but they are not accessible to either other participants or us as analysts. In contrast, inscriptions that offer representational resources associated with prior practices are available to both participants and analysts in the sessions analyzed here.

Our analysis also showed that much of this group's work in mathematics involved the construction of appropriate inscriptions that support the strategy of problem decomposition and translation from inscriptions to formulas by visualizing the decomposition in an appropriate manner. In coordinating these practices, the group works towards a shared understanding of the inscriptions as representations suitable for their task. Thus, the group's practices are *representational practices* in an essential way: the inscriptions are not intrinsically representations, but become representations through the negotiated practices of participants.

This work sheds light on our questions concerning how the breakdown analyzed by Stahl (2007b) could have happened in a group that seemed to be functioning so well, and the manner in which it was resolved. Stahl alludes to facilitator's doubts that participants all understood what each other were doing. Although it was not our focus in this paper, we also see lack of convergence in the data reported here. It is conceivable that a group, "cogitating" in interaction, could produce a solution without any one person internalizing the entire solution. Whether group cognition consists of transformations of distributed representations (Hutchins, 1996) or is enacted in interaction between people (Stahl, 2006), it is not a capability of any one person. Therefore it is not surprising that at the end not everyone is prepared to explicate the solution. Faced with the task of accounting for their work they have to re-enact some of it. Their inscriptions are still available, and their repair indexically invokes these inscriptions while also reconstructing them as representational resources.

The reapplication of prior accomplishments was a participants' concern as well as our concern as analysts, and participants' inscriptions likewise served as a resource for our own work. Organizing the analysis as a sequence of uptake relations (Suthers, 2006) at the level of practices enriches our understanding of how locally contingent interaction unfolds over time. Interaction traces produced from uptake analysis provide a persistent resource for analytical practices—our own immutable mobiles.

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