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## Naturalness Lowers the Upper Bound on the Lightest Higgs boson Mass in Supersymmetry

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### Abstract

We quantify the extent to which naturalness is lost as experimental lower bounds on the Higgs boson mass increase, and we compute the natural upper bound on the lightest supersymmetric Higgs boson mass. We find that it would be unnatural for the mass of the lightest supersymmetric Higgs boson to saturate its maximal upper bound. In the absence of significant fine-tuning, the lightest Higgs boson mass should lie below 120 GeV, and in the most natural cases it should be lighter than 108 GeV. For modest  $\tan\beta$ , these bounds are significantly lower. Our results imply that a failure to observe a light Higgs boson in pre-LHC experiments could provide a serious challenge to the principal motivation for weak-scale supersymmetry.

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# 1 Introduction

The Higgs boson is the last remaining ingredient of a complete standard model. It's persistent elusiveness is perhaps not surprising. Within the framework of the standard model, there are no symmetries which can be invoked to make a fundamental scalar light. The existence of a light scalar degree of freedom which remains fundamental above the weak-scale would argue for supersymmetry since supersymmetry provides the only explicitly known solution to the naturalness problem which accompanies fundamental scalars [1]. Of course, the Higgs boson may not be fundamental at all, and the only testament to its existence may be the eventual unitarization of the longitudinal  $W$  scattering cross section at TeV scale energies. However, although no vestige of the Higgs boson may be seen until the LHC, a failure to observe a Higgs boson in pre-LHC experiments could significantly challenge the principle motivation for weak-scale supersymmetry, at least in its minimal forms.

If nature is supersymmetric above the weak-scale, the allowable range of Higgs boson masses is considerably restricted. In the minimal supersymmetric extension of the standard model (MSSM), the lightest Higgs boson lies below  $m_Z$  at tree level,

$$m_h \leq |\cos 2\beta| m_Z, \quad (1.1)$$

where  $\tan \beta = v_u/v_d$  is the ratio of Higgs boson vacuum expectation values. Quantum corrections can lift the light Higgs boson mass above  $m_Z$  [2], but the magnitude of these corrections are restricted if supersymmetry provides a successful solution to the naturalness problem. Radiative corrections to the light Higgs boson mass in supersymmetry have been calculated by many authors [2, 3, 4]. From these corrections, upper bounds for the lightest Higgs boson mass have been computed either by choosing arbitrary heavy masses for superpartners or by demanding the theory remains perturbative up to some high scale [2, 3, 4]. While these upper bounds reasonably approximate an important, unexceedable upper-limit on the Higgs boson mass, they do not provide a complete picture of our expectations for the mass of the lightest Higgs boson in supersymmetric models. Realistically, we expect the Higgs boson mass to be significantly lighter. To achieve Higgs boson masses as

heavy as these upper-bounds requires some or all superpartner masses to be much heavier than the weak-scale. The appearance of this heavy mass scale in turn requires demonstrably large, unexplained cancellations among heavy masses in order to maintain a light weak-scale. However, avoiding this fine-tuning is the principle reason that supersymmetry was introduced at the weak-scale.

In this article, we observe that it would be quite unnatural for the lightest Higgs boson mass to saturate the maximal upper bounds which have been previously computed. We compute the natural upper bound on the Higgs boson masses in minimal, low-energy supergravity (MLES), and we show the extent to which naturalness is lost as the experimental lower bound on the lightest Higgs boson mass increases. Section two provides a brief review of naturalness and how it is reliably quantified. An analysis of the natural upper bound on the Higgs boson mass follows in section three. We find that for  $m_t < 175$  GeV, if  $m_h > 120$  GeV, minimal low energy supergravity does not accommodate the weak-scale naturally. Moreover, in the *most* natural cases,  $m_h < 108$  GeV when  $m_t < 175$  GeV. For modest  $\tan\beta$ , the natural upper-bound is even more restrictive. In particular, for  $\tan\beta < 2$  and  $m_t < 175$  GeV, if  $m_h > 100$  GeV large fine-tuning is required, while the *most* natural values of the Higgs boson mass lie below  $m_Z$ .

This has important implications for challenging weak-scale supersymmetry at collider experiments. In particular, if the lightest supersymmetric Higgs boson is not observed at CERN's  $e^+e^-$  collider LEP-II, requiring natural electroweak symmetry breaking in MLES will progressively increase the lower bound on  $\tan\beta$  as LEP-II increases in energy. In the *most* natural cases, if the energy of LEP-II is extended to  $\sqrt{s} = 205$  GeV, a light Higgs boson would be observed provided it decays appreciably to  $b\bar{b}$ , but it would not be possible to argue that natural electroweak symmetry breaking is untenable in the minimal supersymmetric standard model if the Higgs boson lies above the kinematic reach of LEP-II. By contrast, the proposed Run-III of Fermilab's Tevatron with  $\mathcal{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1}$  (TeV33) can pose a very serious challenge to the minimal supersymmetric standard model. The projected mass-reach for a standard model Higgs boson at TeV33 is 100 (120) GeV with integrated luminosities of 10 (25)  $\text{fb}^{-1}$  [5]. If the possibility that the light Higgs boson decays primarily to neutralinos can be excluded on the basis of combined searches for superpartners at LEP-II and the Tevatron, natural electroweak symmetry breaking in the minimal supersymmetric standard model will no

longer be possible if TeV33 fails to observe a light Higgs boson.

## 2 Naturalness

The original and principle motivation for weak-scale supersymmetry is naturalness. Supersymmetry provides the only explicitly known mechanism which allows fundamental scalars to be light without an unnatural fine-tuning of parameters. Naturalness also implies that superpartner masses can not lie much above the weak-scale if we are to avoid the fine-tuning which would be needed to keep the weak-scale light. In this section, we recall the principle of naturalness and briefly review how it can be reliably quantified. A more complete discussion of naturalness criteria can be found in Ref. [6]. Although fine-tuning is an aesthetic criterion, once we adopt the prejudice that large unexplained-cancellations are unnatural, a quantitative fine-tuning measure can be constructed and placed on solid footing. For any effective field theory, it is straightforward to identify whether large cancellations occur, and when these fine-tunings are present their severity can be reliably quantified.

In non-supersymmetric theories, light fundamental scalars are unnatural because scalar particles receive quadratically divergent contributions to their masses. Generically, at one-loop, a scalar mass is of the form

$$m_S^2(g) = g^2\Lambda_1^2 - \Lambda_2^2, \quad (2.1)$$

where  $\Lambda_1$  is the ultraviolet cutoff of the effective theory, and  $\Lambda_2$  is a bare term. The divergence in Eq. (2.1) must be almost completely cancelled against the counter term or the fundamental scalar will have a renormalized mass on the order of the cutoff. In supersymmetry, additional loops involving super-partners conspire to cancel these quadratic divergences, but when supersymmetry is broken, the cancellation is no longer complete, and the dimensionful terms in Eq. (2.1) are replaced by the mass splitting between standard particles and their super-partners.

In this toy example, the cancellation is self-evident, and no abstract quantitative prescription is needed to determine when the parameters of the theory must conspire to give a light scalar mass. We are interested in a more complicated example, and this requires a quantitative prescription for identifying instances of fine-tuning. In the toy example, if we examine the sensitivity of

the scalar mass to variations in the coupling  $g$ :

$$\frac{\delta m_S^2}{m_S^2} = c(m_S^2, g) \frac{\delta g}{g}, \quad (2.2)$$

where

$$c(m_S^2; g) = 2 \frac{g^2 \Lambda_1^2}{m_S^2(g)}, \quad (2.3)$$

the scalar mass will be unusually sensitive to minute changes in  $g$  when we arrange for large unexplained-cancellations [7]:

$$c(m_S^2 \ll \Lambda^2) \gg c(m_S^2 \sim \Lambda^2). \quad (2.4)$$

However, the bare sensitivity parameter  $c$ , by itself is not a measure of naturalness. Although physical quantities depend sensitively on minute variations of the fundamental parameters when there is fine-tuning, fine-tuning is not necessarily implied by  $c \gg 1$ . Large sensitivities can occur in a theory even when there are no large cancellations<sup>1</sup>. In particular, this is true for supersymmetric extensions of the standard model, where it is known that bare sensitivity provides a poor measure of fine-tuning [6]. A reliable measure of fine-tuning must compare the sensitivity of a particular choice of parameters  $c$  to a measure of the average, global sensitivity in parameter space,  $\bar{c}$ . The naturalness measure

$$\gamma = c/\bar{c} \quad (2.5)$$

will greatly exceed unity if and only if fine-tuning is encountered [6]<sup>2</sup>. This definition is a quantitative implementation of a refined version of Wilson's naturalness criterion: Observable properties of a system should not be unusually unstable against minute variations of the fundamental parameters.

In supersymmetric extensions of the standard model, as the masses of superpartners become heavy, increasingly large fine-tuning is required to keep

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<sup>1</sup>For example the mass of the proton depends very sensitively on minute variations in the value of the strong coupling constant at high energy, but the lightness of the proton is a consequence of asymptotic freedom and the logarithmic running of the QCD gauge coupling and not the result of unexplained cancellations.

<sup>2</sup>Alternatively, we could define a measure of fine-tuning as the ratio of the amount of parameter space in the theory supporting typical values of  $m_S$  to the amount of parameter space giving a unusually light value of  $m_S$ . This criterion is in fact equivalent to the ratio of sensitivity over typical sensitivity [6].

the weak-scale light. Naturalness places an upper bound on supersymmetry-breaking parameters and superpartner masses. Because the radiative corrections to the Higgs boson mass increase with heavier superpartner masses, naturalness translates into an upper limit on the mass of the lightest Higgs boson. This limit is computed in the following section.

### 3 Analysis

Following the methods of Ref. 6, we have computed the severity of fine-tuning in the minimal supersymmetric standard model. For definiteness, we consider soft supersymmetry breaking parameters with (universal) minimal, low-energy supergravity (MLES) boundary conditions. We quantify the severity of large cancellations, and present our results as upper limits on the Higgs boson mass as a function of the degree of fine-tuning. Although our quantitative results were obtained in a framework with universal soft terms at a scale near  $10^{16}$  GeV, as motivated by MLES, we do not expect our bounds on the Higgs boson mass to significantly increase in models with more general soft supersymmetry breaking masses provided they have minimal particle content at the weak-scale. Because there are enough free parameters in MLES to independently adjust the parameters in the minimal supersymmetric standard model (MSSM) which most significantly increase the Higgs boson mass, more general soft terms could allow one to increase the masses of the squarks from the first two generations above their naturalness limits in MLES, for example, but these new degrees of freedom will not significantly increase the upper limit on the Higgs boson mass. Qualitatively, our results are even more general, if we enlarge the particle content beyond the MSSM, the upper-limit on the lightest Higgs boson mass can be increased [8], but natural values of the lightest Higgs boson mass will lie significantly below any maximal upper-bounds.

Our calculation evolves the dimensionless couplings of the theory at two-loops and includes one-loop threshold contributions and one-loop correction to the Higgs potential. From the resulting weak-scale parameters, we calculate the pole masses for the Higgs bosons at one-loop following standard diagrammatic techniques [3]. The remaining next-to-leading order corrections to the Higgs boson mass arising from the two-loop evolution of dimensionful couplings are small in the natural region of parameter space [3, 4].

Figures 1-3 show the naturalness of the Higgs boson mass as a function of  $\tan\beta$ ,  $m_A$ , and  $m_t$ , respectively. In all three figures ideally natural solutions correspond to  $\gamma = 1$  and fine-tuning is implied by  $\gamma \gg 1$ . Figure 1 shows contours where the severity of fine-tuning -  $\gamma$  exceeds 2.5, 5, 10 and 20 in the  $\tan\beta$ - $m_h$  plane for  $m_t = 175$  GeV. From Fig. 1 we see that the mass of the lightest Higgs boson can not exceed 120 GeV without very significant fine-tuning, while in the most natural cases it lies below 108 GeV. When  $\tan\beta$  is small these limits are even more restrictive. Figure 2 shows naturalness contours for the lightest Higgs boson mass in MLES as a function of the CP-odd Higgs mass,  $m_A$  for  $m_t = 175$  GeV and arbitrary  $\tan\beta$ . If we restrict ourselves to modest or small values of  $\tan\beta$  these curves will become more restrictive in the  $m_h$  direction. Figure 3 shows naturalness contours for the lightest Higgs boson mass in MLES as a function of the top quark mass. The inset in Fig. 3 displays the current uncertainty in the top quark mass, and the projected uncertainties after run-II of Fermilab's Tevatron and after TeV33 [5, 9]. Fine-tuning increases both with increasing superpartner masses and with an increasing top quark Yukawa coupling. Therefore, in contrast to the case of fixed superpartner masses where the corrections to the mass squared of the Higgs boson increases as  $m_t^4$ , for fixed naturalness these corrections increases roughly as  $m_t^2$ .

We can assess the challenge to weak-scale supersymmetry from Higgs boson searches at colliders from the natural regions of parameter space identified in Figs.1-3. The dominant production mechanism for light CP-even Higgs boson at LEP-II is Higgs-strahlung

$$e^+e^- \rightarrow Z^* \rightarrow Z + h \quad (3.1)$$

If Higgs boson decays into light neutralino pairs,  $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ , are kinematically forbidden,  $h$  will decay primarily to  $b\bar{b}$ . An upper bound on the light Higgs mass reach in this mode is set by kinematics and scales as  $m_h < \sqrt{s} - m_Z$  - (a few) GeV. The combined 95% CL exclusion reaches for a standard model (SM) Higgs boson at LEP-II are 83 (98) ((112)) GeV at  $\sqrt{s} = 175$  (192) ((205)) GeV, with integrated Luminosities of 75 (150) ((200))  $\text{pb}^{-1}$ , per experiment [10]. However, it is well known that the observability of the lightest supersymmetric scalar  $h$  can be degraded with respect to the standard model in two respects. First, the  $ZZh$  vertex carries a suppression of  $\sin(\alpha - \beta)$  relative to the standard model vertex, where  $\alpha$  is the mixing angle of the CP-even Higgs scalars. The departure of this factor from unity



can be appreciable for relatively light values of the  $CP$ -odd mass  $m_A$ , but it approaches one as the mass of the  $CP$ -odd Higgs increases. For  $m_A \gtrsim 200$  GeV,  $\cos^2(\beta-\alpha) < .01$ . If the  $CP$ -odd Higgs mass is light it may be produced and seen through associated production  $e^+e^- \rightarrow Ah$ , but this mode provides a less significant challenge to weak-scale supersymmetry because the  $CP$ -odd scalar mass  $m_A$  is much less constrained by naturalness arguments (see Fig. 2). Second, the mass reach for the lightest Higgs  $h$  can also be reduced if  $h$  decays invisibly into a pair of lightest superpartners,  $\tilde{\chi}_1^0\tilde{\chi}_1^0$ . This branching ratio can approach 100% when allowed [11], and this mode becomes more probable as the mass of the lightest Higgs boson increases. In the relatively clean environment of an  $e^+e^-$  collider, a Higgs with such invisible decays could be seen from the acoplanar jet or lepton pair topologies resulting from the decay of the associated  $Z$ , but the Higgs mass reach in this case is reduced to roughly half of the reach when  $h$  decays visibly [10]. When  $\sin^2(\alpha - \beta)BR(h \rightarrow b\bar{b})$  is maximal, in the *most* natural cases, LEP-II operating up to  $\sqrt{s} = 205$  GeV would observe a light Higgs, but this energy is not large enough to argue that natural electroweak symmetry breaking is untenable in minimal supersymmetry if the Higgs boson lies above the kinematic reach of LEP-II.

Kinematically, the proposed Run-III of Fermilab's Tevatron with  $\mathcal{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1}$  (TeV33) [5] can pose a very serious challenge to weak-scale supersymmetry. The best single mode for discovery of a light Higgs boson at the Tevatron is  $q'\bar{q} \rightarrow Wh$ , with  $h \rightarrow b\bar{b}$  [12]. TeV33 can probe a SM Higgs up to 100 (120) GeV with integrated luminosities of 10 (25)  $\text{fb}^{-1}$ . A Higgs boson mass in excess of 120 GeV would be extremely unnatural in the MSSM. However, the  $Wb\bar{b}$  cross section from  $Wh$  production is also reduced by the factor  $BR(h \rightarrow b\bar{b})\sin^2(\alpha - \beta)$ . So the significance of the challenge to weak-scale supersymmetry from light Higgs searches at TeV33 will depend strongly on the ability of searches for neutralinos and charginos at the Tevatron and LEP-II to eliminate the possibility of  $h \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ , by raising the limits on the LSP mass. If this is the case, natural electroweak symmetry breaking in the minimal supersymmetric standard model will no longer be tenable if TeV33 achieves  $\int \mathcal{L}dt = 25\text{fb}^{-1}$  and fails to observe any signal of a Higgs boson.

## 4 Conclusions

Natural choices of parameters in supersymmetric models lead to Higgs boson masses which lie significantly below the maximal upper-bounds determined previously in the literature. We have computed the natural upper bound on the Higgs mass in MLES, and we have quantified the extent to which naturalness is lost as the lower bound on  $m_h$  increases. A Higgs mass above 120 GeV will require very large fine-tuning, while the most natural values of the Higgs mass lie below 108 GeV. The natural values of the lightest Higgs boson mass have important implications for the challenge to weak-scale supersymmetry at colliders. In particular, if the possibility that the Higgs decays predominantly to neutralino pairs can be excluded from neutralino mass limits inferred from other superpartner searches, natural electroweak symmetry breaking will no longer be tenable in the MSSM if TeV33 achieves the projected reach of  $m_h = 120$  GeV and fails to observe signals of a Higgs boson.

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*Figure 1:* Naturalness contours for  $\gamma < 2.5, 5, 10$  and  $20$  in MLES displayed in the  $\tan\beta - m_h$  plane, for  $m_t = 175$  GeV. Ideally natural solutions correspond to  $\gamma = 1$ , while fine-tuning is exhibited by  $\gamma \gg 1$ .

*Figure 2:* Naturalness contours for  $\gamma < 2.5, 5, 10$  and  $20$  in MLES displayed in the  $m_t - m_A$  plane. More restrictive contours will result if  $\tan\beta$  is constrained to be small.

*Figure 3:* Naturalness contours for  $\gamma < 2.5, 5, 10$  and  $20$  in MLES displayed in the  $m_t - m_h$  plane. More restrictive contours will result if  $\tan\beta$  is constrained to be small. The horizontal error bars indicate the current uncertainty in the mass of the top quark.

Figure 1

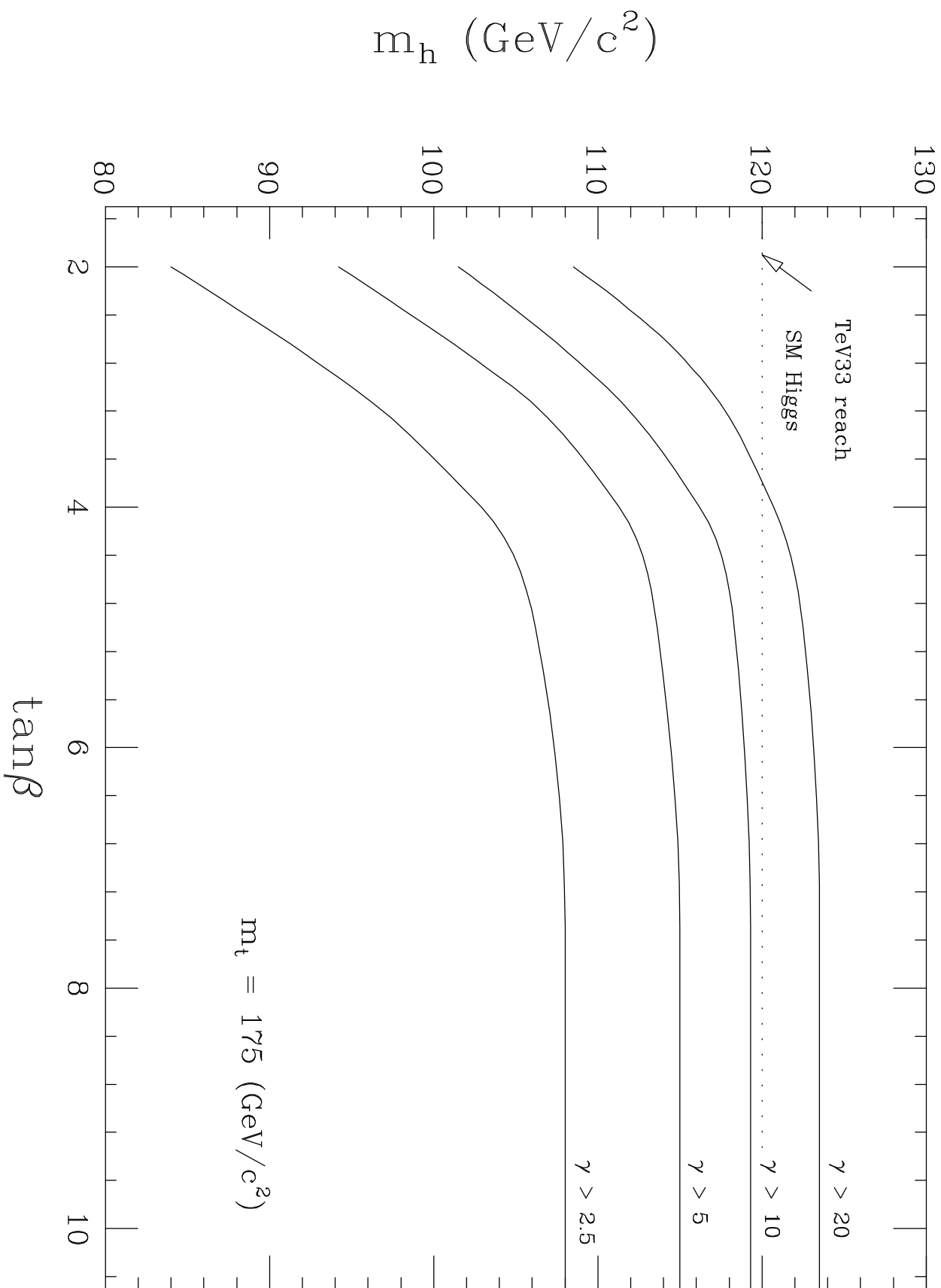


Figure 2

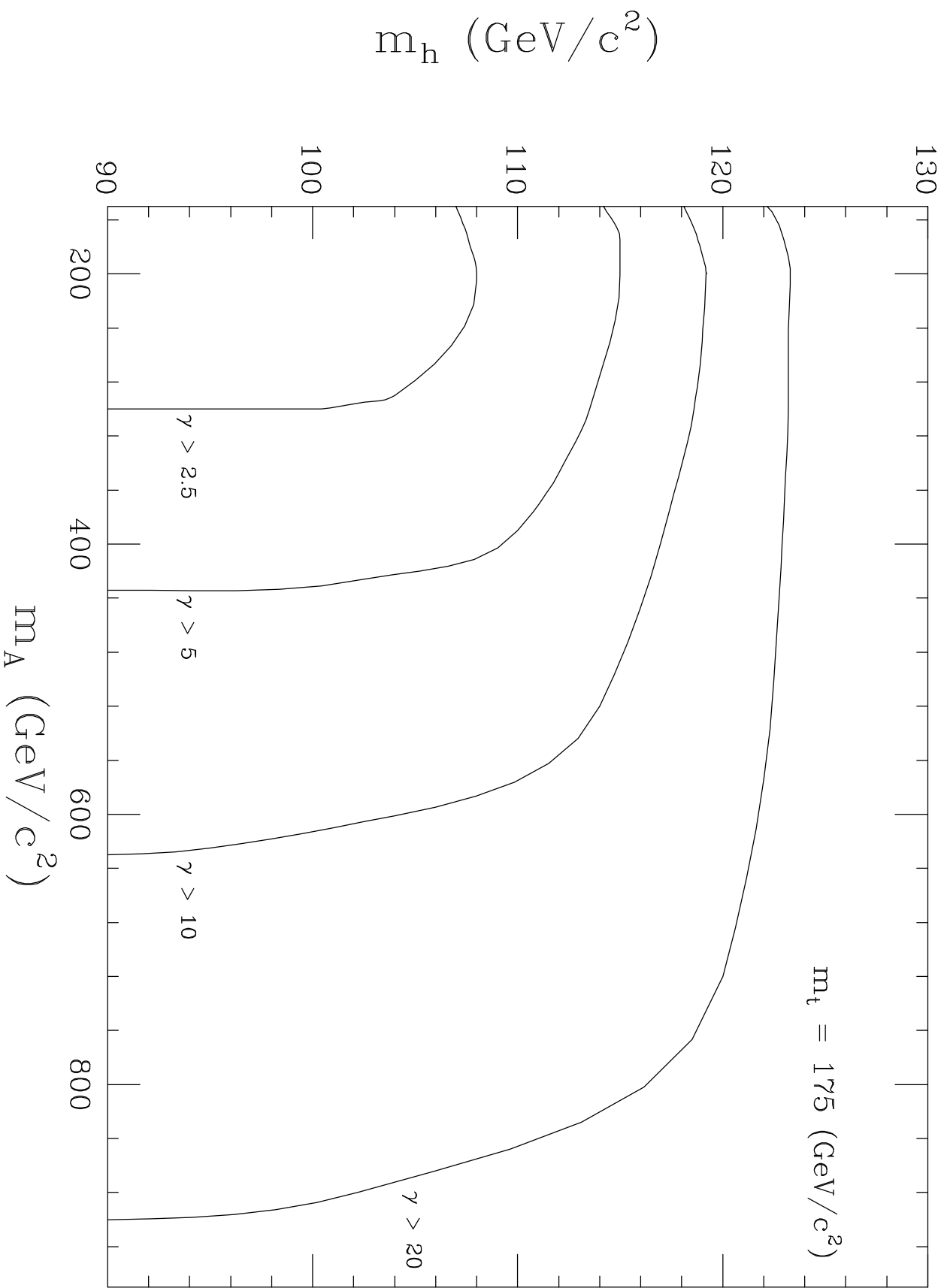


Figure 3

