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Consequences of supergravity with gauged $U(1)_R$ symmetry

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Abstract

The structure of gauged R-supergravity Lagrangians is reviewed, and we consider models with a hidden sector plus light fields of the MSSM. A simple potential for the hidden sector is presented which has a global minimum with zero cosmological constant and spontaneously broken SUSY and R-symmetry. The $U(1)_R$ vector multiplet acquires a Planck scale mass through the Higgs mechanism, and it decouples at low energy. Due to very interesting cancellations, the $U(1)_R$ D-terms also drop out at low energy. Thus no direct effects of the gauging of R-symmetry remain in the low-energy effective Lagrangian, and this result is model independent, requiring only that R-symmetry be broken at the Planck scale and $\langle D \rangle = 0$, where D is the auxiliary field of the $U(1)_R$ vector multiplet. The low-energy theory is fairly conventional with soft SUSY breaking terms for the MSSM fields. As a remnant of the gauging of R-symmetry, it also contains light fields, some required to cancel R-anomalies and others from the hidden sector.

1. Introduction

It is well known that $N = 1$ supersymmetry (SUSY) and supergravity (SG) theories admit a special R-symmetry which distinguishes between bosonic and fermionic superpartners. R-symmetry can appear either as a discrete $\mathbb{Z}_2$ or a continuous $U(1)_R$ group. In the latter form it engenders the chiral rotation $Q_\alpha \rightarrow \left(e^{i\theta\gamma_5} Q \right)_\alpha$ of the Majorana supercharge Q_α. A discrete version of global $U(1)_R$ symmetry is usually incorporated

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in phenomenological models because it forbids terms which would otherwise lead to rapid proton decay. Gauged $U(1)_R$ is only permitted in supergravity, and we discuss this below with the simple motivation that it is generally the gauge form of a symmetry which is most powerful and therefore worth study.

The minimal structure required for gauged R-symmetry is the supergravity multiplet $(e_\mu^a(x), \psi_\mu(x))$ and a vector multiplet $(R_\mu(x), \rho(x))$ containing the R-photon and its superpartner. Gauging $U(1)_R$ produces [1] covariant derivatives

$$D_\mu\psi_\nu = \mathcal{D}_\mu^{\text{grav}}\psi_\nu + igR_\mu\gamma_5\psi_\nu ,$$
$$D_\mu\rho = \mathcal{D}_\mu^{\text{grav}}\rho + igR_\mu\gamma_5\rho , \qquad (1.1)$$

and a shift of the D auxiliary field corresponding to a Fayet–Iliopoulos [2] (FI) parameter $\xi = 2g/\kappa^2$, where $\kappa^2 = 4\pi G_N = 1/M_{\text{Pl}}^2$ is the gravitational coupling. Conversely, coupling a global SUSY theory with a FI term to SG requires the axial gauge interaction in (1.1) with $g = \xi\kappa^2/2$.

In the early 1980's, gauged R-supergravity theories including chiral multiplets were discussed from the viewpoints of superspace [3], Kähler geometry [4], and auxiliary fields [5], and simple models were studied [6]. Surprisingly enough it was only very recently that a paper appeared [7] which addresses the issue of cancellation of the anomalies of the axial R-symmetry and discusses realistic models. The work we present below is similar in spirit to [7], but there are very significant differences.

The general R-invariant model contains $(e_{a\mu}, \psi_\mu)$ and (R_μ, ρ) as previously mentioned, additional vector multiplets (A_μ^a, λ^a) for the other gauged internal symmetries (e.g., those of the standard model or an extension of it), and chiral multiplets (z^α, χ^α). The $U(1)_R$ charges are specified in the covariant derivatives

$$D_\mu\lambda^a = \mathcal{D}_\mu^{\text{grav}}\lambda^a + igR_\mu\gamma_5\lambda^a + \dots ,$$
$$D_\mu\chi^\alpha = \mathcal{D}_\mu^{\text{grav}}\chi^\alpha + ig(r_\alpha - 1)R_\mu\gamma_5\chi^\alpha + \dots , \qquad (1.2)$$
$$D_\mu z^\alpha = \partial_\mu z^\alpha + igr_\alpha R_\mu z^\alpha + \dots ,$$

where $+\dots$ indicates the gauge coupling of the A_μ^a fields. One sees that r_α is the intrinsic R-charge of the chiral multiplet (z^α, χ^α), and that for $r_\alpha = 0$, a chiral multiplet fermion has opposite R-charge to any gaugino or to the gravitino.

From (1.1) and (1.2), one sees that in general all fermions in the theory contribute to anomalous triangle graphs. Although a Green–Schwarz mechanism for cancellation of the R-anomaly has been discussed [8,7], we shall adopt the view that the anomaly should be cancelled by constraining the R-charges of the particles that enter the theory. In Section 4 we discuss these anomalies and the restrictions on the particle content of the theory that are entailed by their cancellation. In particular, anomaly cancellation with gauge group $SU(3)_c \times SU(2)_w \times U(1)_Y$ requires that the minimal extension of the standard model (MSSM) be extended to include new chiral multiplets carrying both non-trivial standard model quantum numbers and R-charges. We choose one particular extension, but there are other possibilities.

A second important ingredient of the models is the superpotential $W(z^\alpha)$ which must have R-charge 2, i.e.

$$\sum_\alpha r_\alpha z^\alpha W_{,\alpha} = 2W \ , \tag{1.3}$$

and we shall assume an additive split between hidden and observable fields $W = W_h + W_o$. W_h and W_o must separately satisfy (1.3). The Kähler potential $K(z^\alpha, \bar{z}^\alpha)$ is assumed to be R-invariant, viz,

$$\sum_\alpha r_\alpha \left(z^\alpha K_{,\alpha} - \bar{z}^\alpha K_{,\bar{\alpha}} \right) = 0 \ , \tag{1.4}$$

and there is a $U(1)_R$ D-term

$$D = \sum_\alpha r_\alpha z^\alpha K_{,\alpha} + \frac{2}{\kappa^2} \tag{1.5}$$

for which the constant shift is just the FI term. The complete scalar potential is then

$$V = e^{\kappa^2 K} \left[D_\alpha W G^{\alpha\bar{\beta}} D_{\bar{\beta}} \overline{W} - 3\kappa^2 W \overline{W} \right] + \frac{1}{2} g^2 D^2 + \dots \ , \tag{1.6}$$

where $G_{\alpha\bar{\beta}} = K_{,\alpha\bar{\beta}}$ is the Kähler metric, $G^{\alpha\bar{\beta}}$ is its inverse,

$$D_\alpha W = W_{,\alpha} + \kappa^2 K_{,\alpha} W \ , \tag{1.7}$$

and ... indicates D-terms for the standard model gauge groups. The potential is constructed by arranging the hidden sector so that it is positive semi-definite with minimum value $V_{\min} = 0$, and such that $D = 0$ at the minimum. This last requirement must be imposed to avoid Planck scale masses for scalar fields in the observable sector, but we shall see that this phenomenological requirement also has important theoretical consequences. Supersymmetry is broken in the vacuum at an adjustable intermediate energy scale which is then related to the mass of the gravitino $m_{3/2}$. R-invariance is broken at the scale M_{Pl}, however, since Eqs. (1.5) and (1.6) generically give vacuum expectation values (VEVs) of this order.

The special features of gauged R-Lagrangians thus include: (i) Field content constrained by R-anomaly cancellation, (ii) superpotential with R-charge 2, and (iii) shifted D-term with $D = 0$ at minimum. Nevertheless our principal result is that the direct effects of gauged R-symmetry cannot be detected at low energy. In part this is obvious, the R-photon mass is of order $g M_{\text{Pl}}$, so photon exchange graphs are negligible at low energy. More surprising is the fact that the net contribution of the light fields in the D-term of (1.6) also cancels when the heavy sector fields are integrated out. For this the condition $\langle D \rangle = 0$ is crucial. So the low-energy effective Lagrangian does not contain the $U(1)_R$ coupling g. It does contain weakly coupled light fields beyond those of the MSSM, some required to cancel anomalies and others from the hidden sector.

In Section 2 we discuss how to obtain the key formulae of gauged R-models presented above from the general component Lagrangian of [4,9]. In Section 3 we present our simple proposal for the hidden sector superpotential. The hidden sector contains an accidental global $U(1)$ symmetry that is spontaneously broken and therefore gives a Nambu–Goldstone (NG) boson. This symmetry can be broken explicitly by modifying

the superpotential, if desired. The R-anomaly conditions are discussed in Section 4, where we determine a particular assignment of the r_α for all fields. In Section 5 we discuss the full theory of coupled hidden and observable sectors. Solutions of the μ-term and gluino mass problems have been incorporated. Section 6 is devoted to the low-energy effective Lagrangian of our gauged R-supergravity model, and some of the special features of its phenomenology are discussed in Section 7. Results are briefly summarized in Section 8, and Appendix A is devoted to a discussion of quadratic divergences.

2. Gauged R-models

The derivation of these models by superspace techniques can be found in [9]. Our discussion is based on the Kähler geometric component Lagrangian of [4,9]. There is no need to present the full Lagrangian, which is complicated. Instead we will discuss only the relevant terms, using the conventions of [1] (but with the $2\kappa^2$ of [1] replaced by κ^2 here, and the $U(1)_R$ coupling e of [1] replaced by $-g$ here).

In the Kähler-geometric viewpoint, the infinitesimal R-transformation of the scalar fields z^α with parameter ϑ defines a holomorphic Killing vector V^α by

$$\begin{aligned}
\delta z^\alpha &= -ir_\alpha z^\alpha \vartheta \equiv V^\alpha \vartheta\,, \\
\delta \bar{z}^\alpha &= +ir_\alpha \bar{z}^\alpha \vartheta \equiv V^{\bar{\alpha}} \vartheta\,.
\end{aligned} \tag{2.1}$$

It is a general mathematical result that a holomorphic Killing vector is the gradient of a real scalar potential $D(z, \bar{z})$,

$$G_{\alpha\bar{\beta}} V^{\bar{\beta}} = iD_{,\alpha}\,, \tag{2.2}$$

and D is unique up to an additive constant for an abelian symmetry. We have made the simplifying assumption that R acts linearly on the coordinates $z^\alpha(x)$ and that the Kähler potential $K(z, \bar{z})$ is invariant (see (1.4)). D is then given by the simple expression

$$D = iK_{,\alpha} V^\alpha + \xi/g\,. \tag{2.3}$$

It is quite striking that the familiar D-terms of SUSY gauge theories have a Kähler-geometric interpretation and that the FI parameter ξ of global SUSY is just the shift ambiguity of the Killing potential D.

If we define the dimensionless constant $c = \xi\kappa^2/2g$, then the $U(1)_R$ covariant derivative of the SUSY partner χ^α of z^α is initially [4,9]

$$D_\mu \chi^\alpha = \left(\mathcal{D}_\mu^{\text{grav}} + ig(r_\alpha - c)R_\mu\gamma_5\right)\chi^\alpha\,. \tag{2.4}$$

The gauge covariance of the superpotential is then expressed by the Kähler covariant condition (see Ref. [4], p. 311)

$$V^\alpha D_\alpha W = -i\kappa^2 D\ W \tag{2.5}$$

with $D_\alpha W$ defined in (1.7). Using (2.3) one sees that this reduces to

$$\sum_\alpha r_\alpha z^\alpha W_{,\alpha} = 2cW \,. \tag{2.6}$$

At this point we can scale the R-charges by $r_\alpha \to cr_\alpha$. One sees that c drops from (2.6), which then reduces to (1.3), and that c can be absorbed by redefinition of the coupling constant $gc \to g$ in (2.4) and *everywhere* in the full Lagrangian of [4,9]. So c (or ξ) is really a superfluous parameter of the SG theory.

We thus reach the conclusion that a gauged U(1) symmetry in SG can appear in the Lagrangian in two discretely different modes: the FI mode in which (1.1)–(1.3) and (1.5) hold, and the conventional mode, which is the one for the U(1) hypercharge of the standard model. In this case the fermion and boson components of a supermultiplet have the same hypercharge, and the superpotential must be invariant, i.e.

$$\sum_\alpha Y_\alpha z^\alpha W_{,\alpha} = 0 \,, \tag{2.7}$$

where Y_α is the hypercharge of z^α. The D-term $D_Y = \sum Y_\alpha z^\alpha K_{,\alpha}$ is unshifted. The low-energy manifestations of the gauge symmetry are also very different. We shall now proceed, with $c = 1$ in all formulae above, as justified by the argument of this section.

3. The hidden sector

For the sake of simplicity, we will work in this section with units $\kappa^2 = 1$, except when a discussion of mass scales is required. Also, let us distinguish between hidden fields $z^\alpha(x)$ and observable fields $y^i(x)$ and assume an additive Kähler potential

$$K = K(z, \bar{z}) + \sum_i \bar{y}^i y^i \,. \tag{3.1}$$

The natural scale of D is the Planck mass, so if $\langle D \rangle$ is not zero, (1.6) contains an unacceptably large mass term [7]

$$g^2 \langle D \rangle \sum_i \bar{y}^i y^i \,. \tag{3.2}$$

For this reason we must arrange the hidden sector so that $\langle D \rangle = 0$.

We can satisfy both $\langle D \rangle = 0$ and $\langle V \rangle = 0$, with a pair of hidden fields z_1, z_2 and the superpotential

$$W = m^{3-a-b} z_1^a z_2^b \,, \tag{3.3}$$

where m is a parameter of intermediate scale $m < M_{\mathrm{Pl}}$. We use subscripted field variables to distinguish between the field index 1 or 2 and the exponent a or b. The Kähler geometry of the hidden sector is that of a product of hyperboloids with Kähler potential

$$K(z, \bar{z}) = -\frac{1}{c_1} \ln(1 - c_1 \bar{z}_1 z_1) - \frac{1}{c_2} \ln(1 - c_2 \bar{z}_2 z_2) \,. \tag{3.4}$$

Each hyperboloid is thus described as the disc $|z_i| < 1/c_i$. W satisfies (1.3) if

$$ar_1 + br_2 = 2. \tag{3.5}$$

We now discuss conditions such that the quantity

$$\widetilde{V} = D_\alpha W G^{\alpha\bar{\beta}} D_\beta W - 3W\overline{W}$$
$$= \rho_1^{a-1}\rho_2^{b-1}\left[\rho_2(a + (1 - ac_1)\rho_1)^2 + \rho_1(b + (1 - bc_2)\rho_2)^2 - 3\rho_1\rho_2\right], \tag{3.6}$$

where $\rho_1 = \bar{z}_1 z_1$ and $\rho_2 = \bar{z}_2 z_2$, has its global minimum at $\widetilde{V} = 0$. If $\langle D \rangle = 0$ also holds, then the full potential V of (1.6) is minimized with zero cosmological constant. We choose $a, b < 1$, so that $\rho_1 = \rho_2 = 0$ is not a minimum. It is then sufficient to require that the quantity in square brackets in (3.6) is minimized with respect to ρ_1 and ρ_2 and vanishes at the minimum. These conditions can be written as

$$\frac{[\ ]}{\rho_1\rho_2} = \frac{(a + (1 - ac_1)\rho_1)^2}{\rho_1} + \frac{(b + (1 - bc_2)\rho_2)^2}{\rho_2} - 3 = 0, \tag{3.7}$$

$$\frac{[\ ]_{,\rho_1}}{\rho_2} = 2(1 - ac_1)(a + (1 - ac_1)\rho_1) + \frac{(b + (1 - bc_2)\rho_2)^2}{\rho_2} - 3 = 0, \tag{3.8}$$

$$\frac{[\ ]_{,\rho_2}}{\rho_1} = 2(1 - bc_2)(b + (1 - bc_2)\rho_2)$$
$$+ \frac{(a + (1 - ac_1)\rho_1)^2}{\rho_1} - 3 = 0. \tag{3.9}$$

Straightforward manipulations then give the conditions

$$\frac{1}{\rho_1} = \left(\frac{1}{a} - c_1\right), \qquad \frac{1}{\rho_2} = \left(\frac{1}{b} - c_2\right), \tag{3.10}$$

$$\frac{a^2}{\rho_1} + \frac{b^2}{\rho_2} = \frac{3}{4}. \tag{3.11}$$

When (3.10) is substituted in (3.11) one finds a simple cubic relation among the four parameters a, b, c_1, c_2. The conditions $2c_1 a < 1$ and $2c_2 b < 1$ are also required so that the geometric constraints $\rho_1 c_1 < 1$ and $\rho_2 c_2 < 1$, respectively, are satisfied.

The conditions above ensure that $\widetilde{V}$ has a stationary point with $\langle \widetilde{V} \rangle = 0$, and one can check that it is a local minimum. We now wish to ensure that the surface $D = 0$ passes through this minimum. Using (1.5) and (3.4) one finds that the $D = 0$ condition is

$$D = \frac{r_1\rho_1}{1 - c_1\rho_1} + \frac{r_2\rho_2}{1 - c_2\rho_2} + 2 = 0. \tag{3.12}$$

Eqs. (3.5), (3.10)–(3.12) constitute five conditions on the eight quantities $a, b, r_1, r_2,$ c_1, c_2, ρ_1, ρ_2. We choose arbitrarily $a = b = \frac{1}{2}$ and $r_1 = 5, r_2 = -1$. The equations can be solved analytically and yield

$$c_1 = \frac{5 - \sqrt{21}}{4}, \qquad \rho_1 = \frac{4}{3 + \sqrt{21}},$$
$$c_2 = \frac{-1 + \sqrt{21}}{4}, \qquad \rho_2 = \frac{4}{9 - \sqrt{21}}, \tag{3.13}$$

which satisfy the geometric constraints. For the parameters a, b, r_1, r_2, c_1, c_2 of this solution, we have obtained computer plots which indicate that $\tilde{V} \geqslant 0$ globally with the minimum at ρ_1, ρ_2 of (3.13).

This solution lies on a three-dimensional hypersurface in the space of parameters. It is easy to explore this surface by choosing other values of a, b, r_1, r_2 which satisfy (3.5) and then find the solution of (3.10)–(3.12). For some values of these input parameters one finds that either c_1 or c_2 or both are negative. From (3.4) one sees that this corresponds to the Kähler geometry of a two-sphere rather than a hyperboloid. However, in all these "would-be-spherical" cases, the ρ_i values were complex, which is unacceptable. So we have partial numerical evidence to suggest that there are no spherical Kähler geometries which satisfy the required physical conditions.

The superpotential (3.3) has an additional accidental U(1) symmetry, which we call S-symmetry, for any pair of charges s_1, s_2 that satisfy

$$as_1 + bs_2 = 0. \tag{3.14}$$

Both R-symmetry and S-symmetry are spontaneously broken, since $\langle z_1 \rangle$ and $\langle z_2 \rangle$ are non-vanishing. The R Nambu–Goldstone boson is absorbed by the R-photon in the Higgs effect, but the S NG boson remains as a massless particle of the hidden sector unless the S-symmetry is explicitly broken. Since the monomial $z_1^{-r_2} z_2^{r_1}$ is R-invariant but not S-invariant, the S-symmetry may be broken by considering the more complicated superpotential

$$W' = m^{2-a-b} z_1^a z_2^b (1 + \gamma' z_1^{-r_2} z_2^{r_1}) . \tag{3.15}$$

We have not studied this case, but since we have added a new parameter, it should be possible to find acceptable vacuum solutions.

As a possible alternative to $W(z_1, z_2)$ of (3.3), we studied the superpotential

$$W'' = z_1 \left(1 + \gamma'' z_1 z_2 \right) , \tag{3.16}$$

which has R-charge 2 if $r_1 = 2$, $r_2 = -2$. With the Kähler potential (3.4), there are three real parameters, and four conditions to determine the values of $|z_1|$, $|z_2|$ at stationary points of V with $\langle D \rangle = 0$. So a count of conditions suggest that there should be a one-parameter family of solutions. However, our numerical exploration was rather unsuccessful. Search programs were numerically unstable, and it took a great deal of work to obtain a solution with parameter values $c_1 = 0.0684$, $c_2 = 30.2$, $\gamma'' = 1$, and $z_1 = 1.05$, $z_2 = 0.181$. The large ratio of the curvatures c_2 to c_1 is unattractive. For these reasons we have not pursued alternatives to (3.3) further.

The next step is to obtain the mass spectrum of the hidden sector particles. We shall consider general values of the parameters a, b, r_1, r_2, although we shall occasionally adopt the specific values for which the explicit vacuum parameters (3.13) were found.

Scalar fields are parameterized as

$$z^\alpha(x) = \frac{1}{\sqrt{2}} \left(v^\alpha + A^\alpha(x) \right) e^{i\phi^\alpha(x)/v^\alpha} \tag{3.17}$$

with real VEVs v^α related to the ρ_α of (3.6) by $(v^\alpha)^2 = 2\rho_\alpha$. The phases $\phi^\alpha(x)$ are linear combinations of the Nambu–Goldstone for the broken R- and S-symmetries. To disentangle them we write the VEV of the Killing vector of (2.1) in terms of its length $V^2 = V^\alpha G_{\alpha\bar\beta} V^{\bar\beta}$ and a real unit vector $\hat V^\alpha$ as

$$V^\alpha = i|V|\hat V^\alpha. \tag{3.18}$$

We then use the orthonormal basis $\left\{ \hat V^\alpha, \hat U^\alpha = G^{\alpha\bar\beta}\varepsilon_{\bar\beta\bar\gamma}\hat V^{\bar\gamma} \right\}$ and define the Higgs bosons for R- and S-invariance as

$$r(x) = \hat V_\beta \phi^\beta(x), \qquad s(x) = \hat U_\beta \phi^\beta(x). \tag{3.19}$$

The latter is R-gauge invariant. It is then straightforward to write the scalar kinetic Lagrangian as

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\sum_{\alpha,\beta} G_{\alpha\bar\beta}\left[\partial_\mu A^\alpha \partial^\mu A^{\bar\beta} + (v^\alpha + A^\alpha)\left(v^{\bar\beta} + A^{\bar\beta}\right) \right.\\
&\quad \left. \times \left(\frac{1}{v^\alpha}\partial_\mu\phi^\alpha + gr_\alpha R_\mu \right)\left(\frac{1}{v^{\bar\beta}}\partial_\mu\phi^{\bar\beta} + gr_\beta R_\mu \right) \right]\\
&\approx \frac{1}{2}G_{\alpha\bar\beta}\partial_\mu A^\alpha \partial^\mu A^{\bar\beta} + \frac{1}{2}\left(\partial_\mu r - \sqrt{2}g|V|R_\mu \right)\left(\partial^\mu r - \sqrt{2}g|V|R^\mu \right) + \frac{1}{2}\partial_\mu s\partial^\mu s.
\end{aligned}
\tag{3.20}
$$

The second form is valid to quadratic order in the fluctuations. One sees that $r(x)$ can be gauged away and that the R-gauge boson acquires the Planck scale mass

$$M^2 = 2g^2|V|^2. \tag{3.21}$$

The scalar mass matrix can be obtained by Taylor expansion of the potential V of (1.6) about its minimum. The result is

$$
\begin{aligned}
V &\approx \frac{1}{2}\left[2g^2|V|^2\hat V_\alpha\hat V_{\bar\beta}A^\alpha A^{\bar\beta} + 4m^{2(3-a-b)}e^K\rho_1{}^a\rho_2{}^b \right.\\
&\quad \left. \times \left((1 - ac_1)^2(A^1)^2 + (1 - bc_2)^2(A^2)^2 \right) \right].
\end{aligned}
\tag{3.22}
$$

The fact that the phases $r(x)$ and $s(x)$ drop out confirms that they are NG fields. The mass matrix is dominated by the D-term contribution, and it is easy to see that one linear combination of A^1 and A^2, predominantly $\hat V_\alpha A^\alpha$, has Planck scale mass $2g^2 M_{\text{Pl}}^2 + \mathcal{O}(m^2(m/M_{\text{Pl}})^{4-2a-2b})$, and the orthogonal combination has mass of order $\mathcal{O}(m^2(m/M_{\text{Pl}})^{4-2a-2b})$.

To analyze the fermion mass spectrum, we need the non-derivative Fermi bilinear terms in the Lagrangian, namely,

$$
\begin{aligned}
&-\sqrt{2}g\bar\lambda\left(V_\alpha L\chi^\alpha + V_{\bar\alpha}R\chi^{\bar\alpha} \right) - \tfrac{1}{2}gD\bar\psi_\mu\gamma^\mu\gamma_5\lambda\\
&-e^{K/2}\left\{ \bar\psi_\mu\sigma^{\mu\nu}(\overline{W}L + WR)\psi_\nu + i\tfrac{1}{2}\bar\psi_\mu\gamma^\mu\left(D_\alpha WL\chi^\alpha + D_{\bar\alpha}\overline{W}R\chi^\alpha \right) \right.\\
&\quad \left. + \tfrac{1}{2}\bar\chi^\alpha\mathcal{D}_\alpha D_\beta WL\chi^\beta + \tfrac{1}{2}\bar\chi^\alpha\mathcal{D}_{\bar\alpha}D_{\bar\beta}\overline{W}R\chi^\beta \right\},
\end{aligned}
\tag{3.23}
$$

where $D_\alpha W$ has been defined in (1.7), and $\mathcal{D}_\alpha D_\beta W$ is the Kähler covariant second derivative

$$\mathcal{D}_\alpha D_\beta W \equiv \partial_\alpha D_\beta W - \Gamma^\gamma_{\alpha\beta} D_\gamma W + K_{,\alpha} D_\beta W. \tag{3.24}$$

We choose the unitary gauge condition

$$\langle D_\alpha W \rangle L\chi^\alpha = 0, \tag{3.25}$$

which is compatible with the $\delta\chi^\alpha$ transformation rule and makes the contribution to the mass matrix of the $\bar\psi \cdot \gamma\chi^\alpha$ term vanish. We can then identify the gravitino mass

$$m_{3/2} = \kappa^2 \left\langle e^{\kappa^2 K/2} W \right\rangle = m^{3-a-b} e^{\kappa^2 \langle K \rangle/2} (\rho_1)^{a/2} (\rho_2)^{b/2} / M_{\text{Pl}}^2. \tag{3.26}$$

For the case $a = b = 1/2$, a gravitino mass of electroweak order implies an intermediate scale $m \sim 10^{10-11}$ GeV.

One should note the orthogonality relation

$$\langle V^\alpha D_\alpha W \rangle = 0, \tag{3.27}$$

which follows immediately from the invariance condition (2.5) in the $\langle D \rangle = 0$ vacuum. The two physical spinors are thus the superpositions of $\lambda(x)$ and $\widehat{V}_\alpha \chi^\alpha(x)$ which diagonalize the mass matrix of (3.23), while the NG spinor is the orthogonal mode $\widehat{U}_\alpha \chi^\alpha \sim \langle D_\alpha W \rangle \chi^\alpha = 0$. Only the $\lambda(x)\widehat{V}_\alpha \chi^\alpha(x)$ mixing term in (3.23) is of Planck scale, and it is easy to see that to leading order, as $M_{\text{Pl}} \to \infty$, the theory contains two Majorana states of mass $M^2 = 2g^2 G_{\alpha\bar\beta} V^\alpha V^{\bar\beta}$. Exact diagonalization of the mass matrix would split these states by an amount of order $m_{3/2}$.

Thus the hidden sector contains the massive spin 1 R-vector boson, with two Majorana spinors and the scalar $A(x) = \widehat{V}_\alpha A^\alpha(x)$, all of mass close to $M^2 = 2g^2 |V|^2$. This is effectively a massive $N = 1$ supersymmetric vector multiplet. The supertrace mass formula of the broken theory is [9]

$$\text{Str}\,\mathcal{M}^2 = \sum_{\text{spins},J} (-1)^{2J} (2J+1) \, \text{Tr}\,\mathcal{M}^2$$

$$= 2\,m_{3/2}^2 - g^2 \langle D^2 \rangle + 2\,g^2 \left\langle G^{\alpha\bar\beta} D_{,\alpha\bar\beta} D \right\rangle - 2m_{3/2}^2 \left\langle R^{\alpha\bar\beta} \frac{D_\alpha W D_{\bar\beta} \overline{W}}{|W|^2} \right\rangle, \tag{3.28}$$

where $R^{\alpha\bar\beta}$ is the Ricci tensor obtained from the Kähler metric. The right-hand side of (3.28) is independent of M_{Pl} because $\langle D \rangle = 0$, and therefore it may be expected that the Planck mass states form massive supermultiplets.

Supersymmetry is spontaneously broken, so there is a massive gravitino with mass $m_{3/2}$ given in (3.26), and there is an additional scalar $B(x) = \widehat{U}_\alpha A^\alpha(x)$ whose mass is of the same order. The graviton remains massless and so does the S NG field $s(x)$ of (3.19). For general values of the parameters a, b of the superpotential, the S-symmetry current has an anomaly, so $s(x)$ is an axion. If $a = b$, however, the S-current is vector-like; there is no anomaly, and $s(x)$ is a massless NG boson. This vector-like property will not hold in the quark sector when the MSSM is included.

One could consider a more complicated hidden sector in which additional chiral multiplets (z^α, χ^α) enter the superpotential $W_h(z^\alpha)$. Due to the finite $\text{Str}\,\mathcal{M}^2$ requirement and the fact that SUSY is broken at an intermediate scale, there is a general constraint that states which acquire mass of order M_{Pl} must occur as massive supermultiplets. However M_{Pl} scale scalar masses can only come from the D-term contribution to the potential V and M_{Pl} scale spinor masses only from the $g\bar{\lambda}\chi$ term in (3.23). But if $\langle D \rangle = 0$ only one scalar acquires a large mass, and there is just one pair of large mass Majorana spinors. It is then a general result that the only M_{Pl} scale states are those of the massive vector multiplet containing the R-photon, while other particles in the hidden sector have masses of order $m_{3/2}$ plus possible massless states from global symmetries. A corollary of this argument is that the minimum size of a hidden sector with M_{Pl} scale R-breaking is the massless R-vector multiplet plus two chiral multiplets. These multiplets contain the three Majorana spinors which form the Goldstino and the two M_{Pl} partners of the R-photon.

The hidden sector model presented above is not consistent as a complete theory because it contains $U(1)_R$ anomalies. The cancellation of anomalies between hidden and observable chiral fermions is the subject of the next section. We will find it necessary to add one additional hidden chiral multiplet (z_3, χ_3). We assume that this does not directly enter the superpotential in order not to disturb the simple analysis of the vacuum which we have made here.

4. Anomalies and the MSSM

In this section we study the anomaly cancellation conditions in a gauged R-supergravity model with hidden sector fields z^α plus the fields of the MSSM which are shown in Table 1. We assume that the MSSM part of the superpotential contains the following conventional Yukawa interactions:

$$W_o = \bar{u}\mathbf{Y}_u \Phi_u Q + \bar{d}\mathbf{Y}_d \Phi_d Q + \bar{e}\mathbf{Y}_e \Phi_d L, \tag{4.1}$$

where the $\mathbf{Y}_{u,d,e}$ are Yukawa coupling matrices. The covariant derivatives (1.1), (1.2) show that $U(1)_R$ is a chiral symmetry which couples to all fermions in the theory, those of chiral multiplets, the gauginos, and the gravitino. There are anomalous triangle graphs with various contributions of external R-photons, standard model gauge bosons, and gravitons.

The anomaly cancellation conditions written in terms of the fermionic R-charges, which are related to the superfield ones by $\tilde{r} = r - 1$, are

$$3\left(\tfrac{1}{6}\tilde{r}_Q + \tfrac{4}{3}r_{\bar{u}} + \tfrac{1}{3}\tilde{r}_{\bar{d}} + \tfrac{1}{2}\tilde{r}_L + \tilde{r}_{\bar{e}}\right) + \tfrac{1}{2}\left(\tilde{r}_{\Phi_u} + \tilde{r}_{\Phi_d}\right) + C_1 = 0, \tag{4.2}$$

$$\tfrac{3}{2}\left(3\tilde{r}_Q + \tilde{r}_L\right) + \tfrac{1}{2}\left(\tilde{r}_{\Phi_u} + \tilde{r}_{\Phi_d}\right) + 2 + C_2 = 0, \tag{4.3}$$

$$\tfrac{3}{2}\left(2\tilde{r}_Q + \tilde{r}_{\bar{u}} + \tilde{r}_{\bar{d}}\right) + 3 + C_3 = 0, \tag{4.4}$$

$$3\left(\tilde{r}_Q^2 - 2\tilde{r}_{\bar{u}}^2 + \tilde{r}_{\bar{d}}^2 - \tilde{r}_L^2 + \tilde{r}_{\bar{e}}^2\right) + \left(\tilde{r}_{\Phi_u}^2 - \tilde{r}_{\Phi_d}^2\right) + C_4 = 0, \tag{4.5}$$

Table 1
MSSM quantum numbers

	Q	$\bar{u}$	$\bar{d}$	L	$\bar{e}$	Φ_u	Φ_d
U(1)$_Y$	$+\frac{1}{6}$	$-\frac{2}{3}$	$+\frac{1}{3}$	$-\frac{1}{2}$	$+1$	$+\frac{1}{2}$	$-\frac{1}{2}$
SU(2)$_w$	2	1	1	2	1	2	2
SU(3)$_c$	3	$\bar{3}$	$\bar{3}$	1	1	1	1

$$3\left(6\tilde{r}_Q^3 + 3\tilde{r}_{\bar{u}}^3 + 3\tilde{r}_{\bar{d}}^3 + 2\tilde{r}_L^3 + \tilde{r}_{\bar{e}}^3\right) + 2\left(\tilde{r}_{\Phi_u}^3 + \tilde{r}_{\Phi_d}^3\right) + 16 + C_5 = 0, \tag{4.6}$$

$$3\left(6\tilde{r}_Q + 3\tilde{r}_{\bar{u}} + 3\tilde{r}_{\bar{d}} + 2\tilde{r}_L + \tilde{r}_{\bar{e}}\right) + 2\left(\tilde{r}_{\Phi_u} + \tilde{r}_{\Phi_d}\right) - 8 + C_6 = 0. \tag{4.7}$$

Eqs. (4.2)–(4.7) correspond, respectively, to the U(1)$_Y^2$ − U(1)$_R$, SU(2)$_W^2$ − U(1)$_R$, SU(3)$_c^2$ − U(1)$_R$, U(1)$_Y$ − U(1)$_R^2$, U(1)$_R^3$, and gravitational mixed anomalies. Here we have taken into account that there are thirteen vector multiplets in the theory, whose fermionic components carry R-charge 1, and that the gravitino contribution to the anomaly is 3 and −21 times the one of a Majorana fermion in (4.6) and (4.7), respectively [10]. We have also assumed three generations of MSSM quark and lepton superfields. The contributions to the different anomalies from any extension to the MSSM as well as from hidden fields are denoted by C_i.

The superpotential (4.1) must have R-charge 2, and this imposes further conditions on some of the R-charges:

$$\tilde{r}_Q + \tilde{r}_{\bar{u}} + \tilde{r}_{\Phi_u} = -1, \tag{4.8}$$

$$\tilde{r}_Q + \tilde{r}_{\bar{d}} + \tilde{r}_{\Phi_d} = -1, \tag{4.9}$$

$$\tilde{r}_L + \tilde{r}_{\bar{e}} + \tilde{r}_{\Phi_d} = -1. \tag{4.10}$$

The MSSM without any extension cannot be anomaly free. This can easily be recognized by realizing that the subsystem of Eqs. (4.2)–(4.4) and (4.8)–(4.10) is only compatible when the relation

$$C_1 + C_2 - 2C_3 = 6 \tag{4.11}$$

is satisfied. Therefore, adding new particles carrying SM quantum numbers is required to cancel some anomalies. This is a necessary but not sufficient condition to make the whole system of equations consistent.

Many possible additions to the MSSM can be considered [7]. Here we choose one particular extension consisting of two new chiral supermultiplets whose SM quantum numbers (SU(3)$_c$, SU(2)$_w$, U(1)$_Y$) are $D = (3, 1, -1/3)$ and $\bar{D} = (\bar{3}, 1, +1/3)$. We also add the two hidden fields responsible for SUSY breaking, as discussed in Section 3, with R-charges $r_1 = 5$ and $r_2 = -1$. This particular extension of the MSSM is motivated by the decomposition of fundamental representations of various larger groups, such as the **27** of E_6 or the **5** of SU(5), under the SM group. In SU(5), (Φ_u, D) and $(\Phi_d, \bar{D})$ correspond to **5** and **5̄** representations, respectively. The D and $\bar{D}$ are hence referred to as color-triplet Higgses. Although we allude to grand unified theory (GUT) groups, it

will become evident that the R-charge assignments are not compatible with the SU(5) structure, for example. The compatibility condition (4.11) implies that the sum of the R-charges of D and $\bar{D}$ is fixed,

$$\tilde{r}_D + \tilde{r}_{\bar{D}} = -9. \tag{4.12}$$

Using (4.2)–(4.5), all light field, fermionic R-charges can be expressed in terms of two of them which we take to be $\tilde{r}_{\bar{u}}$ and $\tilde{r}_{\bar{d}}$ or equivalently, $\sigma = \tilde{r}_{\bar{u}} + \tilde{r}_{\bar{d}} + 2$ and $\delta = \tilde{r}_{\bar{u}} - \tilde{r}_{\bar{d}}$. One then obtains from (4.7) the following relations between σ and δ:

$$\delta = 3\sigma + \omega, \tag{4.13}$$

where $\omega = 2C_6^{(h)}/3 - 30$, and the superscript h denotes the contribution to C_i from the hidden sector. In terms of σ and ω, the fermionic R-charges for all the observable fields are

$$\tilde{r}_Q = -\frac{\sigma}{2} + \frac{3}{2}, \qquad \tilde{r}_L = \frac{3\sigma}{2} - \frac{29}{6},$$

$$\tilde{r}_{\bar{u}} = 2\sigma + \frac{\omega - 2}{2}, \qquad \tilde{r}_{\bar{e}} = -3\sigma - \frac{3\omega - 32}{6},$$

$$\tilde{r}_{\bar{d}} = -\sigma - \frac{\omega + 2}{2}, \qquad \tilde{r}_{\Phi_u} = -\frac{3\sigma}{2} - \frac{\omega + 3}{2},$$

$$\tilde{r}_D = \sigma + \frac{2\omega - 50}{9}, \qquad \tilde{r}_{\Phi_d} = \frac{3\sigma}{2} + \frac{\omega - 3}{2},$$

$$\tilde{r}_{\bar{D}} = -\sigma - \frac{2\omega + 31}{9}. \tag{4.14}$$

Inserting these expressions into (4.6) yields a relation between the terms $C_5^{(h)}$ and $C_6^{(h)}$

$$27\omega^3 + 720\omega^2 + 6480\omega + 54584 - 72C_5^{(h)} = 0. \tag{4.15}$$

This relation is not satisfied for the minimal hidden sector set $\{\tilde{r}_1, \tilde{r}_2\}$ discussed earlier, so the system of equations is incompatible in this case. Adding a third chiral superfield (z_3, χ_3) allows for a solution, albeit irrational. Rationality of the R-charges is however not required in this case, since there is no embedding of $U(1)_R$ in a larger group, and there is no R-charge quantization condition. Rationality is possible if more hidden fields are added [7].

Inserting $\omega = 2\tilde{r}_3/3 - 86/3$ into (4.15), one gets the equation for $\tilde{r}_3$,

$$8\tilde{r}_3^3 + 89\tilde{r}_3^2 - 2647\tilde{r}_3 + 21944 = 0, \tag{4.16}$$

with real solution $\tilde{r}_3 = -27.0823$. There remains one free parameter σ in the determination of the R-charges, but it is not necessary to specify it for our purposes.

Before concluding this section, we will briefly mention that the theory also has a Kähler anomaly [8]. However, since the Kähler manifolds that we are dealing with are topologically trivial, and there is no global or gauge symmetry realized non-linearly on them, the cancellation of this anomaly is not necessary for the consistency of the theory.

5. The complete model

Although the low-energy consequences of gauged R-symmetry are largely model independent, we wish to present a particular model in this section which appears to have a reasonably correct phenomenology. The model contains the 17 chiral and 12 vector multiplets of the MSSM, the SG and R-vector multiplets, plus 3 hidden chiral multiplets and two $SU(3)_c$ triplet chiral multiplets. The model is anomaly free as explained in the last section.

We choose the Kähler potential

$$
K = -\frac{1}{c_1} \ln(1 - c_1|z_1|^2) - \frac{1}{c_2} \ln(1 - c_2|z_2|^2) + |z_3|^2
$$
$$
+ \sum_i |y_i|^2 + \frac{\lambda}{M_{\mathrm{Pl}}} \left(\bar{z}_2 \Phi_u \Phi_d + z_2 \bar{\Phi}_u \bar{\Phi}_d \right) + \frac{\lambda'}{M_{\mathrm{Pl}}^2} |\Phi_u|^2 |\Phi_d|^2 . \tag{5.1}
$$

The first four terms describe a hyperbolic Kähler metric for the fields z_1, z_2, and flat Kähler geometry for z_3 and all other chiral multiplets. The fifth term is a Giudice–Masiero term [11], involving the fields z_2 and the Higgs scalars, which is introduced to solve the μ problem in the model. If this were the only addition, the Kähler metric obtained from K would not be positive definite. Therefore we add the last term, and it is not difficult to show that for $\lambda' > \lambda^2$, the metric is everywhere positive definite.

We assume that the full superpotential is the sum of the term (3.3) for the hidden sector (with $a = b = 1/2$, $r_1 = 5$, $r_2 = -1$) and (4.1) for the observable sector. We now discuss the determination of the vacuum state of the complete theory. It is easy to see that the field configuration $\langle z_3 \rangle = \langle y_i \rangle = 0$ and $\langle z_1 \rangle$ and $\langle z_2 \rangle$ as determined in Section 3 is certainly a local minimum of the full theory with $\langle D \rangle = 0$ and vanishing cosmological constant. However, one cannot be certain that it is the global minimum and that the full potential is positive semi-definite. The same question arises but is rarely discussed [12,13] in most of the other $N = 1$ SG models in the literature. We have examined this issue in the simpler situation of the superpotential

$$
W = m^2 (z_1 z_2)^{1/2} + \lambda'' y^3, \tag{5.2}
$$

in which the observable sector is simulated by the single chiral field y with cubic interaction and flat Kähler potential. Numerical work then shows that the local minimum with $\langle y \rangle = 0$ is in fact the global minimum. The same property has also been shown to hold for the Polonyi potential plus cubic term

$$
W = m^2 (z - \beta) + \lambda'' y^3 \tag{5.3}
$$

with flat Kähler potential.

The CDF lower bound on the gluino mass is approximately 150 GeV. Since the model as so far specified does not contain a classical gluino mass, we modify it by introducing a non-trivial gauge kinetic function [14]. The following two forms

$$
f_{\alpha\beta} = \delta_{\alpha\beta} (1 + \gamma \kappa^6 z_1 z_2^5) , \tag{5.4}
$$

$$\hat{f}_{\alpha\beta} = \delta_{\alpha\beta}(1 + \hat{\gamma}\ln\kappa^6 z_1 z_2^5) \tag{5.5}$$

each generate a gluino mass of order $m_{3/2}$. Both expressions are R-invariant, but they have different behavior under the S-symmetry discussed in Section 3. The first term violates the symmetry explicitly. The second term maintains a non-linear realization of the symmetry and contains an explicit axion coupling, $s(x)F\tilde{F}$. Thus the two terms have different implications for axion physics as we will discuss in Section 7.

6. Low-energy limit

The low-energy limit of a $N = 1$ supergravity theory is obtained by integrating out the heavy fields to get the tree vertices of the low-energy effective Lagrangian. As we will see, this process is a bit more subtle for gauged R-theories than for conventional ones. In principle one should also study loop diagrams, and we will study here a particularly crucial set which threatens to introduce quadratic divergences and spoil the gauge hierarchy which is the major motivation for studying SUSY.

We begin by discussing an effect which we find to be very striking although not directly relevant to the low-energy limit. For every fermion in the theory, one can isolate from the Lagrangian the covariant kinetic term and the Kähler connection term. For the gravitino these are

$$\mathcal{L}_{\psi_\mu} = -\frac{1}{2}\epsilon^{\lambda\rho\mu\nu}\left[\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho - \frac{1}{4}\kappa^2\bar{\psi}_\lambda\gamma_\mu\psi_\rho\left(K_{,\alpha}D_\nu z^\alpha - K_{,\bar{\alpha}}D_\nu\bar{z}^\alpha\right)\right]. \tag{6.1}$$

The second term is a dimension-6 operator whose effects are normally negligible at low energy, but since $\langle K_{,\alpha}z^\alpha\rangle \sim M_{\text{Pl}}^2$, there are induced dimension-4 vertices $\bar{\psi}_\lambda\psi_\rho R_\nu$.

From the covariant derivatives (1.1), (1.2), one finds the net contribution

$$\begin{aligned}
\mathcal{L}_{(\bar{\psi}\psi R)} &= \frac{i}{2}g\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_\mu\psi_\rho R_\nu\left[1 + \frac{1}{4}\kappa^2 r_\alpha\left(K_{,\alpha}z^\alpha + K_{,\bar{\alpha}}\bar{z}^\alpha\right)\right] \\
&= \frac{i}{2}g\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_\mu\psi_\rho R_\nu\left[1 + \frac{1}{2}\kappa^2\left(D - \frac{2}{\kappa^2}\right)\right],
\end{aligned} \tag{6.2}$$

where (1.4), (1.5) have been used. Since $\langle D\rangle = 0$, we see that the minimal coupling of the R-photon to the gravitino actually vanishes in the effective Lagrangian. The same cancellation can be seen to hold for all gauginos $\lambda^{(a)}$, while for chiral fermions there is a partial cancellation, so that the fermion R-charges $(r_\alpha - 1)$ in (1.2) are replaced by r_α. So the "displacement" of the fermion and boson R-gauge couplings, which is one of the most conspicuous features of the initial Lagrangian, cancels. This is a quite robust feature of gauged R-models, independent of the details of the hidden sector and requiring only $\langle D\rangle = 0$. The reason for the cancellation is that, since SUSY is broken through the hidden sector superpotential, the dimension-4 terms in the Lagrangian of fluctuations about the vacuum preserve global SUSY. Dimension-4 couplings to the $U(1)_R$ vector multiplet must be those of a $U(1)$ SUSY gauge theory, so the z^α, χ^α components of any chiral multiplet couple to R_μ with the same strength. Of course,

since the R-photon mass is $\sim gM_{\mathrm{Pl}}$, tree graphs with R-photon exchange are negligible at low energy, whether or not the $U(1)_R$ charge displacement of bosons and fermions cancels.

In conventional SG models one can obtain the low-energy effective Lagrangian of the observable fields simply by replacing hidden fields by their VEVs in the superpotential sector. In our model this is not sufficient because there is a heavy hidden field $A(x)$ which obtains its order gM_{Pl} mass from the large D-terms in the Lagrangian, namely,

$$\frac{1}{2}g^2 D^2 = \frac{1}{2}g^2 \left(\sqrt{2}|V|A + \mathcal{D}^{(2)}(y^\alpha, B) + \ldots \right)^2 . \tag{6.3}$$

The linear term in D was already obtained in the mass matrix calculation of Section 3, and $\mathcal{D}^{(2)}$ denotes all quadratic terms in the light fields. We may simplify the discussion by dropping terms $+\ldots$ in D when $\langle D \rangle = 0$ and also A^2 and Ay_i terms from the superpotential contribution because their low-energy effects are suppressed by the factor $m_{3/2}/M_{\mathrm{Pl}}$ compared to the terms included. At low energy one can also drop $\partial_\mu A$ terms in the Lagrangian. One then sees that all relevant terms in A appear as the perfect square $(\sqrt{2}|V|A + \mathcal{D}^{(2)})^2$. Gaussian integration over $A(x)$, or equivalently, substitution of the solution of its equation of motion, then gives a complete cancellation. In particular the term $(\mathcal{D}^{(2)})^2$, which would have survived if the naive procedure of replacing hidden fields z^α by their VEVs were used, cancels [4]. The condition $\langle D \rangle = 0$ is vital to the above argument. For $\langle D \rangle \neq 0$, some of the terms dropped above must be kept, and substitution of the resulting solution to the equation of motion for $A(x)$ yields residual dimension-4 contact terms in the light fields as well as M_{Pl} masses for these. One can also integrate out the heavy R-photon and its spinor superpartners, and it is easy to see that all residual effects on light fields are suppressed.

We therefore reach the conclusion that all traces of the gauging of R-symmetry disappear from the low-energy effective Lagrangian. This consists of the renormalizable Lagrangian of the supersymmetric gauge theory of the $SU(3)_c \times SU(2)_w \times U(1)_Y$ standard model group, free kinetic terms for the light fields $B(x)$ and $s(x)$ of the hidden sector, scalar potential and Yukawa terms from the superpotential part of the original Lagrangian, and finally dimension-3 and -4 operators from the non-minimal gauge interactions introduced in Section 5 to generate gaugino masses. We now proceed to discuss the scalar potential sector of the Lagrangian.

The low-energy limit in the scalar potential sector of the theory is taken in a conventional way. The superpotential is given by the sum of hidden and observable pieces. The hidden fields z_1, z_2 pick up VEVs of order M_{Pl}. With our choice of the hidden superpotential we have $\langle W_h \rangle \sim m^2 M_{\mathrm{Pl}}$. The gravitino mass is therefore of order $m_{3/2} \sim m^2/M_{\mathrm{Pl}}$. The low-energy limit corresponds to taking $M_{\mathrm{Pl}} \to \infty$ while keeping $m_{3/2}$ fixed. In taking this limit the potential is expanded around the vacuum, and only the terms that are not suppressed by powers of $1/M_{\mathrm{Pl}}$ survive. The resulting potential exhibits the form of a SUSY potential plus soft SUSY-breaking terms

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[4] This result disagrees with that of Ref. [7], where $(\mathcal{D}^{(2)}(y^\alpha))^2$ was included in the low-energy Lagrangian.

$$\sum_i \left(\left| \frac{\partial \tilde{W}_o}{\partial y_i} \right|^2 + m_{3/2}^2 |y_i|^2 \right) + B [\tilde{W}_o]_2 + A [\tilde{W}_o]_3 + \text{h.c.}, \tag{6.4}$$

where $\tilde{W}_o = W_o e^{\kappa^2 \langle K \rangle / 2} + \mu \Phi_u \Phi_d$, and $[\tilde{W}_o]_2$ and $[\tilde{W}_o]_3$ refer to the bilinear and trilinear parts of $\tilde{W}_o$, respectively. The sum extends over all observable scalars of the theory. All these particles acquire a mass of the order of $m_{3/2}$. In our class of models, the effective μ and the soft trilinear and bilinear parameters are given by the following expressions:

$$\mu = \lambda \, m_{3/2} \left(\sqrt{\rho_2} - \frac{2b(1 - c_2 \rho_2)}{\sqrt{\rho_2}} \right) , \tag{6.5}$$

$$A = 2 \, (a + b) \, m_{3/2} , \tag{6.6}$$

$$B = 2 \, \lambda \, m_{3/2}^2 \left(\sqrt{\rho_2} - \frac{b(1 - c_2 \rho_2)}{\sqrt{\rho_2}} \right) / \mu , \tag{6.7}$$

where ρ_2 is assumed dimensionless and equal to its value in (3.13).

The low-energy effects of loop diagrams from the full Lagrangian should be examined. In the main, this study is beyond the scope of the present paper. However it is known that a SUSY gauge theory for gauge groups containing U(1) factors has quadratic divergences [15,16], unless the trace condition $\text{Tr} \, T = 0$ is satisfied for each U(1) generator T. This fact can usually be ignored because the condition $\text{Tr} \, T = 0$ is also required for anomaly cancellation. However in our case, the $\text{Tr} \, R$ condition for anomaly cancellation includes gaugino and gravitino contributions while, as we will explain, that for quadratically divergent scalar mass shifts involves only the chiral spinors, and both conditions cannot hold simultaneously. Since quadratic divergences for the light scalars would spoil the gauge hierarchy, which is normally protected by global SUSY, it is important to examine this situation.

In global SUSY the quadratic divergences emerge from the U(1) D-terms

$$-\frac{1}{2} g^2 D^2 = -\frac{1}{2} \left(\sum r_\alpha \bar{z}^\alpha z^\alpha + \xi + \delta \xi \right)^2 \tag{6.8}$$

(For simplicity we assume a flat Kähler metric to illustrate our point.) The quartic coupling leads to the usual one-loop quadratic mass shift diagram for z^α. Part of the divergence is cancelled by fermion and gauge boson loops, but there is an uncancelled remainder which can be expressed as the counter term

$$\delta \xi \sim g^2 \left(\sum r_\alpha \right) \Lambda^2 \tag{6.9}$$

for the FI parameter (Λ is the ultraviolet cutoff). In our case there is a shift of the fields which makes $\langle D \rangle = 0$, and that turns out to be crucial. In Appendix A, we show that the quadratic divergence cancels for the unshifted (light) scalars, but the shifted (heavy) scalar mass is still divergent. This is enough to show that the gauge hierarchy is not spoiled for a global U(1) SUSY gauge theory if $\langle D \rangle = 0$. In our full supergravity theory, there are additional divergent one-loop mass shift diagrams. For example, those

with a graviton or gravitino in the loop are individually quartic divergent. So we have a possible mass counter term of the form

$$\delta m^2 \sim \frac{1}{M_{\mathrm{Pl}}^2}(\Lambda^4 + m_{3/2}^2 \Lambda^2 + m_{3/2}^4 \ln \Lambda^2) \,. \tag{6.10}$$

We do not study those diagrams here; but our intuition is that the quartic divergence cancels, and the residual quadratic divergence is of no concern for the gauge hierarchy, since one must take a cutoff of the size $\Lambda \sim M_{\mathrm{Pl}}$ in the quantum supergravity theory.

7. Low-energy phenomenology

Although we will not attempt a complete study of all the phenomenological consequences of the model, we shall briefly comment on some selected issues. As discussed in Section 3, the specific model being considered has an accidental chiral global symmetry of the Peccei–Quinn (PQ) type due to the interactions of the super and Kähler potentials. After the spontaneous breaking of supersymmetry, there results a (pseudo) NG boson referred to as an axion, whose decay constant is of order M_{Pl}. Non-perturbative QCD instanton effects result in a mass for the axion, which in this model is too small due to the large scale of symmetry breaking. A very small mass is forbidden by cosmology, since it would lead to overclosing the universe.

As in the MSSM, the simplest solution to this problem is to explicitly break the S, or PQ, symmetry. This can be done by changing W_h as mentioned in Section 3, but it is more interesting to observe that the non-minimal gauge interaction (5.4) that was introduced in Section 4 to solve the gluino mass problem also breaks S-symmetry. The second non-minimal gauge interaction (5.5) leaves the axion unacceptably light, unless the coefficient of this term is tuned to cancel the $s(x)F\tilde{F}$ term from the one-loop quantum anomaly. This would leave a strictly massless NG boson with no connection to the strong CP problem. So we do not pursue this curious, but apparently not useful, possibility.

As in conventional models, the Lagrangian of our model contains conserved currents for the global U(1) symmetries of baryon (B) and lepton (L) number. If only the hidden fields and the Higgs scalar acquire VEVs, then these symmetries are preserved and the proton is stable. However, one should also consider modifications of the superpotential which could lead to the decay of the proton. In particular our model contains color-triplet Higgses, and these may mediate an unacceptable rate of proton decay. The allowed interactions of the color-triplets however are very constrained due to gauged R-symmetry which requires that $r_W = 2$. Given the hidden sector content of the particular model under consideration, all potentially dangerous, renormalizable interactions involving the color-triplets are forbidden independently of σ (see (4.14))

$$QL\overline{D} + \overline{u}\overline{e}D + QQD + \overline{u}\overline{d}\overline{D} \,. \tag{7.1}$$

Indeed, all renormalizable B and L violating terms are also forbidden,

$$\overline{u}\overline{d}\overline{d} + QL\overline{d} + LL\overline{e}\,, \tag{7.2}$$

thus avoiding the problem of rapid proton decay. Models with a gauged discrete symmetry have been proposed to solve the proton decay problem [17]. We have not investigated the interesting possibility that such models are the (discrete) remnants of gauged R-symmetry.

There is a conserved vectorial D-current, so the model contains stable color-triplet states. The R-charges of the color-triplets are such that an explicit mass term in the superpotential is forbidden, as it is for the Higgs isospin doublets. Although the scalar partners of the isosinglet quarks will receive soft breaking contributions to their masses, the isosinglet quarks will remain massless unless, for example, a Giudice–Masiero type term is included for them in the Kähler potential. Given our hidden sector, a possible term would have the form $\Delta K = \lambda_D \kappa^7 z_1^2 z_2^3 \overline{D} D$, and will yield a mass on the order of $m_{3/2}$. This interaction also removes the axion even in the absence of (5.4).

The present model is not consistent with grand unification since, for example, the interactions of isopin-doublets and color-triplet Higgses are independent. Furthermore the R-charges of the chiral fields are GUT-incompatible. Nevertheless it should be pointed out that such a particle content is consistent with superstring phenomenology. Even in the absence of a GUT structure, superstring theories predict gauge unification. However, the model under consideration will be plagued by the light threshold corrections of the color-triplets, and gauge unification will require either new intermediate scale thresholds or a mechanism for generating a large mass for the color-triplets. These possibilities will not be explored any further in this paper.

8. Conclusion

R-symmetry can only be gauged in the context of supergravity, and it is natural to consider the consequences of gauged R-symmetry for phenomenological models. The superpotential is constrained to have R-charge 2, and we have presented a simple hidden sector superpotential for which the vacuum state, with R-symmetry broken at the scale $M_{\rm Pl}$, can be obtained analytically. The requirement that the U$(1)_R$ D-term vanish in this vacuum was imposed initially to avoid $M_{\rm Pl}$ scale masses for scalar particles of the MSSM, but this requirement turns out to have two important consequences for the structure of the models considered. First, all terms involving the U$(1)_R$ gauge coupling g cancel in the low-energy effective Lagrangian, which is then rather conventional with universal soft SUSY-breaking terms involving the MSSM fields. Second, the quadratic divergences which would be expected in a global SUSY theory with ${\rm Tr}\, R \neq 0$ cancel for light fields. In the literature [6,5] there are statements that the flat limit of gauged R-supergravity theories involves $g \to 0$ as a mathematical limit of parameters, and the condition $\langle D \rangle = 0$ is not mentioned. By contrast our proof of the cancellation of terms involving g came from studying the physical low-energy limit of amplitudes in the full SG theory, and $\langle D \rangle = 0$ was a required condition.

Another salient feature of SG theories with gauged R is the constraint on the field content required to avoid triangle anomalies. To cancel anomalies one must add [7] fields which carry standard model quantum numbers but are not present in the MSSM, and one must also add chiral multiplets to the hidden sector beyond the two multiplets which play a role in determining the vacuum.

The principal conclusion that the effects of gauging R-symmetry cannot be directly detected at low energy is disappointing, but it also means that gauged R-symmetry may be a hidden property of the conventional framework of softly broken SUSY. Different low-energy properties could emerge from models in which the gauged R-symmetry is broken at a scale $\ll M_{\text{Pl}}$, and a toy model of this type was considered long ago [6]. It is not immediately clear how to generalize this model to agree with standard model phenomenology, and the issue of quadratic divergences would have to be reexamined since $\langle D \rangle \neq 0$ in such a model. However the investigation of such models is suggested by the present work.

Appendix A

We discuss the cancellation of quadratic divergences in a global SUSY model with $N + 1$ chiral multiplets (ϕ_i, χ_i) coupled to an abelian vector multiplet (A_μ, λ) with an FI term. The ith chiral multiplet has U(1) charge r_i. The Lagrangian is

$$\mathcal{L}_{\text{chiral}} = \sum_{i=0}^{N} (|(\partial_\mu + igA_\mu)\phi_i|^2 + i\overline{\chi}_i\gamma^\mu(\partial_\mu + igA_\mu)L\chi_i) , \tag{A.1}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{i}{2}\overline{\lambda}\not{\partial}\lambda - i\sqrt{2}g\sum_{i=0}^{N}(r_i\phi_i\overline{\lambda}L\chi_i - r_i\overline{\phi}_i\overline{\lambda}R\chi_i) - \tfrac{1}{2}g^2D^2 , \tag{A.2}$$

$$D = \sum_{i=0}^{N} r_i|\phi_i|^2 + \xi . \tag{A.3}$$

We assume that the charge r_0 of ϕ_0 and the FI constant ξ have opposite signs, so there is a supersymmetric ground state in which $\langle\phi_0\rangle = v$ with $v^2 = \xi/r_0$ and $\langle\phi_i\rangle = 0$ for

$i \neq 0$. We then express $\phi_0(x)$ as

$$\phi_0(x) = \frac{1}{\sqrt{2}}(v + A(x) + iB(x)).$$ (A.4)

Quantum computations are performed in a covariant R_ζ gauge with gauge-fixing and ghost Lagrangians

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\zeta}(\partial \cdot A + \zeta g r_0 v y)^2,$$ (A.5)

$$\mathcal{L}_{\text{ghost}} = \partial_\mu \overline{\eta} \partial^\mu \eta - \zeta g^2 r_0^2 v^2 \overline{\eta} \eta - \zeta g^2 r_0^2 v x \overline{\eta} \eta.$$ (A.6)

We will study the two-point function of the unshifted fields and take ϕ_1 for definiteness. Since we are interested only in the quadratic divergence of each diagram, we express results as multiples of the integral $I_2 = \int d^4 k/(2\pi)^4 k^2$.

We find the quadratically divergent contribution to the mass shift from the one-loop, one-particle irreducible (1PI) diagrams with quartic interactions and circulating ϕ_i, A, and the NG boson, B, is

$$\Sigma_{\text{a}} = \left(r_1 \sum_{i=0}^{N} r_i + r_1^2 \right) I_2,$$ (A.7)

There is also a quadratic contribution from three 1PI diagrams, two involving the gauge boson and one a fermion pair λ and χ_1,

$$\Sigma_{\text{b}} = -r_1^2 I_2.$$ (A.8)

Thus the sum of all 1PI diagrams is

$$\Sigma_{\text{1PI}} = r_1 \sum_{i=0}^{N} r_i I_2,$$ (A.9)

which confirms the result of [15,16] that there is a quadratic divergence unless $\text{tr } R = \sum r_i = 0$.

However, in the spontaneously broken theory, there are additional quadratically divergent tadpole diagrams in which the fields ϕ_i, A, B, A_μ, and η circulate in the loop and another in which the fermions λ and χ_0 are coupled by the mass insertion $r_0 v \gamma_5$. We find that the sum of the tadpole graphs is

$$\Sigma_{\text{tadpole}} = -r_1 \sum_{i=0}^{N} r_i I_2,$$ (A.10)

which exactly cancels the 1PI graphs!

Thus there is no quadratically divergent mass shift for the $\phi_i(x)$ fields, with $i \neq 0$. The situation is different for the Higgs field $A(x)$ for which the 1PI and tadpole graphs contribute $r_0 \sum r_i I_2$ and $-3 r_0 \sum r_i I_2$, respectively. The quadratic divergence for $A(x)$ thus cancels only if $\text{Tr } R = 0$.

It should be emphasized that the cancellation between 1PI and tadpole contributions to the mass shift of the ϕ_i fields requires a precise relation between the vertex factors and the mass of the Higgs field. The needed relation is a consequence of the condition $\langle D \rangle = 0$ and therefore reflects the fact that the vacuum is supersymmetric. The same cancellation will occur for the mass shift of the "light" scalars in any of the many possible supersymmetric vacua of the theory.

The model studied in this appendix is considerably simpler than the full gauged R-supergravity theory of the main text. In the latter there are contributions to the R-Higgs scalar mass and vertices both from D-terms and from the superpotential (F-terms). However the effects of the F-terms are suppressed by the ratio $(m_{3/2}/M_{\rm Pl})^2$ compared to the dominant D-terms. So the modification of the quadratic divergences due to the F-terms is of the same order as that of the graviton and gravitino diagrams discussed at the end of Section 6.

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