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Measures of fine tuning [*]

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Abstract

Fine-tuning criteria are frequently used to place upper limits on the masses of superpartners in supersymmetric extensions of the standard model. However, commonly used prescriptions for quantifying naturalness have some important shortcomings. Motivated by this, we propose new criteria for quantifying fine tuning that can be used to place upper limits on superpartner masses with greater fidelity. In addition, our analysis attempts to make explicit the assumptions implicit in quantifications of naturalness. We apply our criteria to the minimal supersymmetric extension of the standard model, and we find that the scale of supersymmetry breaking can be larger than previous methods indicate.

1. Introduction

One of the principle motivations for weak scale supersymmetry is that it provides a framework that stabilizes the hierarchy between the weak scale and the Planck scale, or some other unification scale. In non-supersymmetric models, the mass renormalization of fundamental scalars is quadratically divergent. This divergence must be cancelled, or the fundamental scalar will have a renormalized mass on the order of the cut-off. In the standard model, if the Higgs boson remains a fundamental degree of freedom all the way up to some very heavy scale, we must fine tune a precise cancellation order by order in perturbation theory to maintain the lightness of the weak scale.

Supersymmetry solves this problem because the renormalization effects of superpartners eliminate the quadratic divergences. But supersymmetry is at best a broken symmetry. There are no superpartners degenerate in mass with the particles that have been observed so far. These superpartners can have gauge invariant mass terms if supersymmetry is softly broken, and these masses can be made arbitrarily large provided we increase the scale of supersymmetry breaking. There is a price for this. As the scale of supersymmetry breaking increases the weak scale can only remain light by virtue of an increasingly delicate cancellation. Eventually a point is reached when the model no longer appears to provide a complete explanation of why a light weak scale is stable.

Attempts to pinpoint where and when our understanding of weak scale stability is lost, or becomes incomplete, must of necessity quantify some intuitive notion of naturalness. Such a prescription for quantifying naturalness exists and is widely used in the literature. If we demand that supersymmetric extensions

of the standard model should be "complete" in their explanations of this stability, we can place an upper limit on the scale of supersymmetry breaking. This can be translated into an upper limit on the masses of superpartners.

In this paper, we examine the prescription that is currently used to place upper bounds on superpartner masses [3]. First, we wish to determine if these criteria accurately measure fine tuning. Second, we want to make explicit the assumptions implicit in any attempt to quantify naturalness. Upper limits on sparticle masses obtained from naturalness criteria influence expectations of when and where sparticles will be discovered if supersymmetry is responsible for the stability of the weak scale.

In Section 2 we make a critical examination of fine tuning, and we analyze the prescription now used to quantify naturalness. We critique this traditional method by examining a well known hierarchy. We find that this prescription is not completely satisfactory. The trouble is that the traditional prescription does not distinguish between instances of global sensitivity and real instances of fine tuning. We argue that a reliable measure of fine tuning requires global information about the dependence of certain quantities on their arguments, and we show how the existing prescription can be augmented with this information to yield reliable measures of fine tuning.

In Section 3 we systematically construct a family of prescriptions that coincide with the augmented prescriptions formulated in Section 2. Our construction clarifies the proper normalization of naturalness measures and makes explicit the extent of theoretical prejudice present in any such measure.

In Section 4 we apply our prescription to the minimal supersymmetric standard model (MSSM). We briefly discuss the level of fine tuning the MSSM requires in light of current experimental constraints, and we show how the current situation is much less fine tuned than it previously appeared. A more detailed and extensive application of our criteria to supersymmetric extensions of the standard model is in progress [1].

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[3] Heavy superpartner masses can also be bounded, or at least restricted, by the requirement that the relic density of LSPs does not over close the universe. These constraints provide interesting limits, but they do not provide an absolute upper limit on sparticle masses, and they involve model dependent assumptions concerning conserved R-parity.

2. Traditional measures of fine tuning

When parameters conspire by cancelling or adding in an unusually precise fashion, we think of an atypical quantity that results as fine tuned. In such instances, the quantity, for example M_Z, will exhibit a very strong dependence on its arguments [2]. In supersymmetric extensions of the standard model, the weak scale depends on the soft supersymmetry breaking parameters and other couplings through the renormalization group [3]. In a seminal paper [4], Barbieri and Giudice used these features to place upper bounds on superpartner masses, and they popularized a prescription to quantify fine tuning that is now widely used. They looked for sensitivity in the Z mass to variations in the values of supersymmetry breaking parameters and other couplings. They measured the sensitivity on a general parameter a by

$$c(M_Z^2; a) = \left| \frac{a}{M_Z^2} \frac{\partial M_Z^2}{\partial a} \right| . \tag{2.1}$$

Note that rescaling the derivative by a/M_Z^2 removes the dependence on the overall scale of a and M_Z. Barbieri and Giudice argued that, if supersymmetry is responsible for stabilizing the weak scale, then $c(M_Z^2; a)$ must be less than some upper limit Δ, which they took to be 10. They used this criterion to place upper limits on supersymmetry breaking parameters. This program has been subsequently adopted by many researchers.

The application of Eq. (2.1) in obtaining upper bounds on superpartner masses raises several questions. Do we know the normalization of Eq. (2.1) well enough to say that natural solutions should exhibit $c(M_Z^2, a)$'s below 10 or any other particular value? Should we expect that a simple application of this formula will always give a reliable measure of fine tuning, and if not, can we construct alternative definitions that provide better measures of naturalness? We can apply Eq. (2.1) to a famous hierarchy in order to shed some light on these questions.

The lightness of the proton in comparison to either the Planck scale or the grand unified scale is beautifully explained by the logarithmic running of the QCD coupling, α_3. At one loop, the scale dependence of the strong coupling constant can be expressed as

$$\alpha_3(\mu) = \frac{\alpha_3(M_{\rm Pl})}{1 - (b_3/2\pi)\alpha_3(M_{\rm Pl})\ln(M_{\rm Pl}/\mu)}. \quad (2.2)$$

For simplicity we take $M_{\rm prot} = \Lambda$, where $\alpha_3(\Lambda) = 1$. A straight forward application of Eq. (2.1) to the proton mass yields

$$c(M_{\rm prot}; g_s(M_{\rm Pl})) = \frac{4\pi}{b_3}\frac{1}{\alpha_3(M_{\rm Pl})} \gtrsim 100. \quad (2.3)$$

The large value of $c(M_{\rm prot}, g_3(M_{\rm Pl}))$ occurs because the proton mass is a very sensitive function of $g_3(M_{\rm Pl})$. The lightness of the proton is, of course, not the result of a fine tuning. The proton mass would have exhibited this strong sensitivity no matter what its value was, so it makes no sense to say that a value near 1 GeV is fine tuned. This example illustrates our central point. Eq. (2.1) is really a measure of sensitivity, and sensitivity does not automatically translate into fine tuning. For example, the overestimate of fine tuning would have been even worse had we used Eq. (2.1) to study the naturalness of the technicolour scale with respect to variations in the value of the technicolour gauge coupling at the extended technicolour scale [4].

A reliable measure of fine tuning should give a large value when a quantity is fine tuned and at the same time reduce to something close to unity when it encounters typical sensitivity. This suggests that we divide Eq. (2.1) by some measure of average sensitivity. The resulting ratio will still be large for solutions that are unusually sensitive, but in cases where solutions have a "typical" sensitivity the resulting ratio will be of order one. So a more reliable measure of fine tuning would be

$$\gamma(a) = c(X; a)/\bar{c}, \quad (2.4)$$

where $\bar{c}$ is some average value of $c(X; a)$. For example,

$$\bar{c} = \frac{\int c(a)\,da}{\int da}, \quad (2.5)$$

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[4] In these examples there are no cancellations that we can precisely adjust to create a large fine tuning. However, even in instances of real fine tuning, the largeness of $c(X; a)$ can be, *in part*, due to global sensitivity. As we will show in Section 4, $c(M_Z^2; a)$ overestimates the amount of fine tuning needed to maintain a light Z mass in supersymmetric extensions of the standard model.

or

$$1/\bar{c} = \frac{\int c^{-1}(a)\,da}{\int da}. \quad (2.6)$$

If we apply this new criterion to the lightness of the proton, we find that γ is of order one. It is a simple matter to check that the ratio γ gives a large value in legitimate cases of fine tuning. If we apply Eq. (2.4) to the weak scale hierarchy in a non-supersymmetric model, we get a number of order $\Lambda/M_{\rm weak}$, where Λ is the scale of the cutoff. As we show in the following section, a ratio in a form of Eq. (2.4) can be deduced from very general considerations.

3. Measuring fine tuning

In this section we construct a family of quantitative measures of fine tuning that encompass Eq. (2.4), the augmented prescription we motivated in the previous section. Our purpose is twofold. First, we wish to systematically clarify what measures of fine tuning best quantify our intuitive notion of naturalness and how these measures should be normalized. Second, we wish to make explicit the inherent, discretionary assumptions present in any standard that quantifies naturalness. Any measure of fine tuning that quantifies naturalness can be translated into an assumption about how likely a given set of Lagrangian parameters is. In the absence of a theoretical reason compelling us to choose a certain value, we can consider some sensible distribution of the parameter to study what are the natural predictions of the model. The "theoretical license" at one's discretion when making this choice necessarily introduces an element of arbitrariness to the construction.

Before we proceed to "derive" a quantitative measure of fine tuning some comments are in order. We are motivated to quantify naturalness for tangible theoretical reasons. A model that explains a phenomenon has more predictive power than a model that merely accommodates it. In addition, we understand why the proton can be naturally many orders of magnitude lighter than the Planck scale but the stability of a light scale in a theory of fundamental scalars is mysterious. We would like to understand how the weak scale remains light. Of course, at the level of low energy effective theories, dismissing "unnatural" theories in the

quest for a "natural" explanation of weak scale stability could be misguided. We certainly cannot prove that an explanation of the light weak scale was not butchered by the process in which we constructed our effective theory. For example, one-loop corrections to the cosmological constant from an effective theory with soft supersymmetry breaking generate contributions that are many orders of magnitude greater than the experimental limit. Yet we often entertain the idea that the solution to this problem is not associated with our choice of a low energy Lagrangian. While we cannot elevate the prejudice of searching for natural theories above the level of an axiom, we can hope that its application will lead us to a more complete model that explains the stability of the weak scale. Such models will have testable predictions.

In light of this, we proceed to deduce a measure of fine tuning from general principles. Provided we parametrize our assumptions about the likely distribution for Lagrangian parameters, we should be able to derive a quantitative measure of naturalness. Assume the probability that a Lagrangian parameter lies between a and $a + da$ is

$$dP(a) = \frac{f(a)da}{\int f(a)da}. \tag{3.1}$$

Consider a set of these Lagrangian parameters a_i specified at a renormalization scale that is the high energy boundary of our effective theory (e.g., $\mu = M_{GUT}$). A measurable parameter X (e.g., M_Z^2) will depend on the a_i through the renormalization group equations and possibly on a set of minimization conditions. We can recast Eq. (3.1) as a probability per unit X. Given a probability density $f(a)$, the probability density per unit X is

$$dP = \rho(X)dX, \tag{3.2}$$

where

$$\rho(X) \simeq \frac{1}{Xc(X;a)} \frac{af(a)}{\int f(a)da}. \tag{3.3}$$

In studies of naturalness, we may ask: If the fundamental Lagrangian parameters at our high energy boundary condition are distributed like $f(a)$, how likely is a low energy observable, $X(a)$, to be contained in an interval $u(X)dX$ about X? A quantity X is relatively unlikely to be in an interval proportional to $u(X)dX$ if

$$\frac{\langle u\rho \rangle}{u(X)\rho(X)} \gg 1, \tag{3.4}$$

where $\langle u\rho \rangle = \int da\, u(X)\rho(X)/\int da$.

If we are interested in studying the naturalness of a hierarchy like M_{weak}/M_{GUT}, M_{prot}/M_{Planck}, or M_Z^2/M_{SUSY}^2, the interval that corresponds to the conventional sense of naturalness is [5] $u(X) = X$.

If we define our measure of naturalness as

$$\gamma = \langle X\rho \rangle/X\rho, \tag{3.5}$$

fine tuning corresponds to $\gamma \gg 1$. The definition of γ in Eq. (3.5) necessarily implies that γ is linearly proportional to c. For any realization of γ, we define an average value of $c(X;a)$ by

$$\gamma = c/\bar{c}. \tag{3.6}$$

This definition of $\bar{c}$ corresponds to

$$\bar{c}^{-1} = \frac{\int da\, af(a)c(X;a)^{-1}}{\int da\, af(a)}. \tag{3.7}$$

The similarity between this definition of $\bar{c}$ and the heuristic average posed in Section 2 is apparent.

In order to make practical use of the prescription contained in Eqs. (3.4)–(3.7), we need to specify three things. First, our choice of $f(a)$ reflects our theoretical prejudice about what constitutes a natural value of the Lagrangian parameter a. We will return to this point in Section 4. The two remaining choices are determined by the questions we wish to ask. Our choice of $u(X)$ is determined by the quantity whose naturalness we wish to study. The conventional notion of naturalness for hierarchy problems suggests $u(X) = X$. Finally, our choice for the range of integration for a is related to the broadness of the question we wish to ask. This point will be elaborated upon in Section 4.

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[5] Consider the hierarchy problem in an effective theory with a fundamental scalar defined below some scale Λ_1: $m_S^2 = g^2\Lambda_1^2 - \Lambda_2^2$. The scalar mass can only remain light in comparison to the cutoff scale Λ_1 if we cancel the quadratic divergence against the bare term Λ_2^2. Note that the cancellation we need to place the scalar mass in a 1 GeV window at 10^{16} GeV must be made with the same precision as the cancellation we need to place the scalar mass in a 1 GeV window at a 100 GeV. A small value of the scalar mass is unnatural in the sense that a small change in g leads to a large *fractional* change in m_S^2 so that it is relatively unlikely to be found in an interval $\propto m_S^2 dm_S^2$ around m_S^2.

Before analyzing the naturalness of radiative symmetry breaking in the supersymmetric standard model we specialize Eqs. (3.6) and (3.7) to two examples.

Example I. Let us return to the hierarchy between the proton mass and the Planck scale discussed in Section 2. We will calculate γ for two different choices for $f(a)$. Integrating over $g_s(M_{\mathrm{Pl}})$ in the range $g_- < g < g_+$ we find

$$\gamma_1 = \frac{g_+ + g_-}{4g} \frac{g_+^2 + g_-^2}{g^2},\tag{3.8}$$

for $f(g) = 1$ and

$$\gamma_2 = \frac{1}{3} \frac{g_+^2 + g_+ g_- + g_-^2}{g^2},\tag{3.9}$$

for $f(g) = 1/g$. In each case we see that, if the strong coupling constant at the Planck scale is of order one, our measure indicates that a 1 GeV proton mass arises naturally. We have thus eliminated the problematic overestimate of fine tuning contained in the traditional prescription. In the following example we show that the new prescriptions still registers appropriately large values in real instances of fine tuning.

Example II. Consider the gauge hierarchy problem in a non-supersymmetric theory with fundamental scalars. In this case, the one-loop correction to the scalar mass will be of the form

$$m_S^2(g) = g^2 \Lambda_1^2 - \Lambda_2^2,\tag{3.10}$$

where Λ_1 is the ultraviolet cutoff of our effective theory, and Λ_2 is a bare term chosen to keep the scalar mass light. If we calculate the sensitivity of the Higgs mass with respect to the coupling g, we find

$$c(M_S^2; g) = 2 \frac{g^2 \Lambda_1^2}{m_S^2(g)}.\tag{3.11}$$

Integrating over g in the range $g_- < g < g_+$, we find

$$\bar{c}_1^{-1} = \frac{1}{2} \frac{1}{g(g_+ - g_-)}$$
$$\times \left[\frac{1}{2} \left(g_+^2 - g_-^2 \right) - \frac{\Lambda_2^2}{\Lambda_1^2} \ln \left(\frac{g_+}{g_-} \right) \right],\tag{3.12}$$

for $f(g) = 1$ and

$$\bar{c}_2^{-1} = \frac{1}{2} \left(1 - \frac{1}{g_+ g_-} \frac{\Lambda_2^2}{\Lambda_1^2} \right),\tag{3.13}$$

for $f(g) = 1/g$. In each case $\bar{c}$ is of order one, while $c(m_S^2; g)$ is of order Λ^2/m_S^2. This gives $\gamma \simeq \Lambda^2/m_S^2$, which correctly reproduces the fine tuning needed to maintain light scalar masses. From these examples, we again see that the need to renormalize $c(X; a)$ by $\bar{c}$ is important. When X depends very sensitively on a, $c(X; a)$ will be large even if there is no fine tuning. A largely exaggerated value for the traditional fine-tuning measure, which can occur in the absence of fine tuning, can be removed by rescaling by $\bar{c}$.

4. Naturalness and the MSSM

There are two issues concerning naturalness that should be addressed for radiative electroweak symmetry breaking (EWSB) in supersymmetric extensions of the standard model. The first concerns the natural value of the electroweak scale if electroweak symmetry breaks. The second concerns the naturalness of the EWSB process itself. We will make no attempt to tackle the second problem in this paper, since this would require either knowledge of, or additional assumptions about, a more complete theory.

As already noted, supersymmetry must be broken to reconcile the MSSM with the lack of experimental evidence for superparticles. Since no adequate model of spontaneously broken global SUSY exists, supersymmetry is customarily broken through the introduction of explicit soft terms that do not reintroduce quadratic divergences into the theory. Low energy supergravity provides the motivation for the introduction of these soft breaking terms. The most general form of the soft SUSY breaking potential, including gaugino mass terms, is

$$V_{\mathrm{soft}} = m_{\Phi_u}^2 |\Phi_u|^2 + m_{\Phi_d}^2 |\Phi_d|^2 + B\mu(\Phi_u \Phi_d + \mathrm{h.c.})$$
$$+ m_{\tilde{Q}}^2 |\tilde{Q}_i|^2 + m_{\tilde{L}}^2 |\tilde{L}_i|^2 + m_{\tilde{u}}^2 |\tilde{\bar{u}}|^2 + m_{\tilde{d}}^2 |\tilde{\bar{d}}|^2 + m_{\tilde{e}}^2 |\tilde{\bar{e}}|^2$$
$$+ A_u Y_u \tilde{\bar{u}} \Phi_u \tilde{Q} + A_d Y_d \tilde{\bar{d}} \Phi_d \tilde{Q} + A_e Y_e \tilde{\bar{e}} \Phi_d \tilde{L}$$
$$+ \frac{1}{2} M_l \lambda_l \lambda_l + \mathrm{h.c.}\tag{4.1}$$

A generic feature of these SUGRA inspired models is universality in the soft terms. Therefore, one customarily assumes the following boundary conditions for the masses and trilinears at the gauge coupling unification scale,

$$m_i = m_0 \,, \quad A_u = A_d = A_e = A_0 \,. \tag{4.2}$$

Some universality is important in avoiding unwanted flavor changing neutral current effects. Given the unification of gauge couplings, it is natural to take the gaugino masses equal as well,

$$M_1 = M_2 = M_3 = m_{1/2} \,. \tag{4.3}$$

There are therefore five soft breaking parameters, m_0, A_0, $m_{1/2}$, B_0, and μ_0 in the simplest version of the MSSM. For simplicity and definiteness, we will concentrate on this restricted version of the minimal supersymmetric standard model in this paper, however, our naturalness criteria apply equally well to other scenarios.

In the MSSM, electroweak symmetry breaking proceeds through radiative effects [3,5–7]. The one-loop effective Higgs potential may be expressed as follows,

$$V_{\text{one-loop}}(Q) = V_0(Q) + \Delta V_1(Q) \,, \tag{4.4}$$

where V_0 is the tree level potential, and ΔV_1 represents the one-loop correction[6]. Using the renormalization group, the parameters are evolved to low energies where the potential attains validity. This renormalization group improvement uncovers electroweak symmetry breaking. The exact low energy scale at which to minimize is unimportant as long as the one-loop effective potential is used and the scale is in the expected electroweak range. If we arbitrarily take the minimization scale to be M_Z, then the two minimization conditions may be expressed as follows,

$$\mu^2(M_Z) = \frac{\overline{m}^2_{\Phi_d} - \overline{m}^2_{\Phi_u} \tan^2\beta}{\tan^2\beta - 1} - \tfrac{1}{2}M_Z^2 \,, \tag{4.5}$$

$$B(M_Z) = \frac{(\overline{m}^2_{\Phi_u} + \overline{m}^2_{\Phi_d} + 2\mu^2)\sin 2\beta}{2\mu(M_Z)} \,, \tag{4.6}$$

where $\overline{m}^2_{\Phi_{u,d}} = m^2_{\Phi_{u,d}} + \partial\Delta V_1/\partial v^2_{u,d}$ and $\tan\beta$ is the ratio of the vacuum expectation values of the Higgs fields, v_u/v_d. Demanding correct electroweak symmetry breaking puts constraints on the parameters of the MSSM. For example, the top quark Yukawa coupling is one parameter that has to be large enough in order to achieve the desired radiative breaking. Rewriting

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[6] The effect of including one-loop corrections to the effective potential on the numerical value of the Barbieri–Giudice parameter $c(M_Z^2, y_t)$ was studied in Ref. [8].

Eq. (4.5) yields an equation for M_Z as a function of the parameters of the MSSM,

$$\tfrac{1}{2}M_Z^2 = \frac{\overline{m}^2_{\Phi_d} - \overline{m}^2_{\Phi_u}\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \,. \tag{4.7}$$

In the MSSM, the problem of fine tuning has been commonly treated using the prescription of Ref. [4], although the original bound of $\Delta = 10$ has often been increased to as high as $\Delta = 100$. However, as already discussed, it is difficult to ascertain what constitutes a reasonable bound in the absence of some comparative norm (normalization). A glaring example of this can be found in $c(M_Z^2; a = g_3)$. When applying the criterion of Ref. [4], one typically takes the a-parameter to be a soft breaking mass, such as m_0, $m_{1/2}$, μ_0, etc., or the top Yukawa. However, the strong coupling is also a parameter of the theory, and one can consider $c(M_Z^2; g_3)$. We find that over all the parameter space of the MSSM that we have so far explored, $c(M_Z^2; a)$ is the largest for $a = g_3$. Since all the parameters are ostensibly on equal footing, imposing $c(M_Z^2; g_3) < \Delta = 10$–$100$ may be overly restrictive.

We now apply the realization of γ given in Eqs. (3.5)–(3.7) to the MSSM. To use this prescription we must specify the range of the parameter a. We could simply choose this range by *fiat* (e.g., $0 < m_{1/2} < 10$ TeV), but this seems rather ad hoc. Instead we prefer that the choice of range be dictated by electroweak symmetry breaking. Other choices are possible. We will ask how natural the value $M_Z = 91.2$ GeV is, given that the gauge symmetry breaking occurs correctly. For this choice, the range of a should correspond to values for which $SU(3) \times SU(2) \times U(1)$ is broken to $SU(3) \times U(1)_{\text{em}}$. There are then finite limits to the range of a that come from two conditions on the value of M_Z. The minimum value of M_Z cannot be less than 0, and its maximum value cannot exceed some upper bound, often set by the requirement that sneutrino squared masses be positive.

We display $\gamma(a)$'s computed for two different choices of $f(a)$. γ_1 corresponding to the choice $f(a) = 1$, and γ_2 corresponding to $f(a) = 1/a$. If we adopt 't Hooft's notion of naturalness that Lagrangian parameters should not be small unless setting them to zero increases a symmetry, and we believe that the magnitude of supersymmetry breaking terms should
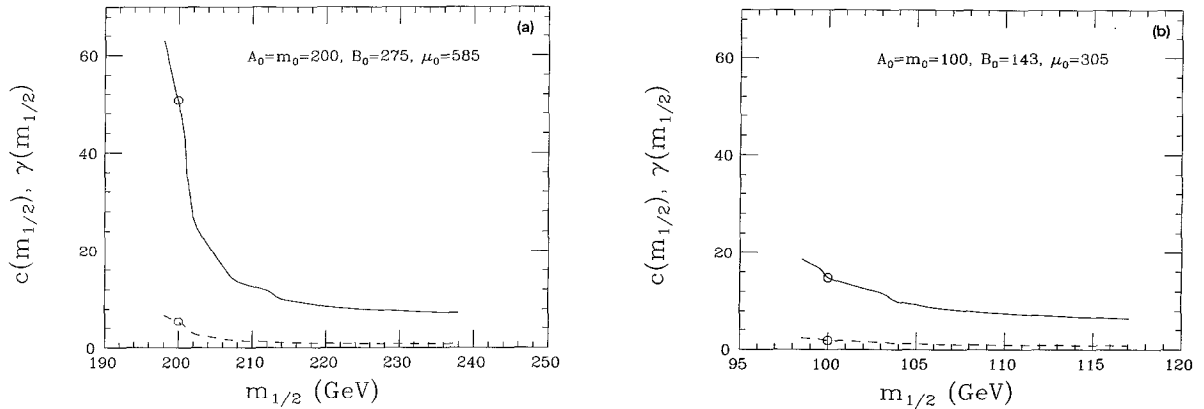
Fig. 1. (a) The fine-tuning parameters $c(m_{1/2})$ (solid) and $\gamma(m_{1/2})$ (dashed) plotted as a function of $m_{1/2}$ for $A_0 = m_0 = 200$ GeV, $B_0 = 275$ GeV and $\mu_0 = 585$ GeV. The circles indicate the point with the correct value of M_Z for this choice of A_0, B_0, m_0, and μ_0. (b) Same as (a) with $A_0 = m_0 = 100$ GeV, $B_0 = 143$ GeV and $\mu_0 = 305$ GeV.
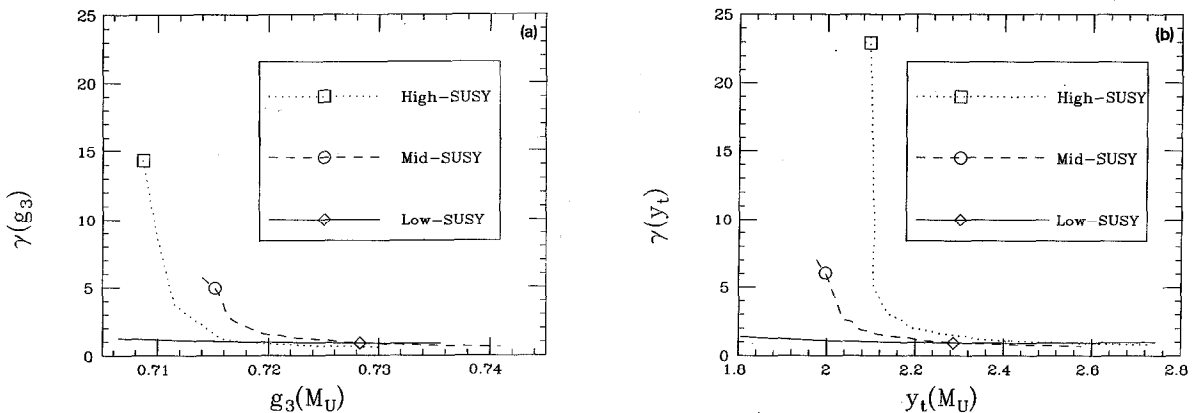

Fig. 2. (a) The fine-tuning parameter $\gamma(g_3)$ plotted as a function of $g_3(M_U)$ for three cases with increasing scale of supersymmetry. The circle, square, and diamond indicate the points with the correct value of M_Z for the three cases. (b) Same as (a) but displays $\gamma(y_t)$ as a function of $y_t(M_U)$ for the same three cases.

be universal, we should choose $f(a) = 1$. However, we also consider $f(a) = 1/a$ to study the sensitivity of our criteria to the choice of $f(a)$ and to allow for non-universality in the magnitude of soft supersymmetry breaking terms (see for example Ref. [9]). Figs. 1–3 show that, in the MSSM, the γ's are very insensitive to which choice of $f(a)$ is made.

In Figs. 1a and 1b, we plot $\gamma(m_{1/2})$ versus $m_{1/2}$ for two choices of the soft supersymmetry breaking parameters A_0, B_0, m_0, and μ_0. On this scale, γ_1 and γ_2 are virtually indistinguishable so we only show γ_1. On the same plots we show, for comparison, the tra-

ditional prescription $c(m_{1/2})$ as well. The range of $m_{1/2}$ corresponds to the values of the common gaugino mass for which $SU(3) \times SU(2) \times U(1)$ is broken to $SU(3) \times U(1)_{em}$. Note that the asymptotic, "natural" value of $c(m_{1/2})$ for large $m_{1/2}$ is order ten and not order one. This is another demonstration why it is necessary to rescale the c's to achieve a sensible measure of fine tuning.

Figs. 2a and 2b show the effect of increasing the overall scale of soft symmetry breaking on fine tuning. In Fig. 2a the fine-tuning parameter $\gamma(g_3)$ is plotted as a function of $g_3(M_U)$, where M_U is the unification
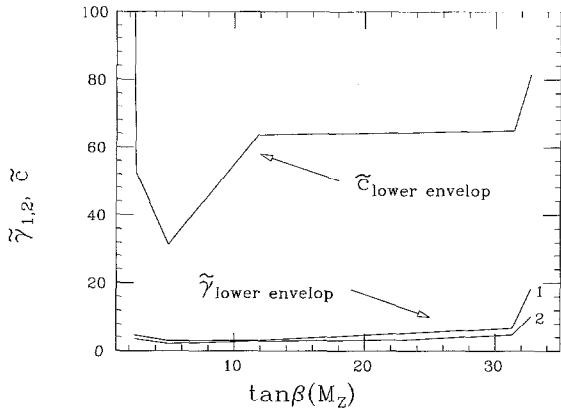
Fig. 3. Curves representing lower envelope of regions defined by plot of $\max\{c(m_{1/2}), c(m_0), c(y_t), c(g_3)\}$ and $\max\{\gamma_{1,2}(m_{1/2}), \gamma_{1,2}(m_0), \gamma_{1,2}(y_t), \gamma_{1,2}(g_3)\}$ versus $\tan\beta(M_Z)$.

scale. We include three choices of A_0, B_0, m_0, and μ_0 with different overall scales of soft symmetry breaking. The square, circle, and diamond in each figure correspond to the point with the correct value of the Z-boson mass for the cases (i) $A_0 = m_0 = m_{1/2} = 400$ GeV, $B_0 = 523$ GeV, $\mu_0 = 1125$ GeV, (ii) $A_0 = m_0 = m_{1/2} = 200$ GeV, $B_0 = 275$ GeV, $\mu_0 = 585$ GeV, and (iii) $A_0 = m_0 = m_{1/2} = 50$ GeV, $B_0 = 90$ GeV, $\mu_0 = 154$ GeV, respectively. The light case has a chargino with a mass less than $M_Z/2$ and therefore is excluded experimentally. Fig. 2b is similar to Fig. 2a but displays $\gamma(y_t)$ versus $y_t(M_U)$.

Fig. 3 displays how much fine tuning the MSSM currently requires in light of some general experimental constraints. We consider a region of our input parameter space defined by the ranges $|A_0| \leq 400$ GeV, $m_0 \leq 400$ GeV, and $|m_{1/2}| \leq 400$ GeV. For values of the soft supersymmetry breaking parameters consistent with a neutral lightest SUSY particle (LSP), with the current LEP measurement of the Z width, with a Higgs mass heavier that 60 GeV, and with chargino masses heavier than $M_Z/2$, we plot

$$\bar{c} = \max\{c(m_{1/2}), c(m_0), c(y_t), c(g_3)\},$$

$$\bar{\gamma}_1 = \max\{\gamma_1(m_{1/2}), \gamma_1(m_0), \gamma_1(y_t), \gamma_1(g_3)\},$$

$$\bar{\gamma}_2 = \max\{\gamma_2(m_{1/2}), \gamma_2(m_0), \gamma_2(y_t), \gamma_2(g_3)\}$$

versus $\tan\beta(M_Z)$. In the figure, we display curves representing the lower envelopes of the resulting regions. Notice that the original Barbieri and Giudice

Table 1

a	$c(a)$	$\bar{c}_1$	$\bar{c}_2$	γ_1	γ_2
$m_{1/2}$	50.8	9.29	10.3	5.47	4.92
m_0	21.8	3.21	4.66	6.79	4.68
g_3	209.	42.3	43.2	4.94	4.84
y_t	32.5	4.92	5.77	6.61	5.63

bound of $c(a) < 10$ has already been exceeded, while the new criteria show that weak scale stability can still arise naturally.

Finally, in Table 1 we display the BG sensitivity parameters $c(a)$ and the fine tuning parameters $\gamma(a)$ for various a in a representative case with $A_0 = m_0 = m_{1/2} = 200$ GeV, $B_0 = 275$ GeV, and $\mu_0 = 585$ GeV. Note that the relative normalization of the sensitivity parameters, $\bar{c}(a)$, can be quite different. This means that we can not adopt a universal measure of fine tuning by appealing only to the $c(a)$'s (e.g., $c < 100$). A relative normalization for each $c(a)$ must be computed in the manner described in Section 3.

5. Conclusions

Naturalness criteria are frequently used to place upper bounds on superpartner masses in supersymmetric extensions standard model. We have analyzed the prescription popularly used to measure fine tuning. This prescription is an operational implementation of Susskind's statement of Wilson's sense of naturalness, "Observable properties of a system should be stable against minute variations of the fundamental parameters." Because this prescription is only a measure of sensitivity, we found that it is not a reliable measure of naturalness. We then constructed a family of prescriptions which measure fine tuning more reliably. Our measure is an operational implementation of a modified version of Wilson's naturalness criterion: Observable properties of a system should not be unusually unstable against minute variations of the fundamental parameters. Our derivation determines the normalization of naturalness measures and makes clear to what extent theoretical prejudice influences these measures. The new prescriptions we construct allow upper bounds on superpartner masses to be placed with greater confidence. By applying our prescriptions to the minimal supersymmetric standard model, we find

that the theory provides a much more natural explanation of weak scale stability than previous methods indicate. More importantly, we find that the scale of supersymmetry breaking can be significantly higher than previous naturalness criteria indicate.

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