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Winter 2016

Transformations

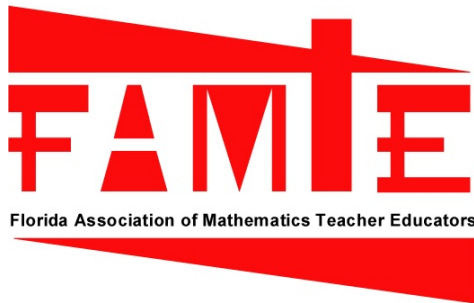
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TRANSFORMATIONS

Table of Contents

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| | |
|---|----|
| President’s Message Hui Fang Huang "Angie" Su, Ed.D., FAMTE President | 3 |
| The Problem-Solving Process in a Mathematics Classroom Enrique Ortiz | 4 |
| Function Concept: Learning from History Ruthmae Sears, Dung Tran, Seoung Woo Lee, Amanda Thomas | 14 |
| Mathematics Anxiety in Society: A Real Phenomena and a Real Solution Joseph Furner, Carol A. Marinas | 24 |
| Integrating Diversity Training into Doctoral Programs in Mathematics Education Ruthmae Sears, Tonisha Lane | 37 |
| When Two Wrongs Made A Right: A Classroom Scenario of Critical Thinking as Problem Solving Hui Fang Huang Angie Su, Frederi A. Ricci, Mamikon Mnatsakanian <i>(Reprinted from Winter 2016, Dimensions in Mathematics Journal)</i> | 46 |
| Mathematics Teacher Educator of the Year Award Graduate Student of the Year Award | 54 |
| FAMTE Membership Application | 56 |

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Winter 2016

Transformations:

A Journal of the Florida Association of Mathematics Teacher Educators

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From the President's Desk . . .

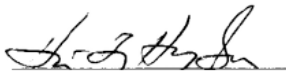
Dear Mathematics Educators:

I am excited that we are launching our inaugural edition of the Transformations Journal - A Publication of Florida Association of Mathematics Teacher Educators. This journal will be made available both in print and online. There will be two issues per year. I encourage you to submit your research articles so that we can share with the mathematics educators around the country. I also invite you to nominate a colleague or self nominate to serve on our Board so that we can help make a difference in the K-22 mathematics education community, both in the state of Florida and in the USA.

As an affiliate of the Florida Council of Teachers of Mathematics (FCTM), I am looking forward to achieve the following goals over the next two years:

1. Create a mini FAMTE conference at FCTM's Annual Conference in order to promote the improvement of Florida's mathematics instructional programs and to promote cooperation and communication among the teachers of mathematics and mathematics teacher educators in Florida.
2. FAMTE Board is represented by a K-12 Mathematics Teacher.
3. FCTM promotes FAMTE activities

With Warm Regards,



*Hui Fang Huang "Angie" Su,
FAMTE President & Editor of
Transformations*

The Problem-Solving Process in a Mathematics Classroom

Enrique Ortiz

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Problem solving provides a working framework to apply mathematics, and well chosen mathematics problems provide students with the opportunity to solidify and extend what they know, and can stimulate students' mathematics learning (NCTM, 2001). Using this framework, students may utilize ways to learn mathematics concepts and skills that are rich with meaning and connections, and pre- and in-service teachers may implement teaching and assessment procedures to establish teaching and assessment environments.

The Four-step Process

Polya's (1957) four-step process has provided a model for the teaching and assessing problem solving in mathematics classrooms: understanding the problem, devising a plan, carrying out the plan, and looking back. Other educators have adapted these steps, but the essence of these adaptations is very similar to what Polya initially developed. The following are two possible variations of Polya's model I have come across: (1) define the problem, develop a plan, implement the plan, and evaluate; or (2) plan, do, act, and check. The implementation of these steps in the classroom is not easy, and, in some cases, could be misused or misleading. In this article, after a discussion of the four-step model and possible connections to the *Common Core State Standards – Standards for Mathematical Practice (CCSS-SMP)* (NGACBP & CCSSO, 2010), I present three possible challenges in implementing this model in K-12 classroom as well as pre- and in-service teachers.

Understanding the problem. At this point, the student should try to understand the problem. This is the step where you want students to engage with the problem or task and want

to actually solve it. These are some possible questions the student could ask (Polya, 1957): Do I know what is the unknown? Do I know what are the data? Do I know what is the condition or conditions involved in the problem? In order to determine the unknown, is the condition sufficient, not sufficient, or redundant? Can I draw a picture to help you understand the problem? Can I introduce suitable notation? Can I separate and write down the various parts of the condition?

On the other hand, pre-service or in-service teacher may ask similar questions to facilitate or assess the understanding of the problem solving process: How has the student demonstrated that she knows what the unknown is? Is the demonstration correct and sufficient? Does she know what are the data and has used the data properly to understand the problem? Can she provide a description of or paraphrase the condition or conditions involved in the problem? In order to determine the unknown, can she identify the condition or conditions as sufficient, not sufficient, or redundant? Can she analyze givens, constraints, relationships, and goals (NGACBP & CCSSO, 2010)? Has the student drawn a picture to help her understand the problem? Has the student considered any suitable notation needed to understand the problem? Has the student separated and written down the various parts of the condition? Are there any specific hints I can provide to help the student understand the problem without giving away the possible plan or answer? Are there any other probing questions I could ask her to develop understanding of the problem and move to the next stage of the process? Are there any resources, materials, information I need to make accessible to the student? Are there any misconceptions or weaknesses related to the content of the problem (including social and experiential background)? Can she start by explaining to herself the meaning of a problem and looking for entry points to its solution and how can I facilitate this process (NGACBP & CCSSO, 2010)? Has she

considered analogous problems, and try special cases and simpler forms of the problem in order to gain insight into its solution (NGACBP & CCSSO, 2010)? NGACBP and CCSSO also indicated that,

Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches (par. 2).

Devising a plan. At this point, the students should try to find connections between the data and the unknown, consider auxiliary problems in an immediate connection cannot be found, and should eventually obtain a plan of the solution (Polya, 1957). Possible questions that the student could ask (Polya): Have I seen the same, similar, or related problem before? Do I know a theorem that I can use? Can I use the results, methods, or strategies of a similar problem? Could I restate the problem? Do I know the vocabulary involved in the problem? Could I solve a more accessible, general, analogous, or special problem? Could I simplify the problem? Could I derive something useful from the data, or think of other data appropriate to determine the unknown? Could I change the unknown or data, or both if necessary? Did I use and taken into account all

the data, essential notions or whole condition? Is the student using improper shortcuts, information or steps to solve the problem?

As before, a pre-service or in-service teacher may ask similar questions to facilitate or assess the student's efforts in devising a plan to solve the problem: How can I help her to see connections with a similar, or related problem? Is there a need to review definition of terms or theorems? Should I remind students of useful results, methods, or strategies of a similar and simpler problem? Can she restate the problem in her own words? Should she solve a more accessible, general, analogous, or special problem? Should I provide some guidance to help her derive something useful from the data, or think of other data appropriate to determine the unknown? How can help the student use and take into account all the data, essential notions or whole condition? Should the student go back to the previous stage of problem solving and try to have a better understanding of the problem? Can she make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt (NGACBP & CCSSO, 2010)?

Carrying out the plan. The student then carries out the plan developed in the previous step. The students should check each step of the solution plan. Possible questions are the following (Polya, 1957): Can I see clearly that the step is correct? Can I prove that it is correct?

Similarly, the pre- and in-service teachers may ask the following questions: What questions could I ask about carrying out the plan and make sure she sees clearly that the step or proof is correct? Should the student go back to the previous steps of the process and check understanding of the problem or the feasibility of the plan? As indicated in the *CCSS-SMP* (NGACBP & CCSSO, 2010), can she make sense of the problem and persevere in solving it?

Looking back. The students should examine the solution obtained. Possible questions are the following (Polya, 1957): Can I check the result, or argument? Can I derive the result differently? Can I see it at a glance? Can I use the result, strategy or method for another problem? Can she understand the approaches of others to solving problems and identify correspondences between different approaches (NGACBP & CCSSO, 2010)?

The pre- and in-service teachers may ask the following questions: Has she checked the result, or argument in a convincing and appropriate manner? Can she derive the result or present another argument? How can she use the result, strategy or method for another problem? How can I make the problem more realistic or general? Can she see the value of solving the problem in different manners?

Furthermore, Polya's model is very useful in the problem solving process as students solve mathematics problems, but it is also very useful in the teaching process. For example, as a teacher assess a students' problem solving solution process, she might notice that the students is working at devising a plan stage of the process. In this case, the teacher could facilitate the process by providing help at that stage, but should not get involved with the carrying out the plan stage, which is next.

Possible challenges in implementing this model

First, we need to keep in mind that learning this four problem-solving steps might not be sufficient to become a better problem solver or mathematician. This approach is mainly a working framework for problem solving. In my experience, an effective problem solver has the ability to work on a problem with flexibility, and following a linear four-step approach might not work all the time. For example, in some cases, you might start to solve a problem without a complete understanding of the problem and this should not stop you from trying to find a

solution. In my opinion, only textbook word problems are found already neatly set up for you. Most real life problems are presented as situations in very messy and random manners. In this type of setting, you might try to solve the “problem” and as you try to solve the “problem” you start understanding the problem. The “problem” evolves and changes as you try to solve it. Slowly, you start realizing what the problem is as you try to understand and solve it. For example, you want to go to college, how do you make it happen? You want to buy or rent an apartment, how do you solve this problem?

Second, sometimes the four-step problem solving process is more useful when you start to organize your arguments. Intuitively, you might feel you know the solution to a problem, but you still need to convince others that you have the correct method or answer. Polya’s (1957) steps could be used to make sure you can present an acceptable and convincing argument. For example, a lawyer might believe the innocence of the client intuitively, but he/she still needs to prove the innocence of the client to the judge or jury. This will involve a proper understanding of the case and laws involved in the case, the development of a proper defense plan, carrying out the defense by presenting arguments and evidence in favor of the client, and looking back to see if more evidence is needed, which in some cases might be arguing for a mistrial, or new trial. Another, example is when a medical doctor believes he/she knows the nature of an illness afflict a patient, but he/she still needs to run some labs to make sure the diagnosis is correct.

Third, the four-step approach could become too methodical or too linear, and might prevent students from being more creative, and to think “out of the box.” As stated in *CCSS-SMP* (NGACBP & CCSSO, 2010), it important that students “monitor and evaluate their progress and change course if necessary” (par. 2). This lack of flexibility could be avoided by

visualizing the problem solving steps as parts of a puzzle (see figure 1). In this view, each part is complementing each other like in a puzzle.

Figure 2 presents another possible diagram of the four areas of the problem-solving process (Polya, 1957) with suggested strategies for each phase. Students could use it to facilitate their problem solving process. Teachers could use it to check the presence of the strategies as students work on problems, provide appropriate hints and guidance, and assess their understanding and progress. The areas are interrelated and should not be used in a linear fashion. Instead, as the arrows illustrate, you could move back and forth from one area to another as needed to solve a problem.

Fourth, teaching of specific problem-solving strategies in isolation could hinder the development of flexibility and the problem-solving process could become an exercise instead. I try to avoid this approach. Instead of teaching specific strategies, I try to help students work with the problem-solving process and the connection between problems. As indicated by Schoenfeld (1985), it is possible to teach learners to use general strategies such as those suggested by Polya, but that is insufficient. It might take several revolutions through the four-step process before finding a solution to a problem.

Concluding Remarks

A true problem solving process will allow students to be flexible, intuitive, and creative. The students should be allowed to move from one step to another, and through many alternatives and strategies. The teachers will also need to be flexible in their assessment of the students and provide many opportunities for discovery and exploration. Finding great problem-solving situation is a challenge, but it is crucial in we want to be effective.

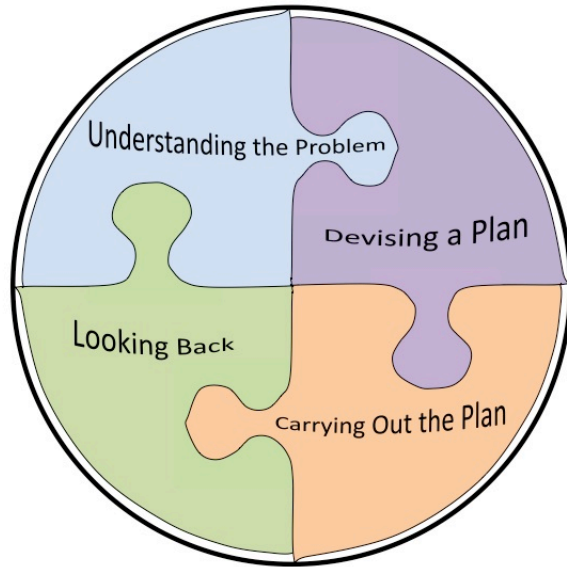


Figure 1. Problem-solving process viewed as a puzzle

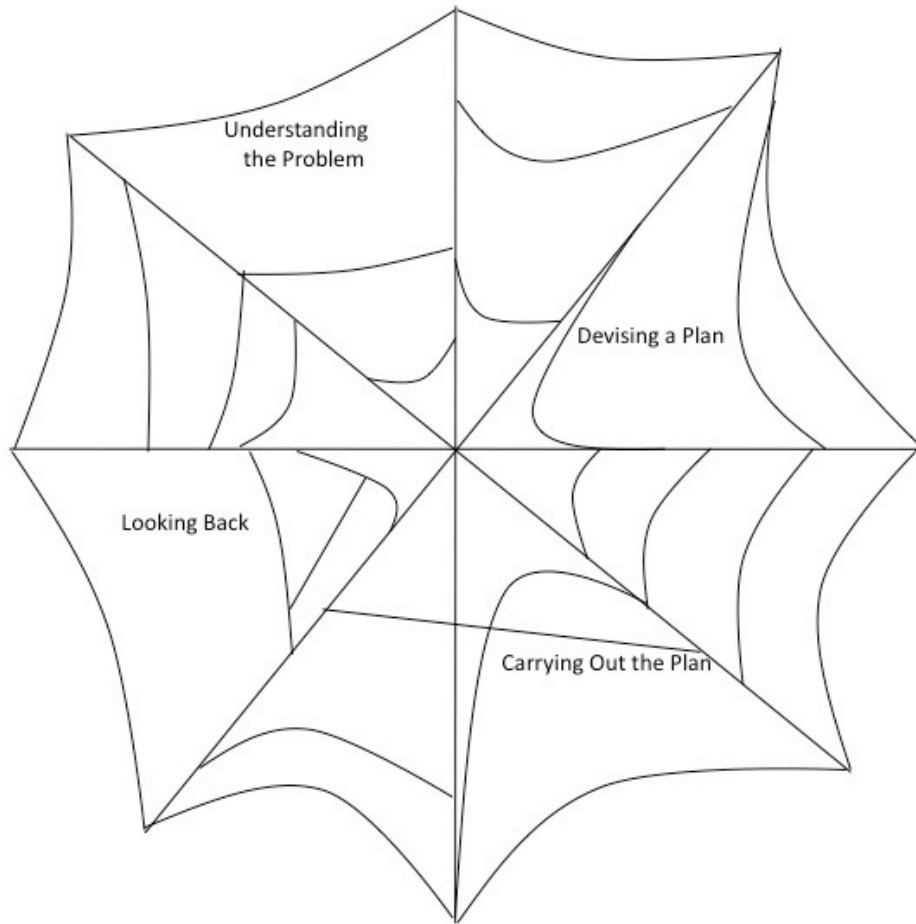


Figure 2. Problem-solving process viewed as a web

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Function Concept: Learning from History

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The importance of functions in school mathematics has grown tremendously within the past century. Functions have progressed from being scantily represented in school mathematics to being a core mathematical topic. C.B. Boyer (1946) acknowledged “The development of the function concept has revolutionized mathematics in much the same way as did the nearly simultaneous rise of non-Euclidean geometry. It has transformed mathematics from a pure natural science- the queen of the sciences- into something vastly large. It has established mathematics as the basis of all rigorous thinking – the logic of all possible relations” (Markovits, Eylor, & Bruckheimer, 1986, p. 18).

Historical speeches and documents, such as Klein’s 1893 Evanston Colloquium, Moore’s 1902 presidential address to the American Mathematical Society, The Reorganization of Mathematics in Secondary Education Report (1923), and The Report of Progressive Education and Joint Committee (1940), advocated that functions and “relational thinking” be a core concept in school mathematics. In fact, Felix Klein considered functions to be the “soul of mathematics”, and advocated that teachers teach functional concepts.

Fortunately, the recommendations made decades ago pertaining to the importance of functions, and the needs to readily integrate the function concept into school mathematics by researchers were not ignored. The recommendations made regarding functions decades ago are evident in today’s curriculum standards. Standards for mathematics require students to be able to define functions, describe functions, identify functions, analyze functions, and recognize patterns in function (NCTM, 2000; Common Core State Standards 2010). Most notably, The Common Core State Standards (2010) has functions as one of five conceptual categories in high school mathematics.

Considering the increased emphasis placed on functions in school mathematics within the past century, we sought to describe how the function concept was presented in secondary mathematics textbooks prior to the “New Math” era.

HISTORICAL OVERVIEW

The function concept has evolved significantly over time: from implicit tools in mathematics and science with no word and no definition to a conceptual category in the Common Core State Standards. Up to the Middle Ages, the concept of function did not

appear with a definition, although, the notion of functional relation existed. During ancient times, tables of squares, tables of square roots, tables of cubics, and table of cubic roots were representations of functions that were visible; as well as geometric figures (Kleiner, 1989; Ponte, 1992). A particular instance of functions in ancient time was the counting of objects; the counting of objects suggests a correspondence between a set of given objects and a sequence of counting numbers (Ponte, 1992).

Felix Klein, a German mathematician, was an advocate for the concept of function being included in school mathematics. Klein accentuated the view that “‘functional thinking’ should be made the binding or unifying principle of school mathematics” (Hamley, 1934a, p.169).

The importance of the function concept was emphasized holistically for secondary mathematics curriculum, in historical mathematics education reports (such as National Committee on Mathematical Requirements 1923 report, and Progressive Education Association 1940 report). Hamley (1934b) noted that the world is becoming “functionally minded”, since functions was increasingly visible in economics, politics, industry and commerce.

Considering the petitions to include functions in school mathematics at the dawn of the twentieth century, our study explored the inclusion of functions in secondary mathematics textbooks during the period 1908-1950, because textbooks were primary resource materials for the teaching of mathematics.

DESIGN OF THE STUDY

We conducted a descriptive study of function concept in school mathematics textbooks. We examined how functions were presented in secondary mathematics textbooks during 1908-1950 in the United States.

We perused the University of Missouri Mathematics Education historical textbooks collection to identify the year functions first appeared. Our initial exploration revealed that the first time the word “functions” was cited, as an independent topic was 1908, hence it was agreed upon that 1908 would be the beginning of the period studied. We restricted the time period to 1950 because it was the last decade before the dawn of the “New Math”, during which more rigid function concept were integrated into school mathematics.

After establishing a time period, we utilized the following criteria to select textbooks: The textbooks must be suitable for high school general mathematics, practical mathematics, or algebra courses; and explicitly reference the word “function” or “functionality”. We collected data from the University of Missouri Mathematics Education historical textbooks collection and from the MOBIUS database. We did not include reprinted books, but kept books that were reprinted with corrections since we were uncertain where the corrections were made. Notwithstanding that 61 of the textbooks examined had a representation of the concept of function, only 36 textbooks provided an explicit definition. Hence for this report, our analysis focuses on 36 of the textbooks in our sample.

Table 1 The number of books in our sample (N =61) that defined functions.

| Year of Publication | Explicit Definition of Function | No Explicit Definition of Function |
|---------------------|---------------------------------|------------------------------------|
| 1908-1920 | 12 | 13 |
| 1921-1930 | 6 | 3 |
| 1931-1940 | 8 | 6 |
| 1941-1950 | 10 | 3 |
| Total | 36 | 25 |

We coded data from each textbook for definitions, representations, and importance of functions. We considered language used to define functions (such as vary, correspond, relate, etc). Additionally, having read the definitions of functions we grouped them into three groups based on leading terms (variables, expressions, and numbers/quantities). Subsequently we analyzed our results descriptively.

RESULTS

We present our results under three major themes: definitions of functions, representations of functions, and the importance of functions.

Definitions

There existed differences in the definitions for the verbs used by textbooks to describe the functional relationships between a dependent and independent variable. The verbs used included depends, relates, corresponds, varies, connects, etc. Additionally, we found that 44% of the definitions described functions in terms of variables, 31% described functions in terms of an expression, and the remaining 25% of the definitions described functions as a quantity/number.

Surprisingly, some definitions in secondary school mathematics textbooks presented the possibility of multiple independent variables, while others provided multivalued functions. Two unique definitions were Brooke and Wilcox (1938) (classified in variables), and Ferrar (1948) (classified in expressions). Brooke and Wilcox considered multivalued functions, and Ferrar (1948) defined both implicit and explicit functions. The respective definitions are as follows:

(I) The variable y is said to be a single-valued function of the variable x for a prescribed range of values which the variable x may assume, when a definite value of y corresponds to each value of x , no matter in what manner the correspondence is specified. (II) The variable y is said to be a multiple-valued function of the variable x for a prescribed range of values, which the variable x may assume, when a definite set of values of y corresponds to each value of x no matter in what manner the correspondence is specified (Brooke & Wilcox, 1938, p. 220).

A function like $y = e^x \sin x$ is sometimes called an EXPLICIT FUNCTION, since the definition of y in terms of x is given directly and explicitly. A function y defined in terms of x by means of an equation that fails to give y

directly in terms of x , such as $1 + xe^y = y$, is sometimes called an implicit function, since the equation implies that y depends on x but does not directly and explicitly define y in terms of x (Ferrar, 1948, p. 51).

Table 2 depicts the percentages of the number of independent variables per dependent variables.

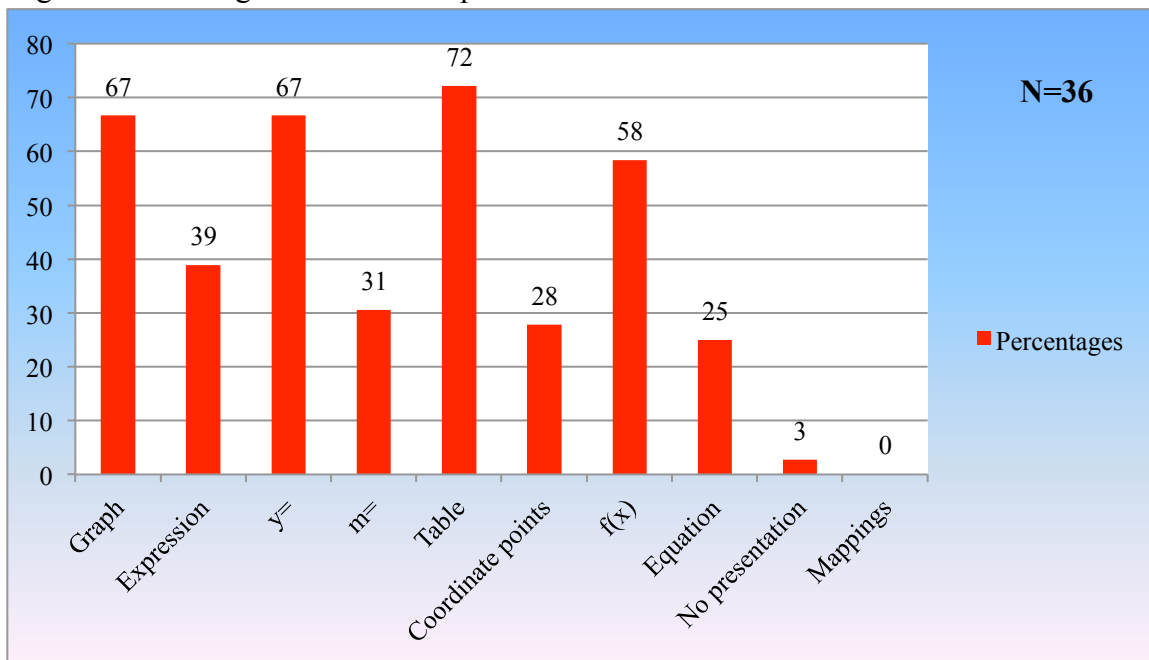
Table 2. The percentages of the number of independent variables in the definitions of functions

| Number of Independent Variable(s) | Percentages (N =36) |
|--|---------------------|
| One variable function only | 83% |
| One variable or multi variables function | 11% |
| Undetermined | 6% |
| Total | 100% |

Representation of Functions

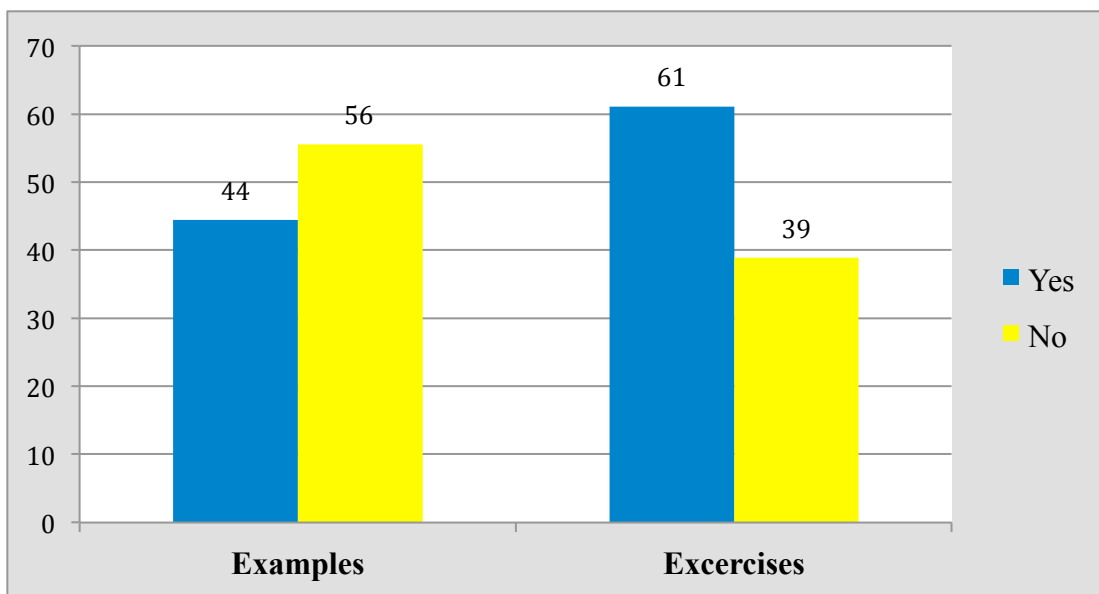
Many textbooks had multiple representations of functions. As depicted in Figure 1, the results highlight that tables were the most popular means to represent functions in textbooks (72%); while only 25% used an equation. None of the mathematics textbooks used mapping to represent functions. Furthermore, only 3% of the textbooks had no representation of functions. Figure 1 depicts the percentage of textbooks that incorporated the various representations.

Figure 1 Percentage of functions representations visible in textbooks.



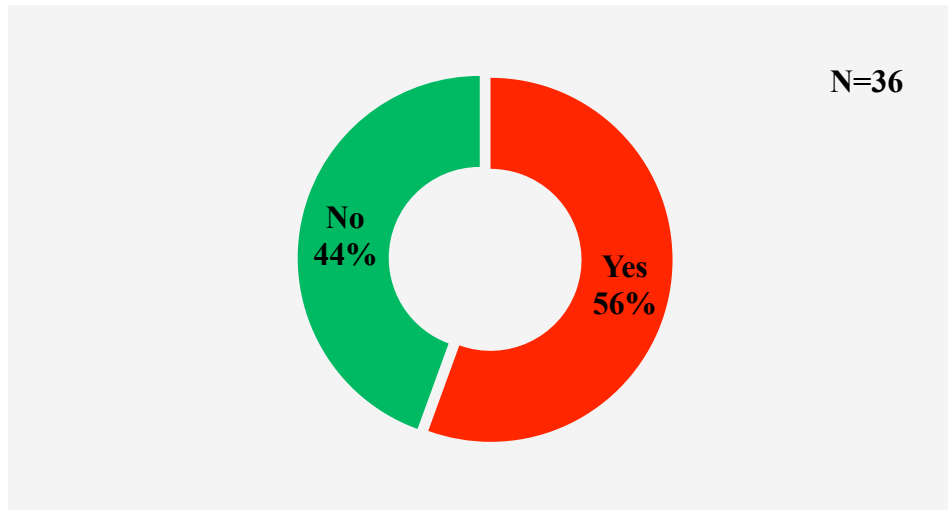
Furthermore, secondary mathematics textbooks during the period 1908-1950 represented the concept of function using real world context more readily in the exercises than in the examples. Figure 2 illustrates that 61% of the textbooks had a real world context in the exercises, while only 44% had a real world context in the examples.

Figure 2 Percentage of functions presented in real world situations in the example and exercises.



Additionally, some books unequivocally stated how to read a functional notation. For example, Crenshaw, Simpson, and Pirenian (1932) wrote, “The symbol does not mean f times x ; it is an abbreviation of the phrase “a function of the variable x ” and is read “ f of x .”” (p.123). How to interpret functional notation were presented in slightly more than half (56%) of the selected sample. Figure 3 illustrates the percentage of textbooks that describes how to read functional notations.

Figure 3 Percentage of textbooks that provided explicit guidance to read functions.



Importance of Functions in Secondary Mathematics Textbooks

During the period 1908-1950, textbooks were not likely to present functions as an independent topic in the Table of Contents. Table 3 reveals that only 17% of our sample had functions as an independent topic (Functions/ Functionality) in the Table of Contents.

Moreover, functions were likely to appear in the first half of the textbooks (Figure 4), and five pages or less were generally allocated explicitly to discuss the functional concept (Figure 5).

Table 3. Chapters in Table of Contents in which functions appeared

| Titles of Chapters in Table of Contents | Number of Books (N=36) |
|--|------------------------|
| Functions and graphs, functions and variables, functions and something | 11 |
| Functions/functional relations | 6 |
| Quadratic equations | 4 |
| Equations | 2 |
| Graphs | 2 |
| Graphic solution | 2 |
| Graph of linear equations | 2 |
| Graphical representation of function and equations | 1 |
| Linear function | 1 |
| Ratio, dependence, proportion | 1 |
| Algebraic expression | 1 |
| Variables and limits | 1 |
| Difference equations and generating functions | 1 |
| Simultaneous equations, graph | 1 |

Figure 4 Percentage of appearance of functions in quartiles of secondary mathematics textbooks.

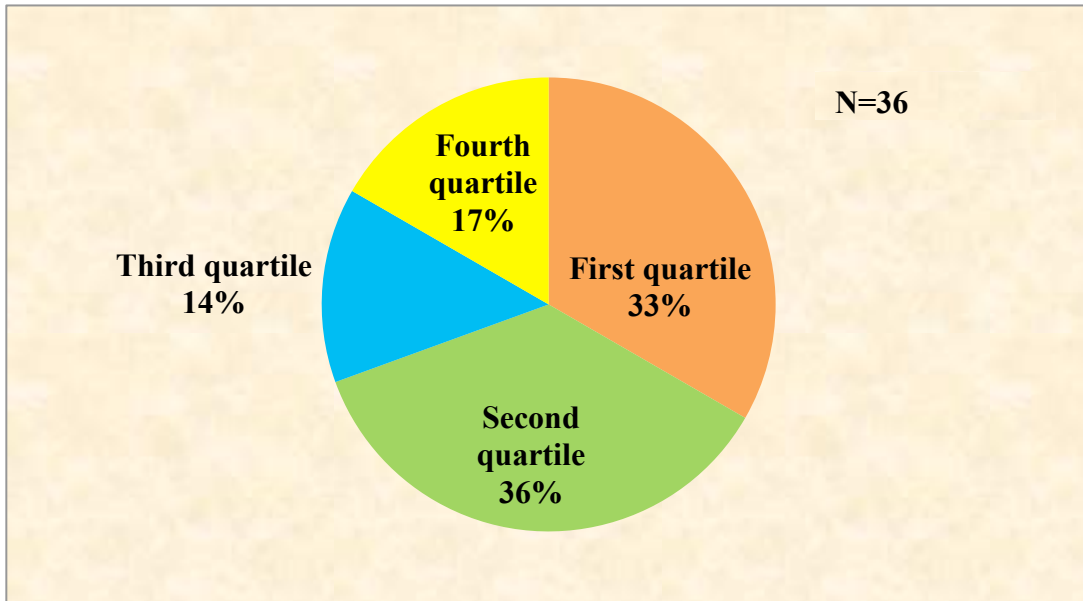
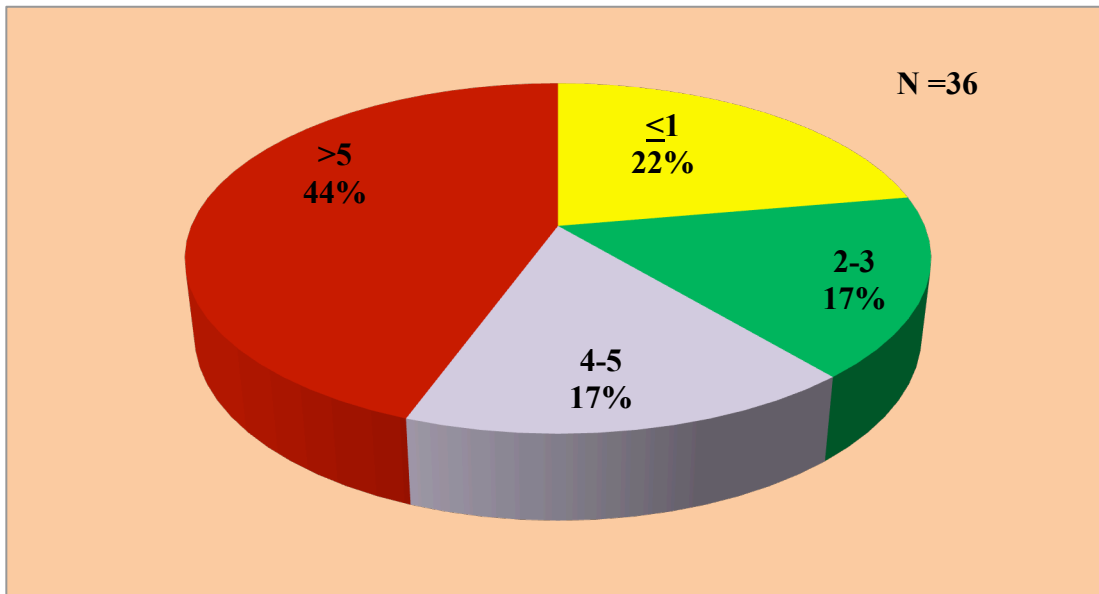


Figure 5 Percentage of number of pages allocated for the concept of function.



SUMMARY

Overall, there exist various definitions and representations of functions in school mathematics textbooks during the period 1908-1950. Furthermore, although functions

were placed in the first half of most textbooks, the concept of function was primarily allotted five or less pages.

The definitions of functions in historical textbooks embodied ancient and modern views. The various definitions aligned with Descartes notion of two dependent variable quantities, Leibnitz and Bernoulli analytical expressions, and Dirchlet correspondence relations between and independent and dependent variable (Kleiner, 1989; Ponte, 1992; Young, Denton, & Mitchell, 1911). Notwithstanding, two textbooks sought to differentiate cases for operational definitions of multivalued functions; the premises reinforced the relations between independent and dependent terms.

The usage of multiple representations to describe the function concept helps students learn mathematical ideas (Brenner, Mayer, Moseley, Brar, Duran, Reed, and Web (1997). Brennen et al. (1997), found that pre-algebra students who learned functions using multiple representations and had meaningful context performed better than students in a control group. Hence, the decision to utilize multiple representations of functions in the textbook during 1908-1950 might have positively influenced students learning of the concept of function.

Finally, the few times that functions appeared as an independent topic, and the average allocation of five or less pages in secondary school mathematics textbooks, reflect the cries that were made to increase functions visibility in school mathematics by The Reorganization of Mathematics in Secondary Education Report (1923) and The Report of Progressive Education and Joint Committee (1940). Although the number of pages allocated to the functions concept was miniscule at the turn of the century, it is apparent that progress was being made.

In conclusion, functions have become more prevalent in mathematics textbooks since the period 1908-1950. Taking into account the increased popularity of functions in school mathematics since 1950, future research needs to examine the concept of function in US secondary mathematics textbooks after the “New Math” era. Moreover, considering the importance placed on functions by the Common Core State Standards for Mathematics (CCSSM), studies ought to document the impact of the CCSSM on the concept of functions in secondary mathematics textbooks; as well as compare the emphasis placed on functions in US mathematics textbooks to other high-performing TIMSS or other international assessment countries. Such variation in historical mathematics textbooks, invokes a need to explore how function is presented in textbooks.

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Mathematics Anxiety in Society: A Real Phenomena and a Real Solution

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Abstract

While math anxiety still remains a real issue affecting student performance and confidence, today it is even more critical with the greater emphasis on producing more students for careers in STEM fields. In an effort to understand ways to ease math anxiety and encourage adaptive achievement behaviors to deal with such anxiety, this paper will explore the topic and provide research-based practices in providing a solution to this existing problem in our schools. There are many studies that show using technology in the teaching of mathematics will help to alleviate math anxiety and encourage students to enjoy learning mathematics. GeoGebra, a dynamic mathematics software, can assist in developing a deeper understanding of geometric/measurement/algebraic concepts in the mathematics classrooms from Grades K-16. Emphasis on addressing math anxiety as a teacher and using technologies like GeoGebra software to teach math are the main foci of the paper.

Keywords: Math Anxiety, Best Practices, STEM, GeoGebra, Technology, Geometry, Common Core State Standards

Introduction

“Of all of our inventions for mass communication, pictures still speak the most universally understood language.”
--- Walt Disney Company



Image 1: The Math Anxiety Issue Exists in Society

The above photograph was taken recently outside a pub in South Florida, USA. The sign reads, “4 out of 3 people have trouble with math.” Obviously, the owner there was making fun of the problem in our society and the sentiment about how we feel about mathematics. Having been mathematics teachers now for almost 30 years each, the authors have heard on numerous occasions that when we tell someone we are math teachers, most people say it was their worst or most despised subject.

In today’s world, it is critical to encourage young people’s confidence in their mathematical ability and their willingness to set goals to pursue math-related academic and professional careers in STEM fields (Science, Technology, Engineering, and Mathematics). The National Council of Teachers of Mathematics (NCTM) in their 1989 Standards, created societal goals for our young people, one being a creation of mathematically confident learners. As Boaler (2008) points out, it is critical to ensure students are confident and well-prepared in mathematics if they are going to compete for high-tech jobs today and in the future. Mathematics anxiety is a real phenomenon in today’s society in many countries around the world. Helping students identify

and address their math anxiety is critical in helping them cope with and overcome such anxiety that otherwise may negatively impact future choices in their academic and professional careers.

Strauss (2015) states that the 2015 scores for the National Assessment of Educational Progress (NAEP) are available and the news is not good for those who think standardized test scores tell us something significant about student mathematics achievement in the USA. She feels that it is difficult to see any real growth across the board since 2011 with math scores backsliding to 2009 levels. Heitin (2015) noted that sixteen states saw declines in 4th grade math scores. Other than Mississippi and Washington, D.C., only the Department of Defense schools had increases in average scores. Unfortunately not a single state had an increase in 8th grade math scores. Twenty-two states had declines in 8th grade math. Heitin feels that NAEP has long been seen as an independent indicator of achievement, and it was not designed to be aligned to a particular set of standards. One thing these educators and the NAEP seem to always neglect to mention is the level of math anxiety that may contribute to student success in mathematics in the USA. Math anxiety is a real phenomenon and needs to be addressed in our schools to increase student achievement and success. More needs to be done to improve math achievement scores and attitudes toward mathematics overall.

Math Anxiety

Math anxiety is defined as feeling of anxiety that one cannot perform efficiently in situations that involve the use of mathematics. Although it is mostly associated with academics, it can apply to other aspects of life (eHow Website, n.d.). Richardson and Suinn (1972) originally defined math anxiety as “a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations” (p. 551). Mathematics anxiety is the "irrational dread of mathematics that interferes with manipulating numbers and solving mathematical problems within a variety of everyday life and academic situations" (Buckley and Ribordy, 1982, p. 1).

Geist (2010) argues that negative attitudes toward mathematics and math anxiety are serious obstacles for young people in all levels of schooling, and an anti-anxiety curriculum is critical in building students' confidence when working with mathematics. Math anxiety is defined as a feeling of panic, helplessness, paralysis, and mental disorganization that arises when some students are confronted with a mathematical task (Núñez-Peña, Suárez-Pellicioni, & Bono, 2013). It is a well-documented phenomenon that has affected our society for over forty years with little being done to address it in our classrooms or the way we teach math. Beilock and Willingham (2014) state that, “Because math anxiety is widespread and tied to poor math skills, we must understand what we can do alleviate it” (p. 29). Educators today need to address this very important issue if they want to see more students have success with mathematics and go into STEM related fields. In its assessment practices, the National Council of Teachers of Mathematics (NCTM) (1989 & 1995) recognized math anxiety as a problem and specifically included it. Standard #10 (NCTM, 1989) prompts teachers to assess their students' mathematical dispositions, such as confidence in using math to solve problems, communicating ideas, and reasoning.

In addition, many people often blame their failures on their lack of a mathematical mind, the notion that men are better than women at math, or that they have poor memories. Sheila Tobias, an expert on the topic of math anxiety since the 1980's, contends that there are two myths about mathematics that need to be eliminated. One is that higher level math is too difficult for otherwise intelligent students to master, and another is that without mathematics you can live a productive intellectual and professional life (Tobias, 1993). Females who believe their math skills are fixed and unchangeable (a fixed or entity view of intelligence that is correlated with performance goals) are less likely to identify with math and show less interest in math than those women who believe their math skills are malleable (growth view of intelligence associated with mastery goals), and those women are more likely to fall prey to the gender gap that currently exists in mathematics (Burkley, Parker, Stermer, & Burkley, 2010). We suggest that these affective factors, including students' attitudes, interest, and potential anxiety for learning math, will affect students' performance and therefore should be taken into account in attempts to improve students' learning processes.

Jackson and Leffingwell (1999) reported that only 7% of the college students in their study did not express math anxiousness. Similarly, Perry (2004) indicated that 85% of students in an introductory college level math class claimed to have experienced anxiety when presented math problems. Even in populations of students where math is a foundational skill (e.g. engineering majors in college), researchers have found math anxiety to be present (Hembree, 1990; Ruffins, 2007). As the STEM fields become more important for our students to study, our schools and teachers need to do more to address math anxiety so that our students are confident to study STEM-related areas (Sparks, 2011).

Both teachers and parents play a critical role in helping to develop positive dispositions toward math. As with most intervention programs, early assessment and action help to develop positive math attitudes. Ooten (2003) outlines a four-step method for managing a person's math anxiety: helping students come to terms with their feelings, challenging their current beliefs, and realizing they are not alone; using intervention strategies to address negative thinking and realize they can be successful at math; reflecting on personal learning styles and modes; and applying learning to mathematical situations as confidence and strategies for doing mathematics increase. These techniques require the teacher's awareness of a student's math anxiety as well as their willingness to support the student in alleviating that anxiety.

Attention to teaching methods that are effective at overcoming math anxiety is important for teacher preparation as well as for in-service math teachers. A student's lack of success with math may be a cause of math anxiety and heightened by any one of several factors: poor math instruction, an insufficient number of math courses in high school, unintelligible textbooks, or misinformation about what math is and what it is not. Research by Oberlin (1982) and Furner (1996) found that some teaching techniques actually cause math anxiety; (a) assigning the same work for everyone, (b) covering the book problem by problem, (c) giving written work every day, (d) insisting on only one correct way to complete a problem, and (e) assigning math problems as punishment for misbehavior. Further, teachers and parents who are afraid of math

can pass on math anxiety to the next generation by modeling behaviors of their own discomfort with the subject.

How to reduce math anxiety:

1. Using “Best Practice” in mathematics such as:

According to Zemelman, Daniels, and Hyde (2015), we need to use best practices in teaching math to make math instruction most effective, things such as:

- Use of manipulatives (concrete math)
- Cooperative group work
- Discussion of math
- Questioning and making conjectures
- Justification of thinking
- Writing in math: thinking, feelings, and problem solving
- Problem-solving approach to instruction
- Content integration and real-life application
- Use of calculators, computers, and all technology
- Being a facilitator of learning
- Assessing learning as a part of instruction

2. Incorporating the NCTM and State/Common Core Math Standards into the curriculum and instruction.
3. Discussing feelings, attitudes, and appreciation of mathematics with students. Most research shows that until a person with math anxiety has confronted this anxiety by some form of discussion/counseling, no “best practices” in math will help to overcome this fear (Furner, 2007).
4. Psychological techniques like anxiety management, desensitization, counseling, support groups, bibliotherapy, and discussions.

As teachers, we need to include activities like a math attitude survey (Appendix A) or read the book *Math Curse* (Scieszka and Smith, 1995) to get students to talk about true feelings toward mathematics. Using surveys and getting our young people to talk about how they feel about mathematics are some of the first steps toward helping them to gain confidence toward math.

Professor Freedman Provides Math Help at: <http://www.mathpower.com>

- Math Teachers’ Ten Commandments
- Math Anxiety Self-Test
- Ten Ways to Reduce Math Anxiety
- Students’ Math Bill of Rights
- Study Skills Tips
- Math Anxiety Code of Responsibilities
- Other Links to Math Help

At the *Mathitudes Online* website (See Appendix B) one can find a multitude of web links related to math anxiety research. A famous quote from W. V. Williams (1988), “Tell me mathematics, and I will forget; show me mathematics and I may remember; involve me...and I will understand mathematics. If I understand mathematics, I will be less likely to have math

anxiety. And if I become a teacher of mathematics, I can thus begin a cycle that will produce less math-anxious students for generations to come.” (p. 101) is a reminder of how critical it is to teach for understanding making things as hands-on and real-world as possible.

“If math teachers do something about helping their students to develop their confidence and ability to do math, we can impact their lives in a positive way forever” and “Our students’ careers and ultimately many of their decisions they will make in life could rest upon how we decide to teach math. We must make the difference for the future of our kids in an ever growing, high-tech, competitive, global world which depends so heavily on mathematics.” (Furner, 2009, p. 27) The authors strongly feel that math teachers need to start out each school year by giving their students a “Mathitudes Survey” of some sort, and there are many online (Appendix A). This will serve as a gauge for teacher to see who and how many are math anxious students in their class and to take the time working with these students to build the necessary confidence in mathematics to be successful in life and a world of STEM.

Technology Use in the Teaching of Mathematics to Address Math Anxiety

The use of technological tools is critical in today’s world. Our students need to work at higher levels of generalization, model and solve complex problems, and focus on decision-making and reasoning (National Council of Teachers of Mathematics (NCTM) 1989, 2000, 2006). NCTM believes mathematical power can arise from technology that includes: increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future.

NCTM (1989) defines technology in school mathematics as:

- * digital tools
- * computers
- * calculators
- * other handheld devices
- * dynamic software
- * podcasts
- * interactive presentation devices
- * spreadsheets
- * Internet-based resources
- * emerging technology and novel uses of technology

NCTM’s focus in using technology is to: promote technology as an essential tool for learning mathematics in the 21st century, integrate the principles and process standards with teaching the content standards, and provide access to all five mathematics content standards for all students. By providing learner-centered strategies that address the diverse needs of all learners of mathematics, NCTM feels that effective teachers maximize the potential of technology to: develop students’ understanding, stimulate their interest, and increase their proficiency in mathematics.

Alday and Panaligan (2013) found in their empirical study that using technology to teach math reduces math anxiety in students while enhancing more positive attitudes toward mathematics. This study found that teaching topics related to circles and parabolas taught using technology, since these topics can best be presented through diagrams that can be shown through animation and visual presentation on a computer would enhance the learning of students with such technology use. The results of this study for the particular math topics that there was a positive

effect on the use of technology use since there was an improved score for the experimental group on the topics considered thus reducing math anxiety.

Iossi (2013) provides a summary of analysis of research on math anxiety research and found that using technology when teaching mathematics can help to foster math understanding while lowering math anxiety. Iossi (2013) claims that technology use to cope with math anxiety is rarely reported in the literature. Iossi reiterated in her paper the research from Goldberg and Waxman (2003) who found as learners experienced successes with using technology their quantiphobia decreased and, students were more confident about enrolling in additional mathematics courses, their study used technologies like Excel to teach statistics and data analysis. There seems to be several reports now demonstrating how technology may be a tool for minimizing such anxiety toward mathematics.

As described by Mishra and Koehler (2006), Technological Pedagogical Content Knowledge (TPCK) is the basis of good teaching with technology and requires not only content knowledge or pedagogical knowledge, but an understanding of the conceptual representation to build on existing knowledge.

Math teachers need to ask themselves some tough questions when it comes to professional development and employing technology into their teaching of mathematics, questions like:

- What role does technology play in providing multiple representations and opportunities for communication to help students develop mathematical understanding?
- How does technology influence your instructional decisions? And, how do your instructional decisions influence your use of technology?
- How can technology increase access to significant mathematics to all students? How do you promote social justice for access to and facility with technology in learning mathematics?
- How are you thinking differently about your use of technology? What are some of the steps you plan to take to promote growth in your own use of technology?

In research by Fahlberg-Stojanovska, & Stojanovski (2009), they found that using GeoGebra motivates and helps students learn at a higher level while exploring conjectures as they draw and measure. Rosen & Hoffman (2009) found that it is very important to integrate both concrete and virtual manipulatives into the math classroom, such as representational models like GeoGebra. Furner & Marinas (2007) found that young people can easily transition from the concrete to the abstract when using manipulatives like geoboards and using geometry sketching software like GeoGebra. The Appendix B provides online websites on resources related to GeoGebra.

Today's mathematics teachers need to check for dispositions toward math and also use best practices like using technology like GeoGebra to help students gain confidence to explore mathematics in positive and exciting ways that appeal to young people today like employing technologies. As seen above from the current research, there is a great deal of research supporting using technology to help address math anxiety when teaching mathematics. The idea of looking closely at math anxiety levels, motivation to learn mathematics, and using technology like GeoGebra to teach and motivate students is critical today in a world of STEM.

GeoGebra

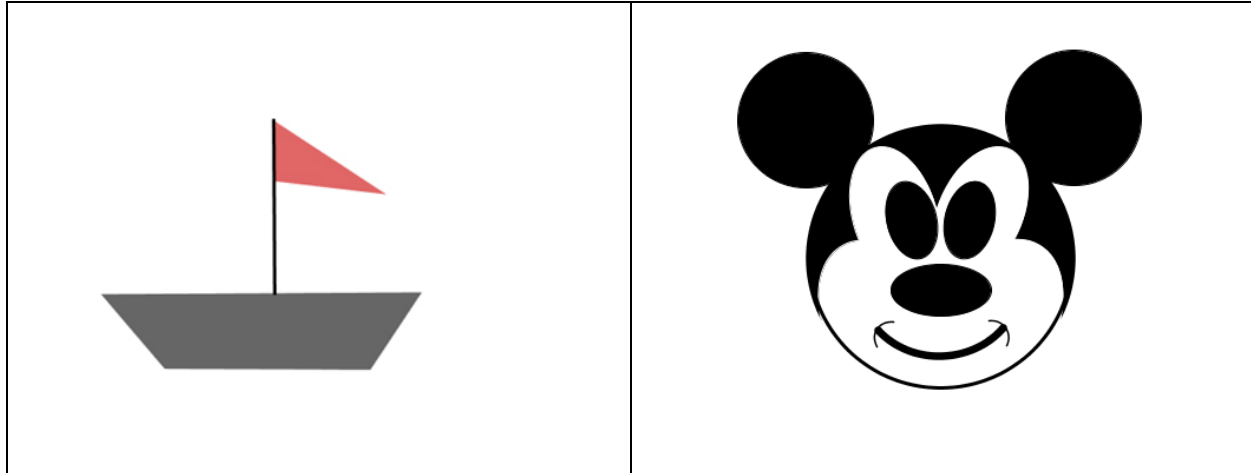


Image 2: GeoGebra was used to create these Images

GeoGebra is free and multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics, and calculus in one easy-to-use package (Hohenwarter, Hohenwarter, & Lavicza, 2009). It can be downloaded for free and accessed at: <http://www.geogebra.org>. The above examples in Image 2 show some drawings students can do with GeoGebra. Students will find using GeoGebra fun and exciting while they learn math as well as allow for some creativity.

GeoGebra was described as raising the enthusiasm for the effective and wise application of technology to the teaching/learning enterprise (Fahlberg-Stojanovska and Stojanovski, 2009; Hewson, 2009). By observing teachers in schools and during the summer workshops, GeoGebra was credited with changing teaching habits. With the availability of dynamic mathematics software, like GeoGebra, teachers are able to make graphical representations of math concepts. As the concepts are introduced with pictorial representations, teachers and their students are able to make the connections between the pictures, the math concepts, and the symbolic representation. When presented with a new concept, students need to think, visualize, and explore relationships and patterns.

Besides GeoGebra, there are many other free online math technology teaching tools (See Appendix B). The Virtual Websites provide great representational understanding for students when learning math concepts at in a positive manner such as using the National Library of Virtual Manipulatives or the National Council of Teachers of Mathematics Illuminations. Both

of these websites are very interactive and used a great deal today in mathematics classrooms. The authors contend that using concrete manipulatives, websites like this that provide representational help, and then going to technologies like GeoGebra can help students transition and advance their mathematical understanding.

Summary

While there are many fearful of mathematics today in our society, using a targeted approach will afford more precise strategies for attenuating students' math anxiety, encouraging more beneficial achievement behaviors for learning math, and inspiring more interest in pursuing careers in math-related STEM fields. In an era of advancing technologies and a push for STEM, it is critical that we address math anxiety and help students develop mathematics confidence. There is a great deal of existing research that supports the use of technology to address math anxiety. One of the best reasons for using dynamic software is that it can even be used for primary-aged students through college, it is fun, easy to use, and students learn a lot about geometry, algebra, measurement and beyond by using this dynamic tool, it can help to prevent and relieve math anxiety and turn young people on to math to explore and feel less intimidated, creating, learning, and having fun while learning mathematics. Math teachers should check for dispositions toward mathematics by using surveys and activities that allow students to talk about how they feel about math. As teachers, we also need to incorporate the best technologies for learning mathematics that can excite and turn students on to math while students create, learn, make sense of, and eventually feel confident in their ability to do math without any apprehension.

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Appendix A:
Mathitudes Survey

Name _____ Grade _____

Math Class _____ Age _____

Career or Career Interest _____

Mathitude Survey

1. When I hear the word math I _____

2. My favorite thing in math is _____

3. My least favorite thing in math is _____

4. If I could ask for one thing in math it would be _____

5. My favorite teacher for math is _____ because _____

6. If math were a color it would be _____

7. If math were an animal it would be _____

8. My favorite subject is _____ because _____

9. Math stresses me out: True or False Explain if you can.

10. I am a good math problem-solver: True or False Explain if you can.

Appendix B

Math Anxiety, GeoGebra, and Virtual Manipulative Websites and Resources for the Mathematics Classroom

| | |
|---|---|
| National Library of Virtual Manipulatives | http://nlvm.usu.edu/ |
| National Council of Teachers of Mathematics | http://www.nctm.org/ccssm/ |
| Cut the Knot | http://www.cut-the-knot.com/Curriculum/ |
| GeoGebra | http://Geogebra.org |
| GeoGebra Wiki Forum | https://wiki.geogebra.org/ |
| GeoGebra Data Files | http://matharoundus.com |
| Math Academy Math Academy | http://www.mathacademy.com/pr/minitext/anxiety/ |
| Mathitudes Online | http://www.coe.fau.edu/centersandprograms/mathitudes/ |

Integrating Diversity Training into Doctoral Programs in Mathematics Education

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There exists a need to promote diversity, equity, and inclusion in mathematics education (Wilson and Franke, 2008). Being cognizant that there are few underrepresented groups that obtain doctoral degrees in mathematical sciences or mathematics education (AMS, 2014; Reys and Dossey, 2008), focused training is needed to prepare doctoral students on diversity issues that may arise in higher education and the means by to address such issues. An advance seminar course or colloquium that would be helpful to mathematics education doctoral students who seek a career position in higher education should be entitled, “Gaining a better perspective of diversity in higher education”. This course would address issues related to establishing and sustaining an equitable and inclusive environment in classroom environments and throughout the university. “Climate can be examined through various components...structural diversity (the number of underrepresented students on a campus), the psychological climate (prejudice), and behavioral dimensions (relations among students, an instructors’ pedagogical approach)” (Hurtado, Milem, Clayton-Pedersen, and Allen, 1999, p. *x*). The climate is often enacted in the hidden curriculum that complements the overt curriculum of the university. Admittedly, diversity courses taught at many universities might address diversity climate issues, however it is not a requirement for a doctorate in mathematics education, and hence most doctoral students in mathematics education never enroll in such courses. Considering that by the year 2044, more than half of the U.S. population will be individuals of color (Colby & Ortman, 2015) and the academy is becoming increasingly diverse, it is imperative that we train educators to work within such diverse contexts. Thus, gaining an understanding of the complexities of diversity, and how to incorporate

it into their practice will be vital to mathematics education doctoral students' success in academia.

Therefore, we propose that an advance seminar course or colloquium in mathematics education be dedicated to the teaching of diversity, equity, and inclusion in higher education: We will first discuss the content that should be covered, and subsequently describe how the training should be organized. By first shedding light on what ought to be learned, faculty members can strategically incorporate pedagogical strategies to promote the learning of the desired content.

Content to be taught

Hurtado, Milem, Clayton Pedersen and Allen (1999) identified four elements that can significantly influence campus climate: *historical legacy of inclusion, structural diversity, psychological climate, and behavioral dimensions*. These four elements will be the core content doctoral students ought to consider in preparing to work within diverse and inclusive higher education contexts.

First, the historical component reflects the tradition of the university. The school goals and policies may be guided by the history in which schools were founded. For example, historically black colleges and universities (HBCU), enrollments have a large percentage of students who are individuals of color. HBCUs were founded to provide education for individuals that were denied access to higher education during segregated eras in education. Additionally, legal challenges can be intertwined into the history of the university. Notwithstanding, many predominantly white universities (PWIs) history promoted segregation, and individuals of color only gained admission because of Supreme court rulings or affirmative action (e.g. Gaines vs. Canada was filed to allow Lloyd Gaines access to the University of Missouri-Columbia in 1938) (Bluford,1959). Hence, the campus climate can be influenced by the school's history for inclusion initiatives (or lack thereof).

Second, structural diversity of the university can significantly impact the climate of diversity at the university. The structural diversity encompasses the ethnicity, race, religion, gender, and sexuality of faculty, staff and students. Researchers have shown that minority ethnic groups may feel repelled when the percentage of minority is small, when compared to the larger population (Hurtado, Milem, Clayton Pedersen and Allen, 1999). Hence universities are challenged to diversify faculty with the intent that it could promote the diversification of the student body. Hurtado, Milem, Clayton Pedersen and Allen (1999) argued that there are five major benefits of diversifying faculty. They noted,

First, faculty of color are able to provide support that benefit students from their particular group...Second a diverse faculty and staff serve as important representatives of the commitment of the institution has to issues of diversity. Third a more diverse faculty and staff serve to create a more comfortable environment ...Fourth, a diverse faculty and staff brings more voices and more diverse perspectives...Fifth, a diverse faculty and staff reflects one measure of institutional success for an educational institution in a pluralistic society. (Hurtado, Milem, Clayton Pedersen and Allen, 1999, p.22)

The benefits of diversifying the faculty populace can have implications for improving campus climate for students of color. Observing faculty in the classroom that is representative of the student population can contribute to positive outcomes for students of color, in addition to majority students (Umbach, 2006). Hence, the structural diversity can significantly impact campus climate within the university setting. Notwithstanding, diversity efforts must seek to ensure that all individuals (both majority and minority) feel valued and a part of the community in which they study, work and socialize.

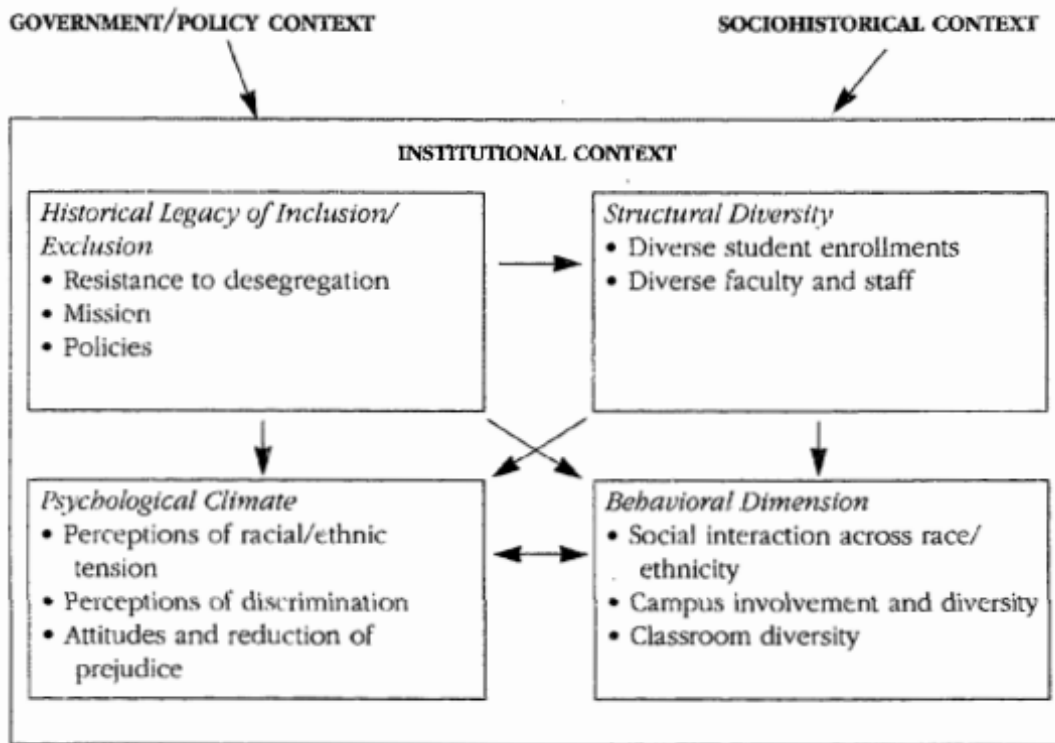
Third, the psychological climate of the university may reflect perception of racism, prejudice or forms of discrimination. Higher education institutions must seek to create inclusive environments such that underrepresented groups feel valued and welcomed. Thus, faculty

members are challenged to ensure that their actions are perceived as equitable and inclusive to all students. An environment that is perceived to be respectful can strengthen relationships between diverse groups, and promote retention of faculty and students at the university. Due to the possibility that a faculty can be accused of acts of discrimination, training is needed to sensitize faculty to differences in cultures and perspectives. Being cognizant of the fact that universities are social agents, the psychological climate of the school will inadvertently impact the wider community. Hence, faculty should promote tolerance for diverse group and respect for all.

The final element of the diversity climate to be discussed is the behavioral dimension. The behavioral dimension considers the interaction of diverse groups in class, clubs, and social interaction. Admittedly, you may have diverse representation on campus, but that does not necessarily mean that the different groups work together in an integrated way, towards a common goal. Within college classes, there exist students who are Black, White, Asian, Muslim etc; a professor is often challenged to find means to showcase the differences in cultures in efforts to positively influence the behavioral dimension. Hence faculty must seek to generate behaviors that promote diversity, and cross culture interaction in efforts to positively contribute to the diversity climate of the class.

Figure 1 depicts Hurtado, Milem, Clayton-Pedersen, and Allen (1999) interrelationship model of *“Elements influencing the climate for racial/ethnic diversity”* (p.4)

Figure 1. Elements influencing the climate for racial/ethnic diversity (Hurtado, Milem, Clayton-Pedersen, and Allen, 1999, p.4)



Prior to the start of the advance seminar or colloquium, doctoral students should be asked to read Hurtado, Milem, Clayton-Pedersen, and Allen (1999) “Enacting diverse learning environments: improving the climate for racial/ethnic diversity in higher education” among other texts. To generate students’ reflection about the reading, doctoral students should be asked to submit a reflection to the following questions: How can the diversity climates of a university impact faculty? How might the diversity climate differ among the following schools: an HBCU, a pre-dominantly white college and a religious university?

The writing assignment before the start of the advance seminar or colloquium, should seek to ignite students to consider challenges they may have as future faculty, based on the diversity climate of the school, and for students to reflect on how the diversity climate may be different pending the type of university they choose to work at. The writing assignment provides

an opportunity for doctoral students to express their perception of variance in the diversity climate pending the school, and will be used as a springboard for the class discussion on diversity climate in higher education.

At the beginning of class doctoral students will be asked to share their responses to the writing assignment: how can the diversity climates of a university impact faculty? Possible responses may consider challenges of assessment, hiring of new faculty, instructional strategies used (such as group work), social clubs in which to participate, etc.

After which, the discussion of the class will shift to Figure 1. Doctoral students should be asked: Is there a particular element(s) that is crucial to the diversity climate, or are all elements of equal significance? Additionally, they should be asked to justify their responses as to whether or not they agree with the unidirectional and bidirectional arrows. Their responses will draw attention to the bureaucracy of the university that may overshadow the campus climate for racial and ethnic diversity.

As a culminating exercise the doctoral students will be asked to participate in *Difficult Dialogues*. According to the University of Missouri *Difficult Dialogues* webpage (2015), the “Program activities provide an environment in which differing views are defended, heard, and considered by those who hold conflicting ideas and values across cultures.” *Difficult Dialogues* creates scenarios in which members of the audience can join the cast and change the outcome of the play. Initially, the audience observes the script unedited, and the second time, they are invited to join the play at anytime to change the outcome of the script. Hence, we proposed that the following difficult dialogues be enacted within a mathematics education context that reflects on employment decisions, class integration and being an agent of change at the university. Hence, the difficult dialogue should depict three themes: whom should we hire? How can we facilitate an integrated and diverse classroom community? And how can we be agents of change within the university?

During the first theme scenario, the cast should depict a search committee trying to decide on whom to hire for a mathematics education position. The candidates will be an African American woman and a White male. The White male is perceived to be more qualified; however the university is challenged to increased faculty diversity. The doctoral students will subsequently be asked to assist with this decision-making, and consider how their decision will influence the climate of structural diversity.

The second theme scenario should focus on promoting cross culture socialization within the class. The script should have a diversity of ethnicity, religion, and social status within the class. The faculty assigns group work but observes individual with similar race, and social class often chooses to work together. So the doctoral students should be asked to play the role of the professor in the script and find means to enhance the behavioral dimension of the climate of diversity in the class.

The third theme scenario should focus on the reenactment of the “Cotton Ball” incident at University of Missouri (Heavin, 2010), in which two young men threw cotton balls in front of the Black Cultural Center and subsequently went and rode the university’s symbolic tiger statue. Once the backdrop of the incidence is set, the cast will then consist of three faculty members who are sitting down for breakfast discussing the recent activities. One faculty would be laughing at the story and suggest that the students were just having fun, another faculty would comment that they should have a white cultural center to complement the black cultural center, while the third faculty states that he hopes this story go away because the school does not need “black drama”, and suggest that the students be removed from the university for this sole reason. Doctoral students should be asked to be a part of this difficult dialogue and extend the breakfast conversation. They should also be asked to consider the implication such an act can have on the psychological climate of diversity at the university and to recommend means to improve the psychological climate of diversity after the incident.

In closing the reading assignment, class activity and difficult dialogues scenarios will seek to promote an understanding of the elements of the diversity climate that exist at institutions of higher education. Having doctoral students' reflection on elements of diversity, an opportunity to participate in situations that may arise regarding diversity can increase their awareness of challenges of diversity that may exist. It is hoped that doctoral students in mathematics education will learn, that saying nothing regarding issues of diversity may actually communicate a message (a negative message) relative to the diversity climate at the university. Hence, this session is needed, because it will provide an opportunity for students to reflect on means that can facilitate the positive climate of diversity at their future place of employment.

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WHEN TWO WRONGS MADE A RIGHT: A Classroom Scenario of Critical Thinking as Problem Solving

Of fate two wrongs infer one right... (C. Ackers for J. Wilford, ed. 1734. "The London Magazine, or, Gentleman's Monthly Intelligencer, Volume 3")

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Critical thinking in Mathematical problem solving

Educators from kindergarten through college often stress the importance of teaching critical thinking within all academic content areas (Foundation for Critical Thinking, 2007, 2013). As indicated by the position statements of the National Council of Teachers of Mathematics, high quality mathematics education before the first grade should use curriculum and teaching practices that strengthen children's problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas" The joint position statement of NAEYC and the National Council of" (NAEYC & NCTM [2002] 2010, 3).

Through the educational and academic institutions critical thinking is identified as an important outcome for achieving the higher orders of learning upon successful completion of a course, a promotion, or a degree (Humphreys, 2013; Jenkins & Cutchens, 2011). Although there are numerous definitions of critical thinking, the authors have selected the definition by Scriven & Paul, 2008 as "the intellectually disciplined process of actively and skillfully conceptualizing, applying, synthesizing, and/or evaluating information gathered from, or generated by,

observation, experience, reflection reasoning, or communication as a guide to belief and action” (Scriven & Paul, 2008). Instructors should teach problem solving within the context of mathematics instruction and engage students in critical thinking by thoughtful questions with discussion of alternative results. Teaching preschool children to problem solve and engage in critical thinking in the context of mathematics instruction requires a series of thoughtful and informed decisions.

MAKING THE CASE IN THE CLASSROOM: An actual historical classroom example

The student made two mistakes, forgetting to extract a square root, adapting to innovation, and flexibility as students usually do. Nevertheless, he immediately realized how to fix them. The result was a simple way to construct Pythagorean triples with an insightful geometric mnemonic rule. We present the following story for accuracy of its mathematical content.

Ms. Angel asked her students to find at least one more Pythagorean triple besides 3, 4, and 5. Nobody came up with one, so Ms. Angel asked Mike to come to the board. She knew Mike was a skilled student, and with her guidance he would be able to construct one such triple.

“Let’s try to find a Pythagorean triple, a right triangle with integer sides,” Ms. Angel said.

Mike was completely confused. He could not remember mathematical concepts well, but utilizing his critical thinking skills, he was not too embarrassed to ask questions: “Why did you call them Pythagorean?” Mike asked.

“They obey the Pythagorean Theorem,” Ms. Angel responded.

Mike continued asking, “What was the Pythagorean Theorem about?”

“The square of the hypotenuse equals the sum of the squares of the legs.”

Mike was not satisfied with the explanation, so he remarked, “This is complicated.”

“Well, we have the simplest example of a Pythagorean triple: 3, 4, 5,” said Ms. Angel.

“That’s really nice. What about 1, 2, 3? May I draw it?”

“You may try, but it’s not going to be a Pythagorean triple.”

“Why not?”

“Because there is no such right triangle with sides 1, 2, 3. It’s not even a triangle, but three overlapping segments.”

“What about 4, 5, 6?”

“No again, it’s not a right triangle.”

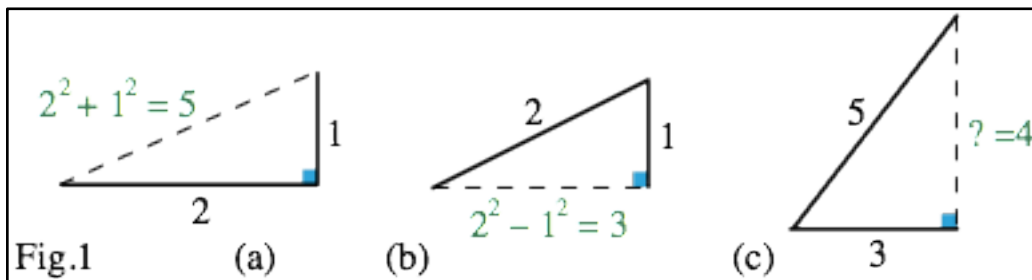
“How should I know that?”

“Because 4 squared plus 5 squared does not equal 6 squared.”

“I know how to square numbers, I just draw a square, with that number of unit squares on the sides, and count the number of unit squares inside it.” Mike eagerly showed what he knew.

“That’s very good, Mike!” Ms. Angel commented. “Back to Pythagorean triples. Do you know another example besides the 3, 4, and 5?”

“Um, let me try. I draw two legs, say, 1 and 2. I square them, 1 and 4, and then add, making 5. That’s the hypotenuse of the Pythagorean 3, 4, 5 triangle.”



“Not really. You forgot to take the square root; the hypotenuse is a square root of 5 which is not an integer. So, try other possible legs.”

Embarrassed, Mike replied, “Sorry, I forgot about the square root.”

By now, Mike got really curious and asked, “What if I take the difference of the squares that we already have? The difference of 4 and 1 is 3. That’s the leg!” Mike exclaimed.

“Again, you forgot to take the square root!”

“But look, earlier I found the hypotenuse, 5. And now I found the leg 3 of the same 3, 4, 5 Pythagorean triple.”

“What about the other leg?” Ms. Angel pressed on.

“I don’t know...isn’t it automatically determined?”

“Yes it is. But it may not give you a Pythagorean triple ... hmm ... but wait a minute. Obviously, it is 4,” Ms. Angel replied.

“Are you sure? That’s really cool!”

“That’s very interesting! You started with two wrong legs, 1 and 2, and obtained two new correct legs, 3 and 4,” smiled Ms. Angel.

“But I had 5 before, so I created the 3, 4, 5 Pythagorean triple by starting with 1 and 2.”

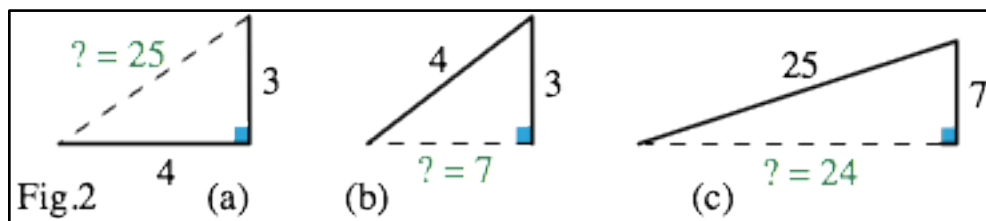
“Yes, but this was just a coincidence. Do you want to try another example?”

Mike got really interested and said, “Okay, give me two numbers for the legs.”

“Take 3 and 4.”

“But, we just did it.”

“No, you started with 1 and 2. Now I want you to start with 3 and 4 and go on your way. You will definitely end up with the hypotenuse not being 5, because 5 is the hypotenuse of the 3, 4, 5 triangle. Your hypotenuse is 25, which is 3 squared plus 4 squared.”



“And that will be wrong?”

“You got it!”

“I am sure you are right, but let me understand.... 25 was the sum of 3 and 4 squared.”

“That’s correct, Mike!”

“Well, 3 squared is 9, and 4 squared is 16; the sum is 25. But, now I will take their difference which is $16 - 9 = 7$. The leg is 7, and not 3 or 4.”

“What do you mean, Mike? Aren’t you constructing the 3, 4, 5 triangle?”

“No, I am not.”

“Then what is the second leg, Mike?”

“We should apply the Pythagorean Theorem.”

“Let’s try it: $25^2 - 7^2 = 625 - 49 = 576$,” Ms. Angel helped to calculate.

“That’s too large for a leg,” announced Mike.

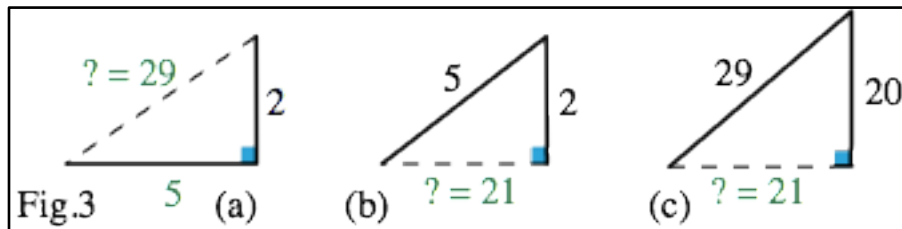
“We didn’t finish yet, Mike. Take the square root ... WOW!”

“Is there something wrong?”

“You are fortunate; it’s 24, a correct value. But something is very wrong here.”

Beginning to feel comfortable, Mike suggested to Ms. Angel, “Let’s try another example, say, 2 and 5. Square them: 2 squared is 4, and 5 squared is 25.”

“The sum is 29, that’s the hypotenuse,” Mike continued. “The difference is 21; that’s a leg. The other leg is determined. In fact, I know how to find it. I just double the product of my initial legs, twice 2 times 5 gives $2 \times 2 \times 5 = 20$,” Mike wrote on the board. “That’s the second leg!”



“How did you come up with the double legs product, Mike?”

“Well, I need the difference of two squares. I factored it in my head into their sum and difference. I knew that one of them is a sum and the other is a difference; I could visualize the terms canceling each other and obtaining the product of doubled squares. Because each term is a square of the starting numbers, I saw the product as the double product squared. I don’t know how to extract square roots, but I know that the square root of a squared whole number is exactly that number itself.”

“That sounds reasonable. Let me check: $20^2 + 21^2$ is 841; that is, 29^2 . That’s correct, Mike.”

“Ms. Angel, I think I did a good job with my homework, right?”

“No, you didn’t. If you did, you would not have made this cute discovery.”

“What discovery? Isn’t this known?”

“The result is known, but not your approach! You made a mistake, or guess, and then fixed it. And you did that not just once, but twice!”

“How many Pythagorean triples are there, Ms. Angel?”

“That number is doubly infinite.”

“Can I get all of them?”

“Using your method? Yes, you can!”

“That sounds great!”

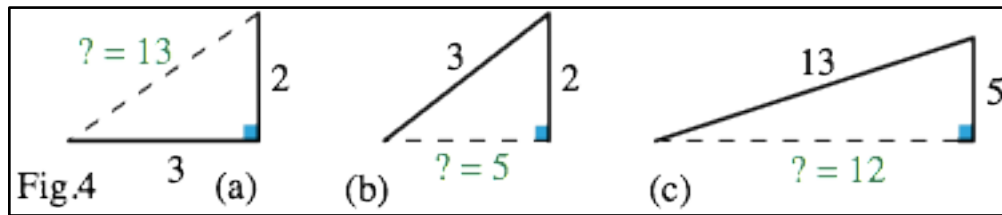
“The question is how to prove that your method always works, Mike!”

“We just check the result.” Mike replied after a short pause.

“Checking is not a proof in mathematics, although...”

“I mean, checking the general case, for any two starting whole numbers for legs.”

“You just gave an idea for a proof. Take two arbitrary integers, call them m and n . The hypotenuse is $m^2 + n^2$. Take $m > n$. Then one of the legs is $m^2 - n^2$. Now, if you square these two values and subtract, you will get the



square of the doubled product, $2mn$. This proves the sufficiency of your method.” Ms. Angel continued, “I don’t know how to prove the necessity, but, I know that this is a known Euclid’s two-parametric representation for Pythagorean triples.”

“I was sure this is known.”

“But your method is also very educational and fun! Isn’t it?” She asked the class.

“Yes, it is!” The entire class shouted. “What we like the most is the clearer visual geometric arguments,” several students commented. “It helps to remember what to do.”

“Then, try this at home. Last time no one came up with any new Pythagorean triple; but now I am asking you to create a dozen of your own Pythagorean triples. It will take less than an hour; it’s such an easy homework assignment!” replied Ms. Angel. “I will check the dozen of the Pythagorean triples known to the ancient Babylonians. Some of them involve huge numbers. I was always curious how they obtained these. With guessing and checking it seems impossible, but with Mike’s guessing and fixing it’s very possible.”

“We can construct really large Pythagorean triples,” commented some students.

Ms. Angel continued, “I am very pleased with today’s work. I also learned some other things that I never knew before: *In Mike-Euclid’s Pythagorean triples, the hypotenuse itself is a sum of two square integers; besides the fact that its square is a sum of squares of two integers.*”

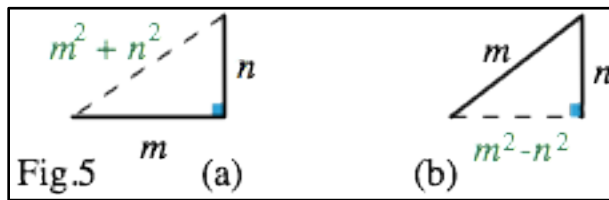
Then she added. “I didn’t realize before that also in *Mike-Euclid’s Pythagorean triples*, one of the legs itself is a difference of two square integers; besides the fact that its square is a difference of squares of two integers.”

“The nice thing is that no square root need be extracted.” Mike commented proudly.

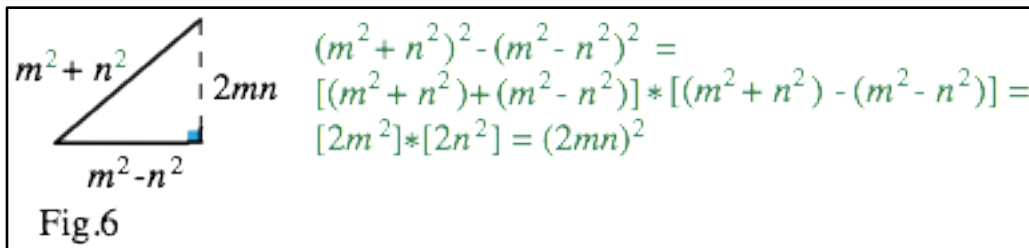
Ms. Angel decided to summarize the discovery; “We still have ten minutes before the end of class. Let me sketch on the board the summary of the things we learned today. And please make your notes in the notebooks.”

Summary: Pairs of wrongs make all right Pythagorean triples.

1. Start with any two integers, and square them.
2. Add the squares to get the hypotenuse, but forget to take the square root (Fig. 5a).



3. Subtract the two squares to get a leg, but forget to take the square root (Fig. 5b).
4. The other leg is determined automatically. It is the double product of the starting numbers.
5. For a proof, check the algebra with general notations, using factoring (Fig. 6).



6. A pseudo-geometric interpretation makes it easy for us to remember the Euclid’s algorithm.
7. The three sides in Fig. 6, together with their multiples, generate all the Pythagorean triples.

Concluding remarks:

Sixty years later Mike extended his insightful geometric vision with doing wrongs and taking no roots to higher dimensional spaces, thus generating correct Pythagorean n-tuples, quadruples, pentuples, sextuples, etc. Their known standard formulas are too complicated to memorize. However, the critical

thinking technique used by the teacher provides the heuristic teaching approach, which trains the student's mind to become an independent thinker.

This scenario included examples of all the critical thinking factors discussed and needed for embracing new thinking for problem-solving, which is a basis for self-empowerment and enhancement of leadership in all nations. Mathematics teachers can change lives by assisting students to become critical thinkers/problem solvers who will be ready to assume the roles of future leaders, change, and innovation in our expanding global networked society. At an increasing speed, globalization is changing work settings and nonwork environments, and it demands new leaders to make decisions and solve problems often and quickly. Chartrand, Ishikawa, and Flander (2009) cited The Conference Board studies, which indicated that 70% of employees with a high school education were deficient in critical thinking skills and even 10% of college graduates lacked critical thinking skills.

There are many activities that demonstrate opportunities to utilize questioning and critical thinking skills within the mathematical courses of instruction. This real-case scenario was written to provide an example on how teachers can incorporate critical thinking into lesson plans, curriculum and classroom activities.

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FAMTE's MATHEMATICS TEACHER EDUCATOR OF THE YEAR AWARD

The Board of Directors of the Florida Association of Mathematics Teacher Educators (FAMTE) has established the **Mathematics Teacher Educator of the Year Award**. The Award will be given on an annual basis with recognition of the recipient at the annual meeting of FAMTE during the Florida Council of Teachers of Mathematics (FCTM) annual conference. The purpose of this award is to recognize excellence in the areas of teaching, research and service.

Eligibility

All mathematics educators who have been employed in a public or private university in the State of Florida for the past two consecutive years and who are members of FAMTE are eligible to apply. Applicants must not have received this award within the past 3 years prior to their application.

Criteria

Nominations are invited that highlight a nominee's involvement at the university, state, and national levels regarding teaching, research, and service. Examples of contributions within each area are included below.

Teaching: For example...

- a. Implementation of effective and innovative teaching practices
- b. Demonstration of innovative teaching methods (e.g. publications, materials, video)
- c. Recipient of awards in teaching from department, college, university, state, and/or national entities
- d. Support of doctoral student development
- e. Textbook authorship

Service: For example...

- a. Active participation in advancing the development and improvement of mathematics teacher education (e.g., membership and leadership roles in state, national, and international organizations)
- b. Unusual commitment to the support of mathematics teachers in the field (e.g., distinctive mentoring experiences)
- c. Participation in editorial boards and/or editorial review of journal manuscripts

Scholarship: For example...

- a. Dissemination of research findings offering unique perspectives on the preparation or professional development of mathematics teachers
- b. Publications useful in the preparation or continuing professional development of mathematics teachers
- c. Acquisition of state and/or nationally funded training and/or research grants
- d. Contribution of theoretical perspectives that have pushed the field of mathematics education forward
- e. Recipient of awards in research from department, college, university, state, and/or national entities

Required Documentation (Maximum of 3 items)

1. A current vita of the nominee
2. A letter of nomination from a FAMTE member documenting evidence related to the indicated criteria that supports the nomination.
3. An (**one**) additional letter of support from an individual active in the educational community (or individuals if letter is co-authored) knowledgeable of the nominee's contributions to mathematics education.

Nomination Process/Deadline: No self-nominations will be accepted. The nomination materials should be sent to FAMTE president, Angie Su at shuifang@gmail.com. Complete nomination packets should be submitted by Friday, October 4, 2016.

FAMTE's DOCTORAL STUDENT OF THE YEAR AWARD

The Board of Directors of the Florida Association of Mathematics Teacher Educators (FAMTE) has established the **Doctoral Student of the Year Award**. The Award will be given on an annual basis with recognition of the recipient at the annual meeting of FAMTE during the Florida Council of Teachers of Mathematics (FCTM) annual conference. The purpose of this award is to acknowledge a mathematics education doctoral student who has shown active involvement in mathematics education at the university, state, and/or national level and who shows potential for success in the field across the areas of teaching, research and service.

Eligibility

All mathematics education doctoral students enrolled in a public or private university in the State of Florida in good academic standing and who are members of FAMTE are eligible for the award. Applicants must not have received this award within the past 2 years prior to their application. Nominees must be enrolled in a doctoral program at the time the award is given.

Criteria

Nominations are invited that highlight a nominee's involvement at the university, state or national level in regards to:

Teaching: For example...

- a. Supervised planning and teaching of undergraduate students
- b. Preparation and delivery of professional development sessions
- c. Assistance to faculty delivery of courses

Service: For example...

- a. Membership in state and/or national professional organizations
- b. Committee participation and/or leadership in state and/or national professional organizations
- c. Participation in editorial review of journal manuscripts or other significant documents

Scholarship: For example...

- a. Collaboration on one or more research projects with peers and/or faculty
- b. Dissemination of research or practitioner-oriented data in journals
- c. Dissemination of research or practitioner-oriented data via conference presentations

Required Documentation (Maximum of 3 items)

1. A current vita of the nominee (specifically indicating institution and doctoral advisor or committee chair)
2. A letter of nomination from a FAMTE member documenting evidence related to the indicated criteria that supports the nomination.
3. An (**one**) additional letter of support from an individual active in the educational community (or individuals if letter is co-authored) knowledgeable of the nominee's contributions to mathematics education.

Nomination Process/Deadline

No self-nominations will be accepted. The nomination materials should be sent to FAMTE president, Angie Su at shuifang@gmail.com. Complete nomination packets should be submitted by Friday, October 4, 2016.

Florida Association of Mathematics Teacher Educators Membership Application (Individual or Affiliate Group)

Florida Association of Mathematics Teacher Educators (FAMTE) – Check membership option and amount.

_____ **One** Year Membership - \$25.00

_____ **Two** Year Membership - \$45.00

_____ **Five** Year Membership - \$100.00

Write your check for the appropriate amount, payable to **FAMTE**, and mail the check and this form to:

Angie Su
FAMTE Membership
2150 Areca Palm Road
Boca Raton, FL 33432-7994

Please complete the following; help us keep our records up to date.

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Transformations
c/o Angie Su,
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