2014

Causal modeling and prediction over event streams

Saurav Acharya
University of Vermont, sacharya@uvm.edu

Causal modeling and prediction over event streams

A Dissertation Presented

by

Saurav Acharya

to

The Faculty of the Graduate College

of

The University of Vermont

In Partial Fullfillment of the Requirements
for the Degree of Doctor of Philosophy
Specializing in Computer Science

October, 2014

Accepted by the Faculty of the Graduate College, The University of Vermont, in partial fulfillment of the requirements for the degree of Doctor of Philosophy specializing in Computer Science.

Dissertation Examination Committee:

Professor Byung S. Lee, Ph.D. — Advisor

Professor Xindong Wu, Ph.D.

Professor Robert Snapp, Ph.D.

Professor Taras I. Lakoba, Ph.D. — Chairperson

Cynthia J. Forehand, Ph.D. — Dean, Graduate College

Date: August 28, 2014

Abstract

In recent years, there has been a growing need for causal analysis in many modern stream applications such as web page click monitoring, patient health care monitoring, stock market prediction, electric grid monitoring, and network intrusion detection systems. The detection and prediction of causal relationships help in monitoring, planning, decision making, and prevention of unwanted consequences.

An event stream is a continuous unbounded sequence of event instances. The availability of a large amount of continuous data along with high data throughput poses new challenges related to causal modeling over event streams, such as (1) the need for incremental causal inference for the unbounded data, (2) the need for fast causal inference for the high throughput data, and (3) the need for real-time prediction of effects from the events seen so far in the continuous event streams.

This dissertation research addresses these three problems by focusing on utilizing temporal precedence information which is readily available in event streams: (1) an incremental causal model to update the causal network incrementally with the arrival of a new batch of events instead of storing the complete set of events seen so far and building the causal network from scratch with those stored events, (2) a fast causal model to speed up the causal network inference time, and (3) a real-time top-k predictive query processing mechanism to find the most probable k effects with the highest scores by proposing a run-time causal inference mechanism which addresses cyclic causal relationships.

In this dissertation, the motivation, related work, proposed approaches, and the results are presented in each of the three problems.

Citations

Material from this dissertation has been published in the following form:

Acharya, S., Lee, B.S.. (2013). *Fast Causal Network Inference over Event Streams*, Proceedings of the 15th International Conference on Data Warehousing and Knowledge Discovery (DaWaK), Prague, Czech Republic, August 26-29.

Acharya, S., Lee, B.S.. (2014). *Incremental Causal Network Construction over Event Streams*, Information Sciences, Elsevier, Volume 261, Pages 32-51, March.

Material from this dissertation has been accepted for publication with minor revisions in Transactions on Large Scale Data and Knowledge Centered Systems, Springer in the following form:

Acharya, S., Lee, B.S.. (2014). *Enhanced Fast Causal Network Inference over Event Streams*, Transactions on Large Scale Data and Knowledge Centered Systems, Springer.

Material from this dissertation has been submitted for publication to Information Sciences, Elsevier in the following form:

Acharya, S., Lee, B.S., Hines, P.. (2014). *Real-time Top-k Predictive Query Processing over Event Streams*, Information Sciences, Elsevier.

To my parents - for everything...

Acknowledgements

I would have never been able to finish my dissertation without the guidance of my committee members, help from friends, and support from my family.

First and foremost, I'd like to express my utmost gratitude to my advisor, Professor Byung S. Lee, for his excellent guidance, care, patience, and for providing me with a good atmosphere for doing research. Thank you for those countless hours of brainstorming, which gave me the very best experience of my PhD life and for contributing not only to my graduate studies but also to my career planning and preparation. I would like to thank Dr. Paul Hines and Dr. Benjamin Littenberg for sharing their valuable insights in problems related to my research. I would like to also thank Dr. Joseph Roure Alcobé for kindly lending me his code to use in one of my research projects.

Many thanks and appreciation to the members of my studies committee, Dr. Xindong Wu, Dr. Robert Snapp - thank you for your insights and advice on my research. Special thanks go to Dr. Taras I. Lakoba, who kindly agreed to chair my dissertation defense committee.

Finally, I would like to thank my parents, sister, and fiancee. They always supported me and encouraged me with their best wishes through good times and bad times.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Modern applications in diverse areas such as patient healthcare, stock market, world wide web, telecommunications, electric grids, and sensor networks produce data continuously and unboundedly at an unprecedented rate. These data-intensive applications need to monitor the event streams continuously to infer the cause of abnormal activities immediately and to predict the likely effects of current activities in real-time, so that informed and timely actions can be taken. In prediction, end-users are particularly interested in the top k most probable effects in the potentially huge answer space. Causal modeling and prediction is thus essential for real-time monitoring, planning, and decision support in these applications.

With the emergence of streaming applications, research on causal modeling and prediction faces new challenges due to three main characteristics of event streams - the continuous and unbounded nature of data, the high throughput of data, and the need for real-time response. Our research addresses these challenges with a primary emphasis on the following aspects:

- *Incremental causal modeling*: The availability of continuous and unbounded data makes it infeasible to reprocess the old data and to rebuild the model from scratch every time new data arrives. However, the existing causal models (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al.,

2006) are designed for an environment where a complete dataset is available all at once. Therefore, a streaming environment necessitates new *incremental* causal modeling approach which revises, instead of rebuilding from scratch, already learned model in the light of new data.

- *Fast causal modeling*: In addition to the unbounded and continuous events, many event streams demand very high throughput processing. The rapid arrival of events makes a fast causal modeling approach imperative. The existing approaches (Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000; Cheng et al., 2002; Heckerman, 1995) are slow, and therefore inapplicable, specifically for two reasons. First, the running time complexity of these approaches increases exponentially with an increase in the number of variables in the event stream. Second, these approaches perform unnecessary computations even when there is no change in the distribution of event streams.

- *Continuous causal prediction*: The causal prediction over event streams need be performed continuously in real-time. In many applications, the users are mostly interested in knowing the most probable k effects. However, there is lack of such top-k prediction algorithms over event streams. One solution is to construct a *traditional* causal network over an event stream and then run a predictive top-k query over it to answer the most likely k effects with the highest scores. However, such a brute force approach has two major limitations. First, the traditional causal model is a directed acyclic graph, and thus does not support cyclic causality. Second, the traditional causal modeling approach employs a conservative approach, which ends up removing significant causal relationships. Furthermore, there exists no top-k predictive query processing mechanism except a naive approach involving an exhaustive searching and sorting of all possible answers, which is inefficient in real-time situations such as event streams.

This dissertation research involves the above three directions (see Figure 1.1). Specifi-

cally, we study three research areas pertaining to *causal modeling* and *prediction* over event streams. First, we study the problem of incremental causal modeling in a continuous and unbounded streaming environment (Research Problem I). Second, we study the problem of faster construction of causal network over event streams (Research Problem II). Third, we study the problem of continuous prediction of top-k effects in real-time given a set of observed events (Research Problem III).

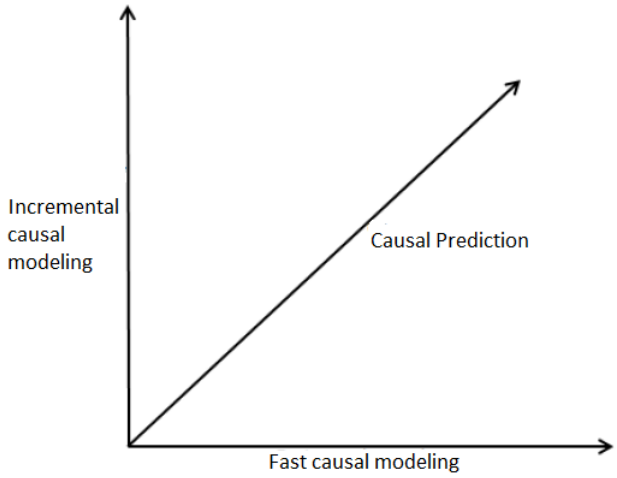


Figure 1.1: Research space of this dissertation

In the rest of this chapter, we provide in Section 1.1 relevant backgrounds on event streams, causality, causal network, and conditional independence tests. Then, in Section 1.2, we introduce the research problems with motivations and contributions. Finally, in Section 1.3, we outline the organization of this dissertation,

1.1 Background

Our work draws from and extends the concepts of causal network modeling to make them applicable over event streams. We provide some backgrounds on the concepts used, based on Chapters 2, 3, 4, and 5.

1.1.1 Event streams

An event stream is a discrete and indefinitely long ordered sequence of event instances. Any timestamped action which has an effect is called an *event instance* (or *event*). The events are created from their prototype called *event type*. Each event instance is created by one event owner. An event type can have many instances, and an event owner can create many instances of any type. Two events are related to each other if they share common attributes such as event owner, location, and time. These attributes are called *common relational attributes* (CRAs). For example, session id might be the CRA in user click stream data.

An event has the following schema: {*timestamp*, *type*, *CRA*, *attribute-set*}. That is, an event has the timestamp at which it was created, the event type it belongs to, the CRA, and a set of additional attributes called the attribute-set. An event is denoted as e_{ij} where i is the value of the CRA and j $(= 1, 2, 3, ...)$ is its event type id (E_j).

Figure 1.2 shows an example of events in a user click stream data where the first half in each line (e.g., e_{11}) represents the actual event instance shown in the second half (e.g., 05/05/11 1:12 pm 1 1 [200s, ...]). The session id serves as the CRA, and the webpage categories (e.g., news, weather, sports, entertainment, etc) serves as the event types for the click stream data.

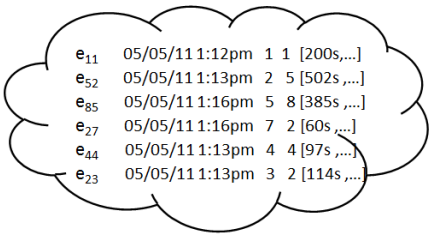
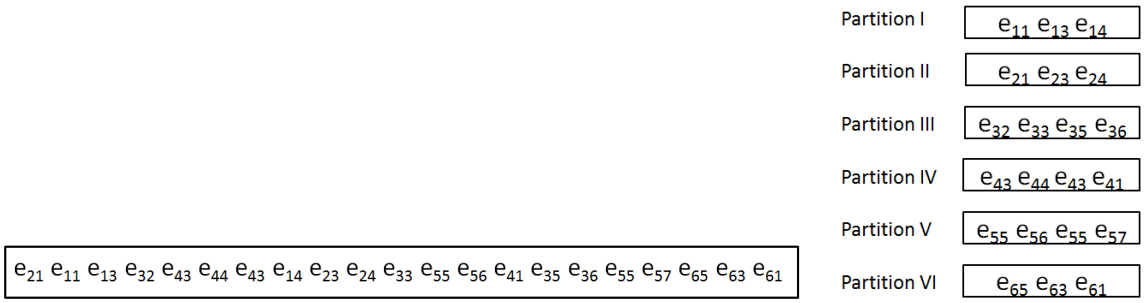


Figure 1.2: Sample of event instances in a stream

We use a sliding window to collect events from a stream during a user-specified observation period T. As a preprocessing step to cluster related events, these events are partitioned by the CRA, creating a *partitioned window* where the events are arranged in the temporal



(a) Raw event stream.

(b) Partitioned window.

Figure 1.3: Event stream

order in each partition. Figure 1.3 shows the partitioned window for the event stream shown in Figure 1.2. Once the observation period expires, the window slides to the next batch of events.

Definition 1 (Partition) *A partition in a partitioned window is defined as the set of observed events arranged in the temporal order over a time period T, that is*

$$W_i = \{e_{ij}(t) | t \leq T, i \in \mathbf{A}, j \in [1, N]\}$$

where $\boldsymbol{A}$ *is the set of CRA values, j is the event type, N is the number of event types, and t is the timestamp.* □

1.1.2 Causality and Causal network

Causality (or causal relationship) is a relationship where a cause leads to an effect. An event can have multiple cause events; similarly, an event can have multiple effect events. Popper defines causality as a relationship between variables that meets three conditions: temporal precedence of a cause over an effect, dependency of the effect on the cause, and absence of other plausible causes (Popper, 1959). The conceptual basis of causality in this dissertation is Popper's definition of causality. Specifically, we use the following notion of causality.

Definition 2 (Causality) *An event type E_i is a cause of another event type E_j ($i \neq j$) if a majority of instances of E_i and a majority of instances of E_j are dependent and a majority of instances of E_i precede a majority of instances of E_j. (The specifics of how many constitute a "majority" is application-dependent.) The causal relationship between these event types is true in the presence of any other event types. In addition, an event instance e_{ki} is said to be a cause of another event instance e_{kj} ($i \neq j$) if they have causality at the event type level and e_{ki} precedes e_{kj}. Note that these two instances share the same CRA (i.e., k).* □

Causality is popularly represented in a directed acyclic graph called causal network (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006). It has a strict requirement that, for every directed edge $\langle u, v \rangle$, the parent node u is a *direct* cause of the child node v. We add to this the temporal ordering, i.e., u should precede v, as another requirement. Figure 1.4 illustrates a causal network of seven event types.

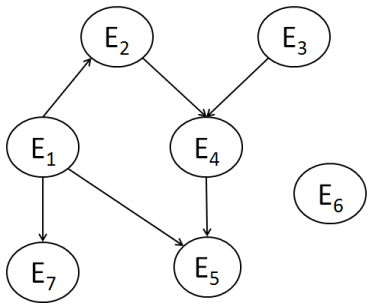


Figure 1.4: Causal network for the sample of seven event types of the events

Causal network should satisfy Causal Markov Condition (Spirtes et al., 1993) which is defined as follows:

Definition 3 (Causal Markov Condition) *Let* **G** *be a causal graph with vertex set* **V** *and* **P** *be a probability distribution over the vertices in* **V** *generated by the causal structure*

represented by **G**. **G** *and* **P** *satisfy the Causal Markov Condition if and only if for every* X *in* **V**, X *is independent of* $\mathbf{V}(\mathbf{Descendants}(\mathbf{X}) \cup \mathbf{Parents}(\mathbf{X}))$ *given* $\mathbf{Parents}(\mathbf{X})$.

Given a set of N event types $\mathbf{E} \equiv \{E_1, ..., E_N\}$, the joint probability distribution of event types in a causal network is specified as

$$P(\mathbf{E}) = \prod_{i=1}^{N} P(E_i|Pa_i)$$

where Pa_i is the set of parent nodes of the event type E_i.

1.1.3 Conditional independence tests

As described in Section 1.1.2, causality requires dependence of effect variables on the cause variables. In other words, as Causal Markov Condition specifies, non-causal variables in a causal network are independent of each other given one or more variables. Conditional independence tests are widely used to determine the independence between variables in the presence of other variables. A popular approach for testing the conditional independence (CI) between two random variables X and Y given a subset of random variables, **C**, is conditional mutual information(CMI) (Cheng et al., 2002; de Campos, 2006). CMI gives the strength of dependency between variables in a measurable quantity, which helps to identify the weak (spurious) causal relationships.

$$\text{CMI}(X, Y|\mathbf{C}) = \sum_{x \in X} \sum_{y \in Y} \sum_{c \in |\mathbf{C}} p(x, y, c) log_2 \frac{p(x, y|c)}{p(x|c)p(y|c)}$$

where p is the probability mass function calculated from the frequencies of variables.

In the traditional CMI, two variables X and Y are said to be independent if $\text{CMI}(X, Y|\mathbf{C}) = 0$, and dependent otherwise. In many cases even though the CMI is positive, we need to distinguish between stronger and weaker dependencies, for example between $\text{CMI}(X, Y|\mathbf{C}) = 10$ and $\text{CMI}(X, Y|\mathbf{C}) = 10^{-3}$. With a higher value of $\text{CMI}(X, Y|\mathbf{C})$, the dependency

between X and Y grows stronger. Thus, to prune out weak dependencies which are more likely to be spurious, we set a threshold CMI value, below which we consider the evidence weak. To do so, we relate CMI with the G^2 test statistics (Bishop et al., 1975; Spirtes et al., 2000) as below.

$$G^2(X, Y|\mathbf{C}) = 2 \cdot N_s \cdot log_e 2 \cdot \text{CMI}(X, Y|\mathbf{C})$$

where N_s is the number of samples (i.e., event instances).

Under the independence assumption, G^2 follows the χ^2 distribution (Kullback, 1968) with the degree of freedom *df* equal to $(n_x - 1)(n_y - 1)\prod_{s \in S} n_s$, where n_x, n_y, and n_s are the number of possible distinct values of X, Y, and S, respectively. So, we perform χ^2 test, which provides a threshold based on *df* and significance level α, to validate the result. We set α as the generally accepted value of 95%.

A boolean function *IsIndependent*$(X, Y, \mathbf{C})$ is defined to test the conditional independence between two variables X and Y, given a condition set of variables $\mathbf{C}$, using the G^2 test statistics. It returns true if these two variables are conditionally independent; otherwise, it returns false.

The unbounded and continuous nature of the event stream warrants an incremental approach and, therefore, the independence test procedure needs to be incremental in our case. So, as a new batch of events arrives, only the record of the frequency of observations should be updated without keeping the old events.

1.2 Research Problems, Motivations, and Contributions

In this journal-format dissertation, as discussed earlier, we investigate three research areas of causal modeling and prediction over event streams. In the first part of the dissertation (Chapter 2), we study the research problem regarding incremental causal modeling. In the

second part of this dissertation (Chapters 3 and 4), we study the research problems to speed up the causal modeling process and to support out-of-order events arrival. In the third part of this dissertation (Chapter 5), we study the problem of continuous causal prediction in real-time over event streams to predict the top k most likely effects given a set of observed events. In these three research areas, we use the temporal precedence information, which is readily available in the event streams, as an important clue for determining causality. According to Barber (Barber, 2012), causality requires explicit temporal information in the model, otherwise only correlation or dependencies can be inferred. A distribution $P(E_1, E_2)$ can be written as either $P(E_1|E_2)P(E_2)$ or $P(E_2|E_1)P(E_1)$ representing E_2 causes E_1 or E_1 causes E_2, respectively. Clearly, without the temporal information, causality is difficult to infer since both these cases represent exactly the same distribution. Note that a cause always precedes its effect (Popper, 1959).

1.2.1 Incremental Causal Modeling over Event Streams

The traditional causal modeling algorithms (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006; Pearl, 2009; Chandra and Kshemkalyani, 2005; Mani et al., 2010) assume that a complete set of data is available at once. For a large number of modern applications, however, the data arrive in a stream continuously and unboundedly. To model causality in such streaming data, the available approach is to rebuild the model from scratch (starting from the first batch of observed data) in light of new data. However, this approach is inefficient in two aspects. First, all the data need to be stored, which is resource intensive. Second, the rebuilding of the model from scratch takes a long time, and therefore is redundant and inefficient. Motivated by these observations, in this dissertation we extend the support for incremental causal modeling over event streams. In an incremental model, with the arrival of new data the model is simply revised instead of being rebuilt completely. The old data can then be discarded.

Unfortunately, there exists no specific work for incremental causal network inference over event streams. While Bayesian networks are very popular and widely used for non-incremental causal modeling (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006; Mani et al., 2010) and there has been some effort towards constructing a Bayesian network incrementally (Alcobé, 2004b; Alcobé, 2004a), the Bayesian network itself is not a causal network for two reasons. First, in a causal network there is a strict requirement that the parent of a node is its direct cause, but the same is not true in a Bayesian network. Second, two or more Bayesian network structures, called equivalence classes (Chickering, 2002), may represent the same probability distribution and, consequently, the causal directions between nodes in the network are often random. The experimental intervention method, most commonly used for experimental static data to alleviate these problems, is unfortunately not applicable in the *stream* environment where the entire dataset is not available at any given time.

Thus, in this research, we propose a new algorithm which incrementally constructs a causal network while addressing the problems mentioned above (Acharya and Lee, 2014b). The key approach is to construct a causal network by merging a temporal network and a Bayesian network. Note that causality requires temporal precedence and dependency between cause and its effect. The temporal precedence direction between nodes is a strong indicator of the causal direction as cause precedes its effect. We model the temporal precedence information incrementally in a temporal network whereas a state-of-the-art incremental Bayesian network algorithm (Alcobé, 2004b; Alcobé, 2005; Alcobé, 2004a) learns the statistical dependencies incrementally. Furthermore, we provide measures to eliminate spurious causalities that do not indicate strong enough causality. In this regard, our approach supports Popper's three conditions for inferring causality, which are temporal precedence, dependency, and no confounding causality (Popper, 1959). We introduce a novel two layered incremental causal network structure to represent causality at both the event type

level and the event instance level. The first layer contains the event types whereas the second layer, which is a virtual layer, contains event instances. The motivations for this two layered causal network model are twofold. First, it allows for an incremental refinement of the network structure at the event type level in light of new event instances. Second, the idea of a virtual layer makes the model flexible enough to add new or discard old event instances (when they expire). In addition to the structural novelty, the causal network is semantically enriched with the notions of causal strength and causal direction confidence associated with each edge.

We conduct two sets of experiments, using real dataset and synthetic datasets, to evaluate the proposed incremental causal network construction algorithm against an existing incremental Bayesian network algorithm. First, the evaluation is focused on the resultant network structure. One evaluation is with respect to the topologies of the resulting networks, and the other evaluation is with respect to the edge directions. In both cases, the networks generated by the two algorithms are compared against a true causal network unknown to the algorithms. Second, the run-time overhead of our incremental algorithm is evaluated against that of the existing Bayesian network algorithm.

1.2.2 Fast causal modeling over event streams

The availability of a large amount of data with high throughput in event streams warrants fast causal inference operations. The existing two approaches – score-based and constraint-based – are slow and their running time increases exponentially with an increase in the number of variables in the event stream. In addition, these approaches do not leverage the changes in the probability distribution of the event streams to avoid unnecessary computations. The first approach, score-based (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006), performs greedy search (usually hill climbing) over all possible network structures to find the best network that represents the data based on

the highest score. This approach, however, has two problems. First, it is slow due to the exhaustive search for the best network structure. An increase in the number of variables in the dataset increases the computational complexity exponentially. Second, two or more network structures, called the equivalence classes (Chickering, 2002), may represent the same probability distribution, and consequently the causal directions between nodes are quite random. Unfortunately, there is no technique for alleviating these problems in a streaming environment. Thus, score-based algorithms are not suitable for streams.

The second approach, constraint-based (Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000; Cheng et al., 2002), does not have the problem of equivalence classes. However, it is slow as it starts with a completely connected undirected graph and thus performs a large number of conditional independence (CI) tests to remove the edges between conditionally independent nodes. Clearly, the number of CI tests increases exponentially with the increase in the number of variables in the dataset. To alleviate this problem, some constraint-based algorithms start with a minimum spanning tree to reduce the initial size of condition sets. However, this idea trades the speed with the accuracy of the causal inference. The constraint-based algorithms such as IC* (Pearl, 2009), SGS (Spirtes et al., 1990), PC (Spirtes et al., 2000), and FCI algorithm (Spirtes et al., 2000) start with a completely connected undirected graph. To reduce the computational complexity, CI tests are performed in several steps. Each step produces a sparser graph than the earlier step, and consequently, the condition set decreases in the next step. However, the number of CI tests is still large. Therefore, these current constraint-based algorithms are not suitable for fast causal inference over streams.

Given these observations, we overcome the drawbacks of the existing approaches as follows. To speed up the causal modeling process, we propose a new time-centric causal modeling approach (Acharya and Lee, 2013). Every causal relationship implies temporal precedence relationship (Popper, 1959). So, temporal precedence information is used as an

important clue in reducing the number of required CI tests and thus maintaining feasible computational complexity. This idea performs fewer computations of CI test due to two factors. First, since causality requires temporal precedence, the nodes with no temporal precedence relationship between them are ignored. Second, in the CI test of an edge, we exclude those nodes from the condition set which do not have temporal precedence relationship with either of the end nodes of the edge. Thus, it reduces the size of the condition set, to the effect of alleviating exponential computational complexity. In addition, the temporal precedence relationship orients the causal edge, unlike the constraint-based algorithms which need a separate set of rules infers the causal direction. The temporal precedence relationships between event types are represented in a temporal network structure, and we propose an algorithm to construct the temporal network applicable in a streaming environment.

We also study the problem of causal inference over out-of-order event streams (Acharya and Lee, 2014a). Since the time-centric causal modeling approach relies on the temporal precedence between events, a larger number of out-of-order events makes such an approach less reliable. In addition, we present a novel change-driven strategy which updates the causal network only when there is strong enough change in te probability distribution of the event streams (Acharya and Lee, 2014a). This idea thus avoids unnecessary causal inference computations.

We demonstrate the utility of the proposed algorithms by conducting two sets of experiments using synthetic and real datasets. The results of the experiments demonstrate the advantages of the proposed algorithm in terms of the running time and the total number of CI tests required for the learning of a causal network by comparing it against the state-of-art constraint-based algorithm for causal network inference. Specifically, we conduct three sets of experiments, first in terms of the accuracies of the resulting causal networks, second the running time, and third the number of CI tests required, to compare our proposed algo-

rithm against the state-of-the-art algorithm. In each set of experiments, we consider both cases of stream being in order and out of order, and also observe the effect of the proposed change-driven causal modeling strategy in reducing the running time. We show that the large number of conditional independence tests is the bottleneck to achieving fast causal modeling.

1.2.3 Continuous prediction over event streams

In this research, we study the problem of continuous prediction of effects over event streams in real-time. Specifically, we address the prediction of top-k most likely effects given a stream of events observed thus far (Acharya et al., 2014). The large answer space and the continuous and unbounded nature of the high throughput streams make such prediction a challenging problem. The brute force approach is to construct a traditional causal network over event stream and then run a predictive top-k query over it to identify the k most probable effects with the highest scores. This approach for causal prediction over the event streams, however, presents new challenges.

The first challenge is the lack of support for cyclic causality in the causal network. In the real world, an instance of type E_1 can cause another instance of type E_2 in one scenario while an instance of type E_2 can cause another instance of type E_1 in another scenario. In the traditional acyclic causal model, only one of these two alternatives, either $E_1 \rightarrow E_2$ or $E_2 \leftarrow E_1$, is possible. This limitation renders the causal network to be an inaccurate prediction model. So, a novel causal modeling approach is necessary to accommodate all possible causal relationships. To address this challenge, we propose a new *event precedence model* which learns every possible temporal precedence relationship, a required criterion for causal relationship, between events to generate a cyclic network structure of the precedence relationships, called the *event precedence network* (EPN).

The second challenge is that the traditional causal model employs a conservative ap-

proach to avoid any suspicious and weak relationships, which exhaustively checks and removes all relationships which are independent in the presence of one or more events. This property of the traditional causal network is the *causal Markov condition*. However, it often ends up removing significant causal relationships. We call it *causal information loss*.

To meet these two challenges, we propose a strategy for inferring causality at runtime from the EPN. The intuition behind such a strategy is that relevant causal inference is possible only after the top-k query variables are known. This approach helps to overcome cyclic causality. Furthermore, since the causal inference is performed at runtime, it only considers the variables which are causally relevant to the query and therefore reduces the causal information loss. In addition, as the EPN retains all precedence relationships (including cyclic relationships), the runtime causal inference overcomes the lack of support for cyclic causality and causal information loss. This approach is naturally adaptive to the changes in the event streams and, therefore, is suitable for streaming environment.

The third challenge is to efficiently process a predictive query in real-time over event streams so that the top-k most likely effects are known. A naive approach is to perform an exhaustive searching and sorting among all possible event types, which is inefficient for real-time event streams. Thus, there is a need for an efficient approach to perform only required search over the possibly huge search space to predict the results. We propose two top-k query processing algorithms to determine the top k effects with the highest scores. One algorithm formalizes the exhaustive search approach, while the other algorithm employs ideas to reduce the search space and terminate as early as possible for real-time query processing.

We conduct extensive experiments with two real datasets. The experiments evaluate the runtime causal inference model and the top-k query processing mechanism in the proposed algorithms. One evaluation is with respect to the accuracy of the top k results, and the other evaluation is with respect to the running time. We demonstrate that the proposed

approach overcomes the two main limitations of the traditional causal inference approach - acyclic causality and causal information loss. In addition, we show the merits of the real-time query processing over exhaustive query processing in terms of the running time.

1.3 Dissertation Outline

This dissertation is organized in a journal format according to the University guidelines. It comprises three parts. The first part (Chapter 2) presents the study of the research area regarding *incremental causal modeling*. It address the problem of constructing a causal network incrementally over event streams.

The second part of the dissertation (Chapters 3 and 4) presents two research problems in supporting fast causal network construction. Chapter 3 addresses the problem of reducing the number of conditional independence tests to speed up the causal modeling process. Chapter 4 extends the work of Chapter 3 to address the problem of redundant causal modeling computations based on a change-driven strategy and also examines the effect of time-centric approach on causal modeling for out-of-order events.

The third part of the dissertation (Chapter 5) presents the research area involving the top-k prediction of possible effects based on the events observed so far. Specifically, the problems involving runtime causal inference and real-time top-k query processing are addressed.

Finally, Chapter 6 summarizes the dissertation and suggests further research.

Chapter 2

Incremental Causal Network Construction over Event Streams

Abstract

This paper addresses modeling causal relationships over event streams where data are unbounded and hence incremental modeling is required. There is no existing work for incremental causal modeling over event streams. Our approach is based on Popper's three conditions which are generally accepted for inferring causality – temporal precedence of cause over effect, dependency between cause and effect, and elimination of plausible alternatives. We meet these conditions by proposing a novel *incremental causal network construction* algorithm. This algorithm infers causality by learning the temporal precedence relationships using our own new *incremental temporal network construction* algorithm and the dependency by adopting a state of the art incremental Bayesian network construction algorithm called the *Incremental Hill-Climbing Monte Carlo*. Moreover, we provide a mechanism to infer only strong causality, which provides a way to eliminate weak alternatives. This research benefits causal analysis over event streams by providing a novel two layered causal network without the need for prior knowledge. Experiments using synthetic and real

datasets demonstrate the efficacy of the proposed algorithm.

2.1 Introduction

People tend to build their understanding of events in terms of cause and effect, to answer such questions as "What caused the IBM stock to drop by 20% today?" or "What caused the glucose measurement of this diabetic patient to increase all of a sudden?". In recent years, there has been growing need for active systems that can perform such causal analysis in diverse applications such as patient healthcare monitoring, stock market prediction, user activities monitoring and network intrusion detection systems. These applications need to monitor the events continuously and update an appropriate causal model, thereby enabling causal analysis among the events observed so far.

In this paper, we consider the problem of modeling causality over *event streams* (not necessarily real-time) with a focus on constructing a *causal network*. The causal network, a widely accepted graphical structure to represent causal relationships, is an area of active research. All the existing works (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006; Pearl, 2009; Chandra and Kshemkalyani, 2005; Mani et al., 2010) in this area have been done for an environment where a complete dataset is available at once. However, event instances in an event stream are unbounded, and in such a case an *incremental* approach is imperative. Thus, the goal of our work is to model causal relationships in a causal network structure incrementally over event streams. To the best of our knowledge, there exists no work done by others with this objective.

Bayesian networks are in popular use for non-incremental causal modeling (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006; Mani et al., 2010). While the Bayesian network encodes dependencies among all variables, it by itself is not the causal network. First, the causal network strictly requires that the parent of a node is its direct cause, but the Bayesian network does not. Second, two

or more Bayesian network structures, called the equivalence classes (Chickering, 2002), can represent the same probability distribution and, consequently, the causal directions between nodes are quite random. There is no technique for alleviating these problems in an event *stream* environment where the entire dataset is not available at any given time.

To overcome this lack of suitable approach for incremental causal modeling over event streams, we propose the *Incremental Causal Network Construction(ICNC)* algorithm. The ICNC algorithm is a hybrid method to incrementally model a causal network, using the concepts and techniques of both temporal precedences and statistical dependencies. Alone, neither dependency nor temporal precedence provides enough clue about causal relationships. The temporal precedence information is learned incrementally in a temporal network with the proposed Incremental Temporal Network Construction (ITNC) algorithm (see Section 2.5.2) whereas the statistical dependencies are learned incrementally with a state of the art algorithm called the Incremental Hill Climbing Markov Chain (IHCMC) (Alcobé, 2004b; Alcobé, 2005; Alcobé, 2004a). There are a few works (Hamilton and Karimi, 2005; Liu et al., 2010) where temporal precedence information is used to identify causal relationships between variables (see Section 2.8), but none of them is for constructing causal networks. In our approach, we further provide measures to eliminate confounding causalities that do not indicate strong enough causality. In this regard, our approach supports Popper's three conditions for inferring causality, which are temporal precedence, dependency, and no confounding causality (Popper, 1959).

We model an incremental causal network with a novel two layered network structure. The first layer is a network of event *types* where an edge between two event types reflects the causality relationship observed between them so far in a stream. The second layer is a network of event *instances*. It is a virtual layer in that there is no explicit link between event instances. Instead, each event type in the first layer maintains a list of its instances which are then connected to instances of another event type through a unique relational attribute

(more on this in Section 2.6.1). The motivations for this two layered causal network model are as follows. First, it allows for an incremental modification of the network structure at the event type level in light of new event instances. Second, the idea of a virtual layer makes the model flexible enough to add new or drop old event instances (drop when the volume of event instances grows too much) while maintaining the overall causal relationships at the event type layer. In addition to the structural novelty, the causal network is semantically enriched with the notions of causal strength and causal direction confidence associated with each edge.

We conduct experiments to evaluate the performance of the proposed *ICNC* algorithm using both synthetic and real datasets. The experiments measure how closely the constructed causal network resembles the true target causal network. Specifically, we compare the Bayesian network produced by IHCMC and the causal network produced by ICNC against a target network. The results show considerable improvements in the accuracy of the causal network over the Bayesian network by the use of temporal precedence relationship between events.

The contributions of this paper are summarized as follows.

- It presents a temporal network structure to represent temporal precedence relationships between event types and proposes an algorithm to construct a temporal network incrementally over event streams.
- It introduces a two-layered causal network with rich causality semantics, and proposes an incremental causal network construction algorithm over event streams. The novelty of the algorithm is in combining temporal precedence and statistical dependency of causality to construct a causal network.
- It empirically demonstrates the advantages of the proposed algorithm in terms of how the temporal network increases the accuracy of the causal network and how close the generated causal network is to the true unknown target causal network.

The rest of the paper is organized as follows. Section 2.2 presents some preliminary concepts. Section 2.3 formulates the specific problem addressed in this paper and outlines the proposed approach. Section 2.4 describes the incremental Bayesian network construction. Section 2.5 and Section 2.6 propose the incremental temporal network construction and the incremental causal network construction, respectively. Section 2.7 evaluates the proposed ICNC algorithm. Section 2.8 discusses related work. Section 2.9 concludes the paper and suggests future work.

2.2 Preliminaries

In this section, we present some key concepts needed to understand the rest of the paper. The concepts are illustrated with a representative use case – diabetic patient monitoring system (Frank and Asuncion, 2010). We select a few important attributes from this real-world case to make the explanations intuitive, and use them in a running example throughout the paper.

2.2.1 Event stream, instance, type

An event stream in our work is a sequence of continuous and unbounded timestamped events. An event refers to any action that has an effect. One event can trigger another event in chain reactions. Each event instance belongs to one and only one event type which is a prototype for creating the instances. We support concurrent events. In this paper an event instance is often called simply an event or an instance if the context makes it clear.

Each event instance is created by one event owner. An event type can have many instances, and an event owner can create many instances of any type. Two event instances are related to each other if they share common attributes such as event owner, location, and time. We call these attributes *common relational attributes(CRAs)*. In Example 1, patient

ID may be the CRA, as the events of the same patient are causally related.

In this paper we denote an event type as E_j and an event instance as e_{ij}, where i indicates the *CRA* and j indicates the event type ID.

Example 1 *Consider a diabetic patient monitoring system in a hospital. There are hundreds of patients admitted to a hospital, and a majority of the actions are related to clinical tests and measurements. Each patient is uniquely identifiable, and each test or action of each patient makes one event instance. For example, a patient is admitted to the hospital, has blood pressure and glucose level measured, and takes medication, creating the instances of the above event types as a result. These instances are repeated for a couple of weeks or months till the patient is discharged. Typical event types from these actions would include NPH-insulin-dose-given (NIDG), regular-insulin-dose-given (RIDG), and hypoglycemic-symptoms-exists (HSE), blood-glucose-measurement-decreased (BGMD), blood-glucose-measurement-increased (BGMI), etc. (more in Table 2.3 in Section 2.7.1.2).*

□

An event type has the following schema: [type ID, type name, event container], where type ID is the primary key and event container is a list of all instances of the type. An event instance has the following schema: [type ID, CRA, timestamp, lifetime, attribute container], where type ID, CRA and timestamp together make the primary key, lifetime is the time duration up to which the event is alive, and attribute container is the set of attribute-value pairs storing any additional information. Note that an event stream contains an indefinitely large number of event instances; hence, an event container, with limited space, cannot store all of them, and therefore an event instance is removed from the event container once its lifetime expires.

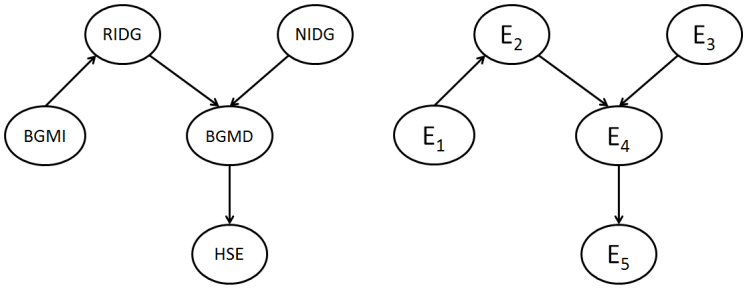
2.2.2 Causality and causal network

Causality (or causal relationship) is a relationship between a cause and an effect. An event can have multiple cause events; similarly, it can have multiple effect events. The conceptual basis of causality in our work is that the effect is dependent on the cause to occur and the cause must precede the effect. More specifically, we use the following notion of causality.

Definition 4 (Causality) *An event type E_i is a cause of another event type E_j ($i \neq j$) if a majority of instances of E_i and a majority of instances of E_j are dependent and a majority of instances of E_i precede a majority of instances of E_j. (The specifics of how many constitute a "majority" is application-dependent.) In addition, an event instance e_{ki} is said to be a cause of another event instance e_{kj} ($i \neq j$) if they have causality at the event type level and e_{ki} precedes e_{kj}. Note that these two instances share the same CRA (k).* □

Causal network is a popularly used data structure for representing causality (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006). It is a directed acyclic graph with a strict requirement that, for every directed edge $\langle u, v \rangle$, the parent node u is a *direct* cause of the child node v. We add to this the temporal ordering, i.e., u should precede v, as another requirement.

Figure 2.1(a) illustrates a causal network of the five event types mentioned in Example 1. The intuitions of the causality among them are as follows – (1) an insulin dose is given to a patient (RIDG or NIDG) to decrease the blood glucose level (BGMD), hence the edge from RIDG and NIDG to BGMD; (2) an increasing blood glucose level (BGMI) triggers the administration of a regular insulin dose (RIDG), hence the edge from BGMI to RIDG; (3) it is common medical knowledge that a decrease in blood glucose level (BGMD) can cause hypoglycemic symptom (HSE), hence the edge from BGMD to HSE. (From here on, we denote BGMI, RIDG, NIDG, BGMD, and HSE with E_1, E_2, E_3, E_4, and E_5, respectively, as shown in Figure 2.1(b).)



(a) Network with the actual event type names.

(b) Network with the simplified event type notations.

Figure 2.1: Causal network for Example 1

2.3 Problem Formulation and the Proposed Approach

2.3.1 Problem

There are three issues that constitute the problem addressed in this paper. First, due to the existence of equivalence classes, the directions of edges in the Bayesian network are not reliable and prone to being incorrect. Therefore, we need a method to learn the correct directions of causal relationships and thereby reduce the number of reversed edges in the causal network. Second, there may be a number of spurious or missing causal relationships in a causal network.[1] The number of spurious edges and the number of missing edges are competing factors bringing a tradeoff in the accuracy of the resulting causal network. Thus, an optimal causal network should have the minimum total number of spurious and missing causal relationships. Third, an event stream is unbounded. Unlike the existing works (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck

[1]The causal network has a strict requirement that the parents of a node are its direct causes. However, the same is not true for a Bayesian network, and, therefore, dependencies between event types detected in a Bayesian network may not be causal relationships. Such dependencies lead to spurious causal relationships in the causal network.

et al., 2006) where a complete dataset is expected to be available for causal modeling, we need an algorithm that constructs a causal network incrementally. In summary, the problem addressed is to construct a causal network incrementally over event streams and while doing so, to reduce the number of reversed edges and minimize the total number of missing and spurious edges in the resultant causal network.

2.3.2 Overview of the approach

To learn a causal network incrementally over event streams, we propose the Incremental Causal Network Construction (ICNC) algorithm (see Section 2.6.1) which models the causal relationships in a two layered network. In the algorithm, we meet Popper's three conditions for inferring causality – temporal precedence, dependency and no confounding causality (Popper, 1959). We propose the *Incremental Temporal Network Construction(ITNC)* algorithm (see Section 2.5) to model the temporal precedences from an event stream into a temporal network incrementally. As mentioned earlier, Bayesian network is reliable to model statistical dependencies and thus we adopt an incremental Bayesian network construction algorithm, the *Incremental Hill-Climbing Monte Carlo (IHCMC)* algorithm (see Section 2.4.2). To resolve the equivalence class problem in Bayesian networks, we use the *DAG-TO-CPDAG* algorithm (Chickering, 2002) to generate a complete partial directed acyclic graph (CPDAG) where unreliable edges are rendered undirected. Then, the ICNC algorithm integrates the temporal network and the CPDAG. During the integration, spurious edges which do not indicate strong enough causality are removed and edges which indicate strong evidence of causality are added with the aim of minimizing the total number of missing and spurious edges, respectively. We rely on the temporal precedence information in the temporal network to identify the correct causal directions of the undirected edges. Section 2.6 describes the rule for this integration and the algorithm for constructing the incremental causal network.

2.4 Incremental Bayesian Network

2.4.1 Bayesian network model

Bayesian network is a directed acyclic graph which encodes a joint probability distribution over a set of random variables. The joint probability distribution of a set of n variables $X \equiv \{X_1, ..., X_n\}$ is specified as

$$P(X) = \prod_{i=1}^{n} P(X_i|Pa_i)$$

where Pa_i is the set of parent nodes of the variable X_i.

Bayesian network encodes the assertions of conditional independence between variables and, thus, is an appropriate network structure to represent statistical dependency between events. Figure 2.2(a) (in Section 2.4.3) shows the Bayesian network corresponding to Example 1.

As already mentioned, however, the same probability distribution can be represented by different Bayesian network structures that are in the same equivalence class (Chickering, 2002). For example in Figure 2.2(a), $E_1 \rightarrow E_2 \rightarrow E_3$ and $E_1 \leftarrow E_2 \rightarrow E_3$ may represent the same joint probability distribution and hence the same network topology. The differences in their network structures are in the directions of the edges. Since the edge direction in a causal network indicates the causal direction between events, the causal meaning becomes entirely different with the change in the edge direction. This ambiguity makes Bayesian networks unsuitable to be used as causal networks. In the example above, the causal direction $E_1 \rightarrow E_2$ (recall E_1 and E_2 are BGMI and RIDG, respectively) makes sense as an increase in blood glucose measurement (BGMI) causes the use of insulin (RIDG). However, the reverse causal direction $E_1 \leftarrow E_2$ (in the Bayesian network of Figure 2.2(a)) is incorrect as the use of insulin does not increase the blood glucose measurement. So, a Bayesian

network cannot be trusted to detect the causal direction between events.

Bayesian variables in the constructed network represent event types. They are boolean variables indicating whether an instance of the represented event type exists or not in the event stream.

2.4.2 Incremental Bayesian network construction algorithm

With the continuous arrival of an event stream with no definite end, it is imperative to construct a Bayesian network incrementally by refining the existing network every time a new batch of data becomes available. Discarding and reconstructing the entire network from scratch every time would be too expensive. As already mentioned, we use the Incremental Hill-Climbing Monte Carlo (IHCMC), an incremental version of the *HCMC* algorithm (Alcobé, 2004b; Alcobé, 2004a). In this subsection we summarize the HCMC and the IHCMC algorithms implemented based on Alcobé's work (Alcobé, 2004b; Alcobé, 2004a).

The HCMC algorithm begins with a network with no edge and, at each iteration, enumerates the neighboring states of the current network state (in the search space) by randomly adding, removing, or reversing edges, and then keeps the network with the highest score. The algorithm terminates when none of the neighboring networks improves the score over the current network. Algorithm 1 summarizes the implemented HCMC algorithm. In the algorithm, the NR (no reversal) neighborhood refers to all DAGs with one arc more or less and do not introduce a directed cycle, whereas the NCR(non-covered reversal) neighborhood refers to the NR neighborhood plus all DAGs with one non-covered edge reversed and does not introduce a directed cycle. (An edge $x \rightarrow y$ in a DAG G is said to be covered in G if $parents(x) \cup x = parents(y)$, that is, all and only the parents of x are the parents of y.)

Initially, the IHCMC algorithm behaves exactly like the HCMC algorithm, which finds a network structure achieving the highest score given the provided data. During this process,

Algorithm 1 HCMC

Require: a dataset D on $\{X_1, ..., X_n\}$ variables, a variable *ncr* indicating whether to use NCR neighorhood or NR neighborhood, a constant MAXTRIALS, a positive integer n

```
1: localMaximum = false, trials = 0;
2: Let G be an edgeless DAG.
3: Calculate the initial neighborhood N(G) (based on ncr) and sufficient statistics for D;
4: while localMaximum is false do
5:    Reverse n randomly chosen edges in G; {//Escape from the local maximum to find the global maximum by randomly reversing edges.}
6:    Let G' be the highest-score DAG in the neighborhood N(G). {//Select the best network structure in the neighborhood.}
7:    if score(G) ≥ score(G') then localMaximum = true; {//There are no network structures with higher score than the current one.}
8:    if localMaximum is false {//The local maximum has not been reached.} then
9:       trials = 0;
10:      G = G';
11:   else if trials < MAXTRIALS {//Repeat the trial up to MAXTRIALS times to find the global maximum.} then
12:      trials = trials + 1;
13:      localMaximum = false;
14:   end if
15: end while
```

the order in which the operators (i.e., add, delete, reverse) coupled with edges are applied is stored for use in the next learning step. Then, with the arrival of a new dataset, the IHCMC algorithm updates the "sufficient statistics" of the old dataset to reflects the new data. (*Sufficient statistics* is a statistical summary of the dataset which contains all information necessary to calculate the scores of the Bayesian network.) Using the new sufficient statistics and the order of operations from the previous step, it determines whether the current network structure should be revised for the new dataset. If a revision is needed, the IHCMC algorithm starts with the highest-score network in the previous step as the initial model. This approach makes the algorithm efficient. When updating the network for the new dataset, IHCMC reduces the search space by restricting the set of operators and edges considered. In order to do that, a certain (used-defined) number of pairs of operators and edges that give the score closest to the best one are stored for the next learning step, and

Algorithm 2 IHCMC

Require: a new dataset D' on $\{X_1, ..., X_n\}$ variables, the highest score network structure G in the old dataset D, the set C_{ij} of candidate parents of each variable X_i and parent X_j from the previous learning step, the ordered set O of operators performed in the previous learning step, and the number n of operators to store in candidate lists. *{ In the first run, G is an edgeless network and C_{ij} and O are empty.}*

Update the sufficient statistics of the old dataset D to reflect the new dataset D';
localMaximum = false; k = 0; run = true;
$G' = G$; *{//Start with the network structure from the old dataset.}*
Calculate neighborhood $N(G')$ with the operators in O;
while run is *true* and $k < |O|$ **do**
  Select the operator, o, that maximizes the score of networks in $N(G')$; *{//Find the best network from the neighborhood $N(G')$.}*
  run = false;
  if o is present within the window of operators being considered in O **then**
    Revise G' by applying the operator o;
    Calculate neighborhood $N(G')$ with the operators in O;
    $k = k + 1$;
    run = true;
  end if
end while
Call the algorithm HCMC with G', sufficient statistics, and the n best candidates in C_{ij};

the search is restricted to only the neighboring networks obtained with these stored pairs. At the end, IHCMC calls the HCMC algorithm so that it continues to build the network structure in light of the new data. Algorithm 2 summarizes the implemented IHCMC algorithm.

2.4.3 DAG-TO-CPDAG algorithm

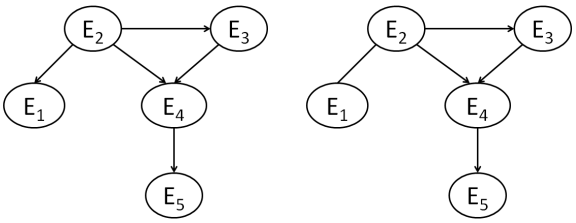
As mentioned earlier, we use the DAG-to-CPDAG algorithm to find the edges with ambiguous direction in a Bayesian network. The algorithm takes a Bayesian network as the input and outputs a completed partial directed acyclic graph (CPDAG) representation of the equivalence class to which that structure belongs. Undirected edges in the CPDAG are the ambiguous edges. We particularly use the implementation proposed by (Chickering, 2002).

Algorithm 3 outlines the three steps in the algorithm. First, it performs a topological sort on the vertices in the input Bayesian network so that, for any pair of vertices x and y, x must precede y if x is an ancestor of y. Second, based on the topological sorting, the edges are sorted first in the ascending order of the incident vertices and then in the descending order of the outgoing vertices. Finally, the ordered edges are labeled either "compelled" or "reversible". The "reversible" edges are made undirected while preserving the edge direction of the "compelled" edges in the final CPDAG.

Algorithm 3 DAG-TO-CPDAG

Require: BayesianNetwork G.

Sort the vertices in G such that x precedes y if and only if x is an ancestor of y;
Based on the topological order of the vertices, sort the edges in G, first in the ascending order of the incident vertices and then in the descending order of the outgoing vertices;
Label every edge in G as "unknown";
while there exists an edge labeled "unknown" in G **do**
  Select the edge of the lowest order, $x \rightarrow y$, that is labeled "unknown";
  run = *true*;
  for every edge $z \rightarrow x$ labeled "compelled" **do**
    if z is not a parent of y {*//If the path is* $z \rightarrow x \rightarrow y$} **then**
      Label $x \rightarrow y$ and every edge incident into y as "compelled";
      run = *false*;
      End this FOR Loop;
    else
      {//If the path is $z \rightarrow x, z \rightarrow y$.*}*
      Label $z \rightarrow y$ as "compelled";
    end if
  end for
  if run is *true* **then**
    if there exists an edge $w \rightarrow y$ such that $w \neq x$ and w is not a parent of x **then**
      Label $x \rightarrow y$ and every "unknown" edge incident to y as "compelled";
    else
      Label $x \rightarrow y$ and every "unknown" edge incident to y as "reversible";
    end if
  end if
end while
Make all "reversible" edges undirected;



(a) Input Bayesian Network (b) Output CPDAG

Figure 2.2: Illustration of DAG-TO-CPDAG.

Figure 2.2 illustrates the Chickering's algorithm (Chickering, 2002). Given the input Bayesian network in Figure 2.2(a), the first step (topological sorting of the vertices) gives E_2, E_3, E_1, E_4, and E_5; then, the second step sorts the edges in the following order: $E_2 \rightarrow E_3$, $E_2 \rightarrow E_4$, $E_2 \rightarrow E_1$, $E_3 \rightarrow E_4$, and $E_4 \rightarrow E_5$; finally, the third step labels the edge $E_2 \rightarrow E_1$ as "reversible" and the remaining edges as "compelled" and, therefore, the "reversible" edge $E_2 \rightarrow E_1$ is made undirected, resulting in the final output CPDAG shown in Figure 2.2(b).

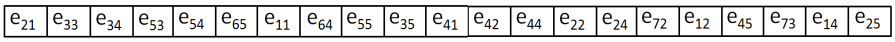
Note that the Chickering's algorithm does not check repeatedly on all edges, hence computationally more efficient than previous rule-based algorithms (Meek, 1995; Pearl and Verma, 1991). In these algorithms, the idea is to undirect every edge in a DAG, except for those edges that participate in a v-structure. (Three nodes x, y and z are said to have a v-structure if their edges form the structure $x \rightarrow y \leftarrow z$.) The rules are applied repeatedly on every edge to determine the edge direction until no rule has any effect on the PDAG, that is, no edge becomes undirected.

2.5 Incremental Temporal Network

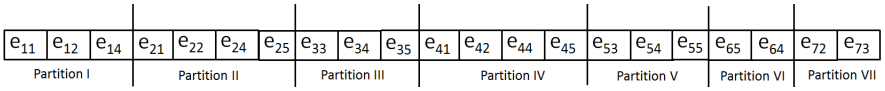
2.5.1 Temporal network model

In this paper the temporal network models the temporal precedences between pairs of events. It is a directed acyclic graph of nodes representing event types. Its construction has to be incremental as well, as it is over a stream. For this purpose, we place a *window* over the stream. The semantics of the window can be dependent on the application, and we use a time-based window here without loss of generality. Typically, the application offers a natural observation period (e.g., day) that makes a window.

As mentioned in Definition 4, causality is defined between events with the same *common relational attribute(CRA)*. So, we arrange the events in a window by CRA as they arrive, producing a *partitioned* window as a result. Events in the same partition have the same CRA and are ordered by the timestamp. Figure 2.3 illustrates it with an event stream from the patient diabetes monitoring system described in Example 1.



(a) Events collected during an observation period (window).



(b) Events in the window partitioned by CRA.

Figure 2.3: Partitioned window of events.

We assume events farther apart temporally are less likely to have a causal relationship between them. So, in a partitioned window, we observe temporal relationships only between events that are near to each other, that is, at most k instances apart. (We refer to k as the *adjacency span* of events.) For example, given a sequence of events $\{e_{i1}, e_{i2}, e_{i3}, e_{i4}\}$, and k specified as 3, we observe the frequencies of event pairs $\{e_{i1}, e_{i2}\}$, $\{e_{i1}, e_{i3}\}$, $\{e_{i1}, e_{i4}\}$,

$\{e_{i2}, e_{i3}\}$, $\{e_{i2}, e_{i4}\}$, and $\{e_{i3}, e_{i4}\}$. These events instances have the same CRA (subscript i) but belong to different event types (E_1, E_2, E_3 and E_4).

With the arrival of a new batch of event instances, we augment each partition in the new window by prefixing it with the last k instances of the partition with the same CRA value in the previous window. This is necessary in order to identify the temporal precedence between instances that are separated into the two consecutive batches.

To determine when an edge, say $E_i \rightarrow E_j$, should be added in a temporal network, a measure providing an evidence of temporal precedence between the event types should be defined. The evidence we use is that the observation of an instance of E_j following an instance of E_i is made frequently enough. So, we use the measure *temporal strength*, as defined below.

Definition 5 (Temporal strength) *Consider an edge $E_i \rightarrow E_j$ ($i \neq j$) in a temporal network. Let f_{ij} be the total number of observations in which an event of type E_i precedes an event of type E_j over all partitions in the* partitioned window*. Then, we define* temporal strength*, s_{ij}, of the edge $E_i \rightarrow E_j$ as*

$$s_{ij} \triangleq \frac{f_{ij}}{\sum_{k=0}^{(N_{ET}-1)} f_{ik}} \tag{2.1}$$

where N_{ET} is the number of event types. □

There are two relevant issues in selecting the edges in a temporal network. First, a *temporal strength threshold* (δ_{s1}) should be provided. Only those edges whose temporal strength is greater than δ_{s1} are included in the temporal network. (There is another threshold, δ_{s2} ($> \delta_{s1}$) used in our work. The temporal strength of an edge higher than δ_{s2} indicates even higher probability of the edge representing a causal relationship (see Definition 4). In Section 2.6, we use δ_{s2} to add a causal relationship missing in the Bayesian network.) Second, a criterion should be set to handle a case in which both an edge and its reverse edge have

temporal strengths higher than δ_{s1}. For this, we use a *gap threshold* (δ_g) which makes sure that when the stronger edge direction is selected, the difference between the two opposite temporal strengths is significant enough. For example, an edge $E_i \rightarrow E_j$ (with frequency f_{ij}) is selected instead of its reverse edge $E_j \rightarrow E_i$ (with frequency f_{ji}) if and only if $\frac{f_{ij}-f_{ji}}{f_{ij}+f_{ji}} > \delta_g$.

2.5.2 Incremental temporal network construction algorithm

The idea behind the *Incremental Temporal Network Construction (ITNC)* algorithm is to collect events from an event stream in a *window* and then use temporal precedence information from the sequence of event pairs in the window to construct a temporal network at the event type level.

The algorithm has three steps. The overall algorithm is centered on a *attenuated frequency matrix*, which is initially empty (i.e., all zero elements) and updated with each new batch of events.

1. Update the attenuated frequency matrix AFM by observing the precedence relationships of event pairs within the adjacency span in the partitioned window (see lines $1-14$ of Algorithm 4). An element f_{ij} in AFM reflects the total number of times events of type E_i precede events of type E_j ($i \neq j$). Each time we observe an event pair (e_{oi}, e_{oj}) in the event stream such that e_{oi} precedes e_{oj}, we increase the value of f_{ij} by $e^{-d\tau}$ where d is the distance between the two events and $\tau \in (0, 1)$ is an attenuation constant. The rationale for the attenuation is that, as the distance between events increases, the probability of them being cause and effect decreases.
2. Calculate the temporal strength of each edge in AFM and store it in a *strength matrix* SM (see lines $15-26$ Algorithm 4). For each pair of an edge and its reversed edge, set the strength of the edge with the lower frequency to zero. The calculated strength of the selected edge, e.g., $E_i \rightarrow E_j$, is stored in the element s_{ij} of SM.

Algorithm 4 Incremental Temporal Network Construction

Require: *window* W, event adjacency span (k), gap threshold (δ_g), temporal strength threshold (δ_{s1}), attenuation constant τ, an edgeless network structure *TN*, *attenuated frequency matrix (AFM).*

Let B_p and B_c be two empty buffers (used to store "parent" events and "child" events, respectively).
for each partition P (corresponding to CRA a) in W {*//Consider one partition at a time as the events of two partitions are unrelated and therefore independent of each other.*} **do**
  for each unique ith timestamp t_i in P **do**
    Clear B_p and Insert all events with timestamp t_i into B_p;
    for $d = 1$ to k such that $i + d \leq$ sizeOf(P) {*//Iterate over all succeeding events within the adjacency span "k" in the same partition.*} **do**
      Clear B_c and Insert all events with timestamp t_{i+d} into B_c;
      for each event instance e_{ac} and e_{ap} in B_c and B_p, respectively **do**
        if type(e_{ac}) $\neq$ type(e_{ap}) {*//There cannot be causal relationships between events of the same type.*} **then**
          Increase the frequency of element $f_{type(e_{ap}),type(e_{ac})}$ in AFM by $e^{-(d-1)\tau}$;
        end if
      end for
    end for
  end for
end for
Let *SM* be an empty strength matrix.
for each pair of elements f_{ij} and f_{ji} in AFM **do**
  //Calculate the temporal strength of only those edges of which the temporal precedence directions are not ambiguous.
  Calculate the evidence of temporal edge direction, $d_{ij} = \frac{f_{ij} - f_{ji}}{f_{ij} + f_{ji}}$;
  if the value of $|d_{ij}|$ is greater than δ_g **then**
    if d_{ij} is positive (i.e., the evidence of f_{ij} is greater than f_{ji}) **then**
      Calculate S_{ij} (see equation 2.1) and set S_{ji} to 0;
    else if d_{ij} is negative (i.e., the evidence of f_{ji} is greater than f_{ij}) **then**
      Calculate S_{ji} (see equation 2.1) and set S_{ij} to 0;
    end if
  end if
end for
for each pair of elements s_{ij} and s_{ji} in SM **do**
  //Add only those edges whose temporal strengths are greater than δ_{s1}.
  if $s_{ij} > \delta_{s1}$ **then**
    Add an edge $E_i \rightarrow E_j$ in *TN*;
  else if $s_{ji} > \delta_{s1}$ **then**
    Add an edge $E_j \rightarrow E_i$ in *TN*;
  end if
  if an edge is added and it introduces cycle in *TN* **then** remove the edge with the lowest temporal strength (in *SM*) in the cycle;
end for

3. Determine the edges of the temporal network using the strength matrix (see lines $27-35$ of Algorithm 4). Only those edges whose temporal strengths are greater than the strength threshold δ_{s1} are added. If a cycle is introduced, we remove the edge with the lowest temporal strength in the cycle.

Let us illustrate the *ITNC* algorithm considering the event stream shown in Figure 2.3. Suppose the adjacency span (k) and the attenuation constant (τ) are set to 2 and 0.5, respectively. Then, in the first step, Algorithm 4 (lines $1-14$) constructs an attenuated frequency matrix shown in Figure 2.4(a) from the event stream. In addition, suppose the gap threshold (δ_g) is set to 10%. Then, in the second step, Algorithm 4 (lines $15-26$) constructs a strength matrix shown in Figure 2.4(b). Note that the edge $E_5 \rightarrow E_4$ fails the gap threshold test and, therefore, its strength is set to 0. In addition, suppose the temporal strength threshold (δ_{s1}) is set to 25%. Then, in the third step, Algorithm 4 (lines $27-35$) constructs from the strength matrix the temporal network shown below in Figure 2.4. Note that the strengths of the edges $E_2 \rightarrow E_3$ and $E_2 \rightarrow E_5$ (21% and 16%, respectively) are lower than the temporal strength threshold and, consequently, are pruned out.

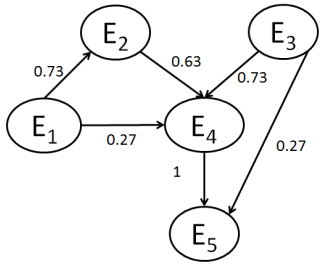
The computational complexity of the ITNC algorithm for a temporal network is polynomial. Let n and n_{cra} be the number of event types and the number of CRAs, respectively. As explained earlier in Section 2.5.1, a partitioned window has n_{cra} partitions. For a new batch of events, the step 1 (attenuated frequency matrix construction) of the ITNC algorithm goes through all of n_{cra} partitions to calculate the attenuated frequency. For each partition P, we find the temporal precedence relationship between each event at a unique timestamp and another event at a later timestamp (within the k adjacency span). The maximum number of events that can occur at any timestamp is n, and the maximum value of k is the size of the partition, $|P|$; so, for one partition the worst case running time is $O(|P| \cdot k \cdot n^2)$, which equals $O(|P|^2 \cdot n^2)$ since $k \leq |P|$. Thus, the running time of the step 1, for n_{cra} partitions, is $O(|P|^2 \cdot n^2 \cdot n_{cra})$. In the step 2 (strength matrix construction),

$$\begin{bmatrix} 0 & 3 & 0 & 1.104 & 0 \\ 0 & 0 & 1 & 3 & 0.736 \\ 0 & 0 & 0 & 2 & 0.736 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) Attenuated frequency matrix

$$\begin{bmatrix} 0 & 0.73 & 0 & 0.27 & 0 \\ 0 & 0 & 0.21 & 0.63 & 0.16 \\ 0 & 0 & 0 & 0.73 & 0.27 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Strength matrix



(c) Output temporal network

Figure 2.4: Illustration of temporal network construction from the event stream in Figure 2.3.

the running time is $O(n^2)$ as the algorithm iterates $n^2/2$ times on the frequency matrix. The step 3 (temporal network construction) iterates $n^2/2$ times on the strength matrix. At each iteration, the algorithm checks for a cycle if an edge is added to the network. In the worst case, every edge in the network is inspected for a cycle to be detected and there are at most $n \cdot (n-1)/2$ edges in the network. So, the running time of the step 3 is $O(n^4)$. Hence, the total running time for the ITNC algorithm is $O(|P|^2 \cdot n^2 \cdot n_{cra} + n^2 + n^4)$ which equals $O(n^2 \cdot (|P|^2 \cdot n_{cra} + n^2))$.

2.6 Incremental Causal Network

2.6.1 Causal network model

As mentioned earlier, we propose to organize the causal network in two layers for compact and yet versatile causal modeling. Representing causality of an unbounded event stream in a single causal network is not only complex but also raises challenges in maintaining

the network with the arrival of new events. So, we prefer a simple network that should be able to represent causality at both the event type level and the event instance level. In the proposed causal network model, the first layer is a network of event types. That means, several thousands of events are aggregated to several dozens of event types, which gives general causal relationships for a majority of the events in the event stream and greatly reduces the size and complexity of the network. The second layer has event instances, and it holds specific causal relationships among them. The event instances are stored in the *event container* of an event type. So, the event instances assume the causal relationship of their event types and an event instance of one event type is causally related to an event instance of another event type through a uniquely identifiable CRA. The temporal precedence relationships between events play a key role in identifying their causal relationship. In short, an event $e_{a_1 i}$ is the cause of another event $e_{a_2 j}$ if and only if (a) they share the same CRA (i.e., $a_1 = a_2$), (b) there exists an edge from E_i to E_j ($i \neq j$) in the causal network (at the type level), and (c) $e_{a_1 i}$ precedes $e_{a_2 j}$ within the adjacency span.

Figure 2.5 illustrates the causal network structure, for the event stream shown in Figure 2.3. The causality among event types are modeled in the first layer. Both E_2 and E_3 are the direct causes of E_4, while E_1 causes E_2 and E_4 causes E_5. The second (virtual) layer holds the causal relationships between event instances. For example, suppose the adjacency span is 2. Then, the event e_{33} is a cause of the event e_{34}, as there is an edge from E_3 to E_4 and e_{33} precedes e_{34} within the adjacency span of the same partition (i.e., under the same *CRA*) (see Figure 2.3); on the other hand, e_{72} does not cause e_{73} even though e_{72} immediately precedes e_{73}, as there is no edge from E_2 to E_3; similarly, e_{42} does not cause e_{45}, as there is no path between them.

Note that the temporal strength gives a measure of the temporal precedence between event types, that is, how often the instances of an event type occur after the instances of another event type. What is deemed more appropriate for causality, however, is a prob-

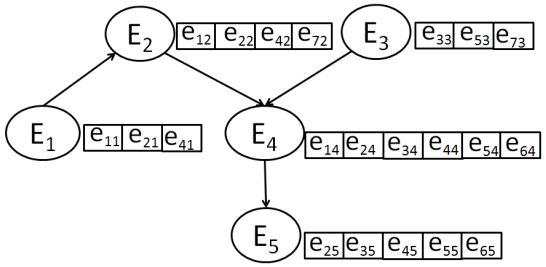


Figure 2.5: Two-layered causal network.

abilistic measure which determines the strength among all likely causes of an event type. This measure is the *causal strength* defined below. The highest causal strength gives the most likely cause of an event type.

Definition 6 (Causal strength) *Consider an edge $E_i \rightarrow E_j$ $(i \neq j)$ in the causal network and its* frequency f_{ij} *in the temporal network. Then, we define the* causal strength*, c_{ij}, of the edge $E_i \rightarrow E_j$ as*

$$c_{ij} \triangleq \frac{f_{ij}}{\sum_{k \in parents(E_j)} f_{kj}}$$

□

In addition to the causal strength, we need a measure of the confidence in the causal direction of an edge.

Definition 7 (Causal direction confidence) *Consider an edge $E_i \rightarrow E_j$ $(i \neq j)$ in the causal network and its* frequency f_{ij} *in the temporal network. If its reversed edge has a* frequency f_{ji}*, then we define the* causal direction confidence*, d_{ij}, of the edge $E_i \rightarrow E_j$ as*

$$d_{ij} \triangleq \frac{|f_{ij} - f_{ji}|}{f_{ij} + f_{ji}}$$

□

Based on the notion of causality in Definition 4, now we propose the rule for merging a temporal network and a Bayesian network.

Rule 1 (Temporal and Bayesian network integration) *Given two event types, we add an edge in the causal network if there exists an edge between them in both the Bayesian network and the temporal network. Moreover, if the temporal strength of an edge in the temporal network is greater than the higher threshold value δ_{s2}, then the edge is added to the causal network even if it does not exist in the Bayesian network. If there are any undirected edges in the Bayesian network (as a result of DAG-to-CPDAG), the direction is set as in the temporal network.* □

Note that an edge direction in the Bayesian network can be ambiguous due to the existence of equivalence classes. Note as well that an edge can be removed or added in the causal network depending on the values of the temporal strength thresholds (δ_{s1}, δ_{s2}) and the gap threshold (δ_g).

2.6.2 Incremental causal network construction algorithm

The algorithm requires an event stream, an observation period (during which a new set of events are collected from the stream) and a list of event types. Initially, an edgeless causal network structure of the given number of nodes is built. Then it is updated incrementally as soon as a new batch of events arrives, as outlined in Algorithm 5.

Recall that the Bayesian network is reliable in judging the existence of statistical dependency between two event types but not the direction of the dependency. So, the key idea of the algorithm is to rely on the statistical dependency given by the Bayesian network and the causal direction given by the temporal network. In addition, the temporal precedence relationships can provide important clues for spurious edge removal or missing edge addition. The algorithm adopts the IHCMC algorithm which finds incrementally a Bayesian network structure achieving the highest score. The Bayesian network thus obtained, however, is one of many possible equivalent network structures and so the edge directions are not reliable. Thus, we run the DAG-To-CPDAG algorithm (Chickering, 2002) which may render some

Algorithm 5 Incremental Causal Network Construction

Require: event stream, a list of event types, temporal strength threshold δ_{s2}

```
1: Construct an edgeless causal network CN with the nodes representing the given event types;
2: for each batch in the event stream do
3:    Run the IHCMC algorithm to update the Bayesian network BN' and then run the DAG-TO-CPDAG algorithm (Chickering, 2002) on BN' to obtain BN; //DAG-TO-CPDAG removes the edge direction of ambiguous edges.
4:    Run the ITNC algorithm to update the temporal network TN;
5:    // Merge TN and BN using Rule 1.
6:    for each pair of event types E_i and E_j (i ≠ j) do
7:       if there is an edge in both BN and TN then
8:          Add an edge in CN; set its direction as in BN if the edge in BN is directed or as in TN if undirected;
9:          Calculate the causal strength as in Definition 6 and the causal direction confidence as in Definition 7;
10:      else if there is no edge in BN but an edge in TN and its temporal strength is greater than δ_s2 then
11:         Add an edge in CN and set its direction as in TN;
12:      else
13:         Add no edge in CN;
14:      end if
15:      if an edge is added and it introduces a cycle in CN then
16:         Remove the edge with the lowest causal strength in the cycle;
17:      end if
18:   end for
19: end for
```

edges undirected if it regards them ambiguous. The temporal network is constructed using the ITNC algorithm (see Section 2.5). The remaining key task is to integrate them according to Rule 1 as they are updated incrementally with a new batch of dataset. If a cycle is introduced in the causal network, we remove the edge with the lowest causal strength in the cycle.

It is well known that the learning of Bayesian network structure, i.e., IHCMC in our case, is a NP hard problem (Chickering et al., 2004). Since the computational complexity of the ITNC algorithm is polynomial (as shown in Section 2.5.2), the computational complexity of the ICNC algorithm is governed by the computational complexity of the IHCMC algorithm.

2.7 Performance Evaluation

We conduct experiments to evaluate the proposed ICNC algorithm against the IHCMC algorithm. The main focus of the evaluation is on the resultant network structure. One evaluation is with respect to the topologies of the resulting networks, and the other evaluation is with respect to the edge directions. In both cases, the networks generated by ICNC and IHMC are compared against a true causal network unknown to the algorithms. In addition, the run-time overhead of ICNC is evaluated against IHCMC. Section 2.7.1 describes the experiment setup, including the evaluation metrics, datasets and the platform used, and Section 2.7.2 presents the experiment results.

2.7.1 Experiment setup

2.7.1.1 Evaluation metrics

Intuitively, the performances of causal network construction algorithms are best evaluated by examining how closely the constructed causal network structures resemble the target causal network. In this regard, we adopt the structural Hamming distance proposed by (Tsamardinos et al., 2006) as the quality metric of the output causal network. The nodes (i.e., event types) are fixed as given to the algorithms, and therefore the network structures are compared with respect to the edges between nodes.

There are three kinds of possible errors in the causal network construction: reversed edges, missing edges, and spurious edges. We use the relative number of the erroneous edges of each kind with respect to the maximum possible number of erroneous edges of that kind as the evaluation metric here. The maximum number of reversed or missing edges is the actual number of edges in the target causal network. On the other hand, the maximum number of spurious edges is given as $\frac{N_{ET}(N_{ET}-1)}{2} - N_{edges}$, where N_{ET} and N_{edges} are the number of nodes (= number of event types) and the number of edges, respectively, in the

Parameter	Meaning
N_O	Number of event owners (with unique ID)
N_{ET}	Number of event types (i.e., nodes)
Max_{NC}	Maximum number of cause events (parents)
Max_{NE}	Maximum number of effect events (children)

Table 2.1: Control parameters for synthetic event stream generation.

target network.

2.7.1.2 Datasets

Experiments are conducted using both synthetic and real datasets.

Synthetic datasets

A synthetic dataset is reverse-engineered from a target causal network. Given control parameters in Table 2.1, the idea is to generate a random causal network, and then convert the causal network to an event stream which reflects the underlying probability distribution of the causal network. Specifically, there are three steps. First, N_{ET} nodes are created and edges are added randomly, and random conditional probabilities are assigned to each edge. Each node can have up to Max_{NC} edges from cause nodes and up to Max_{NE} edges to effect nodes. (We set both Max_{NC} and Max_{NE} to 3 for the experiments presented here.) Second, a joint probability distribution (JPD) table is built from the conditional probabilities assigned to edges of the target causal network. The rows of the JPD table collectively cover all event sequences possible, while each row has its own probability. Third, the probability for each row in the JPD table is multiplied by N_O to calculate the number of repetitions of that event sequence in the dataset. We assume that the event owner is the *CRA* for the dataset.

The size of a JPD table grows exponentially with N_{ET} and therefore we use parallel processing for the event stream generation. The JPD table is divided into multiple partitions and the dataset is created by running parallel processes over each of these partitions. The

Dataset	N_{ET}	N_{edges}	N_O	$N_{instances}$
DS1	4	4	5000	15128
DS2	8	15	30000	124475
DS3	12	22	500000	3173246
DS4	16	39	6553600	50247293
DS5	20	49	52428800	510971687

(N_{edges} is the number of *actual* edges in the network. $N_{instances}$ is the average number of event instances in the datasets of each case.)

Table 2.2: Profiles of the five synthetic datasets.

dataset is thus represented by a collection of files in which the events are shuffled according to the owner ID while preserving the temporal order.

There are five cases of datasets, DS1 through DS5, according to the number of nodes in the represented target causal networks (see their profiles in Table 2.2). The target causal networks have 4, 8, 12, 16 and 20 nodes, respectively, They are created with 1, 2, 16, 64 and 512 parallel processes, respectively, thus consisting of 1, 2, 16, 64 and 512 files, respectively. Each case has 100 different datasets. So, there are a total of 500 different synthetic datasets representing 500 random causal networks. Each row of a synthetic dataset represents one event instance with the schema [type ID, CRA, timestamp, lifetime, attribute container] as discussed in Section 2.2.1.

Real dataset

The real dataset contains diabetes lab test results (Frank and Asuncion, 2010) of 70 different patients over a period ranging from a few weeks to a few months. The dataset has a total 28143 records, about 402 records for each patient. Each record has four fields – date, time, test code, test value. The clinical data of a patient is independent of other patients. Therefore, the patient ID is the *CRA* for this dataset. There are 20 different test codes appearing in the file (shown in the left column of Table 2.3). We define event types of interest from these test codes (shown in the right column of Table 2.3).

Test Code	Event Type
Regular insulin dose	Regular-insulin-dose-given(RIDG)
NPH insulin dose	NPH-insulin-dose-given(NIDG)
UltraLente insulin dose	UltraLente-insulin-dose-given(UIDG)
Unspecified BGM*	Blood-glucose-measurement-increased(BGMI) Blood-glucose-measurement-decreased(BGMD)
Pre-breakfast BGM*	
Post-breakfast BGM*	
Pre-lunch blood BGM*	
Post-lunch BGM*	
Pre-supper BGM*	
Post-supper BGM*	
Pre-snack BGM*	
Hypoglycemic symptoms	Hypoglycemic-symptoms-exist(HSE)
Typical meal ingestion	Typical-meal-ingested(TMI)
More than usual meal ingestion	More-than-usual-meal-ingested(MTUMI)
Less than usual meal ingestion	Less-than-usual-meal-ingested(LTUMI)
Typical exercise activity	Typical-exercise-taken(TET)
More than usual exercise activity	More-than-usual-exercise-taken(MTUET)
Less than usual exercise activity	Less-than-usual-exercise-taken(LTUET)

(Note BGM* : blood glucose measurement)

Table 2.3: Event types defined from the diabetes dataset.

2.7.1.3 Platform

The experiments are conducted on RedHat Enterprise Linux 5 operating system using GCC 4.1.2 in Vermont Advanced Computing Core (VACC) cluster computers. VACC uses the IBM Bluemoon cluster with 364 nodes providing roughly 3,000 computing cores.

2.7.2 Experiment results

We run the ICNC and IHCMC algorithms over each of the five cases of synthethic datasets and the real dataset. First, we compare the topologies of the generated networks against the target causal network and determine how closely they resemble the true causal network. Specifically, we count the number of spurious edges and the number of missing edges. Second, we compare the number of reversed edges in the generated networks with the target causal network to evaluate the causal edge directions. In addition, we compare the

running time of the ICNC and IHCMC algorithms. In these experiments, we choose the median value of $N_{ET}/2$ for the adjacency span (which ranges from 1 to N_{ET}). Similarly, we select the median value of 0.5 for the attenuation constant τ. We assume events in the stream are in temporal order.

We train and test the causal model to evaluate the performance of the ICNC algorithm. In the training phase, we run the simulated annealing algorithm (Kirkpatrick et al., 1983) (SIMULANEALBND function available in MATLAB) to determine the optimal values of temporal strength and gap thresholds (δ_{s1}, δ_{s2}, δ_g) at which the topology of the causal network is closest to the target causal network. As discussed in Section 2.6.1, this topology is influenced by the threshold values. The upper and lower bounds of each threshold are set to 0 and 100%. The optimized thresholds are then used against a new stream of events in the testing phase.

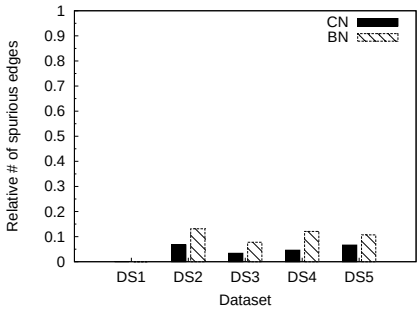
For the synthetic dataset experiment, we randomly divide each dataset into 70% and 30% for training and testing the causal model, respectively. The experiment is repeated ten times for each dataset of each case (DS1 through DS5) to calculate the average relative number of erroneous edges. For the real dataset experiment, we randomly select 70% of the data for training; to test the model, since the dataset is not big enough, we take 50% of the data (30% of the remaining data excluded in the training and 20% of the data used in the training).

2.7.2.1 Comparison of the network topologies from ICNC and IHCMC

The accuracy of the causal network topology is evaluated by the average relative number of spurious and missing edges.

Synthetic dataset experiment

Figures 2.6 and 2.7 show the results of the generated causal network (CN) and Bayesian

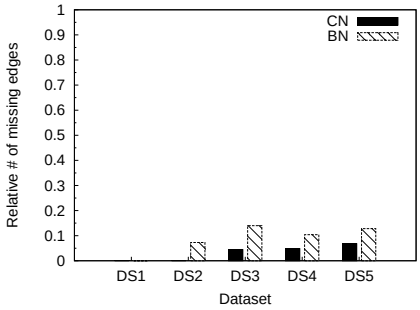


Datasets	CN		BN	
	Mean	**Std. dev**	**Mean**	**Std. dev**
DS_1	0	0	0	0
DS_2	0.067	0.042	0.131	0.080
DS_3	0.034	0.021	0.078	0.046
DS_4	0.046	0.028	0.121	0.073
DS_5	0.066	0.039	0.113	0.066

(a) Mean relative number of spurious edges for each case of dataset.

(b) Mean and Standard deviation of the relative number of spurious edges for each case of dataset.

Figure 2.6: Relative number of spurious edges in the causal network (CN) and the Bayesian network (BN) for the synthetic datasets.



Datasets	CN		BN	
	Mean	**Std. dev**	**Mean**	**Std. dev**
DS_1	0	0	0	0
DS_2	0	0	0.073	0.044
DS_3	0.044	0.026	0.139	0.088
DS_4	0.048	0.028	0.104	0.065
DS_5	0.067	0.041	0.128	0.076

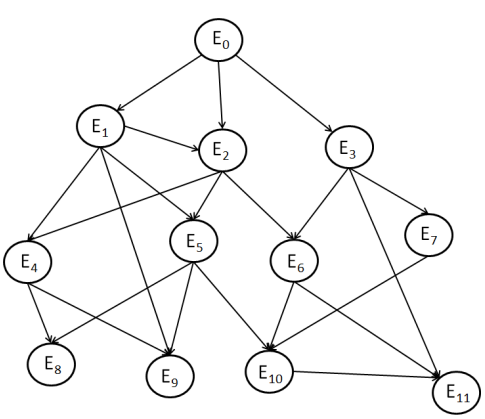
(a) Mean relative number of missing edges for each case of dataset.

(b) Mean and Standard deviation of the relative number of missing edges for each case of dataset.

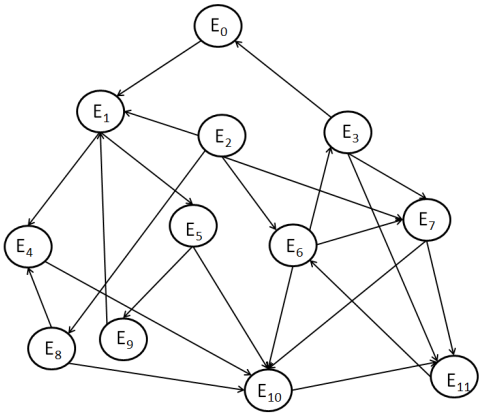
Figure 2.7: Relative number of missing edges in the causal network (CN) and the Bayesian network (BN) for the synthetic datasets.

Dataset	δ_g	δ_{s1}	δ_{s2}
DS_1	18.3	7.1	84.7
DS_2	6.2	0.5	69.8
DS_3	23.3	5.6	91.4
DS_4	33.7	15.1	86.6
DS_5	30.5	3.8	78.4

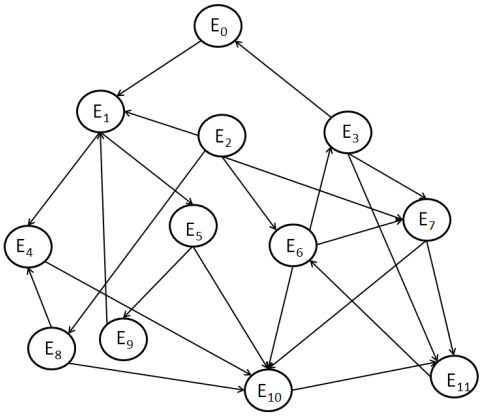
Table 2.4: Optimal threshold values in the synthetic dataset experiments.



(a) True causal network.



(b) Bayesian network from IHCMC.



(c) Causal network from ICNC.

Figure 2.8: Causal and Bayesian networks from the synthetic dataset DS3.

network (BN). (The results are shown in a chart form as well as a tabular form.) The optimal threshold values obtained in the training phase are shown in Table 2.4. In addition, Figure 2.8 shows the true causal network, the causal network generated by ICNC, and the Bayesian network generated by IHCMC, given the dataset DS3. (Those for the other datasets are omitted in the interest of space.)

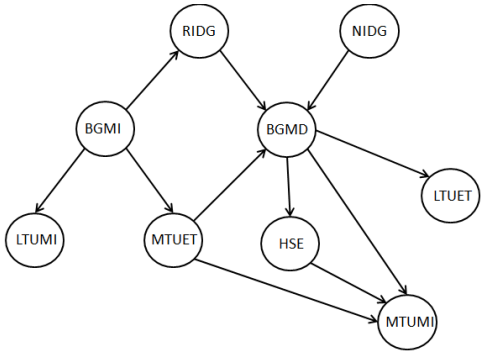
Figure 2.6 shows that the average relative number of *spurious* edges in CN is smaller than that in BN. It is because the ICNC algorithm, through the temporal network, prunes out all edges whose temporal strengths are below δ_{s1} in the temporal network. Figure 2.7 shows that the average relative number of *missing* edges in CN is also smaller than that of BN. This is due to the ICNC algorithm's ability to add edges even if they do not exist in BN when they have temporal strengths greater than δ_{s2} in the temporal network. Moreover, Figures 2.6(b) and 2.7(b) show that the standard deviation in the relative number of spurious edges and missing edges, respectively, is larger for BN than for CN. It is due to the reduction in the number of spurious and missing edges in CN by using the temporal information.

These results confirm the important role of the temporal network and the threshold mechanism to reduce the number of spurious and missing causal relationships.

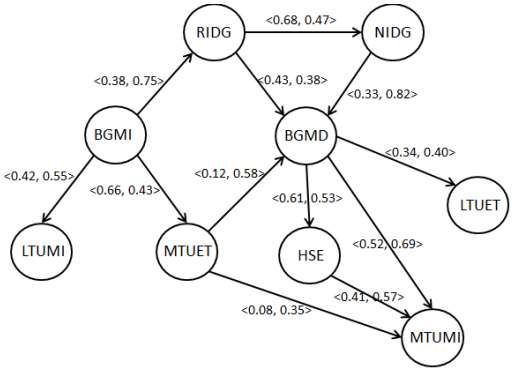
Real dataset experiment

For the diabetes lab test dataset, the optimal values of δ_g, δ_{s1} and δ_{s2} are 26, 23 and 78, respectively. The causal network shown in Figure 2.9(a) is used as the true causal network against which the causal model is trained to determine the optimum threshold values. Figures 2.9(b) and 2.9(c) are the causal network and the Bayesian network generated by ICNC and IHCMC, respectively. Their relative numbers of spurious and missing edges are compared in Figure 2.10.

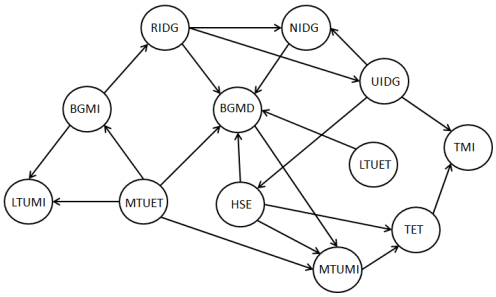
The causal network from ICNC has only one spurious relationship (RIDG causes NIDG) out of the 55 possible spurious relationships. Clearly, there are a higher number of spurious



(a) True causal network.



(b) Causal network from ICNC.



(c) Bayesian network from IHCMC.

Figure 2.9: Causal and Bayesian networks from the real dataset.

edges in the Bayesian network from IHCMC – 12 (i.e, 22% of the possible spurious relationships). It is considered to be due to the fact that the parent of a node in a Bayesian network may not necessarily be its cause (unlike the causal network where the parent of a node is always its cause). The lower number of spurious edges in the causal network is due

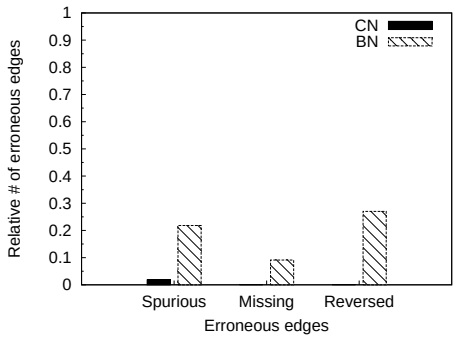


Figure 2.10: Relative number of erroneous edges in the causal network (CN) and the Bayesian network (BN) for the real dataset.

to the temporal strength threshold δ_{s1} and the gap threshold δ_g which prune out the edges with lower temporal strength and narrower gap.

The causal network from the ICNC algorithm has no missing causal relationships out of the 11 causal relationships, whereas the Bayesian network from the IHCMC algorithm has one edge (BGMI causes LTUMI) missing. Evidently, the IHCMC algorithm has failed to identify the causal relationship between BGMI and LTUMI. The ICNC algorithm, on the other hand, has identified the edge as missing and added it, because the edge has temporal strength higher than the threshold δ_{s2} in the temporal network.

The target causal network (Figure 2.9(a)), vetted by a physician, encodes the true causal relationships. We can confirm visually that the causal network from ICNC (Figure 2.9(b)) is an almost exact replica of the true causal network. Note the additional causal strength and causal direction confidence labeled on each edge. The resultant causal network shows that NIDG and RIDG respectively cause BGMD. Indeed, it is well known that insulin decreases the blood glucose level[2]; this also explains why BGMI causes RIDG. BGMI also causes LTUMI. Clearly, patients with high blood glucose level are encouraged for less than usual meal ingestion. In addition, BGMI causes MTEUT, which in turn causes BGMD

[2]UIDG does not show causing BGMD. In fact, Ultralente insulin is not meant to decrease glucose level. Rather, it provides a baseline level of insulin to support minimum metabolism regardless of ingestion or activity. So, this makes sense.

and MTUMI. It is common knowledge that physical activity burns calories, resulting in a decreased blood glucose level (BGMD) and stronger appetite (MTUMI). Note, however, the causal strengths for these two relationships are low, which means MTUET is not a major cause of BGMD. (RIDG and NIDG are the major causes of BGMD.) In addition, BGMD is a strong cause of HSE, LTUET and MTUMI. It is a well-known medical fact that HSE is caused by BGMD and the lower glucose level causes the patients to be prescribed to take less exercise and heavy meal. In fact, HSE, effect of the decrease in blood glucose measurement, is the reason for more than usual meal ingestion(MTUMI).

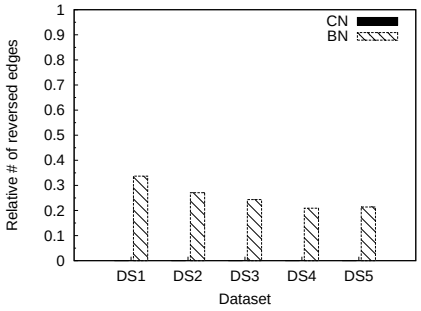
In summary, these synthetic and real dataset experiments confirm that IHCMC alone is not suitable for building an accurate causal network topology. We observe that ICNC is far superior to IHCMC at detecting causally related event types correctly. The results show that the gap threshold and the temporal strength threshold in the ICNC algorithm provides an effective mechanism to identify and remove spurious relationships or add missing causal relationships.

2.7.2.2 Comparison of the edge direction of the networks from ICNC and IHCMC

We compare the causal edge directions of the network structures generated by the ICNC and IHCMC algorithms.

Synthetic dataset experiment

Figure 2.11 shows that the average relative number of *reversed* edges in CN is zero for every dataset. This demonstrates that the ITNC algorithm always detects correct temporal directions from the event stream when generating the temporal network (TN) and the CN generated by the ICNC algorithm inherits these correct temporal directions from TN. In contrast, BN has a higher average relative number of reversed edges for every dataset. In



(a) Mean relative number of reversed edges for each case of dataset.

Datasets	**CN**		**BN**	
	Mean	**Std. dev**	**Mean**	**Std. dev**
DS_1	0	0	0.337	0.203
DS_2	0	0	0.271	0.165
DS_3	0	0	0.243	0.142
DS_4	0	0	0.209	0.119
DS_5	0	0	0.214	0.130

(b) Mean and Standard deviation of the relative number of reversed edges for each case of dataset.

Figure 2.11: Relative number of reversed edges in the causal network (CN) and the Bayesian network (BN) for the synthetic datasets.

addition, the standard deviation of the relative number of reversed edges in BN is particularly larger than those of spurious or missing edges. This is the effect of *equivalence classes* in BN construction which typically involves different edge orientations. It confirms that a Bayesian network cannot be relied upon for the causal direction.

Real dataset experiment

From the causal network and the Bayesian network shown in Figures 2.9(b) and 2.9(c), we can count the relative number of reversed edges as shown in Figure 2.10. Not surprisingly, the Bayesian network has a large number (27%) of reversed causal relationships – for example, the edges between HSE and BGMD, BGMI and MUTET, and BGMD and LTUET. On the other hand, the causal network does not have any reversed causal relationship due to the correct causal direction identified in the temporal network.

Clearly, these synthetic and real dataset experiments confirm that ICNC always gives correct causal directions, thus confirming the crucial merit of temporal precedence relationships in detecting causality. On the other hand, IHCMC can not be relied upon for the causal edge direction.

Datasets	CPU Time (ms)		N_w	CPU time/window (ms)	
	IHCMC	ICNC		IHCMC	ICNC
DS_1	19	22	1	19	22
DS_2	365	412	2	182.50	206
DS_3	17790	20081	16	1111.87	1255.06
DS_4	525268	560536	64	8207.21	8758.37
DS_5	5899505	6280634	512	11522.47	12266.86
DS_R	14853	15094	4	3713.25	3773.5

(N_w and DS_R are the number of partitioned windows observed and the real dataset, respectively.)

Table 2.5: Running time of IHCMC and ICNC algorithms.

2.7.2.3 Comparison of the running time between ICNC and IHCMC

Table 2.5 shows the CPU time spent on processing the complete event stream and the average CPU time per window, for the IHCMC and ICNC algorithms. As expected, running time of the ICNC algorithm is very close to that of the IHCMC algorithm. The running time of the ICNC algorithm is 15.78%, 12.87%, 12.86%, 6.71%, 6.46%, and 16.23% longer than that of the IHCMC algorithm in DS_1, DS_2, DS_3, DS_4, DS_5, and the real dataset, respectively. This confirms that the IHCMC part consumes most of the running time of the ICNC algorithm and that the running time overhead of the ITNC part is negligible.

2.8 Related Work

We discuss related work first with respect to the two main approaches used in our work for causal analysis – dependency-based incremental Bayesian network construction and precedence-based incremental temporal network construction. Then, we discuss other work related to causality over data streams and show the uniqueness of our work.

As already mentioned, we use the IHCMC algorithm (Alcobé, 2004b; Alcobé, 2005) for incremental Bayesian network construction to model the statistical dependencies among events arriving in a stream. Our study concludes that IHCMC is superior to any other existing approaches (Buntine, 1991; Friedman and Goldszmidt, 1997; Lam and Bacchus,

1994). Buntine's approach (Buntine, 1991) can construct an alternative Bayesian networks incrementally given a dataset and a proper ordering of the variables, but is designed to update the posterior probabilities only, not the network structure itself. Therefore, this approach is not applicable as we are concerned about learning the network structure and updating it with an arrival of new batch of events. Lam and Bachhus's approach (Lam and Bacchus, 1994) only refines an already existing network under the assumption that it correctly reflects an accurate model and thus the resulting network is biased toward the existing network structure. So, this approach is not suitable for event streams as the network structure needs to be updated as new events arrive. (Friedman and Goldszmidt, 1997) present three approaches which are not appropriate due to the following reasons. The first approach stores all data items, hence is very memory-inefficient. In contrast, the second approach (called Maximum A Posteriori probability) stores only one single network as the summary of past data, and consequently the learning procedure for new data is biased towards it. The third approach balances between the two, that is, it sacrifices the quality of the constructed network for the reduction in required memory space. Alcobé in his work (Alcobé, 2004b; Alcobé, 2005; Alcobé, 2004a) has converted four well known batch-mode Bayesian network construction algorithms (i.e., CL, K2, B and HCMC) to their incremental versions(i.e., ICL, IK2, IB and IHCMC, respectively). Among these, IHCMC performs the most exhaustive search and yields a Bayesian network of the highest quality within a reasonable time (Alcobé, 2004b).

There is rich literature about temporal modeling or reasoning based on temporal precedence (Glüge et al., 2010; Hamilton and Karimi, 2005; Holme and Saramäki, 2011; Krishna et al., 2010; Liu et al., 2010). The most relevant to our project are TIMERS II algorithm by (Hamilton and Karimi, 2005) and temporal causal graph construction by (Liu et al., 2010), as both propose to use temporal information towards causal analysis. However, neither aims at constructing a causal network as proposed in our work. Specifically, TIMERS II (Hamil-

ton and Karimi, 2005) classifies the relationship between decision attribute and condition attributes into instantaneous, causal, or acausal based on temporal information, but it does not build a causal network and only uses simple temporal conditions and decision tree to determine the nature of causal relationship. The temporal causal graph in Liu et al.'s work (Liu et al., 2010) uses the Granger causality (Granger, 1969), the well known approach for determining causal relationships in a time series data. Given two variables, say X and Y, Granger causality determines that X is a cause of Y if the past value of X can be used (via regression) to predict the future value of Y. This notion of causality is very different from what is proposed in our work in that ours is based on the frequency of the occurrences of the variable values, not the values themselves, and that each variable in our work can represent any event type, not only a time series variable.

The only existing work we find on causal relationships over data streams is by (Kwon and Li, 2009). It is about using temporal, spatial, and spatio-temporal relationships between cause and effect to perform causality join query processing on sensor streams. Similarly, there has been recent work by (Meliou et al., 2010) to support causal query processing in *databases*. However, none of these works addresses causal modeling at all. To the best of our knowledge, there has been no previous work done for causal network modeling by constructing a causal network in a dynamic environment (over event streams). On the other hand, there exists a significant amount of research done on causal analysis in a static environment (Borchani et al., 2007; Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006). Semantically, the work by (Ishii et al., 2010) is closer to the work we are proposing. Their incremental approach is to extract causality for news articles and documents. They employ a lexical approach by comparing subject-verb-object (SVO) tuples for causality detection using natural language processing for each news item. Thus, their work does not infer causality from the observations of the overall data which we are doing in this paper. Our focus is on causality inference over event stream. Besides, it is

not useful in event streams where the lexical approach is not a viable option and the causal relationships need to be inferred from the observation of unbounded events.

2.9 Conclusion and Future Work

In this paper, we focused on the problem of constructing a causal network over continuous event streams incrementally. We proposed a two layered causal network model and presented an algorithm izing temporal precedence relationships and statistical dependency between events in combination. For the temporal precedence relationships, we presented a temporal network model and the temporal network construction algorithm. Then, we combined it with an existing incremental Bayesian network construction algorithm to propose the incremental causal network constructing algorithm, and then through experiments demonstrated that the proposed approach increases the accuracy of causality detection significantly.

In this paper, we assumed the events in a stream are in temporal order. For the future work, we plan to consider the out-of-order event stream which may introduce reversed edges in the temporal network, thereby reducing the reliability of the temporal network. Another interesting direction for the future work is the inclusion of predictive causal query processing. There are applications which can benefit from predicting upcoming events based on the history of prior events.

2.10 Acknowledgments

We thank Joseph Roure Alcobé for providing the initial code base for the IHCMC algorithm and Benjamin Littenberg for his comments on the causal network from the diabetes lab test result data set.

Bibliography

Acharya, S. and Lee, B. (2013). Fast causal network inference over event streams. In *Data Warehousing and Knowledge Discovery*, volume 8057 of *Lecture Notes in Computer Science*, pages 222–235. Springer Berlin Heidelberg.

Acharya, S. and Lee, B. S. (2014a). Enchanced fast causal network inference over event streams. *Transactions on Large Scale Data and Knowledge Centered Systems*.

Acharya, S. and Lee, B. S. (2014b). Incremental causal network construction over event streams. *Information Sciences*, 261(0):32 – 51.

Acharya, S., Lee, B. S., and Hines, P. (2014). Real-time top-k predictive query processing over event streams. *Information Sciences*.

Akdere, M., Çetintemel, U., and Upfal, E. (2010). Database-support for continuous prediction queries over streaming data. *Proc. VLDB Endow.*, 3(1-2):1291–1301.

Alcobé, J. R. (2004a). Incremental hill-climbing search applied to Bayesian network structure learning. In *First International Workshop on Knowledge Discovery in Data Streams, 2004*.

Alcobé, J. R. (2004b). *Incremental Methods for Bayesian Network Structure Learning*. PhD thesis, Department of Computer Science, Universitat Politècnica de Catalunya.

Alcobé, J. R. (2005). Incremental methods for Bayesian network structure learning. *Artificial Intelligence Communications*, 18(1):61–62.

Barber, D. (2012). *Bayesian Reasoning and Machine Learning*. Cambridge University Press.

Barga, R. S., Goldstein, J., Ali, M. H., and Hong, M. (2007). Consistent streaming through time: A vision for event stream processing. In *Proceedings of the Third Biennial Conference on Innovative Data Systems Research*, CIDR'07, pages 363–374.

Bishop, Y. M., Fienberg, S. E., and Holland, P. W. (1975). *Discrete Multivariate Analysis: Theory and Practice*. MIT Press.

Borchani, H., Chaouachi, M., and Ben Amor, N. (2007). Learning causal bayesian networks from incomplete observational data and interventions. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, ECSQARU '07, pages 17–29, Berlin, Heidelberg. Springer-Verlag.

Bowes, J., Neufeld, E., Greer, J., and Cooke, J. (2000). A comparison of association rule discovery and bayesian network causal inference algorithms to discover relationships in discrete data. In Hamilton, H., editor, *Advances in Artificial Intelligence*, volume 1822 of *Lecture Notes in Computer Science*, pages 326–336. Springer Berlin Heidelberg.

Buntine, W. (1991). Theory refinement on Bayesian networks. In *Proceedings of the seventh conference (1991) on Uncertainty in artificial intelligence*, pages 52–60, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Chandra, P. and Kshemkalyani, A. D. (2005). Causality-based predicate detection across space and time. *IEEE Transactions on Computerts*, 54:1438–1453.

Cheng, J. and Druzdzel, M. J. (2000). Ais-bn: An adaptive importance sampling algorithm for evidential reasoning in large Bayesian networks. *J. Artif. Intell. Res. (JAIR)*, 13:155–188.

Cheng, J. and Druzdzel, M. J. (2001). Confidence inference in Bayesian networks. In *Proceedings of the Seventeenth conference on Uncertainty in artificial intelligence*,

UAI'01, pages 75–82, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Cheng, J., Greiner, R., Kelly, J., Bell, D., and Liu, W. (2002). Learning Bayesian networks from data: an information-theory based approach. *Artificial Intelligence*, 137(1-2):43–90.

Chickering, D. M. (2002). Learning equivalence classes of Bayesian-network structures. *Journal of Machine Learning Research*, 2:445–498.

Chickering, D. M., Heckerman, D., and Meek, C. (2004). Large-sample learning of bayesian networks is np-hard. *J. Mach. Learn. Res.*, 5:1287–1330.

Chow, Y. S. and Teicher, H. (1978). *Probability theory : independence, interchangeability, martingales*. Springer-Verlag, New York.

de Campos, L. M. (2006). A scoring function for learning Bayesian networks based on mutual information and conditional independence tests. *J. Machine Learning Research*, 7:2149–2187.

Ellis, B. and Wong, W. H. (2008). Learning causal Bayesian network structures from experimental data. *Journal of the American Statistical Association*, 103(482):778–789.

Elloumi, M. and Zomaya, A. Y. (2013). *Biological Knowledge Discovery Handbook: Preprocessing, Mining and Postprocessing of Biological Data*, volume 23. John Wiley & Sons.

Eppstein, M. and Hines, P. (2012). A "Random Chemistry" algorithm for identifying collections of multiple contingencies that initiate cascading failure. *IEEE Transactions on Power Systems*, 27(3):1698–1705.

Frank, A. and Asuncion, A. (2010). UCI machine learning repository.

Friedman, N. and Goldszmidt, M. (1997). Sequential update of bayesian network structure. In *Proceedings of the Thirteenth conference on Uncertainty in artificial intelli-*

gence, UAI'97, pages 165–174, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Friedman, N., Linial, M., Nachman, I., and Pe'er, D. (2000). Using Bayesian networks to analyze expression data. In *Proceedings of the Fourth Annual International Conference on Computational Molecular Biology*, RECOMB '00, pages 127–135, New York, NY, USA. ACM.

Geiger, D. and Pearl, J. (1988). On the logic of causal models. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 3–14.

Glüge, S., Hamid, O. H., and Wendemuth, A. (2010). A simple recurrent network for implicit learning of temporal sequences. *Cognitive Computation*, 2(4):265–271.

Glymour, C. (2003). Learning, prediction and causal Bayes nets. *Trends in cognitive sciences*, 7(1):43–48.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica*, 37(3):424–438.

Guyon, I., Aliferis, C. F., Cooper, G. F., Elisseeff, A., Pellet, J.-P., Spirtes, P., and Statnikov, A. R. (2008). Design and analysis of the causation and prediction challenge. In *IEEE World Congress on Computational Intelligence Causation and Prediction Challenge*, pages 1–33.

Hamilton, H. J. and Karimi, K. (2005). The timers ii algorithm for the discovery of causality. In *Proceedings of the 9th Pacific-Asia conference on Advances in Knowledge Discovery and Data Mining*, PAKDD'05, pages 744–750, Berlin, Heidelberg. Springer-Verlag.

Heckerman, D. (1995). A Bayesian approach to learning causal networks. In *Proceedings of the Eleventh conference on Uncertainty in artificial intelligence*, UAI'95, pages

285–295, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Heckerman, D. (1999). UCI machine learning repository [http://archive.ics.uci.edu/ml/datasets/msnbc.com+anonymous+web+data].

Henrion, M. (1991). Search-based methods to bound diagnostic probabilities in very large belief nets. In *Proceedings of the Seventh conference on Uncertainty in Artificial Intelligence*, UAI'91, pages 142–150, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Holme, P. and Saramäki, J. (2011). Temporal networks. *CoRR*, abs/1108.1780.

Ishii, H., Ma, Q., and Yoshikawa, M. (2010). An incremental method for causal network construction. In *Proceedings of the 11th international conference on Web-age information management*, WAIM'10, pages 495–506, Berlin, Heidelberg. Springer-Verlag.

Jiang, X.-R. and Gruenwald, L. (2005). Microarray gene expression data association rules mining based on BSC-tree and FIS-tree. *Data & Knowledge Engineering*, 53(1):3 – 29.

Johnson, T., Muthukrishnan, S., and Rozenbaum, I. (2007). Monitoring regular expressions on out-of-order streams. In *Proceedings of the IEEE 23rd International Conference on Data Engineering*, ICDE'07, pages 1315–1319.

Kemeny, J. and Snell, J. (1969). *Finite Markov chains*. University series in undergraduate mathematics. VanNostrand, New York, repr edition.

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220:671–680.

Kłopotek, M. A. (2006). Cyclic Bayesian network–Markov process approach. *Studia Informatica*, 1(2):7.

Krishna, R., Li, C.-T., and Buchanan-Wollaston, V. (2010). A temporal precedence based clustering method for gene expression microarray data. *BMC Bioinformatics*, 11:68.

Kullback, S. (1968). *Information Theory and Statistics*. Dover Publication, 2nd edition.

Kwon, O. and Li, K.-J. (2009). Causality join query processing for data streams via a spatiotemporal sliding window. *Journal of Universal Computer Science*, 15(12):2287–2310.

Lam, W. and Bacchus, F. (1994). Using new data to refine a Bayesian network. In *Proceedings of the Tenth international conference on Uncertainty in artificial intelligence*, UAI'94, pages 383–390, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Li, G. and Leong, T.-Y. (2009). Active learning for causal bayesian network structure with non-symmetrical entropy. In *Proceedings of the 13th Pacific-Asia Conference on Advances in Knowledge Discovery and Data Mining*, PAKDD '09, pages 290–301, Berlin, Heidelberg. Springer-Verlag.

Li, J., Tufte, K., Shkapenyuk, V., Papadimos, V., Johnson, T., and Maier, D. (2008). Out-of-order processing: A new architecture for high-performance stream systems. *Proceegins of the VLDB Endowment*, 1(1):274–288.

Li, M., Liu, M., Ding, L., Rundensteiner, E. A., and Mani, M. (2007). Event stream processing with out-of-order data arrival. In *Proceedings of the 27th International Conference on Distributed Computing Systems Workshops*, ICDCSW '07, pages 67–74, Washington, DC, USA. IEEE Computer Society.

Lin, T. Y., Xie, Y., Wasilewska, A., and Liau, C.-J., editors (2008). *Data Mining: Foundations and Practice*, volume 118 of *Studies in Computational Intelligence*. Springer.

Liu, M., Li, M., Golovnya, D., Rundensteiner, E., and Claypool, K. (2009). Sequence pattern query processing over out-of-order event streams. In *Proceedings of the IEEE 25th International Conference on Data Engineering*, ICDE '09, pages 784–795.

Liu, Y., Niculescu-Mizil, A., Lozano, A. C., and Lu, Y. (2010). Learning temporal causal graphs for relational time-series analysis. In *ICML*, pages 687–694. Omnipress.

MacKay, D. J. C. (1999). Introduction to Monte Carlo methods. In *Learning in graphical models*, pages 175–204. MIT Press, Cambridge, MA, USA.

Mani, S., Aliferis, C. F., and Statnikov, A. R. (2010). Bayesian algorithms for causal data mining. *Journal of Machine Learning Research*, 6:121–136.

Mazlack, L. J. (2004). Mining causality from imperfect data. In *Proceedings of the sixth International FLINS Conference on Applied Computational Intelligence Proceedings*, Applied Computational Intelligence, pages 155–160.

Meek, C. (1995). Causal inference and causal explanation with background knowledge. In *Proceedings of the Eleventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, pages 403–410, San Francisco, CA. Morgan Kaufmann.

Meganck, S., Leray, P., and Manderick, B. (2006). Learning causal bayesian networks from observations and experiments: a decision theoretic approach. In *Proceedings of the Third international conference on Modeling Decisions for Artificial Intelligence*, MDAI'06, pages 58–69, Berlin, Heidelberg. Springer-Verlag.

Meliou, A., Gatterbauer, W., Moore, K. F., and Suciu, D. (2010). Why so? or why no? functional causality for explaining query answers. In *MUD, 2010*, pages 3–17.

Mohammad, Y. and Nishida, T. (2010). Mining causal relationships in multidimensional time series. In Szczerbicki, E. and Nguyen, N., editors, *Smart Information and Knowledge Management*, volume 260 of *Studies in Computational Intelligence*, pages 309–338. Springer Berlin Heidelberg.

Moral, P., Doucet, A., and Jasra, A. (2012). An adaptive sequential Monte Carlo method for approximate Bayesian computation. *Statistics and Computing*, 22:1009–1020.

Pearl, J. (1995). Causal diagrams for empirical research. *Biometrika*, 82:669–710.

Pearl, J. (1998). Graphs, causality, and structural equation models. *Sociological Methods and Research*, 27:226–84.

Pearl, J. (2009). *Causality: Models, Reasoning and Inference*. Cambridge University Press, New York, NY, USA, second edition edition.

Pearl, J. and Verma, T. (1991). A theory of inferred causation. In *KR, 1991*, pages 441–452.

Popper, K. (1959). *The Logic of Scientific Discovery*. Routledge Classics.

Prakasa Rao, B. (2009). Conditional independence, conditional mixing and conditional association. *Annals of the Institute of Statistical Mathematics*, 61(2):441–460.

Rottman, B. M. and Hastie, R. (2014). Reasoning about causal relationships: Inferences on causal networks. *Psychological Bulletin*, 140(1):109–139.

Rudin, C., Letham, B., Salleb-Aouissi, A., Kogan, E., and Madigan, D. (2011). Sequential event prediction with association rules. In *Proceedings of the 24th Annual Conference on Learning Theory*, COLT '11, pages 615–634.

Shachter, R. D. (1990). Evidence absorption and propagation through evidence reversals. In *Proceedings of the Fifth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '89, pages 173–190, Amsterdam, The Netherlands.

Silverstein, C., Brin, S., Motwani, R., and Ullman, J. (2000). Scalable techniques for mining causal structures. *Data Mining and Knowledge Discovery*, 4(2-3):163–192.

Spirtes, P., Glymour, C., and Scheines, R. (1993). *Causation, Prediction, and Search*. Ima Volumes in Mathematics and Its Applications. Springer-Verlag.

Spirtes, P., Glymour, C., and Scheines, R. (2000). *Causation, Prediction, and Search*. MIT Press.

Spirtes, P., Glymour, C. N., and Scheines, R. (1990). Causality from probability. In *Proceedings of the Conference on Advanced Computing for the Social Sciences*, ACSS

'90.

Spirtes, P. and Meek, C. (1995). Learning Bayesian networks with discrete variables from data. In *Proceedings of the First International Conference on Knowledge Discovery and Data Mining*, KDD'95, pages 294–299.

Tsamardinos, I., Brown, L. E., and Aliferis, C. F. (2006). The max-min hill-climbing bayesian network structure learning algorithm. *Mach. Learn.*, 65(1):31–78.

Tulupyev, A. L. and Nikolenko, S. I. (2005). Directed cycles in Bayesian belief networks: probabilistic semantics and consistency checking complexity. In *Proceedings of the 4th Mexican international conference on Advances in Artificial Intelligence*, pages 214–223. Springer.

US-Canada Power System Outage Task Force (2004). Final Report on the August 14, 2003 Blackout in the United States and Canada. Technical report.

Utrera, A. C., Olmedo, M. G., and Callejon, S. M. (2008). A score based ranking of the edges for the pc algorithm. In *Proceedings of the 4th European workshop on probabilistic graphical models*, PGM'08, pages 41 – 48.

Vaiman, M., Bell, K., Chen, Y., Chowdhury, B., Dobson, I., Hines, P., Papic, M., Miller, S., and Zhang, P. (2012). Risk assessment of cascading outages: Methodologies and challenges. *IEEE Transactions on Power Systems*, 27(2):631–641.

Veloso, A. A., Almeida, H. M., Gonçalves, M. A., and Meira Jr., W. (2008). Learning to rank at query-time using association rules. In *Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, SIGIR '08, pages 267–274, New York, NY, USA. ACM.

Verma, T. and Pearl, J. (1988). Causal networks: Semantics and expressiveness. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 69–78.

Wang, K. and Yu, Y. (2013). A query-matching mechanism over out-of-order event stream in iot. *Int. J. Ad Hoc Ubiquitous Comput.*, 13(3/4):197–208.

Young, S. S. and Karr, A. (2011). Deming, data and observational studies. *Significance*, 8(3):116–120.

Zhang, N. L. and Poole, D. (1996). Exploiting causal independence in Bayesian network inference. *J. Artif. Int. Res.*, 5(1):301–328.

Zhang.N.L. and D., P. (1994). A simple approach to Bayesian network computations. In *Proceedings of the Tenth Canadian Conference on Artificial Intelligence*, CCAA '94, pages 171–178.

Zhao, Y. and Strom, R. (2001). Exploitng event stream interpretation in publish-subscribe systems. In *Proceedings of the Twentieth Annual ACM Symposium on Principles of Distributed Computing*, PODC '01, pages 219–228.

Chapter 3

Fast Causal Network Inference over Event Streams

Abstract

This paper addresses causal inference and modeling over event streams where data have high throughput and are unbounded. The availability of large amount of data along with the high data throughput present several new challenges related to causal modeling, such as the need for fast causal inference operations while ensuring consistent and valid results. There is no existing work specifically for such a streaming environment. We meet the challenges by introducing a time-centric causal inference strategy that leverages temporal precedence information to decrease the number of conditional independence tests required to establish the dependencies between the variables in a causal network. Dependency and temporal precedence of cause over effect are the two properties of a causal relationship. We also present the *Temporal Network Inference* algorithm to model the temporal precedence relations into a temporal network. Then, we propose the *Fast Causal Network Inference* algorithm for faster learning of causal network using the temporal network. Experiments using synthetic and real datasets demonstrate the efficacy of the proposed algorithms.

3.1 Introduction

In recent years, there has been a growing need for active systems that can perform causal inference in diverse applications such as health care, stock markets, user activity monitoring, smart electric grids, and network intrusion detection. These applications need to infer the cause of abnormal activities immediately such that informed and timely preventive measures are taken. As a case in point, consider a smart electric grid monitoring application. The failure of a component can cause cascading failures, effectively causing a massive blackout. Therefore, the identification of such cause and effect components in a timely manner enables preventive measures in the case of failure of a cause component, thereby preventing blackouts.

Causal network, a directed acyclic graph where the parent of each node is its direct cause, has been popularly used to model causality (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006; Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000). There are two distinct types of algorithms for learning a causal network: score-based (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006) and constraint-based (Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000; Cheng et al., 2002). Both types of algorithms are slow and, therefore, not suitable for event streams where prompt causal inference is required. Score-based algorithms perform a greedy search (usually hill climbing) to select a causal network with the highest score from a large number of possible networks. With an increase in the number of variables in the dataset, the number of possible networks grows exponentially, resulting in slow causal network inference. On the other hand, constraint-based algorithms (e.g., PC algorithm (Spirtes et al., 2000)) discover the causal structure via a large number of tests on conditional independence(CI). There can be no edge between two conditionally independent variables in the causal network (Pearl, 1995). In a causal network of n variables, two variables X and Y are said to be conditionally

independent given a condition set S if there is at least one variable in S such that X and Y are independent. The condition set S consists of all possible 2^{n-2} combinations of the remaining $n-2$ variables, and therefore the computational complexity grows exponentially as the number of variables increases. So, the current techniques for causal inference are slow and not suitable for event streams which have a high data throughput and where the number of variables (i.e., event types) is large.

With this concern, this paper describes a new time-centric causal modeling approach to speed up the causal network inference. Every causal relationship implies temporal precedence relationship (Popper, 1959). So, the idea is to incorporate temporal precedence information as an important clue to reducing the number of required CI tests and thus maintaining feasible computational complexity. This idea achieves fewer computations of CI test due to two factors. First, since causality requires temporal precedence, we ignore the causality test for those nodes with no temporal precedence relationship between them. Second, in the CI test of an edge, we exclude those nodes from the condition set which do not have temporal precedence relationship with the nodes of the edge. Therefore, it reduces the size of the condition set which is a major cause of the exponential computational complexity. In addition, the temporal precedence relationship intuitively orients the causal edge unlike the constraint-based algorithms where a separate set of rules are needed to infer the causal direction (details in Section 3.3.3).

The contributions of this paper are summarized as follows. First, it presents a temporal network structure to represent temporal precedence relationships between event types and proposes an algorithm, *Temporal Network Inference(TNI)*, to construct a temporal network applicable in streaming environment. Second, it introduces a time-centric causal modeling strategy and proposes an algorithm, *Fast Causal Network Inference(FCNI)*, to speed up the learning of causal network. Finally, it empirically demonstrates the advantages of the proposed algorithm in terms of the running time and the total number of CI tests required

for the learning of causal network by comparing it against the state-of-art algorithm for causal network inference, called the PC algorithm (details in Section 3.3.3).

The rest of this paper is organized as follows. Section 3.2 reviews the existing work on causal network inference. Section 3.3 presents the basic concepts used in the paper. Section 3.4 and Section 3.5 propose the learning of temporal network and faster causal network, respectively. Section 3.6 evaluates the proposed FNCI algorithm. Finally, Section 3.7 concludes the paper and mentions further research.

3.2 Related Work

As explained earlier, there are two main approaches for causal network inference.

The first approach, score-based (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006), performs greedy search (usually hill climbing) over all possible network structures in order to find the network that best represents the data based on the highest score. This approach, however, has two problems. First, it is slow due to the exhaustive search for the best network structure. An increase in the number of variables in the dataset increases the computational complexity exponentially. Second, two or more network structures, called the equivalence classes (Chickering, 2002), may represent the same probability distribution, and consequently the causal directions between nodes are quite random. There is no technique for alleviating these problems in a streaming environment. Thus, score-based algorithms are not viable for streams.

The second approach, constraint-based (Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000; Cheng et al., 2002), does not have the problem of equivalence classes. However, it is slow as it starts with a completely connected undirected graph and thus performs a large number of CI tests to remove the edges between conditionally independent nodes. The number of CI tests increases exponentially with the increase in the number of variables in the dataset. To alleviate this problem, some constraint-based algorithms start with

a minimum spanning tree to reduce the initial size of condition sets. However, this idea trades the speed with the accuracy of the causal inference. The constraint-based algorithms include IC* (Pearl, 2009), SGS (Spirtes et al., 1990), PC (Spirtes et al., 2000), and FCI algorithm (Spirtes et al., 2000). The FCI algorithm focuses on the causal network discovery from the dataset with latent variables and selection bias, which is quite different from the scope of this paper. The PC algorithm is computationally more efficient than IC* and SGS. This is why we evaluate the proposed *FCNI* algorithm by comparing it against the PC algorithm. Like the others, the PC algorithm starts with a completely connected undirected graph. To reduce the computational complexity, it performs CI tests in several steps. Each step produces a sparser graph than the earlier step, and consequently, the condition set decreases in the next step. However, the computational complexity is still $O(n^2 \cdot 2^{n-2})$. (The details are explained in Section 3.3.3.) Therefore, the current constraint-based algorithms are not suitable for fast causal inference over streams.

To the best of our knowledge, there exists no specific work in the causal network inference in a streaming environment. A new approach is needed for faster causal network inference.

3.3 Basic Concepts

This section presents some key concepts needed to understand the paper.

3.3.1 Event streams, type, and instance

An event stream in our work is a sequence of continuous and unbounded timestamped events. An event refers to any action that has an effect and is created by one event owner. One event can trigger another event in chain reactions. Each event instance belongs to one and only one event type which is a prototype for creating the instances. Two event instances are related to each other if they share common attributes such as event owner,

location, and time. We call these attributes *common relational attributes(CRAs)*.

In this paper we denote an event type as E_j and an event instance as e_{ij}, where i indicates the CRA and j indicates the event type.

Example 2 *Consider a diabetic patient monitoring system in a hospital. Each patient is uniquely identifiable, and each clinical test or measurement of each patient makes one event instance. For example, a patient is admitted to the hospital, has their blood pressure and glucose level measured, and takes medication over a period of time. This creates the instances of the above event types as a result. Typical event types from these actions include regular-insulin-dose-given, hypoglycemic-symptoms-exists, blood-glucose-measurement-decreased, increased, etc. Note that the patient ID is the CRA, as the events of the same patient are causally related.* □

3.3.2 Conditional Mutual Information

A popular approach for testing the conditional independence, with respect to the joint probability P, of two random variables X and Y given a subset of random variables S is conditional mutual information(CMI) (Cheng et al., 2002; de Campos, 2006). CMI gives the strength of dependency between variables in a measurable quantity, which helps to identify strong and weak causal relationships in the final causal network.

To test whether X and Y are conditionally independent given S, we compute the conditional mutual information $I_{MI}(X, Y|S)$ as

$$I_{MI}(X, Y|S) = \sum_{x \in X} \sum_{y \in Y} \sum_{s \in S} p_{X,Y,S}(x, y, s) log_2 \frac{p_{X,Y|S}(x, y|s)}{p_{X|S}(x|s) p_{Y|S}(y|s)}$$

where p is the probability mass function calculated from the frequencies of variables.

We only keep the record of these frequencies, not the whole events, by updating them as a new batch of events arrives. Consequently, the independence test procedure is incremental

in our case.

It is said that two variables X and Y are independent when $I_{MI}(X, Y|S) = 0$; otherwise, they are dependent. However, this presents us with the risk of spurious relationships due to weak dependencies (we cannot assume $I_{MI}(X, Y|S) = 10^{-5}$ and $I_{MI}(X, Y|S) = 10$ provide the same degree of confidence in the dependency). With an increase in the value of $I_{MI}(X, Y|S)$, the dependency between the variables X and Y grows stronger. Therefore, to prune out the weak dependencies, we need to set a threshold value of mutual information below which we ignore the evidence as weak. To do so, we relate CMI with G^2 test statistics (Spirtes et al., 2000; Bishop et al., 1975) as below where N_s is the number of samples.

$$G^2(X, Y|S) = 2 \cdot N_s \cdot log_e 2 \cdot I_{MI}(X, Y|S)$$

Under the independence assumption, G^2 follows the χ^2 distribution (Kullback, 1968), with the degree of freedom *df* equal to $(r_x - 1)(r_y - 1)\prod_{s \in S} r_s$, where r_x, r_y, and r_s are the number of possible distinct values of X, Y, and S, respectively. So, we use χ^2 test, which provides a threshold based on *df* and significance level α, to validate the dependency result. We set α as the universally accepted value of 95%.

3.3.3 The PC algorithm

The PC algorithm (Spirtes et al., 2000) (Algorithm 6) starts with a completely connected undirected graph on which the CI tests are performed to remove edges between independent nodes. The key idea is that a causal network has an edge between X and Y in the topology if and only if X and Y are not independent given all condition subsets of the remaining neighbor nodes (Spirtes and Meek, 1995). In Algorithm 6, the topology of the causal network is learned in the steps 1 to 10. The network topology is then assigned causal direction in the steps 11 to 17.

Algorithm 6 PC algorithm

1: Construct the completely connected undirected graph G on the n nodes;
2: Initialize Neighbors(G,X) as the set of nodes adjacent to the node X in G, and SepSet(X,Y), a set of nodes that causes independence between X and Y nodes, as empty;
3: $k \leftarrow 0$;
4: **repeat**
5: **repeat**
6: Select any edge $X - Y$ such that $|Neighbors(G, X)\backslash Y| \geq k$;
7: **repeat**
8: Select any subset S of $Neighbors(G, X)\backslash Y$ such that $|Neighbors(G, X)\backslash Y| = k$;
9: If X and Y are independent given S, remove $X - Y$ from G, remove Y from Neighbors(G,X), remove X from Neighbors(G,Y), and add S to SepSet(X,Y) and SepSet(Y,X);
10: **until** every subset S of $Neighbors(G, X)\backslash Y$ such that $|Neighbors(G, X)\backslash Y| = k$ has been selected.
11: **until** every edge $X - Y$ such that $|Neighbors(G, X)\backslash Y| \geq k$ has been selected.
12: k = k + 1;
13: **until** every edge $X' - Y'$ satisfies $|Neighbors(G, X')\backslash Y'| < k$.
14: **for** each triplet of nodes X, Y, Z such that the edges $X - Y$ and $Y - X$ exist in G but not $X - Z$ **do**
15: Orient $X - Y - Z$ as X→Y←Z if and only if SepSet(X,Z) does not contain Y;
16: **end for**
17: **repeat**
18: **If** there exists $X \rightarrow Y$ and $Y - Z$, but not $X - Z$, **then** orient $Y - Z$ as $Y \rightarrow Z$;
19: **If** there exists $X - Y$ and a directed path from X to Y, **then** orient $X - Y$ as $X \rightarrow Y$;
20: **until** no edge can be oriented.

3.4 Learning Temporal Precedence Relationships

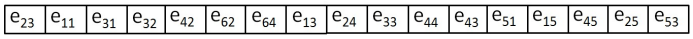
In this section, we describe an incremental approach to model temporal precedence relationships from time-stamped events into a *temporal network*.

3.4.1 Temporal Network Model

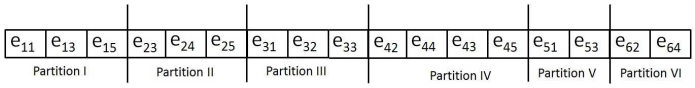
A temporal network is a directed acyclic graph of nodes representing event types where an edge between two nodes represents the temporal precedence relationship between them. To

facilitate the handling of events in a streaming environment, we use a time-based window over the stream. Typically, the application offers a natural observation period (e.g., hour) that makes a window.

As mentioned earlier in Section 3.3.1, two events are related to each other if they share the same *common relational attribute(CRA)*). So, the events in a window are arranged by CRA and ordered by the timestamp as they arrive, producing a *partitioned window* as a result. Figure 3.1 illustrates it.



(a) Events collected during an observation period (window).



(b) Events in the window partitioned by CRA.

Figure 3.1: Partitioned window of events.

With the arrival of a new batch of event instances, we augment each partition in the new window by prefixing it with the last instance of the partition with the same CRA value in the previous window. This is necessary in order to identify the temporal precedence between instances that are separated into the two consecutive batches.

To determine when an edge, say $E_i \rightarrow E_j$, should be added in a temporal network, a measure providing an evidence of temporal precedence between the event types should be defined. The evidence we use is the frequency of the observation of an instance of E_j following an instance of E_i. The *temporal strength* of an edge identified is defined below.

Definition 8 (Temporal strength) *Consider an edge $E_i \rightarrow E_j$ $(i \neq j)$ in a temporal network of n event types. Let f_{ij} be the total number of observations in which an event of type E_i precedes an event of type E_j over all partitions in the* partitioned window. *Then,*

we define temporal strength, s_{ij}, *of the edge* $E_i \rightarrow E_j$ *as*

$$s_{ij} \triangleq \frac{f_{ij}}{\sum_{k=0}^{n-1} f_{ik}}$$

3.4.2 Temporal Network Inference Algorithm

The idea behind the *TNI* algorithm is to collect events from an event stream in a *window* and then use temporal precedence information from the sequence of event pairs in the window to construct a temporal network at the event type level. The overall algorithm is centered on a *frequency matrix*, which is initially empty (i.e., all zero elements) and updated with each new batch of events. The algorithm has two steps for each window, outlined in Algorithm 7.

1. Update the frequency matrix *FM* by observing the precedence relationships of event pairs in the partitioned window (steps 3–13 in Algorithm 7). An element f_{ij} in *FM* reflects the total number of times events of type E_i precede events of type E_j ($i \neq j$). Each time an event pair (e_{oi}, e_{oj}) is observed in the event stream such that e_{oi} precedes e_{oj}, increase the value of f_{ij} by 1.

2. Determine the edges of the temporal network using the frequency matrix (steps 14–24 in Algorithm 7). For each pair of an edge and its reversed edge, select the edge with the greater frequency. Calculate the temporal strength of the selected edge, e.g., $E_i \rightarrow E_j$, and store it in the element s_{ij} of the *strength matrix SM*. Set the strength of the ignored edge with the lower frequency to zero. If a cycle is introduced, remove the edge with the lowest temporal strength in the cycle.

Algorithm 7 Temporal Network Inference (TNI)

Require: an edgeless network structure TN, event stream(s) S

Initialize an empty *frequency matrix (FM)*, an empty strength matrix SM, two empty buffers B_p and B_c (used to store "parent" events and "child" events, respectively);
for each window W in S **do**
  for each partition P (corresponding to CRA a) in W **do**
    for i = 1 to $t_n - 1$ where t_n is the number of unique timestamp in P **do**
      Clear B_p and B_c;
      Insert all events with timestamp t_i and t_{i+1} into B_p and B_c, respectively;
      for all event instances e_{ap} and e_{ac} in B_p and B_c, respectively, **do**
        if type(e_{ac}) $\neq$ type(e_{ap}) {*//There cannot be causal relationships between events of the same type.*} **then**
          Increase the frequency of element $f_{type(e_{ap}),type(e_{ac})}$ in FM by 1;
        end if
      end for
    end for
  end for
  for each pair of elements f_{ij} and f_{ji} in FM **do**
    $s_{ij} \leftarrow 0$, $s_{ji} \leftarrow 0$;
    if $f_{ij} > f_{ji}$ **then**
      Add an edge $E_i \rightarrow E_j$ in TN and set its strength to $s_{ij} = \frac{f_{ij}}{\sum_{k=0}^{n-1} f_{ik}}$;
    else if $f_{ji} > f_{ij}$ **then**
      Add an edge $E_j \rightarrow E_i$ in TN and set its strength to $s_{ji} = \frac{f_{ji}}{\sum_{k=0}^{n-1} f_{jk}}$;
    end if
  end for
  if an edge is added and it introduces cycle in TN **then**
    Remove the edge with the lowest temporal strength (in SM) in the cycle;
  end if
end for

3.5 Learning Causal Network in Reduced Time

In this section, we describe a new approach to reduce the number of CI tests needed to infer the causal structure, thereby speeding up the learning process. The idea is to incorporate temporal precedence relationships to learn the causal network. The correctness of our approach is shown as follows. First, a temporal precedence relationship is a mandatory condition for inferring causality (Popper, 1959). Therefore, causal relationship

subsumes temporal precedence relationship, that is, the causal network is a subgraph of the temporal network. Second, a causal network should satisfy the *Causal Markov Condition (CMC)* (Spirtes et al., 1990; Pearl, 1995; Pearl, 1998) where for every node X in the set of nodes N, X is independent of its non-descendants excluding its parents (i.e., $N \backslash (Descendants(X) \cup Parents(X))$) conditional on its parents. In a temporal network of vertex (or node) set N, a node is temporally independent, and therefore causally independent, of all its non-descendants (except its parents) given its parents. In other words, the temporal network obeys CMC which is a necessary condition for the causal network. Therefore, our idea of considering a temporal network as an initial causal network is correct.

3.5.1 Fast Causal Network Inference Algorithm

The idea behind FCNI algorithm is to reduce the number of CI tests by incorporating temporal precedence information. The algorithm has two steps, as outlined in Algorithm 8.

1. The first step is to construct a temporal network by running the *TNI* algorithm. The temporal network is set as the initial causal network. Note that since temporal precedence is a requirement for a causal relationship, all causal relationships are theoretically guaranteed be in the temporal network.

2. The second step is to adapt the ideas of constraint-based algorithms to learn the final causal network by pruning out the edges between independent nodes. We perform CI tests on every edge between nodes in the initial causal network to verify dependency between them. Conditionally independent nodes are considered to be spurious and hence the edge between them is removed. Steps 2 to 22 perform this step. The main difference from the PC algorithm is the manner in which CI tests are performed. In the PC algorithm, the condition set S for an edge $E_i - E_j$ considers the neighbors of both E_i and E_j whereas in the FCNI algorithm, as the edges are already directed, the condition set S for an edge $E_i \rightarrow E_j$ needs to consider only the parents of E_j (E_j

is independent of the parents of E_i that do not have edge to E_j). Consequently, we need fewer CI tests.

Algorithm 8 Fast Causal Network Inference (FCNI)

Require: *Window W*, *Edgeless Causal Network G = (N, ξ).*{N and ξ are the set of nodes and the set of edges, respectively.}

1: Run the *TNI* algorithm and initialize G with the learned temporal network;
2: **for** each directed edge $(E_i, E_j) \in \xi$ **do**
3: $independent = IsIndependent(E_i, E_j, \phi)$, where ϕ is the empty set; $\{IsIndependent(E_i, E_j, S)$ calculates $I_{MI}(E_i, E_j|S)$ for CI test.$\}$
4: **if** *independent* is true **then**
5: Remove (E_i, E_j) from ξ;
6: **end if**
7: **end for**
8: $k \leftarrow 0$;
9: **repeat**
10: **for** each directed edge $(E_i, E_j) \in \xi$ **do**
11: Construct a set of condition sets, Z, each of cardinality k from the *parents* of E_j excluding E_i;
12: **repeat**
13: Select any subset S from Z;
14: $independent = IsIndependent(E_i, E_j, S)$;
15: Remove S from Z;
16: **until** Z is empty or *independent* is true
17: **if** *independent* is true **then**
18: Remove (E_i, E_j) from ξ;
19: **end if**
20: **end for**
21: $k = k + 1$;
22: **until** number of parents of E'_j in every directed edge $(E'_i, E'_j) \in \xi$ is less than k.

3.5.2 Complexity Analysis

The complexity of the FNCI algorithm for a causal network G is bounded by the largest degree of each node. Let n be the number of nodes (i.e., event types). Then in the worst case, since the causal network inference starts with a temporal network, the number of CI tests required by the algorithm is given as

$$CI_{max} = \sum_{i=1}^{n} d_i 2^{|Z_i|}$$

where $d_i \equiv (i-1)$ is the maximum degree of incoming edges to the node i (there are $\sum_{i=1}^{n} d_i = \frac{n \cdot (n-1)}{2}$ directed edges in the network G) and Z_i is the maximum condition set to each edge involving node i such that $|Z_i| = d_i - 1 = i - 2$. So the computational complexity of *FCNI* algorithm is $O(n \cdot 2^{n-2})$ in the worst case. In contrast, the PC algorithm (described in Section 3.3.3), whose condition set of each node is of cardinality $n-2$ nodes, has the worst case computational complexity of $O(n^2 \cdot 2^{n-2})$. Therefore, in the worst case, the *FCNI* algorithm is n times faster than the PC algorithm.

In the best case, the causal network G takes the form of a minimum spanning tree with $n-1$ edges. In this case, the FCNI algorithm and the PC algorithm require $n-1$ and $4n-6$ CI tests, respectively.

Note that the FCNI algorithm starts with a sparse network as it has only those edges that satisfy the temporal precedence relationships. So, in practice, it starts closer to the best case. In contrast, the PC algorithm starts with a completely connected dense network. So, it starts from the worst case.

3.6 Performance Evaluation

We conducted experiments to compare the proposed FCNI algorithm against the PC algorithm in terms of the accuracy, the running time, and the number of CI tests required on both the algorithms. Section 3.6.1 describes the experiment setup, including the evaluation metrics and the platform used. Section 3.6.2 explains the datasets used and Section 3.6.3 presents the experiment results.

3.6.1 Experiment setup

3.6.1.1 Evaluation metrics.

Intuitively, the quality of causal network learning algorithms are best evaluated by examining how closely the constructed causal network structures resemble the target causal network. In this regard, we adopt the structural Hamming distance proposed by (Tsamardinos et al., 2006) as the quality metric of the output causal network. The nodes (i.e., event types) are fixed as given to the algorithms, and therefore the network structures are compared with respect to the edges between nodes. There are three kinds of possible errors in the causal network construction: reversed edges, missing edges, and spurious edges. We use the number of the erroneous edges of each kind as the evaluation metric.

3.6.1.2 Platform.

The experiments are conducted on RedHat Enterprise Linux 5 operating system using Java(TM) 2 Runtime Environment–SE 1.5.0_07 in Vermont Advanced Computing Core (VACC) cluster computers.

3.6.2 Datasets

Experiments are conducted using both synthetic and real datasets.

Synthetic datasets.

A synthetic dataset is reverse-engineered from a target causal network. Given the control parameters – the number of event owners n_o and the number of event types n, the idea is to generate a random causal network, and then convert the causal network to an event stream which reflects the underlying probability distribution of the causal network. In the interest of space, the details of the event stream generation are not described here. We assume that

the event owner is the *CRA*. The dataset is represented by a collection of files in which the events are shuffled according to the owner ID while preserving the temporal order. We create five datasets (see their profiles in Table 3.1), representing target causal networks of 4, 8, 12, 16 and 20 nodes each.

Dataset	**n**	$\mathbf{n}_{edges}$	$\mathbf{n}_{o}$	$\mathbf{n}_{instances}$
DS_1	4	4	5000	13108
DS_2	8	16	30000	108334
DS_3	12	32	500000	3163237
DS_4	16	46	6553600	49008538
DS_5	20	62	52428800	511972810

(n_{edges} is the number of *actual* edges in the network. $n_{instances}$ is the number of event instances in the dataset.)

Table 3.1: Profiles of the five synthetic datasets.

Real dataset.

The real dataset D_R contains diabetes lab test results (Frank and Asuncion, 2010) of 70 different patients over a period ranging from a few weeks to a few months. The dataset has a total 28143 records, about 402 records for each patient. Each record has four fields – date, time, test code, test value. The clinical data of a patient is independent of other patients. Therefore, the patient ID is the *CRA* for this dataset. There are 20 different test codes appearing in the file from which we define 13 different event types of interest. The details of the event types are omitted due to the space limit.

3.6.3 Experiment results

We ran the FCNI and PC algorithms over each of the five synthetic datasets and the real dataset. We present our evaluation results in three parts. First, we compare the quality of the generated networks against the target causal network and determine how closely they resemble the true causal network. (The details of the true causal networks are omitted due

to the space limit.) Specifically, we count the number of spurious edges, the number of missing edges, and the number of reversed edges. Second, we compare the running time (CPU time) of the two algorithms, and finally, we evaluate the number of CI tests performed on both algorithms. We show that reducing the number of CI tests is the key to reducing the running time of causal network inference. The experiment is repeated ten times for each dataset (DS_1 through DS_5 and D_R) to calculate the average number of erroneous edges, the average running time, and the average number of CI tests. We assume the events are in temporal order.

3.6.3.1 Comparison of the accuracy of the PC and FCNI algorithms.

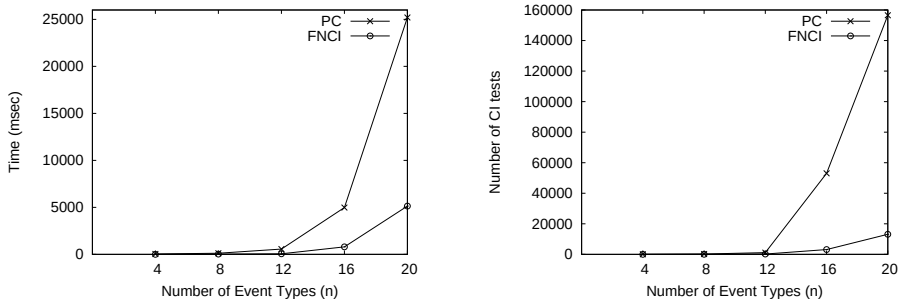
Table 3.2 presents the number of erroneous edges in the causal network produced by the PC and FCNI algorithms. The results show that the quality of the causal network from the FCNI algorithm is similar to that of the PC algorithm. First, the number of missing and spurious edges are comparable. This is due to the reliance of both algorithms on the same test statistics (CMI in our case) to infer the independence of two event types. Second, the number of reversed edges is zero for the FCNI algorithm. Clearly the FCNI algorithm, through the temporal network, is much better at determining the correct causal edge direction. It is because the fact that the cause always precedes its effect is embodied in its temporal precedence relationship. Overall, we conclude that the FCNI algorithm produces almost the same topology as the PC algorithm, while the accuracy of the causal direction is improved.

3.6.3.2 Comparison of the running time of the PC and FCNI algorithms.

Figure 3.2(a) plots the average running time of FCNI and PC algorithms against the number of event types (n) in the synthetic dataset. In all cases, the FCNI algorithm is much faster than the PC algorithm. Clearly, the temporal precedence information helps to reduce the size of condition set and the edges to test. As n increases, the running times of both PC

	Algorithms	DS_1	DS_2	DS_3	DS_4	DS_5	D_R
Missing Edges	*PC*	0	0	0	0	1	1
	FCNI	0	1	0	0	1	1
Reversed Edges	*PC*	0	2	0	2	3	2
	FCNI	0	0	0	0	0	0
Spurious Edges	*PC*	0	3	0	4	3	1
	FCNI	0	3	0	4	3	1

Table 3.2: Number of erroneous edges.



(a) Running time for varying number of event types.

(b) Number of CI tests for varying number of event types.

Figure 3.2: Comparison of the PC and FCNI algorithms

and FCNI algorithms increase. However, the rate of increase of the running time of the PC algorithm is much higher than that of the FCNI algorithm. Therefore, with an increase in n, the ratio of running time between the two algorithms increases. The same observation is made in the real dataset where the running time of the PC and FCNI algorithms are 817 and 118 msecs, respectively. These results verify the important role of temporal precedence relationships to reduce the running time.

3.6.3.3 Comparison of the number of CI tests of the PC and FCNI algorithms.

Figure 3.2(b) shows that the FCNI algorithm performs fewer CI tests than the PC algorithm in all synthetic datasets. The CI tests required are minimized, due to the temporal precedence information, by reducing the size of the condition set and the number of edges

to test. With an increase in the number of event types (n), the rate of increase of the number of CI tests in the PC algorithm is much higher than that in the FCNI algorithm. A similar observation is made in the real dataset where the number of CI tests of the PC and FCNI algorithms are 1239 and 192, respectively. These results confirm the important role of temporal precedence relationships in reducing the number of CI tests. Note the result of CI tests (Figure 3.2(b)) looks almost the same as that of the running time (Figure 3.2(a)). This demonstrates that CI tests are the major performance bottleneck and validates the key idea of our work that reducing the number of CI tests reduces the run time.

3.7 Conclusion and Future Work

In this paper, we presented a novel strategy to incorporate temporal precedence relationships to learn the causal network over event streams. First, we introduced the *Temporal Network Inference* algorithm to model temporal precedence information. Then, we presented the *Fast Causal Network Inference* algorithm to reduce the running time complexity of learning causal network by eliminating unnecessary CI tests. We showed the experiment results to validate our approach by comparing against the state-of-the-art PC algorithm. For the future work, we plan to explore the temporal semantics further for causal network inference over out-of-order event streams.

Bibliography

Acharya, S. and Lee, B. (2013). Fast causal network inference over event streams. In *Data Warehousing and Knowledge Discovery*, volume 8057 of *Lecture Notes in Computer Science*, pages 222–235. Springer Berlin Heidelberg.

Acharya, S. and Lee, B. S. (2014a). Enchanced fast causal network inference over event streams. *Transactions on Large Scale Data and Knowledge Centered Systems*.

Acharya, S. and Lee, B. S. (2014b). Incremental causal network construction over event streams. *Information Sciences*, 261(0):32 – 51.

Acharya, S., Lee, B. S., and Hines, P. (2014). Real-time top-k predictive query processing over event streams. *Information Sciences*.

Akdere, M., Çetintemel, U., and Upfal, E. (2010). Database-support for continuous prediction queries over streaming data. *Proc. VLDB Endow.*, 3(1-2):1291–1301.

Alcobé, J. R. (2004a). Incremental hill-climbing search applied to Bayesian network structure learning. In *First International Workshop on Knowledge Discovery in Data Streams, 2004*.

Alcobé, J. R. (2004b). *Incremental Methods for Bayesian Network Structure Learning*. PhD thesis, Department of Computer Science, Universitat Politècnica de Catalunya.

Alcobé, J. R. (2005). Incremental methods for Bayesian network structure learning. *Artificial Intelligence Communications*, 18(1):61–62.

Barber, D. (2012). *Bayesian Reasoning and Machine Learning*. Cambridge University Press.

Barga, R. S., Goldstein, J., Ali, M. H., and Hong, M. (2007). Consistent streaming through time: A vision for event stream processing. In *Proceedings of the Third Biennial Conference on Innovative Data Systems Research*, CIDR'07, pages 363–374.

Bishop, Y. M., Fienberg, S. E., and Holland, P. W. (1975). *Discrete Multivariate Analysis: Theory and Practice*. MIT Press.

Borchani, H., Chaouachi, M., and Ben Amor, N. (2007). Learning causal bayesian networks from incomplete observational data and interventions. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, ECSQARU '07, pages 17–29, Berlin, Heidelberg. Springer-Verlag.

Bowes, J., Neufeld, E., Greer, J., and Cooke, J. (2000). A comparison of association rule discovery and bayesian network causal inference algorithms to discover relationships in discrete data. In Hamilton, H., editor, *Advances in Artificial Intelligence*, volume 1822 of *Lecture Notes in Computer Science*, pages 326–336. Springer Berlin Heidelberg.

Buntine, W. (1991). Theory refinement on Bayesian networks. In *Proceedings of the seventh conference (1991) on Uncertainty in artificial intelligence*, pages 52–60, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Chandra, P. and Kshemkalyani, A. D. (2005). Causality-based predicate detection across space and time. *IEEE Transactions on Computerts*, 54:1438–1453.

Cheng, J. and Druzdzel, M. J. (2000). Ais-bn: An adaptive importance sampling algorithm for evidential reasoning in large Bayesian networks. *J. Artif. Intell. Res. (JAIR)*, 13:155–188.

Cheng, J. and Druzdzel, M. J. (2001). Confidence inference in Bayesian networks. In *Proceedings of the Seventeenth conference on Uncertainty in artificial intelligence*,

UAI'01, pages 75–82, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Cheng, J., Greiner, R., Kelly, J., Bell, D., and Liu, W. (2002). Learning Bayesian networks from data: an information-theory based approach. *Artificial Intelligence*, 137(1-2):43–90.

Chickering, D. M. (2002). Learning equivalence classes of Bayesian-network structures. *Journal of Machine Learning Research*, 2:445–498.

Chickering, D. M., Heckerman, D., and Meek, C. (2004). Large-sample learning of bayesian networks is np-hard. *J. Mach. Learn. Res.*, 5:1287–1330.

Chow, Y. S. and Teicher, H. (1978). *Probability theory : independence, interchangeability, martingales*. Springer-Verlag, New York.

de Campos, L. M. (2006). A scoring function for learning Bayesian networks based on mutual information and conditional independence tests. *J. Machine Learning Research*, 7:2149–2187.

Ellis, B. and Wong, W. H. (2008). Learning causal Bayesian network structures from experimental data. *Journal of the American Statistical Association*, 103(482):778–789.

Elloumi, M. and Zomaya, A. Y. (2013). *Biological Knowledge Discovery Handbook: Preprocessing, Mining and Postprocessing of Biological Data*, volume 23. John Wiley & Sons.

Eppstein, M. and Hines, P. (2012). A "Random Chemistry" algorithm for identifying collections of multiple contingencies that initiate cascading failure. *IEEE Transactions on Power Systems*, 27(3):1698–1705.

Frank, A. and Asuncion, A. (2010). UCI machine learning repository.

Friedman, N. and Goldszmidt, M. (1997). Sequential update of bayesian network structure. In *Proceedings of the Thirteenth conference on Uncertainty in artificial intelli-*

gence, UAI'97, pages 165–174, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Friedman, N., Linial, M., Nachman, I., and Pe'er, D. (2000). Using Bayesian networks to analyze expression data. In *Proceedings of the Fourth Annual International Conference on Computational Molecular Biology*, RECOMB '00, pages 127–135, New York, NY, USA. ACM.

Geiger, D. and Pearl, J. (1988). On the logic of causal models. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 3–14.

Glüge, S., Hamid, O. H., and Wendemuth, A. (2010). A simple recurrent network for implicit learning of temporal sequences. *Cognitive Computation*, 2(4):265–271.

Glymour, C. (2003). Learning, prediction and causal Bayes nets. *Trends in cognitive sciences*, 7(1):43–48.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica*, 37(3):424–438.

Guyon, I., Aliferis, C. F., Cooper, G. F., Elisseeff, A., Pellet, J.-P., Spirtes, P., and Statnikov, A. R. (2008). Design and analysis of the causation and prediction challenge. In *IEEE World Congress on Computational Intelligence Causation and Prediction Challenge*, pages 1–33.

Hamilton, H. J. and Karimi, K. (2005). The timers ii algorithm for the discovery of causality. In *Proceedings of the 9th Pacific-Asia conference on Advances in Knowledge Discovery and Data Mining*, PAKDD'05, pages 744–750, Berlin, Heidelberg. Springer-Verlag.

Heckerman, D. (1995). A Bayesian approach to learning causal networks. In *Proceedings of the Eleventh conference on Uncertainty in artificial intelligence*, UAI'95, pages

285–295, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Heckerman, D. (1999). UCI machine learning repository [http://archive.ics.uci.edu/ml/datasets/msnbc.com+anonymous+web+data].

Henrion, M. (1991). Search-based methods to bound diagnostic probabilities in very large belief nets. In *Proceedings of the Seventh conference on Uncertainty in Artificial Intelligence*, UAI'91, pages 142–150, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Holme, P. and Saramäki, J. (2011). Temporal networks. *CoRR*, abs/1108.1780.

Ishii, H., Ma, Q., and Yoshikawa, M. (2010). An incremental method for causal network construction. In *Proceedings of the 11th international conference on Web-age information management*, WAIM'10, pages 495–506, Berlin, Heidelberg. Springer-Verlag.

Jiang, X.-R. and Gruenwald, L. (2005). Microarray gene expression data association rules mining based on BSC-tree and FIS-tree. *Data & Knowledge Engineering*, 53(1):3 – 29.

Johnson, T., Muthukrishnan, S., and Rozenbaum, I. (2007). Monitoring regular expressions on out-of-order streams. In *Proceedings of the IEEE 23rd International Conference on Data Engineering*, ICDE'07, pages 1315–1319.

Kemeny, J. and Snell, J. (1969). *Finite Markov chains*. University series in undergraduate mathematics. VanNostrand, New York, repr edition.

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220:671–680.

Kłopotek, M. A. (2006). Cyclic Bayesian network–Markov process approach. *Studia Informatica*, 1(2):7.

Krishna, R., Li, C.-T., and Buchanan-Wollaston, V. (2010). A temporal precedence based clustering method for gene expression microarray data. *BMC Bioinformatics*, 11:68.

Kullback, S. (1968). *Information Theory and Statistics*. Dover Publication, 2nd edition.

Kwon, O. and Li, K.-J. (2009). Causality join query processing for data streams via a spatiotemporal sliding window. *Journal of Universal Computer Science*, 15(12):2287–2310.

Lam, W. and Bacchus, F. (1994). Using new data to refine a Bayesian network. In *Proceedings of the Tenth international conference on Uncertainty in artificial intelligence*, UAI'94, pages 383–390, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Li, G. and Leong, T.-Y. (2009). Active learning for causal bayesian network structure with non-symmetrical entropy. In *Proceedings of the 13th Pacific-Asia Conference on Advances in Knowledge Discovery and Data Mining*, PAKDD '09, pages 290–301, Berlin, Heidelberg. Springer-Verlag.

Li, J., Tufte, K., Shkapenyuk, V., Papadimos, V., Johnson, T., and Maier, D. (2008). Out-of-order processing: A new architecture for high-performance stream systems. *Proceegins of the VLDB Endowment*, 1(1):274–288.

Li, M., Liu, M., Ding, L., Rundensteiner, E. A., and Mani, M. (2007). Event stream processing with out-of-order data arrival. In *Proceedings of the 27th International Conference on Distributed Computing Systems Workshops*, ICDCSW '07, pages 67–74, Washington, DC, USA. IEEE Computer Society.

Lin, T. Y., Xie, Y., Wasilewska, A., and Liau, C.-J., editors (2008). *Data Mining: Foundations and Practice*, volume 118 of *Studies in Computational Intelligence*. Springer.

Liu, M., Li, M., Golovnya, D., Rundensteiner, E., and Claypool, K. (2009). Sequence pattern query processing over out-of-order event streams. In *Proceedings of the IEEE 25th International Conference on Data Engineering*, ICDE '09, pages 784–795.

Liu, Y., Niculescu-Mizil, A., Lozano, A. C., and Lu, Y. (2010). Learning temporal causal graphs for relational time-series analysis. In *ICML*, pages 687–694. Omnipress.

MacKay, D. J. C. (1999). Introduction to Monte Carlo methods. In *Learning in graphical models*, pages 175–204. MIT Press, Cambridge, MA, USA.

Mani, S., Aliferis, C. F., and Statnikov, A. R. (2010). Bayesian algorithms for causal data mining. *Journal of Machine Learning Research*, 6:121–136.

Mazlack, L. J. (2004). Mining causality from imperfect data. In *Proceedings of the sixth International FLINS Conference on Applied Computational Intelligence Proceedings*, Applied Computational Intelligence, pages 155–160.

Meek, C. (1995). Causal inference and causal explanation with background knowledge. In *Proceedings of the Eleventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, pages 403–410, San Francisco, CA. Morgan Kaufmann.

Meganck, S., Leray, P., and Manderick, B. (2006). Learning causal bayesian networks from observations and experiments: a decision theoretic approach. In *Proceedings of the Third international conference on Modeling Decisions for Artificial Intelligence*, MDAI'06, pages 58–69, Berlin, Heidelberg. Springer-Verlag.

Meliou, A., Gatterbauer, W., Moore, K. F., and Suciu, D. (2010). Why so? or why no? functional causality for explaining query answers. In *MUD, 2010*, pages 3–17.

Mohammad, Y. and Nishida, T. (2010). Mining causal relationships in multidimensional time series. In Szczerbicki, E. and Nguyen, N., editors, *Smart Information and Knowledge Management*, volume 260 of *Studies in Computational Intelligence*, pages 309–338. Springer Berlin Heidelberg.

Moral, P., Doucet, A., and Jasra, A. (2012). An adaptive sequential Monte Carlo method for approximate Bayesian computation. *Statistics and Computing*, 22:1009–1020.

Pearl, J. (1995). Causal diagrams for empirical research. *Biometrika*, 82:669–710.

Pearl, J. (1998). Graphs, causality, and structural equation models. *Sociological Methods and Research*, 27:226–84.

Pearl, J. (2009). *Causality: Models, Reasoning and Inference*. Cambridge University Press, New York, NY, USA, second edition edition.

Pearl, J. and Verma, T. (1991). A theory of inferred causation. In *KR, 1991*, pages 441–452.

Popper, K. (1959). *The Logic of Scientific Discovery*. Routledge Classics.

Prakasa Rao, B. (2009). Conditional independence, conditional mixing and conditional association. *Annals of the Institute of Statistical Mathematics*, 61(2):441–460.

Rottman, B. M. and Hastie, R. (2014). Reasoning about causal relationships: Inferences on causal networks. *Psychological Bulletin*, 140(1):109–139.

Rudin, C., Letham, B., Salleb-Aouissi, A., Kogan, E., and Madigan, D. (2011). Sequential event prediction with association rules. In *Proceedings of the 24th Annual Conference on Learning Theory*, COLT '11, pages 615–634.

Shachter, R. D. (1990). Evidence absorption and propagation through evidence reversals. In *Proceedings of the Fifth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '89, pages 173–190, Amsterdam, The Netherlands.

Silverstein, C., Brin, S., Motwani, R., and Ullman, J. (2000). Scalable techniques for mining causal structures. *Data Mining and Knowledge Discovery*, 4(2-3):163–192.

Spirtes, P., Glymour, C., and Scheines, R. (1993). *Causation, Prediction, and Search*. Ima Volumes in Mathematics and Its Applications. Springer-Verlag.

Spirtes, P., Glymour, C., and Scheines, R. (2000). *Causation, Prediction, and Search*. MIT Press.

Spirtes, P., Glymour, C. N., and Scheines, R. (1990). Causality from probability. In *Proceedings of the Conference on Advanced Computing for the Social Sciences*, ACSS

'90.

Spirtes, P. and Meek, C. (1995). Learning Bayesian networks with discrete variables from data. In *Proceedings of the First International Conference on Knowledge Discovery and Data Mining*, KDD'95, pages 294–299.

Tsamardinos, I., Brown, L. E., and Aliferis, C. F. (2006). The max-min hill-climbing bayesian network structure learning algorithm. *Mach. Learn.*, 65(1):31–78.

Tulupyev, A. L. and Nikolenko, S. I. (2005). Directed cycles in Bayesian belief networks: probabilistic semantics and consistency checking complexity. In *Proceedings of the 4th Mexican international conference on Advances in Artificial Intelligence*, pages 214–223. Springer.

US-Canada Power System Outage Task Force (2004). Final Report on the August 14, 2003 Blackout in the United States and Canada. Technical report.

Utrera, A. C., Olmedo, M. G., and Callejon, S. M. (2008). A score based ranking of the edges for the pc algorithm. In *Proceedings of the 4th European workshop on probabilistic graphical models*, PGM'08, pages 41 – 48.

Vaiman, M., Bell, K., Chen, Y., Chowdhury, B., Dobson, I., Hines, P., Papic, M., Miller, S., and Zhang, P. (2012). Risk assessment of cascading outages: Methodologies and challenges. *IEEE Transactions on Power Systems*, 27(2):631–641.

Veloso, A. A., Almeida, H. M., Gonçalves, M. A., and Meira Jr., W. (2008). Learning to rank at query-time using association rules. In *Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, SIGIR '08, pages 267–274, New York, NY, USA. ACM.

Verma, T. and Pearl, J. (1988). Causal networks: Semantics and expressiveness. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 69–78.

Wang, K. and Yu, Y. (2013). A query-matching mechanism over out-of-order event stream in iot. *Int. J. Ad Hoc Ubiquitous Comput.*, 13(3/4):197–208.

Young, S. S. and Karr, A. (2011). Deming, data and observational studies. *Significance*, 8(3):116–120.

Zhang, N. L. and Poole, D. (1996). Exploiting causal independence in Bayesian network inference. *J. Artif. Int. Res.*, 5(1):301–328.

Zhang.N.L. and D., P. (1994). A simple approach to Bayesian network computations. In *Proceedings of the Tenth Canadian Conference on Artificial Intelligence*, CCAA '94, pages 171–178.

Zhao, Y. and Strom, R. (2001). Exploitng event stream interpretation in publish-subscribe systems. In *Proceedings of the Twentieth Annual ACM Symposium on Principles of Distributed Computing*, PODC '01, pages 219–228.

Chapter 4

Enhanced Fast Causal Network Inference over Event Streams

Abstract

This paper addresses causal inference and modeling over event streams where data have high throughput, are unbounded, and may arrive out of order. The availability of large amount of data with these characteristics presents several new challenges related to causal modeling, such as the need for fast causal inference operations while ensuring consistent and valid results. There is no existing work specifically for such a streaming environment. We meet the challenges by introducing a time-centric causal inference strategy which leverages temporal precedence information to decrease the number of conditional independence tests required to establish the causalities between variables in a causal network. (Dependency and temporal precedence of cause over effect are the two properties of a causal relationship.) Moreover, we employ change-driven causal network inference to safely reduce the running time further. In this paper we present the Order-Aware Temporal Network Inference algorithm to model the temporal precedence relationships into a temporal network and then propose the Enhanced Fast Causal Network Inference algorithm for learning a causal network faster using the

temporal network. Experiments using synthetic and real datasets demonstrate the efficacy of the proposed algorithms.

4.1 Introduction

In recent years, there has been a growing need for active systems that can perform causal inference in diverse applications such as health care, stock markets, user activity monitoring, smart electric grids, and network intrusion detection. These applications need to infer the cause of abnormal activities immediately from their event streams, where the event arrival may be in order (e.g., (Barga et al., 2007; Zhao and Strom, 2001)) or out of order (e.g., (Johnson et al., 2007; Li et al., 2007; Liu et al., 2009; Li et al., 2008; Wang and Yu, 2013)) such that informed and timely preventive measures can be taken. As a case in point, consider a smart electric grid monitoring application. The failure of a component can cause cascading failures, effectively causing a massive blackout. The identification of such cause and effect components in a timely manner enables preventive measures in the case of failure of a cause component, thereby preventing blackouts.

Causal network, a directed acyclic graph where the parent of each node is its direct cause, has been popularly used to model causality (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006; Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000). There are two distinct types of algorithms for learning a causal network: score-based (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006) and constraint-based (Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000; Cheng et al., 2002). Both types of algorithms are slow and, therefore, not suitable for event streams where prompt causal inference is required. Score-based algorithms perform a greedy search (usually hill climbing) to select a causal network with the highest score from a large number of possible networks. With an increase in the number of variables in the dataset, the number of possible networks grows exponentially, resulting in slow causal network inference.

On the other hand, constraint-based algorithms (e.g., PC algorithm (Spirtes et al., 2000)) discover the causal structure via a large number of tests on conditional independence(CI). There can be no edge between two conditionally independent variables in the causal network (e.g., (Pearl, 1995; Spirtes and Meek, 1995)). Two variables X and Y are said to be conditionally independent given a condition set S if there is at least one variable in S such that X and Y are independent (e.g., (Chow and Teicher, 1978; Prakasa Rao, 2009)). In a causal network of n variables, the condition set S consists of all possible 2^{n-2} combinations of the remaining $n-2$ variables, and therefore the computational complexity grows exponentially as the number of variables increases. So, the current techniques for causal inference are slow and not suitable for event streams which have a high data throughput and where the number of variables (i.e., event types) is large. Besides, these techniques perform the time-consuming causal inference computations every time a new batch of events arrives even though there may not be significant enough changes in the event stream statistic.

With these concerns, this paper describes a new time-centric causal modeling approach to speed up the causal network inference. Every causal relationship implies temporal precedence relationship (e.g., (Popper, 1959; Hamilton and Karimi, 2005)). So, the idea is to exploit temporal precedence information as an important clue to reducing the number of required CI tests and thus maintaining feasible computational complexity. Four strategies are employed utilizing this idea to achieve fewer computations of CI tests. First, since causality requires temporal precedence, we ignore the causality test for those nodes with no temporal precedence relationship between them. Second, in the CI test of an edge, we exclude those nodes from the condition set which do not have temporal precedence relationship with the nodes of the edge; this strategy reduces the size of the condition set which is a major cause of the exponential computational complexity. Third, we perform the CI tests for weaker edges (i.e., having lower temporal strength) earlier to reduce the size of the condition set of stronger edges, thereby reducing the overall number of CI tests. The rationale for this

is that weaker edges are more likely to be eliminated than stronger edges (Utrera et al., 2008). Fourth, we perform the causal inference computations only if there is a significant enough change in the temporal precedence relationships, which is a necessary condition for a change to occur in the resulting causal relationships. Such a change detection strategy helps to avoid unnecessary causal inference computations, and therefore, saves time.

Due to the reliance on the temporal precedence relationships in an event stream, events arriving *out of order* can bring ambiguities in the resulting causal directions. For instance, the precedence relationships represented in an edge and its reversed edge in a temporal network, which models the temporal precedence relationships, may not be significantly different enough to determine the edge direction. Intuitively, an undirected edge can be used to signify such an ambiguity. Thus, we propose a mechanism to decide between directed and undirected edge in the temporal network in such cases. Note that the constraint-based algorithms like the PC algorithm naturally handle out-of-order event arrivals, as these algorithms do not depend on the temporal ordering of events, and so they can provide a suitable baseline to evaluate the handling of out-of-order events in our proposed method.

The main contributions of this paper are summarized as follows. First, it presents a temporal network structure to represent temporal precedence relationships between event types and proposes an algorithm, *Order-Aware Temporal Network Inference (OATNI)*, to construct a temporal network applicable in the streaming environment. Second, it introduces a time-centric causal modeling strategy and proposes an algorithm, *Enhanced Fast Causal Network Inference (EFCNI)*, to speed up the learning of causal network. Third, it empirically demonstrates the advantages of the proposed algorithm in terms of the accuracy and speed of learning the causal network by comparing it against two state-of-art algorithms, the PC algorithm (details in Sections 4.3.4) and the FCNI algorithm (Acharya and Lee, 2013).

This paper contains the results of a comprehensive study extended from our earlier

work (Acharya and Lee, 2013). The two algorithms in our prior work, the *Temporal Network Inference* (TNI) and the *Fast Causal Network Inference* (FCNI), are extended to the OATNI and the FECNI algorithms, respectively. Specifically, two major extensions have been made. First, the speed of the causal inference mechanism has been increased with two strategies. As the first strategy, the CI tests are performed in the increasing order of the temporal strengths of the edges in order to remove the most probable spurious edge as early as possible, which decreases the condition set size. As the second strategy, presumably unnecessary causal inference computations are avoided by determining whether the changes in temporal precedence information in the event stream are significant enough to warrant such computations. Second, the previous work made an assumption that the event stream is in order. In this paper, the support for fast causal modeling over an out-of-order event stream is added so that the temporal precedence relationships cannot be relied upon as they are. In addition to these two major extensions, the presentation has been extended throughout in many parts of the paper.

The rest of this paper is organized as follows. Section 4.2 reviews the existing work on causal network inference. Section 4.3 presents the basic concepts used in the paper. Section 4.4 and Section 4.5 propose the learning algorithms of temporal network (OATNI) and faster causal network (EFCNI), respectively. Section 4.6 evaluates the proposed EFNCI algorithm. Section 4.7 concludes the paper and suggests further research.

4.2 Related Work

As explained earlier, there are two main approaches for causal network inference.

The first approach, score-based (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006), performs greedy search (usually hill climbing) over all possible network structures in order to find the network that best represents the data based on the highest score. This approach, however, has two problems. First, it is slow due to the

exhaustive search for the best network structure. An increase in the number of variables in the dataset increases the computational complexity exponentially. Second, two or more network structures, called the equivalence classes (Chickering, 2002), may represent the same probability distribution, and consequently the causal directions between nodes are quite random. There is no technique for alleviating these problems in a streaming environment. Thus, score-based algorithms are not suitable for streams.

The second approach, constraint-based (Pearl, 2009; Spirtes et al., 1990; Spirtes et al., 2000; Cheng et al., 2002), does not have the problem of equivalence classes. However, it is slow as it starts with a completely connected undirected graph and thus performs a large number of CI tests to remove the edges between conditionally independent nodes. The number of CI tests increases exponentially with the increase in the number of variables in the dataset. To alleviate this problem, some constraint-based algorithms start with a minimum spanning tree to reduce the initial size of condition sets. However, this idea trades the speed with the accuracy of the causal inference. The constraint-based algorithms include IC* (Pearl, 2009), SGS (Spirtes et al., 1990), PC (Spirtes et al., 2000), and FCI algorithm (Spirtes et al., 2000). The FCI algorithm focuses on the causal network discovery from the dataset with latent variables and selection bias, which is quite different from the scope of this paper. The PC algorithm is computationally more efficient than IC* and SGS. This is why we evaluate the proposed *EFCNI* algorithm by comparing it against the PC algorithm. Like the others, the PC algorithm starts with a completely connected undirected graph. To reduce the computational complexity, it performs CI tests in several steps. Each step produces a sparser graph than the earlier step, and consequently, the condition set decreases in the next step. However, the computational complexity is still $O(n^2 \cdot 2^{n-2})$. (The details are explained in Section 4.3.4.) Therefore, the current constraint-based algorithms are not suitable for fast causal inference over streams.

There have been a number of research works on handing out-of-order event streams (John-

son et al., 2007; Li et al., 2007; Liu et al., 2009; Li et al., 2008; Wang and Yu, 2013). To the best of our knowledge, however, there exists no work applicable to the causal network inference. (Thus, a new approach is needed, and our approach is to allow *undirected edges* in the temporal network.) Johnson et al. (Johnson et al., 2007) propose an algorithm for regular expression matching on streams with out-of-order data, which is not related to causal inference. The works by Li et al. (Li et al., 2007) and Liu et al. (Liu et al., 2009) discuss the problem of processing event pattern queries over event streams that may contain out-of-order data. Li et al. (Li et al., 2008) present a new architecture for stream systems for out-of-order query processing whereas Wang and Yu (Wang and Yu, 2013) propose algorithms for generating and matching queries to raise accuracy and shorten the response time as much as possible over out-of-order events. None of these works is related to causal network inference.

4.3 Basic Concepts

This section presents some key concepts needed to understand the paper.

4.3.1 Event streams

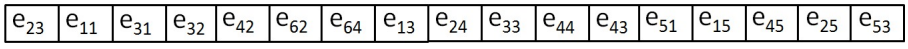
An event stream in our work is a sequence of continuous and unbounded timestamped events. An event refers to any action that has an effect and is created by an event owner. One event can trigger another event in chain reactions. Each event instance belongs to one and only one event type which is a prototype for creating the instances. Two event instances are related to each other if they share common attributes such as event owner, location, and time. We call these attributes *common relational attributes (CRAs).*

In this paper we denote an event type as E_j and an event instance as e_{ij}, where i indicates the *CRA* and j indicates the event type.

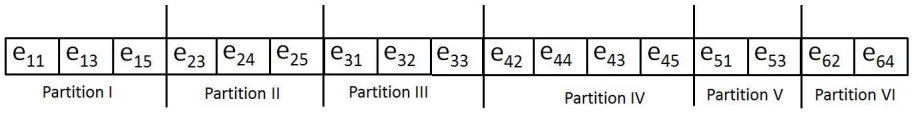
Example 3 *Consider a diabetic patient monitoring system in a hospital. Each patient is uniquely identifiable, and each clinical test or measurement of each patient makes one event instance. For example, a patient is admitted to the hospital, has their blood pressure and glucose level measured, and takes medication over a period of time. This creates the instances of the above event types as a result. Typical event types from these actions include hypoglycemic-symptoms-exists, blood-glucose-measurement-decreased, increased, regular-insulin-dose-given, etc. Note that the patient ID is the CRA, as the events of the same patient are causally related.*

To facilitate the handling of events in a streaming environment, we use a time-based window over the stream. Typically, the application offers a natural observation period (e.g., hour) that makes a window. The causal relationship is only possible between events with the same CRA. Therefore, the events in a window are arranged by CRA and then ordered by the timestamp as they arrive, producing a *partitioned window* as a result. Figure 4.1 illustrates it. (We refer to the *partitioned window* simply as the *window* for the rest of the paper.)

With the arrival of a new batch of event instances, we augment each partition in the new window by prefixing it with the last instance of the partition with the same CRA value in the previous window. This is necessary in order to identify the related instances that are separated into the two consecutive batches.



(a) Events collected during an observation period (window).



(b) Events in the window partitioned by CRA.

Figure 4.1: Partitioned window of events.

We support event streams which may be in order or out of order. An event stream is said

to be in order if and only if every event in every partition arrives in the same temporal order as it was created. In other words, the stream is out of order if any event in any partition arrives in a different temporal order than it was created. The degree of out-of-order, d_{oo}, is given as

$$d_{oo} = \frac{\sum_{k=1}^{n_p} o_k}{n_{ins}} \tag{4.1}$$

where n_p is the number of partitions, o_k is the number of out-of-order events in the k-th partition and n_{ins} is the total number of events in all partitions. Note that d_{oo} is zero for in-order event streams.

4.3.2 Causal networks

Causal network is a popularly used data structure for representing causality (Heckerman, 1995; Ellis and Wong, 2008; Li and Leong, 2009; Meganck et al., 2006; Borchani et al., 2007). It is a graph $G = (N, \xi)$ where N is the set of nodes (representing event types) and ξ is the set of edges between nodes. For each directed edge, the parent node denotes the cause, and the child node denotes the effect.

Consider the event stream of Figure 4.1. The causal relationships among the event types in the stream may be modeled as a causal network like the one shown in Figure 4.2.

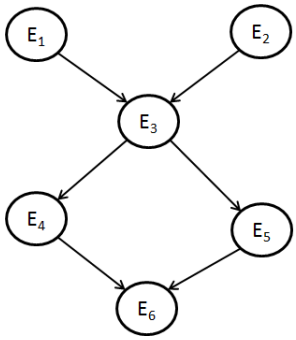


Figure 4.2: Causal network.

The joint probability distribution of a set of n event types $\mathbf{E} \equiv \{E_1, ..., E_n\}$ in a causal

network is specified as

$$P(\mathbf{E}) = \prod_{i=1}^{n} P(E_i|\mathbf{Pa_i})$$

where $\mathbf{Pa_i}$ is the set of the parent nodes of event type E_i.

4.3.3 Conditional mutual information

A popular approach for testing the conditional independence, with respect to the joint probability P, of two random variables X and Y given a subset S of random variables is conditional mutual information(CMI) (e.g., (Cheng et al., 2002; de Campos, 2006)). CMI gives the strength of dependency between variables in a measurable quantity, which helps to identify strong and weak causal relationships in the final causal network.

To test whether X and Y are conditionally independent given S, we compute the conditional mutual information $I_{MI}(X, Y|S)$ as

$$I_{MI}(X, Y|S) = \sum_{x \in X} \sum_{y \in Y} \sum_{s \in S} p_{X,Y,S}(x, y, s) log_2 \frac{p_{X,Y|S}(x, y|s)}{p_{X|S}(x|s) p_{Y|S}(y|s)}$$

where p is the probability mass function calculated from the frequencies of variables.

We only keep the record of these frequencies, not the whole events, by updating them as a new batch of events arrives. Consequently, the independence test procedure is incremental in our case.

It is said that two variables X and Y are independent when $I_{MI}(X, Y|S) = 0$; otherwise, they are dependent. However, this presents us with the risk of spurious relationships due to weak dependencies (we cannot assume $I_{MI}(X, Y|S) = 10^{-5}$ and $I_{MI}(X, Y|S) = 10$ provide the same degree of confidence in the dependency). With an increase in the value of $I_{MI}(X, Y|S)$, the dependency between the variables X and Y grows stronger. Therefore, to prune out weak dependencies, we need to set a threshold value of mutual information below which we ignore the evidence as weak. To do so, we relate CMI with G^2 test statis-

tics (Spirtes et al., 2000; Bishop et al., 1975) as below, where N_s is the sample size.

$$G^2(X, Y|S) = 2 \cdot N_s \cdot log_e 2 \cdot I_{MI}(X, Y|S)$$

G^2 follows the χ^2 distribution (Kullback, 1968), with the degree of freedom *df* equal to $(r_x - 1)(r_y - 1) \prod_{s \in S} r_s$, where r_x, r_y, and r_s are the number of possible distinct values of X, Y, and S, respectively. So, we use χ^2 test, which provides a threshold based on *df* and significance level α, to validate the dependency result. We set α to the universally accepted value of 95%.

4.3.4 The PC algorithm

The key idea of the PC algorithm (Spirtes et al., 2000) is that a causal network has an edge between two nodes, X and Y, if and only if X and Y are not independent given any of the condition subsets of the remaining neighbor nodes (Pearl, 1995; Spirtes and Meek, 1995). Algorithm 9 outlines the PC algorithm.

The algorithm has two parts. In the first part, the algorithm learns the topology of the causal network (Lines 1– 18). It starts with a completely connected undirected graph G of n nodes. Two sets are used for bookkeeping – *Neighbors(G,X)* and *SepSet(X,Y)*. *Neighbors(G,X)* gives the set of nodes adjacent to X, and *SepSet(X,Y)* gives the set of nodes which causes X and Y to be conditionally independent. Initially, *Neighbors(G,X)* has the remaining $n-1$ nodes and *SepSet(X, Y)* is empty. Then, the CI tests are performed between every pair of nodes that have an edge between them to determine whether they are conditionally independent of any other nodes in G. The edge between the conditionally independent nodes is removed. To ensure that all possible combination of nodes are considered in the condition set, the algorithm starts with an empty condition set and ends with the condition set with the maximum possible number of nodes. That is, in the algorithm k refers to the number of nodes in the condition set and is initially set to 0 to denote an

Algorithm 9 PC algorithm

Require: *Window W*

Construct the completely connected undirected graph G of all nodes;
for each node X in G **do**
  Initialize *Neighbors(G,X)* as the set of nodes adjacent to X in G;
end for
for each pair of nodes X and Y in G **do**
  Initialize an empty set *SepSet(X,Y)* as the set of nodes that causes independence between X and Y in G;
end for
$k \leftarrow 0$;
repeat
  repeat
    Select any edge $X - Y$ such that $|Neighbors(G, X)\backslash Y| \geq k$;
    repeat
      Select any subset S of $Neighbors(G, X)\backslash Y$ such that $|Neighbors(G, X)\backslash Y| = k$;
      If X and Y are independent given S, remove $X - Y$ from G, remove Y from Neighbors(G,X), remove X from Neighbors(G,Y), and add S to SepSet(X,Y) and SepSet(Y,X);
    until every subset S of $Neighbors(G, X)\backslash Y$ such that $|Neighbors(G, X)\backslash Y| = k$ has been selected.
  until every edge $X - Y$ such that $|Neighbors(G, X)\backslash Y| \geq k$ has been selected.
  k = k + 1;
until no edge $X' - Y'$ satisfies $|Neighbors(G, X')\backslash Y'| \geq k$.
for each triplet of nodes X, Y, Z such that the edge $X - Y$ and $Y - Z$ exists in G but not $X - Z$ **do**
  Orient $X - Y - Z$ as X→Y←Z if and only if SepSet(X,Z) does not contain Y;
end for
repeat
  If there exists $X \rightarrow Y$ and $Y - Z$, but not $X - Z$, **then** orient $Y - Z$ as $Y \rightarrow Z$;
  If there exists $X - Y$ and a directed path from X to Y, **then** orient $X - Y$ as $X \rightarrow Y$;
until no edge can be oriented.

empty condition set, and then k is gradually increased (by 1 at each iteration) and the CI tests are performed between every pair of nodes with the condition set of size k. This process is repeated until there is no edge left in G whose condition set size is greater than k. Eventually, an undirected network is obtained where an edge between two nodes denotes that these nodes are not conditionally independent in the presence of any of the other $n-2$

nodes.

In the second part, the undirected network topology is assigned causal directions (Lines 19 – 25). It is done in three steps. First, if there are two edges $X - Y$ and $Y - Z$ but not $X - Z$ and *SepSet(X,Z)* does not contain Y, then $X - Y - Z$ is assigned the edge orientations $X \rightarrow Y \leftarrow Z$. The reason is that X and Z are dependent given Y, as the absence of Y in *SepSet(X,Z)* indicates that X and Z are not conditionally independent given Y (Verma and Pearl, 1988; Geiger and Pearl, 1988). Second, if there are edges $X \rightarrow Y$ and $Y - Z$ but not $X - Z$, then $Y - Z$ is oriented as $Y \rightarrow Z$. The absence of an edge between X and Z means that X and Z are not dependent. A directed edge from X to Y and Y to Z with no edge between X and Z makes X and Z independent of Y (Verma and Pearl, 1988; Geiger and Pearl, 1988). Thus, the edge between Y and Z is oriented as $Y \rightarrow Z$. Third, if there are edges $X - Y$ and a directed path from X to Y through any number of nodes, then $X - Y$ is oriented as $X \rightarrow Y$. This is necessary to make the graph acyclic, as the edge direction $X \leftarrow Y$ would make the graph cyclic.

4.4 Learning Temporal Precedence Relationships

In this section, we describe an incremental approach to model temporal precedence relationships from time-stamped events into a *temporal network*.

4.4.1 Temporal network model

A temporal network is a network of nodes representing event types where an edge between two nodes represents the temporal precedence relationship between them. To determine when an edge should be added in a temporal network, a measure providing an evidence of temporal precedence between the event types should be defined. The evidence we use is the frequency of the observation of an instance of E_j following an instance of E_i. We call this

the *precedence frequency.*

Definition 9 (Precedence frequency) *The precedence frequency f_{ij} between two event types E_i and E_j is the total number of observations in which an event of type E_i precedes an event of type E_j over all partitions in the partitioned window.*

$$f_{ij} = \sum_{k=1}^{n_p} n_{(e_{ki}, e_{kj})}$$

where n_p is the number of partitions and $n_{(e_{ki}, e_{kj})}$ is the number of observations in which an event of type E_i precedes an event of type E_j in the k-th partition. □

In our prior work (Acharya and Lee, 2013), we assume that the event stream is in order and the temporal network from such an event stream is directed and acyclic. However, in an event stream with out-of-order event arrivals, the reliance on the temporal order for a definite temporal edge direction between event types may lead to ambiguous scenarios. For instance, the precedence frequencies between two event types E_i and E_j of an edge $E_i \rightarrow E_j$ and its reversed edge $E_i \leftarrow E_j$ may not differ significantly enough to determine an edge direction between them. An undirected edge $E_i - E_j$ is warranted to reflect such ambiguity. Thus, we support undirected edges as well as directed edges in the temporal network model. A directed edge between two nodes reflects a strong temporal precedence relationship between them, whereas an undirected edge reflects an ambiguous temporal precedence relationship between them. A threshold called the *temporal confidence* (θ) is used to select between directed and undirected edges, as presented in Rule 1 below.

Rule 1. Temporal edge direction selection

Suppose the precedence frequencies of an edge $E_i \rightarrow E_j$ and its reversed edge $E_i \leftarrow E_i$ are f_{ij} and f_{ji} such that either $f_{ij} > 0$ or $f_{ji} > 0$, respectively. Then, the edge direction

between these two event types E_i and E_j (i.e., $\Xi(E_i, E_j)$) is selected as follows.

$$\Xi(E_i, E_j) = \begin{cases} E_i \rightarrow E_j & \text{if} \frac{f_{ij}-f_{ji}}{f_{ij}+f_{ji}} > \theta \\ E_i \leftarrow E_j & \text{if} \frac{f_{ji}-f_{ij}}{f_{ij}+f_{ji}} > \theta \\ E_i - E_j & \text{if} |\frac{f_{ij}-f_{ji}}{f_{ij}+f_{ji}}| \leq \theta \end{cases}$$

□

Given a temporal network, we define the edge strength, called the *temporal strength*, as follows.

Definition 10 (Temporal strength) *Consider an edge $E_i \rightarrow E_j$ $(i \neq j)$ in a temporal network of n event types. Let f_{ij} be the precedence frequency from the event type E_i to the event type E_j. Then, we define the* temporal strength*, s_{ij}, of the edge $E_i \rightarrow E_j$ as*

$$s_{ij} \triangleq \frac{f_{ij}}{\sum_{k=1}^{n} f_{ik}}$$

□

That is, the temporal strength of $E_i \rightarrow E_j$ is the precedence frequency of (E_i, Ej) relative to the total precedence frequency over all children nodes of E_i.

4.4.2 Order-aware temporal network inference algorithm

The idea behind the *OATNI* algorithm is to collect events from an event stream in a *window* and then use temporal precedence information from the sequence of event pairs in the window to construct a temporal network at the event type level. The overall algorithm is centered on a *frequency matrix*, which is initially empty (i.e., all zero elements) and updated with each new batch of events.

The algorithm has two steps for each window, as outlined in Algorithm 10.

1. Update the frequency matrix *FM* by observing the precedence relationships of event

Algorithm 10 Order-Aware Temporal Network Inference (OATNI)

Require: *Edgeless network structure* TN*, Event stream(s)* S*, Temporal confidence* θ

1: Initialize an empty frequency matrix FM, an empty strength matrix SM, two empty buffers B_p and B_c (used to store "parent" events and "child" events, respectively);
2: **for** each window W in S **do**
3: **for** each partition P (corresponding to CRA a) in W **do**
4: **for** i = 1 to $t_n - 1$ where t_n is the number of unique timestamps in P **do**
5: Clear B_p and B_c;
6: Insert all events with timestamp t_i and t_{i+1} into B_p and B_c, respectively;
7: **for** each event instance e_{ap} in B_p **do**
8: **for** each event instances e_{ac} in B_c **do**
9: **if** type(e_{ac}) $\neq$ type(e_{ap}) {*//There cannot be causal relationships between events of the same type.*} **then**
10: Increase the frequency of element $f_{type(e_{ap}),type(e_{ac})}$ in FM by 1;
11: **end if**
12: **end for**
13: **end for**
14: **end for**
15: **end for**
16: **for** each pair of elements f_{ij} and f_{ji} such that $f_{ij} > 0$ or $f_{ji} > 0$ in FM **do**
17: $s_{ij} \leftarrow 0$, $s_{ji} \leftarrow 0$;
18: **if** $\frac{f_{ij}-f_{ji}}{f_{ij}+f_{ji}} > \theta$ **then**
19: Add an edge $E_i \rightarrow E_j$ in TN and set its strength to $s_{ij} = \frac{f_{ij}}{\sum_{k=1}^{n} f_{ik}}$;
20: **else if** $\frac{f_{ji}-f_{ij}}{f_{ij}+f_{ji}} > \theta$ **then**
21: Add an edge $E_i \leftarrow E_j$ in TN and set its strength to $s_{ji} = \frac{f_{ji}}{\sum_{k=1}^{n} f_{jk}}$;
22: **else if** $\frac{|f_{ij}-f_{ji}|}{f_{ij}+f_{ji}} \leq \theta$ **then**
23: Add an edge $E_i - E_j$ in TN and set the strengths to $s_{ij} = \frac{f_{ij}}{\sum_{k=1}^{n} f_{jk}}$ and $s_{ji} = \frac{f_{ji}}{\sum_{k=1}^{n} f_{jk}}$;
24: **end if**
25: **end for**
26: **end for**

pairs in the partitioned window (Lines 3–15). An element f_{ij} in FM reflects the total number of times events of type E_i precede events of type E_j ($i \neq j$). Each time an event pair (e_{oi}, e_{oj}) is observed in the event stream such that e_{oi} precedes e_{oj}, increase the value of f_{ij} by 1.

2. Determine the edges of the temporal network using *FM* (Lines 16–25). For each pair of nodes, add an edge according to Rule 1. For a directed edge added, e.g., $E_i \rightarrow E_j$, calculate its temporal strength and store it in s_{ij} of SM. For an undirected edge added, e.g., $E_i - E_j$, calculate the temporal strengths of both directions and store them in s_{ij} and s_{ji} of SM.

Note that when the events arrive in order, the *OATNI* algorithm reduces to the *TNI* algorithm by setting the threshold θ to 0 so that Rule 1 always chooses between the first two cases.

4.5 Learning Causal Network in Reduced Time

In this section, we describe a new approach which reduces the number of CI tests needed to infer the causal structure, thereby speeding up the learning process. Specifically, we explain the key ideas employed and discuss the concrete algorithm. We then prove the correctness of the algorithm and analyze its computational complexity.

4.5.1 Key ideas

Given that the key approach is to exploit temporal precedence relationships to learn the causal network, there are a number of ideas employed to reduce the causal network construction time. We begin by proposing some preliminary lemmas.

Lemma 4.5.1 *A CI test between two event types with no temporal precedence relationship is unnecessary.*

Proof 4.5.2 *Two event types can a have causal relationship only if they have a temporal precedence relationship. Therefore, it is not necessary to perform a CI test (for detecting causality) between two event types which are not temporally related.* □

Lemma 4.5.3 *Event types which do not have temporal precedence relationships with either of the two event types being tested for causality are not needed in the condition set of the CI test.*

Proof 4.5.4 *Consider two event types, E_i and E_j, tested for causality, and consider another event type E_k ($k \neq i, j$). E_k can causally influence E_i (or E_j) only if E_k has a temporal precedence relationship with E_i (or E_k). Therefore, the CI tests between E_i and E_j can safely exclude from the condition set those event types (i.e., E_k) which are not temporally related to either of them.* □

Based on Lemma 4.5.1, the CI tests are performed only for the edges in the temporal network. That is, not every possible edges are considered for the CI tests leading to a reduced number of CI tests. Moreover, since the size of the condition set contributes to the number of CI tests exponentially, we use Lemma 4.5.3 to reduce the condition set size by including only those event types which have temporal relationships, hence possibly causal relationships, to the event types being tested.

We employ another idea to speed up the network inference further, based on Lemma 4.5.5 below.

Lemma 4.5.5 *The number of CI tests performed in the causal network inference decreases if the CI tests between event types are performed in an increasing order of their temporal strength.*

Proof 4.5.6 *Event types with weaker temporal strengths between them have higher likelihood of being conditionally independent than those with stronger temporal strengths. Therefore, if the CI tests are performed between event types with the lowest temporal strength first, then the initial causal network becomes sparser faster and, consequently, the condition sets for the CI tests between event types become smaller faster. This leads to the reduction in the total number of CI tests performed through the causal network inference.* □

Evidently, the reduction in the number of CI tests brings the reduction of running time.

Further, we employ the idea of reducing the overhead of causal network inference by performing it only when there are significant enough changes in temporal precedence relationships in the event stream. The rationale for this is that the causal network tends to absorb changes in the temporal network until the changes are significant enough.

We introduce the temporal *precedence probability* as the measure to normalize the precedence frequencies between event types. The changes in the precedence probabilities give a normalized measure of the changes that have occurred in the temporal network since the last batch of events in the stream.

Definition 11 (Precedence probability) *The precedence probability p_{ij} between two event types E_i and E_j is defined as the ratio of f_{ij} and the summation of all precedence frequencies.*

$$p_{ij} = \frac{f_{ij}}{\sum_{x=1}^{n}\sum_{y=1}^{n} f_{xy}}$$

□

Let PM@t_i and PM@t_{i+1} be the matrices representing the precedence probabilities at the timestamps t_i and t_{i+1}, respectively. Then, the measure of change in the precedence information at t_{i+1}, called *precedence change* (C_p), is calculated as follows.

$$C_p@t_{i+1} = \sum_{x=1}^{n}\sum_{y=1}^{n} |p_{xy}@t_{i+1} - p_{xy}@t_i|$$

where p_{xy} is the element at the position (x, y) (i.e., event types (E_x, E_y)) in PM. Given this change measure, we update the causal network only if the calculated C_p exceeds a certain threshold, called the *precedence change confidence* (δ).

4.5.2 Enhanced fast causal network inference algorithm

Algorithm 11 Enhanced Fast Causal Network Inference (EFCNI)

Require: *Window* W, *Precedence change confidence* δ, *Edgeless causal network* G.

1: Run the *OATNI* algorithm and initialize $G = (N, \xi)$ with the learned temporal network; {N and ξ are the set of nodes and the set of edges, respectively.}
2: Calculate C_p. If $C_p < \delta$, then exit; {Stop if there is no significant change in the event stream.}
3: Sort the edges in ξ in the increasing order of their temporal strength.
4: **for** each edge $(E_i, E_j) \in \xi$ **do**
5: $independent = IsIndependent(E_i, E_j, \phi)$, where ϕ is the empty set; {$IsIndependent(E_i, E_j, S)$ calculates $I_{MI}(E_i, E_j|S)$ for CI test.}
6: **if** *independent* is true **then**
7: Remove (E_i, E_j) from ξ;
8: **end if**
9: **end for**
10: $k \leftarrow 0$;
11: **repeat**
12: **for** each edge $(E_i, E_j) \in \xi$ **do**
13: Construct a set of condition sets, Z, each of cardinality k from the *parents* of E_j excluding E_i;
14: **repeat**
15: Select any subset S from Z;
16: $independent = IsIndependent(E_i, E_j, S)$;
17: Remove S from Z;
18: **until** Z is empty or *independent* is true
19: **if** *independent* is true **then**
20: Remove (E_i, E_j) from ξ;
21: **end if**
22: **end for**
23: $k = k + 1$;
24: **until** there is no E_j in any edge $(E_i, E_j) \in \xi$ with k incident edges.

The algorithm has four steps, as outlined in Algorithm 11.

1. The first step (Line 1) learns a temporal network by running the *OATNI* algorithm. The temporal network, which can have both directed and undirected edges, is set as the initial causal network.

2. The second step (Line 2) checks if there has been a significant enough change in the temporal precedence statistic (i.e., C_p) in the event stream from the last observation

period, and stops if not.

3. The third step (Line 3) sorts the edges of the initial causal network in the increasing order of their temporal strength.

4. The fourth step (Lines 4–24) constructs the final causal network by pruning out the edges between independent nodes. CI tests are performed on every edge between adjacent nodes in the initial causal network to verify dependency between them. Conditionally independent nodes are considered spurious, and hence the edge between them is removed.

The main difference from the PC algorithm is the manner in which the CI tests are performed. In the PC algorithm, the condition set S for an edge $E_i - E_j$ (*undirected*) considers the neighbors of both E_i and E_j whereas in the EFCNI algorithm, the condition set S for an edge $E_i \rightarrow E_j$ (*directed*) needs to consider only the parents of E_j. (E_j is independent of the parents of E_i that do not have edge to E_j.) Consequently, fewer CI tests are needed. In addition, note that the *EFCNI* algorithm reduces to the *FCNI* algorithm by omitting the second and the third steps.

4.5.3 Correctness of the algorithm

To prove the correctness of the algorithm, it suffices to prove the correctness of our approach which starts with a temporal network as the initial causal network and removes edges through CI tests on them. We show the correctness as follows. First, a temporal precedence relationship is a necessary condition for inferring causality (Popper, 1959). Therefore, causal relationship subsumes temporal precedence relationship, that is, the causal network is a subgraph of the temporal network (Lemma 4.5.1). Second, a causal network should satisfy the *Causal Markov Condition (CMC)* (Spirtes et al., 1990; Pearl, 1995; Pearl, 1998) where for every node X in the set of nodes N, X is independent of its non-descendants excluding

its parents (i.e., $N \backslash (Descendants(X) \cup Parents(X))$) given its parents. In a temporal network of vertex (or node) set N, a node is temporally independent, and therefore causally independent, of all its non-descendants (except its parents) given its parents (Lemma 4.5.3).

4.5.4 Complexity analysis

Given n nodes, the computational complexity of the EFCNI algorithm is $O(n^2 \cdot 2^{n-2})$ in the worst case and $O(n)$ in the best case.

Proof 4.5.7 *The computational complexity of the EFCNI algorithm is governed by the total number of possible CI tests which is calculated by summing up the number of CI tests involving each edge. In the worst case, the number of edges in the network is that of a completely connected graph and all edges are undirected. The number of edges in a completely connected graph of n nodes is $\frac{n(n-1)}{2}$. For every edge between two nodes, the remaining $n-2$ nodes are considered in the condition set, as the graph is completely connected and undirected. Therefore, to test conditional independence between a pair of nodes in an edge, there are 2^{n-2} CI tests to perform. Consequently, the total number of CI tests for all edges is $\frac{n(n-1)}{2} \cdot 2^{n-2}$, resulting in the computational complexity of $O(n^2 \cdot 2^{n-2})$.*

In the best case, the initial causal network (i.e., temporal network) is a directed linear graph and the number of edges is the minimum (i.e., $n-1$). In such a graph, there are $n-2$ edges with one incoming edge to either of the nodes and one edge with no incoming edge to either of the nodes. For the edges with one incoming edge, the condition set size is one, and therefore there are two CI tests to perform. For $n-2$ such edges, there are $2n-4$ CI tests. For the remaining one edge with no incoming edge to either of the nodes, there is only one CI test to perform. Therefore, there are $2n-3$ CI tests to perform in the best case, resulting in the computational complexity of $O(n)$. □

The computational complexity of the PC algorithm is $O(n)$ in the best case and $O(n^2 \cdot 2^{n-2})$ in the worst case (Spirtes et al., 2000). Note that, while the computational complexi-

ties are the same, the EFCNI algorithm starts with a sparse network as the use of temporal precedence relationships removes many of the edges. So, it starts closer to the best case. In contrast, the PC algorithm always starts with a completely connected dense network. So, it starts from the worst case. As a result, in practice the EFCNI algorithm shows significant improvement in runtime over the PC algorithm.

The computational complexity of the FCNI algorithm is $O(n)$ in the best case and $O(n \cdot 2^{n-2})$ in the worst case (Acharya and Lee, 2013). FCNI's worst case computation complexity is lower than that of EFCNI by a factor of n. However, unlike the EFCNI algorithm, the FCNI algorithm is not suitable for out-of-order event streams. Moreover, as mentioned earlier, the EFCNI algorithm reduces to the FCNI algorithm when the events are in order and θ is zero. As a result, in practice the EFCNI algorithm is at least as fast and accurate as the FCNI algorithm when the events are in order and preserves the accuracy in the face of out-of-ordered events, compromising the runtime to some extent as an increasing number of events arrive out of order.

4.6 Performance Evaluation

We conducted experiments to compare the proposed EFCNI algorithm against the FCNI and the PC algorithms. There are three sets of experiments – first in terms of the accuracies of the resulting causal networks, second the running time, and third the number of CI tests required. In each set of experiments, we consider both the cases of stream being in order and out of order and also see the effect of the EFCNI's change-driven causal network construction strategy by comparing it with FCNI and PC when there are changes in the event stream statistic. Section 4.6.1 describes the experiment setup, Section 4.6.2 explains the datasets used, and Section 4.6.3 presents the experiment results.

4.6.1 Experiment setup

4.6.1.1 Evaluation metrics.

The evaluation metrics are the speed of the causal network generation and the accuracy of the generated causal network. The running time is the CPU time, and the number of performed CI tests affects the speed. The accuracy is evaluated by examining how closely the constructed causal network structure resembles the target causal network. For this, we adopt the *structural Hamming distance* proposed by Tsamardinos et al. (Tsamardinos et al., 2006) as the measure. The nodes (i.e., event types) are fixed as given to the algorithms, and therefore the network structures are compared with respect to the edges between nodes. There are three kinds of possible errors in the causal network construction: reversed edges, missing edges, and spurious edges. We use the number of erroneous edges of each kind as the evaluation metric.

4.6.1.2 Platform.

The experiments are conducted on RedHat Enterprise Linux 5 operating system using Java(TM) 2 Runtime Environment–SE 1.5.0_07 in Vermont Advanced Computing Core (VACC) cluster computers.

4.6.2 Datasets

Experiments are conducted using both synthetic and real datasets.

Synthetic datasets.

A synthetic dataset is reverse-engineered from a target causal network. Given control parameters in Table 4.1, the idea is to generate a random causal network, and then convert the causal network to an event stream which reflects the underlying probability distribution

Parameter	Meaning
N_O	Number of event owners (with unique ID)
N_{ET}	Number of event types (i.e., nodes)
Max_{NC}	Maximum number of cause events (parents)
Max_{NE}	Maximum number of effect events (children)

Table 4.1: Control parameters for synthetic event stream generation.

of the causal network. Specifically, there are three steps. First, N_{ET} nodes are created and edges are added randomly, and random conditional probabilities are assigned to each edge. Each node can have up to Max_{NC} edges from cause nodes and up to Max_{NE} edges to effect nodes. (We set both Max_{NC} and Max_{NE} to 3 for the experiments presented here.) Second, a joint probability distribution (JPD) table is built from the conditional probabilities assigned to edges of the target causal network. The rows of the JPD table collectively cover all event sequences possible, while each row has its own probability. Third, the probability for each row in the JPD table is multiplied by N_O to calculate the number of repetitions of that event sequence in the dataset. We assume that the event owner is the *CRA* for the dataset.

The size of a JPD table grows exponentially with N_{ET} and therefore we use parallel processing for the event stream generation. The JPD table is divided into multiple partitions and the dataset is created by running parallel processes over each of these partitions. The dataset is thus represented by a collection of files in which the events are shuffled according to the owner ID while preserving the temporal order.

There are five cases of datasets, DS1 through DS5, according to the number of nodes in the represented target causal networks (see their profiles in Table 4.2). The target causal networks have 4, 8, 12, 16 and 20 nodes, respectively. They are created with 1, 2, 16, 64 and 512 parallel processes, respectively, thus consisting of 1, 2, 16, 64 and 512 files, respectively. Each row of a synthetic dataset represents one event instance. To obtain out-of-order event streams, each case of datasets is shuffled randomly up to the required degree of out-of-order (see Equation 4.1). Changes in the event stream statistic is achieved by altering the precedence frequencies of events. Specifically, we generate six batches of the event stream for

Dataset	$\mathbf{N}_{ET}$	$\mathbf{N}_{\mathbf{edges}}$	$\mathbf{N}_{\mathbf{O}}$	$\mathbf{N}_{\mathbf{ins}}$
DS1	4	4	5000	15128
DS2	8	15	30000	124475
DS3	12	22	500000	3173246
DS4	16	39	6553600	50247293
DS5	20	49	52428800	510971687

(N_{edges} is the number of *actual* edges in the network. N_{ins} is the average number of event instances in the datasets of each case.)

Table 4.2: Profiles of the five synthetic datasets.

six observation points (t_1 through t_6) with the C_p values of $14\%, 16\%, 4\%, 6\%, 10\%,$ and12%, respectively, for each case of the datasets. The six batches are equal-sized for each dataset specified in Table 4.2, so the number of instances in a single batch is 2521, 20745, 528874, 8374548, and 85161947 for the dataset D_1, D_2, D_3, D_4, D_5, and D_6, respectively.

Real dataset.

The real dataset D_R contains diabetes lab test results (Frank and Asuncion, 2010) of 70 different patients over a period ranging from a few weeks to a few months. The dataset has a total 28143 records, about 402 records for each patient. Each record has four fields – date, time, test code, test value. The clinical data of a patient is independent of other patients. Therefore, the patient ID is the *CRA* for this dataset. There are 20 different test codes appearing in the file (shown in the left column of Table 4.3) from which we define event types of interest (shown in the right column of Table 4.3).

4.6.3 Experiment results

We run the EFCNI, FCNI and PC algorithms over each type of the five synthetic datasets and the real dataset. We present our evaluation in each of the three sets of experiments. First, we evaluate the accuracy of the generated causal networks against the target causal network and determine how closely they resemble the true causal network. Specifically, we count the number of spurious edges, the number of missing edges, and the number of reversed

Test Code	**Event Type**
Regular insulin dose	Regular-insulin-dose-given(RIDG)
NPH insulin dose	NPH-insulin-dose-given(NIDG)
UltraLente insulin dose	UltraLente-insulin-dose-given(UIDG)
Unspecified BGM*	Blood-glucose-measurement-increased(BGMI) Blood-glucose-measurement-decreased(BGMD)
Pre-breakfast BGM*	
Post-breakfast BGM*	
Pre-lunch blood BGM*	
Post-lunch BGM*	
Pre-supper BGM*	
Post-supper BGM*	
Pre-snack BGM*	
Hypoglycemic symptoms	Hypoglycemic-symptoms-exist(HSE)
Typical meal ingestion	Typical-meal-ingested(TMI)
More than usual meal ingestion	More-than-usual-meal-ingested(MTUMI)
Less than usual meal ingestion	Less-than-usual-meal-ingested(LTUMI)
Typical exercise activity	Typical-exercise-taken(TET)
More than usual exercise activity	More-than-usual-exercise-taken(MTUET)
Less than usual exercise activity	Less-than-usual-exercise-taken(LTUET)

(Note BGM* : blood glucose measurement)

Table 4.3: Event types defined from the diabetes dataset.

edges. Second, we evaluate the running time (CPU time), and third, we evaluate the number of CI tests performed. We show that reducing the number of CI tests is the key to reducing the running time of causal network inference. In each set of experiments, the evaluation covers the scenarios of the event stream being in order and out of order, and, additionally, the scenario of the event stream statistic changing. For the latter scenario, the value of the precedence change confidence δ is set to 9% for all synthetic datasets (DS_1 through DS_5). For the experiments involving in-order event streams, the temporal precedence confidence θ is set to zero (so EFCNI reduces to FCNI) and, for the experiments involving out-of-order event streams, it is set to $24.80\%, 17.23\%, 18.19\%, 21.97\%, 26.50\%$ (each determined after training from 70% of the data) for all synthetic datasets. The experiment is repeated ten times for each dataset (DS_1 through DS_5 and D_R) to calculate the average.

4.6.3.1 Comparison of the accuracies of the PC, FCNI, and EFCNI algorithms

4.6.3.1.1 When the events arrive in order

Table 4.4 presents the number of erroneous edges in the causal network produced by the PC, FCNI, and EFCNI algorithms. The results show that the accuracy of the causal network from the EFCNI algorithm is similar to that of the FCNI and PC algorithms. First, the number of missing and the number of spurious edges are comparable among all three algorithms. This is due to the reliance of the three algorithms on the same test statistics (CMI in our case) to infer the independence of two event types. Additionally, each number is the same between EFCNI and FCNI because EFCNI reduces to FCNI. Second, the number of reversed edges is zero for both the FCNI and EFCNI algorithms. Clearly the FCNI and EFCNI algorithms, through the temporal network, are much better at determining the correct causal edge directions. It is because of the fact that the cause always precedes its effect is embodied in the temporal precedence relationship. Overall, the results show that,

Type of Erroneous Edges	Algorithm	Dataset					
		$\mathbf{DS_1}$	$\mathbf{DS_2}$	$\mathbf{DS_3}$	$\mathbf{DS_4}$	$\mathbf{DS_5}$	$\mathbf{D_R}$
Missing	*PC*	0	0	0	0	1	1
	FCNI	0	1	0	0	1	1
	EFCNI	0	1	0	0	1	1
Reversed	*PC*	0	2	0	2	3	2
	FCNI	0	0	0	0	0	0
	EFCNI	0	0	0	0	0	0
Spurious	*PC*	0	3	0	4	3	1
	FCNI	0	3	0	4	3	1
	EFCNI	0	3	0	4	3	1

Table 4.4: Number of erroneous edges in an in-order event stream.

when the event stream is in order, the EFCNI algorithm produces the same topology as the FCNI algorithm and almost the same topology as the PC algorithm, while the accuracy of the causal directions in the EFCNI algorithm remains the same as the FCNI algorithm and is improved over the PC algorithm.

4.6.3.1.2 When the events arrive out of order

Table 4.5 presents the number of erroneous edges in the causal network produced by the three algorithms for varying degree of out-of-order in the event stream. We show the results for the two datasets DS_4 and DS_5 only; the results from the other datasets are consistent with the results from the two datasets. We make two observations from the results. First, the PC algorithm is more resilient to the out-of-order event arrival than the FCNI or EFCNI algorithm. The number of spurious edges and the number of missing edges are higher in the EFCNI algorithm than in the PC algorithm when the degree of out-of-order is large (i.e., $d_{oo} = 20\%, 25\%$). The reason is that the PC algorithm does not depend on the temporal precedence order for causal network inference at all whereas FCNI and EFCNI do. Second, between the FCNI algorithm and the EFCNI algorithm, EFCNI results in a comparable number of erroneous edges as PC while FCNI results in a larger number of erroneous edges than EFCNI or PC. The FCNI algorithm completely

Type of Erroneous Edges	Algorithm	$\mathbf{d_{oo}}$ for $\mathbf{DS_4}$						$\mathbf{d_{oo}}$ for $\mathbf{DS_5}$					
		0%	**5%**	**10%**	**15%**	**20%**	**25%**	**0%**	**5%**	**10%**	**15%**	**20%**	**25%**
Missing	*PC*	0	0	0	0	0	0	1	1	1	1	1	1
	FCNI	0	1	3	4	4	7	1	4	6	7	11	13
	EFCNI	0	0	0	0	1	2	1	1	1	1	1	2
Reversed	*PC*	2	2	2	2	2	2	3	3	3	3	3	3
	FCNI	0	2	3	5	8	14	0	3	5	6	9	15
	EFCNI	0	0	0	0	0	0	0	0	0	0	0	0
Spurious	*PC*	4	4	4	4	4	4	3	3	3	3	3	3
	FCNI	4	7	9	12	13	17	3	5	9	11	18	23
	EFCNI	4	4	4	4	4	6	3	3	3	3	4	7

Table 4.5: Number of erroneous edges in an out-of-order event stream for different degrees of out-of-order (d_{oo}) (datasets: DS_4 and DS_5).

depends on the temporal order of the events to generate the causal network structure and, consequently, is sensitive to even a small change in the order of the events. In contrast, the EFCNI algorithm employs the OATNI algorithm where the temporal confidence threshold mechanism selects *undirected* edges in the temporal network when the temporal precedence is ambiguous, and this mechanism makes EFCNI more resilient to the changes than FCNI.

4.6.3.1.3 When the event stream has changing temporal precedence statistic

Table 4.6 presents the number of erroneous edges in the causal networks resulting from the three algorithms for the event stream with changing temporal precedence statistic. As expected, the number of erroneous edges from the FCNI or PC algorithm is not affected by these changes because the causal network inference is run every time a new batch of events arrives. In contrast, for the EFCNI algorithm, the number of erroneous edges increases when C_p is lower than δ (at t_3 and t_4). (Note that in such cases the causal network inference is not run.) Additionally, the errors are larger at t_4 than t_3. This is because the higher value of C_p results in a greater difference between the causal network constructed and the true causal network and, more importantly, because the accuracy of the resulting causal network keeps on degrading as we keep on skipping the causal inference. On the other hand, for a batch of events with C_p greater than δ (i.e., at t_1, t_2, t_5, and t_6), the EFCNI algorithm

Type of Erroneous Edges	Algorithm	DS_4						DS_5					
		t_1	t_2	t_3	t_4	t_5	t_6	t_1	t_2	t_3	t_4	t_5	t_6
Missing	*PC*	0	0	0	0	0	0	1	1	1	1	1	1
	FCNI	0	0	0	0	0	0	1	1	1	1	1	1
	EFCNI	0	0	1	2	0	0	1	1	2	4	1	1
Reversed	*PC*	2	2	2	2	2	2	3	3	3	3	3	3
	FCNI	0	0	0	0	0	0	0	0	0	0	0	0
	EFCNI	0	0	0	1	0	0	0	0	1	2	0	0
Spurious	*PC*	4	4	4	4	4	4	3	3	3	3	3	3
	FCNI	4	4	4	4	4	4	3	3	3	3	3	3
	EFCNI	4	4	5	7	4	4	3	3	5	8	3	3

Table 4.6: Number of erroneous edges in a changing event stream over the six observation points t_1 through t_6 (datasets: DS_4 and DS_5).

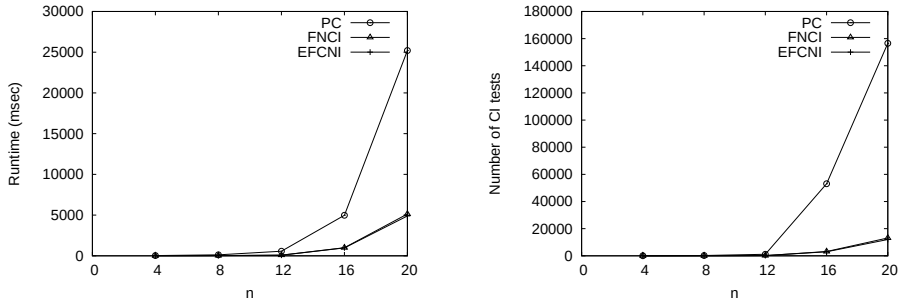
rebuilds the causal network and, consequently, the resulting causal network reflects the true causal network representing the event stream seen thus far. Therefore, at these time points the number of erroneous edges remains the same as if the event stream had no change.

4.6.3.2 Comparison of the running time of the PC, FCNI, and EFCNI algorithms.

4.6.3.2.1 When the events arrive in order

Figure 4.3(a) shows the average running time of the EFCNI, FCNI, and PC algorithms for varying number of event types in the synthetic datasets. In all cases, the running time of the EFCNI algorithm is the shortest while the running time of the PC algorithm is the longest. Clearly, the temporal precedence information helps to reduce the size of condition set and the number of edges for CI tests in both the FCNI and EFCNI algorithms. In addition, the EFCNI algorithm sorts the edges based on their temporal strength and then tests the conditional independence of the weaker edges, which are more likely to fail the tests, earlier and therefore further reduces the running time. As the number (n) of event types increases, the running time increases in all three algorithms, but the rate of increase is the highest for the PC algorithm and the lowest for the EFCNI algorithm. The same observation is made in the real dataset where the running time of the PC, FCNI, and EFCNI algorithms are

817, 118, and 112 msecs, respectively. These results verify the important role of temporal precedence relationships in reducing the running time.



(a) Running time for varying number of event types.

(b) Number of CI tests for varying number of event types.

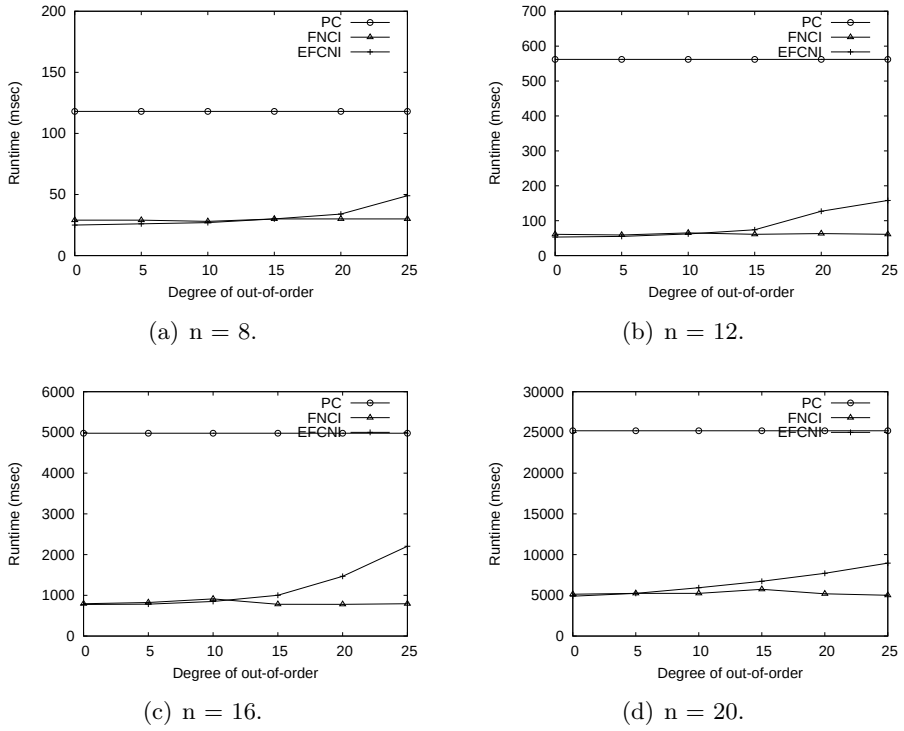
Figure 4.3: Comparison of the running time of the PC, FCNI and EFCNI algorithms for in-order event streams.

4.6.3.2.2 When the events arrive out of order

Figure 4.4 shows that the EFCNI algorithm performs the fastest causal network inference among the three algorithms when the event stream is in order (i.e., degree of out-of-order $d_{oo} = 0$). As d_{oo} increases, the running time of the EFCNI algorithm increases rapidly. It is due to the strategy that renders the edges with temporal strength lower than θ in the temporal network undirected. Consequently, the number of CI tests increases, resulting in an increase in the running time. On the other hand, as seen in the figure, the running time of FCNI algorithm remains short for the out-of-order event arrivals. However, the FCNI algorithm compromises the accuracy in such an event stream as discussed in Section 4.6.3.1. In addition, the result shows that the running time of the PC algorithm is constant as it is not affected by the out-of-order event arrivals.

4.6.3.2.3 When the event stream has changing temporal precedence statistic

Figure 4.5 shows that the EFCNI algorithm performs the fastest causal network inference among the three algorithms over the event stream with changing temporal precedence



(a) n = 8. (b) n = 12.

(c) n = 16. (d) n = 20.

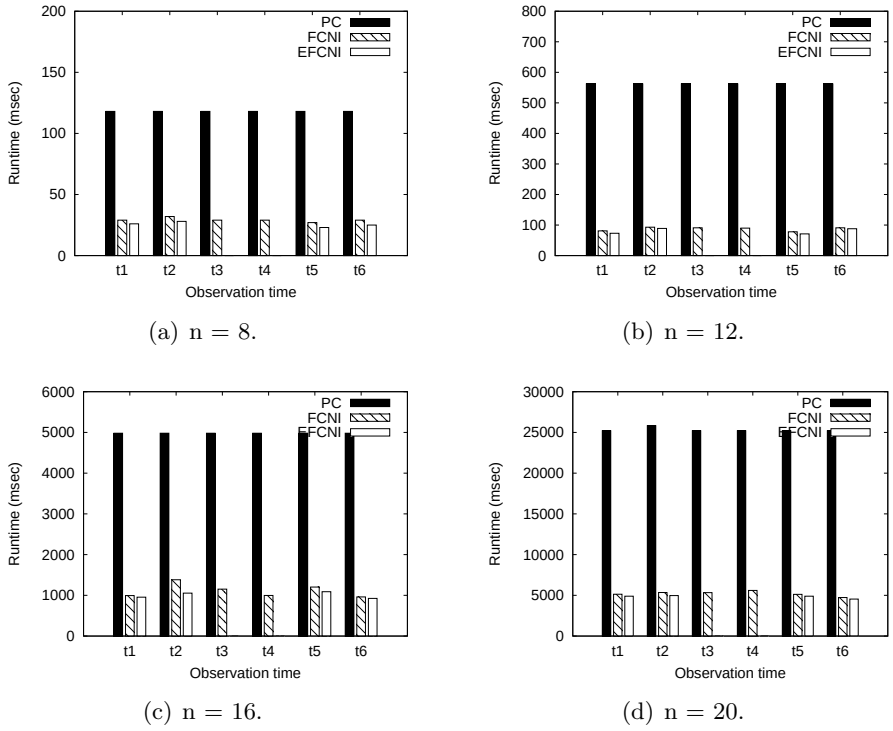
Figure 4.4: Comparison of the running time of the PC, FCNI, and EFCNI algorithms for out-of-order event streams.

statistic. In the figure, for all values of n, the FCNI and PC algorithms perform the CI tests for the causal network inference every time a new batch of events arrives. On the other hand, the EFCNI algorithm performs it only when the precedence statistic changes significantly enough in the event stream. (C_p is greater than δ at t_1, t_2, t_5, and t_6.) The EFCNI algorithm skips the causal inference at t_3 and t_4, which helps to reduce the overall running time.

4.6.3.3 Comparison of the number of CI tests of the PC, FCNI, and EFCNI algorithms.

4.6.3.3.1 When the events arrive in order

Figure 4.3(b) shows that the EFCNI algorithm performs fewer CI tests than the PC and



(a) n = 8. (b) n = 12.

(c) n = 16. (d) n = 20.

Figure 4.5: Comparison of the running time of the PC, FCNI, and EFCNI algorithms for changing event streams.

FCNI algorithms in all synthetic datasets. The CI tests are decreased by reducing the size of the condition set and the number of edges to test with the help of the temporal precedence information. In addition, the sorting of the edges (based on the their temporal strengths) helps to reduce the number of CI tests. With an increase in the number of event types (n), the rate of increase in the number of CI tests of the PC algorithm is much higher than that of the EFCNI and FCNI algorithms. A similar observation is made in the real dataset where the number of CI tests of the PC, FCNI, and EFCNI algorithms are 1239, 192, and 176, respectively. These results confirm the important role of temporal precedence relationships in reducing the number of CI tests. Note the result of CI tests (Figure 4.3(b)) looks almost the same as that of the running time (Figure 4.3(a)). This demonstrates that CI tests are the major performance bottleneck and validates the key idea of our work that reducing the

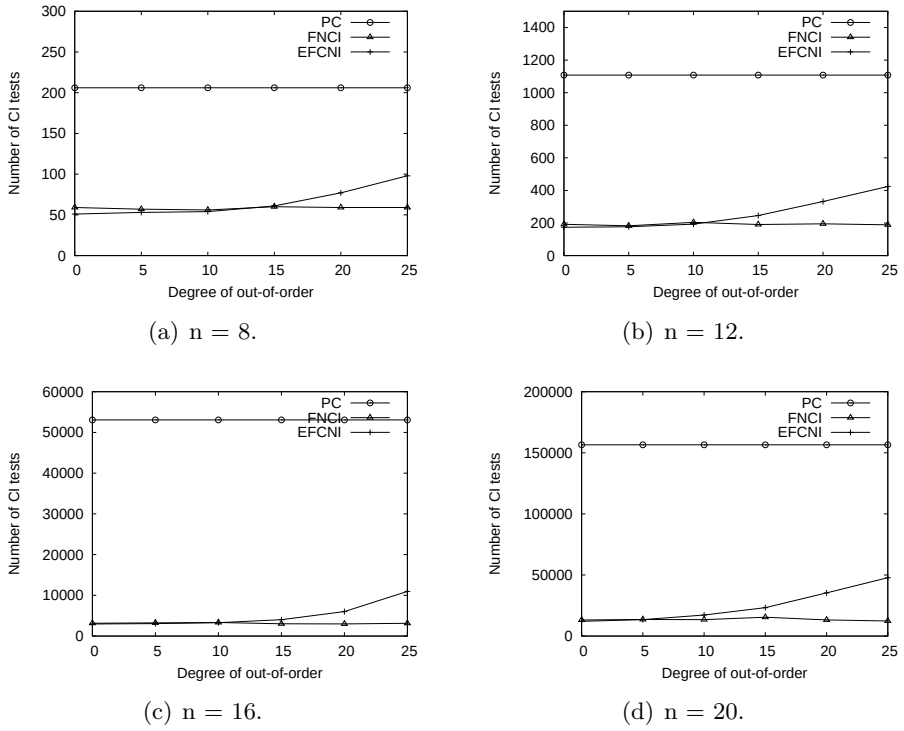
number of CI tests reduces the run time.

4.6.3.3.2 When the events arrive out of order

Figure 4.6 shows that, as the degree of out-of-order increases, the number of CI tests of the EFCNI algorithm increases. A higher degree of out-of-order leads to the temporal strengths of more edges lower than the temporal confidence threshold (i.e., θ), and this results in rendering more edges undirected and therefore performing more CI tests. Consequently, as the degree of out-of-order increases, the number of CI tests of the EFCNI algorithm becomes closer to that of the PC algorithm. Note that the PC algorithm is not affected by the out-of-order event arrivals and, thus, the number of CI tests does not change for varying degree of out-of-order. The results also show that the FCNI algorithm performs the smallest number of CI tests when the events arrive out of order. However, its accuracy is the worst among the three algorithms as discussed in Section 4.6.3.1.

4.6.3.3.3 When the event stream has changing temporal precedence statistic

Figure 4.7 shows that the EFCNI algorithm performs the smallest number of CI tests among the three algorithms over the event stream with changing temporal precedence statistic. As discussed earlier, the FCNI and PC algorithms regenerate the causal network with the arrival of every new batch of events while the EFCNI algorithm does it only when the change in the event stream (i.e., C_p) is greater than δ (as seen at t_1, t_2, t_5, and t_6). As a result, the EFCNI algorithm skips causal inference computations involving a large number of CI tests at t_3 and t_4 where the value of C_p is less than δ. As expected, the number of CI tests is highest for the PC algorithm at every observation point t_1 through t_6 due to its highest computational complexity.



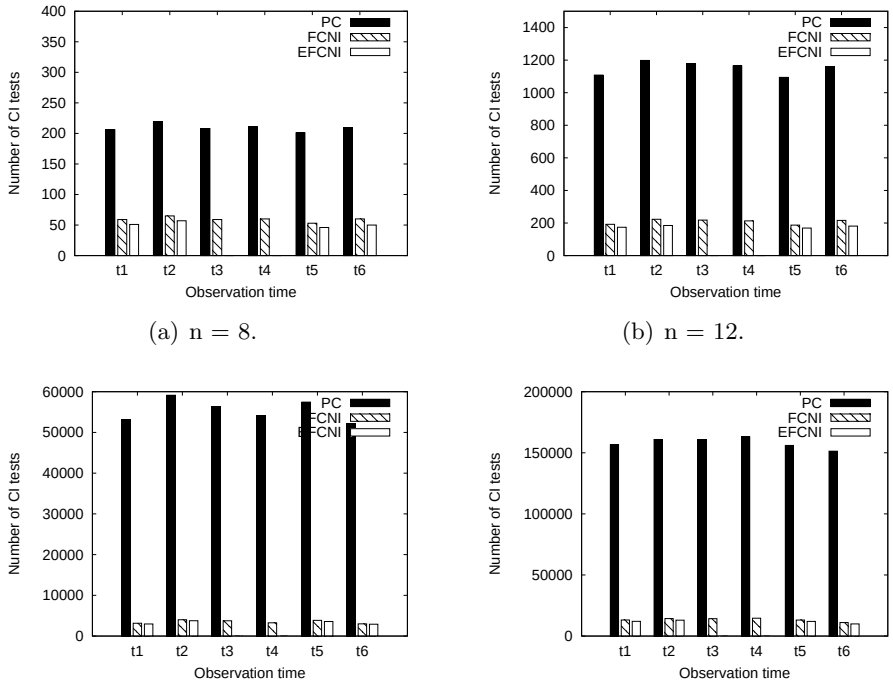
(a) n = 8. (b) n = 12.

(c) n = 16. (d) n = 20.

Figure 4.6: Comparison of the number of CI tests of the PC, FCNI, and EFCNI algorithms for out-of-order event streams.

4.6.3.4 Summary of experiment results

The EFCNI algorithm is faster than the FCNI algorithm for an event stream where the events arrive in order. The EFCNI algorithm enhances the FCNI algorithm with two additional strategies to reduce the number of CI tests. It has been demonstrated that the CI tests are the performance bottleneck and thus the reduction in the number of CI tests is the key to decreasing the running time of the algorithm. Moreover, unlike the FCNI algorithm which requires events to arrive in order, the EFCNI algorithm can perform the causal network inference accurately even when the events arrive out of order. As the degree of out-of-orderness d_{oo} increases, the accuracy of the EFCNI algorithm comes at the expense of the running time (i.e., the number of CI tests) to some extent.



(a) n = 8.

(b) n = 12.

(c) n = 16.

(d) n = 20.

Figure 4.7: Comparison of the number of CI tests of the PC, FCNI, and EFCNI algorithms for changing event streams.

The EFCNI algorithm is much faster than the PC algorithm in all experiments. In some scenarios (e.g., medical applications like patient health tracking), the accuracy of the result may be more important than the runtime. In case the accuracy of the EFCNI algorithm is not satisfactory in such scenarios – for example, if the event stream has many out-of-order events (i.e., with large d_{oo}) – then the OATNI algorithm can be tweaked to increase the EFCNI algorithm's accuracy by setting θ to a larger value (e.g., toward 100%). A larger θ value forces the edges more to be undirected and, therefore, makes the EFCNI algorithm behave more like the PC algorithm, thus achieving higher accuracy.

Furthermore, the EFCNI algorithm saves time by avoiding causal inference computations when there is not significant enough changes in the statistic of the event stream.

4.7 Conclusion and Future Work

In this paper, we presented a novel strategy to exploit temporal precedence relationships to learn the causal network over event streams. First, we introduced the *Order-Aware Temporal Network Inference* algorithm to model temporal precedence information. Then, we presented the *Enhanced Fast Causal Network Inference* algorithm to reduce the running time of learning causal network by reducing the number of performed CI tests significantly. These algorithms efficiently handle the event streams even if the events are out of order and saves the running time further by performing causal inference only if the temporal precedence statistic changes significantly enough. We showed the experiment results to validate our approach by comparing against the state-of-the-art PC and FCNI algorithms.

There are a number of future work in the plan. First, we plan to support cyclic causality. Second, we plan to investigate the effect of concept drift in the causal network inference so that the computations are performed only among the event types affected by the changes. Third, we plan to perform the experiments on datasets with a much larger number (i.e., hundreds to thousands) of event types to show the practicality of our proposed algorithms in a big data environment.

Bibliography

Acharya, S. and Lee, B. (2013). Fast causal network inference over event streams. In *Data Warehousing and Knowledge Discovery*, volume 8057 of *Lecture Notes in Computer Science*, pages 222–235. Springer Berlin Heidelberg.

Acharya, S. and Lee, B. S. (2014a). Enchanced fast causal network inference over event streams. *Transactions on Large Scale Data and Knowledge Centered Systems*.

Acharya, S. and Lee, B. S. (2014b). Incremental causal network construction over event streams. *Information Sciences*, 261(0):32 – 51.

Acharya, S., Lee, B. S., and Hines, P. (2014). Real-time top-k predictive query processing over event streams. *Information Sciences*.

Akdere, M., Çetintemel, U., and Upfal, E. (2010). Database-support for continuous prediction queries over streaming data. *Proc. VLDB Endow.*, 3(1-2):1291–1301.

Alcobé, J. R. (2004a). Incremental hill-climbing search applied to Bayesian network structure learning. In *First International Workshop on Knowledge Discovery in Data Streams, 2004*.

Alcobé, J. R. (2004b). *Incremental Methods for Bayesian Network Structure Learning*. PhD thesis, Department of Computer Science, Universitat Politècnica de Catalunya.

Alcobé, J. R. (2005). Incremental methods for Bayesian network structure learning. *Artificial Intelligence Communications*, 18(1):61–62.

Barber, D. (2012). *Bayesian Reasoning and Machine Learning*. Cambridge University Press.

Barga, R. S., Goldstein, J., Ali, M. H., and Hong, M. (2007). Consistent streaming through time: A vision for event stream processing. In *Proceedings of the Third Biennial Conference on Innovative Data Systems Research*, CIDR'07, pages 363–374.

Bishop, Y. M., Fienberg, S. E., and Holland, P. W. (1975). *Discrete Multivariate Analysis: Theory and Practice*. MIT Press.

Borchani, H., Chaouachi, M., and Ben Amor, N. (2007). Learning causal bayesian networks from incomplete observational data and interventions. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, ECSQARU '07, pages 17–29, Berlin, Heidelberg. Springer-Verlag.

Bowes, J., Neufeld, E., Greer, J., and Cooke, J. (2000). A comparison of association rule discovery and bayesian network causal inference algorithms to discover relationships in discrete data. In Hamilton, H., editor, *Advances in Artificial Intelligence*, volume 1822 of *Lecture Notes in Computer Science*, pages 326–336. Springer Berlin Heidelberg.

Buntine, W. (1991). Theory refinement on Bayesian networks. In *Proceedings of the seventh conference (1991) on Uncertainty in artificial intelligence*, pages 52–60, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Chandra, P. and Kshemkalyani, A. D. (2005). Causality-based predicate detection across space and time. *IEEE Transactions on Computerts*, 54:1438–1453.

Cheng, J. and Druzdzel, M. J. (2000). Ais-bn: An adaptive importance sampling algorithm for evidential reasoning in large Bayesian networks. *J. Artif. Intell. Res. (JAIR)*, 13:155–188.

Cheng, J. and Druzdzel, M. J. (2001). Confidence inference in Bayesian networks. In *Proceedings of the Seventeenth conference on Uncertainty in artificial intelligence*,

UAI'01, pages 75–82, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Cheng, J., Greiner, R., Kelly, J., Bell, D., and Liu, W. (2002). Learning Bayesian networks from data: an information-theory based approach. *Artificial Intelligence*, 137(1-2):43–90.

Chickering, D. M. (2002). Learning equivalence classes of Bayesian-network structures. *Journal of Machine Learning Research*, 2:445–498.

Chickering, D. M., Heckerman, D., and Meek, C. (2004). Large-sample learning of bayesian networks is np-hard. *J. Mach. Learn. Res.*, 5:1287–1330.

Chow, Y. S. and Teicher, H. (1978). *Probability theory : independence, interchangeability, martingales*. Springer-Verlag, New York.

de Campos, L. M. (2006). A scoring function for learning Bayesian networks based on mutual information and conditional independence tests. *J. Machine Learning Research*, 7:2149–2187.

Ellis, B. and Wong, W. H. (2008). Learning causal Bayesian network structures from experimental data. *Journal of the American Statistical Association*, 103(482):778–789.

Elloumi, M. and Zomaya, A. Y. (2013). *Biological Knowledge Discovery Handbook: Preprocessing, Mining and Postprocessing of Biological Data*, volume 23. John Wiley & Sons.

Eppstein, M. and Hines, P. (2012). A "Random Chemistry" algorithm for identifying collections of multiple contingencies that initiate cascading failure. *IEEE Transactions on Power Systems*, 27(3):1698–1705.

Frank, A. and Asuncion, A. (2010). UCI machine learning repository.

Friedman, N. and Goldszmidt, M. (1997). Sequential update of bayesian network structure. In *Proceedings of the Thirteenth conference on Uncertainty in artificial intelli-*

gence, UAI'97, pages 165–174, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Friedman, N., Linial, M., Nachman, I., and Pe'er, D. (2000). Using Bayesian networks to analyze expression data. In *Proceedings of the Fourth Annual International Conference on Computational Molecular Biology*, RECOMB '00, pages 127–135, New York, NY, USA. ACM.

Geiger, D. and Pearl, J. (1988). On the logic of causal models. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 3–14.

Glüge, S., Hamid, O. H., and Wendemuth, A. (2010). A simple recurrent network for implicit learning of temporal sequences. *Cognitive Computation*, 2(4):265–271.

Glymour, C. (2003). Learning, prediction and causal Bayes nets. *Trends in cognitive sciences*, 7(1):43–48.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica*, 37(3):424–438.

Guyon, I., Aliferis, C. F., Cooper, G. F., Elisseeff, A., Pellet, J.-P., Spirtes, P., and Statnikov, A. R. (2008). Design and analysis of the causation and prediction challenge. In *IEEE World Congress on Computational Intelligence Causation and Prediction Challenge*, pages 1–33.

Hamilton, H. J. and Karimi, K. (2005). The timers ii algorithm for the discovery of causality. In *Proceedings of the 9th Pacific-Asia conference on Advances in Knowledge Discovery and Data Mining*, PAKDD'05, pages 744–750, Berlin, Heidelberg. Springer-Verlag.

Heckerman, D. (1995). A Bayesian approach to learning causal networks. In *Proceedings of the Eleventh conference on Uncertainty in artificial intelligence*, UAI'95, pages

285–295, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Heckerman, D. (1999). UCI machine learning repository [http://archive.ics.uci.edu/ml/datasets/msnbc.com+anonymous+web+data].

Henrion, M. (1991). Search-based methods to bound diagnostic probabilities in very large belief nets. In *Proceedings of the Seventh conference on Uncertainty in Artificial Intelligence*, UAI'91, pages 142–150, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Holme, P. and Saramäki, J. (2011). Temporal networks. *CoRR*, abs/1108.1780.

Ishii, H., Ma, Q., and Yoshikawa, M. (2010). An incremental method for causal network construction. In *Proceedings of the 11th international conference on Web-age information management*, WAIM'10, pages 495–506, Berlin, Heidelberg. Springer-Verlag.

Jiang, X.-R. and Gruenwald, L. (2005). Microarray gene expression data association rules mining based on BSC-tree and FIS-tree. *Data & Knowledge Engineering*, 53(1):3 – 29.

Johnson, T., Muthukrishnan, S., and Rozenbaum, I. (2007). Monitoring regular expressions on out-of-order streams. In *Proceedings of the IEEE 23rd International Conference on Data Engineering*, ICDE'07, pages 1315–1319.

Kemeny, J. and Snell, J. (1969). *Finite Markov chains*. University series in undergraduate mathematics. VanNostrand, New York, repr edition.

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220:671–680.

Kłopotek, M. A. (2006). Cyclic Bayesian network–Markov process approach. *Studia Informatica*, 1(2):7.

Krishna, R., Li, C.-T., and Buchanan-Wollaston, V. (2010). A temporal precedence based clustering method for gene expression microarray data. *BMC Bioinformatics*, 11:68.

Kullback, S. (1968). *Information Theory and Statistics*. Dover Publication, 2nd edition.

Kwon, O. and Li, K.-J. (2009). Causality join query processing for data streams via a spatiotemporal sliding window. *Journal of Universal Computer Science*, 15(12):2287–2310.

Lam, W. and Bacchus, F. (1994). Using new data to refine a Bayesian network. In *Proceedings of the Tenth international conference on Uncertainty in artificial intelligence*, UAI'94, pages 383–390, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Li, G. and Leong, T.-Y. (2009). Active learning for causal bayesian network structure with non-symmetrical entropy. In *Proceedings of the 13th Pacific-Asia Conference on Advances in Knowledge Discovery and Data Mining*, PAKDD '09, pages 290–301, Berlin, Heidelberg. Springer-Verlag.

Li, J., Tufte, K., Shkapenyuk, V., Papadimos, V., Johnson, T., and Maier, D. (2008). Out-of-order processing: A new architecture for high-performance stream systems. *Proceegins of the VLDB Endowment*, 1(1):274–288.

Li, M., Liu, M., Ding, L., Rundensteiner, E. A., and Mani, M. (2007). Event stream processing with out-of-order data arrival. In *Proceedings of the 27th International Conference on Distributed Computing Systems Workshops*, ICDCSW '07, pages 67–74, Washington, DC, USA. IEEE Computer Society.

Lin, T. Y., Xie, Y., Wasilewska, A., and Liau, C.-J., editors (2008). *Data Mining: Foundations and Practice*, volume 118 of *Studies in Computational Intelligence*. Springer.

Liu, M., Li, M., Golovnya, D., Rundensteiner, E., and Claypool, K. (2009). Sequence pattern query processing over out-of-order event streams. In *Proceedings of the IEEE 25th International Conference on Data Engineering*, ICDE '09, pages 784–795.

Liu, Y., Niculescu-Mizil, A., Lozano, A. C., and Lu, Y. (2010). Learning temporal causal graphs for relational time-series analysis. In *ICML*, pages 687–694. Omnipress.

MacKay, D. J. C. (1999). Introduction to Monte Carlo methods. In *Learning in graphical models*, pages 175–204. MIT Press, Cambridge, MA, USA.

Mani, S., Aliferis, C. F., and Statnikov, A. R. (2010). Bayesian algorithms for causal data mining. *Journal of Machine Learning Research*, 6:121–136.

Mazlack, L. J. (2004). Mining causality from imperfect data. In *Proceedings of the sixth International FLINS Conference on Applied Computational Intelligence Proceedings*, Applied Computational Intelligence, pages 155–160.

Meek, C. (1995). Causal inference and causal explanation with background knowledge. In *Proceedings of the Eleventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, pages 403–410, San Francisco, CA. Morgan Kaufmann.

Meganck, S., Leray, P., and Manderick, B. (2006). Learning causal bayesian networks from observations and experiments: a decision theoretic approach. In *Proceedings of the Third international conference on Modeling Decisions for Artificial Intelligence*, MDAI'06, pages 58–69, Berlin, Heidelberg. Springer-Verlag.

Meliou, A., Gatterbauer, W., Moore, K. F., and Suciu, D. (2010). Why so? or why no? functional causality for explaining query answers. In *MUD, 2010*, pages 3–17.

Mohammad, Y. and Nishida, T. (2010). Mining causal relationships in multidimensional time series. In Szczerbicki, E. and Nguyen, N., editors, *Smart Information and Knowledge Management*, volume 260 of *Studies in Computational Intelligence*, pages 309–338. Springer Berlin Heidelberg.

Moral, P., Doucet, A., and Jasra, A. (2012). An adaptive sequential Monte Carlo method for approximate Bayesian computation. *Statistics and Computing*, 22:1009–1020.

Pearl, J. (1995). Causal diagrams for empirical research. *Biometrika*, 82:669–710.

Pearl, J. (1998). Graphs, causality, and structural equation models. *Sociological Methods and Research*, 27:226–84.

Pearl, J. (2009). *Causality: Models, Reasoning and Inference*. Cambridge University Press, New York, NY, USA, second edition edition.

Pearl, J. and Verma, T. (1991). A theory of inferred causation. In *KR, 1991*, pages 441–452.

Popper, K. (1959). *The Logic of Scientific Discovery*. Routledge Classics.

Prakasa Rao, B. (2009). Conditional independence, conditional mixing and conditional association. *Annals of the Institute of Statistical Mathematics*, 61(2):441–460.

Rottman, B. M. and Hastie, R. (2014). Reasoning about causal relationships: Inferences on causal networks. *Psychological Bulletin*, 140(1):109–139.

Rudin, C., Letham, B., Salleb-Aouissi, A., Kogan, E., and Madigan, D. (2011). Sequential event prediction with association rules. In *Proceedings of the 24th Annual Conference on Learning Theory*, COLT '11, pages 615–634.

Shachter, R. D. (1990). Evidence absorption and propagation through evidence reversals. In *Proceedings of the Fifth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '89, pages 173–190, Amsterdam, The Netherlands.

Silverstein, C., Brin, S., Motwani, R., and Ullman, J. (2000). Scalable techniques for mining causal structures. *Data Mining and Knowledge Discovery*, 4(2-3):163–192.

Spirtes, P., Glymour, C., and Scheines, R. (1993). *Causation, Prediction, and Search*. Ima Volumes in Mathematics and Its Applications. Springer-Verlag.

Spirtes, P., Glymour, C., and Scheines, R. (2000). *Causation, Prediction, and Search*. MIT Press.

Spirtes, P., Glymour, C. N., and Scheines, R. (1990). Causality from probability. In *Proceedings of the Conference on Advanced Computing for the Social Sciences*, ACSS

'90.

Spirtes, P. and Meek, C. (1995). Learning Bayesian networks with discrete variables from data. In *Proceedings of the First International Conference on Knowledge Discovery and Data Mining*, KDD'95, pages 294–299.

Tsamardinos, I., Brown, L. E., and Aliferis, C. F. (2006). The max-min hill-climbing bayesian network structure learning algorithm. *Mach. Learn.*, 65(1):31–78.

Tulupyev, A. L. and Nikolenko, S. I. (2005). Directed cycles in Bayesian belief networks: probabilistic semantics and consistency checking complexity. In *Proceedings of the 4th Mexican international conference on Advances in Artificial Intelligence*, pages 214–223. Springer.

US-Canada Power System Outage Task Force (2004). Final Report on the August 14, 2003 Blackout in the United States and Canada. Technical report.

Utrera, A. C., Olmedo, M. G., and Callejon, S. M. (2008). A score based ranking of the edges for the pc algorithm. In *Proceedings of the 4th European workshop on probabilistic graphical models*, PGM'08, pages 41 – 48.

Vaiman, M., Bell, K., Chen, Y., Chowdhury, B., Dobson, I., Hines, P., Papic, M., Miller, S., and Zhang, P. (2012). Risk assessment of cascading outages: Methodologies and challenges. *IEEE Transactions on Power Systems*, 27(2):631–641.

Veloso, A. A., Almeida, H. M., Gonçalves, M. A., and Meira Jr., W. (2008). Learning to rank at query-time using association rules. In *Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, SIGIR '08, pages 267–274, New York, NY, USA. ACM.

Verma, T. and Pearl, J. (1988). Causal networks: Semantics and expressiveness. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 69–78.

Wang, K. and Yu, Y. (2013). A query-matching mechanism over out-of-order event stream in iot. *Int. J. Ad Hoc Ubiquitous Comput.*, 13(3/4):197–208.

Young, S. S. and Karr, A. (2011). Deming, data and observational studies. *Significance*, 8(3):116–120.

Zhang, N. L. and Poole, D. (1996). Exploiting causal independence in Bayesian network inference. *J. Artif. Int. Res.*, 5(1):301–328.

Zhang.N.L. and D., P. (1994). A simple approach to Bayesian network computations. In *Proceedings of the Tenth Canadian Conference on Artificial Intelligence*, CCAA '94, pages 171–178.

Zhao, Y. and Strom, R. (2001). Exploitng event stream interpretation in publish-subscribe systems. In *Proceedings of the Twentieth Annual ACM Symposium on Principles of Distributed Computing*, PODC '01, pages 219–228.

Chapter 5

Real-time Top-K Predictive Query Processing over Event Streams

Abstract

This paper proposes and compares new methods for predicting the k events that are most likely to occur next, given real-time and historical event streams. Existing approaches to causal prediction queries have a few limitations. First, they exhaustively search over an acyclic causal network to find the most likely k effect events; unfortunately, however, data from real event streams frequently reflect cyclic causality. Second, they are so conservative in determining the causal relationships between events that they end up omitting many significant causal links. We overcome these limitations by proposing a novel *event precedence model* and a *run-time causal inference* mechanism. The event precedence model is a network, constructed incrementally over event streams, representing every temporal precedence relationship among events. The temporal precedence relationship is a necessary condition for causality. The proposed run-time causal inference mechanism learns causal relationships dynamically during query processing. Specifically, this paper presents two query processing algorithms – one performs the *exhaustive search* and the other performs

a more efficient *reduced search with early termination*. Experiments using two real datasets verify the effectiveness of the probabilistic top-k prediction queries and the efficiency of the algorithms.

5.1 Introduction

Causal prediction (e.g. (Glymour, 2003; Guyon et al., 2008; Spirtes et al., 2000)) is emerging as an essential field for real-time monitoring, planning and decision support in diverse applications such as stock market analysis, electric power grid monitoring, sensor network monitoring, network intrusion detection, and web click-stream analysis. There is a need for active systems to continuously monitor the event streams from these applications to allow for the prediction of future effect events in real time. Specifically, given a sequence of potentially causal events, many applications would benefit from good algorithms to predict the next most likely (namely, top k) effect events. The potentially huge answer space, however, and the unknown dynamics as well as the unbounded and evolving nature of event streams make such top-k prediction a challenging task.

Consider the following two scenarios as motivating examples.

Example 4 *Web page click stream:* Consider web-based online systems. A majority of them display the same content for everyone. However, the user experience can be more productive with a dynamic system where content is displayed based on real-time prediction of users' most likely activities, given historical data. One can use the results (i.e., the web pages/links most likely to be visited next) to display the most relevant links, contents, and advertisements at each step of the user activity. Such an arrangement may help to retain the user longer by displaying the most relevant information, thereby increasing the content consumption (e.g., sales, page visits, ad clicks). □

Example 5 *Electric power grid:* Consider an electric power grid. When components of

a power grid fail, as a result of a storm, malfunction or cyber-attack, a cascading sequence of subsequent component failures may result, which may lead to a very large blackouts (e.g., (Vaiman et al., 2012)). Thus, a timely prediction of the components that are most likely to fail next, given a list of a few components that have failed, may enable operators to take mitigating actions (like shutting down sections of the power grid) before a large-scale blackout occurs. Cascading blackouts typically progress slowly (minutes to tens of minute) in the initial stages; a few seconds delay to compute and implement emergency controls is generally sufficient. □

In this paper, we consider the problem of continuously predicting the *top-k* most probable next effects over *event streams* in real time. To the best of our knowledge, there exists no efficient top-k query processing mechanism suitable for event streams for continuous causal prediction. In addition, much of the previous work on the prediction of effect events given one or more cause events is based on exhaustive search over a causal network (e.g., (Akdere et al., 2010; Zhang and Poole, 1996)). A causal network represents the cause and effect relationships, called causality, in a directed acyclic graph. This *traditional* causal network model (e.g., (Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006; Pearl, 2009; Spirtes et al., 2000; Spirtes et al., 1990)) has three major limitations and, therefore, can not be trusted for causal prediction. First, since it is acyclic, it cannot have loops, such as $A \rightarrow B \rightarrow C \rightarrow A$ or bidirectional relationships such as $A \leftrightarrow B$, and consequently, does not support cyclic causality (e.g., (Friedman et al., 2000; Rottman and Hastie, 2014)). However, the event streams from many applications do show cyclic relationships. For example, a visitor to a news web site may visit the home page, proceed to read an article, and then return to the home page, creating a cyclic relationship between these vertices in the graph. Second, the causal Markov condition, often considered an essential property of traditional causal networks, is conservative in the causal inference, and as a result removes many important causal relationships from the causal network (Pearl,

1995; Pearl, 1998; Rottman and Hastie, 2014; Spirtes et al., 1990). That is, the causal Markov condition calls for the removal of those relationships which could potentially be independent in the presence of one or more events. The rationale for this is to avoid any suspicious and weak relationships. It, however, often backfires and ends up removing significant causal relationships (Rottman and Hastie, 2014). We call this phenomenon the *causal information loss*. Third, the evolving and unbounded nature of event streams warrants an adaptive causal modeling approach, which is not readily available from the traditional causal networks.

To overcome this lack of a suitable approach for top-k prediction over event streams, we propose the following novel techniques: (1) *event precedence modeling*, (2) *run-time causal inference*, and (3) two *top-k* query processing algorithms – *Exhaustive Search* (ES) and *Reduced Search Early Termination* (RSET). The event precedence model learns every possible temporal precedence relationship, a required criterion for causal relationship, between events and represents them in a network structure, called the *event precedence network* (EPN). The run-time causal inference allows for inferring causality at run time from the EPN. This run-time approach is naturally adaptive to the changes in the event streams and, therefore, is suitable for streaming environment. Furthermore, as the EPN retains all precedence relationships (including cyclic relationships), the run-time causal inference can overcome the two limitations of the traditional causal model discussed earlier (i.e., lack of support for cyclic causality and loss of many important causal relationships). The two top-k query processing algorithms (i.e., ES and RSET) determine the top k effects with the highest scores. The ES algorithm formalizes the exhaustive search approach, while the RSET algorithm presents a partial search approach with an early termination condition for faster query processing.

We conduct experiments to evaluate the performance of the proposed *ES* and *RSET* algorithms using two real datasets. For each dataset we perform two sets of experiments

to evaluate their accuracies and runtimes, respectively. In each evaluation, there are two objectives. The first objective is to compare the run-time causal inference mechanism of the proposed algorithms (i.e., ES, RSET) against the state-of-the-art traditional causal inference mechanism called the Fast Causal Network Inference (FCNI) algorithm (Acharya and Lee, 2013). The FCNI algorithm is essentially inapplicable to our problem due to its lack of ability to handle cyclic causality and run-time causal inference, but is the best available in the state of the art. The second objective is to compare the query processing mechanisms between the ES algorithm and the RSET algorithm.

The contributions of this paper are summarized as follows.

1. It presents an event precedence model to represent the temporal precedence relationships between event types and proposes an algorithm to construct an event precedence network incrementally over event streams.

2. It introduces a run-time causal inference mechanism to infer the causal relationships in real time, and proposes two query processing algorithms: Exhaustive Search and Reduced Search Early Termination, to continuously predict the top-k next effects over event streams.

3. It empirically demonstrates the advantages of the proposed run-time causal inference mechanism and the query processing algorithms in terms of the prediction accuracy and the runtime.

The rest of the paper is organized as follows. Section 5.2 discusses the related work, and Section 5.3 presents some preliminary concepts. Section 5.4 formulates the specific problem addressed in this paper and outlines the proposed framework. Sections 5.5 and 5.6 describe the event precedence model and the query processing model, respectively. Section 5.7 evaluates the proposed query processing algorithms. Section 5.8 concludes the paper and suggests future work.

5.2 Related Work

This section first discusses conventional causal inference techniques and then describes how this paper makes unique contributions relative to other work related to causal prediction.

There are two approaches for constructing a traditional causal network. The first approach, search and score based (e.g., (Ellis and Wong, 2008; Heckerman, 1995; Li and Leong, 2009; Meganck et al., 2006)), performs greedy search (usually hill climbing) over all possible causal networks of the data to select the network with the highest score. This approach, however, has two limitations. First, the computational complexity increases exponentially as the number of variables in the causal network increases. Second, the problem of equivalence classes (Chickering, 2002), where two or more network structures represent the same probability distribution, makes the causal direction between nodes quite random and therefore unreliable. The second approach, constraint-based (e.g., (Cheng et al., 2002; Pearl, 2009; Spirtes et al., 2000; Spirtes et al., 1990)), which performs a large number of conditional independence tests between variables to construct a causal network, does not have the problem of equivalence classes. The state-of-the-art Fast Causal Network Inference (FCNI) algorithm (Acharya and Lee, 2013) presents a traditional constraint-based causal network inference mechanism over event streams. Thus, we consider the FCNI algorithm as the representative of the traditional causal network approach in this paper.

In addition, there has been some work (e.g., (Elloumi and Zomaya, 2013; Kłopotek, 2006; Tulupyev and Nikolenko, 2005)) to support *cyclic* Bayesian network which aims to handle the cyclic causality in Bayesian networks. This work, however, still carries the drawbacks inherent in the Bayesian network approach – that is, the ambiguity of equivalence classes and the inability to meet the requirement of a causal network that the parent node in the network should always represent the direct cause – and hence is not useful in our work.

The existing body of work on prediction only addresses inference of the likelihood of

occurrence of an effect variable given a cause variable (e.g., (Cheng and Druzdzel, 2000; Cheng and Druzdzel, 2001; MacKay, 1999; Moral et al., 2012; Shachter, 1990; Zhang.N.L. and D., 1994)), while the prediction of top k effects requires finding the most likely k effects among all possible effect variables. Therefore, the only way to find the top-k next effects is to construct a traditional causal network, which ignores cyclic causality and suffers from causal information loss, over event streams and then infer the top k effects of the cause exhaustively(e.g., (Akdere et al., 2010; Henrion, 1991; Zhang and Poole, 1996)). To the best of our knowledge, there is no solution to address cyclic causality, mitigate the causal information loss, and perform only necessary partial search to find the top-k effects of the given causes over event streams.

The well-established association rule mining algorithms (e.g., (Jiang and Gruenwald, 2005; Rudin et al., 2011; Veloso et al., 2008)) are extensively used for prediction and recommendation. However, association does not necessarily imply causation (e.g., (Bowes et al., 2000; Lin et al., 2008; Mazlack, 2004; Mohammad and Nishida, 2010; Silverstein et al., 2000; Young and Karr, 2011)). Therefore, they are not useful to our problem due to the exclusion of the fundamental concept of causality. That is, two variables that are associated require stronger conditions, such as temporality and strength, to be considered causally related.

5.3 Preliminaries

In this section, we introduce the concepts that are central to the techniques explained in the paper.

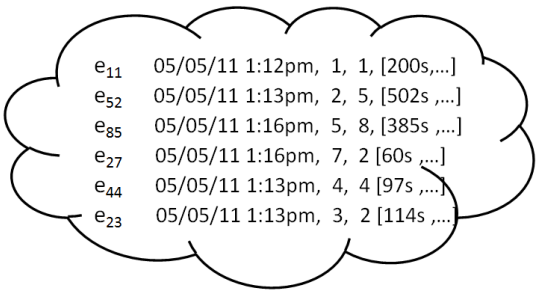


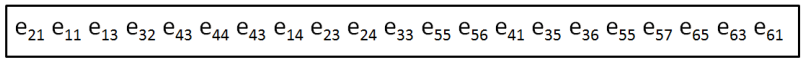
Figure 5.1: Sample of event instances in a stream from Example 4.

5.3.1 Event Streams

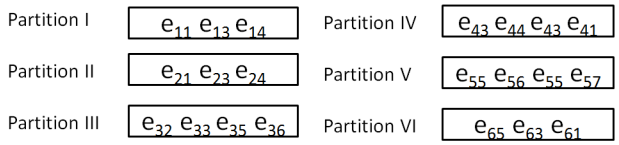
An event stream is a discrete, indefinitely long sequence of event instances. An *event instance* (or event) refers to a timestamped action which may have an effect. A prototype for creating events is called an *event type*. Each event instance is created by one event owner. An event type can have many instances, and an event owner can create many instances of any type. Two events are related to each other if they share common attributes such as event owner, location, and time. These attributes are called *common relational attributes* (CRAs). In Example 4 and Example 5, the CRAs are the session id and the blackout id, respectively .

An event has the following schema: $\langle$*timestamp*, *type*, *CRA*, *attribute-set*$\rangle$. That is, an event has the timestamp at which it was created, the event type it belongs to, the CRA value, and a set of additional attributes called the attribute-set. An event is denoted as e_{ij} where i is the value of the CRA and j (=1, 2, 3, ...) is its event type id (E_j).

Example 6 Figure 5.1 shows an illustrative example of events in a user click event stream of Example 4. The first field in each line (e.g., e_{52}) denotes the actual event instance shown in the remainder of the line (e.g., $\langle$ 05/05/11 1:13 pm, 2, 5, [502s, ...]$\rangle$). The session id serves as the CRA and the webpage categories (e.g., news, weather, sports, entertainment) are the event types. For instance, in the event instance e_{52}, 2 is the event type and 5 is the CRA. Note that the event type is represented by a numerical equivalent of the original



(a) Raw Event Stream.



(b) Partitioned Window.

(e_{ij}s are abbreviations of actual event instances such as shown in Figure 5.1.)

Figure 5.2: Event stream.

event type (e.g., news = 1, weather = 2, sports = 3, entertainment = 4). □

We use a sliding window, specifically called *partitioned window* (Acharya and Lee, 2013), to accumulate the events from the stream for a user-specified observation period T. As a preprocessing step to cluster related events in the window, these events are partitioned by the CRA and then arranged in temporal order in each partition. Figure 5.2 shows what a partitioned window looks like for the event stream shown in Figure 5.1. Once the observation period expires, the window slides to the next batch of events.

Definition 12 (Partition) *A partition W_i in a partitioned window is defined as a set of observed events sharing the same CRA value i and arranged in the temporal order over a time period T, that is*

$$W_i = \{e_{ij}(t) | t \leq T, i \in \mathbf{A}, j \in [1, N]\}$$

where t is the timestamp, $\mathbf{A}$ is the set of all possible CRA values, j is the event type id, and N is the number of event types. □

5.3.2 Causal Networks

A causal network (or causal *Bayesian* network) is a directed acyclic graph $G = (V, \Xi)$ to encode causality, where V is the set of nodes (representing event types) and Ξ is the set of

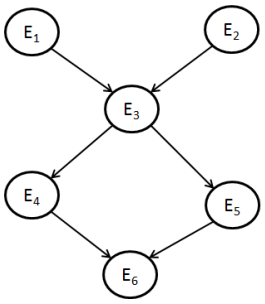


Figure 5.3: Causal network.

edges between nodes. For each directed edge, the parent node denotes the cause, and the child node denotes the effect.

The joint probability distribution of a set of N event types $\mathbf{E} \equiv \{E_1, ..., E_N\}$ in a causal network is specified as

$$P(\mathbf{E}) = \prod_{i=1}^{N} P(E_i|\mathbf{Pa_i})$$

where $\mathbf{Pa_i}$ is the set of the parent nodes of event type E_i.

Consider the event stream of Figure 5.2. The causal relationships among the event types in the stream may be modeled as a causal network like the one shown in Figure 5.3.

5.3.3 Conditional Independence Tests

A popular approach for testing the conditional independence (CI) between two random variables X and Y in a set of random variables, $\mathbf{C}$, is *conditional mutual information* (CMI) (e.g., (Cheng et al., 2002; de Campos, 2006)).

$$\text{CMI}(X, Y|\mathbf{C}) = \sum_{x \in X} \sum_{y \in Y} \sum_{c \in \mathbf{C}} P(x, y, c) log_2 \frac{P(x, y|c)}{P(x|c)P(y|c)}$$

where P is the probability mass function calculated from the frequencies of variables. CMI gives the strength of dependency between variables in a measurable quantity, which helps to identify the weak (or spurious) causal relationships.

In the traditional CMI, two variables X and Y are said to be independent if $\text{CMI}(X, Y|\mathbf{C}) = 0$, and dependent otherwise. This criterion itself offers no distinction between weak and strong dependencies. With a higher value of $\text{CMI}(X, Y|\mathbf{C})$, the dependency between X and Y should be considered stronger. Thus, to prune out weak dependencies, we need a threshold CMI value, below which we consider the evidence "too weak". To do so, we relate CMI with the G^2 test statistics (Bishop et al., 1975; Spirtes et al., 2000) as below.

$$G^2(X, Y|\mathbf{C}) = 2 \cdot N_s \cdot log_e 2 \cdot \text{CMI}(X, Y|\mathbf{C})$$

where N_s is the number of samples (i.e., event instances).

Under the independence assumption, G^2 follows the χ^2 distribution (Kullback, 1968) with the degree of freedom *df* equal to $(n_x - 1)(n_y - 1)\prod_{s \in S} n_s$, where n_x, n_y, and n_s are the number of possible distinct values of X, Y, and S, respectively. So, we perform the test of independence between X and Y given $\mathbf{C}$ by using the calculated G^2 test statistics as the χ^2 test statistics in a χ^2 test, which provides the threshold based on *df* and significance level α, to validate the result. We set α as the generally accepted value of 95%.

We define a Boolean function *IsIndependent*$(X, Y, \mathbf{C})$ to test the conditional independence between two variables X and Y given a condition set of variables $\mathbf{C}$ using the G^2 test statistics. It returns true if these two variables are conditionally independent; otherwise, it returns false.

The unbounded and continuous nature of event streams of interest makes it infeasible to store all of the historical data. Therefore, we use an incremental approach such that when a new batch of events is processed, we only update the record of the frequency of observations without keeping the old events.

Symbols	Definitions
N_p	Number of partitions
N_{ei}	Number of event instances in the i-th partition
N_e	Total number of event instances in all partitions
E_i	Event type with id i
e_{ij}	Event instance of type j and CRA i
N	Number of event types
C_i	Cause event type at position i
T_i	Effect event type at position i
O	Causal search order
S_i	Event type at the i-th position in O
C_δ	The most recent cause event type
$\mathbf{E}$	Set of N event types in the data $\{E_1, E_2, ..., E_N\}$
R_k	Ranked list of the *top-k* event types
$N_{instances}$	Number of event instances
N_{CRA}	Number of common relational attributes
$f(E_i, E_j)$	Number of observations of the instances of type E_i followed by the instances of type E_j

Table 5.1: Definitions of main symbols.

5.4 Problem Formulation and Proposed Approach

In this section, we formulate the problem, identify the research issues and then propose a framework to solve these issues. Table 5.1 summarizes the key notations used in this paper.

5.4.1 Problem Formulation

Causal prediction of the top k effects in an event stream presents two problems. First, it requires inference of the causal relationships among event types, based on the events observed thus far, at run time. Such an approach is necessary to adapt to an evolving stream of events and to overcome the limited causal inference power (i.e., lack of support for cyclic causality and causal information loss) of the traditional causal modeling methods. Second, there is a need for an efficient and continuous query processing method to perform only the necessary *partial* search over the possibly huge search space to predict the results.

The naive approach to determine the top most likely k event types by performing exhaustive searching and sorting among all possible event types is inefficient in a real-time situation.

Based on the above facts, we derive from the top-k predictive query problem the following three central research issues.

1. How can we model the relationships among events to avoid causal information loss and to adapt to changes in event streams?

2. How can we address cyclic causality in the causal model?

3. How can we efficiently run a causal predictive query on this causal model to continuously forecast the top-k probable effects?

In our work, the predictive query is a standing continuous query, so the ranked result list may change every time a new event is observed. The events which are being predicted are *effect events* while the events which are used for prediction are *cause events*. For clarity in the rest of the paper, we denote the cause event type and the effect event type as $C_i \in \mathbf{E}$ and $T_j \in \mathbf{E}$, respectively, where i and j are the positions of the events in each sequence. Note that C_i and E_i are not necessarily the same, and T_j and E_j are not, either.

Definition 13 (Top-K Predictive Query) *Given a set of event types* $\mathbf{E} = \{E_1, E_2, ...\}$ *and a set of observed events* $\varphi_{\mathbf{e}} = \{e_{ij}, e_{ik}, ...\}$ *such that* $(\forall k) e_{ik}.type \in \mathbf{E}$*, the top-k predictive query* $\boldsymbol{Q}(\varphi_{\mathbf{e}}, \mathbf{E})$ *is a conjunctive query that returns the most probable k next event types paired with the top k highest scores, that is, returns* $\{(T_1, S_1), (T_2, S_2), ..., (T_k, S_k)\}$ *where* $T_j \in \mathbf{E}$ $(j = 1, 2, ..., k)$ *is the event type in the j-th position in the top-k results and* S_j $(j = 1, 2, ..., k)$ *is the probabilistic score of* T_j *such that* $S_1 \geq S_2 \geq .. \geq S_k$. □

Given the uncertain nature of an event stream, a *probabilistic* scoring function is needed for prediction. We now define the effect scoring function which gives a measure of how likely the effect is to occur.

Definition 14 (Effect Scoring Function) *Given a set of cause event types* $\mathbf{C} = \{C_1, C_2, ..., C_m\}$, *we define the effect scoring function as the probability of the occurrence of an effect event type,* T_i, *as follows.*

$$P(T_i|\mathbf{C}) = \frac{P(T_i, \mathbf{C})}{P(\mathbf{C})}$$

where P *is the probability mass function formulated from the frequencies of the instances of the argument event types.* □

5.4.2 Proposed Approach

We propose a framework, illustrated in Figure 5.4, to address the three research issues identified in Section 5.4.1.

First, we propose an *event precedence model (EPM)* (Section 5.5) to capture all the precedence relationships between events in a cyclic network structure. (Note that the basic requirement of the cause and effect relationship is the temporal precedence of cause over effect (Popper, 1959).) This inclusion of all temporal precedence – hence, likely causal– relationships help to avoid the causal information loss. EPM takes the partitioned window (collected from the event stream) as an input and incrementally builds a model to reflect all precedence relationships in the input data. The actual data is discarded once a new batch of events arrives. Such an adaptive approach is essential for a streaming environment with continuous and unbounded data.

Second, since the precedence relationship alone does not necessarily imply a casual relationship, based on the EPM the causal inference is performed "on the fly" during query processing (Section 5.6). Such a *run-time causal inference* mechanism is needed to handle the cyclic causal relationships and to avoid the causal information loss. It is only after each query run is finished that the set of relevant cause events, from which causal prediction is done, is known and then the causal direction in a cyclic causality is selected based on the cause events.

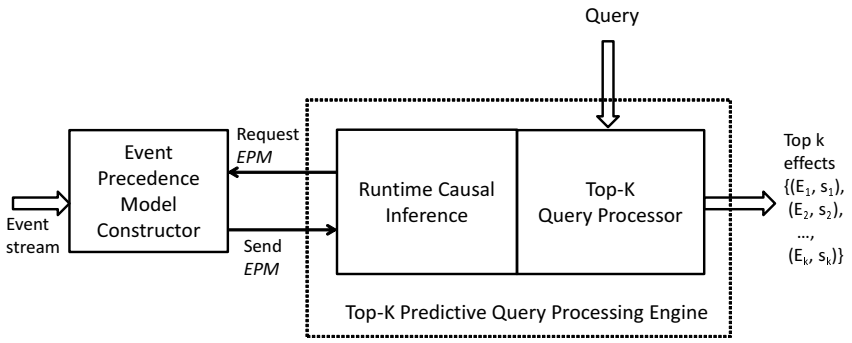


Figure 5.4: Top-k real-time causal prediction framework.

Third, we present two query processing algorithms – the *Exhaustive Search* algorithm (Section 5.6.2) and the *Reduced Search Early Termination* algorithm (Section 5.6.3) – to continuously predict the top k event types with the highest scores based on the inferred causal relationships.

5.5 Event Precedence Model

In this section, we introduce a novel incremental mechanism to model the precedence relationships between events in a network structure.

5.5.1 Model

To overcome the limitations of the existing causal models described in Section 5.4.1, we propose the *event precedence model* (EPM). It represents the temporal precedence relationships in a graphical network structure, called *event precedence network (EPN)*, over which further analysis is done to predict the probable effect events based on the observed cause events. Note that the temporal precedence is a required criterion of a causal relationship. To avoid any information loss, evidence of the precedence between every two events in the stream is preserved.

We make the following assumptions in the event precedence model.

- Given an ordered sequence of cause event types $\{C_0, C_1, ..., C_\delta\}$, an instance of the effect event type T_0 can not occur without the occurrence of an instance of the most recent cause event type C_δ. Moreover, while all past events influence the future events, the strongest influence is exerted by the immediately preceding event of the effect event. With this in mind, the precedence relationships only between every two consecutive events are considered.

- There can not be a causal relationship between events of the same type and, therefore, such precedence relationships are ignored.

- The cause and effect events should share the same CRA value. As described in Section 5.3.1, the events are grouped into partitions based on their *CRA* values (e.g., session id and blackout id in Examples 4 and 5, respectively). In other words, two events are not related to each other if they have different CRA values.

The proposed EPM is a *first order absorbing Markov chain* (Kemeny and Snell, 1969) where an observation is independent of all previous observations except the most recent one and every state can reach an absorbing (a.k.a., terminating) state. Thus, the probability of occurrence of an effect event given past cause events is given as follows.

$$P(T_0|C_0, C_1, ..., C_\delta) = P(T_0|C_\delta).$$

This can be further written as below.

$$P(T_0|C_\delta) = \frac{P(T_0, C_\delta)}{P(C_\delta)} \tag{5.1}$$

which can be estimated as

$$\frac{P(T_0, C_\delta)}{P(C_\delta)} \equiv \frac{f(T_0, C_\delta)}{\sum_{E_j \in children(C_\delta)} f(E_j, C_\delta)} \tag{5.2}$$

where $f(E_i, E_j)$ denotes the number of observations in which instances of the type E_i precedes instances of the type E_j.

In summary, *EPM* allows us to automatically build a tractable probabilistic graphical model from the events, discovering the existing dependencies among the event types in the event stream. These dependencies are represented by a graph, as illustrated in Figure 5.5(b), where the conditional probabilities are stored at the node level.

Algorithm 12 *Event Precedence Model*

Require: A partitioned window P for a batch of new events.

Observation:

for each partition $W_k \in P$ where k is the CRA value **do**
  for each pair of consecutive events (of type E_i and E_j such that $i \neq j$) in W_k **do**
    $f(E_i, E_j)$++ i.e., increase the observed frequency by 1;
  end for
end for

Graph Generation:

Construct an edgeless network $G = (V, \Xi)$;
for each pair of event types, E_i and E_j such that $i \neq j$, **do**
  if $f(E_i, E_j) > 0$ **then**
    $\Xi \leftarrow \{\Xi \cup \{E_i \rightarrow E_j\}\}$ i.e., add an edge $E_i \rightarrow E_j$;
    $P(E_j|E_i) \leftarrow \frac{f(E_i,E_j)}{\sum_{E_k \in children(E_i)} f(E_i,E_k)}$;
  else
    $P(E_j|E_i) \leftarrow 0$;
  end if
  if $f(E_j, E_i) > 0$ **then**
    $\Xi \leftarrow \{\Xi \cup \{E_j \leftarrow E_i\}\}$ i.e., add an edge $E_j \rightarrow E_i$;
    $P(E_i|E_j) \leftarrow \frac{f(E_j,E_i)}{\sum_{E_k \in children(E_j)} f(E_j,E_k)}$;
  else
    $P(E_i|E_j) \leftarrow 0$;
  end if
end for

5.5.2 Algorithm

Algorithm 12 outlines the event precedence network construction algorithm. It has three steps: *observation*, *graph generation*, and *evidence inscription*. These steps are discussed below.

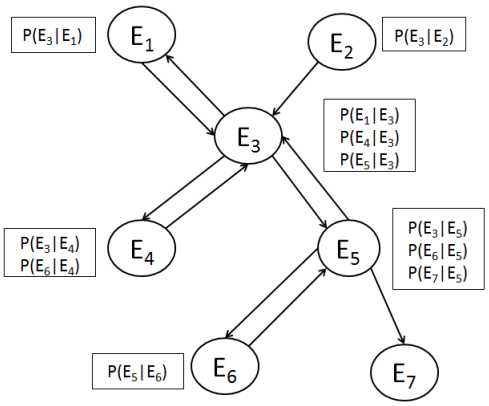
1. *Observation :* This step observes the adjacent neighbor events in each partition of the window to learn the precedence relationships. Note that, based on the assumptions stated earlier, the precedence relationships should be between events in the same partition and between events of different types. Suppose E_i and E_j are the event types of two adjacent events. Then, their count f(E_i, E_j) is increased by 1.

2. *Graph Generation :* This step starts with an edgeless graph $G = (V, \Xi)$ where V is the set of nodes (event types) and Ξ is an empty set of edges. Then, for any evidence of the precedence relationship between event types E_i and E_j (i.e., f$(E_i, E_j) > 0$), an edge is added between the two nodes representing these event types. Note that the graph supports anti-parallel edges between nodes; in addition, a cyclic loop of edges is also supported. Thus, the graphical model offers the flexibility to incorporate all possible types of relationships, unlike in the traditional systems where only directed edges are supported. In addition, for every edge added in the graph, the probability of an event type given its parent event type is calculated using Equation 5.2. The calculated probabilities are then stored in the nodes.

Example 7 Let us illustrate Algorithm 12 considering the event stream shown in Figure 5.2. First, as explained earlier, the observation step counts the number of observations of the precedence relationships between the types of consecutive events in every partition of the partitioned window. These counts are reflected in a matrix form in Figure 5.5(a). Second, for those pairs of event types whose precedence counts in Figure 5.5(a) are greater than zero, we connect them with directed edges as shown in Figure 5.5(b). Clearly, the fig-

$$\begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Observation



(b) Graph Generation

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.50 & 0 & 0 & 0 & 0 \\ 0.333 & 0 & 0 & 0.50 & 0.167 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0.50 & 0.25 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) Probabilities of the edges

Figure 5.5: Illustration of event precedence network construction from the event stream in Figure 5.2.

ure shows that the precedence network supports cyclic relationships, thereby allowing cyclic causality on further causal inference. In addition, the EPN encodes all possible evidences of the precedence relationship, whether weak or strong. Therefore, the causal information loss can be avoided in subsequent causal inference. The conditional probability of each node in the network given its parent is calculated using Equation 5.2. The result is shown in a matrix form in Figure 5.5(c). □

5.5.3 Complexity Analysis

The running time complexity of the algorithm is polynomial with the total number of events that have arrived thus far and the number of event types.

Runtime Analysis. The observation step counts every pair of consecutive events in every partition of the window. Clearly, for each partition, the number of the counts is always one less than the number of events in it. If N_e and N_p are the number of events and the number of partitions, respectively, then the running time complexity is given as $\mathrm{O}(\sum_{i=1}^{N_p}(N_{ei}-1)) \approx \mathrm{O}(N_e)$, where N_{ei} is the number of events in the i-th partition.

The graph generation phase checks for the evidence of the precedence relationships between every pair of event types. In the worst case, the event precedence network is completely connected (including cyclic edges and self referencing edges) and has N^2 edges. So, the running time complexity is given as $\mathrm{O}(N^2)$. So, the running time complexity of this step is proportional to $N(N-1)$ or $\mathrm{O}(N^2)$.

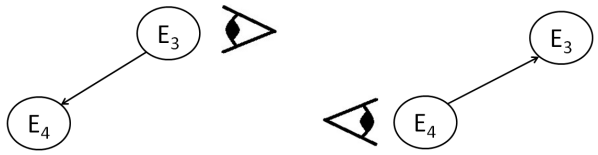
Hence, the total running time is given as $\mathrm{O}(N_e) + \mathrm{O}(N^2) = \mathrm{O}(N_e + N^2)$.

5.6 Top-K Predictive Query Processing

In this section, we first describe the predictive query processing model and then present the two top-k continuous predictive query processing algorithms – *Exhaustive Search* (ES) algorithm and the more efficient *Reduced Search Early Termination* (RSET) algorithm.

5.6.1 Predictive Query Processing Model

The predictive query processing problem can be formulated as a search problem to find the possible effects of a given set of observed events in a causal network. However, the traditional causal network cannot be used for query processing due to the causal information loss and its lack of support for cyclic causality. To address this issue, since we already know



(a) E_3 as EOP (b) E_4 as EOP

Figure 5.6: Views from the EOPs for Figure 5.5(b).

that every causal relationship is always a temporal precedence relationship, we propose to infer causality during query processing from the event precedence network to determine the possible effects.

The idea is to explore the event precedence network (EPN), which represents all the precedence relationships (including cyclic precedence relationships) among the event types, to answer the predictive queries when evidence is available. Indeed, the effect events are always the descendants of the cause events. Therefore, an outward breadth first search on the EPN is required to find the effect events. In situations where a visited node is encountered again, as EPN is cyclic, we ignore it. As discussed in Section 5.5, the next effect events can not occur without the existence of the most recent cause event. Therefore, the starting point for exploring the EPN is always the event type C_δ of the most recent cause event. We call this event type the *effect observation point* (EOP). For instance, in Figure 5.5(b), consider the two event types E_3 and E_4. E_3 is the effect of E_4 when E_4 is the EOP whereas E_4 is the effect of E_3 when E_3 is the EOP, as illustrated in Figure 5.6.

However, there are two issues which make EPN not directly usable to answer a predictive query. First, a precedence relationship is not necessarily a causal relationship. So, we have to remove from the precedence relationships those that are not causal. Second, two variables may have causal relationship in the absence of other variables, but may not exhibit causality in the presence of certain condition variables. For example, rain and wet ground are dependent variables, as rain causes wet ground. However, they are independent in the presence of a roof over the ground (which is a conditional variable), as rain does not cause

wet ground, given the existence of a roof. Therefore, we test causal relationships to resolve these issues during query processing for finding possible effects. To determine causality, the conditional independence tests, as described in Section 5.3.3, are performed between two event types with an edge in the EPN.

The ranking score of the predicted effect event type E_i, given a set of causal event types $\mathbf{C}$, is calculated as $P(E_i|\mathbf{C})$ (Definition 14). Note that the conditional probability of every child node given its parent node is recorded in the EPN. To calculate the scores in our graphical model, we use the multiplicative property of the conditional probability (i.e., $P(X|Z) = P(X|Y) \cdot P(Y|Z)$), so that the score of a node can be computed based on the score of its parents, as follows.

$$P(E_i|\mathbf{C}) = \sum_{E_p \in S_{parents}(E_i)} P(E_i|E_p)P(E_p|\mathbf{C}) \tag{5.3}$$

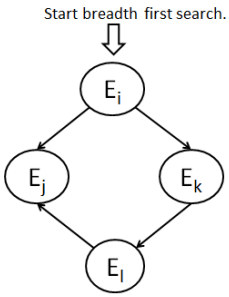
where $S_{parents}(E_i)$ is the set of the parents of E_i.

Lemma 5.6.1 *In a causal network $G = (V, \Xi)$ constructed so far during query processing at runtime, given a set of cause event types $\mathbf{C}$, the ranking score of a parent node (E_p) is never lower than that of its child node (E_c). i.e., $P(E_c|\mathbf{C}) \leq P(E_p|\mathbf{C})$.*

Proof 5.6.2 *The proof is straightforward. From Equation 5.3, we know that*

$$P(E_c|\mathbf{C}) = \sum_{E_p \in S_{parents}(E_c)} P(E_c|E_p)P(E_p|\mathbf{C})$$

where $S_{parents}(E_c)$ is the set of the parents of E_c. As described earlier (Section 5.5.1), we assume that the causal influence for an effect event type E_c is exerted only by its immediately preceding event type E_p. Therefore, we obtain $P(E_c|\mathbf{C}) = P(E_c|E_p) \cdot P(E_p|\mathbf{C})$. Since $P(E_c|E_p) \leq 1$ always holds, we further obtain $P(E_c|\mathbf{C}) \leq P(E_p|\mathbf{C})$. □



Score of E_j is $P(E_j|E_i) \cdot P(E_i|\mathbf{C}) + P(E_j|E_l) \cdot P(E_l|\mathbf{C})$. Note that E_l is explored only after E_j. Therefore, $P(E_l|\mathbf{C})$ is unknown during the calculation of E_j's score

Figure 5.7: Need for a causal search order.

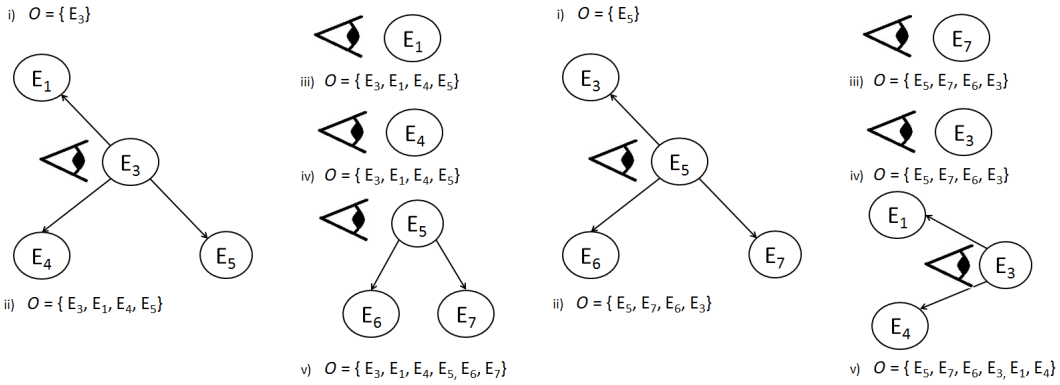
5.6.2 Exhaustive Search Algorithm

5.6.2.1 Approach

The most straightforward solution to the top-k prediction problem is to search for all possible effects exhaustively during the run-time causal inference over EPN and then sort them, according to their scores, in non-increasing order to determine the k effects with the top scores. We call this solution the *Exhaustive Search* (ES) approach.

The ES approach should have a robust strategy for exploring the EPN to infer effects. As discussed earlier, an outward breadth first search may be run over the EPN for run-time causal inference. However, the score calculation of the effects is a bit challenging. To apply Equation 5.3, the scores of the parents of an event type should be known before its score can be calculated, but it is not always possible as demonstrated in Figure 5.7. Therefore, we define a search order, called *causal search order*, before exploring the EPN for run-time causal inference.

Definition 15 ***(Causal Search Order).*** *The causal search order O is an ordered set of event types $\{S_1, S_2, ..., S_N\}$ observed during the outward breadth first search of the EPN such that S_{i+j} is never an ancestor of S_i, where $j > 0$ and $i + j \leq N$.* □



(a) E_3 as EOP (b) E_5 as EOP

Figure 5.8: Causal search order from EOPs in the EPN of Figure 5.5(b).

In addition to guiding the search for possible effects, the causal search order provides us with an effective strategy to calculate the ranking score. It gives an order of the event types such that the probabilities of parents are always known before calculating the probabilities of their children.

Example 8 Let us illustrate the *causal search order* considering the EPN shown in Figure 5.5(b). As described earlier, we run outward breadth first search in EPN from the EOP. Suppose E_3 is the EOP. Initially, E_3 is added to O and is explored. Then, the children of E_3 are added, so O becomes $\{E_3, E_1, E_4, E_5\}$. Then, since E_3 has already been explored, the next unexplored node in O, E_1, is explored. However, no new nodes are added to O as E_1 has no child. Then, we consider the next unexplored node (E_4) in O. Similar to E_1, no new nodes are added to O as E_4 has no child. Now, the only remaining unexplored node in O is E_5. So, the children of E_5 are added to O, which then becomes $\{E_3, E_1, E_4, E_5, E_6, E_7\}$. The recently added unexplored nodes E_6 and E_7 do not have any children and, therefore, no new nodes are added to O. So, the final causal search order O is $\{E_3, E_1, E_4, E_5, E_6, E_7\}$. These steps are shown in Figure 5.8(a). Similarly, when EOP is E_5, the causal search order O may be determined to be $\{E_5, E_7, E_6, E_3, E_1, E_4\}$, as shown in Figure 5.8(b). □

5.6.2.2 Algorithm

Algorithm 13 outlines the ES algorithm. The input to the algorithm is the size of the result k, the event precedence network G, and the set of the recently observed δ cause event types arranged in temporal order $\{C_1, C_2, ..., C_\delta\}$.

The four main steps of the algorithm are given as follows.

1. First, an outward breadth first search over G from EOP (the most recent cause event type C_δ) is run to determine the *causal search order* O. Line 1 shows this step.

2. Second, the *marginal independence tests* (i.e., CI tests with an empty condition set ϕ) are performed between the event types during the search to remove any weak relationships (lines 2- 7).

3. Third, G is searched to find the effects of every unexplored node E_j based on the ordering of the event types in O. Lines 8 - 19 shows this step. The CI tests between E_j and each of its parents are performed as only the parents of E_j can have effect on it. These tests are required to make sure that the event types being considered are not independent in the presence of other event types. In case of independence between E_j and its parent, the edge representing their precedence relationship in G is removed. Lines 9 - 16 shows this step.

   Then, the score of the node E_j is calculated and stored, as shown in lines 17 and 18.

4. Finally, the ranking scores of all event types explored in non-increasing order are sorted and then the k event types with the top scores are selected (line 20).

Example 9 Let us illustrate the ES algorithm considering the EPN shown in Figure 5.5(b). Suppose **C** is $\{E_2, E_3\}$ and k is 2.

1. First, the causal search order O, from the EOP (i.e., E_3), is determined as $\{E_3, E_1, E_4, E_5, E_6, E_7\}$.

Algorithm 13 *ES Algorithm*

Require: A temporally ordered set of recently observed δ cause event types $S = \{C_1, C_2, ..., C_\delta\}$, the size of the result k, an empty buffer B_T to store the effect event types and their scores, and the event precedence network $G = (V, \Xi)$.

1: Determine *causal search order,* O, with the outward breadth first search in G from the EOP (C_δ).
2: **for** every edge $E_i \rightarrow E_j \in \Xi$ **do**
3: $isIndependent \leftarrow IsIndependent(E_i, E_j, \phi)$);
4: **if** *isIndependent* is true **then**
5: $\Xi \leftarrow \Xi - \{E_i \rightarrow E_j\}$ (//remove the weak relationship.)
6: **end if**
7: **end for**
8: **for** every node $E_j \in (O - C_\delta)$ **do**
9: $S_{parents} \leftarrow$ parents of E_j;
10: **for** every parent node $E_i \in S_{parents}$ **do**
11: $isIndependent \leftarrow I_{CMI}(E_i, E_j, S_{parents} - \{E_i\})$
12: **if** *isIndependent* is *true* **then**
13: $\Xi \leftarrow \Xi - \{E_i \rightarrow E_j\}$;
14: $S_{parents} \leftarrow S_{parents} - \{E_i\}$;
15: **end if**
16: **end for**
17: $P(E_j|\mathbf{C}) \leftarrow \sum_{E_p \in S_{parents}} P(E_j|E_p)P(E_p|\mathbf{C})$;
18: Insert the pair $(E_j, P(E_j|\mathbf{C}))$ into B_T;
19: **end for**
20: Sort the nodes in B_T in the non-increasing order of the score and return the top k results from B_T.

2. Then, the marginal independence tests are performed on each edge in EPN. For simplicity in illustration, we assume that these tests fail to remove any edges.

3. Now, the score of each event type in O is calculated and stored into the buffer B_T based on their ordering in O.

   (a) The score of the first unexplored event type E_1 is calculated and updated in B_T as follows.

   - Determine the parents of E_1, $S_{parents} = \{E_3\}$
   - Perform CI tests between every parent of E_1 (i.e., $S_{parents}$) and E_1. Suppose the CI test succeeds, and thus no edge is removed.

- Calculate the score of E_1: $P(E_1|\mathbf{C}) = P(E_1|E_3) \cdot P(E_3|\mathbf{C}) = 0.333$.
- Update B_T as $\{(E_1, 0.333)\}$.

(b) The score of the next unexplored event type E_4 is calculated similarly as above, and B_T is updated to $\{(E_1, 0.333), (E_4, 0.50)\}$.

(c) Following the same step as above, B_T is updated to $\{(E_1, 0.333), (E_4, 0.50), (E_5, 0.167)\}$ for the next event type E_5.

(d) Next, B_T is updated to $\{(E_1, 0.333), (E_4, 0.50), (E_5, 0.167), (E_6, 0.0835)\}$ for the event type E_6.

(e) B_T is updated to $\{(E_1, 0.333), (E_4, 0.50), (E_5, 0.167), (E_6, 0.0835), (E_7, 0)\}$ for the event type E_7. Note that the score $P(E_7|\mathbf{C})$ of E_7 is assigned as zero. For the parent event type E_5, the probability of its child E_7 is much lower (half) than the probability of its other child E_6. Thus, for this illustration, we assume the CI test between E_7 and E_5 fails. Consequently, there is no edge between them.

4. Finally, B_T is sorted in non-increasing order of the score as $\{(E_4, 0.50), (E_1, 0.333), (E_5, 0.167), (E_6, 0.0835), (E_7, 0)\}$. Then, the top-2 predicted next event types are selected from B_T as $\{(E_4, 0.50), (E_1, 0.333)\}$.

□

5.6.3 Reduced Search Early Termination Algorithm

5.6.3.1 Approach

The ES algorithm searches exhaustively in the EPN during query processing to determine the top-k results and, therefore, may well be slow. Naturally there is a need for an alternative method that is faster. There are two issues to deal with for that purpose.

1. *Running time* : With the exhaustive search of ES on EPN to find all possible results, the search space increases with the number of variables and it performs unnecessary computations. A good query processing method should avoid redundant and unnecessary computations to reduce the running time.

2. *Accuracy* : Usually, the tradeoff for reducing running time is the loss in the accuracy of the results. The running time is decreased by reducing the search space, which may skip the correct effect events. The query processing method should avoid such an incident as much as possible.

To achieve faster running time while predicting accurate and consistent results, we propose a new algorithm called the Reduced Search Early Termination (RSET) algorithm with the following strategies.

1. *To reduce the running time.* There are two ideas to reduce the running time. First, we reduce the search space by exploring only the descendants of the nodes in the current top-k during query execution. The nodes not in the top-k or their descendants, due to Lemma 5.6.1, have lower scores, thereby disqualified from being top k candidates. Second, we use a priority-based breadth-first search with an early termination criterion such that the query execution is stopped as soon as it is certain that the top k results have been found. The priority-based breadth-first search always chooses the unexplored descendant node with the highest score to explore its children. The early termination criterion is met if there is no change in the list of event types in the top-k; that means, there is no more descendant node whose score can be greater than those in the current top-k. Consequently, the search space is only partially explored. For this reason, even though the causal inference is done at runtime, it incurs only a small overhead.

2. *To predict accurate results.* In an effort to achieve the same level of accuracy as the

exhaustive search, we employ two strategies. First, we utilize the evidence from all explored nodes to calculate the ranking score. Second, we perform the breadth-first search of the EPN in such a way that the events with greater score are processed earlier. It is worth noting that the ancestors always have higher or the same (in the worst condition) score as their descendants (see Lemma 5.6.1).

5.6.3.2 Algorithm

Algorithm 14 outlines the RSET algorithm and can be described as follows.

1. First, two empty buffers, B_C and B_k, are created to store the event types explored during query processing and the top k effect event types computed, respectively. B_k can hold maximum k event types. Line 1 states this step.

2. Second, we add the EOP, C_δ, with 1 as its score to both these buffers. C_δ has the probability of 1 since the event type has already been observed. Line 2 states this step.

3. Then, the algorithm employs the following strategies to (a) reduce the search space, (b) predict accurate results, and (c) terminate as early as possible.

    - **To reduce the search space.** Two ideas are employed to reduce the search space. First, it explores only the children of EOP or the event types in the buffer B_k for further computation. Line 3 reflects this strategy. For any event type E_c not in B_k (i.e., top-k), E_c and its descendants are ignored, thus reducing the search space. Lemma 5.6.1 ensures that the probabilities of the children of E_c remain much lower than that of E_c, or equal even in the (rare) best case. Second, the buffer B_k is always sorted in non-increasing order of the score after a new event type is added to it (lines 18–19 and 23–24). So, the priority for network exploration is always given to the *unvisited* node with the highest score.

This means that we consider the fact that its children might have higher ranking scores than the unvisited nodes already in B_k. Consequently, the nodes with the lowest probability (E_{lowest}) in B_k can be removed if B_k is full, which further reduces the search space. Lines 3 and 23 reflect this mechanism.

- **To predict accurate results.** It keeps in B_c the record of all event types visited. Basically, the event types in EPN are explored in the breadth-first search. Therefore, the parents of a node have already been explored by the time the algorithm considers the child node. While doing so, the causal relationships are tested in lines 7 – 14. Although the first strategy above reduces the search space, the algorithm still keeps all ancestors in B_c by recording all visited nodes so far. Due to this search strategy, only the nodes with lower scores are not visited. So, the ranking score calculation is not affected by the reduced search space. Consequently, the accuracy of prediction result is not degraded. Lines 7, 15, and 16 make use of this strategy.
- **To terminate as early as possible.** An early termination is possible if there is no change in the list of event types in B_k after exploring their children. It means, due to Lemma 5.6.1, there are no more event types further down the current level of exploration that can have higher probability than those already in B_k. Checking only unvisited nodes in line 3 reflects this strategy.

Example 10 Let us illustrate the RSET algorithm considering the EPN shown in Figure 5.5(b). Suppose **C** is $\{E_2, E_3\}$ and k is 2.

1. First, two empty buffers B_C and B_k are created to store all the event types explored so far and to store the current top k predicted event types, respectively.
2. Then, the search starts with the EOP (E_3) and updates B_C to $\{(E_3, 1)\}$.

Algorithm 14 *RSET Algorithm*

Require: A temporally ordered set of recently observed δ cause event types $\mathbf{C} = \{C_1, C_2, ..., C_\delta\}$, the size of the result k, and event precedence network $G = (V, \Xi)$.

$B_C \leftarrow \{\}$, $B_k \leftarrow \{\}$ i.e., create two buffers B_C and B_k;
$B_C \leftarrow B_C \cup \{(C_\delta, 1)\}$;
for every *unvisited* node $E_c \in (\{C_\delta\} \cup$ (set of event types in B_k)) with the highest score **do**
  Mark E_c as *visited*.
  $S_{children} \leftarrow$ children of E_c;
  for each node $E_j \in S_{children}$ **do**
    "*Visited*" parents $S_{parents} \leftarrow$ (set of parents of E_j) $\cap B_C$;
    for every node $E_i \in S_{parents}$ **do**
      $isIndependent \leftarrow I_{CMI}(E_i, E_j, S_{parents} - \{E_i\})$
      if *isIndependent* is *true* **then**
        $\Xi \leftarrow \Xi - \{E_i \rightarrow E_j\}$;
        $S_{parents} \leftarrow S_{parents} - \{E_i\}$;
      end if
    end for
    $P(E_j|\mathbf{C}) \leftarrow \sum_{E_p \in S_{parents}} P(E_j|E_p)P(E_p|\mathbf{C})$;
    $B_C \leftarrow B_C \cup \{(E_j, P(E_j|\mathbf{C}))\}$;
    if $|B_k| \leq k$ **then**
      $B_k \leftarrow B_k \cup \{(E_j, P(E_j|\mathbf{C}))\}$;
      Sort the event types in B_k in non-increasing order of their scores;
    else
      Find the entry with the lowest score, (E_{lowest}, P_{lowest}), in B_k.
      if $P(E_j|\mathbf{C}) >$ the lowest value in B_k **then**
        $B_k \leftarrow (B_k - \{(E_{lowest}, P_{lowest})\}) \cup \{(E_j, P(E_j|\mathbf{C}))\}$;
        Sort the event types in B_k in non-increasing order of their scores;
      end if
    end if
  end for
end for

3. Now, every *unvisited* event type is explored as follows. For simplicity in illustration, we assume that the CI tests return false and hence the edges are not removed.

   (a) The first unvisited event type, E_3, is marked as visited and its children, $S_{children}$ $(= \{E_1, E_4, E_5\})$, are explored.

      i. The score of the first unexplored child, E_1, is calculated and added to the

two buffers as follows.

- Determine the parents of E_1; $S_{parents}$ i set to $\{E_3\}$.
- Perform CI test of the edge between E_1 and E_3 given $S_{parents}$.
- Calculate the score of E_1 as $P(E_1|\mathbf{C}) = P(E_1|E_3)P(E_3|\mathbf{C}) = 0.333$.
- Update B_C to $\{(E_3, 1), (E_1, 0.333)\}$.
- Update and sort B_k to $\{(E_1, 0.333)\}$.

ii. The same steps as above are followed for the next unexplored child E_4. The two buffers B_C and B_k are updated to $\{(E_3, 1), (E_1, 0.333), (E_4, 0.50)\}$ and $\{(E_4, 0.50), (E_1, 0.333)\}$, respectively.

iii. B_C and B_k are updated to $\{(E_3, 1), (E_1, 0.333), (E_4, 0.50), (E_5, 0.167)\}$ and $\{(E_4, 0.50), (E_1, 0.333)\}$, respectively, for the next unexplored child E_5.

(b) Now, the next unvisited event type, E_4 in B_k, is marked as visited and its children are explored. However, there are no child of E_4, i.e., $S_{children}$ is empty. Therefore, there is no computation to be performed.

(c) The same result is seen for the next event type, E_1, as well. As it has no child (i.e., $S_{children}$ is empty), there is no computation to be performed.

4. The top-k result in B_k is obtained as $\{(E_4, 0.50), (E_1, 0.333)\}$.

For the same $\mathbf{C}$, the RSET algorithm considered only four event types – E_3, E_1, E_4, E_5 and the ES algorithm considered all event types – $E_3, E_1, E_4, E_5, E_6, E_7$. Note that in this example, RSET produced the same result (i.e., $B_k = \{(E_4, 0.50), (E_1, 0.333)\}$) as ES (which is typical unless the value of k is significantly large). It shows the merit of the early terminating reduced search approach of the RSET algorithm against the exhaustive search approach of the ES algorithm. □

5.7 Performance Evaluation

We conduct experiments to evaluate the run-time causal inference model and the top-k query processing mechanism in the proposed *RSET algorithm* and the *ES algorithm*. One evaluation is with respect to the accuracy of the top-k results, and the other evaluation is with respect to the runtime. Section 5.7.1 describes the experiment setup, including the evaluation metrics, datasets and the platform used, and Section 5.7.2 presents the experiment results.

5.7.1 Experiment Setup

5.7.1.1 Evaluation Metrics

Intuitively, the performances of the *top-k query processing* algorithms are best evaluated by examining two important evaluation criteria – accuracy and runtime.

1. **Accuracy.** Suppose R_k is the ranked list of the top k effects predicted from the set of cause event types, $\mathbf{C}$, observed so far in the test sequence. To measure accuracy, the next event type observed in the test sequence, E_o ($o \in [1, N]$), is checked against the predicted ranked list R_k. If E_o exists in R_k, we say the prediction is correct (hit); otherwise we say the prediction is incorrect (miss).

   There are two methods for deciding the correctness of a top-k predicted result. First, in some scenarios, a hit may be a sufficient condition for correctness. In such a case, we say the accuracy is 100% for a hit and 0% for a miss. We call this perspective *hit-or-miss* (*non-weighted*). Second, in other scenarios, the rank of the predicted result may also play an important role in determining the accuracy. Clearly, if the algorithm predicts E_o as the most likely effect (i.e., at the top of R_k with the highest probability), then the accuracy is 100%. As we go down the list in R_k, the accuracy

decreases. Therefore, to reflect this point, we take the probability of each event in R_k into consideration when calculating the prediction accuracy. We call this perspective *weighted*.

***Hit-or-miss* accuracy.** Let n_{hits} and n_{misses} be the number of hits and the number of misses, respectively out of n_{tests} tests. Then, the hit-or-miss accuracy of the result, α_{hm}, is calculated as follows.

$$\alpha_{hm} = \frac{n_{hits}}{n_{tests}} = \frac{n_{hits}}{n_{hits} + n_{misses}} \tag{5.4}$$

Weighted accuracy. Suppose $P(E_o)$ is the score of E_o in R_k. As discussed earlier, the rank of E_o in R_k contributes towards the calculation of the prediction accuracy. The rank is based on the score; therefore, we normalize the score such that the prediction accuracy decreases gradually with the decrease in the rank of E_o in R_k. The accuracy of E_o, in the case of a miss, is 0% whereas the accuracy of E_o, in the case of a hit, is $\frac{P(E_o)}{max\{P(E_j)|E_j \in R_k\}}$, where the denominator is the highest probability among all event types in R_k. (Note that this measure gives the top event type the accuracy of 100%.)

Let n_{hits} and n_{misses} be the same as above. Then, the weighted accuracy of the result, $\alpha_{weighted}$, is computed as follows.

$$\alpha_{weighted} = \frac{\sum_{i=1}^{n_{tests}} \frac{P(E_{o_i})}{max\{P(E_{j_i})|E_{j_i} \in R_{k_i}\}}}{n_{tests}}$$

where E_{o_i} is the i-th observed event type in the test data.

As mentioned earlier, the accuracy of a miss is 0%. Therefore, we can consider only

Dataset	N	$N_{instances}$	CRA	N_{CRA}
MSNBC	17	4698795	session id	989818
Power grid	565	94339	blackout id	4492

(N_{CRA} is the number of different CRA values. $N_{instances}$ is the number of event instances in the dataset.)

Table 5.2: Profiles of the datasets.

the hits in the numerator.

$$\alpha_{weighted} = \frac{\sum_{h=1}^{n_{hits}} \frac{P(E_{oh})}{max\{P(E_{j_h})|E_{j_h} \in R_{k_h}\}}}{n_{hits} + n_{misses}} \tag{5.5}$$

where E_{o_h} is the h-th observed event type which has the result hit in the test data.

2. **Runtime.** The *runtime* is the CPU time taken during query processing. Note that the event precedence network construction is not part of the query processing mechanism. Therefore, we do not include it in the measurement of the runtime. In the query processing with the RSET algorithm and the ES algorithm, there is an overhead of run-time causal inference and it is included in the runtime. In contrast, the query processing with the traditional causal inference (i.e., FCNI algorithm) does not include the causal network construction time in the runtime as the causal inference is performed only once (during the causal network construction) prior to query processing. In our work, latency is the interval between the arrival of a new event and the identification of its top k effect events. However, as the time for EPN update is insignificant (see the polynomial runtime in Section 5.5.3) compared to the time for query processing (which is exponential), latency is essentially the query processing time.

5.7.1.2 Datasets

Experiments are conducted using two real-world datasets (summary in Table 5.2) to evaluate the proposed algorithms.

5.7.1.2.1 Electric power grid dataset. This dataset contains simulated temporal sequences of cascading electric power grid component outages, such as those that can lead to very large blackouts (e.g., (US-Canada Power System Outage Task Force, 2004)). The sequences were generated using a model of the Polish power network, which is described in (Eppstein and Hines, 2012). Each sequence represents the order in which the components failed, as well as the time of the failure. Each grid component is considered an event type, whereas a component failure is an event instance. This dataset includes 4492 cascade sequences and 565 distinct event types.

In the original dataset, each file, representing one blackout, has a list of the components that failed in that blackout. The original schema of the power grid data is as follows: $\langle$*event indicator*, *timestamp*, *component id*$\rangle$. The event indicator can be one of 0, 1, and -1, which refer to an initiating event, a dependent event, and a stop event, respectively. There is always at least one *initiating event* at the beginning of each component failure sequence (with 0 as its starting time). Since these events are always at the beginning of the sequence, there is no inward edge towards them in the event precedence network. A *dependent event* is the result of the initiating event or another dependent event. A blackout sequence always has at least one dependent event. We treat both an initiating event and a dependent event in the same way. On the other hand, a *stop event* denotes the end of the blackout and is not a real event. Therefore, we ignore stop events. The *timestamp* and the *component id* are respectively the starting time of an event and the attribute that uniquely identifies a grid component.

To create an event stream, we modify the schema and randomly mix the data from the files while preserving the temporal order of the component failures in each blackout. The modified schema, $\langle$*timestamp*, *component id*, *blackout id*, *event indicator*$\rangle$, has an additional tag *blackout id* to identify the blackout to which the component failure belongs to. So, the blackout id is the CRA in the power grid dataset.

5.7.1.2.2 MSNBC.com web dataset. This dataset consists of click-stream data of 989818 sequences obtained from the UCI repository. Each sequence reflects the browsing activities, arranged in temporal order, in one user session. The dataset gives a random sample of the length of visits of users browsing the msnbc.com web site on the whole day of September 28, 1999. The length of the visit is an estimate of the total number of clicks or pages seen by each user and is based on the "Internet Information Server (IIS) logs for msnbc.com and news-related portions of msn.com" (Heckerman, 1999). A webpage category is an event type, and a webpage visit is an event instance. The session id of the visit is the CRA for its event instance.

The number of distinct event types is 17. That is, a sequence can have web activities related to 17 different webpage categories. These event types (i.e., webpage categories) are frontpage, news, technology, local, opinion, on-air, miscellaneous, weather, msn-news, health, living, business, msn-sports, sports, summary, bbs, and travel. The total number of event instances (i.e., page visits) is 4698795.

To create an event stream, we randomly mix the events while preserving the temporal order of the events for each session. The schema of the events is $\langle$*timestamp*, *webpage category*, *session id*, $\Phi\rangle$, where Φ denotes an empty attribute set.

5.7.1.3 Experiment Platform

The experiments are conducted on a 2.3 GHz Intel Core i5 machine with 4GB of memory, running Windows 7. The algorithms are implemented in Java 1.7.0.

5.7.2 Experiment Results

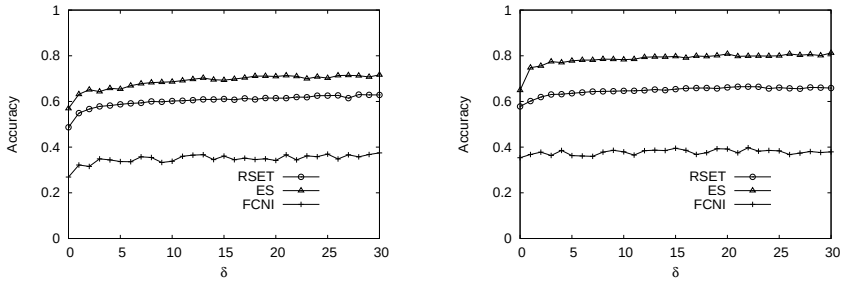
We perform two sets of experiments to evaluate the RSET and ES algorithms. One set of experiments is to evaluate the prediction accuracy, and the other set of experiments is to evaluate the runtime. There are two objectives in each set of experiments. The first objective

is to compare the query processing with the run-time causal inference mechanism (of RSET and ES) against the query processing with the traditional causal inference mechanism (of the FCNI algorithm). For a fair comparison, as the goal is only to compare the causal inference mechanisms, the query processing mechanism of either RSET algorithm or ES algorithm can be used in the FCNI algorithm, and, in this experiment, we choose the RSET algorithm. The second objective is to compare the query processing mechanism between the RSET algorithm and the ES algorithm. In addition, the effects of k on the proposed algorithms are studied in each set of experiments.

We randomly divide each dataset into 70% and 30% for training and testing the proposed algorithms, respectively. From the training dataset, EPN is constructed as an input to the RSET and ES algorithms, whereas a causal network is constructed as an input to the FCNI algorithm. The testing data simulates event stream. As soon as a new event arrives, it is added to the partitioned window. Then, the top-k prediction query execution is triggered in response to the most recent event at position δ (called the *EOP index*) in the sequence of cause events in the same partition. Note that the RSET and ES algorithms perform query processing over the EPN whereas the FCNI algorithm does so over the causal network. Upon the arrival of a new event, the measurements of prediction accuracy and runtime are repeated and the calculated average accuracy and average runtime are reported.

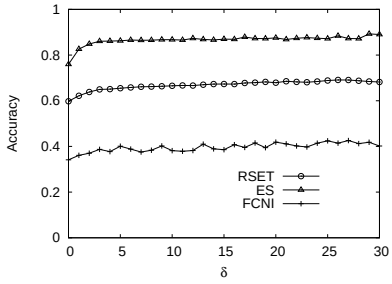
5.7.2.1 Accuracy

Figures 5.9 and 5.10, respectively, show the hit-or-miss and the weighted accuracies for the MSNBC dataset and Figures 5.11 and 5.12 show them for the power grid dataset. In these figures, the accuracies of the RSET algorithm, the ES algorithm, and the FCNI algorithm are compared for different values of the EOP index (δ) over the sequence of events in the condition set. In addition, Figures 5.13 and 5.14 show the average hit-or-miss and weighted accuracies for different values of k in the MSNBC dataset and the power grid dataset,

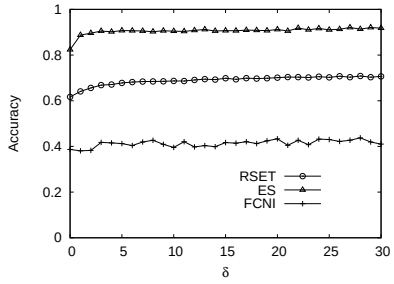


(a) k =1

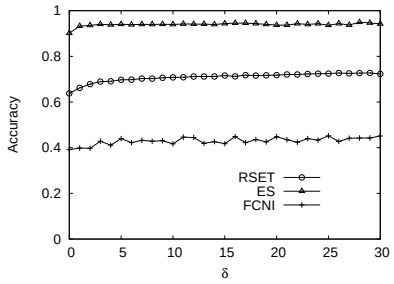
(b) k = 3



(c) k = 5



(d) k = 7

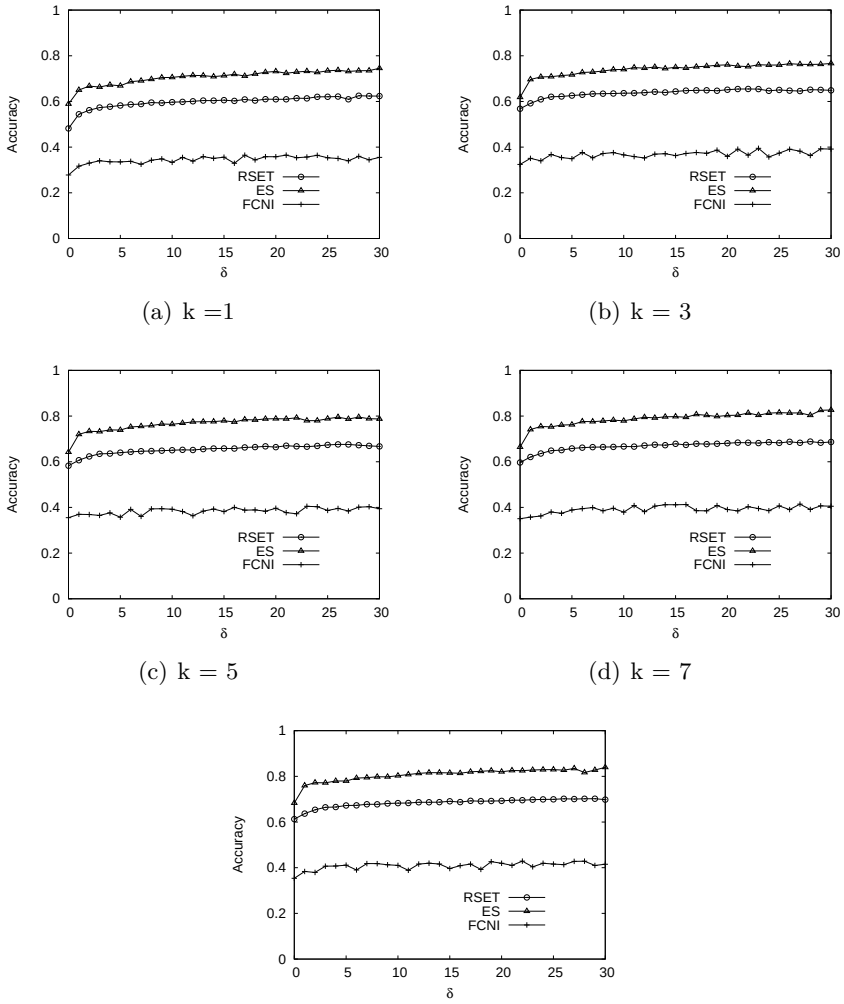


(e) k = 9

Figure 5.9: Hit-or-miss accuracies of the RSET, ES, and FCNI algorithms w.r.t EOP index (δ) for MSNBC dataset. (Note that the EOP index δ is the position of the most recent event in the sequence of cause events in the same partition.)

respectively.

As expected, for all cases, the hit-or-miss accuracy is never lower than the weighted accuracy. Clearly, a hit in the hit-or-miss accuracy always receives the score of 1 while a hit in the weighted accuracy receives a score lower than 1 unless the observed event type in the test data has the highest score in the ranked list. When k is one, the size of the



(a) k =1 (b) k = 3

(c) k = 5 (d) k = 7

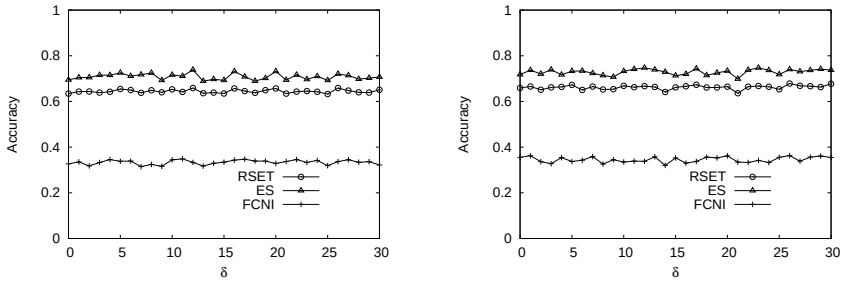
(e) k = 9

Figure 5.10: Weighted accuracies of the RSET, ES, and FCNI algorithms w.r.t EOP index (δ) for MSNBC dataset.

result ranked list is one, hence Equation 5.5 reduces to Equation 5.4, and therefore the two accuracy measures give the same value.

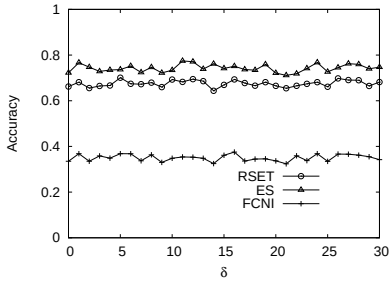
Comparison of the causal inference mechanisms

All results show that the prediction accuracies of both ES and RSET algorithms are significantly higher than that of the FCNI algorithm at every EOP index (δ). This difference

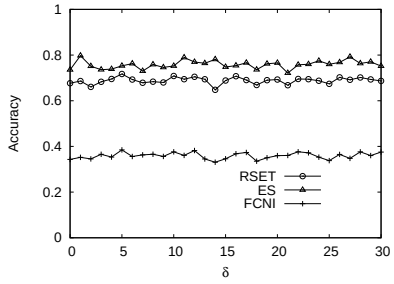


(a) k =1

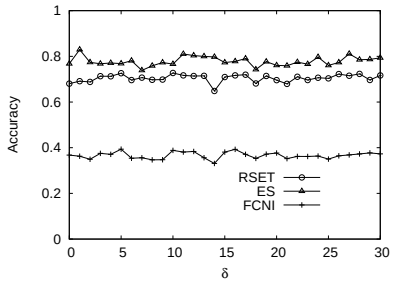
(b) k = 5



(c) k = 10



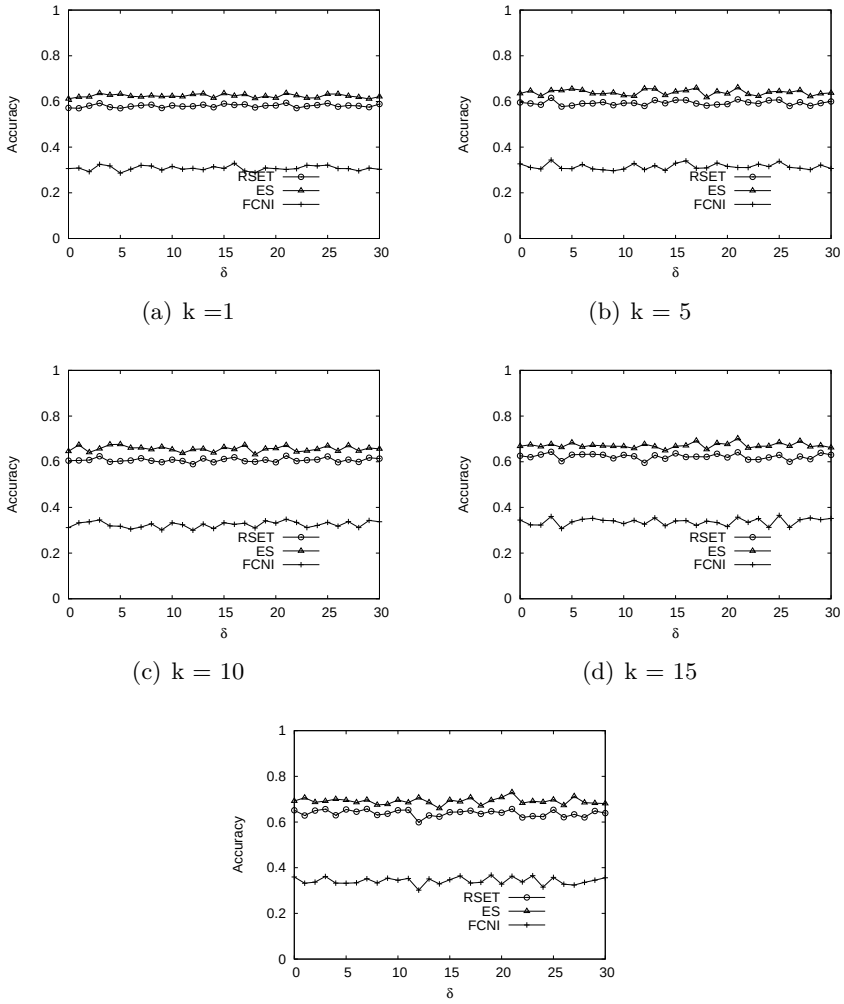
(d) k = 15



(e) k = 20

Figure 5.11: Hit-or-miss accuracies of the RSET, ES, and FCNI algorithms w.r.t EOP index (δ) for the power grid dataset.

in accuracy comes from the difference in their causal models. (Recall that for fairness the FCNI algorithm uses the same query processing mechanism used in the RSET algorithm.) Thus, it confirms the expectation that the traditional causal model (of the FCNI algorithm) is so limited due to its lack of support for cyclic causality and the loss of causal information that the prediction accuracy is compromised significantly. On the other hand, the RSET and ES algorithms both use run-time causal inference which can handle cyclic causality and



(a) k =1 (b) k = 5

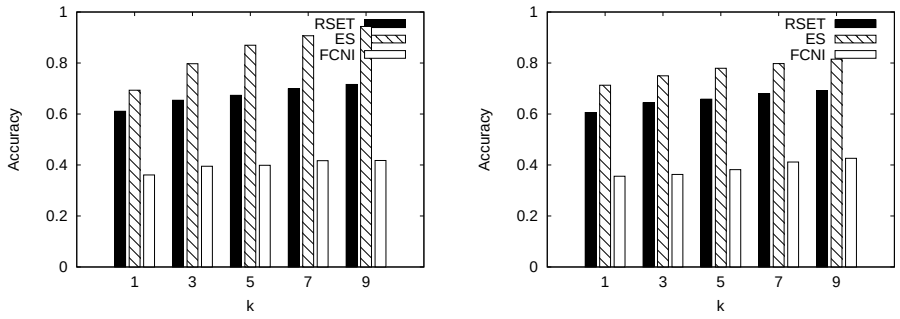
(c) k = 10 (d) k = 15

(e) k = 20

Figure 5.12: Weighted accuracies of the RSET, ES, and FCNI algorithms w.r.t EOP index (δ) for the power grid dataset.

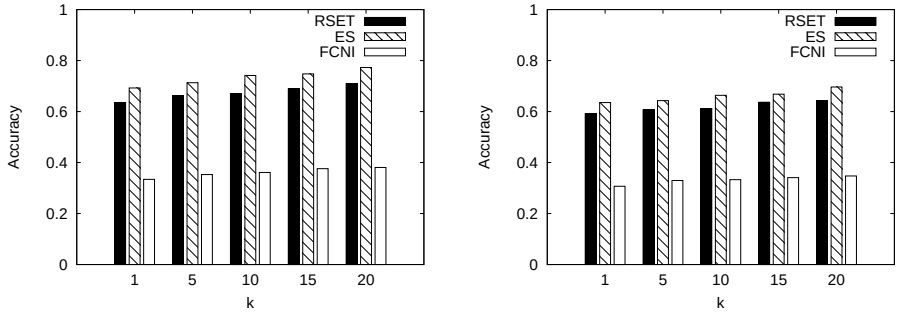
causal information loss, thereby achieving higher prediction accuracies.

The results also show that the accuracy of all three algorithms increases with the increase of *k*. The reason for this is that more event types are considered as the probable next effects. Moreover, the accuracy in the ES and the RSET algorithms is always higher than in the FCNI algorithm. This indicates that the deficiencies of the FCNI algorithm (i.e., acyclic causality and causal information loss) leads to excluding many important causal



(a) hit-or-miss (b) weighted

Figure 5.13: Accuracies of the RSET, ES, and FCNI algorithms w.r.t k for MSNBC dataset.



(a) hit-or-miss (b) weighted

Figure 5.14: Accuracies of the RSET, ES, and FCNI algorithms w.r.t k for the power grid dataset.

relationships and, as a result, the accuracy is always much lower than that of the ES and the RSET algorithms regardless of the value of k.

Comparison of the query processing mechanisms

Three observations are made from the results. First, all results show that the prediction accuracy of the ES algorithm is always higher than that of the RSET algorithm. This is evident from the fact that the ES algorithm performs an exhaustive search whereas the RSET algorithm performs only a partial search.

Second, the accuracy of the RSET algorithm is more comparable to that of the ES algorithm in the power grid dataset than in the MSNBC dataset. This can be explained

as follows. When the ratio of k to N is larger, both algorithms have higher probabilities of making correct predictions but the gap between their accuracies is larger because ES performs exhaustive search while RSET performs partial. Note that k/N is smaller in the power grid dataset (the largest k/N considered is 0.035) than in the MSNBC dataset (the largest k/N considered is 0.53), and therefore the gap is smaller for the power grid dataset.

Third, as discussed earlier, as the value of k increases, the accuracies of all three algorithms increase. Here we add further evaluation on the ES algorithm and the RSET algorithm with a focus on their search mechanisms. As k increases, the search space of the RSET algorithm increases, leading to a higher gain in the accuracy. In the case of the ES algorithm, the search space remains constant regardless of k, but the number of candidate effects from which the highest k is selected increases and, consequently, the accuracy still increases. Intuitively, the rate of increase in the accuracy is higher for the hit-or-miss accuracy metric than the weighted accuracy metric in both algorithms.

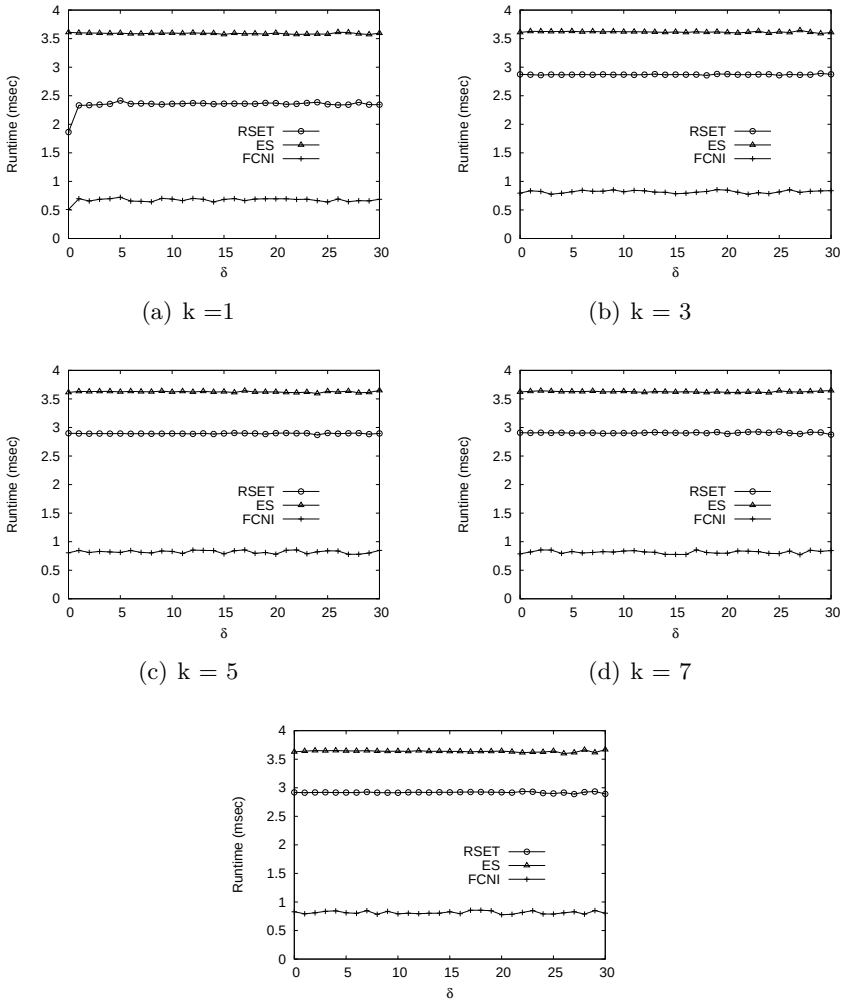
5.7.2.2 Runtime

In this experiment, we compare the runtime among the three algorithms (RSET, ES, FCNI). In addition, we analyze the effects of k on the runtime.

Figures 5.15 and 5.16 show the runtime for the MSNBC dataset and the power grid dataset, respectively. In these figures, the runtime of the three algorithms are compared for different values of the EOP index (δ) over the sequence of events in the condition set. In addition, Figure 5.17 shows the runtime for different values of k in the MSNBC and the power grid datasets.

Comparison of the causal inference mechanisms.

The results show that the runtimes of the RSET and ES algorithms are longer than that of the FCNI algorithm at every EOP index for every value of k. As discussed in Section 5.6.3, the RSET and ES algorithms have an overhead of the run-time causal inference during



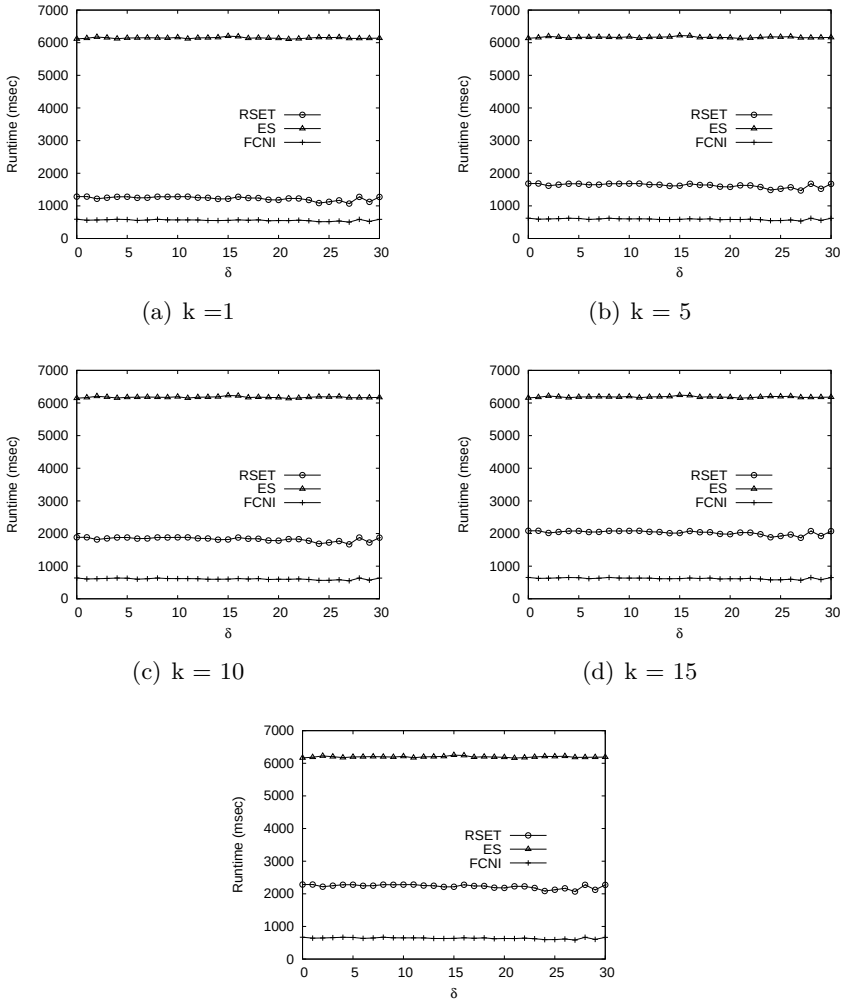
(a) k =1 (b) k = 3

(c) k = 5 (d) k = 7

(e) k = 9

Figure 5.15: Runtime of the RSET, ES, and FCNI algorithms w.r.t EOP index (δ) for MSNBC dataset.

query processing while FCNI algorithm does not as it uses a pre-built causal network for prediction. Therefore, the runtimes for the ES and RSET algorithms are always longer than that of the FCNI algorithm. Interestingly, the runtimes of the three algorithms are longer in the power grid dataset than the MSNBC dataset. This is due to a larger number of event types (i.e., N) in the power grid dataset.



(a) k =1 (b) k = 5

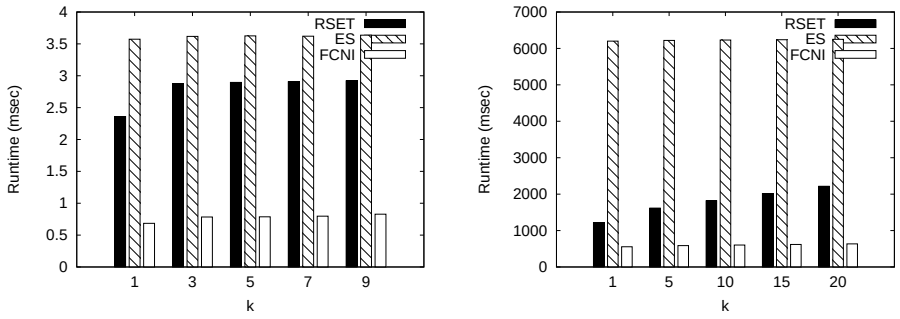
(c) k = 10 (d) k = 15

(e) k = 20

Figure 5.16: Runtime of the RSET, ES, and FCNI algorithms w.r.t EOP index (δ) for the power grid dataset.

Comparison of the query processing mechanisms.

The results suggest three observations. First, as expected, the runtime of the RSET algorithm is always shorter than the ES algorithm. The main reason is in the differen search scope (i.e., exhaustive in ES and partial in RSET) as discussed in Section 5.6. In addition, there is an overhead in the ES algorithm, unlike the RSET algorithm, to calculate the causal



(a) MSNBC (b) Powergrid

Figure 5.17: Runtime of the RSET, ES, and FCNI algorithms w.r.t k.

search order.

Second, the runtime difference between the RSET algorithm and the ES algorithm is smaller for the MSNBC dataset (Figure 5.15) than for the power grid dataset (Figure 5.16). This is due to the larger network size (i.e., N) in the power grid dataset (than in the MSNBC dataset). That is, a larger network size results in a larger search space (which is typical over event streams) and thus requires longer runtime for query processing.

Third, the runtime of the ES algorithm does not change with an increase in k. The ES algorithm always runs an exhaustive search, irrespective of the value of k, and uses k only to filter out the top-k event types out of N event types at the end, which has insignificant effect on the overall runtime. On the other hand, the runtime of the RSET algorithm does increase with an increase in k. The search space covered by the RSET algorithm increases with k, leading to the increased runtime.

5.7.2.3 Discussion of experiment results

The proposed run-time causal inference mechanism, in the RSET and ES algorithms, handles the cyclic causality and avoids the causal information loss, and thus improves the prediction accuracy significantly. The FCNI algorithm, on the other hand, performs worse than both the RSET and ES algorithms as it cannot handle cyclic causality. The ratios

of the number of cycles over the number of edges in the EPN are 0.69 and 0.85 for the power grid dataset and the MSNBC dataset, respectively. Intuitively, the accuracy of the FCNI algorithm would suffer increasingly more as the number of cycles increases. Thus, despite its reduced runtime, the FCNI algorithm is not suitable for top-k predictive query processing over event streams.

The accuracy of the RSET algorithm is comparable to the accuracy of the ES algorithm. Therefore, the RSET algorithm, as it is much faster than the ES algorithm, is more suitable for *real-time* continuous top-k query processing over event streams whereas the ES algorithm is more suitable when the time is of lesser importance.

Between the two datasets, the runtime for the power grid dataset is much longer than the runtime for the MSNBC dataset. This is due to the difference in the numbers of event types in these two datasets. That is, the much larger number of event types in the power grid dataset leads to much more conditional independence tests during the run-time causal inference, thus resulting in slower query execution. In our work, the runtime measurements were made on a low-end laptop. The use of a more powerful computational setup (e.g., parallel processing) would help to further reduce the runtime.

5.8 Conclusion

In this paper, we focused on the problem of continuous top-k prediction over event streams. We proposed a novel causal inference mechanism, called run-time causal inference, to support the cyclic causal relationships and overcome the causal information loss. Then, we proposed two query processing algorithms, called the RSET algorithm and the ES algorithm, which use the concept of the run-time causal inference to predict top k effects continuously. Finally, through experiments, we demonstrated that the proposed approach overcomes the two main limitations of the traditional causal inference approach – acyclic causality and causal information loss. We showed that the proposed RSET and ES algorithms greatly

improved the causal inference power for real data seen these days in various applications.

In this paper, we assumed the events in a stream are in temporal order. For the future work, we plan to consider out-of-order event streams which may introduce erroneous relationships in the event precedence network, thereby increasing the speed of the prediction.

Bibliography

Acharya, S. and Lee, B. (2013). Fast causal network inference over event streams. In *Data Warehousing and Knowledge Discovery*, volume 8057 of *Lecture Notes in Computer Science*, pages 222–235. Springer Berlin Heidelberg.

Acharya, S. and Lee, B. S. (2014a). Enchanced fast causal network inference over event streams. *Transactions on Large Scale Data and Knowledge Centered Systems*.

Acharya, S. and Lee, B. S. (2014b). Incremental causal network construction over event streams. *Information Sciences*, 261(0):32 – 51.

Acharya, S., Lee, B. S., and Hines, P. (2014). Real-time top-k predictive query processing over event streams. *Information Sciences*.

Akdere, M., Çetintemel, U., and Upfal, E. (2010). Database-support for continuous prediction queries over streaming data. *Proc. VLDB Endow.*, 3(1-2):1291–1301.

Alcobé, J. R. (2004a). Incremental hill-climbing search applied to Bayesian network structure learning. In *First International Workshop on Knowledge Discovery in Data Streams, 2004*.

Alcobé, J. R. (2004b). *Incremental Methods for Bayesian Network Structure Learning*. PhD thesis, Department of Computer Science, Universitat Politècnica de Catalunya.

Alcobé, J. R. (2005). Incremental methods for Bayesian network structure learning. *Artificial Intelligence Communications*, 18(1):61–62.

Barber, D. (2012). *Bayesian Reasoning and Machine Learning*. Cambridge University Press.

Barga, R. S., Goldstein, J., Ali, M. H., and Hong, M. (2007). Consistent streaming through time: A vision for event stream processing. In *Proceedings of the Third Biennial Conference on Innovative Data Systems Research*, CIDR'07, pages 363–374.

Bishop, Y. M., Fienberg, S. E., and Holland, P. W. (1975). *Discrete Multivariate Analysis: Theory and Practice*. MIT Press.

Borchani, H., Chaouachi, M., and Ben Amor, N. (2007). Learning causal bayesian networks from incomplete observational data and interventions. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, ECSQARU '07, pages 17–29, Berlin, Heidelberg. Springer-Verlag.

Bowes, J., Neufeld, E., Greer, J., and Cooke, J. (2000). A comparison of association rule discovery and bayesian network causal inference algorithms to discover relationships in discrete data. In Hamilton, H., editor, *Advances in Artificial Intelligence*, volume 1822 of *Lecture Notes in Computer Science*, pages 326–336. Springer Berlin Heidelberg.

Buntine, W. (1991). Theory refinement on Bayesian networks. In *Proceedings of the seventh conference (1991) on Uncertainty in artificial intelligence*, pages 52–60, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Chandra, P. and Kshemkalyani, A. D. (2005). Causality-based predicate detection across space and time. *IEEE Transactions on Computerts*, 54:1438–1453.

Cheng, J. and Druzdzel, M. J. (2000). Ais-bn: An adaptive importance sampling algorithm for evidential reasoning in large Bayesian networks. *J. Artif. Intell. Res. (JAIR)*, 13:155–188.

Cheng, J. and Druzdzel, M. J. (2001). Confidence inference in Bayesian networks. In *Proceedings of the Seventeenth conference on Uncertainty in artificial intelligence*,

UAI'01, pages 75–82, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Cheng, J., Greiner, R., Kelly, J., Bell, D., and Liu, W. (2002). Learning Bayesian networks from data: an information-theory based approach. *Artificial Intelligence*, 137(1-2):43–90.

Chickering, D. M. (2002). Learning equivalence classes of Bayesian-network structures. *Journal of Machine Learning Research*, 2:445–498.

Chickering, D. M., Heckerman, D., and Meek, C. (2004). Large-sample learning of bayesian networks is np-hard. *J. Mach. Learn. Res.*, 5:1287–1330.

Chow, Y. S. and Teicher, H. (1978). *Probability theory : independence, interchangeability, martingales*. Springer-Verlag, New York.

de Campos, L. M. (2006). A scoring function for learning Bayesian networks based on mutual information and conditional independence tests. *J. Machine Learning Research*, 7:2149–2187.

Ellis, B. and Wong, W. H. (2008). Learning causal Bayesian network structures from experimental data. *Journal of the American Statistical Association*, 103(482):778–789.

Elloumi, M. and Zomaya, A. Y. (2013). *Biological Knowledge Discovery Handbook: Preprocessing, Mining and Postprocessing of Biological Data*, volume 23. John Wiley & Sons.

Eppstein, M. and Hines, P. (2012). A "Random Chemistry" algorithm for identifying collections of multiple contingencies that initiate cascading failure. *IEEE Transactions on Power Systems*, 27(3):1698–1705.

Frank, A. and Asuncion, A. (2010). UCI machine learning repository.

Friedman, N. and Goldszmidt, M. (1997). Sequential update of bayesian network structure. In *Proceedings of the Thirteenth conference on Uncertainty in artificial intelli-*

gence, UAI'97, pages 165–174, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Friedman, N., Linial, M., Nachman, I., and Pe'er, D. (2000). Using Bayesian networks to analyze expression data. In *Proceedings of the Fourth Annual International Conference on Computational Molecular Biology*, RECOMB '00, pages 127–135, New York, NY, USA. ACM.

Geiger, D. and Pearl, J. (1988). On the logic of causal models. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 3–14.

Glüge, S., Hamid, O. H., and Wendemuth, A. (2010). A simple recurrent network for implicit learning of temporal sequences. *Cognitive Computation*, 2(4):265–271.

Glymour, C. (2003). Learning, prediction and causal Bayes nets. *Trends in cognitive sciences*, 7(1):43–48.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica*, 37(3):424–438.

Guyon, I., Aliferis, C. F., Cooper, G. F., Elisseeff, A., Pellet, J.-P., Spirtes, P., and Statnikov, A. R. (2008). Design and analysis of the causation and prediction challenge. In *IEEE World Congress on Computational Intelligence Causation and Prediction Challenge*, pages 1–33.

Hamilton, H. J. and Karimi, K. (2005). The timers ii algorithm for the discovery of causality. In *Proceedings of the 9th Pacific-Asia conference on Advances in Knowledge Discovery and Data Mining*, PAKDD'05, pages 744–750, Berlin, Heidelberg. Springer-Verlag.

Heckerman, D. (1995). A Bayesian approach to learning causal networks. In *Proceedings of the Eleventh conference on Uncertainty in artificial intelligence*, UAI'95, pages

285–295, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Heckerman, D. (1999). UCI machine learning repository [http://archive.ics.uci.edu/ml/datasets/msnbc.com+ anonymous+web+data].

Henrion, M. (1991). Search-based methods to bound diagnostic probabilities in very large belief nets. In *Proceedings of the Seventh conference on Uncertainty in Artificial Intelligence*, UAI'91, pages 142–150, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Holme, P. and Saramäki, J. (2011). Temporal networks. *CoRR*, abs/1108.1780.

Ishii, H., Ma, Q., and Yoshikawa, M. (2010). An incremental method for causal network construction. In *Proceedings of the 11th international conference on Web-age information management*, WAIM'10, pages 495–506, Berlin, Heidelberg. Springer-Verlag.

Jiang, X.-R. and Gruenwald, L. (2005). Microarray gene expression data association rules mining based on BSC-tree and FIS-tree. *Data & Knowledge Engineering*, 53(1):3 – 29.

Johnson, T., Muthukrishnan, S., and Rozenbaum, I. (2007). Monitoring regular expressions on out-of-order streams. In *Proceedings of the IEEE 23rd International Conference on Data Engineering*, ICDE'07, pages 1315–1319.

Kemeny, J. and Snell, J. (1969). *Finite Markov chains*. University series in undergraduate mathematics. VanNostrand, New York, repr edition.

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220:671–680.

Kłopotek, M. A. (2006). Cyclic Bayesian network–Markov process approach. *Studia Informatica*, 1(2):7.

Krishna, R., Li, C.-T., and Buchanan-Wollaston, V. (2010). A temporal precedence based clustering method for gene expression microarray data. *BMC Bioinformatics*, 11:68.

Kullback, S. (1968). *Information Theory and Statistics*. Dover Publication, 2nd edition.

Kwon, O. and Li, K.-J. (2009). Causality join query processing for data streams via a spatiotemporal sliding window. *Journal of Universal Computer Science*, 15(12):2287–2310.

Lam, W. and Bacchus, F. (1994). Using new data to refine a Bayesian network. In *Proceedings of the Tenth international conference on Uncertainty in artificial intelligence*, UAI'94, pages 383–390, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Li, G. and Leong, T.-Y. (2009). Active learning for causal bayesian network structure with non-symmetrical entropy. In *Proceedings of the 13th Pacific-Asia Conference on Advances in Knowledge Discovery and Data Mining*, PAKDD '09, pages 290–301, Berlin, Heidelberg. Springer-Verlag.

Li, J., Tufte, K., Shkapenyuk, V., Papadimos, V., Johnson, T., and Maier, D. (2008). Out-of-order processing: A new architecture for high-performance stream systems. *Proceegins of the VLDB Endowment*, 1(1):274–288.

Li, M., Liu, M., Ding, L., Rundensteiner, E. A., and Mani, M. (2007). Event stream processing with out-of-order data arrival. In *Proceedings of the 27th International Conference on Distributed Computing Systems Workshops*, ICDCSW '07, pages 67–74, Washington, DC, USA. IEEE Computer Society.

Lin, T. Y., Xie, Y., Wasilewska, A., and Liau, C.-J., editors (2008). *Data Mining: Foundations and Practice*, volume 118 of *Studies in Computational Intelligence*. Springer.

Liu, M., Li, M., Golovnya, D., Rundensteiner, E., and Claypool, K. (2009). Sequence pattern query processing over out-of-order event streams. In *Proceedings of the IEEE 25th International Conference on Data Engineering*, ICDE '09, pages 784–795.

Liu, Y., Niculescu-Mizil, A., Lozano, A. C., and Lu, Y. (2010). Learning temporal causal graphs for relational time-series analysis. In *ICML*, pages 687–694. Omnipress.

MacKay, D. J. C. (1999). Introduction to Monte Carlo methods. In *Learning in graphical models*, pages 175–204. MIT Press, Cambridge, MA, USA.

Mani, S., Aliferis, C. F., and Statnikov, A. R. (2010). Bayesian algorithms for causal data mining. *Journal of Machine Learning Research*, 6:121–136.

Mazlack, L. J. (2004). Mining causality from imperfect data. In *Proceedings of the sixth International FLINS Conference on Applied Computational Intelligence Proceedings*, Applied Computational Intelligence, pages 155–160.

Meek, C. (1995). Causal inference and causal explanation with background knowledge. In *Proceedings of the Eleventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, pages 403–410, San Francisco, CA. Morgan Kaufmann.

Meganck, S., Leray, P., and Manderick, B. (2006). Learning causal bayesian networks from observations and experiments: a decision theoretic approach. In *Proceedings of the Third international conference on Modeling Decisions for Artificial Intelligence*, MDAI'06, pages 58–69, Berlin, Heidelberg. Springer-Verlag.

Meliou, A., Gatterbauer, W., Moore, K. F., and Suciu, D. (2010). Why so? or why no? functional causality for explaining query answers. In *MUD, 2010*, pages 3–17.

Mohammad, Y. and Nishida, T. (2010). Mining causal relationships in multidimensional time series. In Szczerbicki, E. and Nguyen, N., editors, *Smart Information and Knowledge Management*, volume 260 of *Studies in Computational Intelligence*, pages 309–338. Springer Berlin Heidelberg.

Moral, P., Doucet, A., and Jasra, A. (2012). An adaptive sequential Monte Carlo method for approximate Bayesian computation. *Statistics and Computing*, 22:1009–1020.

Pearl, J. (1995). Causal diagrams for empirical research. *Biometrika*, 82:669–710.

Pearl, J. (1998). Graphs, causality, and structural equation models. *Sociological Methods and Research*, 27:226–84.

Pearl, J. (2009). *Causality: Models, Reasoning and Inference*. Cambridge University Press, New York, NY, USA, second edition edition.

Pearl, J. and Verma, T. (1991). A theory of inferred causation. In *KR, 1991*, pages 441–452.

Popper, K. (1959). *The Logic of Scientific Discovery*. Routledge Classics.

Prakasa Rao, B. (2009). Conditional independence, conditional mixing and conditional association. *Annals of the Institute of Statistical Mathematics*, 61(2):441–460.

Rottman, B. M. and Hastie, R. (2014). Reasoning about causal relationships: Inferences on causal networks. *Psychological Bulletin*, 140(1):109–139.

Rudin, C., Letham, B., Salleb-Aouissi, A., Kogan, E., and Madigan, D. (2011). Sequential event prediction with association rules. In *Proceedings of the 24th Annual Conference on Learning Theory*, COLT '11, pages 615–634.

Shachter, R. D. (1990). Evidence absorption and propagation through evidence reversals. In *Proceedings of the Fifth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '89, pages 173–190, Amsterdam, The Netherlands.

Silverstein, C., Brin, S., Motwani, R., and Ullman, J. (2000). Scalable techniques for mining causal structures. *Data Mining and Knowledge Discovery*, 4(2-3):163–192.

Spirtes, P., Glymour, C., and Scheines, R. (1993). *Causation, Prediction, and Search*. Ima Volumes in Mathematics and Its Applications. Springer-Verlag.

Spirtes, P., Glymour, C., and Scheines, R. (2000). *Causation, Prediction, and Search*. MIT Press.

Spirtes, P., Glymour, C. N., and Scheines, R. (1990). Causality from probability. In *Proceedings of the Conference on Advanced Computing for the Social Sciences*, ACSS

'90.

Spirtes, P. and Meek, C. (1995). Learning Bayesian networks with discrete variables from data. In *Proceedings of the First International Conference on Knowledge Discovery and Data Mining*, KDD'95, pages 294–299.

Tsamardinos, I., Brown, L. E., and Aliferis, C. F. (2006). The max-min hill-climbing bayesian network structure learning algorithm. *Mach. Learn.*, 65(1):31–78.

Tulupyev, A. L. and Nikolenko, S. I. (2005). Directed cycles in Bayesian belief networks: probabilistic semantics and consistency checking complexity. In *Proceedings of the 4th Mexican international conference on Advances in Artificial Intelligence*, pages 214–223. Springer.

US-Canada Power System Outage Task Force (2004). Final Report on the August 14, 2003 Blackout in the United States and Canada. Technical report.

Utrera, A. C., Olmedo, M. G., and Callejon, S. M. (2008). A score based ranking of the edges for the pc algorithm. In *Proceedings of the 4th European workshop on probabilistic graphical models*, PGM'08, pages 41 – 48.

Vaiman, M., Bell, K., Chen, Y., Chowdhury, B., Dobson, I., Hines, P., Papic, M., Miller, S., and Zhang, P. (2012). Risk assessment of cascading outages: Methodologies and challenges. *IEEE Transactions on Power Systems*, 27(2):631–641.

Veloso, A. A., Almeida, H. M., Gonçalves, M. A., and Meira Jr., W. (2008). Learning to rank at query-time using association rules. In *Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, SIGIR '08, pages 267–274, New York, NY, USA. ACM.

Verma, T. and Pearl, J. (1988). Causal networks: Semantics and expressiveness. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 69–78.

Wang, K. and Yu, Y. (2013). A query-matching mechanism over out-of-order event stream in iot. *Int. J. Ad Hoc Ubiquitous Comput.*, 13(3/4):197–208.

Young, S. S. and Karr, A. (2011). Deming, data and observational studies. *Significance*, 8(3):116–120.

Zhang, N. L. and Poole, D. (1996). Exploiting causal independence in Bayesian network inference. *J. Artif. Int. Res.*, 5(1):301–328.

Zhang.N.L. and D., P. (1994). A simple approach to Bayesian network computations. In *Proceedings of the Tenth Canadian Conference on Artificial Intelligence*, CCAA '94, pages 171–178.

Zhao, Y. and Strom, R. (2001). Exploitng event stream interpretation in publish-subscribe systems. In *Proceedings of the Twentieth Annual ACM Symposium on Principles of Distributed Computing*, PODC '01, pages 219–228.

Chapter 6

Concluding Remarks

In this dissertation research, we conducted studies on causal modeling and prediction over event streams. With the emergence of streaming data in many real-world applications, the results of this research advances our understanding and enhances the state-of-the-art in this research area.

6.1 Summary

In the first part of this dissertation, we conducted research on the problem of incremental causal modeling over event streams where the data is unbounded and continuous. The research studied the fundamental properties of causality in the context of such streaming environment which led us to propose two novel algorithms. First, we proposed an incremental temporal network construction algorithm to learn the temporal precedence relationships between the event types in the stream. Second, we proposed a hybrid method for incremental causal network construction by combining the state-of-the-art incremental Bayesian network algorithm and the proposed incremental temporal network construction algorithm. We demonstrated the necessity of such a hybrid approach for causal modeling in observational data such as event streams where the experimental intervention, most commonly used

in the conventional Bayesian methods to determine causality in experimental static data, is not feasible. We demonstrated the importance of the research we engaged by showing the utility of the proposed algorithm in real world application. Our performance study showed the merits of our approach in terms of accurate causal modeling.

In the second part of this dissertation, we dealt with three detailed studies regarding fast construction of causal networks over event streams. First, we investigated and successfully identified the bottleneck in the conventional causal modeling approach that was preventing the fast causal modeling process. Based on this study, we proposed a novel algorithm using temporal precedence semantics to reduce the number of conditional independence tests required for the causal modeling. Second, we studied the effect of changes in probability distribution of the event streams in the learning of causal network structure and leveraged it to propose a change-driven approach to further speed up the learning process. Third, we added the support for out-of-order events in our time-centric causal modeling approach. Results from this study led us to better understand the factors affecting the speed of the causal modeling process.

In the third part of this dissertation, we focused on the problem of top-k causal prediction in real-time over event streams. Specifically, we addressed two research areas of causal prediction - causal modeling and top-k predictive query processing. For causal modeling, we proposed a runtime causal inference strategy to address cyclic causality and causal information loss. For top-k predictive query processing, we proposed two algorithms to answer causal query in real-time by reducing the search space and terminating as soon as the top k answers are found. Our performance study showed the merits of the runtime causal inference and verified the real-time speed of our query processing algorithms.

6.2 Future Work

Our studies centered on causal modeling and prediction over event streams open new directions to several interesting future works, including the following ones. First, while we support causal modeling of discrete variables, the work can be further extended to support continuous variables. In many applications, it is necessary to quantify the value of the effect variables depending on the value of the cause variables. Second, the temporal information used in our work to model the causality can be extended to include interval based time. There may be many scenarios in which both the start time and the end time need to be considered to correctly identify cause and effect relationships. Third, our work can be further extended to address causality in time series data. In such a case, the pattern of temporal relationships between two variables need to be identified over a period of time to establish causal relationships between them. Fourth, the work can be extended to event streams with non-stationary data distribution. Finally, a stopping criteria can be introduced in our work so that the algorithm stops as the arrival of new event instances does not alter the existing causal model.

For fast causal modeling over data streams, potential future work includes the following. The change-driven strategy we proposed to update a causal model either completely revises the model or completely avoids the revision. Instead of a complete revision of the model, our work can be extended further be revise only those causal relationships affected by the changes in the stream. The objective is to minimize the number of conditional independence tests by reducing the number of variables to consider.

Another interesting application for causal prediction is to identify the top-k influencers in a social network. The goal is to monitor the user activities, such as tweets, retweets, or likes, to build an influence graph and predict how a user, called influencer, can influence the activities of other users, called followers. It can be used for many real-world problems

such as targeted advertisements.

Furthermore, one promising direction for future work is to broaden the scope of causality across different domains. In a complex system, an event in one domain may cause an event in another domain. In such a case, there should be causal models maintained across the domains as well as in each domain. Another interesting future work would be to perform the experiments on datasets with a much larger number (i.e., hundreds to thousands) of event types to show the practicality of our proposed algorithms in a big data environment.

Bibliography

Acharya, S. and Lee, B. (2013). Fast causal network inference over event streams. In *Data Warehousing and Knowledge Discovery*, volume 8057 of *Lecture Notes in Computer Science*, pages 222–235. Springer Berlin Heidelberg.

Acharya, S. and Lee, B. S. (2014a). Enchanced fast causal network inference over event streams. *Transactions on Large Scale Data and Knowledge Centered Systems*.

Acharya, S. and Lee, B. S. (2014b). Incremental causal network construction over event streams. *Information Sciences*, 261(0):32 – 51.

Acharya, S., Lee, B. S., and Hines, P. (2014). Real-time top-k predictive query processing over event streams. *Information Sciences*.

Akdere, M., Çetintemel, U., and Upfal, E. (2010). Database-support for continuous prediction queries over streaming data. *Proc. VLDB Endow.*, 3(1-2):1291–1301.

Alcobé, J. R. (2004a). Incremental hill-climbing search applied to Bayesian network structure learning. In *First International Workshop on Knowledge Discovery in Data Streams, 2004*.

Alcobé, J. R. (2004b). *Incremental Methods for Bayesian Network Structure Learning*. PhD thesis, Department of Computer Science, Universitat Politècnica de Catalunya.

Alcobé, J. R. (2005). Incremental methods for Bayesian network structure learning. *Artificial Intelligence Communications*, 18(1):61–62.

Barber, D. (2012). *Bayesian Reasoning and Machine Learning*. Cambridge University Press.

Barga, R. S., Goldstein, J., Ali, M. H., and Hong, M. (2007). Consistent streaming through time: A vision for event stream processing. In *Proceedings of the Third Biennial Conference on Innovative Data Systems Research*, CIDR'07, pages 363–374.

Bishop, Y. M., Fienberg, S. E., and Holland, P. W. (1975). *Discrete Multivariate Analysis: Theory and Practice*. MIT Press.

Borchani, H., Chaouachi, M., and Ben Amor, N. (2007). Learning causal bayesian networks from incomplete observational data and interventions. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, ECSQARU '07, pages 17–29, Berlin, Heidelberg. Springer-Verlag.

Bowes, J., Neufeld, E., Greer, J., and Cooke, J. (2000). A comparison of association rule discovery and bayesian network causal inference algorithms to discover relationships in discrete data. In Hamilton, H., editor, *Advances in Artificial Intelligence*, volume 1822 of *Lecture Notes in Computer Science*, pages 326–336. Springer Berlin Heidelberg.

Buntine, W. (1991). Theory refinement on Bayesian networks. In *Proceedings of the seventh conference (1991) on Uncertainty in artificial intelligence*, pages 52–60, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Chandra, P. and Kshemkalyani, A. D. (2005). Causality-based predicate detection across space and time. *IEEE Transactions on Computerts*, 54:1438–1453.

Cheng, J. and Druzdzel, M. J. (2000). Ais-bn: An adaptive importance sampling algorithm for evidential reasoning in large Bayesian networks. *J. Artif. Intell. Res. (JAIR)*, 13:155–188.

Cheng, J. and Druzdzel, M. J. (2001). Confidence inference in Bayesian networks. In *Proceedings of the Seventeenth conference on Uncertainty in artificial intelligence*,

UAI'01, pages 75–82, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Cheng, J., Greiner, R., Kelly, J., Bell, D., and Liu, W. (2002). Learning Bayesian networks from data: an information-theory based approach. *Artificial Intelligence*, 137(1-2):43–90.

Chickering, D. M. (2002). Learning equivalence classes of Bayesian-network structures. *Journal of Machine Learning Research*, 2:445–498.

Chickering, D. M., Heckerman, D., and Meek, C. (2004). Large-sample learning of bayesian networks is np-hard. *J. Mach. Learn. Res.*, 5:1287–1330.

Chow, Y. S. and Teicher, H. (1978). *Probability theory : independence, interchangeability, martingales*. Springer-Verlag, New York.

de Campos, L. M. (2006). A scoring function for learning Bayesian networks based on mutual information and conditional independence tests. *J. Machine Learning Research*, 7:2149–2187.

Ellis, B. and Wong, W. H. (2008). Learning causal Bayesian network structures from experimental data. *Journal of the American Statistical Association*, 103(482):778–789.

Elloumi, M. and Zomaya, A. Y. (2013). *Biological Knowledge Discovery Handbook: Preprocessing, Mining and Postprocessing of Biological Data*, volume 23. John Wiley & Sons.

Eppstein, M. and Hines, P. (2012). A "Random Chemistry" algorithm for identifying collections of multiple contingencies that initiate cascading failure. *IEEE Transactions on Power Systems*, 27(3):1698–1705.

Frank, A. and Asuncion, A. (2010). UCI machine learning repository.

Friedman, N. and Goldszmidt, M. (1997). Sequential update of bayesian network structure. In *Proceedings of the Thirteenth conference on Uncertainty in artificial intelli-*

gence, UAI'97, pages 165–174, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Friedman, N., Linial, M., Nachman, I., and Pe'er, D. (2000). Using Bayesian networks to analyze expression data. In *Proceedings of the Fourth Annual International Conference on Computational Molecular Biology*, RECOMB '00, pages 127–135, New York, NY, USA. ACM.

Geiger, D. and Pearl, J. (1988). On the logic of causal models. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 3–14.

Glüge, S., Hamid, O. H., and Wendemuth, A. (2010). A simple recurrent network for implicit learning of temporal sequences. *Cognitive Computation*, 2(4):265–271.

Glymour, C. (2003). Learning, prediction and causal Bayes nets. *Trends in cognitive sciences*, 7(1):43–48.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica*, 37(3):424–438.

Guyon, I., Aliferis, C. F., Cooper, G. F., Elisseeff, A., Pellet, J.-P., Spirtes, P., and Statnikov, A. R. (2008). Design and analysis of the causation and prediction challenge. In *IEEE World Congress on Computational Intelligence Causation and Prediction Challenge*, pages 1–33.

Hamilton, H. J. and Karimi, K. (2005). The timers ii algorithm for the discovery of causality. In *Proceedings of the 9th Pacific-Asia conference on Advances in Knowledge Discovery and Data Mining*, PAKDD'05, pages 744–750, Berlin, Heidelberg. Springer-Verlag.

Heckerman, D. (1995). A Bayesian approach to learning causal networks. In *Proceedings of the Eleventh conference on Uncertainty in artificial intelligence*, UAI'95, pages

285–295, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Heckerman, D. (1999). UCI machine learning repository [http://archive.ics.uci.edu/ml/datasets/msnbc.com+anonymous+web+data].

Henrion, M. (1991). Search-based methods to bound diagnostic probabilities in very large belief nets. In *Proceedings of the Seventh conference on Uncertainty in Artificial Intelligence*, UAI'91, pages 142–150, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Holme, P. and Saramäki, J. (2011). Temporal networks. *CoRR*, abs/1108.1780.

Ishii, H., Ma, Q., and Yoshikawa, M. (2010). An incremental method for causal network construction. In *Proceedings of the 11th international conference on Web-age information management*, WAIM'10, pages 495–506, Berlin, Heidelberg. Springer-Verlag.

Jiang, X.-R. and Gruenwald, L. (2005). Microarray gene expression data association rules mining based on BSC-tree and FIS-tree. *Data & Knowledge Engineering*, 53(1):3 – 29.

Johnson, T., Muthukrishnan, S., and Rozenbaum, I. (2007). Monitoring regular expressions on out-of-order streams. In *Proceedings of the IEEE 23rd International Conference on Data Engineering*, ICDE'07, pages 1315–1319.

Kemeny, J. and Snell, J. (1969). *Finite Markov chains*. University series in undergraduate mathematics. VanNostrand, New York, repr edition.

Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220:671–680.

Kłopotek, M. A. (2006). Cyclic Bayesian network–Markov process approach. *Studia Informatica*, 1(2):7.

Krishna, R., Li, C.-T., and Buchanan-Wollaston, V. (2010). A temporal precedence based clustering method for gene expression microarray data. *BMC Bioinformatics*, 11:68.

Kullback, S. (1968). *Information Theory and Statistics*. Dover Publication, 2nd edition.

Kwon, O. and Li, K.-J. (2009). Causality join query processing for data streams via a spatiotemporal sliding window. *Journal of Universal Computer Science*, 15(12):2287–2310.

Lam, W. and Bacchus, F. (1994). Using new data to refine a Bayesian network. In *Proceedings of the Tenth international conference on Uncertainty in artificial intelligence*, UAI'94, pages 383–390, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Li, G. and Leong, T.-Y. (2009). Active learning for causal bayesian network structure with non-symmetrical entropy. In *Proceedings of the 13th Pacific-Asia Conference on Advances in Knowledge Discovery and Data Mining*, PAKDD '09, pages 290–301, Berlin, Heidelberg. Springer-Verlag.

Li, J., Tufte, K., Shkapenyuk, V., Papadimos, V., Johnson, T., and Maier, D. (2008). Out-of-order processing: A new architecture for high-performance stream systems. *Proceegins of the VLDB Endowment*, 1(1):274–288.

Li, M., Liu, M., Ding, L., Rundensteiner, E. A., and Mani, M. (2007). Event stream processing with out-of-order data arrival. In *Proceedings of the 27th International Conference on Distributed Computing Systems Workshops*, ICDCSW '07, pages 67–74, Washington, DC, USA. IEEE Computer Society.

Lin, T. Y., Xie, Y., Wasilewska, A., and Liau, C.-J., editors (2008). *Data Mining: Foundations and Practice*, volume 118 of *Studies in Computational Intelligence*. Springer.

Liu, M., Li, M., Golovnya, D., Rundensteiner, E., and Claypool, K. (2009). Sequence pattern query processing over out-of-order event streams. In *Proceedings of the IEEE 25th International Conference on Data Engineering*, ICDE '09, pages 784–795.

Liu, Y., Niculescu-Mizil, A., Lozano, A. C., and Lu, Y. (2010). Learning temporal causal graphs for relational time-series analysis. In *ICML*, pages 687–694. Omnipress.

MacKay, D. J. C. (1999). Introduction to Monte Carlo methods. In *Learning in graphical models*, pages 175–204. MIT Press, Cambridge, MA, USA.

Mani, S., Aliferis, C. F., and Statnikov, A. R. (2010). Bayesian algorithms for causal data mining. *Journal of Machine Learning Research*, 6:121–136.

Mazlack, L. J. (2004). Mining causality from imperfect data. In *Proceedings of the sixth International FLINS Conference on Applied Computational Intelligence Proceedings*, Applied Computational Intelligence, pages 155–160.

Meek, C. (1995). Causal inference and causal explanation with background knowledge. In *Proceedings of the Eleventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-95)*, pages 403–410, San Francisco, CA. Morgan Kaufmann.

Meganck, S., Leray, P., and Manderick, B. (2006). Learning causal bayesian networks from observations and experiments: a decision theoretic approach. In *Proceedings of the Third international conference on Modeling Decisions for Artificial Intelligence*, MDAI'06, pages 58–69, Berlin, Heidelberg. Springer-Verlag.

Meliou, A., Gatterbauer, W., Moore, K. F., and Suciu, D. (2010). Why so? or why no? functional causality for explaining query answers. In *MUD, 2010*, pages 3–17.

Mohammad, Y. and Nishida, T. (2010). Mining causal relationships in multidimensional time series. In Szczerbicki, E. and Nguyen, N., editors, *Smart Information and Knowledge Management*, volume 260 of *Studies in Computational Intelligence*, pages 309–338. Springer Berlin Heidelberg.

Moral, P., Doucet, A., and Jasra, A. (2012). An adaptive sequential Monte Carlo method for approximate Bayesian computation. *Statistics and Computing*, 22:1009–1020.

Pearl, J. (1995). Causal diagrams for empirical research. *Biometrika*, 82:669–710.

Pearl, J. (1998). Graphs, causality, and structural equation models. *Sociological Methods and Research*, 27:226–84.

Pearl, J. (2009). *Causality: Models, Reasoning and Inference*. Cambridge University Press, New York, NY, USA, second edition edition.

Pearl, J. and Verma, T. (1991). A theory of inferred causation. In *KR, 1991*, pages 441–452.

Popper, K. (1959). *The Logic of Scientific Discovery*. Routledge Classics.

Prakasa Rao, B. (2009). Conditional independence, conditional mixing and conditional association. *Annals of the Institute of Statistical Mathematics*, 61(2):441–460.

Rottman, B. M. and Hastie, R. (2014). Reasoning about causal relationships: Inferences on causal networks. *Psychological Bulletin*, 140(1):109–139.

Rudin, C., Letham, B., Salleb-Aouissi, A., Kogan, E., and Madigan, D. (2011). Sequential event prediction with association rules. In *Proceedings of the 24th Annual Conference on Learning Theory*, COLT '11, pages 615–634.

Shachter, R. D. (1990). Evidence absorption and propagation through evidence reversals. In *Proceedings of the Fifth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '89, pages 173–190, Amsterdam, The Netherlands.

Silverstein, C., Brin, S., Motwani, R., and Ullman, J. (2000). Scalable techniques for mining causal structures. *Data Mining and Knowledge Discovery*, 4(2-3):163–192.

Spirtes, P., Glymour, C., and Scheines, R. (1993). *Causation, Prediction, and Search*. Ima Volumes in Mathematics and Its Applications. Springer-Verlag.

Spirtes, P., Glymour, C., and Scheines, R. (2000). *Causation, Prediction, and Search*. MIT Press.

Spirtes, P., Glymour, C. N., and Scheines, R. (1990). Causality from probability. In *Proceedings of the Conference on Advanced Computing for the Social Sciences*, ACSS

'90.

Spirtes, P. and Meek, C. (1995). Learning Bayesian networks with discrete variables from data. In *Proceedings of the First International Conference on Knowledge Discovery and Data Mining*, KDD'95, pages 294–299.

Tsamardinos, I., Brown, L. E., and Aliferis, C. F. (2006). The max-min hill-climbing bayesian network structure learning algorithm. *Mach. Learn.*, 65(1):31–78.

Tulupyev, A. L. and Nikolenko, S. I. (2005). Directed cycles in Bayesian belief networks: probabilistic semantics and consistency checking complexity. In *Proceedings of the 4th Mexican international conference on Advances in Artificial Intelligence*, pages 214–223. Springer.

US-Canada Power System Outage Task Force (2004). Final Report on the August 14, 2003 Blackout in the United States and Canada. Technical report.

Utrera, A. C., Olmedo, M. G., and Callejon, S. M. (2008). A score based ranking of the edges for the pc algorithm. In *Proceedings of the 4th European workshop on probabilistic graphical models*, PGM'08, pages 41 – 48.

Vaiman, M., Bell, K., Chen, Y., Chowdhury, B., Dobson, I., Hines, P., Papic, M., Miller, S., and Zhang, P. (2012). Risk assessment of cascading outages: Methodologies and challenges. *IEEE Transactions on Power Systems*, 27(2):631–641.

Veloso, A. A., Almeida, H. M., Gonçalves, M. A., and Meira Jr., W. (2008). Learning to rank at query-time using association rules. In *Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, SIGIR '08, pages 267–274, New York, NY, USA. ACM.

Verma, T. and Pearl, J. (1988). Causal networks: Semantics and expressiveness. In *Proceedings of the Fourth Annual Conference on Uncertainty in Artificial Intelligence*, UAI '88, pages 69–78.

Wang, K. and Yu, Y. (2013). A query-matching mechanism over out-of-order event stream in iot. *Int. J. Ad Hoc Ubiquitous Comput.*, 13(3/4):197–208.

Young, S. S. and Karr, A. (2011). Deming, data and observational studies. *Significance*, 8(3):116–120.

Zhang, N. L. and Poole, D. (1996). Exploiting causal independence in Bayesian network inference. *J. Artif. Int. Res.*, 5(1):301–328.

Zhang.N.L. and D., P. (1994). A simple approach to Bayesian network computations. In *Proceedings of the Tenth Canadian Conference on Artificial Intelligence*, CCAA '94, pages 171–178.

Zhao, Y. and Strom, R. (2001). Exploitng event stream interpretation in publish-subscribe systems. In *Proceedings of the Twentieth Annual ACM Symposium on Principles of Distributed Computing*, PODC '01, pages 219–228.