The Probability of Coalition Formation: Spatial Voting Power Indices

by

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Abstract

This paper aims to include actor's policy preferences into a probabilistic definition of two common indices of relative voting power: the Shapley-Shubik index and the (normalized) Banzhaf index. While we acknowledge the validity of standard indices and their strength in analyzing relative influence in institutions in an a priori, or 'constitutional', perspective, we develop upon them here in an attempt to explicitly account for players' policy preferences. Hence, we aim to partially combine spatial approaches of voting with voting power analysis. After elaborating a respective model for both uni-dimensional and multi-dimensional policy spaces, we suggest ways to test the predictive capacity of such models, either by directly incorporating data on preferences or by estimating such preferences, for example on the basis of theories of endogenous policy formation.

1. Introduction

Theories of coalition formation encompass both 'policy-blind' and 'policy-oriented' approaches[2]. Among the traditional approaches are the theory of minimal winning and minimal size coalitions (Riker, 1962) and Leiserson's bargaining theory (Leiserson, 1966). Among the traditional 'policy-oriented' approaches are de Swaan's minimal range theory (de Swaan, 1973) and Axelrod's theory of minimal connected winning coalitions (Axelrod, 1970). However, it should be noted that the latter approaches implicitly build upon 'policy-blind' theories by focusing their

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[2] For a helpful overview of various approaches, including more 'traditional' theories of coalition-formation and newer extensions, see van Deemen (1997).

attention on winning coalitions[3]. Most approaches use information both as regards the relative weight of players and their policy position as factors determining which coalitions form in reality. Generally, central to research on coalition formation are voting mechanisms, policy issues at stake, interaction among players, and players' policy preferences. Policy-oriented theories of coalition formation can rely on either uni- or multi-dimensional models[4].

By comparison, studies on relative voting power have tended to be 'policy blind' in the sense of emphasizing the difference between nominal voting weights and effective voting power of the players within a committee. Intuitively, the strength of an (implicit or explicit) coalition depends on which players it encompasses. Given a specific voting rule, a player can render a coalition winning or losing when it is 'critical' (or 'pivotal') to the fate of the coalition. When players hold different voting weights, as in the case of the member states within the Council of the EU for example, larger players (i.e. players with more votes) appear to enjoy an absolute advantage over smaller ones. The recent re-weighting of votes in the Council, naturally, affects this balance[5]. Yet, because of the option of defection, only some players -- even though they may be small -- can play a critical role within coalitions. Voting power is attributed to these pivotal players, as they are decisive to the success of a coalition. Hence, factors such as voting weights and the decision threshold crucially affect the difference between voting weights and voting power within a committee.

In recent years, a lively debate has emerged, mainly among political scientists analyzing decision-making in the EU, over the practical relevance and theoretical foundation of voting power indices.[6] Critiques raised against such approaches encompass the assumed inability of voting power models to take complex institutional arrangements and processes into account (which might have detrimental implications when dealing with entities such as the EU[7]). Another criticism rests on the *a priori* nature of voting power indices as they tend to abstract from players' policy preferences.

Accordingly, some scholars have suggested ways to incorporate institutional procedures into voting power analysis, in particular in the context of analyses of EU decision-making (see e.g. Widgrén, 1996; Nurmi and Meskanen, 1999, Laruelle and Widgrén, 2000). Similarly, while stressing the usefulness of a priori voting power models in addressing constitutional issues

[3] In reality, for example in the analysis of government-formation in advanced industrial societies, it has been found that minority coalitions form rather frequently (as is the case, for example, in Denmark and Ireland). For a respective overview and approaches aiming to explain 'viable' minority coalitions, usually referring to the spatial location of political parties in a two-dimensional policy space, e.g. see Laver and Schofield (1990). Extensions to multi-dimensional policy spaces can be found, for example, in Laver and Shepsle (1996).

[4] Among the multi-dimensional approaches are Grofman's protocoalition formation theory (Grofman 1982, 1996) and Schofield's heart theory (Schofield 1993, 1995). For an overview of various spatial theories of coalition formation, e.g. see de Vries (1999).

[5] On possible effects, see e.g. Baldwin et al. (2000), Felsenthal and Machover (2001), Leech (2001).

[6] See for example articles published on voting power in volumes 11 (No.3) and 13 (No.1) of the *Journal of Theoretical Politics* (1999, 2001).

[7] It may be mentioned, however, that the very complexity of EU decision-making procedures – both in the sense of different procedures being in effect and the relative elaborateness, for example, of the co-decision procedure - complicates the construction of meaningful models to capture the interaction among EU institutions and assessments of their relative influence in EU decision-making. For an overview of how different model specifications critically affect predictions as regards institutions' relative influence – occasionally leading to diametrically opposed findings - - see for example Steunenberg and Selck (2000).

(where coalition formation can be viewed as a 'random process' in the long run – with players acting behind a so-called 'veil of ignorance'), several contributions have attempted to include the preferred policy positions of players into the determination of relative voting power. For example, Kirman and Widgrén (1995) consider fixed, pre-determined coalitions based on the assumed policy position of member states, in order to calculate their distribution of policy-specific voting power in the Council. Similarly, Colomer and Hosli (1997) and Bilal and Hosli (1999) have restricted coalition formation to players that are contingent, i.e. have adjacent preferred positions, on a uni-dimensional ideological (or 'policy preference') scale, the former study focusing on political parties in the European Parliament and the latter on policy preferences of member states in the Council.

The aim of this paper is to integrate elements of the spatial theory of voting into voting power analysis, by developing spatial indices of voting power based on the respective preferences of the various players[8], in both an uni-dimensional and multi-dimensional setting. In particular, Section 2 introduces explicit probability weightings into two standard voting power indices. Drawing on spatial models of coalition formation, Section 3 presents various ways to identify the probability of a coalition on a single policy dimension. Section 4 shows how this new framework allows to tackle multi-dimensional policy issues as well as single issues that are affected by several factors (i.e. multi-factor voting decisions). We should note, however, that we do not explicitly account for EU inter-institutional dynamics, as this would largely complicate the approach as presented here. Finally, Section 5 offers suggestions on how to derive meaningful values for spatial power indices either on the basis of empirical data or information derived by theories of endogenous policy formation.

2. *Probabilistic power indices*

The two most commonly used indices to measure players' voting power are the Banzhaf (1965) and Shapley-Shubik (1954) indices. The purpose of this section is to move away from the assumption of random coalition formation inherent in such indices (which is, however, suitable for analyzing constitutional issues[9]). Instead, we propose a revised version of these traditional indices, introducing the notion of probabilistic coalition formation dependent on players' (policy) preferences.

2.1. The normalized Banzhaf power index: a probabilistic version

In order to illustrate respective calculations, consider 3 players, *a, b* and *c*, in the following weighted voting game:

$$\begin{matrix}[3;\, 2,\, 1,\, 1]\\ \quad a,\ b,\ c\end{matrix}$$

Following Banzhaf (1965), the winning coalitions in this game are

$$\{a,b\},\ \{a,c\}\ \text{and}\ \{a,b,c\},$$

[8] For another recent attempt to construct probabilistic power indices, see Laruelle and Valenciano (2001).

[9] See for example Lane and Mæland (2000).

where the swing voters (i.e. those who by changing their vote could 'swing' the coalition from winning to losing) are denoted in bold italics. The individual Banzhaf indices for *a*, *b* and *c* hence are in the ratio 3:1:1. Once normalized, the following indices are derived:

$$\beta_a = 3/5,\ \beta_b = 1/5 \ \text{ and } \ \beta_c = 1/5,$$

where the normalized Banzhaf index is defined as

$$\beta_i = \mu_i \Big/ \sum_{j=1}^{n} \mu_j$$

with μ_i denoting the number of winning coalitions to which player *i* belongs and in which it is critical (with $i \in \{1,\dots,n\}$).

The Banzhaf index takes coalitions that are equally probable into account. When considering policy-driven coalition formation, it seems natural, however, to assign a weight to each coalition that reflects how likely a coalition is to form. Taking the respective weight *W* of each coalition into account, the revised normalized Banzhaf indices are:

$$R\beta_a = \eta_a/(\eta_a + \eta_b + \eta_c)$$

where

$$\eta_a = [W(a,b) + W(a,c) + W(a,b,c)],\ \eta_b = W(a,b),\ \eta_c = W(a,c)\ .$$

The weight *W* of a coalition reflects the probability with which it is formed. The prefix "R" is used throughout this paper to indicate "revised".

In the general case, with *n* players, the revised normalized Banzhaf index for voter *i* is then defined as

$$R\beta_i = \eta_i \Big/ \sum_{j=1}^{n} \eta_j$$

where η_i is the sum of the weights of the coalition in which player *i* is a swing voter.

2.2. The Shapley-Shubik power index: a probabilistic version

Consider the same weighted voting game, where

$$\underset{\quad a,\ b,\ c}{[3;\ 2,\ 1,\ 1]}$$

but in this case, players vote in order, as suggested by Shapley (1953) and Shapley and Shubik (1954). In this framework, all coalition orderings are taken into account, and the pivotal player is defined as the one turning a losing coalition into a winning one. The total number of possible orders, *3!*, leads to the following set of feasible coalitions:

$$\{a,\ \boldsymbol{b}, c\},\ \{a,\ \boldsymbol{c},\ b\},\ \{b,\ \boldsymbol{a},\ c\},\ \{b,\ c,\ \boldsymbol{a}\},\ \{c,\ \boldsymbol{a},\ b\},\ \{c,\ b,\ \boldsymbol{a}\},$$

(pivotal players are indicated in bold italics). For *n* players, the standard Shapley-Shubik index is defined as

$$\phi_i = \sum_{S \subset N} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S-i)]$$

where *s* denotes the position of the pivotal player i in the ordered coalition *S*. The function *v* has value 1 for winning coalitions and 0 otherwise. In our example, this leads to the following Shapley-Shubik indices for players *a, b and c*:

$$\phi_a = 4/6 = 2/3,\ \phi_b = 1/6,\ \phi_c = 1/6.$$

Since all winning coalitions, irrespective of their permutations, are taken into account in the Banzhaf index, they may include several decisive players. By comparison, in the framework of the standard Shapley-Shubik index, where all player orderings have equal probability (i.e. $1/n!$), the crucial aspect in determining players' voting power is the number of orderings in which they occupy the position of the pivotal voter. To each given ordering of *n* players corresponds exactly one critical winning coalition, namely the one consisting of the players up to the pivotal voter (i.e. of a ranking lower or equal to the pivotal player). Of course, several orderings project onto the same critical winning coalition. Note that ϕ_i is non-zero only if the coalition is critical, i.e. winning with player i included, and not winning without player i. To summarise, the voting power index is given by the number of times a player is pivotal in all orderings of the players, divided by the number of all such orderings (see the appendix in Shapley and Shubik, 1954).

The occurrence of a particular coalition can also be assessed on the basis of a probabilistic perspective. The number of possible orderings is n!. The number of orderings that project onto *S* is $(s-1)!\ (n-s)!$ multiplied by the number of pivotal players in *S*. The quantity $Q(S,i)$ can be introduced as the probability of coalition *S*, irrespective of whether that coalition is winning or not, and is equal to

$$Q(S,i) = \frac{(s-1)!(n-s)!}{n!}$$

The standard Shapley-Shubik index can, therefore, be expressed as

$$\phi_i = \sum_{S \subset N} Q(S,i)\,[v(S) - v(S-i)]$$

Let us now consider the weight W of a coalition (to be defined in the next section). We will use the weight W of a coalition to influence ('bias') the quantity $Q(S,i)$, thus accounting for the preferences of each player. Such a 'bias' may be defined as:

$$\hat{Q}(S,i) = \frac{Q(S,i)\,W(S)}{\sum_{T \subset N} Q(T)\,W(T) \big| \{ i \in T; i \text{ pivotal} \} \big|}$$

where T runs through every coalition. Consequently, the revised Shapley-Shubik index takes the general form

$$R\phi_i = \sum_{S \subset N} \hat{Q}(S,i)\,[v(S) - v(S-i)]$$

In short, the revised voting power indices presented in this paper aim to represent a generalisation of the standard Banzhaf and Shapley-Shubik voting power indices. Instead of assuming that all coalitions are equally probable, revised voting power indices can take the probability into account that some coalitions are more likely to form than others. In this formulation, the probability of a coalition depends on the respective preferences of players. To calculate such revised Banzhaf and Shapley-Shubik indices, the probability of coalition formation must thus be determined. This is the objective of the next section.

3. The probability of forming a coalition in one dimension

Several methods to determine the probability of coalition-formation may be envisaged. In this paper, we will use spatial models.[10] The approach consists in representing the preferred position ('ideal point') of each player on a policy scale. Drawing on Axelrod (1970), the intuition of conflict minimisation in policy-driven coalitions is that the smaller the distance between the ideal points of players on a policy scale, the closer are their preferences, and therefore, the more likely they are to form a coalition with each other.

Formally, consider a set $N = 1,2,\ldots,n$ of players, also called voters. The preference, or ideal point, of each player, x_i (with $i = 1,\ldots,n$), is assumed to be distributed on a uni-dimensional (normalized) cardinal scale in an Euclidean space, such that $0 \le x_1 \le x_2 \le \ldots \le x_n \le 1$. The difference in preferences between two players, i and j, can hence be measured by the distance on the cardinal scale separating the preferences of the two players: $d_{i,j} = |x_j - x_i|$.[11] Coalitions are more likely to form among players sharing similar preferences. Hence, the closer the preferences (i.e. the smaller the distance) between players, the more likely they are to reach an agreement.

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[10] For an overview of spatial theories, see e.g. Enelow and Hinich (1985) and Hinich and Munger (1997). A recent overview of respective approaches can be found in de Vries (1999).

[11] Alternatively, it is possible to use Hamming distances between preferences. See van Deemen and Hosli (1999).

Based on this general principle, the probability of a coalition may be formalised in various ways, depending on the underlying assumptions made as regards the mechanisms of coalition-formation.

First, let us consider a very simple approach, where the probability that a coalition will form depends on the distance between the (extreme) players of that coalition exclusively. The weight $W(i,j)$, defined by

$$W(i,j) = \exp(-u\, d_{i,j}),$$

determines the probability that players i and j reach an agreement in a negotiation process, the chance of a positive outcome being greater if the respective positions are close to each other on the preference scale. The parameter $u \geq 0$ is a measure of the flexibility in the negotiation between players. The higher the value of u, the less likely is agreement for all players. But generally, players close to each other on the scale are more likely than players distant from each other to reach an agreement. At the limit, when $u \rightarrow \infty$, only players with an identical position (i.e. $d_{i,j} = 0$) will reach an agreement. When $u = 0$, any two players will agree irrespective of their particular preferences, as if compelled by some external factor. Hence, $u \rightarrow \infty$ corresponds to infinite rigidity and $u = 0$ to infinite flexibility. In general, in the absence of prior information, it will be assumed that $u = 1$. Note also that for $u \geq 0$ fixed, the probability of reaching agreement only depends on the respective preferences of the players. Hence, two players i and j with identical preferences have an identical ideal point, and will therefore always reach an agreement: $W(i,j) = 1$ when $x_i = x_j$.

While alternative formulations may be envisaged, the elegance of this approach is that, by adopting the exponential, the additivity property is preserved.[12] That is, assuming that players a and b on the one hand, and players b and c on the other hand, positioned as $x_a < x_b < x_c$, negotiate independently, the probability of a successful outcome in both cases depends on the weight

$$\begin{aligned} W(a,b) \cdot W(b,c) &= \exp(-ud_{a,b}) \cdot \exp(-ud_{b,c}) \\ &= \exp(-u(d_{a,b} + d_{b,c})) = \exp(-ud_{a,c}) = W(a,c) \end{aligned}$$

which is the probability of players a and c reaching an agreement.

Let us now discuss the case of a coalition involving several players. For a set of N players, ranked on a scale ranging from 1 to n such that $x_1 \leq x_2 \leq \ldots \leq x_n$, the weight of a coalition C, where $C \subset N$, is determined by the probability that the two extreme players of the coalition, e_{min} and e_{max}, reach an agreement:

$$W(C) = W(e_{min}, \ldots, e_{max}) = W(e_{min}, e_{max})$$

where $x_{emin} < z < x_{emax}$ for any $z \in C$, or, in other words, $W(C)$ is determined by $\max_{i,j \in C}\{d_{i,j}\}$ (the diameter of C, which is, in essence, the 'spread' between the members in coalition C). The weight

[12] Alternatively, scholars of spatial theory and coalition formation often assume that a player's utility is inversely proportional to the squared distance between players' preferred positions; see e.g. Riker and Ordeshook (1973), Enelow and Hinich (1984) or Schofield (1995).

W(*C*) also corresponds to the joint probability that each player in coalition *C* forms an alliance with his immediate neighbours in *C*. It is worth emphasising that the additivity property of the coalition weight is preserved only with respect to the ranking of player preferences. The order in which players enter a coalition does not matter in this approach. The empty coalition obtains a weight of zero. It is also important to point out that we are considering probability in a dichotomous way, i.e. the coalition *C* will form or not form, each event occurring with probability W(*C*) and 1 − W(*C*), respectively. Thus, the weight, to some extent, can be considered to be a measure of the viability of a coalition.

From the notion of a coalition's weight, we can derive a probability on the set of all coalitions (viewed as the set of all unordered subsets of *N*). The probability that, among all possible coalitions, coalition *C* will form is given by

$$P(C) = \frac{W(C)}{\sum_{D \subset N} W(D)}$$

which is simply a probability proportional to weight, with $\sum_{D \subset N} P(D) = 1$. Note that if we set the parameter $u = 0$, the probability above reduces to a uniform probability over the set of coalitions.

This formalisation is very generic in the sense that it does not take the actual possible policy preference of the coalition into account, but only relies on the respective positions of the players. The minimum requirement adopted here is that a coalition is viable only if the extreme players of the coalition (i.e. the players whose preferences differ the most within the coalition) can reach an agreement.

A somewhat more realistic assumption would be to explicitly consider the preferred policy position of the coalition. This 'coalition ideal point' could be identified as the (weighted) average of the preferred position (i.e. ideal point) of each player in the coalition.[13] That is, the preferred policy position of a coalition *C* could be expressed as

$$x_C = \frac{\sum_{z \in C} \omega_z x_z}{\sum_{z \in C} \omega_z}$$

where x_z stands for the ideal point of player *z* in the coalition *C* ($z \in C$), and ω_z -- the weight of a player in the coalition -- represents the relative size of player *z* within this coalition.[14]

The probability that coalition *C* will form could then depend on the likelihood that each member of the coalition indeed joins this coalition. This in turn can be determined by the distance between the ideal point of each player in the coalition and the (jointly preferred) position of the coalition, $d_{z,C} = |x_C - x_z|$. Formally, the weight of the coalition *C* depends on the joint weights of each player *z* joining the coalition:

[13] See de Swaan (1973) for seminal insights into such approaches.

[14] In the case of coalition-formation within the Council of the EU, ω_z would be determined by the number of votes (i.e. the weight) that each member state holds.

$$W(C) = \prod_{z \in C} W(z,C),$$

where $W(z,C) = \exp(-u\, d_{z,C})$. Using the additivity property of the exponential, the weight of a coalition can be rewritten as

$$W(C) = \exp\left(-u \sum_{z \in C} d_{z,C}\right)$$

That is, the weight of a coalition is a function of the sum of the individual distances between each member of the coalition and the expected preferred position of the coalition.[15] In terms of probabilities, where the set of all coalitions is taken into account, the probability of a coalition C becomes

$$P(C) = \frac{\prod_{z \in C} W(z,C)}{\sum_{D \subset N} W(D)}$$

An alternative setting would consist in considering coalition-formation resulting from a policy proposal initiated either by a player or by an external actor.[16] The probability that a coalition C_I forms to support a (fixed) policy initiative I, located at point x_I on the policy scale, then is

$$P(C_I) = \frac{\prod_{z \in C_I} W(z,I)}{\sum_{D \subset N} W(D)},$$

where, as previously defined, $W(z,I) = \exp(-u\, d_{z,I})$.

4. A multi-dimensional spatial analysis of probabilistic voting power

The concept of the probability of forming a coalition, as described in the previous section, can be extended to encompass multi-dimensional issues in coalition-formation.[17] This has the advantage of allowing the model to address two common phenomena in voting decisions. The first one concerns 'package deals'. In situations where a single vote is required for a legislative package that contains several issues (also referred to as 'omnibus bills'), the ultimate position (i.e. ideal point) of a player is the result of her preference on each single issue. As some issues may be of more relevance to a player than others, her aggregate preference, which will determine her position on the package deal -- and thus her probability of joining a coalition over another -- can be seen as a weighted average of her issue specific preferences. The second instance of multi-

[15] This sum of individual distances between each player and the coalition ideal point, $\sum_{z \in C} d_{z,C}$, may be referred to as 'the distance of a coalition' (see de Vries, 1999, p.93).

[16] In the EU, the Commission could be seen as such an external actor with the right of initiative in many areas as provided by the EC Treaty, the players in our framework being the member states in the Council.

[17] For a survey of theories of spatial coalition games, see e.g. Owen (1995, ch.16) or de Vries (1999, ch.4).

dimensionality in voting decisions arises when a vote (e.g. on a single bill) is influenced by several factors. In this case, the position of a player is not primarily determined by a single factor (represented on a one-dimensional cardinal preference scale in our model), but is influenced by a string of factors, such as possibly ideological considerations, economic and social factors, or geographical location.

In both instances, package deals and multi-factor decisions, the aggregate preference of a player can be identified by a weighted average of the position of the player on each respective (normalized) cardinal scale of preferences represented in an Euclidean space. For m issues there will be m cardinal scales indicating the preference of each player on each single issue, as in the case of package deals. Similarly, for m factors influencing a single decision (i.e. a vote), there will be m preference scales. As some issues/factors may be more important than others, the weighting coefficient should be issue or factor specific.

In the formal framework developed in the previous sections, consider the general case of preferences in m dimensions. Each player's preference can be placed on a linear scale and represents a coordinate in the m-dimensional Cartesian space $\boldsymbol{R}^m$. Hence, each axis corresponds to a preference scale (or policy dimension). The overall preference of a player is now represented as a point in $\boldsymbol{R}^m$ and each individual issue can be retrieved by projecting onto the corresponding axis. The probability that players form a coalition depends on the chance that they reach an agreement on each issue. It follows that the weight of a coalition C takes the form

$$W^{Tot}(C) = \sum_{k=1}^{m} p^k W^k(C)$$

where p^k indicates the weighting factors for issue k ($k = 1,\dots,m$), with $\sum p^k = 1$. If the probability of a coalition depends on the distance between its (extreme) players, $W^k(C)$ is given by

$$W^k(C) = \exp\left(-u^k \left|x^k_{e_{\max}} - x^k_{e_{\min}}\right|\right)$$

whereas when an agreement is reached among players depending on the expected preferred position of the coalition (determined by the individual distance between each player and the preferred aggregate position of the coalition), the weight of a coalition on issue k can be written as

$$\begin{aligned} W^k(C) &= \prod\nolimits_{z\in C} W^k(z,C) \\ &= \prod\nolimits_{z\in C} \exp\left(-u^k \left|x^k_C - x^k_z\right|\right) \\ &= \exp\left(-u^k \sum\nolimits_{z\in C} \left|x^k_C - x^k_z\right|\right) \end{aligned}$$

Again, as in the uni-dimensional case, we can derive a probability on the set of all coalitions. The probability that, among all possible coalitions, the multi-dimensional coalition C will form is given by

$$P^{Tot}(C) = \frac{W^{Tot}(C)}{\sum_{D\subset N} W^{Tot}(D)}$$

which is simply a probability proportional to total weight.

In a m-dimensional game, the revised voting power indices suggested here must integrate the multi-dimensional probabilities W^{Tot}. The formulas for the revised Banzhaf and Shapley-Shubik indices remain the same, but with W^{Tot} instead of W.

The voting power indices predict that players with more weight (i.e. higher number of votes) are more likely to have more absolute voting power than smaller players. Yet, introducing probabilities of coalition-formation based on players' preferences suggests that, in general, central players in a coalition are also more likely to exert voting power than players with extreme preferences. Consequently, a group of players having similar preferences may also gain greater joint voting power by acting collectively in coalition games.[18]

5. Conclusions and suggestions for empirical applications

If information on player's preferences is available – e.g. as assessed through respective interviews and data collection efforts – such information can be directly incorporated into the measures of relative voting power as presented in this paper. Hence, it is possible to extend the two traditional indices of voting power analyzed here, the (normalized) Banhaf index and the Shapley-Shubik index, to instances in which preferences of players are known and can be represented in a uni-dimensional or multi-dimensional policy space. We hence adapt traditional indices by including probabilities that players, according to the spatial location of their policy preferences, aim to form coalitions. This approach allows us to account for both players' policy preferences and their voting weights in determining their relative voting leverage. Changing preference distributions, or adapted voting weights, will both affect the values of these spatial voting power indices.

If straightforward information regarding preferences is lacking, however, other ways to proceed can be envisaged. We outline one possible approach here, but it should be kept in mind that several other strategies may be interesting to pursue.

In the case of research as regards decision-making in the European Union, for example, in the absence of direct information on players' preferences, insights can possibly be gained by using a political-economy approach where economic policies are considered to be determined endogenously.[19] Such an approach has the merit of integrating institutional and political aspects into standard economic analysis, in an attempt to explain how economic policies are decided upon. Hence, the adoption of a policy Y may be (at least partially) explained by a string of factors X_k.[20]

[18] The desire to maximize voting power can reinforce incentives to seek alliances among players. This could be the case in the EU for instance for the Franco-German axis, the Benelux group or close cooperation among Nordic countries. E.g. see Hosli (1996).

[19] On endogenous policy theory, e.g. see Magee *et al.* (1989), Moravcsik (1997) or Persson and Tabellini (2000), and the references they provide.

[20] Such an approach has been used extensively to analyse trade policy (and to a lesser extent fiscal policies). For instance, in our case, Y can be a tariff or more generally the rate of protection, dependent on factors X_k such as import penetration, the capital/labour ratio, the share of intermediary goods produced, consumer and industry concentration,

In order to illustrate such an approach, consider a simple linear regression analysis such as

$$Y = a_o + a_1 X_1 + a_2 X_2 + \dots + a_m X_m + e$$

where Y is the endogenous dependent variable, the X_i's denote exogenous explanatory variables and the a_k stand for the regression coefficients, with $k = 1,\dots,m$ (and e simply being a stochastic error term).

In terms of the model developed in this paper (section 4), the parameters of the regression function may then be interpreted as follows: the m factors influencing the (single) policy issue Y are captured by m explanatory variables X^k (with $k = 1,\dots,m$). Hence, an explanatory variable X^k corresponds to each dimension k. The (normalized) value of variable X_i^k for each player i also indicates her ideal point on the preference scale k, x_i^k. The weight of each factor k, denoted by p^k, can be derived (after normalization) from the regression coefficient a^k.

In short, after some transformations, it seems possible to extrapolate information from regression analysis on (endogenous) policy determination, in order to estimate not only the preferences of the players, but also the relative importance of the factors that influence their policy decisions. This information may, subsequently, be used to calculate revised (i.e. probabilistic) voting power indices.

As mentioned above, this is only one possible avenue to estimate players' preferences as encompassed in our measurements of relative voting power. We aim to incorporate information on players' preferences – either as obtained by data collection efforts or estimated indirectly – in order to apply our probabilistic spatial indices of relative voting power in empirical analyses.

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