Proceedings of the 5th International Conference on Nonlinear Dynamics
ND-KhPI2016
September 27-30, 2016, Kharkov, Ukraine

The Nonlinear Deformation of the Body System Under Electromagnetic Field Action

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Abstract
The paper discusses issues concerning the deformation of a system of conductive bodies under the action of the electromagnetic field. Problem of nonlinear deformation of technological system for electromagnetic forming is considered as a practical application. The problem is solved by the finite element method. Spatial-temporal distributions of the main components of the electromagnetic field are obtained. The ability to review the problem of deformation in the quasi-stationary formulation is justified. The distribution of the main component of the stress-strain state is presented. The influence of the current magnitude at the maximum stresses is evaluated.

Keywords
Electromagnetic field, nonlinear deformation, finite element method, electromagnetic forming

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Introduction

Electromagnetic field (EM-field) is an integral part of work for many elements of structures and machines. They include elements of power equipment, power conversion system (transformers, generators), devices designed to protect against lightning and electromechanical energy converters High-intensity EM-fields cause substantial energy levels in electro-conductive bodies, which can lead to failure. Therefore, approaches to determine stress–strain states (SSS) of electro-conductive bodies are required to estimate the strength. Such methods should be based on suitable models of continuum thermo-mechanics. Historically, the formulation of such coupled models was motivated by the development of:

– the theory of piezoelectricity (piezomagnetism and piezosemiconductivity) bodies due to the wide usage of electromechanical energy converters;

conductive bodies in a strong magnetic field;

– the theory of propagation of waves in a deformable body taking into account the relationship between the mechanical and the electromagnetic fields (for problems of geophysics and seismology as well as for the development of nondestructive testing methods in structural elements);

– the theoretical foundations of heat processing of bodies using external electromagnetic radiation.

Theoretical fundamentals describing models of continuum mechanics, which take into account the effect of the coupled fields of different physical nature (including electromagnetic fields) are presents by Truesdell and Toupin, Pidstryhach (Podstrigach), Sedov, Eringen, Ambarcumyan, Maugin, Nowacki among others. The influence of external EM-field effects on the thermo-mechanical state of the body is taken into account in the equilibrium conditions by electromagnetic forces and moments. The electromagnetic field in vacuum and in a moving deformable body is described by Maxwell’s equations.

To characterize the EM-field, five vector quantities are introduced including the intensity and the inductance of the electric and magnetic fields as well as the density of the surface charges. To find these characteristics, several electromagnetic properties including conductivity, polarization, and magnetization are required. Within the framework of thermo-mechanics, there are two classes of theories: the dynamic theory and the quasi-static one. The latter is used for slowly moving bodies with the speed much less than the speed of light in vacuum, which ensures the invariance of the equations of electrodynamics and mechanics relative to the Galilean transformation.

For studies in which the main objective is the analysis of the structural strength the theory of magnetoelasticity can be used. Fundamentals of the theory of magnetoelasticity with coupling EM-field and mechanical stresses and strains in a moving electro-conductive body (in the general case, the body is polarized and magnetized) are given by Knopoff [1] and Chadwick [2]. Here the propagation of elastic waves taking into account the Earth's magnetic field is analyzed. Dunkin and Eringen [3] formulated the dynamic problems of magneto-elasticity in the case of vibrations of bodies and distribution of magneto-elastic waves. Pidstryhach et al. [4], and Ambarcumyan et al. [5] developed a general theory of magneto-elasticity. They formulated general asymptotic solutions of the three-dimensional equations of magneto-elasticity for shells and plates and proposed a theory of magneto-elasticity for thin bodies. Wave propagations, oscillations, control, and other problems in the theory of magnetoelastic solids are considered in [6].

Many manufacturing forming processes for conductive solids, for example bending or induction heating, apply strong electromagnetic fields. Process simulations are usually based on the coupled electromagnetic, thermal, and mechanical analysis, for example [7]. It is documented that within the range of high temperatures material properties change essentially. For example, electrical conductivity, yield strength, and other properties during the heating may vary by several times. Furthermore, inelastic flow at high temperature is accompanied by changes in microstructure, such that hardening, softening, recovery, and damage processes should be taken into account within a constitutive model with internal state variables [8] or by application of enhanced continuum theories with higher-order gradients and phase field variables [9]. Therefore, there is a practical need for the mathematical formulation of problems of non-isothermal elastoplastic deformation of bodies under the influence of external electromagnetic fields, taking into account the mutual influence of fields of different nature in a wide ranges of temperature, and taking into account the nonlinearity of electromagnetic, thermal, and mechanical properties of materials.

1. Problem statement

One of the most striking examples of the power of influence of the EM-field at the conductive bodies is an electromagnetic process electromagnetic metal forming (EMF). EMF is a dynamic, high-strain-rate forming method. In this process, deformation of the workpiece is driven by the interaction of a current generated in the workpiece with a magnetic field generated by an inductor (coil) adjacent to the workpiece. In particular, the interaction of these two fields results in a material body force, i.e., the electromagnetic force, representing an additional supply of momentum to the material resulting in deformation. EMF is one of a number of high-deformation-rate forming methods which offer certain advantages over other forming methods such as increase in formability for certain kinds of materials, reduction in wrinkling, and the ability to combine forming and assembly operations. The stamping and forming of metal and nonmetallic workpiece by exposing the workpiece powerful electromagnetic pulse is studied in [10]. Modeling of processes of forming and stamping blanks is discussed in [11]. In these papers, the finite element and boundary element methods are applied to solve coupled problems of magneto-thermo-elastic with regard to high-strain-rate deformation.

The levels of the developed electromagnetic forces can lead to failure of the inductor under EMF. Therefore, the stress analysis is required in order to assess the strength. It is also necessary to investigate the processes of elastic–plastic deformation of workpiece for optimal performance of the process. Papers [12, 13] concern the issues regarding the structural analysis of inductors intended for attraction of ferromagnetic workpieces. These works consider the task in non-stationary and quasi-stationary statements. For the attraction of non-ferromagnetic workpieces, we can use the inductors with the assistant screen. In this case, there are many issues associated with the structural analysis of the inductor as a whole and its constituent parts. Fig. 1 provides a model variant of composed single-turn inductor with an assistant screen. Such an inductor can be used for straightening dents in thin-walled structural elements [14].

Conditions of loading, fixing and geometry allow us to consider the problem in the axisymmetric formulation. The fig. 2 shows the design scheme of the inductor with the workpiece. The gap introduced between the inductor and the workpiece, simulates a dent in the real case. The system is considered together with environment (air) in order to correctly specify the conditions of attenuation of the electromagnetic field on the distance from the source. Current, uniformly distributed over the cross section of the turn of the inductor, considered as a source of EM-field.

It is necessary to consider the deformation of the system under given conditions, and then figure out how to change the state of the system with increasing of the current magnitude. For the analysis in this case, we will rely on the problem statement and the solution method proposed in the paper [12].

The general mathematical formulation of problems of non-isothermal elasto-plastic flow of electrically conductive bodies under the influence of an external electromagnetic field is discussed in [12]. To determine characteristics of the electromagnetic field for domains occupied by the body and the environment, the Maxwell equations are formulated. To describe the stress and strain states of the body, constitutive equations for non-isothermal elasto-plastic flow are applied. The influence of electromagnetic fields on the heat transfer and deformation is described by the equations of heat transfer and dependencies for electromagnetic forces. Interactions of electromagnetic and thermal fields, and distributions of electromagnetic forces in the bodies and on the contact surfaces are discussed. The finite element method (FEM) is used as numerical method of analysis. Its specific implementation is based on the principle of minimum of total energy of the system.

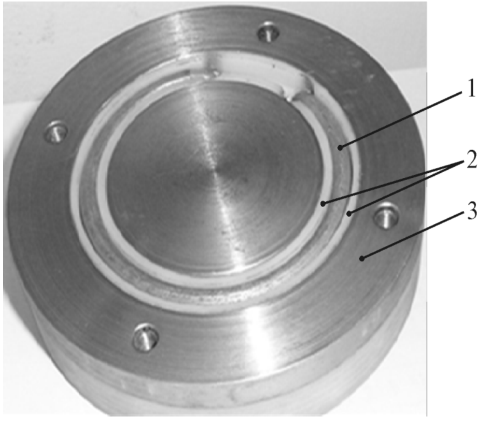

Figure 1. The single-turn inductor. 1 – wireway of the inductor, 2 – insulation, 3 – assistant screen.

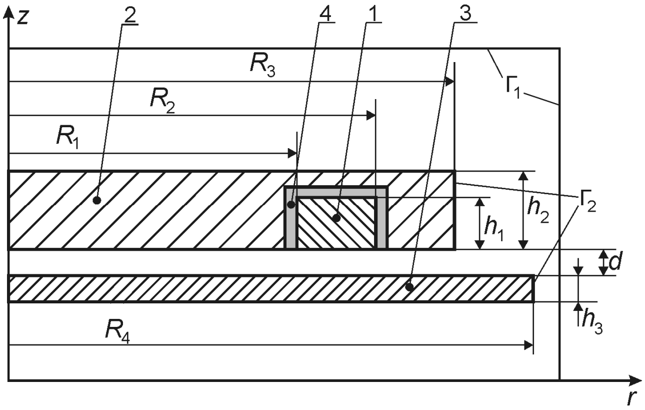

Figure 2. Inductor – workpiece system (axisymmetrical model). 1 – wireway of the inductor, 2 – assistant screen, 3 – workpiece, 4 – insulation.

2. Discussion of the results

The problem was solved with the following geometrical sizes: $R_1 = 150\,\text{mm}$, $R_2 = 167\,\text{mm}$, $R_3 = 175\,\text{mm}$, $R_4 = 200\,\text{mm}$, $h_1 = 10\,\text{mm}$, $h_1 = 15\,\text{mm}$, $h_3 = 1\,\text{mm}$, $d = 1\,\text{mm}$, thickness of the insulation – $1\,\text{mm}$. The size of the environment was varied to meet the conditions of attenuation of the EM-field at a distance from the field source. Material properties: wireway – copper, $\mu_r = 1$, $\gamma = 7\times10^7\,(\Omega\text{m})^{-1}$, $E = 180\,\text{GPa}$, $\nu = 0{,}33$, $\sigma_Y = 180\,\text{GPa}$; assistant screen – steel, $\mu_r = 1$, $\gamma = 0{,}2\times10^7\,(\Omega\text{m})^{-1}$, $E = 215\,\text{GPa}$, $\nu = 0{,}27$, $\sigma_Y = 270\,\text{GPa}$; workpiece – steel, $\mu_r = 1$, $\gamma = 0{,}2\times10^7\,(\Omega\text{m})^{-1}$, $E = 200\,\text{GPa}$, $\nu = 0{,}29$, $\sigma_Y = 220\,\text{GPa}$; insulation – glass fiber, $\mu_r = 1$, $\gamma = 0\,(\Omega\text{m})^{-1}$, $E = 2{,}5\,\text{GPa}$, $\nu = 0{,}3$, $\sigma_+ = 70\,\text{GPa}$, $\sigma_- = 90\,\text{GPa}$; environment – air, $\mu_r = 1$, $\gamma = 0\,(\Omega\text{m})^{-1}$. Here: μ_r – relative magnetic permeability, γ – electrical conductivity, E – modulus of elasticity, ν – Poisson's ratio, σ_Y – yield stress, σ_+ – tensile strength, σ_- – compressive strength.

The current density is changed in time according to the law $j(t) = I_m e^{-\delta 2\pi f t}\cdot\sin(2\pi f t)$, where magnitude of current $I_m = 40\,\text{kA}$, frequency $f = 2\,\text{kHz}$, the relative damping coefficient $\delta = 0.3$. A finite element model was created using four-node axisymmetric finite element with bilinear approximation of displacements and circumferential components of the magnetic vector potential [23,24]. The finite elements of this type automatically satisfy the conditions on the boundaries of media with different physical properties. To account the mechanical contact it was introduced the layers of special contact finite elements [15]. The boundary conditions, that model the attenuation of EMF on the distance from the source and fixing of the ends of the inductor and workpiece, were considered: $A\big|_{\Gamma_1}=0; u_r\big|_{\Gamma_2}=0; u_z\big|_{\Gamma_2}=0.$

The spatiotemporal distribution of components of the electro-magnetic fields was obtained at the first stage of the solution. Also, we performed a series of calculations, which varied the dimensions of the surrounding area. Analysis of the results showed that the components of the EMFs are reduced approximately five times when the distance from the field source to the boundary of the environment is comparable to the double thickness of the auxiliary screen. Further, the value component of the electromagnetic force acting on the workpiece surface was determined. A comparison with the results presented in [14], which were obtained semi-analytical means showed that the maximum discrepancy does not exceed 12%.

Next, let us consider the main results of the analysis of the SSS of the system “inductor – workpiece”. Analysis of the electromagnetic field main components distribution has shown that there is a significant decrease in their value over time. Therefore, the electromagnetic force acting on the workpiece will decrease as fast as the components of the electromagnetic field. Therefore, the deformation problem can be considered in the formulation of the quasi-stationary, with the values of the electromagnetic field components corresponding to the maximum time. The distribution component of SSS has been received for the temporary high EM-field. Analysis of the displacements shows that the workpiece is deformed to a much greater extent than the inductor. This fact is explained by the peculiarities of the geometry of the objects involved. The inductor is much more massive body than the workpiece. Maximum of workpiece displacements are observed in the vicinity of its center. In general it can be noted that the workpiece is attracted to the inductor, i.e., we can say, that this induction system can be used for pulling dents in thin-walled structural elements.

The fig.3 shows the pattern of distribution of intensity of the Von Mises equivalent stress in the elements of the system. Note that the maximum stresses in the inductor are observed directly in the turn and its surroundings. The stresses in the workpiece have two peaks: in the area of fixation and in the vicinity of the center of the workpiece. In reality, the size of the inductor, most often, much less than the dimensions of the structural member. Here two cases are possible: the action is about fixation and the action on removal of the fixation. In the first case it is necessary to analyze the processes occurring in the vicinity of the fixation.

The stresses in the inductor exceed the stresses levels in the workpiece. Overall, it should be noted that when the magnitude of the current equals 40 kA, neither in the workpiece nor in the inductor stresses does not exceed dangerous values. Although, the stresses in the vicinity of the coil (insulation material) are approaching threat values.

Next, we conducted a series of calculations, which varied the magnitude of the current. Note that the qualitative characteristics of the stress distribution in the elements of the system persist for all values of the magnitude. In the fig.4 are plotted the maximum values of the Von Mises equivalent stress in the inductor and the workpiece versus the amplitude of the current.

The maximum stress in the workpiece while all the considered values of the current magnitude do not reach the yield strength, i.e., the workpiece didn’t begin to deform plastically. So, when this embodiment of the inductor of the conditions of technological operations will not be achieved. Stresses in the inductor are higher than in the workpiece. The stresses in the insulation material when, the current magnitude equals 60 kA, exceed the limit values. Insulation may deteriorate, which in this case is not valid.

It should also be noted that when current flows, heat generation occurs in the conductor. In our study, issues concerning non-stationary distribution of the temperature field due to heat generation were not considered. It is obvious that the increase in the amplitude of the current will increase the temperature of the system elements that must be considered in the analysis of SSS.

Obviously in this case, the method of increasing the number of turns of the inductor is more promising, because, it is known that the generalized magnetic pressure is proportional to the number of turns.

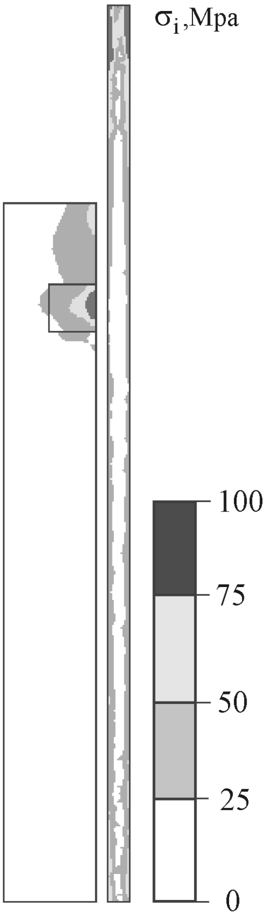

Figure 3. Distribution of the von Mises equivalent stress

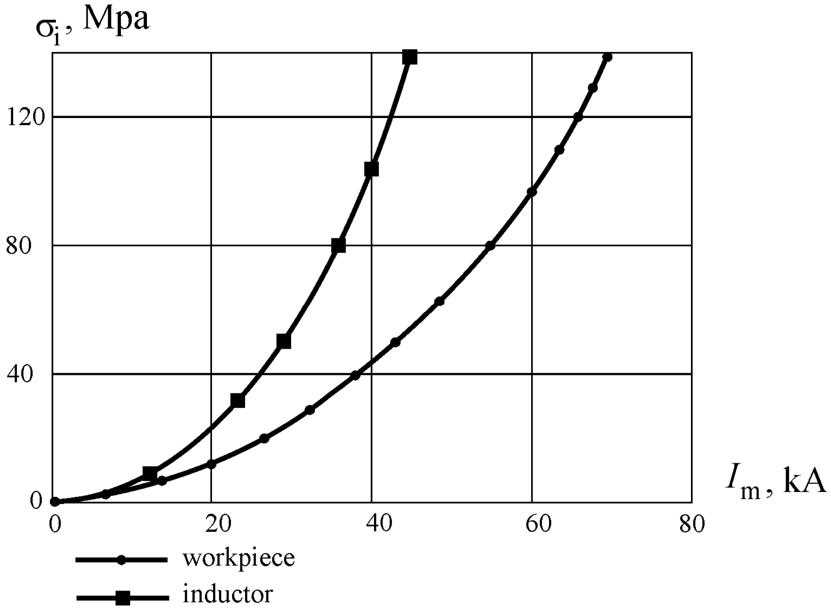

Figure 4. Maximum von Mises equivalent stress versus magnitude of the current

Conclusions

The paper deals with the issues devoted to the study of elastic-plastic deformation of conductive bodies under the action of the electromagnetic field. The feasibility of using numerical methods of analysis is justified. It is consider the practical task of the analysis of the deformation of the technological system for electromagnetic forming of materials. The distribution of stresses in the elements of the system is obtained. The change in maximum stress with increase in the current magnitude is analyzed.

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