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FE-ANALYSIS OF DYNAMIC CREEP-DAMAGE IN THIN-WALLED STRUCTURES

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Abstract

The models for description of creep-damage behaviour in materials and thin shallow shells and plates deforming in conditions of joint action of static and fast cyclic load are given. The properties of the proposed material model were established by comparison of experimental and numerical data. The method for numerical simulation by in-house code of a dynamic creep and long-term strength of shallow shells and plates is created on the basis of the FEM. New laws of dynamic creep influence on stress-strain state, shaping and fracture of thin-walled elements of structures had been established by numerical calculations. With the purpose of verification of the created method of dynamic creep numerical simulation of rectangular plates made of an aluminium alloy were carried in order to verify the method of calculation.

Key words: creep, damage, dynamic, FM-analysis, thin-walled

1 Introduction

It is known from the experience of operation of different structures, that the conditions of their loading in many cases correspond to the joint action of static and cyclical loads. In high-temperature area at such loading in a material of structures the irreversible creep strains and damage evolution conditioned by mechanisms of cyclical strains are developed.

Also the level of temperature occurs the essential influence on behaviour of metal materials under cyclical load. So resistance of a fatigue, processes of damage accumulation, mackrocrack initiation and propagation determine the long-term strength of a material at low and normal temperatures. In conditions of high-temperature action not only structural modifications in a material have been noticeably intensified, but also the character of cyclical deformation varies. So in the field of high temperatures ($\geq 0.5T_{\text{melt}}$) athermal softening processes at a fatigue are substituted on thermally activated. It is exhibited by presence of inflection points on a fatigue curve aside of a sharp drop of a resistance to a fatigue fracture. Besides in conditions of elevated temperatures the joint action of static and cyclical loads leads to an accumulation of irreversible creep strains. It is known Refs [1-5] that the fracture of a material as a result of a cyclical creep occurs for a limit number of cycles and is determined by joint processes of a fatigue and creep. And to separate pure fatigue fracture from cyclical creep one it is impossible. At the same time it is possible to select areas of preferential development of different failure processes. The quantitative measure for separation of processes which are carrying by different fracture mechanisms due to experimental data can be selected the stress cycle asymmetry coefficient $A_{\text{cr}} = \sigma_{\text{cycl}}/\sigma_{\text{stat}}$. It can be regarded as invariant to time fracture initiation depending from temperature and physical and chemical properties of the material. From the physical point of view magnitude A_{cr} corresponds to a condition of a stick-slip variation of a fracture activation energy that determines its estimation from the analysis of thermal activation parameters of a fatigue and creep Ref. [6].

The investigations of the stress-strain state of creeping structures in conditions high cyclic loading refer to a problem of a dynamic creep. The phenomenon of a dynamic creep occurs in conditions of superposition on constant stress of a high-frequency ($> 1..3$ Hz) a periodically varying stress with amplitude which is not exceeding a values of average stress – $A \leq A_{cr}$ Ref. [5]. Investigations of a dynamic creep (vibrocreep) in pure metals, for example, J. Greenwood's and J. Kennedy's experiences on lead samples, A. Meleka's on copper samples, M. Manjoine's on aluminum-copper ones have shown the essential acceleration of the creep process in comparison with the static loading and with saving the regularities of stationary creep. The essential reduce of long-time strength and intensive softening by A.E. Johnson and N.E. Frost, B.J. Lasan, J. Guarnieri and L.A. Erkovich, F.H. Vitovec are marked for different high-strength steels. It was established, that the frequency of cyclical load after some value (3..10 Hz), which is characteristic for each material, does not influence properties of a dynamic creep.

B. Lasan and afterwards J. Vidal, R. Koterazawa and S. Taira, Yu.N. Rabotnov had done the description of material behaviour at a dynamic creep conditions on the basis of the concept of equivalent static stress σ_{eq}, defined on parameters of loading cycle. In Refs [1,7,8] the magnitude of static stress caused a creep strain is equal for cyclical one have been used. The magnitude of constant stress is equal to the sum of the average stress and additional component, depending from amplitude of cyclical stress was presented in Ref. [4]. The connection between cyclic and static strain rates in conditions of action by only static component of cycle was considered by A. Nadai in Ref. [9]. The generalization of laws between parameters of static and dynamic creep processes and their experimental verification for high-temperature nickel-based alloys was presented in Ref. [10].

The estimation of long-term strength at a dynamic creep can be done on the basis of the equations of durability as the correlation of limiting stress σ_{cycl} and σ_{stat} depending on the assumptions about damage accumulation due to fatigue and creep Ref. [11], as well as with using of phenomenological rupture models. For the last one the laws of long-term static strength are obtained by analogy with creep at some equivalent stress calculated for the cycle of loading Refs [4,5].

The numerical analysis of the dynamic creep in structures is slightly presented in a literature. It is possible to mark only some papers for case of a uniaxial state. In Ref. [5] the problem of the determination stress-strain state in a beam loaded by static axial and cyclical bending load is solved. The solutions of the dynamic creep problem for a system of two beams loaded by a variable concentrated force for the cantilever beam with a rectangular cross-section under load with the variable moment are obtained by J. Boyle and J. Spense Ref. [12]. Non-stationary alternating cyclic loading by the moment for the cantilever beam with round cross-section was studied in Ref. [13]. A creep damage problem for a beam at pure bending subjected by external cyclic moment has been solved by L.M. Kachanov Ref. [14].

The cyclic creep-damage problems in case of multiaxial stress state are solved in Refs [15-18] on the basis of the procedure of asymptotic expansions in two time scales. The general solution of the cyclic creep-damage problem Ref. [15] was used for a two-dimensional stress state Ref. [16] and for shells of revolution Ref. [17]. The applicable constitutive equations were suggested in Ref. [18].

The aim of this paper is, at first, the extension of the general dynamic creep constitutive equations on the case of multiaxial stress state taken place in thin shallow shells and plates, at second, the elaboration the numerical method for the simulation of a dynamic creep-damage process in thin-walled structural elements. Results had been obtained by using the presented numerical method will be considered in the paper. The results of numerical calculation and experimental data of dynamic creep of rectangular plate with cut will be compared and discussed in order to verify the proposed method. The conclusions about influence of high-frequency external loading on the long-term strength in thin-walled structural elements at high-temperature conditions will be made on the basis of data obtained in the paper.

2 Mathematical problem statement

2.1 Method of asymptotical expansions

Let the mathematical statement of the creep problem for thin shallow shells loaded by joint action of constant and fast varying external loading will be done on the basis of the general theory of the three-dimensional dynamic creep problem formulated in Ref. [15].

Let us present the external surface load in a shell as the sum of two components - main slowly varied in time or constant action and oscillated action: $p = p^0 + p^1\Phi\,(\Omega t)$. The function $\Phi(\Omega t)$ varies under the law of a single-periodic harmonic function with constant frequency Ω and period $T = 2\pi/\Omega$ is essentially smaller, than time before rupture t_*. Let us suggest the existence of the small parameter $\mu = T/t_\star \ll 1$ under the accepted conditions of loading. The incommensurability of frequencies of main and oscillation actions allows to enter two time scales, such as 'slow' t and 'fast' $\xi = t/\mu$. Then the initial system of equations of thin shallow shells dynamic creep problem on the basis of asymptotic expansions on the small parameter μ for unknown functions, which depending on coordinates and two time scales and their averaging on a period of simple cycle can be reduced to recurrent system of equations which are adequate to two initial-boundary value problems. First one is the usual creep problem, where the creep evolution performs in a scale of 'slow' time at an action of constant external loading. This problem determines the main, global creep-damage process. The unknowns of this problem we shall mark by an index “0”. The second problem corresponds to the forced elastic harmonic oscillations are developed in a scale of 'fast' time. In this case the unknown functions we shall mark by an index “1”.

Using the equations of Kirchhoff-Love type theory for thin shallow shells Ref. [19] by saving the usually accepted in the theory unknowns we can write a system of equations for the stationary creep of shell problem as follows

$$\begin{aligned} T^0_{ij,j} &= 0 \\ M^0_{ij,ij} + k_{ij}T^0_{ij} + p^0 &= 0 \\ Q^0_i &= M^0_{ij,j} \qquad (i,j=1,2) \end{aligned} \tag{1}$$

where T_{ij} designate the membrane forces, Q_i correspond to shear forces, M_{ij} are the bending and twisting moments, k_{ij} are principal curvatures in the middle surface of shell.

Geometrical the strain-displacement relations including nonlinear terms for consideration finite deflection, is used quadratic terms only, can be written as

$$\begin{aligned} \varepsilon^0_{ij} &= C_{ijkl}\sigma^0_{kl} + c^0_{ij} = \varepsilon^{0\,m}_{ij} + z \cdot \chi^0_{ij} \\ \chi^0_{ij} &= -w^0_{,ij} \\ \varepsilon^{0\,m}_{ij} &= \frac{1}{2}\left(u^0_{i,j} + u^0_{j,i} + w^0_{,i}w^0_{,j}\right) - k_{ij}w^0 \qquad (i,j=1,2) \end{aligned} \tag{2}$$

Here ε_{ij} are the components of full strain tensor, c_{ij} are the components of creep strain tensor; ε^m_{ij} are the strain components and χ_{ij} are the components of the changes of curvatures and twisting of the middle surface; u_i – are the displacement in plane and w presents the deflection in middle surface of shell.

The creep constitutive equations are written in framework of the theory of shallow shells including the additional creep components

$$\begin{aligned} T^0_{ij} &= b_{ijkl}\,\varepsilon^{0\ m}_{kl} - T^C_{ij} \\ M^0_{ij} &= d_{ijkl}\chi_{kl} - M^C_{ij} \\ b_{ijkl} &= \frac{Eh}{1-\nu^2} \cdot \left[\delta_{ik}\delta_{jl} \cdot \frac{(1-\nu)}{2} + \nu \cdot \delta_{ij}\delta_{kl}\right] \\ d_{ijkl} &= \frac{Eh^3}{12(1-\nu^2)} \cdot \left[\delta_{ik}\delta_{jl} \cdot \frac{(1-\nu)}{2} + \nu \cdot \delta_{ij}\delta_{kl}\right] \end{aligned}$$

$$T_{ij}^{C} = \int\limits_{-h/2}^{h/2} \bar{b}_{ijkl} \cdot c_{kl}\, \mathrm{d}z\,, \qquad \bar{b}_{ijkl} = \frac{b_{ijkl}}{h}$$
$$M_{ij}^{C} = \int\limits_{-h/2}^{h/2} \bar{d}_{ijkl} \cdot c_{kl} \cdot z\, \mathrm{d}z\,, \qquad \bar{d}_{ijkl} = \frac{12 d_{ijkl}}{h^3} \tag{3}$$

The motion of shells in a scale of 'fast' time at small forced vibrations can be expressed by following known equations Ref. [20]

$$\begin{aligned} T_{ij,j}^{1} &= \rho h u_{i,\xi\xi}^{1} \\ M_{ij,ij}^{1} + k_{ij} T_{ij}^{1} + p^{1} &= \rho h w,_{\xi\xi}^{1} \\ Q_{i}^{1} &= M_{ij,j}^{1}, \qquad (i,j=1,2) \end{aligned} \tag{4}$$

The system of equations Eqs (1,4) cannot be considered as independent. The constitutive equations of a stationary creep contain the creep strains c_{ij}^{0}, which are determined from the dynamic creep constitutive equations. They include amplitude stresses, defined only after solution of the forced vibrations problem.

2.2 Constitutive equations

Within the framework of the accepted approach Ref. [15] the dynamic creep constitutive equations can be obtained from constitutive equations of a stationary creep law after its modification using the asymptotic expansions and averaging in a cycle. The obtained relations will differ from the laws of a stationary creep only by functions depending on the stress cycle asymmetry coefficient. This coefficient is equal to the ratio of the amplitudes of von Misses equivalent stress (solution in 'fast' scale) to the creep von Misses equivalent stress (solution in 'slow' scale) in a point of a shell: $A = \sigma_i^1/\sigma_i^0$.

The relations of the classical incremental Yu. N. Rabotnov's creep theory with structural parameters Ref. [4] for building the stationary creep constitutive equations are used. The material deterioration processes due to creep were described by scalar damage parameter ω, based on the concept of reduction of the cross-section area of damaged material Refs [4,5]. Then the constitutive equations for multiaxial stress state of the main global creep coupled with damage process (neglecting by an index "0") can be noted as

$$\begin{cases} \dot{c}_{ij} = \Lambda\left(\sigma_e, \omega\right) \cdot S_{ij} \\ \dot{\omega} = \Theta\left(\sigma_e^{\omega}, \omega\right)\,, \qquad \omega(0) = 0, \qquad \omega(t_\star) = \omega_\star \end{cases} \tag{5}$$

where $\dot{c}$ are the components of creep strains rate tensor; $\omega_\star$ is the critical damage value specified for the material; S_{ij} are the components of the stress deviator: $S_{ij} = \sigma_{ij} - \frac{1}{3} \cdot (\sigma_{ij}\delta_{ij}) \cdot \delta_{ij}$; $\Lambda\left(\sigma_{\mathrm{e}}, \omega\right)$, $\Theta\left(\sigma_{\mathrm{e}}^{\omega}, \omega\right)$ are empirical functions have to be determined by basic experimental investigations from uniaxial creep curves and long-term strength curves respectively; σ_e is the equivalent stress accepted in the creep theory for generalization of the experimental data from a uniaxial creep curve to multiaxial state; σ_e^ω is the equivalent stress accepted in the theory of long-term strength at extention to multiaxial state. For most metal materials the dynamic creep state equations and the damage evolution equation can be formulated as (Leckie, Hayhurst Ref. [21])

$$\begin{cases} \Lambda\left(\sigma_\mathrm{i},\omega\right) = \dfrac{B\sigma_i^{n-1}}{\left(1-\omega^r\right)^m} \\ \Theta\left(\sigma_\mathrm{e}^\omega,\omega\right) = \dfrac{D\left(\sigma_\mathrm{e}^\omega\right)^k}{\left(1-\omega^r\right)^l}, \qquad \sigma_\mathrm{e}^\omega = \sigma_i\alpha + (1-\alpha)\sigma_I \end{cases} \tag{6}$$

Here B, n, r, m, D, k, l are the material constants have been obtained from experimental creep data and long-term strength data; $\sigma_e = \sigma_i$, $\sigma_i = \left(\dfrac{3}{2}S_{ij}S_{ij}\right)^{1/2}$ is the von Misses equivalent stress; $\sigma_e = \sigma_e^\omega$ is the equivalent stress for damage evolution equation, $\sigma_I = \max\{\sigma_1,\sigma_2,\sigma_3\}$ is the maximum principal stress. Constant α defines the sensitivity of the material to the failure mode and depends on physical-mechanical properties of the material.

Constitutive dynamic creep damage equations for multiaxial stress state with respect to the relations Eq. (6) can be represented as follows

$$\begin{cases} \dot{c}_{ij} = \dfrac{3}{2}\dfrac{B\sigma_i^{n-1}}{\left(1-\omega^r\right)^m} S_{ij}\cdot H(A) \\ \dot{\omega} = \dfrac{D\left(\sigma_e^\omega\right)^k}{\left(1-\omega^r\right)^l}\cdot G(A), \qquad \omega(0)=0, \qquad \omega(t_\star)=\omega_\star \end{cases} \tag{7}$$

The influence functions $H(A)$ and $G(A)$ in Eq (7) are introduced in the form: $H(A) = \int_0^1 \left[1 + A\Phi(\xi)\right]^n \mathrm{d}\xi$, $G(A) = \int_0^1 \left[1 + A\Phi(\xi)\right]^k \mathrm{d}\xi$. In the case of harmonic law of forced vibrations the influence functions $H(A)$ and $G(A)$ value can be calculated numerically for integer n and k as

$$\begin{aligned} H(A,n) &= \int_0^1 \left(1 + A\sin 2\pi\xi\right)^n \mathrm{d}\xi \cong 1 + \frac{n(n-1)}{4}A^2\left(1 + \frac{(n-2)(n-3)}{16}A^2\right) \\ G(A,k) &= \int_0^1 \left(1 + A\sin 2\pi\xi\right)^k \mathrm{d}\xi \cong 1 + \frac{k(k-1)}{4}A^2\left(1 + \frac{(k-2)(k-3)}{16}A^2\right) \end{aligned} \tag{8}$$

The dynamic creep damage material models proposed in this paper for uniaxial stress state correspond to well known dynamic constitutive equation of Lasan-Taira-Rabotnov Refs [4,5,7].

The results of dynamic creep experimental investigation using the specimens made for aluminum alloy D16AT at 300 $^\circ$ C Ref. [22] due to creep time 7.5 h loaded by the constant stress $\sigma^0 = 27$ MPa and at amplitude stress $\sigma^1 = 13.5$ MPa with frequency of vibration

290 Hz are shown on Fig. 1.a). Here points correspond to averaging experimental data, lines as well as for creep curves calculated by Eqs (7,8) for cases: 1 - $A = 0$ and 2 - $A = 0.5$. The uniaxial creep curves with respect of last stage before the failure initiation of a material obtained numerically for both cases of loading are presented on Fig. 1.b). Let's mark a satisfactory accuracy of the dynamic creep model is proposed in Ref. [18].

2.3 Variational problem statement

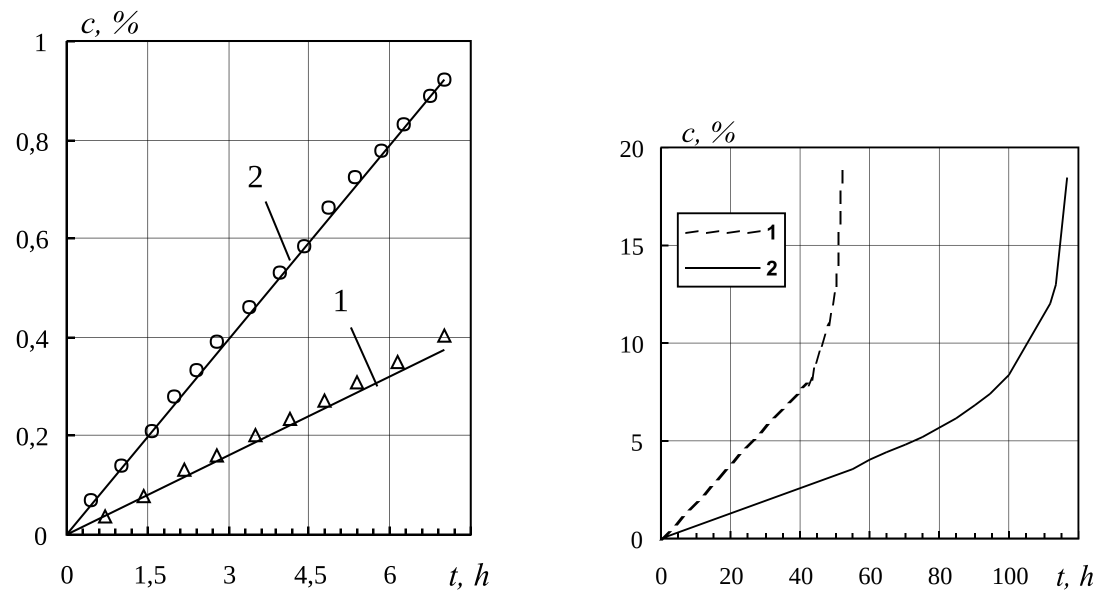

Fig. 1. Dynamic creep curves for specimens made of alloy D16AT: a) comparison of experimental (points) and numerical (lines) data; b) numerically obtained dynamic creep curves with respect of last stage before the failure initiation.

For solving the dynamic creep problem for shallow shells let us use the approach permitting to come from the operator form of the basic differential equations Eqs (1-4) relatively the displacements $\underline{u}$

$$L\underline{u} = \underline{p}$$

to variational functional based on the principal of virtual displacements

$$(L\underline{u}, \underline{u})_H - \left(\underline{p}, \underline{u}\right) = 0,$$

such expression at the case of the symmetric and positive operator $L\underline{u}$ coincides with an energy functional

$$F(\underline{u}) = (L\underline{u}, \underline{u}) - \left(\underline{p}, \underline{u}\right) = 2\int_v W(\underline{u})\, \mathrm{d}v - \int_S \underline{p}\underline{u}\, \mathrm{d}S$$

where $(L\underline{u}, \underline{u})$ designates the usually scalar product, H is the Banach space, $\left(\underline{p}, \underline{u}\right)$ is the linear functional, $W(\underline{u})$ is specific potential energy.

Applying the proposed approach for a dynamic creep problem of shallow shells and assuming that a creep strains are known at each time step, we shall formulate a variational functional given on displacements. The stationary condition of this functional will correspond to the equilibrium equations Eq. (1) at fixed time moment and to variational equation of the Lagrange type which includes additional work by pseudo-forces. The last ones are defined by irreversible creep strains (underline terms in equation). The variational equation for any time moment can be formulated as

$$\begin{aligned} \delta J^0 =& \frac{1}{2}\int\limits_S \left(T_{ij}^{0\,\prime}\, \delta \varepsilon_{ij}^{0\ m} - M_{ij}^{0\,\prime}\, \delta \chi_{ij}^0 \right) \mathrm{d}S - \int\limits_S p^0 \delta w^0 \,\mathrm{d}S - \\ & \underline{\frac{1}{2}\int\limits_S T_{ij}^C \delta\ \varepsilon_{ij}^{m\,0}\ \mathrm{d}S} - \underline{\frac{1}{2}\int\limits_S M_{ij}^C \delta \chi_{ij}^0 dS} = 0 \end{aligned} \tag{9}$$

where $T_{ij}^{0\,\prime} = b_{ijkl}\ \varepsilon_{kl}^{0\ m},\ M_{ij}^{0\,\prime} = d_{ijkl} \chi_{kl}^0$.

The principal field of unknown displacements corresponds to a minimum point of the variational functional Eq. (9), defined on kinematically suitable displacements. Creep strain tensor components, temperature and the external slowly varying forces are assumed as known at each time step and have to be determined from previous time step in equation Eq. (9).

The variational problem statement for shell small forced vibrations is well known. Variational equation in this case at ‘fast’ scale time ξ can be written as

$$\delta J^1 = \int\limits_S \left(b_{ijkl}\ \varepsilon_{ij}^{1\ m}\, \delta \varepsilon_{ij}^{1\ m} - d_{ijkl} \chi_{kl}^1 \delta \chi_{ij}^1 \right) \mathrm{d}S - \int\limits_S \rho h u_{,\xi\xi}^1 \delta u^1 \,\mathrm{d}S - \int\limits_S p^1 \cdot \delta w^1 \,\mathrm{d}S = 0 \tag{10}$$

where u^1 and w^1 are unknown displacements fields of shell middle surface at the problem of forced vibration.

3 Numerical methods

From the mathematical point of view the dynamic creep problem of shallow shells refers to physically nonlinear initial-boundary value problem of mathematical physics. For it solving let us usually use time-step discretization methods. Ones to it the initial nonlinear creep problem reduces to the sequence of linear boundary-value problems with fixed creep-load components as Eq. (9) at each time step.

The constitutive equation and damage evolution equation as Eq. (7) are integrated by the following iteration scheme:

$$\begin{aligned} \underline{c}_{(k+1)} &= \underline{c}_{(k)} + \left(\frac{1+\gamma_k}{2} \dot{\underline{c}}_{(k+1)} + \frac{1-\gamma_k}{2} \dot{\underline{c}}_{(k)} \right) \Delta_k t, \\ \underline{\omega}_{(k+1)} &= \underline{\omega}_{(k)} + \left(\frac{1+\gamma_k}{2} \dot{\underline{\omega}}_{(k+1)} + \frac{1-\gamma_k}{2} \dot{\underline{\omega}}_{(k)} \right) \Delta_k t, \end{aligned} \tag{11}$$

The obtained values of creep strains and damage parameter are used in Eq. (9). Weighting coefficient γ_k (where k is the number and Δ_k is the time-step size) can vary to assure the accuracy, convergence and stability of the iteration scheme Eq. (11).

The solutions of linear boundary-value problem were performed by a finite element method at each time step. FE-analysis is preferable, as allows to take into calculation complex geometry of shells and plates and various boundary conditions without a modification of the common solution scheme of boundary-value problems.

For the meshing we use the three-node plate simplex element with 18 degrees of freedom. It have the shape functions are linear relatively homogeneous coordinates for approximation the membrane displacements and cubic polynomials for deflection of a shell Ref. [23]. This element is based on the Kirchhoff-Love type shell theory and contains 10 Gauss points in the plane and 9 integration points through the thickness. Further let us use the usual FEM definitions in the vector-matrix representations for main unknowns and modify the integration on shell area by the sum of integrals on finite elements. After that we shall reduce the variational equation Eq. (9) at the time step $t \geq 0$ to following variational equation relatively the vector of nodal principal displacements of finite element $\{q^{0\beta}\}$

$$\delta J^0 = \sum_\beta \delta J^{0\beta} = \sum_\beta \left\{ \int_{S^\beta} \delta \left\{q^{0\beta}\right\}^{\mathrm{T}} [D_0]^{\mathrm{T}} [E] [D_0] \, \delta \left\{q^{0\beta}\right\} \, \mathrm{d}S - \int_{S_I^\beta} \delta \left\{q^{0\beta}\right\}^{\mathrm{T}} [B]^{\mathrm{T}} \left\{p^0\right\} \, \mathrm{d}S - \right.$$
$$\left. \int_{S^\beta} \delta \left\{q^{0\beta}\right\}^{\mathrm{T}} [D_0]^{\mathrm{T}} [P] \{c_m\} \, \mathrm{d}S - \int_{S^\beta} \delta \left\{q^{0\beta}\right\}^{\mathrm{T}} [D_L]^{\mathrm{T}} [P] \{c_m\} \, \mathrm{d}S \right\} + \tag{12}$$
$$\sum_\beta \int_{S^\beta} \left\{q^{0\beta}\right\}^{\mathrm{T}} \left([D_L]^{\mathrm{T}} [E] [D_0] + [D_0]^{\mathrm{T}} [E] [D_L] + [D_L]^{\mathrm{T}} [E] [D_L]\right) \left\{q^{0\beta}\right\} \mathrm{d}S$$

where β is the number of element, $[B]$, $[E]$, $[D] = [D_0] + [D_L]$ and $[P]$ are the shape, elastic, strain (linear and nonlinear) and creep strain matrixes respectively, writing in usual definitions of FEM.

After the FEM summing procedure the variational equation Eq. (12) has been reduced to the solution of the following system of linear equations

$$[K] \cdot \left\{q^0\right\} = \{F_p\} + \{F_c\}\,, \tag{13}$$

where $[K] = [K_0] + [K_L]$ is the global stiffness matrix:

$$[K] = \sum_\beta \int_{S^\beta} [D_0]^{\mathrm{T}} [E] [D_0] \, \mathrm{d}S + \sum_\beta \int_{S^\beta} \left([D_L]^{\mathrm{T}} [E] [D_0] + [D_0]^{\mathrm{T}} [E] [D_L] + [D_L]^{\mathrm{T}} [E] [D_L]\right) \mathrm{d}S\,,$$

$\{F_p\}$ is the generalized external force vector:

$$\{F_p\} = \sum_\beta \int_{S_1^\beta} [B]^{\mathrm{T}} \cdot \left\{p^0\right\} \, \mathrm{d}S\,,$$

$\{F_c\}$ is the generalized pseudo-force vector is defined by irreversible creep strains:

$$\{F_C\} = \sum_{\beta} \int\limits_{S^\beta} [D]^{\mathrm{T}} \cdot [P] \cdot \{c_m\} \ \mathrm{d}S \,.$$

The global stiffness matrix has been built by the known rules defined in Ref. [23] and assuming the approach about compatibility of strains and equilibrium of forces in nodes. Quadratic nonlinear terms $[K_L]$ in a global stiffness matrix were transferred to a right part of the equation Eq. (13) and it was presented by the vector-column of pseudo-forces are defined by the elastic nonlinear strains. Nonlinear loads are assumed as known from the previous time step of the integration of the basic equation Eq. (13). The components of stiffness matrix $[K]$, generalized nodal forces vector $\{F_p\}$, generalized nodal pseudo-forces vector $\{F_c\}$ defined by creep strains $\{c_m\}$ and generalized nodal pseudo-forces vector from nonlinear elastic strains have been calculated using the quadrature formula. So, the integration in a plane of a finite element was performed by Hammer's formula and through the thickness using the Newton–Cotes one. The solution of system Eq. (13) is carried out by Cholesky's method. This numerical procedure uses only nonzero diagonal members of a matrix $[K]$ that is rather effective in practical calculations by FE-method. With the purpose of a regularization of nodal stresses in considered FEM scheme the conjugated approximation method was realized. Last one supposes the smoothing of nodal stresses by polynomials coordinated with the shape functions of principal unknowns.

The forced oscillations problem of shallow shells was solved by FE-method and is reduced to a solution of the following equation

$$\left(\int\limits_{V} [D_0]^{\mathrm{T}} [E] [D_0] \ \mathrm{d}V \right) \left\{ q^1 \right\}^{\beta} = \int\limits_{S_1} [B]^{\mathrm{T}} \left\{ p^1 \right\} \ \mathrm{d}S - \int\limits_{V} [B]^{\mathrm{T}} \rho h \left\{ q^1_{,\xi\xi} \right\} \ \mathrm{d}V$$

The distribution of amplitude stresses is obtained for mesh, which is coordinated with the solution of the creep problem. The main equation of the forced oscillations problem with respect to $\{q(t)\} = \{q^1\} \cdot \cos(\Omega t + \alpha)$ are expressed as

$$\left([K] - \Omega^2 \cdot [M] \right) \cdot \left\{ q^1 \right\} = \left\{ F^1 \right\}, \tag{14}$$

where $[M]$, $[K]$ are the mass and the stiffness matrix, $\{F^1\}$ is an amplitude external forces vector. The mass matrix $[M]$ is obtained similarly to ensemble procedure of stiffness matrix Ref. [23]. For the accuracy of calculations of the system equations Eq. (14) the mass matrix of a element ($[H]$ is the thickness matrix)

$$[M] = \rho h \cdot \int\limits_{S^\beta} [B]^{\mathrm{T}} \cdot [H] \cdot [B] \ \mathrm{d}S \,,$$

had been derived to the diagonal form. The method of selection of a principal diagonal and the numerical integration procedure of Euler's scheme was used.

Using the vector of nodal amplitude displacements $\{q^1\}$ which is known after solution Eq. (14) the components of amplitude stress vector were determined as follows

$$\left\{\sigma^1\right\} = [D_0] \cdot [B] \cdot \left\{q^1\right\}.$$

The components of equivalent von Misses stress vector we can calculate from

$$\underline{\sigma}_i^1 = \left(\underline{\sigma}^{1\,\mathrm{T}} \cdot \underline{\underline{I}} \cdot \underline{\sigma}^1\right)^{1/2}; \qquad \underline{\underline{I}} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

The constitutive equations of dynamic creep damage for multiaxial stress state in shell will be written as follows

$$\begin{aligned}
&\underline{\dot{c}} = \frac{B\left(\sigma_e\right)^{n-1}}{\left(1-\omega^r\right)^m} \cdot H(A) \cdot \underline{\underline{G}} \cdot \underline{S}\,; \qquad \underline{\underline{G}} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}, \\
&\dot{\omega} = \frac{D\left(\sigma_e^{\omega}\right)^k}{\left(1-\omega^r\right)^l} \cdot G(A)\,, \qquad \omega(0) = 0\,, \quad \omega(t_*) = \omega_*\,, \qquad (15) \\
&\sigma_e^{\omega} = \sigma_i \alpha + (1-\alpha)\,\sigma_I\,, \qquad \sigma_I = \max\left\{\sigma_{(1,2)} = \frac{1}{2} \cdot \left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]\right\}.
\end{aligned}$$

Thus FEM procedure of the dynamic creep-damage problem of thin shallow shells and plates includes a solving of the equations Eq. (12) at each time step. The resulting vector of principal unknowns and of vector creep strain, damage and nonlinear elastic strain vector are used for the calculation at the next time step. At the initial time moment both problem of shells the elastic deformation and forced vibration are solved by FE methods. On the basis of the suggested algorithm the in-house engineer software realizing numerical calculation of the creep-damage problem of thin plates and shallow shells in conditions of static and dynamic loading had been created. The numerical investigations have established new laws of shaping and failure of thin shallow shells and plates in conditions of static and dynamic creep.

4 Numerical investigations

Creep-damage processes in shallow shells and plates at static and cyclical loading were studied. The laws of a dynamic creep in thin-walled structures are established by comparison of results of calculation for both cases of loading. The calculations show that cyclical loading, at first, even at small amplitudes of cyclic load essentially influences on shaping and life-time before the failure initiation. Secondly, the amplitude and frequency of the external loading can essentially influence the long-term strength of thin-walled structures. So, loading with frequency which is less than the first eigen frequency of a plate or a shell reduces the life-time before the failure initiation and this influence becomes more significant, than more the frequency of forced oscillations come nearer to the first eigen frequency of a plate or shell. With growth of amplitude of load the established regularity becomes more expressed. This conclusion is illustrated by the creep curves on Fig. 2. Here time-dependent accumulation of the creep irreversible strain c_{11} on the outside surface in a central point of a square plate with clamped edge loaded at its center with the varying transversal force $P = P^0(1 + A \sin \Omega t)$ with frequency is equal to 20 Hz and $P^0 = 300$ kN are shown. The sizes of the plate are accepted equal: plate length $a = b = 1$ m, thickness $h = 0.05$ m, material Ti-6Al-2Gr-2Mo, $T = 400\,^\circ$C Ref. [24]. By the initial computation was defined the lowest eigen frequency of the plate, which was equal to 419 Hz.

The numerical results of the static creep calculations with $P = P^0(1+A)$ and $A = 0.2$ are illustrated on Fig. 2 by the curve "$P = 360$ kN". The numerical results of dynamic creep with $A = 0.2$ and static creep ones with load is equal to its maximum value in cycle (curve "$P = 360$ kN") are shown on Fig. 2. The results for both cases are differ significantly.

Thus dynamic creep-damage process cannot be described using for the simulation only static loading the which is equal to a maximum level of load in cycle. So, as can be seen on Fig. 2 the time before failure initiation in case only of maximum static loading in cycle is significantly underestimated, and as well as rate of damage accumulation and of maximum stresses relaxation are overestimated.

The next example illustrates the time dependent solution of clamped square plate with frequencies of external loading situated between eigen frequencies are more than first eigen one. These numerical investigations show, that for frequencies of external loading with frequencies are more than the first eigen one both a fall and a rise of a shaping level, a rate of stresses relaxation and a long-term strength are possible. And, the marked regularities are defined by character of distribution in the plate volume of a stress cycle asymmetry coefficient.

The accumulation of a creep strain in a central point of considered above plate is presented on Fig. 3 for a case, when the frequency of cyclical component of external loading is equal to: 1 - 0 Hz, 2 - 1000 Hz, 3 - 1700 Hz and 4 - 2200 Hz. It can be seen on Fig. 3, that the solutions in this case have the difference for life-time predictions and insignificant difference for the accumulated creep strains values.

The character of the stress cycle asymmetry coefficient's distribution, determined on a outside surface of a plate with central cut for above cases of loading is presented on Fig. 4. Let's mark the significant difference in distributions of coefficient A in cut of a plate for different frequencies of external loading. However, the maximum of coefficient A for all cases takes place at the centre of a plate, where the calculations are defined the place of the failure initiation. At the same time, depending on a level of forced frequency, the time

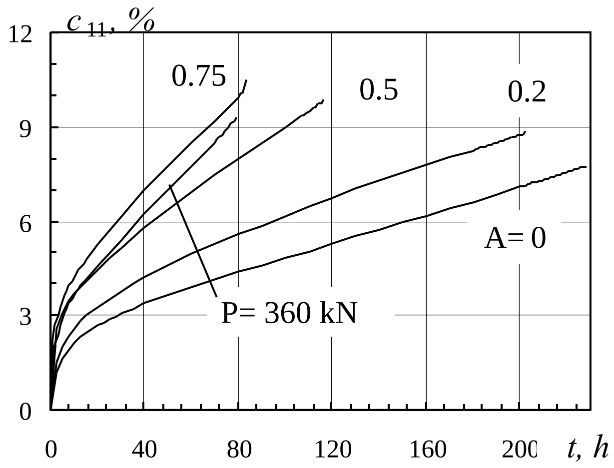

Fig. 2. Time variation of maximum creep strain c_{11} on an outside surface in a central point of a clamped edge square plate.

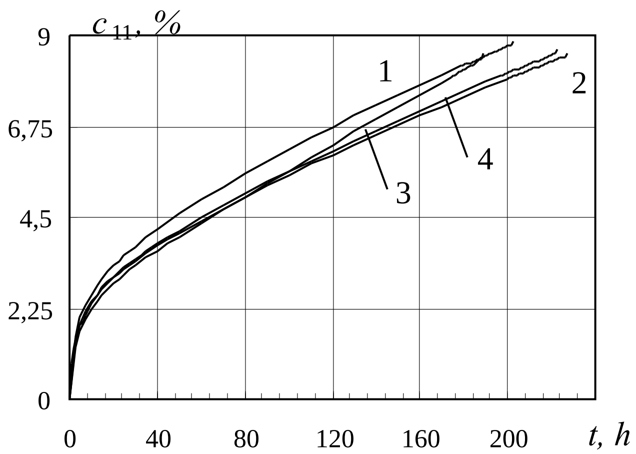

Fig. 3. Influence of the frequency of external loading on the time variation for maximum creep strain c_{11} in a central point of the outside surface of a clamped edge square plate.

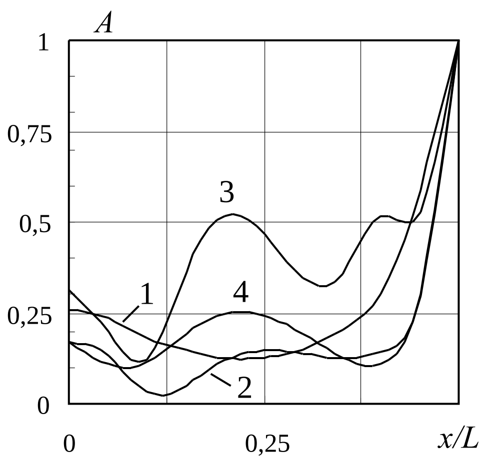

Fig. 4. Distribution of the stress cycle asymmetry coefficient on an outside surface of central cut of a plate for different forced frequencies.

before the failure initiation and rate of damage accumulation in this point of the plate are various (see Fig. 3).

The lack of definiteness in obtained laws of dynamic creep behaviour raises a role and practical significance of numerical simulation in the applied problems with a various combination of loading. With this purpose the proposed solution method can be recommended for the analysis of a stress-strain state and estimation of long-term strength in numerical investigations of a dynamic creep of thin-walled structural members for design or a prediction of their properties.

5 Experimental investigations

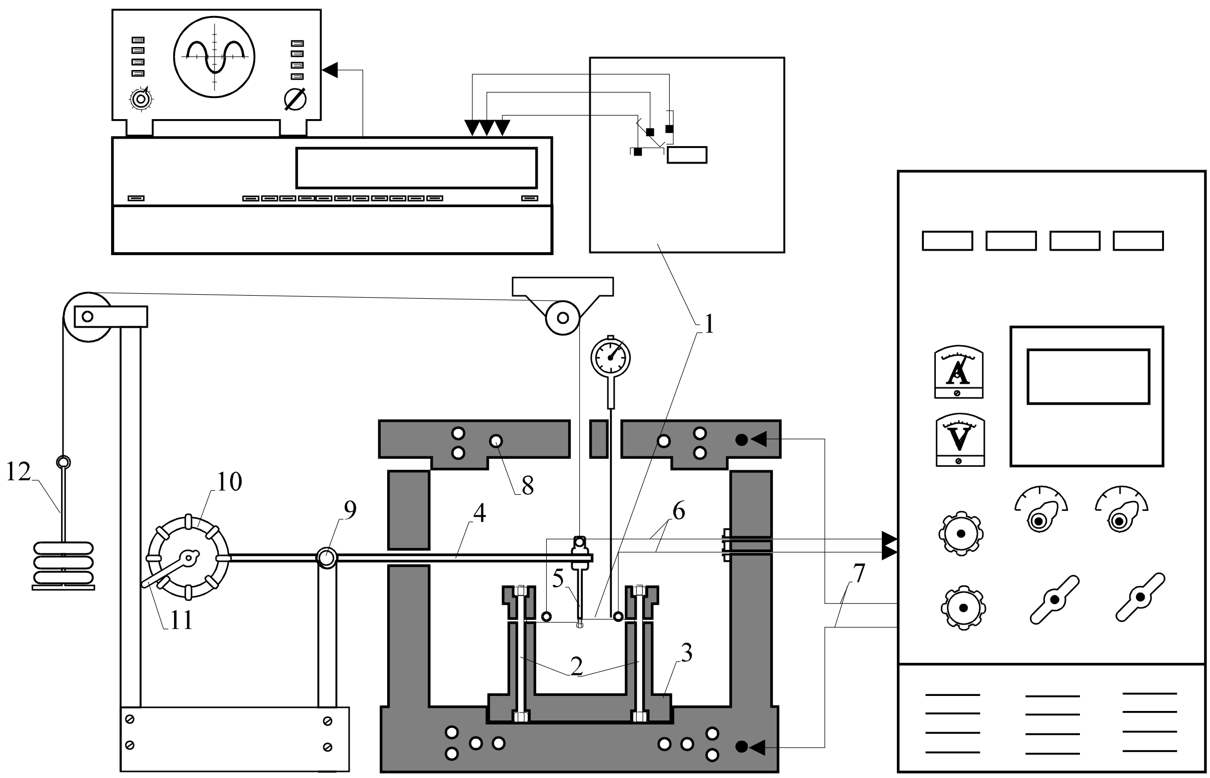

Fig. 5. The scheme of the experimental equipment for investigations of a dynamic and static creep of plates under transversal loading.

In order to verify the proposed method of dynamic creep calculation for thin-walled structural members the experimental investigations were carried out. The dynamic creep of plates with a rectangular cut of an aluminum structural alloy D16AT at temperature 300 $^\circ$C Ref. [25] was studied experimentally. The scheme of experimental equipment is presented in Fig. 5. Here let us present the unit's main parts: testing plate model (1); load system; electric furnace; information and measurement system SIIT-3; cathode oscillograph S1-77; temperature control system automatic compensate potentiometer KSP-4; thermocouples output (6) and input (7). The plate model have linear size is equal to $a = b = 80$ mm, thickness equal to $h = 1$ mm and size of cut is equal to $c = 10$ mm,

$d = 5$ mm. The plate was loaded at a center the concentrated force with static component $P^0 =$ 100 N.

Static loading was carried by means a cable, some blocks and a massive detail of a required mass, which it was suspended on the special support (12). The cable was incorporated to a plate by means of the steel shift (5), which was fixed in plate central part by the screw such that the head of the screw had small area of a touch with edges of a technological orifice. Plate was located in assembly (3) with catches (2) realizing a boundary condition of clamped edge plate. The electric motor (10) with eccentric (11) had excited simple harmonic vibrations, and the rod (4) fixed in a cylindrical joint (9) had transmitted these oscillations directly to centre of a plate with a frequency depending on an amount of turnovers of the shaft (25 Hz) and with amplitude, depending from the eccentric inertial properties ($P^1 = 0.2P^0$). The values of static and cyclic loads beforehand were tested by the calibrated experiences. System calibration was made for determination the values of losses of dynamic load are connected with influence of inertial characteristics of massive details. Strain gauges with resistance value $R = 100\ \Omega$ and with base $5 \cdot 10^{-3}$ m were cemented on the specimen's surface. Signals were feed into the digital and analog outputs of the apparatus SIIT-3. From the analog output the signal was feed to cathode oscillograph S1-77. The heat of a plate was made by furnace with potency 3.5 kW with autocontrol of temperature by the controlling thermocouples (7). The temperature measurement is carried by two chromel-alumel thermocouples (6) are installed on operating part of plate and connected to potentiometer KSP-4. The deviation from specified temperature in a whole time of experiment didn't exceed 1-2 $^\circ$C. Temperature measurement's results are written on the moving diagram tape. The deflection of a plate has been registered by the indicating gauges with a resolving capacity in 0.01 m. They have been installed in a x-shaped sawcut of the furnace cap in points of plate remote from center on distances 10 mm and 20 mm on perpendicular directions. These points of gauging have been denoted on the Fig. 6 as curves 1 and 2 accordingly. Here the experimental data by a point are presented and numerical data are shown by full line. The initial failure due to creep in a plate was observed through a technological orifice in the furnace cap.

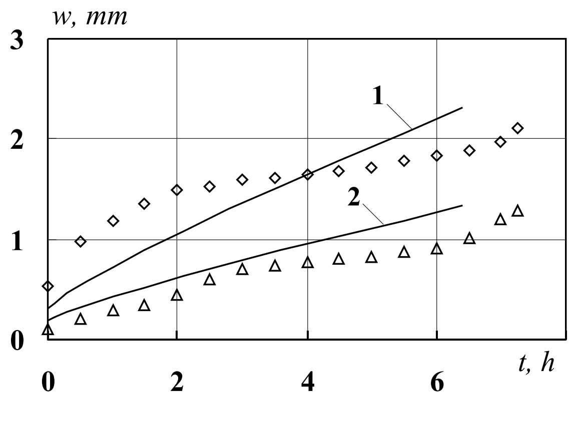

Fig. 6. Comparison of numerical (full lines) and experimental (points) data. Time-dependent deflection in points of gauging (1, 2) of the plate with a cut.

The results of comparison of time-dependent deflection in sectional views of a plate with cut obtained experimentally and computationally are represented on a Fig. 7.

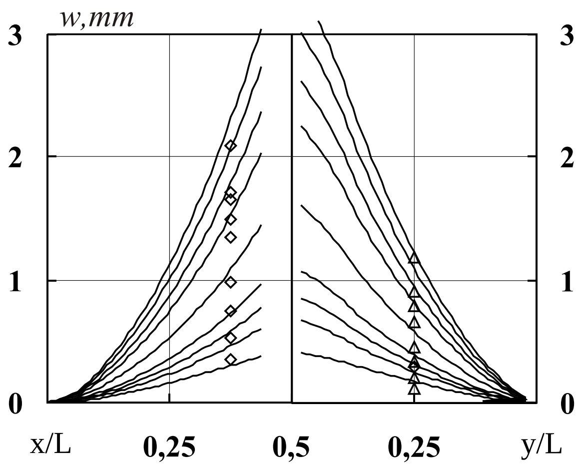

Fig. 7. Comparison of time-dependent deflection in sectional views of a plate with a cut: left – along axis OX; right – along axis OY.

The data of the central sectional view of plate along of the greater side of a cut are considered in the left part on Fig. 7, in right part these data are considered along smaller side. The lines show results of numerical simulation, points contain the experimental data registered by indicators.

Let's mark good agreement of the numerical solution with the experimental data. The results of the numerical simulation overestimate the deflection value in relation to experimental data time to fracture. The obtained data differ in a limits 20 %. At the same time the results on long-term strength of the plate are sufficiently near. So, the time before rupture initiation at experimental investigations has obtained equal to 7.5 h, and the numerical value is equal to 7 h.

Thus, the proposed calculation method of a creep and long-term strength of thin-walled structural members allows to describe with an adequate accuracy their stress strain state and to determine time before fracture initiation due to the creep process. Consequently we made a conclusion that method may be used for a solution of the applied engineering tasks.

6 Conclusions

The kinetics of stress strain state of thin shallow shells at combined static and cyclical loading was studied in conditions of a high-temperature creep.

The variant of dynamic creep constitutive equations of a material is proposed. They were obtained by modification of the known equations of a static creep with using of asymptotic expansions on the small parameter and their averaging for cycle. The obtained equations are differed from the laws of a static creep only by influence functions depending on a stress cycle asymmetry coefficient $A(t)$. The proposed constitutive equations were generalized on a case of a multiaxial stress state which takes place in shells. The equivalent stress in the law of a creep corresponds to the von Misses stress such stress in the kinetic equation for damage evolution was accepted using the Pisarenko-Lebedev's long-term strength criterion.

Within the framework of the theory of asymptotic expansions, using separation of time on the 'fast' and 'slow', the dynamic creep problem of thin shells and plates is reduced to solving both problems of a stationary creep and elastic forced vibrations. These problems are connected by the dynamic creep constitutive equations. The solution of both problems is carried out by FE method.

The laws of a static creep and time before failure initiation of plates and shells and new laws a shaping and a long-term strength connected with cyclic loading are established on the basis of calculated and experimental data. Comparison of results of calculations for static and cyclic loading have shown, that cyclic loading, at first, even at small amplitudes of load essentially changes data on shaping and life-time; secondly, the parameters of external loading, such as the amplitude and frequency essentially influence the long-term strength.

References

[1] Vidal J., About a cyclic creep of high-temperature alloys. In: *High-temperature alloys at varying temperatures and stresses*, Trans. from Engl., Gosenergoizdat, Moscow, Leningrad, p. 156-175, 1960. (In Russian).

[2] Garoffallo F., *Creep laws and long-term strength of metals and alloys*, Trans. from Engl., Metallurgia, Moscow, 1968. (In Russian).

[3] Lasan B.J., Fatigue of constructional materials at high temperatures, In: *Problems of high temperatures in airal structures*, Trans. from Eng., Izd-vo inostr. lit., Moscow, p. 233-256, 1961. (In Russian).

[4] Rabotnov Yu.N., *Creep problems in structural member*, Amsterdam: Noth Holland, 1969.

[5] Taira S., Ohtani ., *Theory of high-temperature strength of materials*, Trans. from Jap., Metallurgia, Moscow, 1986. (In Russian).

[6] Dorn J.E., The spectrum of a creep activation energies, In: *Creep and recovering*, Trans. from Engl., Metallurgia, Moscow, p. 291-325, 1961. (In Russian).

[7] Lasan B.J., Dynamic creep and rupture properties of temperature-resistant materials under tensile fatigues stress, *Proc. ASTM*, 49, p. 757-787, 1949.

[8] Taira S., Koterazawa R., Dynamic creep and fatigue of an 18Mo-Cb steel at elevated temperature, *Bull. JSME*, 5, No 17, p. 15-20, 1962.

[9] Nadai A., *Plasticity and rupture of solid bodies*, Mir, Moscow, 1969. (In Russian).

[10] *Cyclic creep of high-temperature nickel-based of alloys*, Edited by Golub V.P., Nauk. dumka, Kiev, 1983. (In Russian).

[11] Freudenthal A.M., Aspects of fatigue damage accumulation at elevated temperatures, *Acta metal*, 11, 7, p. 753-758, 1963.

[12] Boyle J.T., Spence J., *Stress analysis for creep*, Butterworths, London, 1983.

[13] Sosnin O.V., Gorev B.V., Nikitenko A.F., *Power variant of creep theory*, Novosibirsk, 1986. (In Russian).

[14] Kachanov L.M., *Introduction to continuum damage mechanics*, Dordrecht: Kluwer, 1986.

[15] Morachkovsky O.K., Non-linear creep problems of bodies under action of fast oscillated field, *Prikl. mechanika*, V.28, No 8, p. 17-23, 1992. (In Russian).

[16] Breslavsky D., Morachkovsky O., A new model of nonlinear dynamic creep, *IUTAM Symposium on Anisotropy, Inhomogenity and Nonlinearity in Solid Mechanics*, Dordrecht: Kluwer Academic Publisher, p. 161-166, 1995.

[17] Altenbach H., Breslavsky D., Morachkovsky O., Naumenko K., Cyclic creep-damage in thin-walled structures, *Journal of Strain Analysis for Engineering Design*, Suffolk, UK, Vol. 35, No 1, p. 1-11, 2000.

[18] Breslavsky D., Morachkovsky O., Cyclic creep constitutive equations with consideration of creep-fatigue interaction, *Proc. of 1 st International Conference on Mechanics of Time Dependent Materials*, Bethel: SEM, p. 61-66, 1995.

[19] Mushtari H.M., Galimov K.Z., *The non-linear theory of elastic shells*, Tatknigoizdat, Kazan, 1959. (In Russian).

[20] Reissner E., On transverse vibration of thin shallow shells, *Quarterly of Appl. Math.*, 13, N 2, 1955.

[21] Leckie F., Hayhurst D., Constitutive equations for creep rupture, *Acta Metall.*, 25, p. 1059-1070, 1977.

[22] Breslavsky D.V., Morachkovsky O.K., Shipulin S.A., Creep and rupture of notched plates under fast cyclic load, *Proc. 17th Symp. on Experimental Mechanics of Solids*, Warsaw, p. 118-123, 1996.

[23] Zienkiewicz O., Taylor R., *The Finite Element Method*, McGraw-Hill, London, 1991.

[24] Bodnar A., Chrzanowski M., Cracking of Creeping plates in terms of continuum damage mechanics, *J. of Theoretical and Applied Mechanics*, 1, 32, p. 31-41, 1994.

[25] Konkin V.N., Morachkovskij O.K., Long-term strength of light alloys with anisotropic properties, *Probl. Prochn.*, 6, p. 38-42, 1987. (In Russian).