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ASYMPTOTIC METHODS AND FINITE ELEMENT METHOD IN CYCLIC CREEP-DAMAGE PROBLEMS

The paper presents the description of methods, algorithms and software were developed for numerical simulation of two- and three-dimensional creep-damage problems. Special attention is stressed on the problems of solid's behavior at conditions of cyclic varying of stress and temperature fields which is widely spread in modern aerospace and power industry.

The full mathematical problem statement of solid's creep in the case of joint action of static and cyclically varying forces and temperatures is presented [1]. We regard the solid initially isotropic body with volume V which is fixed in a surface part S_1. The body is loaded by volume forces f_i, the traction p_i acts on the surface part S_2. The case of constant in time volume forces is regarded: $f = f(x_i), x_i \in V, i = 1,2,3.$ Traction has constant value $p_i^{(0)} = p_i^{(0)}(x_i)$ and varies in a time through the cycle $\Phi_i(t)$: $p_i = p_i^{(0)} + \Phi_i(t)$, $x_i \in S_2$, $\Phi_i(t) = p_i^{\max}\sum_{k=1}^{\infty} Z_k \sin(\Omega_k t + \beta_k) + p_i^a \sin(\Omega t)$, where $p_i^{\max}$, p_{ia} are the amplitudes of appropriate components of traction; $Z_k = \sqrt{a_k^2 + b_k^2}$; $\Omega_k = 2\pi k / T$; $\Omega = 2\pi / T_2$.

$$\varepsilon_{ij} = e_{ij} + p_{ij} + c_{ij} + \varepsilon_{ij}^T,\ \sigma_{ij,j} + f_i = \rho\frac{\partial^2 u_i}{\partial t^2},\ \varepsilon_{ij} = \frac{u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}}{2},\ x_i \in V; \qquad (1)$$

$$\sigma_{ij} = D_{ijkl}(\varepsilon_{kl} - p_{rl} - c_{kl} - \varepsilon_{kl}^T);\ \sigma_{ij}n_j = p_i^{(0)} + \Phi_i(t),\ x_i \in S_2;\ u_i = \bar{u}_i,\ x_i \in S_1;$$

$$u_i(x_i,0) = c_{ij}(x_i,0) = 0$$

where n_j are the components of the unit vector which is normal to body's surface, $j = 1,2,3;$ $\{D\}$ is a tensor of elastic properties of the material; u_i are known values of the displacements on the surface S_1, which are constant in a time; ρ is the material density. Total strain consists of elastic e_{ij}, temperature ε_{ij}^{T}, creep c_{ij} and instantaneous plastic p_{ij} components. The last ones are determined in necessary cases by solution of separate elastic-plastic problem.

The non-stationary heat conduction problem is stated additionally in order to determine the temperature distribution in each time moment [1].

$$a_T^2 \Delta \overline{T} + Q_T = \dot{\overline{T}}\ ,\ a_T^2 = k_T / (c_T \rho),\ Q_T = F / (c_T \rho) \qquad (2)$$

where Δ is Laplace's operator; c_t, k_t, are the coefficients of specific heat and heat conduction; F is the density of thermal sources.

The direct integration of the system of nonlinear differential equations (1) meets a number of numerical problems. From the other hand the state equations of cyclic behavior of materials at different programs of stress and temperature varying demonstrate the essential acceleration of creep strain and damage accumulation rates, which demands separate description in state equations. These reasons demand the use of special procedure for the solution of stated problem.

The method of asymptotic expansions jointly with the method of averaging over of the periods of stress or temperature varying is used for the transformation of original system of equations. These procedures led to two systems of equations, one of which describes the motion of a system in the slow time scale as well as the second simulates the fast [1-3]. The constitutive creep-damage equations were derived by use of similar asymptotic and averaging procedures [1-2] are added to the first system. Only this first system has to be solved for any time moment, as well as the second supplies the amplitude values for constitutive equations.

This solution is organized by use of the Finite Element Method (FEM) for boundary problems with addition of time-step integration methods like predictor-corrector for initial value ones. The above approach was realized for three principal problems: two- and three dimensional stress-strain state and thin closed and unclosed shells of revolution at arbitrary loading. Developed software include three finite elements: three-nodal triangle for problems like plane stress state, eight-nodal brick for three-dimensional modeling and four-nodal element of thin shell of revolution with 7 degrees of in each node. Three application packages were developed for solution of above mentioned creep-damage problems. The software include the graphical pre- and postprocessors and mathematical processors. Examples of numerical modeling of creep-damage processes at cyclic loading and heating can be found in [1-3].

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