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Simultaneous determination of position and mass in the cantilever sensor using transfer function method

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We present the simultaneous measurement of mass and position of micro-beads attached to the cantilever-based mass sensors using the transfer function method. 10 μm diameter micro-beads were placed on micro-cantilevers and the cantilevers were excited by lead-zirconate-titanate through low-pass filtered random voltages. The cantilever vibration was measured via a laser Doppler vibrometer before and after applying the beads. From the measured transfer function, the bead position was identified using its influence on the cantilever kinetic energy. The bead mass was then obtained by analyzing the wave propagation near the beads. The predicted position and mass agreed well with actual values. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4813839]

In the last few decades, a large number of cantilever-based sensors have been developed to detect proteins, DNA, cells, bacteria, and viruses for a variety of applications, such as homeland security, clinical diagnostics, food safety analysis, and environmental monitoring.[1,2] They are shown to be highly sensitive to detect zeptograms in vacuum[3] and they enable real-time detection in a label-free manner.[1,2] The cantilever sensors generally measure the change in surface stress or resonant frequency shifts to find the mass of target analytes.[1,2]

In the latter type of the cantilever sensors, it is commonly assumed that the target materials are uniformly distributed on the cantilever or they are positioned at the tip of the cantilever. If they are not placed at the tip, their positions on the cantilever need to be found because the resonant frequency shift due to the addition of mass changes with its position.[4] Although it is critical to find both the position and the mass for ultrasensitive and highly accurate detection, only a few studies have been conducted. An approximate solution based on the Rayleigh-Ritz method was obtained on a vibrating cantilever with attached beads[5,6] and a perturbation technique was used to find the functional relationship between the frequency response and the mass at a particular position.[7] These methods are based on the assumption that the mass ratio of the attached beads to the cantilever is very small.

In this paper, we present an alternative technique to simultaneously find the position and the mass of micro-beads attached on a cantilever using the transfer function method. When a bead is attached to a cantilever, its vibration characteristics change due to mass discontinuity. To monitor this change, the equivalent mass of the cantilever is measured from the piezoelectrically actuated vibration. The kinetic energy is used to calculate the sensitivity of the wave propagation to the bead position. Using the change of the equivalent mass and the sensitivity, the position and the mass can be identified in an exact manner regardless of the mass ratio of beads to the cantilever.

The equation of motion for a vibrating cantilever is $D\frac{\partial^4 w}{\partial x^4} + M_b \frac{\partial^2 w}{\partial t^2} = 0$, where D is the bending stiffness per unit length, M_b is the mass per unit length, and w is the deflection of the cantilever.[8] The internal damping of the cantilever reduces the vibration amplitude. To model the vibration dissipation, the complex stiffness is used as $\hat{D} = D(1 + \mathrm{i}\eta)$ where η is the loss factor and $\mathrm{i} = \sqrt{-1}$. Assuming harmonic vibration as $w(x,t) = \mathrm{Re}\{\hat{w}(x)\mathrm{e}^{\mathrm{i}\omega t}\}$, the cantilever vibration response can be expressed as

$$\hat{w}(x) = \hat{A}_1 \sin\hat{k}_b x + \hat{A}_2 \cos\hat{k}_b x + \hat{A}_3 e^{\hat{k}_b(x-L)} + \hat{A}_4 e^{-\hat{k}_b x}, \quad (1)$$

where $\hat{k}_b = (\omega^2 M_b/\hat{D})^{1/4}$ is the wavenumber. With the boundary conditions given as

$$\hat{w}(0) = w_0, \quad \frac{\partial \hat{w}(0)}{\partial x} = \frac{\partial^2 \hat{w}(L)}{\partial x^2} = \frac{\partial^3 \hat{w}(L)}{\partial x^3} = 0, \quad (2)$$

where w_0 is the displacement input from the piezo actuator and L is the cantilever length, the vibration response can be obtained. The transfer function is determined as

$$\Lambda e^{\mathrm{i}\phi} = \hat{w}(x_1)/w_0\,, \quad (3)$$

where Λ is the amplitude and ϕ is the phase of the transfer function. When the transfer function is measured experimentally, Eq. (3) is a function of the wavenumber, $\hat{k}_b$, and can be solved numerically.[9,10] The equivalent mass of the cantilever is consequently obtained as

$$\hat{M}_b' = \hat{k}_b{}^4 \hat{D}/\omega^2. \quad (4)$$

This equivalent mass increases with the bead attachments compared to that of the bare cantilever, M_b.

When a bead is attached at a position of large kinetic energy, the rate of increase in mass is significant. The sensitivity S of the equivalent mass to the bead position depends

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on the relative magnitude of the kinetic energy at the bead position to average kinetic energy of the cantilever and is computed as

$$S(x,\omega) = L\left(\frac{\partial w(x,\omega)}{\partial t}\right)^2 \Big/ \int_0^L \left(\frac{\partial w(x,\omega)}{\partial t}\right)^2 dx. \quad (5)$$

The position indicator is calculated using the correlation as follows:

$$\rho(x) = \mathrm{cov}(\Delta M'_b, S) \Big/ \sqrt{\mathrm{var}(\Delta M'_b)\mathrm{var}(S)}, \quad (6)$$

where $\Delta M'_b = \hat{M}'_b - M_b$. The maximum value of ρ shows the position of the beads, x_a.

To find the actual mass of the beads at the identified position, the cantilever response needs to be determined after taking into account of the bead attachment. The cantilever vibration response can then be given as

$$\begin{aligned}\hat{w}_1(x) = &\left[\hat{A}_1 \sin \hat{k}_b x + \hat{A}_2 \cos \hat{k}_b x + \hat{A}_3 e^{\hat{k}_b(x-x_a)} + \hat{A}_4 e^{-\hat{k}_b x}\right][1 - H(x-x_a)] \\ &+ \left[\hat{A}_5 \sin \hat{k}_b (x-x_a) + \hat{A}_6 \cos \hat{k}_b (x-x_a) + \hat{A}_7 e^{\hat{k}_b(x-L)} + \hat{A}_8 e^{-\hat{k}_b(x-x_a)}\right][H(x-x_a)], \quad (7)\end{aligned}$$

where $H(x)$ is the Heaviside step function. The effects of bead mass m can be analyzed with the boundary condition at $x = x_a$,

$$EI\left(\frac{\partial^3 \hat{w}_+}{\partial x^3} - \frac{\partial^3 \hat{w}_-}{\partial x^3}\right) = m\omega^2 \hat{w}. \quad (8)$$

In addition, free boundary conditions at $x = L$, displacement boundary conditions at $x = 0$, and continuity of displacements, slopes, and curvature at $x = x_a$ are applied. The predicted response is compared with the measured transfer function in Eq. (3) to find the mass of the bead.

Fig. 1 shows the schematic of the experimental setup. The cantilevers (ALOAL-TL, Probes, Korea) are 500 μm long, 30 μm wide, and 2.7 μm thick with an aluminum coating on the top. The fundamental frequency of the cantilevers is 15 kHz. Polystyrene micro-beads (72986, Sigma-Aldrich, UK) 10 μm in diameter were then placed on the cantilevers using a micro-positioner. Random noises with 10 MHz bandwidth generated by a function generator (33210A, Agilent, USA) were applied to the PZT (lead-zirconate-titanate) ($10 \times 10 \times 2$ mm, ITB Corporation, Korea) through a low-pass filter (cut-off frequency: 1 MHz). The vibration velocities of the cantilevers excited by the PZT were measured at $x = x_1$, where first four bending modes of vibration are measurable, via a single-point laser Doppler vibrometer (Polytec, Germany). The vibration velocities at $x = 0$ were also measured using the amplitude signal going into the PZT. These two velocities were measured before and after placing the beads.

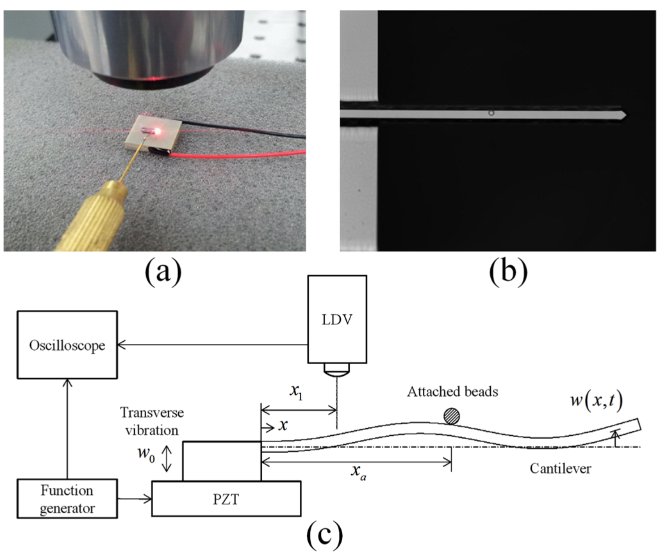

FIG. 1. (a) Image of a cantilever chip excited by the PZT. (b) Image of a 10 μm polystyrene micro-bead attached on a micro-cantilever. (c) A schematic of experimental set-up, which consists of a micro-cantilever, a single-point laser Doppler vibrometer, a function generator and PZT. The vibration velocity of the cantilever was measured at $x = x_1$ via the laser Doppler vibrometer.

Fig. 2(a) shows the resulting transfer functions when the cantilevers were excited by the random vibration input ($w_0 = 3.45$ mm/s rms) induced from the PZT. The measured responses of the cantilevers were compared with the predicted values, and there was an excellent agreement. The bending stiffness of the cantilevers without beads was also calculated. The measured resonance frequencies decreased with bead attachments. Figs. 2(b) and 2(c) shows the change in the obtained equivalent mass compared with the bare cantilever. From the bead attachment, the obtained equivalent mass increased and exhibited cyclic variation with frequency. The measured magnitude of mass variation agreed well with that of the sensitivity.

To determine the exact bead position, the position indicator ρ was obtained using Eq. (6) (Fig. 3(a)). The optically measured and predicted bead positions are shown in Table I and they have relative differences of 1.6–4.7%. After determining the bead position, the mass of the beads was identified (Fig. 3(b)). The obtained values showed frequency dependent variations. The predicted masses at 95% confidence interval were in a good agreement with actual masses in the selected frequency range (Table I).

In conclusion, we presented an experimental method to identify the position and the mass of beads attached to a cantilever. The effects of the beads on the cantilever kinetic energy were computed from the standing wave pattern and

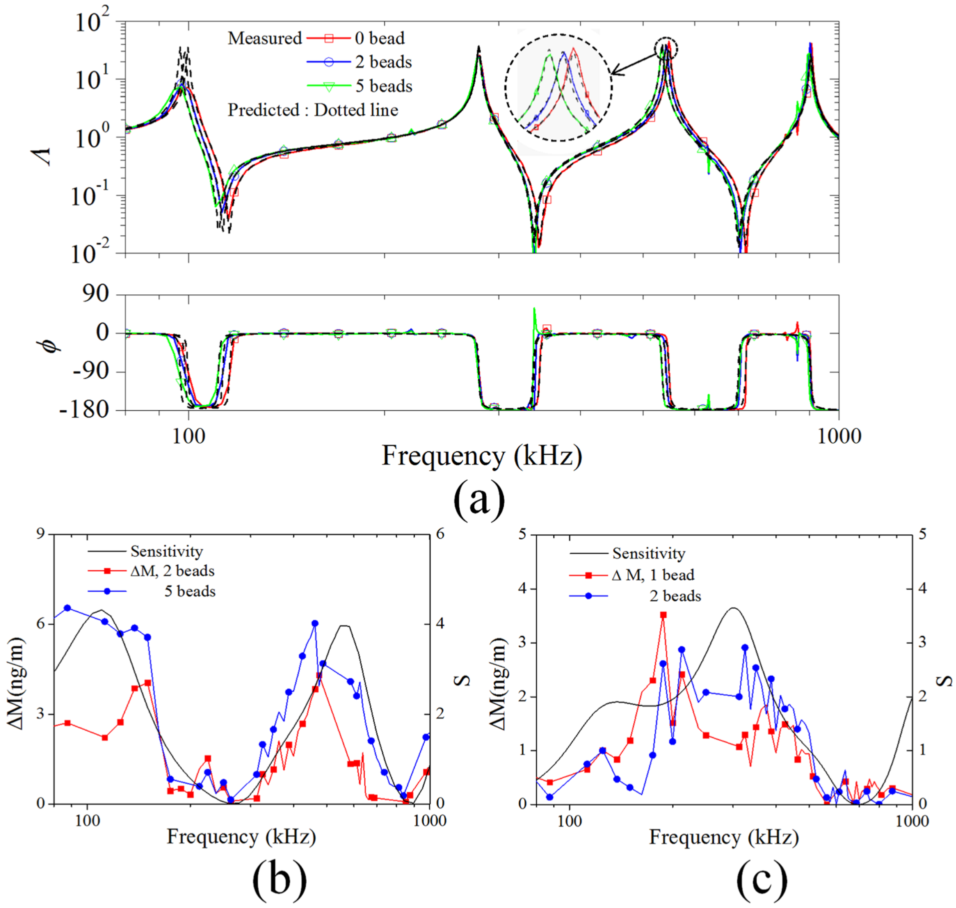

FIG. 2. (a) Comparison of predicted and measured transfer functions for vibration of the micro-cantilevers with/without the beads. Variation of the sensitivity with ΔM_b calculated using the vibration characteristics for (b) $x_a = 246\,\mu$m and (c) $x_a = 342\,\mu$m.

used to determine the bead position. After analyzing the transverse wave propagation near the beads, the mass was identified. The predicted values were in a close agreement with actual values. These results suggest that the proposed algorithm can provide an accurate methodology in identifying the positions and the masses of analytes attached on the cantilever sensors, particularly when the targets are cells, viruses, or bacteria.

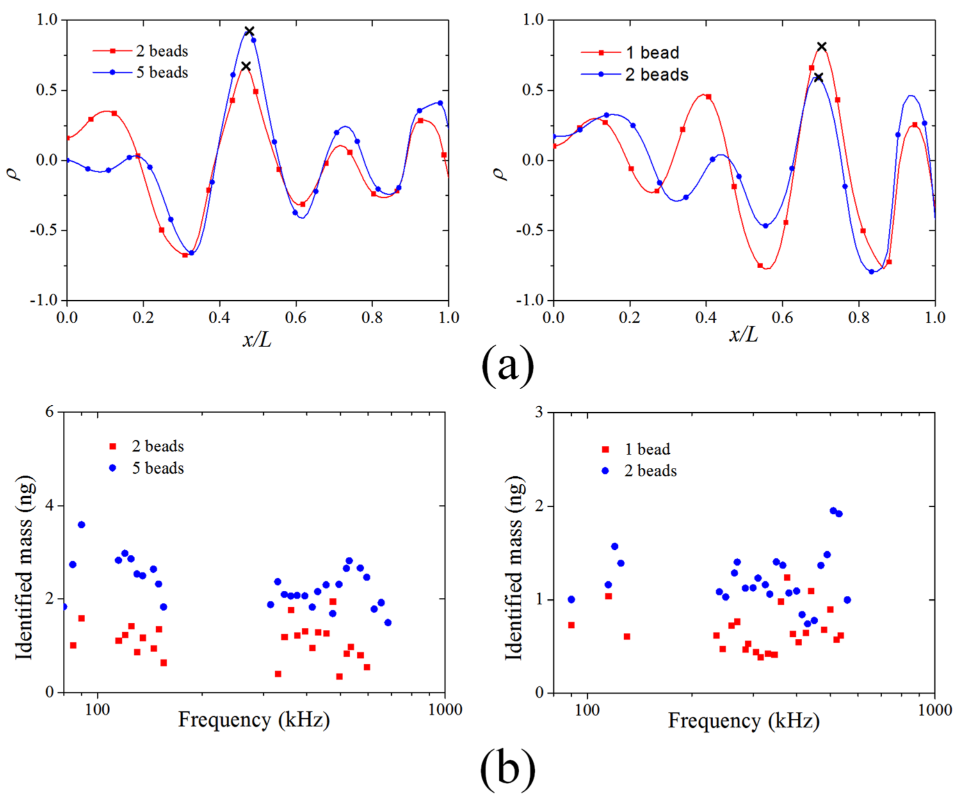

FIG. 3. (a) Position indicators calculated by using correlation between ΔM_b and the sensitivity function for beads located at (left) $x_a = 246\,\mu$m and (right) $x_a = 342\,\mu$m. (b) Experimentally obtained masses of the beads for (left) $x_a = 246\,\mu$m and (right) $x_a = 342\,\mu$m.

TABLE I. Identified position and mass of the beads for different positions and numbers of beads.

Measured values			Predicted values		
				Bead mass (ng)	
No. of beads	Bead position (μm)	Bead mass (ng)	Bead position (μm)	80 ~ 160 kHz	300 ~ 560 kHz
2	246	1.100 ± 0.066	242	1.213 ± 0.280	1.107 ± 0.134
5	246	2.750 ± 0.165	237	2.770 ± 0.487	2.219 ± 0.099
1	342	0.550 ± 0.033	358	0.792 ± 0.547	0.639 ± 0.085
2	342	1.100 ± 0.066	357	1.280 ± 0.397	1.211 ± 0.105

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