Key Substitution in the Symbolic Analysis of Cryptographic Protocols (extended version)

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Abstract. Key substitution vulnerable signature schemes are signature schemes that permit an intruder, given a public verification key and a signed message, to compute a pair of signature and verification keys such that the message appears to be signed with the new signature key. Schemes vulnerable to this attack thus permit an active intruder to claim to be the issuer of a signed message.
A digital signature scheme is said to be vulnerable to *destructive exclusive ownership property (DEO)* If it is computationaly feasible for an intruder, given a public verification key and a pair of message and its valid signature relatively to the given public key (m, s), to compute a pair of signature and verification keys and a new message m' such that s is a valid signature of m' relatively to the new verification key.
In this paper, we investigate and solve positively the problem of the decidability of symbolic cryptographic protocol analysis when the signature schemes employed in the concrete realisation have this two properties.

1 Introduction

According to *West's Encyclopedia of American Law*, a *signature* is

> "A mark or sign made by an individual on an instrument or document to signify knowledge, approval, acceptance, or obligation... [Its purpose] is to authenticate a writing, or *provide notice of its source*[1]..."

We will not deal any further with legal considerations, but it is interesting to note that while digital signatures are primarily employed to authenticate a document, *i.e.* ensure that the signer endorses the content of the document, they can also be employed to prove the origin of a document, *i.e.* ensure that only one person could have signed it. Indeed, most of the cryptographic work on digital signatures has aimed at certifying that no-one could sign a document in the place of someone else.

The analysis of digital signature primitives has however focused on the former authentication property. Formally speaking, the yardstick security notion for assessing the robustness of a digital signature scheme is the existential enforceability against adaptative chosen-message attacks (UNF-CCA) [10]. This

[1] We have emphasised

notion states that, given a signing key/verification key pair, it is infeasible for someone ignorant of the signing key to forge a message that can pass the verification with the public verification key, and this even when messages devised by the attacker are signed beforehand. The security goal provided by this property is the impossibility (within given computing bounds) to impersonate a legitimate user (*i.e.* one that does not reveal its signature key) when signing a message.

We note that this robustness does not address the issue of the identification of a source of a message. However, this latter concept is also pertaining to digital signatures when they are employed in a non-repudiation protocol. While one would not differentiate the two properties at first glance, they are different since the authentication property requires the existence of the participation of the signer in the creation of the message, while the latter mandates the unicity of a possible creator of a message.

The two notions of message authentication and source authentication collapse in the *single-user* setting when there exists only one pair of signature/verification keys. They may however be different in a *multi-user* setting. We believe that the first work in this direction was the discovery of a flaw on the Station-to-Station protocol by Blake-Wilson and Menezes [12], where the authors show how it is possible to confuse a participant into thinking it shares a key with another person than the actual one. The attack consisted in the creation, by the attacker, of a signature/verification key pair dependent upon messages sent in the protocol. Defining a signature scheme to have the Duplicate Signature Key Selection (DSKS) property if it permits such a construction with non-negligible probability, they showed that several standard signature schemes (including RSA, DSA, ECDSA and ElGamal) had this property, but also that a simple countermeasure (signing the public key along with the message) existed in all cases, but was rarely implemented. This DSKS property was formally defined as *Key substitution* in [2], where it is also discussed, after a review of what could be called an attack on a signature scheme in the multi-user setting. It was also later presented independently in [8] as *Conservative Exclusive Ownership*. The companion property of *Destructive Exclusive Ownership* by which an intruder may also change arbitrarily the signed message is also introduced and they showed that the usual signature algorithms (such as RSA and DSS) have this property. While the same attacks as in [2] are exhibited, the authors also demonstrate how this can be used in practice to poison a badly implemented PKI with fake CRLs (T. Pornin, personal communication).

Automated validation of security protocols. Cryptographic protocols have been applied to securing communications over an insecure network for many years. While these protocols rely on the robustness of the employed security primitives, their design is error-prone. This difficulty is reflected by the repeated discovery of logical flaws in proposed protocols, even under the assumption that cryptographic primitives were perfect. As an attempt to solve the problem, there has been a sustained effort to devise formal methods for specifying and verifying the security goals of protocols. Various symbolic approaches have been proposed to represent protocols and reason about them, and to attempt to verify security

properties such as confidentiality and authenticity, or to discover bugs. Such approaches include process algebra, model-checking, equational reasoning, constraint solving and resolution theorem-proving (e.g., [14,9,16,1]).

Our goal is to adapt the symbolic model of concrete cryptographic primitives in order to reflect inasmuch as possible their imperfections that could be used by an attacker to find a flaw on a protocol. The work described in this paper relies on the compositionality result obtained in [17] that permits us to abstract from other primitives and consider protocols that only involve a signature scheme having DSKS property (resp. vulnerable to DEO property).

Outline. In Section 2 we will present an attack by Baek *et al.* demonstrating how an actual intruder can use the DSKS property of a signature scheme to attack a protocol. We then describe in Section 3 the formalism in which we will analyse cryptographic protocols. In Section 4 we present how we model the possible actions of an intruder taking advantage of the DSKS property of a signature scheme and in section 5, we present how we model the possible actions of an intruder taking advantage of the vulnerability of signature scheme to DEO property. We present in Section 7 an algorithm that permits to reduce the analysis to an analysis in the empty equational theory, and give in Section 8 a decision procedure for the reachability problem in these protocols. We conclude in Section 9.

2 An example of attack

We do not present here the original attack on the station-to-station protocol, but one that we believe to be simpler, and given by *Baek et al.* [3] on the KAP-HY (*Key Agreement protocol*, proposed by *Hirosi* and *Yoshida* in [11]).

Presentation of the KAP-HY protocol. This protocol relies on a redundant signature scheme to provide key confirmation at the end of a key exchange. The signature of a message m by agent A is denoted $s_A(m)$. Abstracting the details of the Diffie-Hellman key construction with messages u_A and u_B, and of the signature scheme, the protocol reads as follows:

$$A \to B : u_A, A$$
$$B \to A : u_B, s_B(u_A), B$$
$$A \to B : s_A(s_B(u_A), u_B)$$

An unknown key share (UKS) attack on a key agreement protocol is an attack whereby two entities A and B participating in a key agreement protocol may end the protocol successfully, but with a wrong belief on who shares a key with who. In [3], *Baek et al.* showed that the redundant signature scheme employed in the KAP-HY protocol possesses the DSKS property, and elaborate on this to show that the KAP-HY is vulnerable to a UKS attack. In this attack, the intruder E waits that A initiates a session with him:

(1) $A \to E : u_A, A$ (2) $E \to A : u_B, s_B(u_A), E$
(1′) $E \to B : u_A, A$ (3) $A \to E : s_A(s_B(u_A), u_B)$
(2′) $B \to E(A) : u_B, s_B(u_A), B$ (3′) $E \to B : s_A(s_B(u_A), u_B)$

In this attack, the intruder E records, but passes unchanged, the first message, and initiates a session as A with B. It then intercepts the second message, and builds from the public key of B and from the message $s_B(u_A)$ a signature/verification key pair, and registers this key pair. E then passes the signature, but this time accompanied by its identity (2'). The main point is that when A checks the signature of the incoming message, it accepts it on the ground that it seems to originate from E. At the end of this execution, A believes that the key is shared with E whereas it is actually shared with B.

The computation of the new pair of keys (P_E, S_E) proceeds as follows. At the end of flow (2), the intruder knows the signature of u_A made by Bob using his public key, then, by using DSKS property of the used signature scheme, he creates the new pair of keys (P_E, S_E). The crucial point, common to all DSKS attacks, is the construction of a new key pair from a public verification key and from a signed message. We will model this operation with appropriate deduction rules, and prove that protocol analysis remains decidable.

3 Formal setting

3.1 Basic notions

We consider an infinite set of free constants C and an infinite set of variables $\mathcal{X}$. For any signature $\mathcal{G}$ (*i.e.* sets of function symbols not in C with arities) we denote $\mathrm{T}(\mathcal{G})$ (resp. $\mathrm{T}(\mathcal{G}, \mathcal{X})$) the set of terms over $\mathcal{G} \cup \mathrm{C}$ (resp. $\mathcal{G} \cup \mathrm{C} \cup \mathcal{X}$). The former is called the set of ground terms over $\mathcal{G}$, while the latter is simply called the set of terms over $\mathcal{G}$. The arity of a function symbol g is denoted by $\mathrm{ar}(g)$. Variables are denoted by x, y, terms are denoted by s, t, u, v, and finite sets of terms are written $E, F, ...$, and decorations thereof, respectively. We abbreviate $E \cup F$ by E, F, the union $E \cup \{t\}$ by E, t and $E \setminus \{t\}$ by $E \setminus t$. The *subterms* of a term t are denoted $\mathrm{Sub}(t)$ and are defined recursively as follows. If t is an atom (*i.e.* $t \in \mathcal{X} \cup \mathrm{C}$) then $\mathrm{Sub}(t) = \{t\}$. If $t = g(t_1, \ldots, t_n)$ then $\mathrm{Sub}(t) = \{t\} \cup \bigcup_{i=1}^{n} \mathrm{Sub}(t_i)$. The *positions* in a term t are sequences of integers defined recursively as follows, ε being the empty sequence representing the root position in t. We write $p \leq q$ to denote that the position p is a prefix of position q. If u is a subterm of t at position p and if $u = g(u_1, \ldots, u_n)$ then u_i is at position $p \cdot i$ in t for $i \in \{1, \ldots, n\}$. We write $t_{|p}$ the subterm of t at position p. We denote $t[s]$ a term t that admits s as subterm. The size $\|t\|$ of a term t is the number of distinct subterms of t. The notation is extended as expected to a set of terms.

A substitution σ is an involutive mapping from $\mathcal{X}$ to $\mathrm{T}(\mathcal{G}, \mathcal{X})$ such that $\mathrm{Supp}(\sigma) = \{x | \sigma(x) \neq x\}$, the *support* of σ, is a finite set. The application of a substitution σ to a term t (resp. a set of terms E) is denoted $t\sigma$ (resp. $E\sigma$) and is equal to the term t (resp. E) where all variables x have been replaced by the term $\sigma(x)$. A substitution σ is *ground* w.r.t. $\mathcal{G}$ if the image of $\mathrm{Supp}(\sigma)$ is included in $\mathrm{T}(\mathcal{G})$.

An *equational presentation* $\mathcal{H} = (\mathcal{G}, \mathcal{A})$ is defined by a set $\mathcal{A}$ of equations $u = v$ with $u, v \in \mathrm{T}(\mathcal{G}, \mathcal{X})$ and u, v without free constants. For any equational

presentation $\mathcal{H}$ the relation $=_{\mathcal{H}}$ denotes the equational theory *generated* by $(\mathcal{G}, \mathcal{A})$ on $\mathrm{T}(\mathcal{G}, \mathcal{X})$, that is the smallest congruence containing all instances of axioms of A. Abusively we shall not distinguish between an equational presentation $\mathcal{H}$ over a signature $\mathcal{G}$ and a set $\mathcal{A}$ of equations presenting it and we denote both by $\mathcal{H}$. If the equations of $\mathcal{A}$ can be oriented from left to right, we write the equations in $\mathcal{A}$ with an arrow, $l \to r$. The equations can then only be employed from left to right, and $\mathcal{A}$ is called a rewrite system. An equational theory can in this case be defined by a rewrite system. An equational theory $\mathcal{H}$ is said to be *consistent* if two free constants are not equal modulo $\mathcal{H}$ or, equivalently, if it has a model with more than one element modulo $\mathcal{H}$.

Let $\mathcal{A}$ be a set of rewrite rules $l \to r$. The rewriting relation $\to_{\mathcal{A}}$ between terms is defined by $t \to_{\mathcal{A}} t'$ if there exists $l \to r \in \mathcal{A}$ and a substitution σ such that $l\sigma = s$ and $r\sigma = s'$, $t = t[s]$ and $t' = t[s \leftarrow s']$. $\mathcal{A}$ is convergent if and only if it is terminating and confluent. In this case, all rewriting sequences starting from t are finite and have the same limit, and this limit is called the *normal form* of t. We denote this normal form $(t)\downarrow_{\mathcal{A}}$, or $(t)\downarrow$ when the considered rewriting system is clear from the context. A substitution σ is in normal form if for all $x \in \mathrm{Supp}(\sigma)$, the term $\sigma(x)$ is in normal form.

3.2 Unification systems

In the rest of this section, we let $\mathcal{H}$ be an equational theory on $\mathrm{T}(\mathcal{G}, \mathcal{X})$ and $\mathcal{A}$ be a *convergent* rewriting system generating $\mathcal{H}$.

Definition 1. *(Unification systems) Let $\mathcal{H}$ be an equational theory on $\mathrm{T}(\mathcal{G}, \mathcal{X})$. A* $\mathcal{H}$-unification system *$\mathcal{S}$ is a finite set of pairs of terms in $\mathrm{T}(\mathcal{G}, \mathcal{X})$ denoted by $\{u_i \stackrel{?}{=}_{\mathcal{H}} v_i\}_{i \in \{1,\dots,n\}}$. It is satisfied by a substitution σ, and we note $\sigma \models_{\mathcal{H}} \mathcal{S}$, if for all $i \in \{1, \dots, n\}$ we have $u_i\sigma =_{\mathcal{H}} v_i\sigma$. In this case we call σ a* solution *or a* unifier *of $\mathcal{S}$.*

When $\mathcal{H}$ is generated by $\mathcal{A}$, the confluence implies that if σ is a solution of a $\mathcal{H}$-unification system, then $(\sigma)\downarrow$ is also a solution of the same unification system. Accordingly we will consider in this paper only solutions in normal form of unification systems. A *complete set of unifiers* of a $\mathcal{H}$-unification system $\mathcal{S}$ is a set Σ of substitutions such that, for any solution τ of $\mathcal{S}$, there exists $\sigma \in \Sigma$ and a substitution τ' such that $\tau =_{\mathcal{H}} \sigma\tau'$. The unifier τ is a *most general unifier* of $\mathcal{S}$ if the substitution τ' in the preceding equation must be a variable renaming.

In the context of unification modulo an equational theory, standard (or syntactic) unification will also be called unification in the empty theory. In this case, it is well-known that there exists a unique most general unifier of a set of equations. This unifier is denoted $mgu(\mathcal{S})$, or $mgu(s, t)$ in the case $\mathcal{S} = \left\{ s \stackrel{?}{=}_{\emptyset} t \right\}$.

Unifiability Problem

Input: A $\mathcal{H}$-unification system $\mathcal{S}$.
Output: SAT iff there exists a substitution σ such that $\sigma \models_{\mathcal{H}} \mathcal{S}$.

Let us now introduce the notion of narrowing, that informally permits to instantiate and rewrite a term in a single step.

Definition 2. *(Narrowing) Let s and t be two terms. We say $t \rightsquigarrow s$ iff there exists $l \to r \in \mathcal{A}$, a position p such that $t_{|p} \notin \mathcal{X}$ and $s = t\sigma[p \leftarrow r\sigma]$, where $\sigma = mgu(t_{|p}, l)$. We denote by $\rightsquigarrow$ the narrowing relation.*

Assume $t \rightsquigarrow t'$ with a rule $l \to r$ applied at a position p in t. A basic position in t' is either a non-variable position of t not under p or a position $p \cdot q$ where q is a non-variable position in r. Basic narrowing is a restricted form of narrowing where only terms at basic positions are considered to be narrowed. In the rest of this paper, we denote $t \rightsquigarrow_{\text{b.n.}} t'$ a basic narrowing step.

3.3 Intruder deduction systems

The notions that we give here have been defined in [17]. These definitions have since been generalised to consider a wider class of intruder deduction systems and constraint systems [15]. Although this general class encompasses all intruder deduction systems and constraint systems given in this paper, we have preferred to give the simpler definitions from [17] which are sufficient for stating our problem. We will refer, without further justifications, to the model of [15] as *extended* intruder systems and *extended* constraint systems. The latter correspond to symbolic derivations in which a most general unifier of the unification system has been applied on the input/output messages.

In the context of a security protocol (see *e.g.* [5] for a brief overview), we model messages as ground terms and intruder deduction rules as rewrite rules on sets of messages representing the knowledge of an intruder. The intruder derives new messages from a given (finite) set of messages by applying deduction rules. Since we assume some equational axioms $\mathcal{H}$ are satisfied by the function symbols in the signature, all these derivations have to be considered *modulo* the equational congruence $=_{\mathcal{H}}$ generated by these axioms. In the setting of [17] an intruder deduction rule is specified by a term t in some signature $\mathcal{G}$. Given values for the variables of t the intruder is able to generate the corresponding instance of t.

Definition 3. *An* intruder system $\mathcal{I}$ *is given by a triple $\langle \mathcal{G}, S, \mathcal{H} \rangle$ where $\mathcal{G}$ is a signature, $S \subseteq \mathrm{T}(\mathcal{G}, \mathcal{X})$ and $\mathcal{H}$ is a set of equations between terms in $\mathrm{T}(\mathcal{G}, \mathcal{X})$. To each $t \in S$ we associate a* deduction rule $\mathrm{L}^t : \mathrm{Var}(t) \twoheadrightarrow t$ *. The set of rules $\mathrm{L}_{\mathcal{I}}$ is defined as the union of L^t for all $t \in S$.*

Each rule $l \twoheadrightarrow r$ in $\mathrm{L}_{\mathcal{I}}$ defines an intruder deduction relation $\twoheadrightarrow_{l \twoheadrightarrow r}$ between finite sets of terms. Given two finite sets of terms E and F we define $E \twoheadrightarrow_{l \twoheadrightarrow r} F$ if and only if there exits a substitution σ, such that $l\sigma =_{\mathcal{H}} l'$, $r\sigma =_{\mathcal{H}} r'$, $l' \subseteq E$ and $F = E \cup \{r'\}$. We denote $\twoheadrightarrow_{\mathcal{I}}$ the union of the relations $\twoheadrightarrow_{l \twoheadrightarrow r}$ for all $l \twoheadrightarrow r$ in $L_{\mathcal{I}}$ and by $\twoheadrightarrow^*_{\mathcal{I}}$ the transitive closure of $\twoheadrightarrow_{\mathcal{I}}$. Note that by definition, given sets of terms E, E', F and F' such that $E =_{\mathcal{H}} E'$ and $F =_{\mathcal{H}} F'$ by definition

we have $E \twoheadrightarrow_{\mathcal{I}} F$ iff $E' \twoheadrightarrow_{\mathcal{I}} F'$. We simply denote by $\twoheadrightarrow$ the relation $\twoheadrightarrow_{\mathcal{I}}$ when there is no ambiguity about $\mathcal{I}$.

A *derivation* D of length n, $n \geq 0$, is a sequence of steps of the form $E_0 \twoheadrightarrow_{\mathcal{I}} E_0, t_1 \twoheadrightarrow_{\mathcal{I}} \cdots \twoheadrightarrow_{\mathcal{I}} E_n$ with finite sets of terms $E_0, \ldots E_n$, and terms $t_1, \ldots, t_n$, such that $E_i = E_{i-1} \cup \{t_i\}$ for every $i \in \{1, \ldots, n\}$. The term t_n is called the *goal* of the derivation. We define $\overline{E}^{\mathcal{I}}$ to be equal to the set of terms that can be derived from E. If there is no ambiguity on the intruder deduction system $\mathcal{I}$ we write $\overline{E}$ instead of $\overline{E}^{\mathcal{I}}$.

3.4 Simultaneous constraint satisfaction problems

We now introduce the constraint systems to be solved for checking protocols. It is presented in [17] how these constraint systems permit to express the reachability of a state in a protocol execution.

Definition 4. *($\mathcal{I}$-Constraint systems) Let $\mathcal{I} = \langle \mathcal{G}, S, \mathcal{H} \rangle$ be an intruder system. An* $\mathcal{I}$-constraint system *$\mathcal{C}$ is denoted $((E_i \rhd v_i)_{i \in \{1,\ldots,n\}}, \mathcal{S})$ and is defined by a sequence of pairs $(E_i, v_i)_{i \in \{1,\ldots,n\}}$ with $v_i \in \mathcal{X}$, $E_i \subseteq \mathrm{T}(\mathcal{G}, \mathcal{X})$ for $i \in \{1, \ldots, n\}$, and $E_{i-1} \subseteq E_i$ for $i \in \{2, \ldots, n\}$, and $\mathrm{Var}(E_i) \subseteq \{v_1, \ldots, v_{i-1}\}$ and by an $\mathcal{H}$-unification system $\mathcal{S}$.*

An $\mathcal{I}$-Constraint system *$\mathcal{C}$ is satisfied by a substitution σ if for all $i \in \{1, \ldots, n\}$ we have $v_i\sigma \in \overline{E_i\sigma}^{\mathcal{I}}$ and if $\sigma \models_{\mathcal{H}} \mathcal{S}$. We denote that a substitution σ satisfies a constraint system $\mathcal{C}$ by $\sigma \models_{\mathcal{I}} \mathcal{C}$.*

Constraint systems are denoted by $\mathcal{C}$ and decorations thereof. Note that if a substitution σ is a solution of a constraint system $\mathcal{C}$, by definition of deduction rules and unification systems the substitution $(\sigma)\downarrow$ is also a solution of $\mathcal{C}$. In the context of cryptographic protocols the inclusion $E_{i-1} \subseteq E_i$ means that the knowledge of an intruder does not decrease as the protocol progresses: after receiving a message a honest agent will respond to it, this response can then be added to the knowledge of the intruder who listens to all communications. The condition on variables stems from the fact that a message sent at some step i must be built from previously received messages recorded in the variables $v_j, j < i$, and from the ground initial knowledge of the honest agents.

Our goal will be to solve the following decision problem for the intruder deduction system modelling a signature scheme having the DSKS property.

$\mathcal{I}$-Reachability Problem

Input: An $\mathcal{I}$-constraint system $\mathcal{C}$.
Output: SAT iff there exists a substitution σ such that $\sigma \models_{\mathcal{I}} \mathcal{C}$.

4 Symbolic model for key substitution attacks

A digital signature scheme is defined by three algorithms: the signing algorithm, the verification algorithm and the key generation algorithm. The last algorithm

generates for each user a pair of keys, one of them will be used as signing key and will be kept secret, while the other is public and will be used as a verifying key. We abstract the key generation algorithm with two functions, PK(_) and SK(_) denoting respectively the verification and signature keys of an agent. We assume it is not possible, given an agent's name A, to *compute* $\mathrm{PK}(A)$ or $\mathrm{SK}(A)$. The signature of a message m with signature key k is a public algorithm $\mathrm{Sig}(\_,\_)$, and the resulting signed message is denoted $\mathrm{Sig}(m,k)$. We consider signatures with appendix, where the verification algorithm $\mathrm{Ver}(\_,\_,\_)$ –which is available to everyone– takes in its input a message m, a signature s and the public verification key k. The application of the algorithm is denoted $\mathrm{Ver}(m,s,k)$, and its outcome can be 0 (s is not the signature of m with the signature key associated with the verification key k) or 1 (s is a valid signature).

In addition to these functions, we add two new functions, $\mathrm{P'K}(\_,\_)$ and $\mathrm{S'K}(\_,\_)$, which are public and take as argument a signed message s and a verification key k corresponding to this signed message, and output respectively a verification and a signature key denoted $\mathrm{P'K}(s,k)$ and $\mathrm{S'K}(s,k)$. The verification of s with the verification key $\mathrm{P'K}(s,k)$ succeeds.

Given this informal description, the equational theory $\mathcal{H}_{\mathcal{DSKS}}$ to which these operations abide by is presented by the following set $\mathcal{A}_{\mathcal{DSKS}}$ of equations:

$$\mathcal{A}_{\mathcal{DSKS}} = \begin{cases} \mathrm{Ver}(x,\mathrm{Sig}(x,\mathrm{SK}(y)),\mathrm{PK}(y)) = 1 \\ \mathrm{Ver}(x,\mathrm{Sig}(x,\mathrm{S'K}(y_1,y_2)),\mathrm{P'K}(y_1,y_2)) = 1 \\ \mathrm{Sig}(x,\mathrm{S'K}(\mathrm{PK}(y),\mathrm{Sig}(x,\mathrm{SK}(y)))) = \mathrm{Sig}(x,\mathrm{SK}(y)) \end{cases}$$

The public operations defined above are now translated into an intruder system $\mathcal{I}_{\mathrm{DSKS}} = \langle \mathcal{G}_{\mathcal{DSKS}}, \mathcal{L}_{\mathcal{DSKS}}, \mathcal{H}_{\mathcal{DSKS}} \rangle$ with:

$$\begin{cases} \mathcal{G}_{\mathcal{DSKS}} = \{\mathrm{Sig},\mathrm{Ver},\mathrm{S'K}(\_,\_),\mathrm{P'K}(\_,\_),0,1,\mathrm{SK},\mathrm{PK}\} \\ \mathcal{L}_{\mathcal{DSKS}} = \{\mathrm{Sig}(x,y),\mathrm{Ver}(x,y,z),\mathrm{S'K}(x,y),\mathrm{P'K}(x,y),0,1\} \end{cases}$$

Note that the presentation $\mathcal{A}_{\mathcal{DSKS}}$ is not convergent, and thus we cannot apply results on basic narrowing as is. To this end we introduce a rewriting system $\mathcal{R}_{\mathcal{DSKS}}$ which is convergent and obtained by Knuth-Bendix [6] completion on $\mathcal{A}_{\mathcal{DSKS}}$, and such that two terms have the same normal form for $\mathcal{R}_{\mathcal{DSKS}}$ iff they are equal modulo $\mathcal{H}_{\mathcal{DSKS}}$.

Lemma 1. *$\mathcal{H}_{\mathcal{DSKS}}$ is generated by the convergent rewriting system:*

$$\mathcal{R}_{\mathcal{DSKS}} = \begin{cases} \mathrm{Ver}(x,\mathrm{Sig}(x,\mathrm{SK}(y)),\mathrm{PK}(y)) \to 1 \\ \mathrm{Ver}(x,\mathrm{Sig}(x,\mathrm{S'K}(y_1,y_2)),\mathrm{P'K}(y_1,y_2)) \to 1 \\ \mathrm{Ver}(x,\mathrm{Sig}(x,\mathrm{SK}(y)),\mathrm{P'K}(\mathrm{PK}(y),\mathrm{Sig}(x,\mathrm{SK}(y)))) \to 1 \\ \mathrm{Sig}(x,\mathrm{S'K}(\mathrm{PK}(y),\mathrm{Sig}(x,\mathrm{SK}(y)))) \to \mathrm{Sig}(x,\mathrm{SK}(y)) \end{cases}$$

Proof. The application of the *Knuth-Bendix* completion procedure [6] to $\mathcal{H}_{\mathcal{DSKS}}$ gives us the convergent rewriting system $\mathcal{R}_{\mathcal{DSKS}}$. This rewriting system generates $\mathcal{H}_{DSKS}$, this conclude the proof.

□

5 Symbolic model for DEO attacks

A digital signature scheme is vulnerable against *destructive exclusive ownership (DEO)* if it is computationaly feasible for the intruder, given K_{pub} and a pair (m, s) such that $\mathrm{Ver}(K_{pub}, m, s) = 1$, to produce values K'_{pub}, K'_{priv}, m' and s' such that $K'_{pub} \neq K_{pub}$, K'_{priv} matches K'_{pub}, $s' = s$, $m' \neq m$ and $\mathrm{Ver}(K'_{pub}, m', s) = 1$.

A digital signature scheme is defined by three algorithms: the signing algorithm, the verification algorithm and the key generation algorithm. These algorithms have the same properties as above (section 4). We abstract the signature scheme with the following functions symbols: PK(_), SK(_), Sig(_, _) and Ver(_, _, _). In order to model DEO attacks, we introduce three functions symbols, P"K(_, _), S"K(_, _) and f(_, _) which are public and take as argument a signed message s and a public verification key k_{pub} corresponding to this signed message, and output respectively a new verification key, a new signature key and a new message denoted $\mathrm{P"K}(s, k)$, $\mathrm{S"K}(s, k_{pub})$ and $\mathrm{f}(s, k_{pub})$. The verification of s with the new public key $\mathrm{P"K}(s, k_{pub})$ and the message $\mathrm{f}(s, k_{pub})$ succeeds.

Given this informal description, the equational theory $\mathcal{H}_{\mathcal{DEO}}$ to which these operations abide by is presented by the following set $\mathcal{A}_{\mathcal{DEO}}$ of equations:

$$\mathcal{A}_{\mathcal{DEO}} = \begin{cases} \mathrm{Ver}(x, \mathrm{Sig}(x, \mathrm{SK}(y)), \mathrm{PK}(y)) = 1 \\ \mathrm{Ver}(x, \mathrm{Sig}(x, \mathrm{S"K}(y_1, y_2)), \mathrm{P"K}(y_1, y_2)) = 1 \\ \mathrm{Sig}(\mathrm{f}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))), \mathrm{S"K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y)))) = \mathrm{Sig}(x, \mathrm{SK}(y)) \end{cases}$$

The public operations defined above are now translated into an intruder system $\mathcal{I}_{\text{DEO}} = \langle \mathcal{G}_{\mathcal{DEO}}, \mathcal{L}_{\mathcal{DEO}}, \mathcal{H}_{\mathcal{DEO}} \rangle$ with:

$$\begin{cases} \mathcal{G}_{\mathcal{DEO}} = \{\mathrm{Sig}, \mathrm{Ver}, \mathrm{S"K}(\_, \_), \mathrm{P"K}(\_, \_), \mathrm{f}, 0, 1, \mathrm{SK}, \mathrm{PK}\} \\ \mathcal{L}_{\mathcal{DEO}} = \{\mathrm{Sig}(x, y), \mathrm{Ver}(x, y, z), \mathrm{S"K}(x, y), \mathrm{P"K}(x, y), \mathrm{f}(x, y), 0, 1\} \end{cases}$$

Note that the presentation $\mathcal{A}_{\mathcal{DEO}}$ is not convergent, and thus we cannot apply results on basic narrowing as is. To this end we introduce a rewriting system $\mathcal{R}_{DEO}$ which is convergent and obtained by Knuth-Bendix [6] completion on $\mathcal{A}_{\mathcal{DEO}}$, and such that two terms have the same normal form for $\mathcal{R}_{DEO}$ iff they are equal modulo $\mathcal{H}_{\mathcal{DEO}}$.

Lemma 2. *$\mathcal{H}_{\mathcal{DEO}}$ is generated by the convergent rewriting system:*

$$\mathcal{R}_{DEO} = \begin{cases} \mathrm{Ver}(x, \mathrm{Sig}(x, \mathrm{SK}(y)), \mathrm{PK}(y)) \to 1 \\ \mathrm{Ver}(x, \mathrm{Sig}(x, \mathrm{S"K}(y_1, y_2)), \mathrm{P"K}(y_1, y_2)) \to 1 \\ \mathrm{Ver}(\mathrm{f}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))), \mathrm{Sig}(x, \mathrm{SK}(y)), \mathrm{P"K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y)))) \to 1 \\ \mathrm{Sig}(\mathrm{f}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))), \mathrm{S"K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y)))) \to \mathrm{Sig}(x, \mathrm{SK}(y)) \end{cases}$$

6 Decidability of unifiability

It can easily be shown, using the criterion of termination of basic narrowing on the right-hand side of rules of $\mathcal{R}_{DSKS}$ (resp. $\mathcal{R}_{DEO}$), that basic narrowing

terminates when applied with the rules of $\mathcal{R}_{DSKS}$ (resp. the rules of $\mathcal{R}_{DEO}$). The main result of [7] then implies the following proposition, when applying basic narrowing with $\mathcal{R}_{DSKS}$ (resp. $\mathcal{R}_{DEO}$) non-deterministically on the two sides of an equation modulo $\mathcal{R}_{DSKS}$ (resp. $\mathcal{R}_{DEO}$) and terminates with unification modulo the empty theory.

Proposition 1. *Basic narrowing is a sound, complete and terminating procedure for finding a complete set of most general $\mathcal{H}_{\mathcal{DSKS}}$-unifiers (resp. $\mathcal{H}_{\mathcal{DEO}}$ unifiers).*

One can actually be more precise, and we will need the following direct consequence of Hullot's unification procedure, that states that applying basic narrowing permits one to "guess" partially the normal form of a term t.

Lemma 3. *Let t be any term and σ be a normalised substitution. There exists a term t' and a substitution σ' in normal form such that $t \rightsquigarrow^*_{\text{b.n.}} t'$ and $t'\sigma' = (t\sigma)\downarrow$ where $\rightsquigarrow_{\text{b.n.}}$ represent a basic narrowing relation modulo $\mathcal{H}_{\mathcal{DSKS}}$ (resp. modulo $\mathcal{H}_{\mathcal{DEO}}$).*

While this presentation by a convergent rewrite system ensures the decidability of unification modulo $\mathcal{H}_{\mathcal{DSKS}}$ (resp. modulo $\mathcal{H}_{\mathcal{DEO}}$), we prove bellow that the unifiability problem, as well as the partial guess of a normal form, is in fact in NPTIME.

Complexity of unification

Theorem 1. *Let t be a term and (D) be a basic narrowing derivation modulo $\mathcal{H}_{\mathcal{DSKS}}$ (resp. modulo $\mathcal{H}_{\mathcal{DEO}}$) starting from t. Then, the length of (D) in bounded by $\|t\|$.*

Proof. Let us prove the theorem for the basic narrowing derivations modulo $\mathcal{H}_{\mathcal{DSKS}}$. Let t be a term and D be a basic narrowing derivation starting from t, $D : t = t_0 \rightsquigarrow_{b.n} t_1 \rightsquigarrow_{b.n} \ldots \rightsquigarrow_{b.n} t_n$. $\mathcal{R}_{DSKS}$ is convergent and any basic narrowing derivation starting from the right members of the rules of $\mathcal{R}_{DSKS}$ terminates, then, by [7], (D) terminates. Let us prove that $\|D\| \leq \|t\|$. Let $P_0 = \mathrm{P}(t_0)$ be the number of distinct subterms of t_0 where we can apply the basic narrowing. We note that if the basic narrowing can be applied on a term s at a position p and if there exists another subterm of s at position q such that $t_{|p} = t_{|q}$, we apply the basic narrowing at the positions p and q at the same time. Let $t_i \rightsquigarrow_{b.n} t_{i+1}$ be a step in (D) and let $l_i \to r_i \in \mathcal{R}_{DSKS}$ be the applied rule. For any $l \to r \in \mathcal{R}_{DSKS}$, r is not narrowable. By the fact that r_i is not narrowable and by the definition of basic narrowing [7], we have $P_{i+1} < P_i$. We deduce that $\|D\| \leq P_0$, but $P_0 \leq \|t_0\|$ then $\|D\| \leq \|t\|$.

The case of derivations modulo $\mathcal{H}_{\mathcal{DEO}}$ is analogoes. □

Corollary 1. *The $\mathcal{H}_{\mathcal{DSKS}}$-unifiability (resp. $\mathcal{H}_{\mathcal{DEO}}$-unifiability) can be decided in NPTIME.*

PROOF. Let us prove the corollary for $\mathcal{H}_{\mathcal{DSKS}}$-unifiability. Let P and Q be two terms. $\mathcal{R}_{DSKS}$ is convergent and any basic narrowing derivation starting from the right members of the rules of $\mathcal{R}_{DSKS}$ terminates then, there exists an $\mathcal{H}_{DSKS}$-unification algorithm (proposition 1). Let us prove that this algorithm runs in NPtime. Suppose $M = H(P, Q)$, (H is a new function symbol representing the cartesian product), and $m = \|M\| = \|P\| + \|Q\| + 1$. For any basic narrowing derivation D starting from M, we have $\|D\| \leq m$ (Theorem 1). Suppose that our algorithm always explore the right branch, then, starting from any term M, the algorithm will be perform at most $\|M\|$ steps before halting. Then, we have the corollary.

By the same reasoning, we prove that $\mathcal{H}_{\mathcal{DEO}}$-unifiability can be decided in NPTIME. □

7 Saturation

7.1 Construction

Let $\mathcal{H}$ be an equational theory presented by a convergent rewrite system $\mathcal{R}$. The saturation of the set of deduction rules $\mathcal{L}$ defined modulo the equational theory $\mathcal{H}$ is the output of the application of the saturation rules of Figure 1 starting with $\mathcal{L}' = \mathcal{L}$ until any added rule is subsumed by a rule already present in $\mathcal{L}'$.

$$\textsf{Subsumption}: \quad \frac{l_1 \twoheadrightarrow r \in \mathcal{L}' \quad l_2 \twoheadrightarrow r \in \mathcal{L}'}{\mathcal{L}' \leftarrow \mathcal{L}' \setminus \{l_2 \twoheadrightarrow r\}} \; l_1 \subseteq l_2$$

$$\textsf{Closure}: \quad \frac{l_1 \twoheadrightarrow r_1 \in \mathcal{L}', \quad (t, l_2) \twoheadrightarrow r_2 \in \mathcal{L}'}{\mathcal{L}' \leftarrow \mathcal{L}' \cup \{(l_1, l_2 \twoheadrightarrow r_2)\sigma\}} \; \begin{array}{c} t \notin \mathcal{X} \\ \sigma = mgu_{\emptyset}(r_1, t) \end{array}$$

$$\textsf{Narrow}: \quad \frac{l \twoheadrightarrow r \in \mathcal{L}' \quad (l, r) \rightsquigarrow_{\text{b.n.}} (l', r')}{\mathcal{L}' \leftarrow \mathcal{L}' \cup \{l' \twoheadrightarrow r'\}}$$

Fig. 1. System of saturation rules.

The application of the saturation rules on $\mathcal{L}_{\mathcal{DSKS}}$ (resp. on $\mathcal{L}_{\mathcal{DEO}}$) terminates, and yields the following sets of rules:
$\mathcal{L}_{\mathcal{DSKS}}{}' = \mathcal{L}_{\mathcal{DSKS}} \cup x, \mathrm{SK}(y) \twoheadrightarrow \mathrm{Sig}(x, \mathrm{SK}(y)) \cup x, \mathrm{S'K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))) \twoheadrightarrow \mathrm{Sig}(x, \mathrm{SK}(y))$ and
$\mathcal{L}_{\mathcal{DEO}}{}' = \mathcal{L}_{\mathcal{DEO}} \cup \mathrm{f}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))), \mathrm{S"K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))) \twoheadrightarrow \mathrm{Sig}(x, \mathrm{SK}(y))$

We define four new *extended* intruder systems: $\mathcal{I}'_{\text{DSKS}} = \langle \mathcal{G}_{\mathcal{DSKS}}, \mathcal{L}_{\mathcal{DSKS}}{}', \mathcal{H}_{DSKS} \rangle$, $\mathcal{I}_{\emptyset 1} = \langle \mathcal{G}_{\mathcal{DSKS}}, \mathcal{L}_{\mathcal{DSKS}}{}', \emptyset \rangle$, $\mathcal{I}'_{\text{DEO}} = \langle \mathcal{G}_{\mathcal{DEO}}, \mathcal{L}_{\mathcal{DEO}}{}', \mathcal{H}_{DEO} \rangle$ and $\mathcal{I}_{\emptyset 2} = \langle \mathcal{G}_{\mathcal{DEO}}, \mathcal{L}_{\mathcal{DEO}}{}', \emptyset \rangle$. These intruder systems do not satisfy the requirements that the left-hand side of deduction rules have to be variables. The deduction relation, the derivations and the set of reachable terms are defined as usual from ground instances of deduction rules.

7.2 Properties of a saturated system

In the rest of this paper, we suppose $\mathcal{H}$, $\mathcal{R}$, $\mathcal{L}$, $\mathcal{L}$', $\mathcal{I} = \langle \mathcal{G}, \mathcal{L}, \mathcal{H} \rangle$, $\mathcal{I}' = \langle \mathcal{G}, \mathcal{L}', \mathcal{H} \rangle$ and $\mathcal{I}_\emptyset = \langle \mathcal{G}, \mathcal{L}', \emptyset \rangle$ to be either respectively $\mathcal{H}_{\mathcal{DSKS}}$, $\mathcal{R}_{\mathcal{DSKS}}$, $\mathcal{L}_{\mathcal{DSKS}}$, $\mathcal{L}'_{\mathcal{DSKS}}$, $\mathcal{I}_{\text{DSKS}}$, $\mathcal{I}'_{\text{DSKS}}$or respectively $\mathcal{H}_{\mathcal{DEO}}$, $\mathcal{R}_{\mathcal{DEO}}$, $\mathcal{L}_{\mathcal{DEO}}$, $\mathcal{L}'_{\mathcal{DEO}}$, $\mathcal{I}_{\text{DEO}}$, $\mathcal{I}'_{\text{DEO}}$.

Let us first prove that the deduction system obtained after saturation gives exactly the same deductive power to an intruder.

Lemma 4. *For any set of normal ground terms E and any normal ground term t we have: $E \twoheadrightarrow^*_{\mathcal{I}} t$ if and only if $E \twoheadrightarrow^*_{\mathcal{I}'} t$.*

Proof. First, let us assume that $t \in \overline{E}^{\mathcal{I}}$, that is, there exists a $\mathcal{I}$-derivation (D) starting from E of goal t, and let us prove that there exists a $\mathcal{I}'$-derivation starting from E of goal t. If there exists a step in the derivation D which uses a rule $l \twoheadrightarrow r \in \mathcal{L}$ but not in $\mathcal{L}'$, then, by construction of $\mathcal{L}'$, there exists another rule $l_1 \twoheadrightarrow r$ in $\mathcal{L}'$ such that $l_1 \subseteq l$ and thus that can be applied instead of $l \twoheadrightarrow r$. We conclude that $E \twoheadrightarrow^*_{\mathcal{I}'} t$.

For the reciprocal, let us assume that there exists a $\mathcal{I}'$-derivation starting from E of goal t, and let us prove that there exists a $\mathcal{I}$-derivation starting from E of goal t. We begin by defining an arbitrary order on the rules of $\mathcal{L}$, and we extend this order to the rules of $\mathcal{L}' \setminus \mathcal{L}$ as follows: the rules of $\mathcal{L}$ are smaller than the rules of $\mathcal{L}' \setminus \mathcal{L}$ and the rules of $\mathcal{L}' \setminus \mathcal{L}$ are ordered according to the order of their construction during the saturation. Let $\mathrm{M}(D)$ be the multiset of deduction rules applied in D. Let $\Omega(E,t) = \{D \mid D : E \twoheadrightarrow^* t\}$. Since $t \in \overline{E}^{\mathcal{I}'}$, $\Omega(E,t) \neq \emptyset$. Let D be a derivation in $\Omega(E,t)$ having the minimal $\mathrm{M}(D)$, and let us prove that D does not use rules in $\mathcal{L}' \setminus \mathcal{L}$. By contradiction, suppose that D uses a rule $l \twoheadrightarrow r \in \mathcal{L}' \setminus \mathcal{L}$. Since $l \twoheadrightarrow r \notin \mathcal{L}$, it has been constructed according to the rules of saturation. Let us review the possible cases:

- If $l \twoheadrightarrow r$ has been constructed by the third rule of saturation, there exists a rule $l_1 \twoheadrightarrow r_1 \in \mathcal{L}'$ such that $(l_1, r_1) \rightsquigarrow^*_{b.n} (l, r)$. By definition of deductions, $l_1 \twoheadrightarrow r_1$ can be applied instead of $l \twoheadrightarrow r$. Let (D') be the derivation where $l' \twoheadrightarrow r'$ replaces $l \twoheadrightarrow r$, (D') is in $\Omega(E,t)$. Since $l_1 \twoheadrightarrow r_1$ has an order smaller than the order of $l \twoheadrightarrow r$, we have $\mathrm{M}(D') < \mathrm{M}(D)$, which contradicts the minimality of $\mathrm{M}(D)$.
- If $l \twoheadrightarrow r$ has been constructed by the second rule of saturation, there exists two rules $l_1 \twoheadrightarrow r_1$ and $s, l_2 \twoheadrightarrow r_2$ in $\mathcal{L}'$ such that $\mu = mgu(r_1, s)$, $s \notin \mathcal{X}$, $l = ((l_1, l_2)\mu)\downarrow$ and $r = (r_2\mu)\downarrow$. suppose that $l \twoheadrightarrow r$ is applied on the set of terms F, $F \twoheadrightarrow_{l \twoheadrightarrow r} F, g$. Since $(l\sigma)\downarrow \subseteq F$ and $(r\sigma)\downarrow = g$ for a substitution σ, we have $(l_1\mu\sigma)\downarrow \subseteq F$ and $((s, l_2)\mu\sigma)\downarrow \subseteq F \cup (r_1\mu\sigma)\downarrow$, this implies that $F \twoheadrightarrow_{l_1 \twoheadrightarrow r_1} F, (r_1\mu\sigma)\downarrow \twoheadrightarrow_{l_2, s \twoheadrightarrow r_2} F, (r_1\mu\sigma)\downarrow, g$. Let (D') be the derivation where $l_1 \twoheadrightarrow r_1$ and $s, l_2 \twoheadrightarrow r_2$ replace $l \twoheadrightarrow r$. (D') is in $\Omega(E,t)$. Since $l_1 \twoheadrightarrow r_1$ and $s, l_2 \twoheadrightarrow r_2$ have an order smaller than the order of $l \twoheadrightarrow r$, we have $\mathrm{M}(D') < \mathrm{M}(D)$ which contradicts the minimality of $\mathrm{M}(D)$.

We conclude that (D) does not use rules in $\mathcal{L}' \setminus \mathcal{L}$, then, we have the reciprocal of the lemma. □

Moreover, we can prove that when considering only deductions on terms in normal form and yielding terms in normal form, it is sufficient to consider derivations modulo the empty theory (Corollary 2).

Lemma 5. *Let E (resp. t) be a set of terms (resp. a term) in normal form. We have: $E \twoheadrightarrow_{\mathcal{I}'} E, t$ if and only if $E \twoheadrightarrow_{\mathcal{I}_\emptyset} E, t$.*

Proof. Assume first $E \twoheadrightarrow_{\mathcal{I}'} E, t$. There exists a rule $l \twoheadrightarrow r \in \mathcal{L}'$ and a substitution σ in normal form such that $(l\sigma)\downarrow \subseteq E$ and $t = (r\sigma)\downarrow$. By Lemma 3, there exists a set of terms l', r', and a substitution σ' in normal form such that $l \rightsquigarrow^*_{\text{b.n.}} l'$, $r \rightsquigarrow^*_{\text{b.n.}} r'$, and $(l\sigma)\downarrow = l'\sigma'$, and $(r\sigma)\downarrow = r'\sigma'$. By the saturation, we have added at some point $l' \twoheadrightarrow r'$ to $\mathcal{L}'$. Either this rule is present in the final $\mathcal{L}$' and can be applied, or it is subsumed by a rule that can be applied on E. The converse is left to the reader. □

Corollary 2. *Let E (resp. t) be a set of terms (resp. a term) in normal form. We have: $E \twoheadrightarrow^*_{\mathcal{I}'} E, t$ if and only if $E \twoheadrightarrow^*_{\mathcal{I}_\emptyset} E, t$.*

Next lemma states that if a term in the left-hand side of a deduction rule of the saturated system is not a variable, then we can assume it is not the result of another saturated deduction rule.

Lemma 6. *Let E (resp. t) be a set of terms (resp. a term) in normal form. If $t \in \overline{E}^{\mathcal{I}_\emptyset}$, then there exists a $\mathcal{I}_\emptyset$-derivation starting from E of goal t such that: for all $\mathcal{I}_\emptyset$ rules $l \twoheadrightarrow r$ applied with substitution σ, for all $s \in l \setminus \mathcal{X}$, we have $s\sigma \subseteq E$.*

Proof. Let us prove by induction on the length n of a derivation D starting from E of goal t that either D satisfies the property or their exists another $\mathcal{I}_\emptyset$-derivation D' of length smaller to n starting from E of goal t which satisfies the property.
The case $n = 1$ is obvious.
Suppose that the lemma is true for derivations of length $\leq n$ and let us prove it for derivations D of length $n + 1$.
$D : E = E_0 \twoheadrightarrow^{i-1} E_{i-1} \twoheadrightarrow E_{i-1}, t_i \twoheadrightarrow \ldots \twoheadrightarrow E_n \twoheadrightarrow E_n, t$. Suppose that D does not satisfy the property, there exists a step i in D where the rule $l \twoheadrightarrow r$ is applied with the substitution σ, and there exists $s \in l \setminus \mathcal{X}$ such that $s\sigma \notin E$. Since $s\sigma \notin E$, it has been constructed at some step $j < i$. We have:
$D : E \twoheadrightarrow^{j-1} E_{j-1} \twoheadrightarrow E_{j-1}, s\sigma \twoheadrightarrow \ldots \twoheadrightarrow E_i \twoheadrightarrow E_i, t_i \twoheadrightarrow \ldots \twoheadrightarrow E_n \twoheadrightarrow E_n, t$.
Let $l_j \twoheadrightarrow r_j \in \mathcal{L}'$ be the rule applied, with the substitution τ, to construct $s\sigma$. Since $r_j\tau = s\sigma$, r_j and s are unifiable with $\mu = mgu(r_j, s)$. Then the rule $(l_j, l \setminus s \twoheadrightarrow r)\mu$ has been constructed. Since μ is a substitution most general than σ, it can be applied on E_i to yield t_i. This implies that we can reduce D to:
$D' : E \twoheadrightarrow^{j} E_j \twoheadrightarrow E_{j+2} \twoheadrightarrow \ldots \twoheadrightarrow E_i \twoheadrightarrow E_i, t_i \twoheadrightarrow \ldots \twoheadrightarrow E_n \twoheadrightarrow E_n, t$ where the construction of $s\sigma$ is spell and the applied rule at the step i is $(l_j, l \setminus s)\mu \twoheadrightarrow r\mu$. We note that $\|D'\| < \|D\|$, then $\|D'\| \leq n$. By induction, either D' satisfies the property or there exists another $\mathcal{I}_\emptyset$-derivation D'' starting from E of goal t which satisfies the property. Then, we have the lemma for derivations of length n+1 and this concludes the proof. □

8 Decidability of reachability

The main result of this paper is the following theorem.

Theorem 1 *The $\mathcal{I}_{\text{DSKS}}$-Reachability (resp. $\mathcal{I}_{\text{DEO}}$-Reachability) problem is decidable.*

The rest of this paper is devoted to the presentation of an algorithm for solving $\mathcal{I}_{\text{DSKS}}$-Reachability (resp. $\mathcal{I}_{\text{DEO}}$-Reachability) problems and to a proof scheme of its completeness, correctness and termination. This decision procedure comprises three different steps.

Let $\mathcal{C}$ be an $\mathcal{I}$-constraint system.

8.1 First step: guess of a normal form

Step 1. Apply non-deterministically basic narrowing steps on all subterms of $\mathcal{C}$. Let $\mathcal{C}_0 = \left\{(E_i^0 \rhd v_i^0)_{i\in\{1,\ldots,n\}}, \mathcal{S}^0\right\}$ be the resulting constraint system.

Remark. Let σ be a solution of the original constraint system, with σ in normal form. This first step will non-deterministically transform each $t \in \text{Sub}(\mathcal{C})$ into a term t' such that, according to Lemma 3 we will have $(t\sigma)\!\downarrow = t'\sigma'$.

8.2 Second step: resolution of unification problems

Step 2. Solve the unification system $\mathcal{S}^0$ modulo the empty theory, and apply the obtained unifier on the deduction constraints to obtain a constraint system $\mathcal{C}' = \left\{(E_i' \rhd t_i')_{i\in\{1,\ldots,n\}}\right\}$

Remarks. We prove below that if there exists a solution to the original constraint system, then there exists a solution of $\mathcal{C}'$ for the extended intruder system $\mathcal{I}_\emptyset$. $\mathcal{C}$' itself is not a constraint system, but an *extended* constraint system.

Lemma 7. *If σ is a substitution in normal form such that $\sigma \models_{\mathcal{I}} \mathcal{C}$, there exists a $\mathcal{C}'$ at Step 2 and a substitution σ' in normal form such that $\mathcal{C} \rightsquigarrow^*_{\text{b.n.}} \mathcal{C}'$ and $\sigma' \models_{\mathcal{I}_\emptyset} \mathcal{C}'$.*

Proof. By definition $\sigma \models_{\mathcal{I}} \mathcal{C}$ implies that for all $i \in \{1,\ldots,n\}$ we have $\sigma \models_{\mathcal{I}} (E_i \rhd t_i)$. Thus there exists by Lemma 4 an $\mathcal{I}'$-derivation starting from $(E_i\sigma)\!\downarrow$ to $(t_i\sigma)\!\downarrow$. Since σ is in normal form, by lemma 3, there exists E_i', t_i' and σ' in normal form such that $E_i \rightsquigarrow^*_{\text{b.n.}} E_i'$, $t_i \rightsquigarrow^*_{\text{b.n.}} t_i'$, $(E_i\sigma)\!\downarrow = E_i'\sigma'$ and $(t_i\sigma)\!\downarrow = t_i'\sigma'$ for all $i \in \{1,\ldots,n\}$. By Lemma 5 $(E_i\sigma)\!\downarrow \rightarrow^*_{\mathcal{I}'} (t_i\sigma)\!\downarrow$ then implies $(E_i\sigma)\!\downarrow \rightarrow^*_{\mathcal{I}_\emptyset} (t_i\sigma)\!\downarrow$ Since $(E_i\sigma)\!\downarrow = E_i'\sigma'$ and $(t_i\sigma)\!\downarrow = t_i'\sigma'$ then, $\sigma' \models_{\mathcal{I}_\emptyset} (E_i' \rhd t_i')$ for all $i \in \{1,\ldots,n\}$ and thus we have the lemma. □

$$\textsf{Apply}: \quad \frac{\mathcal{C}_\alpha, E \rhd t, \mathcal{C}_\beta}{(\mathcal{C}_\alpha, (E \rhd y)_{y \in l_x}, \mathcal{C}_\beta)\sigma} \begin{array}{c} l_x, l_1, \ldots, l_n \twoheadrightarrow r \in \mathcal{L}' \text{ and } l_x \subseteq \mathcal{X}, t \notin \mathcal{X} \\ e_1, \ldots, e_n \in E \text{ and } \sigma = mgu(\left\{(e_i \stackrel{?}{=} l_i)_i, r \stackrel{?}{=} t\right\}) \end{array}$$

$$\textsf{Unif}: \quad \frac{\mathcal{C}_\alpha, E \rhd t, \mathcal{C}_\beta}{(\mathcal{C}_\alpha, \mathcal{C}_\beta)\sigma} \begin{array}{c} u, t \notin \mathcal{X} \\ u \in E,\ \sigma = mgu(u, t) \end{array}$$

Fig. 2. System of transformation rules.

8.3 Third step: Transformation in solved form

Step 3. To simplify the constraint system, we apply the transformation rules of Figure 2. Our goal is to transform $\mathcal{C}$' into a constraint system such that the right-hand sides of deduction constraints (the t_i) are all variables. When this is the case, we say that the constraint system is in *solved form*. It is routine to check that a constraint system in solved form is satisfiable.

Lemma 8. *Let $\mathcal{C} = \{\mathcal{C}_\alpha, E \rhd t, \mathcal{C}_\beta\}$ be such that $\mathcal{C}_\alpha$ is in solved form. Then, for all substitution σ, $\sigma \models \mathcal{C}$ if and only if $\sigma \models \{\mathcal{C}_\alpha, (E \setminus \mathcal{X}) \rhd t, \mathcal{C}_\beta\}$.*

Proof. It suffices to prove that if $x \in E \cap \mathcal{X}$ and σ is a substitution such that $\sigma \models \mathcal{C}$, then we have $\sigma \models \{\mathcal{C}_\alpha, (E \setminus \{x\}) \rhd t, \mathcal{C}_\beta\}$. Given $x \in E$ there exists a set of terms $E_x \subseteq E$ such that $E_x \rhd x \in \mathcal{C}_\alpha$. Since $\sigma \models \mathcal{C}$ we have $\sigma \models E_x \rhd x$, and by the fact that $E_x \subseteq E \setminus \{x\}$ we have $\sigma \models E \setminus \{x\} \rhd x$. Since we also have $\sigma \models (E \rhd t)$ this implies $\sigma \models E \setminus \{x\} \rhd t$. The reciprocal is obvious since $E \setminus \{x\} \subseteq E$. □

It also can be proved that the lazy constraint solving procedure terminates. This lemma also helps us to prove the completeness of lazy constraint solving (stated in Lemma 11).

Lemma 9. *(DSKS-termination.) Let $\mathcal{C}$ be an $\mathcal{I}_{DSKS}$-constraint system. The application of transformation rules of the* algorithm *using $\mathcal{L}'_{\mathcal{DSKS}}$ rules terminates.*

Proof. Let $\text{nbv}(\mathcal{C}) = |\text{Var}(\mathcal{C})|$ be the number of variables in $\mathcal{C}$, and $\mathcal{M}(\mathcal{C})$ denote the multiset of the right-hand side of deduction constraints in $\mathcal{C}$. Let us prove that after any application of a transformation rule on a constraint system $\mathcal{C} = (\mathcal{C}_\alpha, E \rhd t)$ (where $\mathcal{C}_\alpha$ is in solved form), either $\text{nbv}(\mathcal{C})$ decreases strictly, or the identity substitution is applied on $\mathcal{C}$ during the transformation and $\mathcal{M}(\mathcal{C})$ strictly decreases.

The first point will ensure that after some point in a sequence of transformations the number of variables will be stable, and thus from this point on $\mathcal{M}(\mathcal{C})$ will strictly decrease. The fact that no more unification will be applied and that the extension of the subterm ordering on multisets is well-founded will then imply that there is only a finite sequence of different constraint systems, and thereby the termination of the constraint solving algorithm.

This fact is obvious if the Unif rule is applied, since it amounts to the unification of two subterms of $\mathcal{C}$. It is then well-known that if the two subterms

are not syntactically equal, the number of variables in their most general unifier is strictly less than the union of their variables, which is included in $\mathrm{Var}(\mathcal{C})$. If they are syntactically equal, then no substitution is applied, and thus denoting $\mathcal{C}$' the result of the transformation, we have $\mathcal{M}(\mathcal{C}) = \mathcal{M}(\mathcal{C}') \cup \{t\}$, and thus $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$.

Let us now consider the case of the Apply rule, and let $\mathcal{C}$' be the obtained constraint system. If the underlying intruder deduction rule is in $\mathcal{L}_{\mathcal{DSKS}}$, the fact that t is not a variable implies that the variables of the right-hand side of the rule will be instantiated by the strict maximal subterms $t_1, \ldots, t_k$ of t. We will thus have:

$$\mathcal{M}(\mathcal{C}') = \mathcal{M}(\mathcal{C}) \cup \{t_1, \ldots, t_n\} \setminus \{t\}$$

and thus $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$.

It now suffices to prove the Lemma for the two rules in $\mathcal{L}'_{\mathcal{DSKS}} \setminus \mathcal{L}_{\mathcal{DSKS}}$:

rule $x, \mathrm{SK}(y) \twoheadrightarrow \mathrm{Sig}(x, \mathrm{SK}(y))$**:** The substitution σ applied is the most general unifier of the unification system $\left\{\mathrm{Sig}(x, \mathrm{SK}(y)) \stackrel{?}{=} t, \mathrm{SK}(y) \stackrel{?}{=} u\right\}$ for some $u \in E$. Since this is syntactic unification and since we can assume neither u (by Lemma 8) nor t (by definition of the Apply rule) are variables, we must have $u = \mathrm{SK}(u')$ and $t = \mathrm{Sig}(t_1, t_2)$. The second equation thus yields $y = u'$, with $u' \in \mathrm{Sub}(\mathcal{C})$. Replacing in the first equation, σ is the most general unifier of the equation $\mathrm{Sig}(x, \mathrm{SK}(u')) \stackrel{?}{=} \mathrm{Sig}(t_1, t_2)$, which reduces into the set of equations $\left\{x \stackrel{?}{=} t_1, \mathrm{SK}(u') \stackrel{?}{=} t_2\right\}$. The first equation implies that x is instantiated by a strict subterm t_1 of t. If the second equation is trivial we have $\mathcal{M}(\mathcal{C}') = \mathcal{M}(\mathcal{C}) \cup \{t_1\} \setminus \{t\}$, and thus $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$. Otherwise, since $\mathrm{Var}(\mathrm{SK}(u')) \cup \mathrm{Var}(t_2) \subseteq \mathrm{Var}(\mathcal{C})$ we have $\mathrm{nbv}(\mathcal{C}') < \mathrm{nbv}(\mathcal{C})$.

rule $x, \mathrm{S'K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))) \twoheadrightarrow \mathrm{Sig}(x, \mathrm{SK}(y))$**:** The substitution σ applied is the most general unifier of the unification system $\left\{\mathrm{Sig}(x, \mathrm{SK}(y)) \stackrel{?}{=} t, \mathrm{S'K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))) \stackrel{?}{=} u\right\}$ for some $u \in E$. Since this is syntactic unification and since we can assume neither u (by Lemma 8) nor t (by definition of the Apply rule) are variables, we must have $u = \mathrm{S'K}(u'_1, u'_2)$ and $t = \mathrm{Sig}(t_1, t_2)$. If σ is the identity on $\mathcal{C}$, we are done, since in this case we have $\mathcal{M}(\mathcal{C}') = \mathcal{M}(\mathcal{C}) \cup \{t_1\} \setminus \{t\}$ and thus $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$. Otherwise let us examine how the unification system is solved. It is first transformed into:

$$\left\{x \stackrel{?}{=} t_1, \mathrm{SK}(y) \stackrel{?}{=} t_2, \mathrm{PK}(y) \stackrel{?}{=} u'_1, \mathrm{Sig}(x, \mathrm{SK}(y)) \stackrel{?}{=} u'_2\right\}$$

Resolving the first equation yields (note that $x \notin \mathrm{Var}(\mathcal{C})$):

$$\left\{\mathrm{SK}(y) \stackrel{?}{=} t_2, \mathrm{PK}(y) \stackrel{?}{=} u'_1, \mathrm{Sig}(t_1, \mathrm{SK}(y)) \stackrel{?}{=} u'_2\right\}$$

Let us consider two cases, depending on whether both u'_1 and t_2 are variables:

- If they are both variables, then solving the first equation removes t_2 from $\mathrm{Var}(\mathcal{C})$ but adds a variable y. The second equation will also remove

u'_1, but since the variable y is already present, it will not add another variable. Since $\mathrm{PK}(y)$ and $\mathrm{SK}(y)$ are not unifiable, we note that we must have $t_2 \neq u'_1$, and thus we have removed two variables and added one by solving the two first equations. The remaining equation contains only variables of the "intermediate" constraint system, and thus will not add any new variable. In conclusion, in this case, the number of variables of $\mathcal{C}$ decreases by at least 1.

- If say t_2 is not a variable, and thus $t_2 = \mathrm{SK}(t'_2)$, with $t'_2 \in \mathrm{Sub}(\mathcal{C})$. Resolving the first equation and injecting the solution in the remaining equations yields the unification system:

$$\left\{\mathrm{PK}(t'_2) \stackrel{?}{=} u'_1, \mathrm{Sig}(t_1, \mathrm{SK}(t'_2)) \stackrel{?}{=} u'_2\right\}$$

  Note that up to this point the substitution σ that we built does not affect any variable of $\mathcal{C}$. If this remaining unification system is trivial, then the substitution applied on $\mathcal{C}$ is the identity, we are done (see above). Otherwise, since all the variables in this system are in $\mathrm{Var}(\mathcal{C})$, it strictly reduces $\mathrm{nbv}(\mathcal{C})$. This terminates the proof of this case.

Thus, if this rule is applied, either no substitution is applied on $\mathcal{C}$ and $\mathcal{M}(\mathcal{C})$ strictly decreases, or the number of variables in the resulting constraint system $\mathcal{C}$' is strictly smaller than the number of variables in $\mathcal{C}$.

□

Lemma 10. *(DEO-termination.) Let $\mathcal{C}$ be an $\mathcal{I}_{DEO}$-constraint system. The application of transformation rules of the* algorithm *using $\mathcal{L}_{\mathcal{DEO}'}$ rules terminates.*

Proof. Let $\mathcal{C} = \left\{(E_i \rhd t_i)_{i\in\{1,\ldots,n\}}\right\}$ be an $\mathcal{I}_{DEO}$-constraint system not in solved form and let the complexity of $\mathcal{C}$ be a couple ordered lexicographically with the following components:

- $\mathrm{nbv}(\mathcal{C})$, the number of distinct variables in $\mathcal{C}$,
- $\mathcal{M}(\mathcal{C})$ the multiset of the right-hand side of deduction constraints in $\mathcal{C}$.

We have to show that each rule reduces the complexity. The fact is obvious if the $Unif$ rule is applied, since it amounts to the unification of two subterms of $\mathcal{C}$. If is then well-known that if two subterms are not syntactically equal, then the number of variables in their most general unifier is strictly less than the union of their variables, which is included in $\mathrm{Var}(\mathcal{C})$. If their are syntactically equal, then no substitution is applied, and thus denoting $\mathcal{C}$' the result of transformation, we have $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$.

Let us now consider the case of *Apply* rule, and let $\mathcal{C}$' be the obtained constraint system. If the underlying intruder deduction rule is in $\mathcal{L}_{DEO}$, the fact that t is not a variable implies that the right-hand side of the rule will be instantiated by the strict maximal subterms $t_1, \ldots, t_k$ of t. we will thus have $\mathcal{M}(\mathcal{C}') = \mathcal{M}(\mathcal{C}) \cup \{t_1, \ldots, t_k\} \setminus \{t\}$ and thus $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$.

It is now suffices to prove the Lemma for the rule in $\mathcal{L}_{\mathcal{DEO}'} \setminus \mathcal{L}_{\mathcal{DEO}}$:

the applied rule is: $\mathrm{f}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))), \mathrm{S"K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))) \twoheadrightarrow \mathrm{Sig}(x, \mathrm{SK}(y))$. The substitution σ is the most general unifier of the unification system $\left\{t \stackrel{?}{=} \mathrm{Sig}(x, \mathrm{SK}(y)), e_1 \stackrel{?}{=} \mathrm{f}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y))), e_2 \stackrel{?}{=} \mathrm{S"K}(\mathrm{PK}(y), \mathrm{Sig}(x, \mathrm{SK}(y)))\right\}$ for some $e_1,\ e_2 \in E$. since it is syntactic unification and since we can assume neither e_1, neither e_2 (by Lemma 8) nor t (by definition of the Apply rule) are variables, we must have $t = \mathrm{Sig}(t_1, t_2),\ e_1 = \mathrm{f}(v_1, v_2)$, and $e_2 = \mathrm{S"K}(v_3, v_4)$.

- If $t_2 \in \mathcal{X}$, we have $\sigma(x) = t_1,\ \sigma(t_2) = \mathrm{SK}(y)$ and the unification system is then transformed into:
  $\left\{v_1 \stackrel{?}{=} \mathrm{PK}(y), v_2 \stackrel{?}{=} \mathrm{Sig}(t_1, \mathrm{SK}(y)), v_3 \stackrel{?}{=} \mathrm{PK}(y), v_4 \stackrel{?}{=} \mathrm{Sig}(t_1, \mathrm{SK}(y))\right\}$.
  By the fact that t_3 is replaced by y, $x, y \notin \mathrm{Var}(\mathcal{C})$, and the number of variables in σ is strictly less than the union of variables of the unification system, we deduce that $\mathrm{nbv}(\mathcal{C}') < \mathrm{nbv}(\mathcal{C})$.
- If $t_2 \notin \mathcal{X}$ then $t_2 = \mathrm{SK}(t_3)$. We have $\sigma(x) = t_1,\ \sigma y = t_3$ and the unification system is then transformed into:
  $\left\{v_1 \stackrel{?}{=} \mathrm{PK}(t_3), v_2 \stackrel{?}{=} \mathrm{Sig}(t_1, \mathrm{SK}(t_3)), v_3 \stackrel{?}{=} \mathrm{PK}(t_3), v_4 \stackrel{?}{=} \mathrm{Sig}(t_1, \mathrm{SK}(t_3))\right\}$.
  If the unification system is obvious, that is σ is the identity substitution, we have $\mathcal{C}' = \mathcal{C} \setminus (E \rhd t)$, and then $\mathcal{M}(\mathcal{C}') = \mathcal{M}(\mathcal{C}) \setminus t$ which implies that $\mathcal{M}(\mathcal{C}') < \mathcal{M}(\mathcal{C})$. Else, we have $\mathrm{nbv}(\mathcal{C}') < \mathrm{nbv}(\mathcal{C})$.

This concludes the proof. □

Lemma 11. *If $\mathcal{C}'$ is satisfied by a substitution σ', it can be transformed into a system in solved form by the rules of Figure 2.*

Proof. Let $\mathcal{C}$ be a deterministic constraint system not in solved form and let i be the smallest integer such that $t_i \notin \mathcal{X}$, then $\mathcal{C} = \{\mathcal{C}_\alpha, E_i \rhd t_i, \mathcal{C}_\beta\}$ where $\mathcal{C}_\alpha$ is in solved form. Let σ be a substitution such that $\sigma \models_{\mathcal{I}_\emptyset} \mathcal{C}$, and let us prove that $\mathcal{C}$ can be reduced to another satisfiable constraint system $\mathcal{C}'$ by applying the transformation rules given in the algorithm. $\sigma \models_{\mathcal{I}_\emptyset} \mathcal{C}$, then $\sigma \models_{\mathcal{I}_\emptyset} \{\mathcal{C}_\alpha, E_i \setminus \mathcal{X} \rhd t_i, \mathcal{C}_\beta\}$ (Lemma 8) and then, $(E_i \setminus \mathcal{X})\sigma \to^*_{\mathcal{I}_\emptyset} t_i\sigma$. We have two cases:

- If $t_i\sigma \in (E_i \setminus \mathcal{X})\sigma$, there exists a term $u \in (E_i \setminus \mathcal{X})$ such that $u\sigma = t_i\sigma$. Let μ be the most general unifier of u and t_i, then $\sigma = \theta\mu$, and we can simplify $\mathcal{C}$ by applying the first transformation rule Unif, $\mathcal{C} \Longrightarrow \mathcal{C}' = \{\mathcal{C}_\alpha\mu, \mathcal{C}_\beta\mu\}$. We have $\sigma \models_{\mathcal{I}_\emptyset} \mathcal{C}_\alpha$ and $\sigma \models_{\mathcal{I}_\emptyset} \mathcal{C}_\beta$, then $\theta \models_{\mathcal{I}_\emptyset} \{\mathcal{C}_\alpha\mu, \mathcal{C}_\beta\mu\}$.
- If $t_i\sigma \notin (E_i \setminus \mathcal{X})\sigma$ there exists a derivation starting from $(E_i \setminus \mathcal{X})\sigma$ of goal $t_i\sigma$, and then from $E_i\sigma$ of goal $t_i\sigma$. By lemma 6, there exists a derivation starting from $E_i\sigma$ of goal $t_i\sigma$ such that for all steps in the derivation such that $l \to r$ is the applied rule with the substitution σ, for all $s \in l$ and $s \notin \mathcal{X}$, we have $s\sigma \subseteq E_i\sigma$. This implies that we can reduce $\mathcal{C}$ to $\mathcal{C}'$ by applying the Apply rule of transformation and $\theta \models_{\mathcal{I}_\emptyset} \mathcal{C}'$.

We deduce that for all satisfiable constraint systems $\mathcal{C}$ such that $\mathcal{C}$ is not in solved form, $\mathcal{C}$ can be reduced to another satisfiable constraint system $\mathcal{C}'$ by applying the transformation rules. When applying the transformation rules to a constraint system, we reduce its complexity (Lemmas 9 and 10), this implies that when we

reduce $\mathcal{C}$, we will obtain at some step a satisfiable constraint system which can not be reducible, this constraint system is in solved form. This concludes the proof. □

Lemma 12. *(Correctness.) Let* $\mathcal{C} = \{(E_i \rhd t_i)_{i\in\{1,\ldots,n\}}\}$ *and* $\mathcal{C}' = \{(E'_i \rhd t'_i)_{i\in\{1,\ldots,n\}}\}$ *such that* $\mathcal{C}'$ *is obtained by applying the basic-narrowing on the terms of* $\mathcal{C}$*. For every substitution* σ' *such that* $\sigma' \models_{\mathcal{I}_\emptyset} \mathcal{C}'$*, there exists a substitution* σ *such that* $\sigma \models_{\mathcal{I}} \mathcal{C}$*.*

Proof. We have $\mathcal{C} = \{(E_i \rhd t_i)_{i\in\{1,\ldots,n\}}\}$, $\mathcal{C} \rightsquigarrow^*_{b.n} \mathcal{C}'$ and $\mathcal{C}' = \{(E'_i \rhd t'_i)_{i\in\{1,\ldots,n\}}\}$. Let θ be the composition of substitutions applied in the basic-narrowing derivation, for all $i \in \{1,\ldots,n\}$ we have $(E_i\theta)\downarrow = E'_i$ and $(t_i\theta)\downarrow = t'_i$. Let σ' be a substitution such that $\sigma' \models_{\mathcal{I}_\emptyset} \mathcal{C}'$, for all $i \in \{1,\ldots,n\}$ we have $t'_i\sigma' \in \overline{E'_i\sigma'}^{\mathcal{I}_\emptyset}$, this implies that for all $i \in \{1,\ldots,n\}$ $t'_i\sigma' \in \overline{E'_i\sigma'}^{\mathcal{I}'}$ (Corollary 2), and then, for all $i \in \{1,\ldots,n\}$ $t'_i\sigma' \in \overline{E'_i\sigma'}^{\mathcal{I}}$ (Lemma 4). From the fact that $(E_i\theta_n)\downarrow = E'_i$ and $(t_i\theta_n)\downarrow = t'_i$ for all $i \in \{1,\ldots,n\}$, we deduce that $t_i\theta\sigma' \in \overline{E_i\theta\sigma'}^{\mathcal{I}}$ for all $i \in \{1,\ldots,n\}$ and this conclude the proof. □

9 Conclusion

Besides the actual decidability result obtained in this paper, we believe that the techniques developed to obtain this result, while still at an early stage, are promising and of equal importance. Several recent work [4,13] have proposed conditions on intruder systems ensuring the decidability of reachability with respect to an active or passive intruder. In a future work we plan to research whether the given conditions imply the termination of the saturation procedure and the termination of the symbolic resolution.

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