# About periodicity and signal to noise ratio - The strength of the autocorrelation function.

Nadine Martin, Corinne Mailhes

# About periodicity and signal to noise ratio - The strength of the autocorrelation function

Nadine Martin
GIPSA-lab, INPG/CNRS
Department Images Signal
Grenoble Campus
BP 46 - 961, rue de la Houille Blanche
F-38402 Saint Martin d'Hères – France
Phone 33 (0)4 76 82 62 69
Fax 33 (0)4 76 57 47 90
nadine.martin@gipsa-lab.grenoble-inp.fr

Corinne Mailhes
IRIT-TESA, INP – ENSEEIHT
2, rue Charles Camichel
B.P. 7122
31071 Toulouse Cedex 7 – France

**Abstract**

In condition monitoring a part of the information necessary for decision-making comes from scrutinizing a time measure or a transform of this measure. Frequency domain is commonly exploited; lag domain is not, albeit advantages of the autocorrelation function have long been known. In this paper, we dwell on the autocorrelation function in order to extract some interesting properties of the measure. We propose two indicators in order to characterize the periodicity of a signal. First is based on the non-biased autocorrelation function and indicates a fundamental periodicity rate. Second is based on the biased autocorrelation and gives a dominant-power periodicity rate. The study of the 2D-plane defined by these two indicators allows the definition of regions attached to one type of periodicity from periodic to aperiodic through almost-periodic and quasi-periodic. Combined with an estimation of the correlation support, a final decision about the periodicity of the signal is given. In case of a periodic signal, a way of estimating the global signal ratio is proposed. These new outputs are valuable for initializing more complex processing. All the algorithms proposed are fully automatic, one click use! Relevance of these indicators is shown on real-world signals, current and vibration measures mainly.

## 1. Introduction

In condition monitoring, a part of the information necessary for decision-making comes from scrutinizing a time measure or a transform of this measure. In this context, the frequency domain is commonly exploited. Oddly enough, it is not usual to consider the lag domain, whereas properties of this domain have long been known, specially for periodic signals(1). The autocorrelation or intercorrelation functions are most often used in system identification so as to extract the modes when the system is driven by a known input. For example, in the method of the random decrement vibration signature[2], [3], the autocorrelation function is used because of its exact proportionality to the free vibration decay. Modal parameters can be estimated from cross-correlations[4] .The correlation dimension of a nonlinear dynamical system provides an estimation of its number of degrees of freedom[5].

In the lag domain, resonant modes are not separated as in the frequency domain but noise is separated from the different harmonics. Noise is here defined as a very wide band spectral signal in which the components of interest are embedded. Investigate this domain offers the opportunity of characterizing the type of harmonicity of the signal which presents a link with the property of periodicity.

This paper focuses upon this interesting property of the autocorrelation function, its ability to characterize the periodicity property of a signal, and upon the link with the oscillating systems.

Section 2 makes an attempt to clear up the notion of periodicity relative to harmonic oscillations with an extension to random processes. Beyond a classification, the object of this section is to highlight the key role of the autocorrelation function. In section 3, two indicators are proposed in order to characterize the periodicity of a signal. First is based on the non-biased autocorrelation function and indicates a fundamental periodicity rate. Second is based on the biased autocorrelation and gives a dominant-power periodicity rate. Section 4 studies a 2D-plane defined by these two indicators, which allows the definition of regions attached to one type of periodicity from periodic to aperiodic through almost-periodic and quasi-periodic. Combined with an estimation of the correlation support, a final decision about the periodicity of the signal is given. In case of a periodic signal, a way of estimating the global signal ratio is proposed. These new outputs are valuable for initializing more complex processing. All the algorithms proposed are fully automatic, one click use! Section 5 shows relevance of these indicators on real-world signals, current and vibration measures mainly.

## 2. What is periodicity?

### *2.1 Periodic signals*

Mathematically, a continuous signal $s(t)$ locally defined on the set $L^2(\Re)$ of finite energy signals is fully periodic with period *T*, when the signal exactly satisfies

$$s(t) = s(t+T). \tag{1}$$

For periodic signals, all the physical quantities, as amplitude and frequency, repeat at equal intervals.

As examples, a simple harmonic oscillation referred to as one sinusoidal signal

$$s(t) = A\sin(2\pi\nu t + \varphi), \tag{2}$$

with $A$ the amplitude, $\nu=1/T$ the frequency and $\varphi$ the phase, is periodic. A sum of $m$ harmonic oscillations of period $T_i$ is also periodic if the $T_i$ are rationally linearly dependent. Then, all oscillating systems without energy dissipation are concerned.

Other examples of periodic signals are inharmonic oscillations, such as relaxation waves, that can be square wave or sawtooth, i.e. non-sinusoidal signals which can be decomposed in sum of sinusoidal or harmonic oscillations of pulsations multiple of a given pulsation refered to as the fundamental pulsation.

### *2.2 Quasi-periodic signals*

Always in the case of continuous functions locally defined on the set $L^2(\Re)$ of finite energy signals, quasi-periodic signals are a generalization of periodic signals (see Figure 1). A signal $s_{qp}(t)$ is quasiperiodic[(6)] with $m$ periods $T_1, .., T_m$ when

$$s_{qp}(t) = g\left\{s_1(t), s_2(t), ..., s_m(t)\right\}, \tag{3}$$

where $g$: $\Re^m \rightarrow \Re$ is continuously differentiable and the $m$ signals $s_i(t)$ are continuous periodic signals with respect to each period $T_i$. All the periods are required to be strictly positive and to be rationally linearly independent. This last constraint is the fundamental property of the quasi-periodic signals, which differentiates from periodic signals.

Quasi-periodic signals possess the following properties: addition and multiplication of quasi-periodic signals yield quasi-periodic ones. If $m=2$, the signal is said bi-periodic.

Therefore, an obvious example of quasi-periodic signals is the sum of $m$ harmonic signals defined by (2) with all ratios of period $T_i$ being irrational.

### *2.3 Almost-periodic and pseudo-periodic signals*

In the case of continuous functions, almost-periodic signals are a generalization of quasi-periodic signals and then of periodic signals (see Figure 1). A signal $s_{ap}(t)$ is almost-periodic[(7)] if every sequence $s_{ap}(t+T_\varepsilon)$ of translations of $s_{ap}(t)$, for all $\varepsilon>0$, has a subsequence that converges uniformly for $T_\varepsilon$ in $(-\infty,+\infty)$, that is

$$\forall \varepsilon > 0, \exists T_\varepsilon \text{ such that} \left|s_{ap}(t+T_\varepsilon) - s_{ap}(t)\right| < \varepsilon. \tag{4}$$

There are several ways of defining classes of almost-periodic functions, based on notions of closure, of an almost-period and of translation. Each of these classes can be obtained as a closure, with respect to some metric, of the set of all finite trigonometric sums. (4) is the original definition proposed by H. Bohr (1947).

In oscillating systems, a quasi-sinusoidal oscillation is a usual denomination for oscillator variation close to a sine variation. According to definition (4) and in spite of its name, such an oscillation is not quasi-periodic but almost-periodic. That is the case of a Wien bridge oscillator.

Another example of almost-periodic signals is given by signals referred to as being pseudo-periodic signals (See Figure 1). A pseudo-periodic signal[8] is defined locally on the set $L^2(\mathfrak{R})$ of finite energy signals, such that all linear combinations of the translated signals have equivalent norm in $L^2(I)$, and $L^2(J)$, as soon as intervals $I$ and $J$ are long enough. The lower bound of $I$ and $J$ is referred to as the pseudo-period. Paley and Wiener has shown that a pseudo-periodic signal is an almost-periodic function[8].

In other words, a pseudo-periodic signal can be decomposed into a periodic function along with a set of parameters that define the deviations of the pattern from true periodicity[9],[10]. Let $s(t)$ be a real valued continuous function with compact support in $[0,T]$, a pseudo-periodic signal $s_{pp}(t)$ is

$$s_{pp}(t) = \sum_i m_i s(\alpha_i t + \beta_i) \quad (5)$$

where $s(t)$ is called the template function for $s_{pp}(t)$.The $\alpha_i$ are called the stretching parameters or scale frequencies, and represent the lengthening or shortening of the periods. The $\beta_i$ are called the translation parameters in time, and allow nonuniform timing of the process, for instance, an acceleration or deceleration in a system. The $m_i$ adjust the amplitude. In a musical context, the $\alpha_i$ correspond to the pitch of the waveform while the $\beta_i$ correspond to the rhythm in which the waveforms appear. The template function plays a role analogous to that of a mother wavelet, while the stretching parameter is analogous to a scale factor. However, the template functions do not need to be orthogonal and do not form a basis, rather, they form a frame[10].

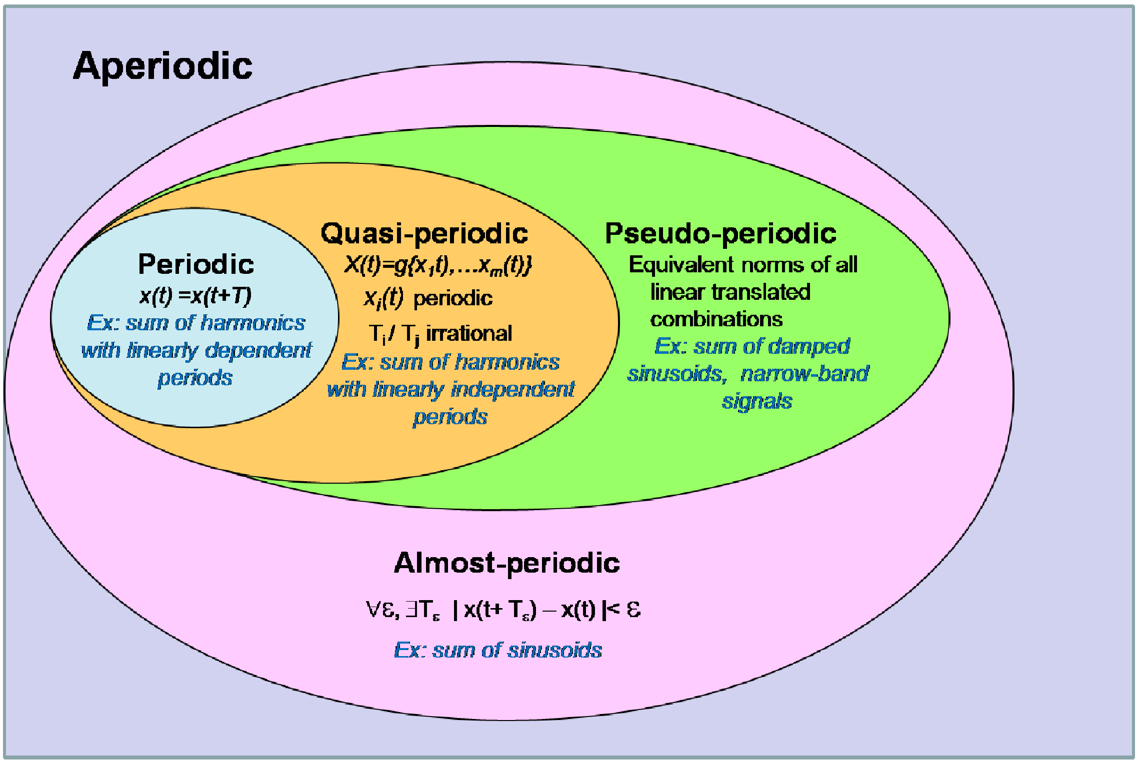


**Figure 1. Periodic, quasi-periodic, almost and pseudo-periodic signals.**

The most simple example is a damped harmonic oscillation also referred to as pseudo-sinusoidal oscillation,

$$s_{pp}\left(t\right) = Ae^{-t/\tau}\sin\left(2\pi\nu t + \varphi\right), \tag{6}$$

without scaling factor and time translation but with a damping parameter 1/τ, which modifies the amplitude of the periodic sine preventing signal $s_{pp}(t)$ to be periodic. All oscillating systems with energy dissipation are concerned.

### *2.4 Periodicity for stochastic processes*

The previous definitions valid for deterministic signals can be extended to stationary stochastic processes but the equalities are true in a mean square sense[(11)]. Let *x(t)* be a stationary stochastic process. *x(t)* is fully periodic with period *T*, when it satisfies

$$E\left\{x(t+T) - x(t)\right\}^2 = 0, \tag{7}$$

where *T* is the smallest number satisfying (7), *E{.}* being the expected value. If (7) holds, then

$$R_x\left(\tau + T\right) = R_x\left(\tau\right), \tag{8}$$

for every lag τ, $\tau \neq 0$, of its autocorrelation function $R_x\left(\tau\right) = E\left\{x\left(t\right)x^*\left(t-\tau\right)\right\}$, the superscript * denoting the conjugate. Therefore, it is of interest to observe that the autocorrelation function of a periodic stochastic process is also periodic pointing out that $R_x(\tau)$ is a deterministic function. It is then straightforward to extend all previous definitions to stochastic processes by the following: definition of the periodicity in time in a mean square sense and equivalent definition than for deterministic processes with true equalities in lag domain.

As example, a laser light is a stochastic process, whose emitted frequency is dirturbed by a phase noise due to the finite spectral linewidth of the laser. This process having a narrow band spectrum relatively to the emitted frequency is pseudo-periodic.

In oscillating systems as rotating systems, vibrations contain sums of periodic signals which are not linearly dependent or rational in general. These signals are most often almost periodic. When measuring one vibration of a part of a system, the measure can be periodic. But, in many cases, even if the physical system leads to deterministic signals, a random noise often disturbed the measure, hence the interest of considering the autocorrelation function in order to define periodicity indicators.

## 3. Two periodicity indicators

Two indicators computed on the autocorrelation function are proposed. Besides the interesting property of the autocorrelation function face to periodicity as noticed in the previous section, another one is attractive for our purpose. When a signal is embedded in a noise uncorrelated with the signal and with a spectral band wider than the signal of interest, in the lag domain, signal is alone for every lag greater than the noise correlation support, a helpful property for estimating the signal periodicity. In addition, estimating the autocorrelation function from a time signal requires no smoothing as it is in the dual frequency domain.

Let *r(t)* be a stationary stochastic process, the sum of either a stochastic or deterministic signal *x(t)* embedded into noise *n(t)*,

$$r(t) = x(t) + n(t), \tag{9}$$

*n(t)* being white or a lightly colored. For $\tau = 0$, $R_r(\tau)$ is the sum of the power of signal *x(t)*, $P_x$, and of noise *n(t)*, $P_n$,

$$R_r(0) = R_x(0) + R_n(0) = P_x + P_n, \tag{10}$$

If *n(t)* is white and *x(t)* is fully periodic with period *T*:

- For $\tau > 0$, $R_r(\tau)$ is fully periodic with period *T*

$$R_r(\tau) = R_x(\tau) = R_x(\tau + kT), k \text{ an integer} . \tag{11}$$

- For $\tau = kT$, $R_r(\tau)$ is equal to the signal power $P_x$

$$R_r(kT) = R_x(0) = P_x \tag{12}$$

- For $\tau \neq kT$ and $\tau \neq 0$, $R_r(\tau)$ is equal to $R_x(\tau)$ and lower to $P_x$

$$R_r(\tau) = R_x(\tau) < P_x \tag{13}$$

The periodicity or aperiodicity of *x(t)* is unknown and has to be determined. Making use of the autocorrelation function leads to estimate first a fundamental periodicity of signal *x(t)* and second a dominant-power periodicity. An estimation of the signal to noise ratio of the signal is also deduced.

### *3.1 Fundamental-frequency test*

*3.1.1 The general principle*

From (12), lags multiple of the periodicity period of a signal match to the maxima of the autocorrelation function of the noisy observation. The fundamental-frequency test proposed is based upon this property and consists of detecting autocorrelation function maxima at regular intervals and with equal amplitude. The autocorrelation function being estimated, equation (12) becomes an approximation and the detection should include the statistical dispersion of the autocorrelation estimation.

The non-biased estimate of the autocorrelation function is used in order to define a ratio $C_{fun}$, which will be equal to *1* for a full periodic signal and *0* for an aperiodic signal. Finally, the fundamental frequency of signal *r(t)* is estimated with the confidence of $C_{fun} \times 100$ % and, in case of a periodic signal, a signal to noise ratio is estimated.

*3.1.2 The algorithm*

Let $\hat{R}_r[m]$ be the non-biased discrete estimation of $R_r(\tau)$,

$$\hat{R}_r[m] = \frac{1}{N-m}\sum_{n=m}^{N} r[n]\, r^*[n-m] \quad 0 \le m \le N-1, \tag{14}$$

with *N* the length of signal *r[n]* sampled at $F_s$. In order to avoid both noise lag support of a possible non-white noise and high variance of the estimation in the high lag values, the test is applied within a lag support defined at each step of the algorithm.

The algorithm steps are:

*1*. Compute $\hat{R}_{r\max}$ the maximum of the autocorrelation function, within $[m_a, m_b]$ support,

$\hat{R}_{r\max} = \hat{R}_r[m_{\max}] \quad / \quad m_{\max} = \arg\max_{m\in[m_a, m_b]} \hat{R}_r[m].$

The choice of values $m_a$ and $m_b$ is in 3.1.4.

*2*. Set $\hat{P}_{x\ init}$ and $\hat{P}_{n\ init}$, initial settings of $P_x$ and $P_n$ respectively, and check that $\hat{R}_{r\max}$ is not associated to noise,

$\hat{P}_{x\ init} = \hat{R}_{r\max}$ from (12) and $\hat{P}_{n\ init} = \hat{R}_r(0) - \hat{P}_{x\ init}$ from (10),

$$\hat{R}_{r\max} \underset{r[n]\ random\ noise}{\overset{r[n]\ not\ aperiodic}{\gtrless}} c_b \times \sigma_{\max} \quad \text{with } \sigma_{\max} = \frac{\sqrt{N}}{|N-m_{\max}|}\hat{P}_{b\ init}$$

The choice of value $c_b$ is in 3.1.4.

3. Compute the lag set $\xi$ such that autocorrelation values are close to $\hat{P}_{x\ init}$, i. e; in an amplitude interval defined from the standard deviation of the estimator, and within $[m_c, m_d]$ support,

$$\xi = \left\{ m_j \in [m_c, m_d] \ / \ \hat{P}_{x\ init} - c\,\sigma[m_j] - c_{ech}\hat{P}_{x\ init} \le \hat{R}_r(m_j) \le \hat{P}_{x\ init} + c\sigma[m_j] - c_{ech}\hat{P}_{x\ init} \right\}$$

with $\sigma[m] = \sqrt{\mathrm{var}(\hat{R}_r[m])} \approx \frac{4N-6m}{(N-m)^2}\hat{P}_{n\ init}\hat{P}_{x\ init} + \frac{N}{(N-m)^2}\hat{P}^2_{n\ init}$, standard deviation for all lags multiple of the signal period, given in [13] when noise is white from statistics [12] of estimator (14).

The set $\xi$ makes sense only *if card* $\xi = J \ge J_{min}$.

The choice of parameters $m_c$, $m_d$, *c*, $c_{ech}$ *and* $J_{min}$ is discussed in 3.1.4.

4. Evaluate the fundamental-periodicity coefficient $C_{fun}$ defined as the percentage of elements of $\xi_d$ being close to the presumed signal fundamental period $1/\hat{f}_{fun}$, subject to a ± *toler* tolerance, $\xi_d$ being defined as the set of distances between consecutive lags of $\xi$, and $\hat{f}_{fun}$ being estimated from the median $Med_{\xi_d}$ of $\xi_d$,

$$C_{fun} = \frac{1}{J-1}\sum_{j=1}^{J-1}\delta\left(\left|d_j - Med_{\xi_d}\right|\right) \quad \text{with } \delta(u) = \begin{cases} 1 & \text{if } u < toler\ Med_{\xi_d} \\ 0 & otherwise \end{cases}$$

and $\xi_d = \left\{ d_j \,/\, d_j = m_{j+1} - m_j,\ m_j \in \xi \ \ and \ \ 1 \le j \le J-1 \right\}$ $\hat{f}_{fun} = F_s / Med_{\xi_d}$

The choice of tolerance parameter *toler* is discussed in 3.1.4.

*3.1.3 A by-product: estimation of a global signal ratio*

When the fundamental frequency has been estimated, the estimation of signal and noise powers, $\hat{P}_x$ and $\hat{P}_n$ respectively, are finally adjusted from elements of $\xi$,

$$\hat{P}_x = \frac{1}{J}\sum_{j=1}^{J}\hat{R}_r\left[m_j\right] \quad \text{and} \quad \hat{P}_n = \hat{R}_r\left[0\right] - \hat{P}_x \text{, with } m_j \in \xi. \tag{15}$$

Hence, the signal to noise ratio *SNR* of the observed signal is estimated as

$$SNR = 10\log\left(\hat{P}_x / \hat{P}_n\right), \tag{16}$$

this ratio having a sense only if the confidence ratio $C_{fun}$ is close to 100%.

*3.1.4 The by-default values*

Fundamental-frequency test needs the setting of 9 parameters which has been chosen according to the investigation of real-world and synthetic signals in a significant data bank:

- $\left[m_a, m_b\right]$, a lag support within which $\hat{R}_{r\max}$ is computed: $m_a = 5\%N$; $m_b = 50\%N$; in any case, $m_a > 0$ and $m_b \le 50\%N$.
- $c_b$, a tolerance factor applied to $\sigma_{\max}$ of $\hat{R}_{r\max}$ : $c_b = 3$.
- $\left[m_c, m_d\right]$, a lag support within which $\xi$ is computed: $m_c = 5\%N$; $m_d = 60\%N$; in any case, $\left[m_a, m_b\right] \subset \left[m_c, m_d\right]$.
- $c$, a tolerance factor applied to $\sigma\left[m\right] = \sqrt{\text{var}\left(\hat{R}_r\left[m\right]\right)}$ in order to estimate $\xi$: $c=2$.
- $c_{ech}$, a tolerance factor applied to $\hat{P}_{x\ init}$ in order to take sampling effects into account: $c_{ech}=0.05$; in any case $0.05 \le c_{ech} \le 0.1$.
-$J_{min}$, the minimal number of detected maxima : $J_{min} = 3$.
-*toler*, a tolerance factor applied to the median of $\xi_d$ , i.e; to the presumed period, in order to compute the fundamental-periodicity coefficient: *toler = 10%.*

### *3.2 Dominant frequency test*

*3.2.1 The general principle*

This second indicator, in addition to the first one, aims at estimating the periodicity of the dominant-power signal part. The theoretical principle is to determine the periodicity of the smoothed autocorrelation function, the smoothing being set so as to highlight the dominant-power harmonic. Unlike the first test, the autocorrelation function estimator should have a low variance so the biased estimator is prefered.

The autocorrelation function is filtered by a low-pass filter bank. For each filtered function, a periodicity ratio is computed; the suitable filter is the one given the first a maximum ratio for a decreasing filter order (more and more narrow filter). A final one denoted as $C_{dom}$ is selected such that the dominant-power frequency of signal *r(t)* is estimated with the confidence of $C_{dom} \times 100\,\%$.

*3.2.2 The algorithm*

For sake of simplicity, the notation $\hat{R}_r[m]$ is kept while designing in this algorithm the biased estimation of the autocorrelation. Let $\hat{R}_r^k[m]$ be the filtered biased discrete estimation of $R_r(\tau)$,

$$\hat{R}_r^k[m] = FFT^{-1}\left[\frac{1}{N\,F_s}\left|\sum_{n=0}^{N-1} r[n]\,\mathrm{e}^{-2\pi \mathrm{j} f n/F_s}\right|^2 \times W^k(f)\right], \qquad (17)$$

with $N$ the length of signal $r[n]$ sampled at $F_s$, $f$ the frequency variable, and $W^k(f)$ a filter bank defined from a window $W(f)$ raised to power $k$ belonging to a filter power set. The choice of window $W(f)$ and of the filter power set is in *3.2.3.*

The algorithm steps are:

1. Compute for each $k$ the lag set $\xi^k$ corresponding to minimal autocorrelation values within $[0,\, m_a^{'}]$ support,

$$\xi^k = \left\{ m_j = \underset{m_j \in [0,\, m_a^{'}]}{\arg\min}\, \hat{R}_r^k[m] \right\}.$$

The set $\xi^k$ makes sense only *if card* $\xi^k = J' \geq J_{\min}^{'}$

The choice of parameters $m_a^{'}$ and $J_{\min}^{'}$ is in 3.2.3.

2. Evaluate for each $\xi^k$ a ratio $C_{dom}(k)$ defined as the percentage of elements of $\xi_d^k$ being close to the presumed signal dominant-power period $1/\hat{f}_{dom}$, subject to a ± *toler'* tolerance, $\xi_d^k$ being defined as the set of distances between consecutive lags of $\xi^k$, and $\hat{f}_{dom}$ being estimated from the mean $Mean_{\xi_d^k}$ of $\xi_d^k$,

$$C_{dom}(k) = \frac{1}{J'-1}\sum_{j=1}^{J'-1} \delta\left(\left|d_j - Mean_{\xi_d^k}\right|\right) \text{ with } \delta(u) = \begin{cases} 1 & \text{if } u < toler'\, Mean_{\xi_d^k} \\ 0 & \text{otherwise} \end{cases}$$

and $\xi_d^k = \left\{ d_j \,/\, d_j = m_{j+1} - m_j,\ m_j \in \xi^k \ \ and \ \ 1 \leq j \leq J'-1 \right\}$, $\hat{f}_{dom}(k) = F_s / Mean_{\xi_d^k}$

The choice of tolerance parameter *toler'* is discussed in 3.2.3.

3. Determine the dominant-power periodicity coefficient $C_{dom}$ by selecting the maximum of $C_{dom}(k)$ at the lowest filter order $k$, and then the corresponding periodicity frequency,

$$C_{dom} = \max_{\min(k)} C_{dom}(k) \quad \text{and} \quad \hat{f}_{dom} = \hat{f}_{dom}\left(\arg\max_{\min(k)} C_{dom}(k)\right).$$

*3.2.3 The by-default values*

As for the previous one, the dominant-power frequency test needs the setting of 5 parameters which has been chosen according to the investigation of real-world and synthetic signals in a significant data bank:

1. *W(f)*, the window function for the filter bank is a Blackman-Harris 4T.
2. Filter power set = [0, 1, 2, 3, 4, 6, 8, 11, 15, 20, 30, 40, 60, 100, 200, 400];
3. $\left[0, m_a^{'}\right]$, a lag support within which $\min \hat{R}_r^k\left[m\right]$ are computed: $m_a$ = *1%,N*; it could be 100% but the computation time can then be very long.
4. $J_{\min}^{'}$, the minimal number of detected maxima : $J_{\min}^{'}$ = 4.
5. *toler'*, a tolerance factor applied to the mean of $\xi_d^k$, i.e; to the presumed period, in order to compute the periodicity coefficient: *toler' = 10%;*

# 4. A 2D plane for a final decision

### *4.1 Correlation tests and periodicity*

Signals in condition monitoring are complex and can exhibit many type of periodicities. The aim of this paper is to assess the periodicity property of a signal with a simple and fast algorithm. The method proposed is a classification, which classes are characteristic regions in a 2D space defined from the two periodicity coefficients suggested in section 3. Figure 2 shows this space where the region definition comes from the investigation of real-world and synthetic signals in a significant data bank.

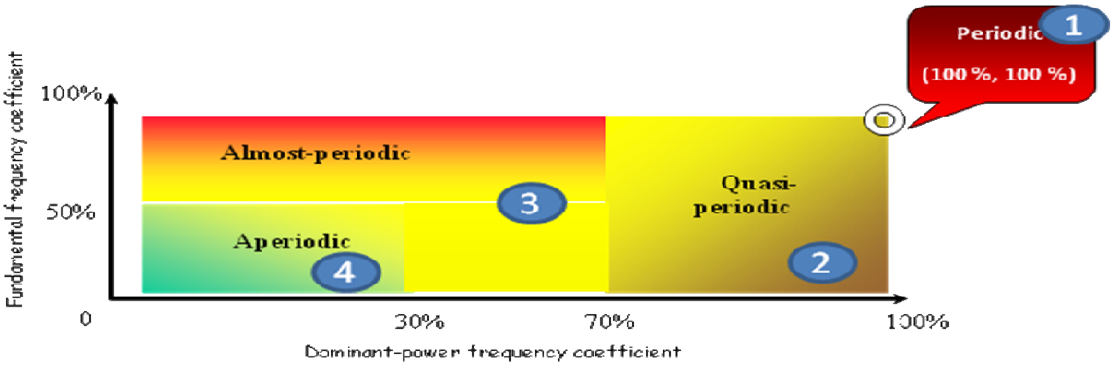


**Figure 2. A first hypothesis about the periodicity of a signal.**

### *4.2 The meaning of a correlation support*

From the non-biased estimator of the autocorrelation function (see (14)), the correlation support is estimated as the maximum lag being greater than a threshold defined from the standard deviation of the estimator corresponding to a white noise only,

$$\begin{aligned} m_{cor\_\sup} &= \max_{m\in[0,N]}\left\{m/\ \hat{R}_r\left[m\right] > C\ \sigma\left[m\right]\right\}, \\ \text{if}\quad m_{cor\_\sup} &\geq 0{,}7N \text{ then } \quad m_{cor\_\sup} = N \end{aligned} \tag{18}$$

with $\sigma^2\left[\tau\right] \approx \frac{N}{\left(N-\tau\right)^2}\hat{R}_r\left[0\right]^2$ as in section 3.1.2 with $\hat{P}_x = 0$, see also (15). The so-defined threshold $C\ \sigma\left[m\right]$ represents a limit above which the estimated autocorrelation no longer corresponds to the white noise hypothesis. The duration $m_{cor\_\sup}$ corresponds then to a rough estimate of the correlation support for the positive lags. Due to the high variance of the autocorrelation estimate, a correlation support greater than 70%N is set to N. In the same way as in 3.1.4 and 3.2.3, the C by-default value has been set to 4.

Figure 3 shows along the lag axis how we have defined the signal spectral band relatively to the correlation support. Thus, five complementary intervals have been defined along this axis.

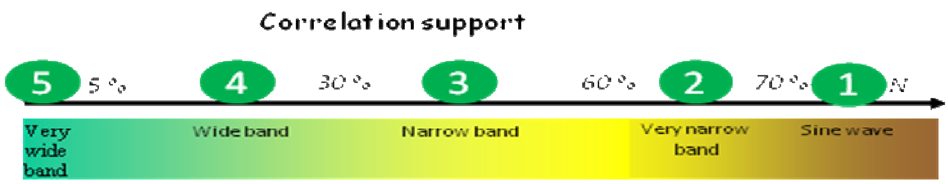


**Figure 3. Correlation support and size of the signal spectral band.**

### *4.3 The final decision*

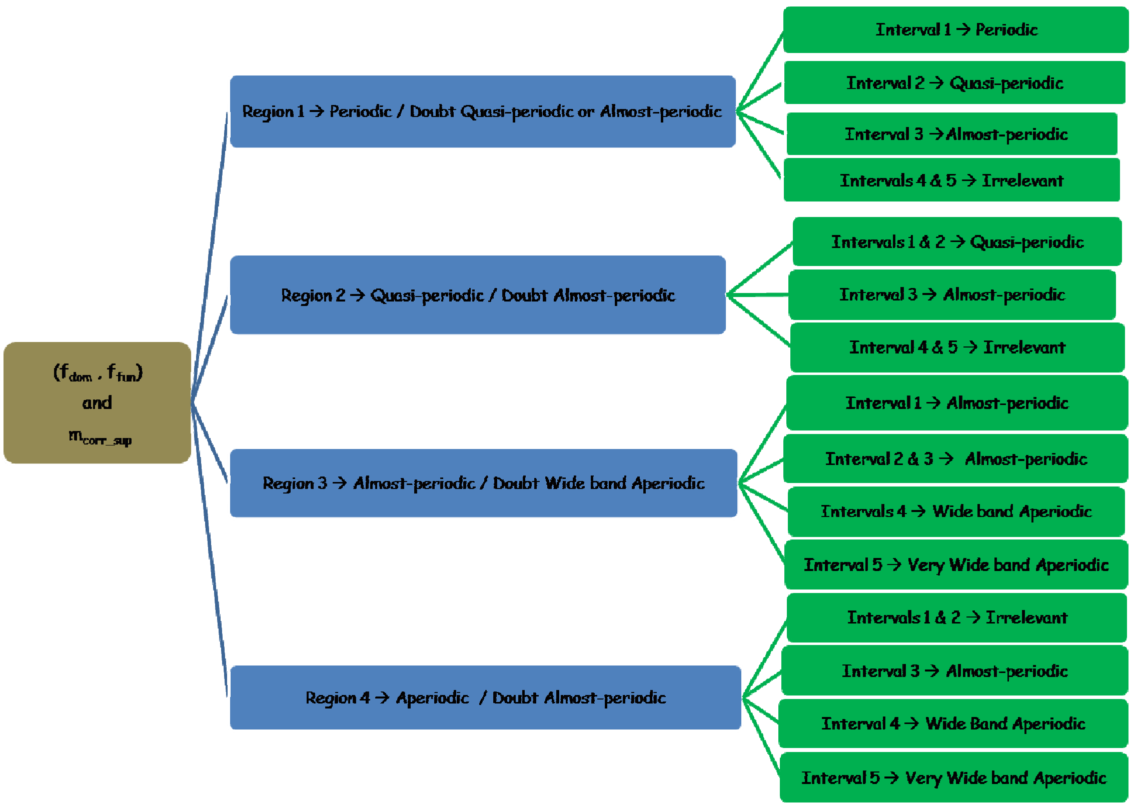


**Figure 4. Classification of signals according to the periodicity test proposed. Region numbers come from the periodicity 2D-space (Figure 2) and interval numbers from the correlation support (Figure 3).**

The addition of the correlation support information allows us to refine the estimation of the prevailing periodicity of the analysed signal. Figure 4 shows the final classification proposed in order to link the 4 regions defined in figure 2 and the 5 intervals defined in figure 3. As previously stated, the validation has been investigated in a significant data bank of real-world and synthetic signals, as shown in the next section.

## 5. Results on simulations & real-world signals

The aim of this section is to apply the indicators proposed on simulated and real-world signals in order to estimate the signal periodicity in accordance with the definitions of section 2. All results are summed up in Table 1 and Table 2.

For all simulated signals, the classification is independent of the noise level, this result was expected according to the autocorrelation support of a white noise. It is less true for very low signal to noise ratio, around -10 dB, and for the damped sine. For the harmonic family, the fundamental frequency is estimated even if the fondamental is not present.

For real-world signals, the results obtained are complying with expert point of view. The fundamental frequency and power-dominant one can be similar or within the same range, which is the case for signals 6 and 7. In other cases the results are complementary.

**Table 1. Results on simulations & real-world signals**

| *Signal* | | $F_s$ | *N* | *Periodicity hypothesis* | *n°* |
|---|---|---|---|---|---|
| ***Room Acoustics*** | Impulse response | 16 000 Hz | 32 762 | Wide band aperiodic and embedded in significant noise | 1 |
| ***Lidar current*** | Heterodyne current-coherent laser radar | 40 500 Hz | 81 200 | Very wide band aperiodic and embedded in significant noise | 2 |
| ***Magnetism*** | Barkhausen noise | 200 000 Hz | 40 000 | Wide band aperiodic and embedded in significant noise | 3 |
| ***Bioacoustics*** | Dolphin whistles | 15 324 Hz | 270 336 | Very wide band aperiodic and embedded in significant noise | 4 |
| ***Hydroacoustics*** | Boat passing | 5 000 Hz | 100 000 | Almost -periodic | 5 |
| ***Vibratory 1*** | Hydraulic noise in nuclear reactor pump | | 102 400 | Periodic and embedded in significant noise | 6 |
| ***Vibratory 2*** | Hydraulic noise in oil station | | 50 000 | Quasiperiodic and embedded in significant noise | 7 |
| ***Vibratory3*** | Vehicle cell | 20 491 Hz | 420 045 | Quasiperiodic and embedded in significant noise | 8 |
| ***Vibratory 4*** | Rotating machine | 6 365 Hz | 65 536 | Quasiperiodic and embedded in low noise. | 9 |
| ***Simulated*** | 2_SW_periodic_80dB | 1 000 Hz | 100 000 | Periodic and embedded in low noise | 10 |
| | 2_SW_periodic_0dB | 1 000 Hz | 100 000 | Periodic and embedded in significant noise | 11 |
| | 2_SW_quasi_no_noise | 1 000 Hz | 100 000 | Quasiperiodic and embedded in low noise | 12 |
| | 2_SW_quasi_-10dB | 1 000 Hz | 100 000 | Quasiperiodic and embedded in significant noise | 13 |
| | Damped_sine_80dB | 1 Hz | 10 000 | Almost-periodic embedded in significant noise | 14 |
| | Damped_sine_0dB | 1 Hz | 10 000 | Wide band periodic and embedded in significant noise | 15 |
| | Harmonics_no_noise_freq_fun_50Hz | 1000 Hz | 10 000 | Periodic and embedded in low noise. | 16 |
| | Harmonics_10dB_freq_fun_50Hz | 1000 Hz | 10 000 | Periodic and embedded in significant noise | 17 |

## 6. Conclusions

A signal classification according to the signal periodicity is proposed. Autocorrelation domain is exploited in order to estimate characteristic frequencies with a confident level. Classes are currrently chosen from the investigation of real-world and synthetic signals in a significant data bank. An improvement could be obtained by making profit of Monte Carlo simulations. Another lead to pursue would be the using of fuzzy logics in order to avoid nonlinear thresholds in the class definition.

**Table 2. Items which lead to conclusions of Table 1.**

| n° | $f_{fun}$ | $C_{fun}$ | $f_{dom}$ | $C_{dom}$ | SNR | Region in 2D periodicity space | $m_{cor\_sup}$ | Interval along lag axis |
|---|---|---|---|---|---|---|---|---|
| 1 | 492,31 Hz | 6,9% | 1656,96 Hz | 31,3% | 17,46 dB | 3 | 1,6% | 5 |
| 2 | 1760,87 Hz | 8,1% | 5389,93Hz | 40,2% | -18 dB | 3 | 0,0% | 5 |
| 3 | 26 666,67 Hz | 14,3% | 13 123,36 Hz | 40,0% | -12,33 dB | 3 | 25,4% | 4 |
| 4 | 901,41 Hz | 5,4% | 273,43 Hz | 58,7% | -21,28 dB | 3 | 3,3% | 5 |
| 5 | undefined | 0,0% | 472,08 Hz | 48,4% | undefined | 3 | 60,5% | 2 |
| 6 | 2857,14 Hz | 100,0% | 2821,78 Hz | 100,0% | -7,10 dB | 1 | 100,0% | 1 |
| 7 | 46,88 Hz | 59,3% | 46,77 Hz | 100,0% | 3,22 dB | 2 | 100,0% | 1 |
| 8 | 1,33 Hz | 41,2% | 351,76 Hz | 91,6% | 2,41 dB | 2 | 100,0% | 1 |
| 9 | 344,05 Hz | 14,2% | 46,97 Hz | 100,0% | 12,99 dB | 2 | 100,0% | 1 |
| 10 | 111,11 Hz | 100,0% | 109,88 Hz | 100,0% | 19,06 dB | 1 | 100,0% | 1 |
| 11 | 111,11 Hz | 100,0% | 109,99 Hz | 100,0% | -0,09 dB | 1 | 100,0% | 1 |
| 12 | 22,22 Hz | 69,0% | 109,98 Hz | 100,0% | 16 dB | 2 | 100,0% | 1 |
| 13 | 38,46 Hz | 21,6% | 109,98 Hz | 100,0% | -10,2 dB | 2 | 100,0% | 1 |
| 14 | 0,29 Hz | 46,2% | 0,30 Hz | 69,0% | 3,75 dB | 3 | 36,0% | 3 |
| 15 | 0,29 Hz | 46,0% | 0,30 Hz | 69,0% | -2,86 dB | 3 | 26,6% | 4 |
| 16 | 50,00 Hz | 100,0% | 49,38 Hz | 100,0% | 73,35 dB | 1 | 100,0% | 1 |
| 17 | 50,00 Hz | 100,0% | 49,38 Hz | 100,0% | 9,99 dB | 1 | 100,0% | 1 |

## References


1. Bendat J. S., Piersol A. G., Random data analysis and measurement procedures. Wiley, 1986.
2. Vandiver J.K., Dunwoody A.B., Campbell R.B., Cook M;F., A math. basis for the random decrement vibration signature analysis technique, J. of Mechanical, 1982.
3. Ibrahim S. R., Random decrement technique for modal identification of structures. J. Spacecraft Rockets 14, 696-700, 1977.
4. Shen F., Zheng M., Feng Shi D., Xu F.**,** Using the cross-correlation technique to extract modal parameters on response-only data, Journal of Sound and Vibration, Volume 259, Issue 5, Pages 1163-1179, 30 January 2003.
5. Logan D., Mathew J., Using the correlation dimension for vibration fault diagnosis of rolling element bearings. MSSP, vol. 10, no3, pp. 241-250, 1996.
6. Benoist-Gueutal P., Mathématiques pour la physique, Ed. De Boeck &Larcier, 2005.
7. Corduneanu C., Gheorghiu N. and Barbu V., Almost periodic functions, Chelsea publishing Company, 1989.
8. Paley R. and Wiener N., Fourier transforms in the complex domain, New-York, American Mathematical Society, 1934.
9. Wong H., Sethares W. A., Estimation of pseudo-periodic signals, IEEE ICASSP, Montreal, Quebec, Canada, May 17-21, 2004.
10. Sethares W. A, Repetition and pseudo-periodicity, Tatra Mt. Math. Publ. 23, pp. 1-16, 2001).
11. Papoulis A., Probability, Random variables and stochastic processes, McGraw-Hill Book Company, 1965.
12. Oppenheim A. V., Schafer R. W., Digital Signal processing, prentice hall Editions, Englewood Cliffs, N.J., 1975.
13. Durnerin M., A strategy for interpretation in spectral analysis- Detection and characterization of spectrum components (in French), PhD, Grenoble INP, September 21, 1999.