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Management of the risk of backorders in a MTO–ATO/MTS context under imperfect requirements

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A B S T R A C T

Production systems are often classified according to the way production is released, e.g. make-to-stock (MTS), make-to-order (MTO), assembly-to-order (ATO) or engineer-to-order (ETO). The choice of a type of production depends on the decoupling point between customer and supplier. In some supply chains, like in the aeronautical sector, a customer may work according to a MTO process (since his product is highly specific) while his supplier works with a MTS process (since he delivers variants of standards components). This situation sets specific problems that are seldom considered in the literature, especially when collaboration between actors is required for an efficient management of the supply chain, which is the case when uncertainties are present. In this paper, we propose a method based on fuzzy modelling allowing a customer to choose a plan taking into account the uncertainty on his requirements when he works in MTO–ATO while his supplier is in MTS.

1. Introduction

Nowadays, companies are not anymore competing as independent entities but as a part of collaborative supply chains. Due to various phenomena, among which the bullwhip effect is the best known, the uncertainty on the demand creates risks of backorders or obsolete inventory in the supply chain. To reduce these risks, different approaches exist, among which an increased coordination between customer and supplier or the explicit integration of the uncertainty into the planning process (see for instance [1]).

The coordination of the supply chain can be performed using a "vertical" or "horizontal" approach. The "vertical" approach promotes a centralised synchronization of the supply chain, through an APS (Advanced Planning System) [2], using other centralised approaches, like Multi level scheduling Lot Sizing [3], or using inventory policy approaches [4]. The "horizontal" approach refers to collaborative planning, required when the supply chain is composed of independent entities [5]. Various kinds of industrial collaborative processes have been standardized for implementing cooperation between retailers and manufacturers, like the "Collaborative Planning, Forecasting and Replenishment" (CPFR®) [6], which aims at creating short and reactive decision loops between customers and suppliers in order to cope with the growing uncertainty on demand forecasting, due to the shortening of the product life cycle and to customers' versatility.

Within supply chains made of independent entities, the collaborative processes are usually characterised by a set of point-to-point customer/supplier relationships with partial information sharing [1]: one or several procurement plans are built and propagated through the supply chain using negotiation processes.

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Taking into account explicitly the uncertainty of the demand in the planning processes can also help to make more informed decisions [7–9]. The uncertainty can then be integrated into the cooperative planning processes built by the customer and his suppliers, but a specific difficulty occurs when the customer and his supplier do not work according to the same production process. A customer may for instance work according to a MTO (Make-to-Order) process (since his product is highly specific), while his supplier works with a MTS (Make-to-Stock) process (since he delivers variants of standards components). This situation, quite common in aeronautic supply chains, sets interesting specific problems, but has received until now poor attention from researchers.

In this specific context, we suggest to explicitly model the imperfection on the data (demand, process, supplies), taking into account the customer's knowledge on the capacity of his suppliers, also considered as imprecise.

In order to solve this problem, we suggest that as a first step, the customer computes a set of possible plans, then using the information he has on the maximal capacity of the supplier, he chooses the plan that minimizes the risk of supplier backordering or excess of input inventory. In that purpose, three sub-problems can be identified (see Fig. 1):

- suggest a model for imperfect data using possibility theory,
- calculate the possibility of backordering for each plan,
- define a decision making process allowing to select the less risked plan.

Our objectives are so here to propose criteria to evaluate the risk of a plan in terms of backordering, and a method to calculate the maximal backordering level.

This article is organised as follows: Section 2 presents a state of the art on the application of possibility theory to production planning problems. In Section 3 are reminded some theoretical points needed to solve the problem. In Section 4, a model of the imperfections on the data is suggested, within the framework of possibility theory. A method for computing the backordering level is described in Section 5, while a decision process based on this technique is suggested in Section 7. A numerical example illustrating the general method is presented in Section 8.

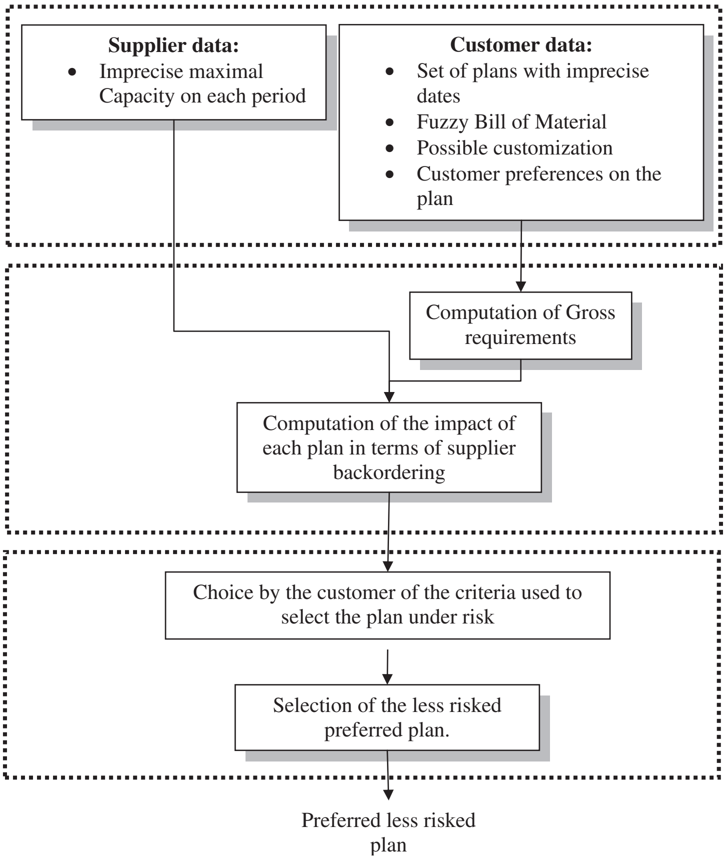

Fig. 1. Position of the method in the production planning process.

2. Literature review

In order to remain competitive, companies have to propose more and more customised products on the market [10]. This customization impacts the type of production process that has to be chosen: make-to-order (MTO), assembly-to-order (ATO), engineer-to-order (ETO), etc. [11–14]. In spite of this, the management of supply chains where the actors have different production processes (customer in ATO and suppliers in MTS for instance) is seldom considered in the literature [3], even if in the case of a supply chain grouping independents actors, the centralised approaches cannot be used for addressing this problem.

In the literature, three different sources of uncertainty are usually distinguished: on the *demand*, on the *process* and on the *supplies* (see [15] for a review). These uncertainties cannot always be modelled using stochastic approaches, due to the difficulty to have access to historical data allowing to determine a probability distribution. The theory of fuzzy sets [16] and the theory of possibility [17] are often used to model uncertainty in that case [7,15]. In this article we propose to take into account the uncertainty on the demand (including on the customization of the product), on the process (task duration and quantity required to assemble the product), and on the supplies (delivery quantity of the suppliers). Since we shall use fuzzy set and theory of possibility to model uncertainty, we have to solve a problem of decision making using fuzzy parameters.

In the literature, there are three popular families of approaches for coping with fuzzy decision parameters. In the first family, the decision maker chooses one of the possible solutions using either the *Defuzzification then optimisation* approach (for example using the Yager index [18,15]), or the *Maximisation of the possibility of optimality of the solution* (suggested by Bellman and Zadeh [19] and used in [20] and [21] for instance). These two approaches can be qualified as "optimistic" and are appropriate in the case of flexible parameters. Nevertheless, the applicability of these approaches is limited in a context of uncertainty, because it does not take into account all the possible impacts of a decision but look for an optimal solution for one possible scenario [22]. The third approach, which can be called *Robust optimisation* [22], maximises the degree of necessity (resp. certainty) that the cost of the plan satisfies a given fuzzy goal. The difficulty of this approach is to define the fuzzy goal of the decision maker, especially if there is more than one decision maker. In this article, we propose to use two robust criteria that do not need a formal definition of the goal: the minimization of the maximal expected value of backordering and the minimization of the risk of backordering. To evaluate a risk level, a risk matrix [23] is often used, since it is an efficient and user-friendly tool. Therefore, we suggest to use the risk matrix to represent the risk of supplier backordering to support the decision maker in the choice of a criterion between the two proposed.

In the literature, planning under uncertainty in the framework of possibility theory has often been applied to MTS production processes [24,15,25,8,9] or to MTO production processes [26,27] but the situation MTO–ATO/MTS is not often considered even if this case may arise in reality for customised products. In [7] is suggested a method to compute a set of possible gross requirements from a plan while in [24] is described a decision support-oriented approach for building a procurement plan from a set of possible gross requirements (see Table 1 which gives a panorama of the literature according to the type of uncertainty and the considered production process). With a complementary view, this article proposes a method for choosing a plan within an elementary partnership of a collaborative supply chain, composed of one customer and *n* suppliers, when the customer is in MTO–ATO and the suppliers in MTS, taking into account the imperfections on the customer's gross requirements. The customer (for example Airbus or Boeing) assembles a customised product in which customization is performed at a late stage of manufacturing, an element of customization being associated to a supplier (choice of a given component, like the engines for instance).

3. Theoretical background

In this section, we give the basis of the calculation of gross requirements (using the MRP method) using crisp data. We then present the notions of the possibility theory that are used in this article and introduce the risk matrix.

3.1. Calculation of a supply plan using MRP

Within supply chains, production management is usually performed through a cascade of MRP2 modules (Manufacturing Resource Planning, see for instance [28]), included in all the ERP (Enterprise Resource Planning) systems, used for managing the majority of nowadays companies [29]. Using MRP2, and for each partner of the supply chain, forecasts and firmed long

Table 1
Data imperfection and production process.

References	Production process	Uncertainty/imprecision
Chen and Huang [26], Balasubramanian and Grossmann [27]	MTO-ATO	Task duration
Guillaume et al. [22], Peidro et al. [30], Mula et al. [25], Fargier and Thierry [9], Grabot et al. [8]	MTS	On demand
Guillaume et al. [7], Guillaume et al. [24]	MTO-ATO supplier MTS	On demand on task duration

term programs are used as inputs for building a Sales and Operation Plan (SOP) which plans what should be sold and what will be produced by period (e.g. per week or month) on a long horizon (1–3 years). A more precise Master Production Schedule (MPS) can then be deduced at lower term (6 months to 1 year). On the base of the obtained sequenced requirements on the final products, the bills of materials are used (Material Requirement Planning step) for generating a Production Plan describing what should be internally produced at short term (set of production orders) and a Supply Plan concerning the components to buy from the suppliers (set of purchase order).

MRP uses crisp data: it is easy to combine firm orders, which due dates are precisely known, with forecasts based on quantities by periods: it is only needed to check in which period an order is located.

If the positioning of an order through time is imprecise, the problem is much more difficult, since it is necessary to calculate the probability or possibility that an order belongs to a period.

In Section 3.2, we define a possibility distribution and show how to model imprecision on knowledge in the framework of possibility theory. We then define the fuzzy operators required to compute the gross requirements: sum and intersection. Thirdly, we present a criterion for decision-making under an "uncertain" scenario (scenario without knowledge on the chance of realisation) and two criteria for decision under a "possible" scenario uncertainty (scenarios modelled by possibility distributions).

3.2. Representation of imprecision

An imprecise information may be defined as $v \in A$ where A is a subset of S which contains more than one element. The imprecision may be expressed by a disjunction of values [31] defined by a possibility distribution on S. $v \in A$ means that all the values from v outside A are supposed to be impossible.

A possibility distribution π_v of v quantifies the plausibility of the information v. π_v is a function of S in L such as $\forall s \in S, \pi_v(s) \in L,$ *and* $\exists s, \pi_v(s) = 1$ with v denoting an ill-known value in S, and L the scale of plausibility ([0, 1] for the theory of possibility).

A possibility distribution can be modelled by an interval where the lower and upper bound are gradual real numbers. A gradual real number (or gradual number for simplification) $\tilde{r}$ is defined by an assignment function $A_{\tilde{r}}$: (0, 1] → ℝ [32].

3.3. Selected operators of possibility theory

Let us first recall some results from the possibility theory [16,17].

3.3.1. Sum

In order to describe events by possibility distributions, trapezoidal distributions (cf. Fig. 2), denoted (a, b, c, d, h), can be used without important loss of generality, since these sets intend to model an expertise suggesting a global shape rather than a precise function.

The sum of two trapezoidal distributions A_i and A_j defined by the quintuplets A_i (a_i, b_i, c_i, d_i, h_i) and A_j (a_j, b_j, c_j, d_j, h_j) is defined in [17] as:

$$\begin{aligned} A_i(+)A_j &= (a,b,c,d,h)\,\textit{with} \\ h &= \min(h_i,h_j) \\ c &= h\cdot\left(\frac{c_i}{h_i}+\frac{c_j}{h_j}\right) \\ d &= h\cdot\left(\frac{d_i}{h_i}+\frac{d_j}{h_j}\right) \\ a &= a_i+a_j-c_i-c_j+c \\ b &= b_i+b_j+d_i+d_j-d \end{aligned} \tag{1}$$

Within the MRP framework, calculating gross requirements consists in allocating quantities of components to periods. If the date of the requirement is imprecise, we need to compute the possibility that a set (the requirement) belongs to a given interval of time (the period).

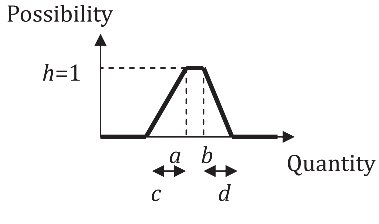

Fig. 2. Trapezoidal distribution of possibility.

3.3.2. Membership measure in the possibility theory

If A is an event, modelled by a possibility distribution, and F a fuzzy set denoting an imprecise category, the degree of membership of A–F (between 0 and 1) is evaluated with two measures in possibility theory [17]: the possibility degree $\Pi(A \in F)$ (Equation (2)) and the necessity degree $N(A \in F)$ (Equation (3)). The possibility is the upper bound and the necessity is the lower bound of the compatibility between A and F. These two measure are linked by the dual relation $\Pi(A \in F) = 1 - \mathrm{N}(\bar{A} \in F)$, $\bar{A}$ denoting the complement of A.

$$\Pi(A \in F) = \sup_u(\min(\pi_A(u), \mu_F(u))) \tag{2}$$

$$\mathrm{N}(A \in F) = \sup_u(\min(1 - \pi_A(u), \mu_F(u))) \tag{3}$$

The possibility $\Pi(A \in F)$ and the necessity $N(A \in F)$ are respectively the upper bound and the lower bound of probability of $\Pr(A \in F)$ (Equation (4)).

$$\mathrm{N}(A \in F) \leqslant \Pr(A \in F) \leqslant \Pi(A \in F) \tag{4}$$

The result of Equation (2) when F is an interval is illustrated in Fig. 3.

3.4. Decision criteria

A decision problem can be defined as a situation where a Decision Maker (DM) has to choose between several possibilities. A part of the theory of decision [33] focuses on the case when the result of the choice is uncertain. Many models allow to represent the uncertainty as a probability distribution, as a possibility distribution or as a set of scenarios. The choice of the model depends of the quality of the knowledge on the uncertainty: the probability distribution requires a "perfect" knowledge, while a possibility distribution may deal with an incomplete knowledge (partial ignorance may be modeled) and sets of scenarios may describe possible situations with a total ignorance on their possibility of occurrence. The consequence of a choice (decision) may be described by a degree of satisfaction of the decision maker.

Depending of the model of uncertainty used and on the characteristic of the decision (pessimistic or optimistic), the literature proposes different criteria allowing to model the preferences of the decision maker. In this section, we present first a criterion aiming at making robust decisions under uncertain scenarios and two criteria aiming at making a robust decision under possible scenarios uncertainty modeled by possibility.

3.4.1. Leximin

This criterion aims at choosing the decision that has the higher minimal satisfaction level in the set of the possible satisfaction levels. Let x_i the utility of decision x for the scenario $i \in I$ and $a \in \Re$, we define $J(a,x) = \{i \in I | x_i \leqslant a\}$ and $|J(a,x)|$ the cardinality of $J(a,x)$. We write $x \succ_{Lm} y$ if decision x is preferred to the decision y using the leximin criterion. The leximin is defined as follows [34]:

$$x \succ_{Lm} y \leftrightarrow \exists a \, such \; that \, |J(a,x)| < |J(a,y)| \& (\forall b < a) |J(b,x)| = |J(b,y)|$$

3.4.2. Expected value of possibility distribution

Knowing that the possibility is the upper bound of probability and the necessity the lower bound, so the expected value is ill-known and belongs to an interval which lower bound is the expected value for the possibility measure and the upper bound is the expected value of the necessity measure. The robustness of a decision can be defined as the minimisation of the maximal negative impact. So, minimizing an expected value with a "robust" meaning is minimizing the maximal possible expected value. The maximal expected value is given by Eq. (5) [35].

$$E^* = \int_{-\infty}^{+\infty} x d\mathrm{N}(] - \infty, x[) \tag{5}$$

with $N(]-\infty, x[= 1 - \Pi([x, +\infty[)$.

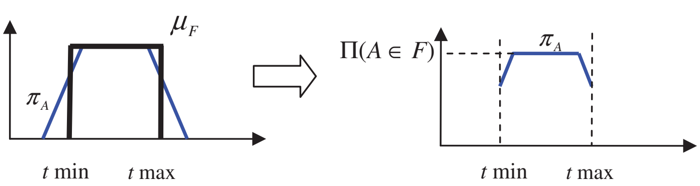

Fig. 3. Computation of the possibility levels that an element belongs to an interval.

The risk is often defined as the plausibility of an event multiplied by its impact (consequence) [23]. In the possibility context, the plausibility of the event is measured by the possibility degree. Therefore, the expected value measures the mean risk of a decision.

3.4.3. Shilkret integral

The *Shilkret* integral of d denotes the maximal risk of decision d in the possibilistic context with a quantitative utility function (u: $x \to \mathbb{R}$ such that $\forall x, y \in X, \quad x \succ y \iff u(x) > u(y)$). In fact, this integral calculates the maximal value of possibility of an event multiplied by the consequence of the event [36].

$$Sh(d) = \max_x(\pi^d(x) \times u(x),\ \forall \pi^d(x),\ u(x) \in [0;1]) \quad (6)$$

Therefore the *Shilkret* integral measures the maximal risk of a decision.

3.5. Risk matrix

Risk is a concept that reflects both a range of possible outcomes and the chance (possibility of occurrence) of the outcomes. Risk is often quantified using a risk matrix (Fig. 4) [23]. On the x-axis is represented the impact of the outcomes, and on y-axis the "chance" of the outcomes. Typically, Impact and Chance are modelled by four values each, which allows to quantify the resulting risk as denoted in Fig. 4.

4. Model of imperfections and preferences for the considered problem

For the rest of the paper, the following notations are adopted: let p be a plan with $p \in P$, s a supplier with $s \in S$, t a period with $t \in H$ and o an order with $o \in O$.

4.1. Model of customer data

4.1.1. Product data

Each order o has a possible customization, each customization being the result of buying a component from a different supplier (for instance, buying the engine of an aircraft from Rolls-Royce or from SNECMA). The DM (Decision Maker) gives a possibility level $\Pi(o, s)$ to each possible customization concerning order o from the supplier s, based on his knowledge of the expectations of the final customer (Fig. 5), respecting the constraints $\max_{s \in S}(\Pi(o, s)) = 1\ \forall o \in O$.

Moreover, for taking into account possible scraps, the DM may consider an imprecision on the number of components required to assemble the product i of the order o through a bill of materials with a fuzzy required quantity, linked to the customization: the number of components needed to assemble a product is not certain but imprecise, due to the possible discard or components damaged during the assembly process. To take into account this imprecision, we also use a possibility level of the quantity. In that case, the classical bill of materials becomes a Fuzzy-bill of materials (see Fig. 6, where "around" 5 components c_1 are needed and "around" 22 components c_m are needed for the assembly).

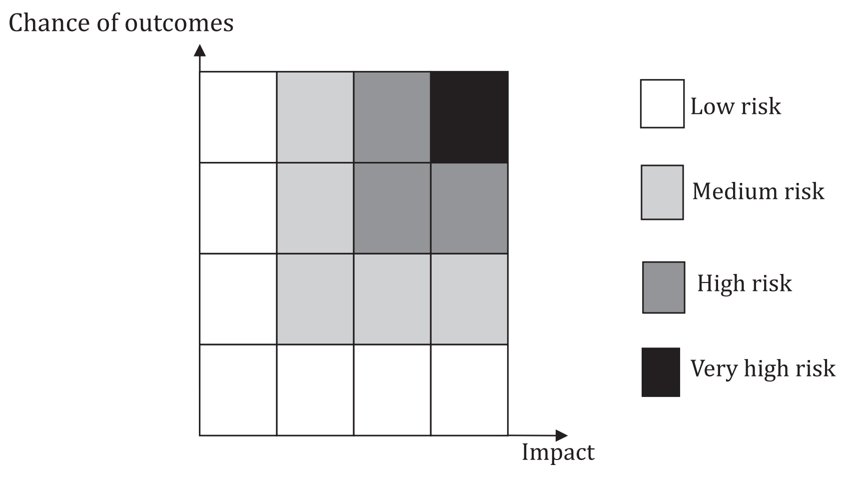

Fig. 4. Risk matrix.

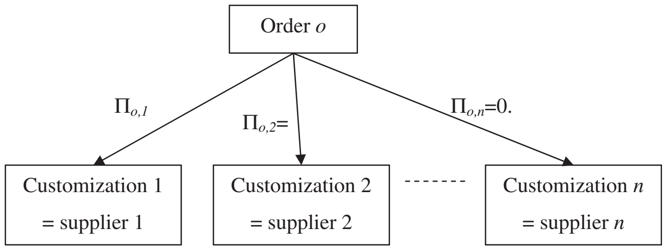

Fig. 5. Model of the possible customization.

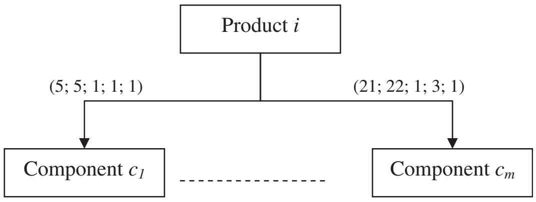

Fig. 6. Fuzzy bill of materials.

4.1.2. Definition of a plan

In the context of integration of uncertainty in the planning process, we assume that the customer uses a plan, which takes into account the imprecision on the demand and on the duration of the tasks [26,27]. Therefore, the plan becomes a fuzzy plan (Fig. 7), where each order is modelled by a possibility distribution related to its date.

4.2. Preferences on the plan

We consider that the customer selects a set of possible plans. The preferences on the plan may come from two considerations:

1. the customer gives his preference on the set of plans (he can classify the plans),
2. the customer gives his preferences on the fact that a given order is planned before or after another in a plan. More formally, he gives preferences on the disjunctive constraints between orders (the DM is able to class each pair of orders).

In the first case, the customer defines a strict order over the plans: P_i, plan number i with $i = 1, \ldots, I$ thus that $P_1 \succ \ldots \succ P_I$

In the second case, the customer gives his preferences on the disjunctive constraints; these preferences are a relation $r(i, j)$ so that:

- if $r(i, j) = 1$ and $r(j, i) = 0$ i before j is strictly preferred than j before i.
- if $r(k, h) = 1$ and $r(h, k) = 0.9$ k before h is a little bit preferred to h before k.

In order to classify the production plans according to this set of preferences on disjunctive constraints, we may use the *Leximin* classification, which is the most discriminating criterion [31]. Nevertheless, it is still possible that two plans are equal. In this case, we ask the decision maker to class these equal plans in order to have a complete order over the plans.

Fig. 7. Fuzzy plan.

4.3. Model of the supplier data

The maximal capacity of the supplier allocated to the order is often imperfectly known by the customer, since it depends on short terms load variations. In the context of risk minimization of backordering, the lower bound of the supplier capacity has a critical impact (the lower the capacity, the higher the risk of backordering). To simplify the model, we only take into account the lower bound of the maximal capacity of the supplier. So, we can model the maximal capacity allocated by the supplier s for each period t by a gradual number $\widetilde{C}_t^s$ (see §3.2). For each period t, the supplier gives the most possible maximal capacity and the maximal capacity in the worst case (minimal possible value), as shown in Fig. 8.

If a supplier does not provide information on his maximal capacity, the customer can model his own supposition on the possible maximal capacity of his supplier, or may consider an infinite capacity.

5. Computation of the backordering level

In this section is shown how to evaluate the impact of each possible plan in terms of backordering, in order to choose the less risked plan. To evaluate the maximal possible level of backordering, we need to compute the maximal cumulative gross requirements over the planning horizon.

Let us note $B_t = \max\left(0; BB_t - \sum_{i=1}^{t} C_i\right)$ BB_t being the cumulative gross requirements for horizon t, C_i the maximal capacity for period i. C_i is a constant, so $\max\left(0; BB_t - \sum_{i=1}^{t} C_i\right)$ is maximal when BB_t is maximal.

The first step is to build the requirements from the bill of materials and the possible customization. The second step is to compute the maximal cumulative gross requirements over the horizon. The third step consists in calculating the possibility level of backordering for each period, on the base of the maximal cumulative gross requirements and of the maximal capacity of the supplier.

5.1. Model of the requirement quantity

On the base of the two inputs – possibility level of the possible customization $\Pi(o,s)$ and fuzzy bill of material $(a; b; c; d; 1)$ – we can build the required quantity of orders for each customization $\widetilde{R}_o^s = (a; b; c; d; \Pi(o,s))$.

5.2. Computation of the maximal cumulative gross requirements

In this section, we present the method for a given supplier and a given plan, so $\widetilde{R}_o^s = (a; b; c; d; \Pi(o,s))$ becomes $\tilde{R}_o = (a; b; c; d; \Pi(o))$

To compute the maximal cumulative gross requirements BB_t, we have (1) to find the set of possible combinations of the requirements, together with their possibility levels, (2) to compute for each possible combination the maximal cumulative quantity. Algorithm 1 calculates the set of orders belonging to horizon t with a possibility level π (noted $\boldsymbol{C}_t^{\pi}$).

Algorithm 1: Computation of the set scenarios by dates

Input: set of orders O, set of horizon sizes $\{H_t\}_{t=1,\ldots,T}$
Output: set of scenarios by dates of the maximal cumulative gross requirement $\boldsymbol{C}_t^{\pi}$
For $t = 1$ **to** T **do**
  For all o **in** O **do**
    $\Pi_o^t = \Pi(o \in [0; H_t])$ // *possibility level that order o belongs to horizon* H_t
  End
End
For all t **in** H **do**
  $a = 0$
  While $a \neq 1$ **do**
    $a = \min_{o \in \boldsymbol{O}}(\Pi_o^t | \Pi_o^t \neq 0)$
      If $a = 1$ **then**
        $\boldsymbol{C}_t^1 = \{o | \Pi_o^t \neq 0\}$
      Else
        $\boldsymbol{C}_t^a = \{o | \Pi_o^t \geqslant 0\}$ // *a scenario with possibility level* α *is the set of orders that has a possibility level to belong to* H_t *greater than* α
      End
    $O = O - \{o | \Pi_o^t = a\}$ // *we remove the order which has the minimal possibility level to belong to* H_t
  End
End

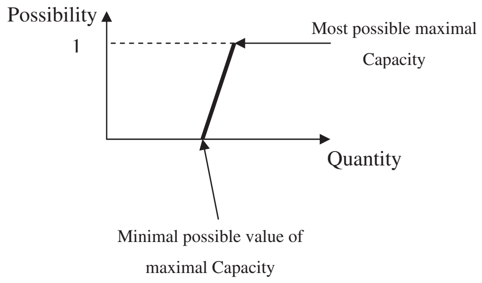

Fig. 8. Model of supplier capacity.

From the set of scenarios by dates of the maximal cumulative gross requirements and according to the uncertainty on the customization, we calculate the possibility of the maximal cumulative gross requirements with Algorithm 2.

Algorithm 2: Computation of the maximal cumulative gross requirements

---

Input: set of scenarios by dates of the maximal cumulative gross requirement $\{\boldsymbol{C}_t^{\pi}\}_{t=1,\ldots,T}^{\pi\in \boldsymbol{PI}}$, uncertain required quantity $\widetilde{R}_o$ with o ∈ O
Output: set of maximal cumulative gross requirements $\tilde{BB}_t$ for t = 1, . . ., T
  For t = 1 **to** T **do**
    For each $\pi \in$ **PI** in increasing order **do**
      $A_{\pi} = \{\Pi(o) | b < \Pi(o) \leqslant \pi\}_{o \in \boldsymbol{C}_t^{\pi}}$
      For each $\alpha \in A_{\pi}$ **do**
        $\boldsymbol{S}_t^{\alpha,\pi} = o \in \boldsymbol{C}_t^{\pi} | \Pi(o) \geqslant \alpha$
      End
      $\tilde{BB}_t^{\alpha,\pi} = \bigoplus_{o \in bi S_t^{\alpha,\pi}} \tilde{R}_o$
      $\tilde{BB}_t^{\alpha,\pi} = \tilde{BB}_t^{\alpha,\pi} \oplus (0;0;0;0;\pi)$
      $\{c\} = \{c\} + (\alpha, \pi)$
  End
  $\tilde{BB}_t = \bigcup_{\forall c} \tilde{BB}_t^{\alpha,\pi}$
  $b = \pi$
End

5.3. Computation of possible backorders

The maximal backordering level is linked to the maximal value of the cumulative gross requirements. Let us note $\tilde{BB}_t^+$ the maximal fuzzy bound of the maximal cumulative gross requirement $\tilde{BB}_t$ thus, $\tilde{BB}_t = [\tilde{BB}_t^-; \tilde{BB}_t^+]$. $\tilde{BB}_t^+$ is a gradual number and the maximal backordering level $\tilde{B}_t$ is also a gradual number. The maximal backordering level is calculated according to Eq. (7):

$$\tilde{B}_t = \max \left(\tilde{BB}_t^+ - \sum_{i=1}^{t} \tilde{C}_i;\ 0 \right) \tag{7}$$

In fact, the maximal mean risk (mean of possibility for product backordering) and the maximal risk (maximal value of the possibility of product backordering) depend on the upper bound of the possible backordering. So, we only have to calculate the upper bound $\tilde{B}_t$, which is a gradual number.

6. Selection of the less risked plan

From the previous calculation, the DM has the information on the couple impact/possibility of each production plan. With this information and the preferences of the DM, we propose in this section a method to choose a plan. The first step consists in supporting the decision maker to choose a criterion using a risk matrix; the second in selecting a set of plans which minimises the chosen criterion, and as a consequence maximises the satisfaction of the DM.

6.1. Choice of the criterion

Depending on the level of risk, we propose two different decision criteria to select a plan giving satisfaction to the DM. Fig. 9 shows two examples of risk representation in the framework of possibility theory using the risk matrix (the qualification of the level of risk by the DM(s) is represented on the matrix). The maximal backordering level represents the worst scenario. According to its possibility, the DM(s) can judge whether there is a scenario that is too risked (i.e. possibility $\times$ backordering level is too high) or not. Two different situations may appear:

1. the possible backordering is considered as non critical by the DM(s) (Fig. 9(a)),
2. the possible backordering is considered as critical by the DM(s) (Fig. 9(b)).

In case 1, the possible backordering is not critical, so we recommend to choose a plan that minimizes the maximal expected value of backordering level. On the other hand, in case 2, the possible backordering is critical, so we recommend to choose a plan which minimizes the maximal risk (possibility of the event multiplied by the backordering level, so using the *Shilkret* integral). If the set of selected plans is composed by more than one plan, we may improve the selection using the second criterion. So, in the first case, we choose the plan with the minimal risk, knowing that the plan has the minimal maximal expected value of backordering level. In the second case, we choose the plan with the minimal maximal expected value of backordering level, knowing that the plan has the minimal risk.The process of selection of a criterion is therefore composed of three steps:

1. build the risk matrix,
2. ask the DM for the criticality of the possible backordering level,
3. select the corresponding criterion (minimization of expected value or *Shilkret* integral).

6.2. Decision process

In Fig. 10 is shown the flowchart of the process of selection of a plan. On the left side is shown the method for choosing the plan that minimizes the average value of risk. From the set of plans, we choose those that have the minimal expected value of backordering. We then ask the DM if he wants to choose inside this set the plans that minimize the Shilkret integral (maximal value of risk). If not, we select in the set of plans the preferred one; if yes, we select the sub-set that has the minimal value of the Shilkret integral, then we select in this sub-set the preferred plan.

On the right side of Fig. 10 is shown the method allowing to choose the plan according to the minimization of the maximal value of the risk. The method is similar to the previous one: we only replace the expected value by the Shilkret integral in the first selection process, then the Shilkret integral by the expected value in the second. Then, we select the set of plans that are equivalent for the *Leximin* criterion. Within this set, we select the preferred plan.

6.2.1. Minimization of maximal expected backordering level

For each plan ($p \in P$) we calculate the maximal expected backordering level with Eqs. (8) and (9).

Let us consider $\Pi_{\tilde{B}_{t,s,p}}$ the possibility measure for the distribution $\tilde{B}_{t,s,p}$.

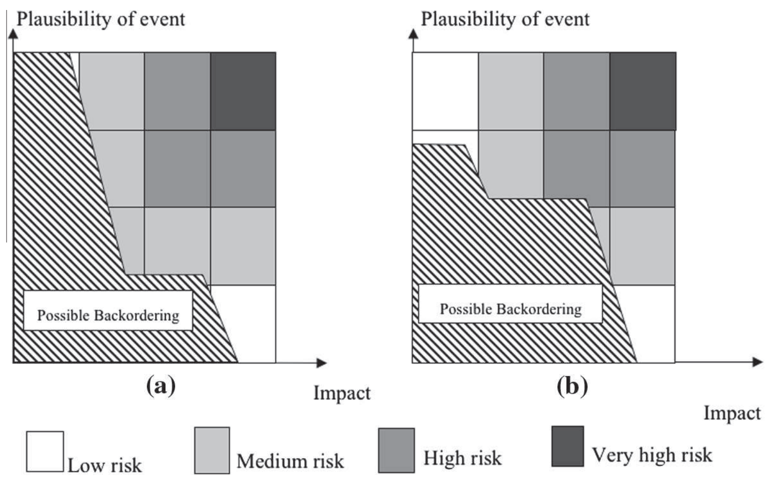

Fig. 9. Representation of the possible risk.

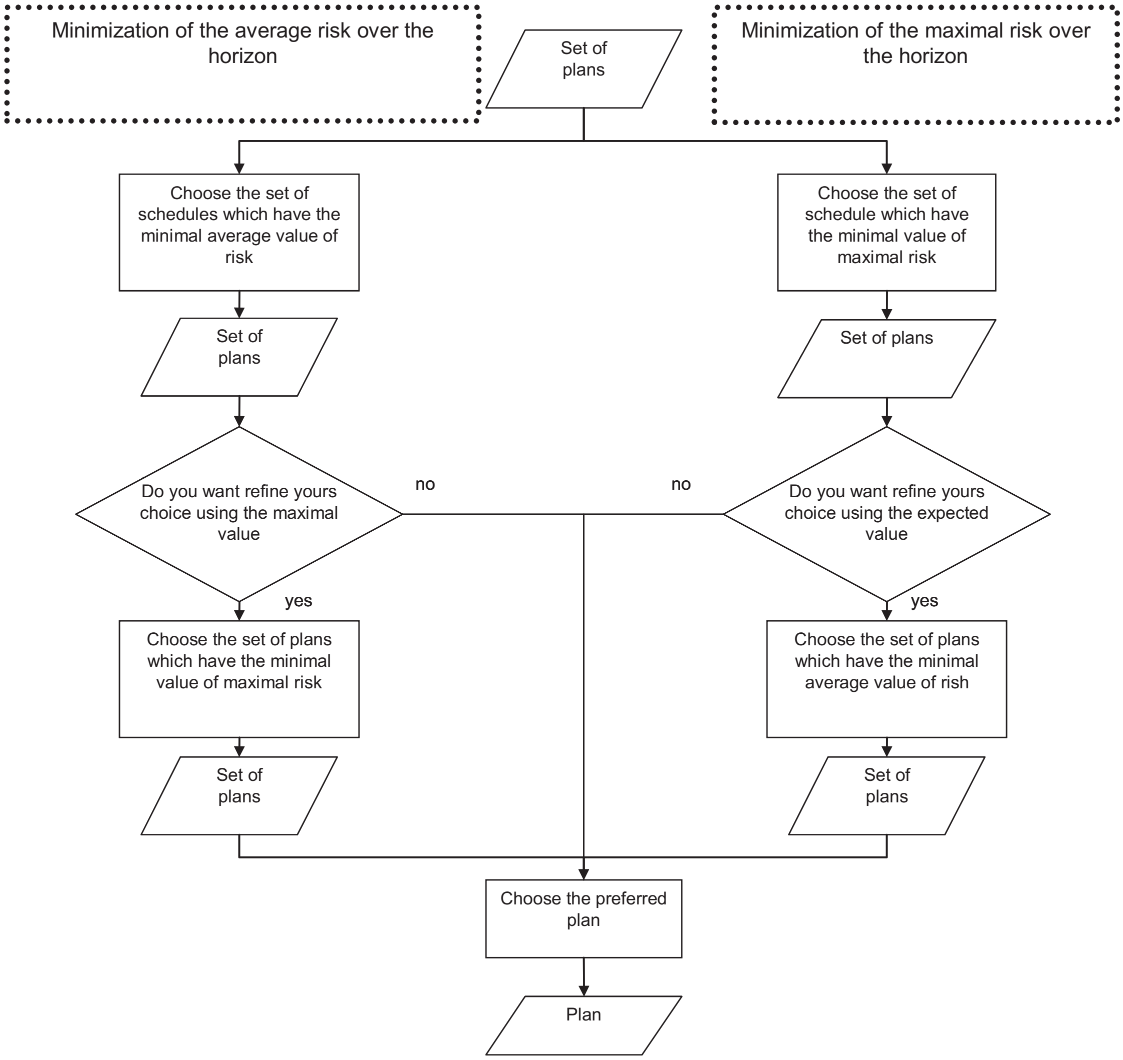

Fig. 10. Flow chart of the methods for selecting a plan.

$$Er_{p,s} = \sum_{t=1}^{T} E^{*}\left[\tilde{B}_{t,s,p}\right] = \sum_{t=1}^{T} \int_{-\infty}^{+\infty} x d\Pi_{\tilde{B}_{t,s,p}}([x, +\infty[) \quad \forall p \in P, \; s \in S \tag{8}$$

$$Er_p = \sum_{s \in S} Er_p^s \quad \forall p \in P \tag{9}$$

We then select the plan allowing that the maximal expected backordering level is minimal depending on the supplier (Eqs. (10) and (11)).

$$Er = \arg\min_{p \in P}(Er_p) \tag{10}$$

$$P_{avg} = p | Er_p = Er \tag{11}$$

Depending on the decision of the DM, we choose the preferred plan in the set P_{avg} or we apply the selection process «minimization of maximal risk» (Section 6.2.2) to the set P_{avg}. We then obtain the set $P_{avg,\max} \subseteq P_{avg}$ and we choose the preferred plan in this set.

6.2.2. Minimization of the maximal risk

In this article, we use a cost function $f(x)$ which is the impact in terms of backordering level, in place of the qualitative utility function $u(x) \in [0;1]$ so that the decision maker does not need to formalize his/her utility. As a consequence, the Shilkret integral becomes:

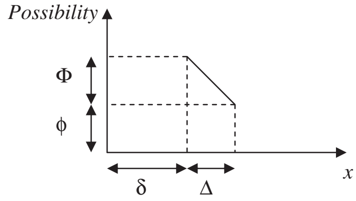

Fig. 11. Linear function of backordering.

$$Sh(d) = \max_x(\pi^d(x) \times f(x), \quad \forall \pi^d(x), \quad f(x) \in [0; +\infty)) \tag{12}$$

In order to select the set of plans that minimizes the maximal risk, we calculate the minimal maximal risk level over the plans ($p \in P$), over the suppliers ($s \in S$) and periods ($t \in H$) and we select the plan allowing that the maximal risk level is minimal (Eqs. (13)–(15)).

$$Mr_p = \max_{s \in S, t \in H}\left(sh\left(\tilde{B}_{t,s,p}\right)\right) = \max_{s \in S, t \in H}\max_x(x \times \Pi_{\tilde{B}_{t,s,p}}(x)) \tag{13}$$

$$Mr. = \min_{p \in P}(Mr_p) \tag{14}$$

$$P_{\max} = p \middle| Mr_p = Mr \tag{15}$$

Algorithm 3 calculates $\max_x(x \times \Pi_{\tilde{B}_{t,s,p}}(x))$.

Algorithm 3: Computation of Shilkret integral

Input: set of linear functions (Fig. 11) $f_i(x) = -\frac{\Phi_i}{\Delta_i}x + \frac{\Phi_i \delta_i}{\Delta_i} + \Phi_i + \varphi_i$ with $i = 1, \ldots, I$ of backordering $\tilde{B}_{t,s,p}$
Output: $sh(\tilde{B}_{t,s,p})$
$sh(\tilde{B}_{t,s,p})$ = **0**
For $i = 1$ **to** I **do**
  $x_0 = \frac{1}{2}(\Delta_i + \frac{\Delta_i \varphi_i}{\Phi_i} + \delta_i)$
  If $x_0 > (\delta_i + \Delta_i)$ **then**
    $A = (\delta_i + \Delta_i) \times f_i(\delta_i + \Delta_i)$
  End
  If $x_0 < \delta_i$ **then**
    $A = \delta_i \times f_i(\delta_i)$
  End
  If $x_0 \in [\delta_i; \delta_i + \Delta_i]$ **then**
    $A = x_0 \times f_i(x_0)$
  End
  $sh(\tilde{B}_{t,s,p}) = \max(sh(\tilde{B}_{t,s,p}); A)$
End

Proof: $\Pi_{\tilde{B}_{t,s,p}}(x)$ is decomposed in a set of i linear functions ($f_i(x) = a_i x + b_i$) with negative coefficients ($a_i < 0$) for $x \in [\delta_i; \delta_i + \Delta_i] = X_i$ (cf. Fig. 11). So $sh(\tilde{B}_{t,s,p}) = \max_i \max_{x \in X_i}(x \times f_i(x))$ and $x \times f_i(x)$ is maximal for $x_0 = \frac{1}{2}(\Delta_i + \frac{\Delta_i \varphi_i}{\Phi_i} + \delta_i)$. If $x_0 \in [\delta_i; \delta_i + \Delta_i]$,$\max_{x \in X_i}(x \times f_i(x)) = x_0 \times f_i(x_0)$. Moreover, the function $x \times f_i(x)$ is decreasing on $]x_0; +\infty[$ and increasing on$]-\infty; x_0[$. So, if $x_0 > (\delta_i + \Delta_i)$ then $\max_{x \in X_i}(x \times f_i(x)) = (\delta_i + \Delta_i) \times f_i(\delta_i + \Delta_i)$ and if $x_0 < \delta_i$ then $\max_{x \in X_i}(x \times f_i(x)) = \delta_i \times f_i(\delta_i)$.

Depending on the decision of the DM, we choose the preferred plan in the set $P_{\max}$ or we apply the selection process «minimization of average risk» (see Section 6.2.1) to the set. We then obtain the set $P_{\max,avg} \subseteq P_{\max}$ and we choose the preferred plan in this set.

7. Classical approach vs. suggested method versus: an illustrative example

We illustrate in this section the main differences between the suggested method and the classical approach which would be used in an industrial context, based on grouping the orders, then performing a Material Requirement Planning calculation as defined in MRP2 (which is not devoted to the MTO logic, but within which specific orders may be considered). Of course, we do not perform here a strict comparison between the two approaches, since MRP2 does not take into account the knowledge included in the possibilistic framework, and therefore cannot provide as rich results as the suggested method. The objective is only here to show, on an example close to an industrial case, that the additional complexity needed by the suggested method may really allow to better support the decision of the human actors.

We consider a plan composed of ten orders (order 1 to order 10). Orders 1, 3, 5, 7, 9 concern Product 1, whereas orders 2, 4, 6, 8, 10 concern Product 2. Product 1 includes 8 units of the component, whereas Product 2 requires 22 units of the same component. Scraps may occur during the assembly process, making that more components may be necessary to obtain the right number of final products.

Two variants of the component may be chosen by the customers, each provided by a different supplier. Nevertheless, this choice is not yet sure when the plan has to be built. The two suppliers have a limited capacity, which should be taken into account when building the plan.

7.1. Classical approach

In a classical approach, a precise plan by period is built according to the demand, supposed to be precisely known. A MRP calculation is then performed, taking into account increased quantities due to scraps. For coping with customization, the decision maker would assess the probability associated to each configuration for each order, on the base of subjective knowledge (exchanges with the customer) or on a statistical base using historical data. As often done empirically in companies, "probable" quantities required for each component would be calculated by multiplying the quantity of a component by the probability to get this variant. The cumulated gross requirements can then be calculated and compared with the cumulated capacity of each supplier. The feasibility of the plan is so checked, and the plan is adopted if it is feasible. Otherwise, the plan is modified, most of the time by moving some orders or asking the suppliers to punctually increase their capacity.

Let us illustrate this process with numerical values. The starting dates of the orders according to one of the considered plan (Plan 1) are given in Table 2 (1 for day 1, 2 for day 2, etc).

According to the bill of materials, 8 components are required for Product 1 and 22 for Product 2. Nevertheless, up to 6 components may be scrapped during the assembly of Product 1, and up to 9 for Product 2. With a pessimistic attitude, the decision maker would increase the number of required components of the maximum level of scraps, leading to requirements of 14 components for each Product 1, and 31 for each Product 2. The probabilities for having the version of the components provided by each supplier are given in Table 3 for each order. Getting such information is mandatory but difficult, since such precise data are seldom known.

It is now possible to build a plan by periods (here, weeks) in order to have a global view on the feasibility of Plan 1 (see Table 4). The orders have been allocated to their period in lines 1 and 2 of Table 4. The required quantity of each component is then calculated according to the BOMs of Products 1 and 2 (increased by their maximum scrap) and the probability to have each variant confirmed. Only Supplier 1 is here considered as an example. His nominal capacity being 41 parts by period, he will be able to provide the required quantity in each period: the plan is considered as feasible (see Table 4).

Nevertheless, it is clear that several points may set into question the real feasibility, but also the efficiency of this plan:

- the BOMS have been increased by the maximal value of the scraps, which makes highly probable that all the ordered components will not be used. Nevertheless, no indicator is provided here on the risk taken if the quantity is decreased.
- the probabilities provided in Table 3 are far from being certain. What is the robustness of the plan, or the risk taken if these probabilities are false, is not known (the probabilities only give an image of an "average plan").
- in order to build the plan, the dates of requirements have been considered as precise whereas they may be slightly adapted in order to improve the plan (e.g. in the aeronautic sector, delivery can be done at due date ±7 days). This degree of freedom could be used to optimise a plan, but can hardly be explicitly mentioned in Table 4: using a classical approach, a new precise plan with other dates would have to be separately built, then assessed with a "test and trial" approach, which may require a lot of time.

Table 2
Crisp dates of the orders – Plan 1.

Order	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8	Order 9	Order 10
Crisp dates	7	5	11	6	15	9	21	22	24	28

Table 3
Probability to need the version of each supplier.

	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6	Order 7	Order 8	Order 9	Order 10
Supplier 1	0,7	0,7	0,6	0,3	0,4	0,6	0,2	0,2	0,5	0,6
Supplier 2	0,3	0,3	0,4	0,7	0,6	0,4	0,8	0,8	0,5	0,4

Table 4
Analysis of feasibility of Plan 1.

	Week 1 (1–7)	Week 2 (8–14)	Week 3 (15–21)	Week 4 (22–28)	Week 5 (29–35)
Product 1	O1	O3	O5, O7	O9	0
Product 2	O2, O4	O6	0	O8, O10	0
Component 1	41	27	17	32	0
Component 2	36	18	0	45	0
Cumulated GR Supp. 1	41	68	85	117	117
Cum. Cap. supplier 1	42	84	126	168	210
Cumulated GR Supp. 2	36	54	54	99	99
Cum. Cap. supplier 2	42	84	126	168	210

– a close point is that the plan does not take into account the fact that the date when an order will be processed may be precisely known at short term, but becomes more and more imprecise when the order is planned far in the future (because of the cumulative effect of the disturbances which will necessarily occur). In Table 4, all the processing dates are considered as precisely positioned in a given week.
– finally, a customer may have punctually some extra-capacity, for instance using extra hours. This is not taken into account in the plan but may be considered as additional information by the decision maker, using his own expertise.

As a conclusion, the plan provided in Table 4 is very simple, but is in fact an "ideal" view of what may happen, without any measure of the associated confidence. We shall illustrate in Section 7.2 how the suggested possibilistic framework may allow to model the available degrees of freedom, and may provide measures of the risk taken if a given plan is adopted, resulting in a much more complex but more informative assessment of possible plans.

7.2. Suggested "possibilistic" approach

7.2.1. Modelling of imprecise data

The same orders than in Section 7.1 are considered, but the degrees of freedom which may be used on the starting dates are modelled by fuzzy numbers, as suggested in Section 4.1.2. Let us consider that the decision maker (DM) proposes four plans (see Table 5). We consider here that the position of the orders in the plans is not anymore a crisp value like in Table 2 but a fuzzy number. It can be seen in Table 5 and in the graphical representation of Plan 1 provided in Fig. 12 that the imprecision on the positioning of an order increases through time. The preferences of the DM over the plans are the followings: $P1 \succ P2 \succ P3 \succ P4$.

It is now possible to explain more subtly that scraps are possible during the assembly of the products: let us consider for instance that Product 1 needs (10; 12; 2; 2; 1) components (i.e. at least 8 and no more than 14, but most likely between 10 and 12) while product 2 needs (25; 28; 3; 3; 1) components. Similarly, the possibility levels concerning the choice of a possible supplier, linked to the customization, are represented in Table 6. The information is here weaker (since the two values are independent) but much more robust (for the same reason) than when using probabilities as in Table 3.

From the fuzzy bill of materials of Product 1 and Product 2, and from the possibility levels concerning customization, we can build the required quantity for each order and each supplier. Again, we only consider here the first supplier:

Required quantity for supplier 1 of order 1: (10; 12; 2; 2; 1); possibility level is 1 so $\tilde{R}_1^1 = (10; 12; 2; 2; 1)$.

In the same way: $\tilde{R}_2^1 = (25; 28; 3; 3; 1)$ $\tilde{R}_3^1 = (10; 12; 2; 2; 1)$ $\tilde{R}_4^1 = (25; 28; 3; 3; 0.5)$ $\tilde{R}_5^1 = (10; 12; 2; 2; 0.75)$ $\tilde{R}_6^1 = (25; 28; 3; 3; 1)$ $\tilde{R}_7^1 = (10; 12; 2; 2; 0.25)$ $\tilde{R}_8^1 = (25; 28; 3; 3; 0.25)$ $\tilde{R}_9^1 = (10; 12; 2; 2; 1)$ $\tilde{R}_{10}^1 = (25; 28; 3; 3; 1)$.

7.2.2. Computation of the maximal cumulative gross requirements $\tilde{B}B_t^+$

The first step is to compute the set of scenarios by dates of the maximal cumulative gross requirements $\boldsymbol{C}_t^{\pi}$. In that purpose, we apply Algorithm 1:

Table 5
Plan 1 with fuzzy starting dates.

	Plan 1	Plan 2	Plan 3	Plan 4
Order 1	(9; 9; 2; 1)	(6; 6; 1; 1)	(6; 6; 1; 1)	(5; 5; 2; 1)
Order 2	(6; 6; 1; 1)	(9; 9; 1; 1)	(24; 24; 2; 2)	(19; 19; 2; 4)
Order 3	(15; 15; 4; 1)	(10; 10; 2; 2)	(26; 26; 4; 1)	(10; 11; 1; 2)
Order 4	(8; 9; 2; 2)	(12; 13; 2; 2)	(28; 28; 3; 3)	(15 ; 16 ; 2; 2)
Order 5	(17; 17; 2; 3)	(25; 27; 3; 4)	(17; 17; 2; 3)	(8; 8; 1; 2)
Order 6	(10; 11; 1; 1)	(17; 18; 3; 3)	(10; 11; 1; 1)	(18; 18; 2; 2)
Order 7	(26; 27; 5; 1)	(16; 16; 1; 3)	(20; 21; 3; 2)	(24; 24; 2; 2)
Order 8	(23; 24; 1; 2)	(23; 24; 1; 2)	(23; 24; 1; 2)	(34; 34; 4; 4)
Order 9	(26; 28; 2; 2)	(26; 28; 2; 2)	(26; 28; 2; 2)	(26; 26; 3; 2)
Order 10	(30; 35; 2; 0)	(28; 30; 4; 3)	(28; 30; 4;3)	(32; 32; 4; 2)

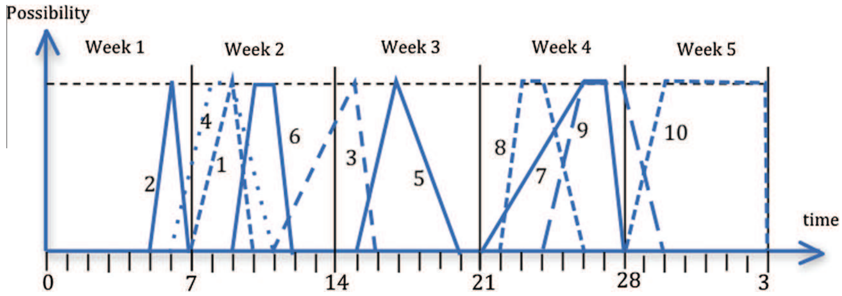

Fig. 12. Illustration of plan 1.

Table 6
Possibility levels linked to customization.

Orders:	1	2	3	4	5	6	7	8	9	10
Supplier 1	1	1	1	0.5	0.75	1	0.25	0.25	1	1
Supplier 2	0.5	0.5	0.75	1	1	0.75	1	1	1	0.75

- Compute the possibility level of each order to belong to each horizon $t = 1, \ldots, 5$ (Table 7),
- Build $\boldsymbol{C}_t^{\pi}$ (Table 8).

From Table 7, we know that order 2 belongs to horizon 1 with the possibility level 1 and order 4 belongs to horizon 1 with possibility level 0.5. For horizon 1, we have then to consider two possible scenarios by date (one with $\pi = 1$ and the second with $\pi = 0.5$): {2} and {2, 4}. In the same way, we build Table 8 according to the data mentioned in Table 7.

From Table 8 and $\tilde{R}_0^1$, we calculate the gross requirements (Algorithm (2)) by:

- computing the fuzzy quantity for each possible scenario (Table 9),
- merging the fuzzy quantities and extracting the maximal possible quantity (Table 10).

For example, the fuzzy quantity of scenario $\boldsymbol{C}_1^{0.5} = \{2, 4\}$ is composed by one combination $c = (\alpha, \pi) = (0.5, 0.5)$ because the set $\{2, 4\}$ does not have requirements with a possibility level lower than 0.5 (1 for 2 and 0.5 for 4):

$$\tilde{B}B_1^{0.5,0.5} = \tilde{R}_2^1 \oplus \tilde{R}_4^1 \oplus (0;0;0;0;0.5) = (25;28;3;3;1) \oplus (25;28;3;3;0.5) \oplus (0;0;0;0;0.5)$$

$$\tilde{B}B_1^{0.5,0.5} = (48.5\ ,\ 57.5\ ,\ 4.5\ ,\ 4.5\ ,\ 0.5)$$

Some scenarios $\boldsymbol{C}_t^{\pi}$ are composed of more than one combination $c = (\alpha, \pi)$, for example $\boldsymbol{C}_2^1 = \{1, 2, 4, 6\}$ but the maximal possibility of requirement 4 is 0.5, so we have two combinations: (0.5, 1) and (1, 1).

Horizon 1 contains two fuzzy quantities: $\tilde{B}B_1^{1,1} = (25,\ 28\ ,\ 3\ ,\ 3\ ,\ 1)$ and $\tilde{B}B_1^{0.5,0.5} = (48.5\ ,\ 57.5\ ,\ 4.5\ ,\ 4.5\ ,\ 0.5)$.

$\tilde{B}B_1 = \bigcup_{c\in\{(1,1);(0.5,0.5)\}} \tilde{B}B_1^c = (25,\ 28\ ,\ 3\ ,\ 3\ ,\ 1) \cup (48.5\ ,\ 57.5\ ,\ 4.5\ ,\ 4.5\ ,\ 0.5)$. To compute the maximal backordering level, we only need the maximal possible cumulative gross requirement. Fig. 13 shows how to extract the maximal cumulative gross requirement ($\tilde{B}B_1^+$ in black) from the fuzzy maximal cumulative gross requirement ($\tilde{B}B_1$ in grey dotted line).

On Fig. 14 we represent graphically the five maximal cumulative gross requirements (for horizon 1–5) that are detailed in Table 10. For horizon $t = 1$, the maximal gross required quantity for possibility level 1 is 28, the quantity increases and the

Table 7
Possibility level of each order to belong to each horizon t.

Horizon/orders	$t = 1$: [0;7]	$t = 2$: [0;14]	$t = 3$: [0;21]	$t = 4$: [0;28]	$t = 5$: [0;35]
1	0	1	1	1	1
2	1	1	1	1	1
3	0	0.75	1	1	1
4	0.5	1	1	1	1
5	0	0	1	1	1
6	0	1	1	1	1
7	0	0	0	1	1
8	0	0	0	1	1
9	0	0	0	1	1
10	0	0	0	0	1

Table 8
Resulting scenarios $\boldsymbol{C}_t^{\pi}$.

Possibility π	Horizon t				
	1	2	3	4	5
1	2	1,2,4,6	1,2,3,4,5,6	1,2,3,4,5,6,7,8,9	1,2,3,4,5,6,7,8,9,10
0.75		1,2,3,4,6			
0.5	2,4				

Table 9
Fuzzy quantity for each possible scenario $c = (\alpha, \pi)$.

[0; 7]	[0; 14]	[0; 21]	[0; 28]	[0; 35]
$c = (1,1)$ (25; 28; 3; 3; 1)	$c = (1,1)$ (60; 68; 8; 8; 1)	$c = (1,1)$ (80; 92; 12; 12; 1)	$c = (1,1)$ (80; 92; 12; 12; 1)	$c = (1,1)$ (105; 120; 15; 15; 1)
$c = (0.5,0.5)$ (48.5; 57.5; 4.5; 4.5; 0.5)	$c = (0.5,1)$ (81; 100; 7; 7; 0.5)	$c = (0.75,1)$ (87; 107; 11; 11; 0.75)	$c = (0.75,1)$ (87; 107; 11; 11; 0.75)	$c = (0.75,1)$ (111.25; 135.75; 13.25; 13.25; 0.75)
	$c = (0.75,0.75)$ (67.5; 82.5; 7.5; 7.5; 0.75)	$c = (0.5,1)$ (108.33; 138.66; 10.33; 10.33; 0.5)	$c = (0.5,1)$ (108.33; 138.33; 10.33; 10.33; 0.5)	$c = (0.5,1)$ (131.83; 168.16; 11.83; 11.8; 0.5)
	$c = (0.5,1)$ (90; 113; 8; 8; 0.5)		$c = (0.25,1)$ (138.16; 183.83; 10.16; 10.16; 0.25)	$c = (0.25,1)$ (160.91; 214.09; 10.91; 10.91; 0.25)

Table 10
Maximal cumulative quantity of gross requirements.

$t = 1$	Quantity	28	29.5	57.5	62				
	Possibility	1	0.5	0.5	0				
$t = 2$	Quantity	68	70	82.5	85	113	121		
	Possibility	1	0.75	0.75	0.5	0.5	0		
$t = 3$	Quantity	92	95	107	110.66	138.66	149		
	Possibility	1	0.75	0.75	0.5	0.5	0		
$t = 4$	Quantity	92	95	107	110.66	138.66	143.5	183.83	194
	Possibility	1	0.75	0.75	0.5	0.5	0.25	0.25	0
$t = 5$	Quantity	120	123.25	135.75	140.16	168.16	174	214	225
	Possibility	1	0.75	0.75	0.5	0.5	0.25	0.25	0

possibility decreases until 29.5 for a possibility level 0.5; the quantity increases till 57.5 with possibility 0.5, and the quantity increases and the possibility decreases until 0.

7.2.3. Computation of the possible backordering

From the maximal cumulative gross requirements and the maximal capacity of the supplier (Table 11), we calculate the maximal backordering level (Table 12) for each horizon t using Eq. (7). For example, we do not have backorders until possibility 0.5. Thirteen backorders are possible with a possibility level 0.5. Backorders increase up to 20 with possibility 0.

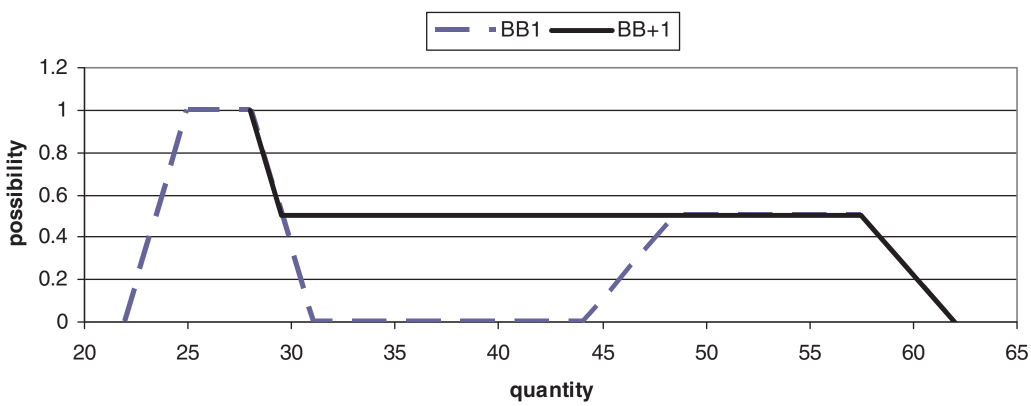

Fig. 13. Representation of $\tilde{BB}_1$ and $\tilde{BB}_1^+$.

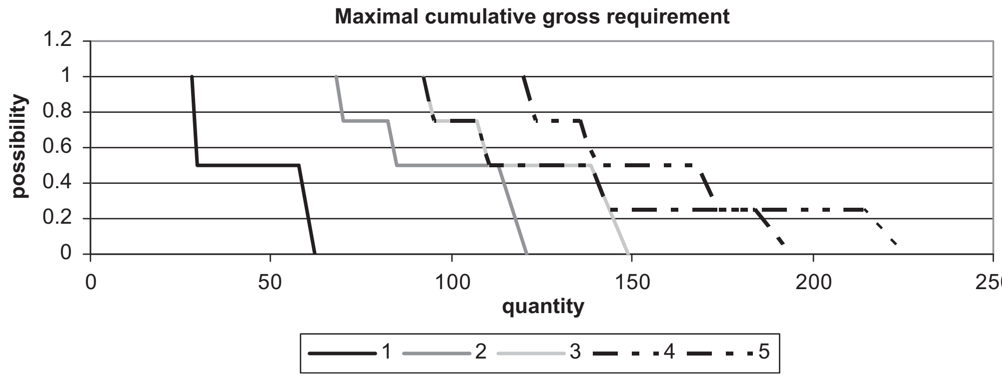

Fig. 14. Maximal cumulative gross requirements.

Table 11
Maximal capacity of the supplier.

	Period									
	1		2		3		4		5	
Quantity	42	47	42	47	42	47	42	47	42	47
Possibility	0	1	0	1	0	1	0	1	0	1

Table 12
Maximal Backordering level.

$t = 1$	Quantity	0	0	13	20
	Possibility	1	0.5	0.5	0
$t = 2$	Quantity	0	0	24	37
	Possibility	1	0.5	0.5	0
$t = 3$	Quantity	0	0	5.16	23
	Possibility	1	0.5	0.5	0
$t = 4$	Quantity	0	0	10.83	26
	Possibility	1	0.25	0.25	0
$t = 5$	Quantity	0	0	15	
	Possibility	1	0.216	0	

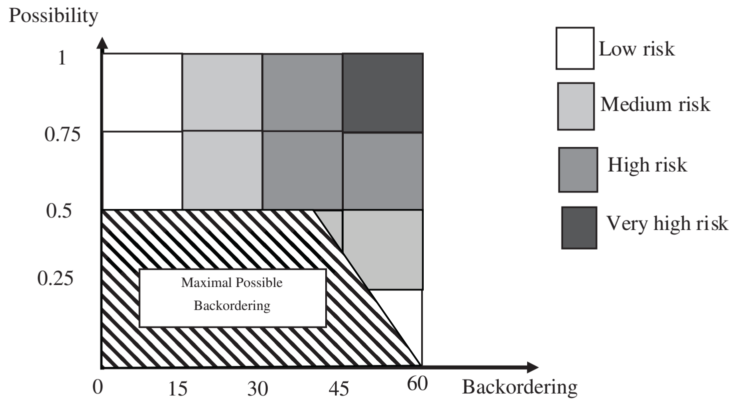

Fig. 15. Evaluation of the risk of plan 1.

7.2.4. Selection of the less risked plan

The first step of selection of the less risked plan is the choice of the criterion to minimize. In that purpose, we place the possible level of backordering (which is the union of all possible backordering for each plan, period and supplier) in the risk matrix built by the DM(s). The result is illustrated in Fig. 15.

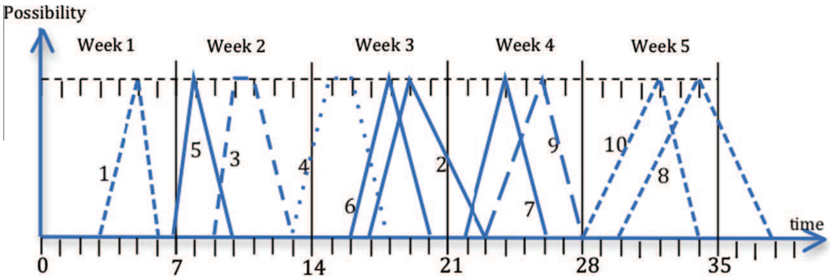

Fig. 16. Chosen plan (plan 4).

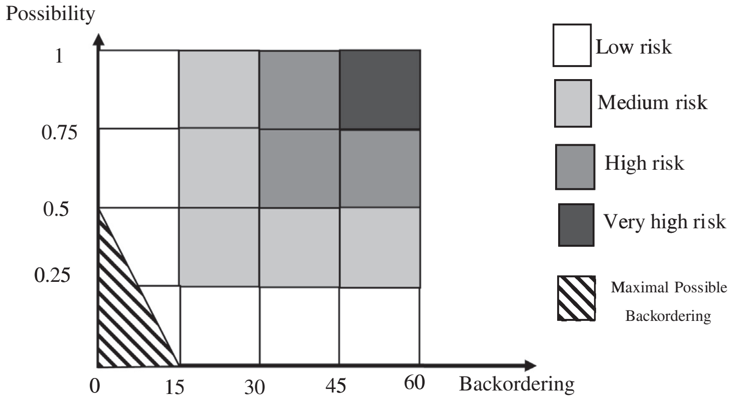

Fig. 17. Risk evaluation of the chosen plan.

From Fig. 15, we see that the maximal risk is close to the "high level", so the DM chooses the minimization of the Shilkret integral to minimize the maximal risk.

We compute the value of Shilkret integral for all the possible plans using Algorithm 4. The results are:

- *P1:* 13.08
- *P2:* 19.58
- *P3:* 16.8
- *P4:* 1.875

The best plan is *P4* knowing that the set $P_{max} = \{P4\}$. The plan *P4* is represented on Fig. 16, and the maximal backordering level of this plan on Fig. 17.

It can be seen with this example that the suggested method takes into account much more information than a classical approach, and uses the possibilistic framework for defining indictors related to the risk of a candidate plan, which cannot be obtained from a classical method.

8. Conclusion and perspectives

In this paper, we suggest, in the context of collaborative planning within a supply chain, a new method to calculate the maximal cumulative gross requirements, so that a general method to choose the less risked supply plan. This method allows the customer to take into account his knowledge on the possible customization and on the imprecision on the dates and quantities of the requirements. We suggest five possible decision processes based on two measures: the average risk and the maximal risk. These two measures provide complementary views that the DM may consider separately or may combine for defining a new utility function. This study will support the decision maker in the choice of the optimisation criteria. As a perspective, from the academic point of view, it may now be interesting to study the impact on the supplier of the strategy to give or not his maximal capacity to the customer. More generally, testing this approach on real situations requires to find a way to "hide" most of the mathematical formalism to potential users, while focusing on the visualisation of the possible risks of each plan. Contacts have been made with industrial partners in that purpose.

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