A Generalized Semi-Analytical model for delay/Doppler altimetry

Abderrahim Halimi, Corinne Mailhes, Jean-Yves Tourneret, François Boy, Thomas Moreau

A GENERALIZED SEMI-ANALYTICAL MODEL FOR DELAY/DOPPLER ALTIMETRY

A. Halimi[(1)]*, C. Mailhes*[(1)]*, J.-Y. Tourneret*[(1)]*, F. Boy*[(2)] *and T. Moreau*[(3)]

[(1)] University of Toulouse, IRIT-ENSEEIHT-TéSA, Toulouse, France
[(2)] Centre national d'études spatiales (CNES), Toulouse, France
[(3)] Collecte Localisation Satellite (CLS), Ramonville Saint-Agne, France
{abderrahim.halimi, corinne.mailhes, jean-yves.tourneret}@enseeiht.fr, Francois.Boy@cnes.fr,tmoreau@cls.fr

ABSTRACT

This paper introduces a new model for delay/Doppler altimetry, taking into account the effect of antenna mispointing. After defining the proposed model, the effect of the antenna mispointing on the altimetric waveform is analyzed as a function of along-track and across-track angles. Two least squares approaches are investigated for estimating the parameters associated with the proposed model. The first algorithm estimates four parameters including the across-track mispointing (which affects the echo's shape). The second algorithm uses the mispointing angles provided by the star-trackers and estimates the three remaining parameters. The proposed model and algorithms are validated via simulations conducted on both synthetic and real data.

***Index Terms*—** Delay/Doppler altimetry, antenna mispointing, Cryosat-2, least squares estimation

1. INTRODUCTION

Delay/Doppler altimetry (DDA) is a new technology whose principle has been introduced in [1]. It aims at reducing the measurement noise and increasing the along-track resolution in comparison with conventional pulse limited altimetry. An interesting semi-analytical model was recently proposed in [2] for DDA. However, this model is valid when there is no mispointing angle or when the mispointing angle can be neglected. This paper proposes a generalized semi-analytical model for DDA taking into account the antenna mispointing angle. More precisely, we derive an analytical expression for the flat surface impulse response (FSIR) considering antenna mispointing angles, a circular antenna pattern, no vertical speed effect and a uniform scattering. The two dimensional delay/Doppler map is then obtained by a numerical convolution between this analytical FSIR expression, the probability density function (PDF) of the heights of the specular scatterers and the time/frequency point target response (PTR) of the radar. After defining the proposed model, the effect of the antenna mispointing on the waveform is analyzed. The parameters of the generalized delay/Doppler model are then estimated using a least squares approach as in [2]. The proposed model and estimation algorithm are validated on both synthetic and real waveforms. The obtained results are very promising and show the accuracy of this generalized model with respect to the previous model assuming absence of antenna mispointing.

2. A NEW GENERALIZED DELAY/DOPPLER MODEL

The mean power of a delay/Doppler echo can be expressed as the convolution of three terms [3, 4]. The power term depends on two dimensions, i.e, time and Doppler frequency as follows

$$P(t,f) = \text{FSIR}(t,f) * \text{PDF}(t) * \text{PTR}(t,f) \tag{1}$$

$$\text{with} \quad \begin{cases} \text{PDF}(t) &= \frac{1}{\sqrt{2\pi}\sigma_s}\exp\left(-\frac{t^2}{2\sigma_s^2}\right) \\ \text{PTR}(t,f) &= \text{PTR}_T(t)\text{PTR}_F(f) \\ \text{PTR}_T(t) &= \left|\frac{\sin\left(\pi\frac{t}{T}\right)}{\pi\frac{t}{T}}\right|^2 \\ \text{PTR}_F(f) &= \left|\frac{\sin\left(\pi\frac{f}{F}\right)}{\pi\frac{f}{F}}\right|^2 \end{cases} \tag{2}$$

where t is the two-way incremental ranging times, i.e., $t = t' - \frac{2h}{c}$, with t' is the travel time of the echo from the instant of transmission, h is the altitude of the satellite, c is the speed of light, T is the sampling period, F is the frequency resolution and σ_s is linked to the significant wave height SWH by $\sigma_s = \frac{\text{SWH}}{2c}$. The proposed analytical expression for the generalized FSIR (that considers the antenna mispointing angles) is obtained by integrating the reflected energy at each time instant t from each Doppler beam n (see Fig. 1). The resulting FSIR is given by

$$\begin{aligned} \text{FSIR}(t,n) &= \frac{P_u}{\pi}\left(1+\frac{ct}{2h}\right)^{-3} U(t) \\ &\times \exp\left[-\frac{4}{\gamma}\left(1-\frac{\cos^2(\xi)}{1+\epsilon^2(t)}\right)+\frac{b}{2}\right] I_0\left(\frac{b}{2}\right) \\ &\times \left[I_0(a)\left(\phi_{t,n+1}-\phi_{t,n}\right)+\sum_{k=1}^{m}\frac{1}{k}I_k(a)\,h_{k,n}(\tilde{\phi})\right] \end{aligned} \tag{3}$$

where

$$h_{k,n}(\tilde{\phi}) = \begin{cases} 2\cos\left(k\tilde{\phi}\right)\left[\sin\left(k\phi_{t,n+1}\right) - \sin\left(k\phi_{t,n}\right)\right] \\ \text{for even } k \\ \\ -2\sin\left(k\tilde{\phi}\right)\left[\cos\left(k\phi_{t,n+1}\right) - \cos\left(k\phi_{t,n}\right)\right] \\ \text{for odd } k \end{cases} \tag{4}$$

and γ is an antenna beam width parameter, P_u is the waveform amplitude, $\phi_{t,n} = \text{Re}\left[\arctan\left(\frac{y_n}{\sqrt{\rho^2(t)-y_n^2}}\right)\right]$, $y_n = \frac{h\lambda}{2v_s} f_n$ is the coordinate of the nth along-track beam, λ is the wavelength, v_s is the satellite velocity, $f_n = (n - 32N_f - 0.5)\frac{F}{N_f}$ is the nth Doppler frequency (with $n \in \{1, \cdots, 64N_f\}$), N_f is the frequency oversampling factor, $\epsilon(t) = \frac{\rho(t)}{h} = \sqrt{\frac{ct}{h}}$, $a(t,\xi) = \frac{4\epsilon(t)}{\gamma}\frac{\sin(2\xi)}{1+\epsilon^2(t)}$, $b(t,\xi) = \frac{4\epsilon^2(t)}{\gamma}\frac{\sin^2(\xi)}{1+\epsilon^2(t)}$, ξ (resp. $\tilde{\phi}$) is the mispointing angle with respect to the z axis (resp. the x axis [2]), $U(.)$ is the Heaviside function and I_k is the kth order modified Bessel function of the first kind.

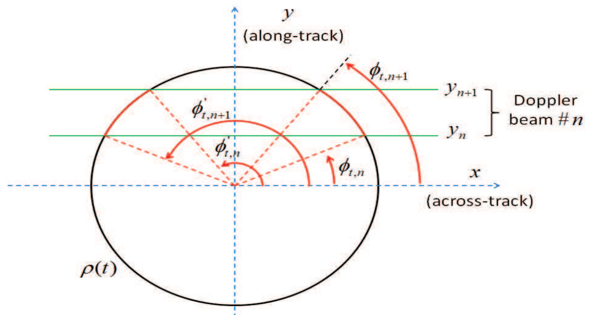


Fig. 1: Integrating angles for specific circle of propagation and Doppler beam.

The resulting signal is shifted by the time instant τ which is known as the epoch parameter (related to the range between the satellite and the observed surface). Note that the final altimetric signal depends on the following parameter vector $\boldsymbol{\theta} = (\text{SWH}, P_u, \tau, \xi_{\text{ac}}, \xi_{\text{al}})^T$ where $\xi_{\text{al}} = \xi\sin\left(\tilde{\phi}\right)$ and $\xi_{\text{ac}} = \xi\cos\left(\tilde{\phi}\right)$ are the across-track and along-track mispointing angles.

The reflected power $P(t,f)$ (known as a delay/Doppler map) is finally obtained by a numerical computation of the double convolution in (1) using the FSIR formula given in (3). In order to obtain a "multi-look" altimetric waveform, a delay compensation operation is applied to each Doppler beam followed by the sum of these beams [5, 6]. The resulting multi-look delay/Doppler signal can be written $s(t) = \sum_{n=1}^{N} P\left(t - \delta t_n, f_n\right)$, where δt_n is the delay compensation expressed in seconds.

Figures 2(a) and 2(b) show the behavior of the multi-look echo as functions of the along-track and across-track mispointing angles, respectively. Fig. 2(a) shows that the along-track mispointing reduces the amplitude of the multi-look echo (top figure) while it does not change the shape of the waveform as observed in the normalized waveforms (bottom figure). Fig. 2(b) shows that the across-track mispointing reduces the amplitude of the multi-look echo (top figure) but it also affects the shape of the waveform as observed in the normalized waveforms (bottom figure). The obtained results show that the shape of the multi-look echo depends on the parameter vector $\widetilde{\boldsymbol{\theta}} = (\text{SWH}, \tau, P_u, \xi_{\text{ac}})$ while ξ_{al} mainly affects the amplitude of the waveform already considered in P_u. Therefore, we propose to estimate the four parameters in $\widetilde{\boldsymbol{\theta}}$ and to assume $\xi_{\text{al}} = 0°$ in the following. Note finally that the discrete multi-look echo is gathered in the vector $\boldsymbol{s} = (s_1, \cdots, s_K)^T$, where $K = 128$ samples (so called "gates") and $s_k = s(kT)$.

3. PARAMETER ESTIMATION

As in [2], we propose to estimate the parameters of the generalized DDA model by using the following least-squares strategy

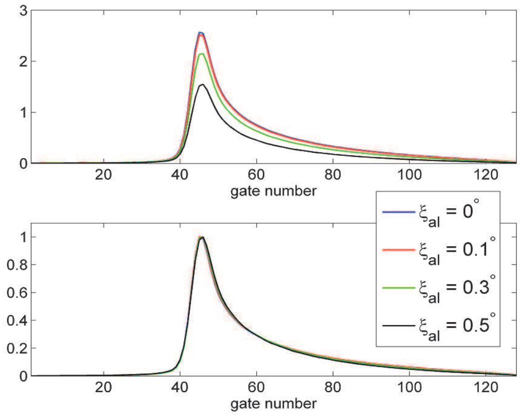
$$\hat{\boldsymbol{\theta}}_{\text{LS}} = \underset{\boldsymbol{\theta}}{\text{argmin}}\,\frac{1}{2}\sum_{k=1}^{K} g_k^2(\boldsymbol{\theta}) \tag{5}$$

where $g_k(\boldsymbol{\theta}) = y_k - s_k(\boldsymbol{\theta})$ is the vector of residues, $\boldsymbol{y} = (y_1, \ldots, y_K)^T$ is a noisy version of $\boldsymbol{s}(\boldsymbol{\theta}) = [s_1(\boldsymbol{\theta}), \ldots, s_K(\boldsymbol{\theta})]^T$ which depends on the parameter vector of interest $\boldsymbol{\theta}$. In this paper, we propose to solve (5) using the Levenberg-Marquardt algorithm [7]. This algorithm uses a gradient descent approach to update the vector of parameters $\boldsymbol{\theta}$ as follows

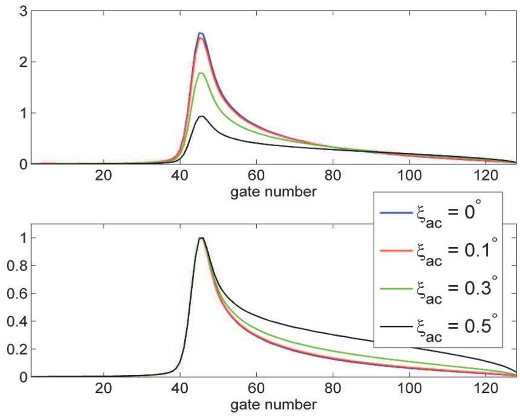
$$\boldsymbol{\theta}^{(i+1)} = \boldsymbol{\theta}^{(i)} - \left[\boldsymbol{J}^T\boldsymbol{J} + \mu\boldsymbol{I}_J\right]^{-1}\boldsymbol{J}^T\boldsymbol{g}\left(\boldsymbol{\theta}^{(i)}\right) \tag{6}$$

where $\boldsymbol{\theta}^{(i)}$ is the estimate of $\boldsymbol{\theta}$ at the ith iteration, $\boldsymbol{J} = \boldsymbol{J}\left(\boldsymbol{\theta}^{(i)}\right) = \left[\frac{\partial\boldsymbol{g}\left(\boldsymbol{\theta}^{(i)}\right)}{\partial\theta_1}, \cdots, \frac{\partial\boldsymbol{g}\left(\boldsymbol{\theta}^{(i)}\right)}{\partial\theta_J}\right]$ is a $K \times J$ matrix such that $\frac{\partial\boldsymbol{g}(\boldsymbol{\theta})}{\partial\theta_j} = \left[\frac{\partial g_1(\boldsymbol{\theta})}{\partial\theta_j}, \cdots, \frac{\partial g_K(\boldsymbol{\theta})}{\partial\theta_j}\right]^T$, J is the number of parameters to estimate, $\boldsymbol{I}_J$ is the $J \times J$ identity matrix and μ is a regularization parameter.

There are different ways to take advantage of the proposed model. The first approach, denoted by DDA4, estimates four altimetric parameters $\widetilde{\boldsymbol{\theta}} = (\text{SWH}, P_u, \tau, \xi_{\text{ac}})^T$ while considering $\xi_{\text{al}} = 0°$ (since this parameter mainly affect the echo's amplitude). In the second approach, we introduce the values of ξ_{al} and ξ_{ac} estimated by the star-tracker of the satellite in the proposed model and we only estimate the three parameters $\boldsymbol{\theta} = (\text{SWH}, P_u, \tau)^T$. The resulting approach is denoted by generalized DDA3 (G-DDA3). The next section compares these approaches with the method introduced in [2] neglecting mispointing angles (denoted by DDA3).



(a) Variation of ξ_{al} ($\xi_{\text{ac}} = 0°$).



(b) Variation of ξ_{ac} ($\xi_{\text{al}} = 0°$).

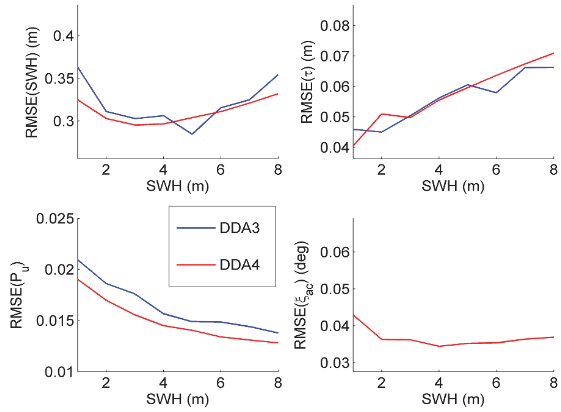
Fig. 2: Effect of (a) the along-track mispointing and (b) the across track mispointing on (top) the multi-look echoes and (bottom) the normalized multi-look echoes (obtained with $P_u = 1, \tau = 44$ gates and SWH $= 3$ meters).

4. SIMULATION RESULTS

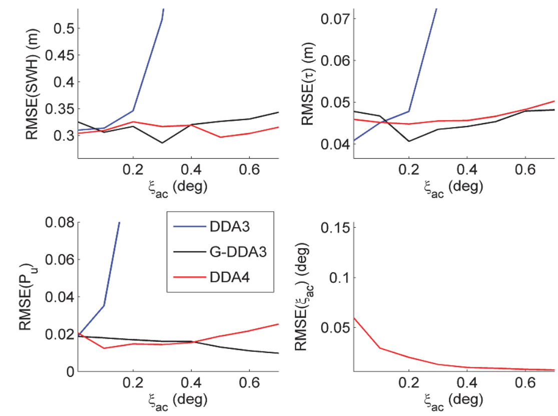
4.1. Synthetic waveforms

The performance of the proposed model and estimation algorithms has been evaluated by comparing the root mean square errors (RMSEs) of the estimated parameters when considering synthetic data with known parameter values. For real data, we compare the means and the standard-deviations (STDs) of the estimated parameters obtained with the different approaches. The first experiment considers synthetic echoes without antenna mispointing with varying SWH in the interval $[1, 8]$ meters. Fig. 3 (a) shows a similar performance for DDA3 and DDA4 algorithms (G-DDA3 is the same as DDA3 since $\xi_{\text{ac}} = \xi_{\text{al}} = 0°$).

The second set of experiments evaluates the performance of the proposed algorithms on synthetic data when varying ξ_{ac} in the interval $[0, 0.7]$ degrees. Fig. 3 (b) shows that DDA3 (proposed in [2]) is sensitive to ξ_{ac} contrary to both DDA4 and G-DDA3 algorithms. Thus, DDA4 and G-DDA3 should be preferred to DDA3 for large values of ξ_{ac}.



(a) versus SWH ($\xi_{\text{ac}} = 0°$).



(b) versus ξ_{ac} (SWH $= 2$ m).

Fig. 3: Parameter RMSEs versus (a) SWH (b) ξ_{ac} for DDA3, G-DDA3 and DDA4 algorithms (500 Monte-Carlo runs, $P_u = 1, \tau = 31$ gates, $\xi_{\text{al}} = 0°$).

4.2. Real Cryosat-2 waveforms

This section evaluates the performance of the proposed model for 400 seconds of real Cryosat-2 waveforms. Fig. 4 shows an excellent fit between a real Cryosat-2 echo (in blue) and its estimates using DDA4 (in red). This fit can be quantified by the normalized reconstruction error (NRE) criteria defined as

$$\text{NRE} = \sqrt{\frac{\sum_{k=1}^{K}\left[y_k - s_k\left(\hat{\boldsymbol{\theta}}\right)\right]^2}{\sum_{k=1}^{K} y_k^2}}. \tag{7}$$

		τ (m)	SWH (m)	P_u	ξ_{ac} (deg)	ξ_{al} (deg)
Means	DDA3	16.274	2.289	90.213	-	-
	G-DDA3	16.270	2.247	92.803	0.083	0.0572
	DDA4	16.262	2.234	96.993	0.146	-
STDs (20 Hz)	DDA3	0.0843	0.355	1.933	-	-
	G-DDA3	0.0845	0.354	1.987	1.01×10^{-4}	7×10^{-4}
	DDA4	0.0827	0.351	1.871	0.031	-

Table 1: Means and standard deviations for DDA3, G-DDA3 and DDA4 algorithms. Best results (in green) and second best results (in red). Note that the means and STDs of G-DDA3 for ξ_{ac} and ξ_{al} are given by the star-trackers.

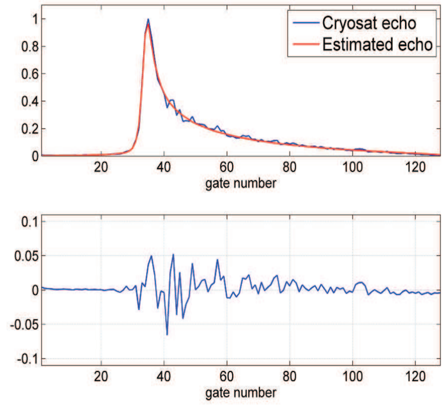


Fig. 4: Example of estimated Cryosat-2 echo using the proposed DDA4 model (NRE = 0.065). (top) real Cryosat-2 echo superimposed with its estimation, (bottom) difference between the real Cryosat-2 echo and its estimation.

Fig. 4 shows a low NRE that is mainly due to the noise corruption of the observed echo. The means and STDs of the estimated parameters are shown in Table 1. This table shows similar means for the estimated altimetric parameters when considering the three approaches. It also shows that real Cryosat-2 echoes present an across-track mispointing angle close to $0.14°$ justifying the necessity to consider the generalized model. Finally, Table 1 shows that DDA4 presents the best performance since it provides the lowest parameter STDs. This result confirms the importance of estimating the antenna mispointing angle since it improves the estimation quality.

5. CONCLUSIONS

This paper defined a generalized semi-analytical model for delay/Doppler altimetry, taking into account the effect of antenna mispointing. The parameters of this model were estimated by considering two different least squares algorithms depending on the knowledge or the absence of knowledge about mispointing angles. Both algorithms showed promising results for synthetic and real data. Future work include the generalization of the proposed model by considering an elliptical antenna as studied in [8] for the case of conventional altimetry.

6. REFERENCES

[1] R. K. Raney, "The delay/Doppler radar altimeter," *IEEE Trans. Geosci. Remote Sens.*, vol. 36, no. 5, pp. 1578–1588, Sept. 1998.

[2] A. Halimi, C. Mailhes, J.-Y. Tourneret, P. Thibaut, and F. Boy, "A semi-analytical model for delay/Doppler altimetry and its estimation algorithm," *IEEE Trans. Geosci. Remote Sens.*, 2013, to appear.

[3] C. Martin-Puig and G. Ruffini, "SAR altimeter retracker performance bound over water surfaces," in *Proc. IEEE Int. Conf. Geosci. Remote Sens. (IGARSS)*, Cape Town, South Africa, July 12-17, 2009, pp. 449 –452.

[4] L. Phalippou and F. Demeestere, "Optimal re-tracking of SAR altimeter echoes over open ocean: from theory to results for SIRAL2," in *Proc. Ocean Surf. Topogr. Sci. Team Meet.*, San Diego, California, Oct. 2011, pp. 1–18.

[5] D. Wingham, L. Phalippou, C. Mavrocordatos, and D. Wallis, "The mean echo and echo cross product from a beamforming interferometric altimeter and their application to elevation measurement," *IEEE Trans. Geosci. Remote Sens.*, vol. 42, no. 10, pp. 2305 – 2323, Oct. 2004.

[6] A. Halimi, C. Mailhes, and J.-Y. Tourneret, "Cramér-Rao bounds and estimation algorithms for delay/Doppler and conventional altimetry," in *Proc. EUSIPCO*, Marrakech-Marocco, Sept. 2013.

[7] D. P. Bertsekas, *Nonlinear programming*. Belmont, Massachusetts: Athena Scientific, 1995.

[8] D. J. Wingham and D. W. Wallis, "The rough surface Impulse Response of a Pulse-Limited Altimeter With an Elliptical Antenna Pattern," *IEEE Antennas and Wireless Propagation Letters*, vol. 9, pp. 232–235, Sept. 2010.