The First Steps in Fuzzy Set Theory in France Forty Years Ago

Didier Dubois, Henri Prade

The first steps in fuzzy set theory in France forty years ago (and before) [*]

Didier Dubois and Henri Prade

IRIT-CNRS, Université Paul Sabatier, 31062 Toulouse Cedex 09, France

Abstract

At the occasion of the fiftieth anniversary of the founding article "Fuzzy sets" by L. A. Zadeh, we briefly outline the beginnings of fuzzy set research in France, taking place some ten years later, pointing out the pioneering role of Arnold Kaufmann and few others in this emergence. Moreover, whe also point out that the French counterpart of the name "fuzzy set" had appeared some 15 years before Zadeh's paper, in a paper written in French by the very person who also invented triangular norms in the 1940's. [1]

Keywords: fuzzy set; fuzzy logic; history.

1   Before the beginning

Strangely enough, the phrase "ensembles flous" (the French translation of "fuzzy sets") first appeared in a paper published in French in the Compte-Rendus of the French Academy of Sciences in 1951 [68] by Karl Menger (1902-1985). He was an Austrian mathematician [90] who emigrated to the USA before the second World War. He is the son of the well-known economist Carl Menger (1840-1921) himself one of the fathers of the theory of subjective utility value. Karl Menger was an active member of the Vienna Circle; later he was at the origin of triangular norms with the paper "Statistical metrics" [67], where triangular norms emerge in stochastic geometry from the generalization of the classical triangle inequality when distances between two elements of a metric space are represented by probability distributions rather than by numbers. His spectrum of interest, which was very wide [70], included logic. He especially proposed a "logic of the doubtful" where (italics are from Menger himself):

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[1]This paper is a translated and expanded version of a conference paper [29].

> we divide the propositions into three mutually exclusive classes of modality: μ_+ consisting of the *asserted*, μ_0 consisting of the *doubtful*, μ_- consisting of the *negated* propositions. $[\cdots]$ In contrast to the traditional 3-valued logic, *the modality of a compound is not determined by the modalities of the components.*" [66]

He was thus making it clear very early that uncertainty is not compositional.

What is truly remarkable in Menger's 1951 paper is not only the use of the French counterpart to "fuzzy sets", but also its application to a notion closely related to Zadeh's idea: in fuzzy set terminology, the paper is about max-product transitive fuzzy relations! However, in Menger's paper, we can read (p. 2002):

> Nous appellerons cette fonction même un *ensemble flou* et nous interpréterons $\Pi_F(x)$ comme la probabilité que x appartienne à cet ensemble. Si Π_F ne prend que les valeurs 1 et 0, il s'agit essentiellement d'un sous-ensemble de U au sens classique et nous parlerons d'ensemble *rigide*. [2]

Moreover, Menger was soon aware of the emergence of fuzzy sets since one year after the publication of Zadeh's seminal paper [98], he wrote [69]:

> In 1951, I suggested that, besides studying well-defined sets, it might be necessary to develop a theory in which the element-set-relation is replaced by *the probability an element belonging to a set.* In a Paris note [Ref] [3], I called such an object, in contrast to an ordinary or rigid set, *ensemble flou* (= hazy set)." In a slightly different terminology, this idea was recently expressed by Bellman, Kalaba and Zadeh [Ref][4] under the name of *fuzzy set.* (These authors speak of the degree rather than than the probability an element belonging to a set.)

Thus, the distinction between probability and degree of membership was very clear for Menger from the beginning; see [23, 93] for further discussions.

A worth noticing coincidence took place on May 28, 1951, the day when the French mathematician Arnaud Denjoy (1884-1974) transmitted Karl Menger's communication on "ensembles flous" to the French Academy of Sciences. Indeed, Arnaud Denjoy also transmitted on the same day a communication by Gustave Choquet [15] whose abstract was

> En vue d'une théorie des fonctions non additives d'ensembles, on définit et l'on étudie la classe des sous-ensembles des espaces séparés

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[2] In English: "We shall call such a function a *fuzzy set* and we shall interpret $\Pi_F(x)$ as the probability that x belongs to this set. If Π_F only takes the values 1 and 0, it is essentially a subset of U in the classical sense and we shall speak of *crisp* set."

[3] Reference [68].

[4] R. Bellman, R. Kalaba, L. Zadeh. Abstraction and pattern classification. J. of Mathematical Analysis and Applications, 13 (1), 1-7, January 1966.

engendrée à partir des compacts par réunions ou intersections dénombrables et par applications continues"[5]

Thus on the same day where the phrase "ensemble flou" appeared, elements towards the theory of capacities (i.e., "fuzzy measures" in Sugeno's terminology [94]) and Choquet integrals [16] (which would appear later as the quantitative counterpart of Sugeno integrals) were also presented. The fuzzy future thus began on May 28, 1951, even if several decades and a significant research effort would still be necessary before the full landscape could be put together.

Besides, another piece of early work, apparently written independently from Zadeh's pioneering paper, is also worth mentioning. It is an article in French published in 1968 by a French linguist, Yves Gentilhomme (b. in 1920) in a Romanian journal [38]. In this paper, Gentilhomme calls "ensemble flou" a nested pair of subsets, one gathering what he regards as "the central elements", while the second larger subset also includes "peripheral elements". Gentilhomme motivates his proposal by an example of "hypergrammaticality" in texts (illustrated by a poem by Alphonse Allais where the author intentionally makes an abusive use of the imperfect tense of the subjunctive mood in order to produce a comical effect) and by an example of more or less credible words in French built from the same root. Then Gentilhomme provides a formal set-theoretic apparatus for combining his "ensembles flous", and he proposes to assign a degree of membership equal to 1/2 to peripheral elements (those that are not central). It is in the 1974 pioneering research monograph by Negoita and Ralescu [75] (who were working in Romania at that time) that Gentilhomme's paper is first reported and put in relation with Zadeh's work; "ensembles flous" are translated by "flou sets" in the English version of the book the year after [76].

2   Arnold Kaufmann

Edwin Diday (b. 1940) seems to have published the first journal paper in France influenced by the fuzzy set idea [22] in 1972. The paper presents a new approach to (fuzzy) clustering, called the dynamical cloud method (in French, "méthode des nuées[6] dynamiques"), and cites Zadeh [98] and Ruspini [86].

However, it is Arnold Kaufmann (1911-1994) [27, 28] who unquestionably introduced fuzzy set theory in France. He was an applied mathematician, author, or co-author of a long series of books covering many areas in engineering mathematics, including automatic control and operations research. His books, many of which were translated into English, were not only covering standard applied mathematics, but also many advanced topics in relation with current research

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[5]In English: "In view of a theory of non additive set functions, one defines the class of the subsets of separated spaces, generated from compacts by denumerable unions or intersections and by continuous mappings."

[6]the use of this word , which means "clouds" reminds us that at the beginning Zadeh was hesitating between the words "fuzzy" and "cloudy": Indeed, he wrote in 1962: "we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions." in [97]

at that time[19, 20, 21, 39, 59, 40, 41, 42, 43, 57, 58, 60, 44, 46, 47, 48, 61][7]. As he was in contact with Lotfi Zadeh, he heard about fuzzy sets very early, and was quickly enthusiastic about this challenging way of thinking. He was the first in the world to publish a monograph on the theory of fuzzy (sub)sets in 1973 [49]. It was translated into English two years later [52][8]. The first 1973 volume was soon followed by three other ones [50, 51, 53], and by a book of exercises [56] (with Michel Cools and Thierry Dubois). Altogether, this series of five volumes mainly cover fuzzy set theoretic operations, fuzzy relations, and their applications to many fields: classification and pattern recognition, automata and systems, multicriteria decision, as well as linguistics, logic, topology, matroïds, etc. These books, and in particular the first one, had a great impact on the dissemination of fuzzy set theory in France. Continuing to write books on fuzzy logic-related topics, his strong interest for the topic never waned until the end of his life.

Let us quote the last paragraph of the conclusion of his first fuzzy set book [49]:

> Je voudrais exprimer un souhait très sincère. Je voudrais que mes lecteurs, initiés et intéressés par mon modeste travail, puissent aller plus loin, beaucoup plus loin, encore plus loin. Les sciences humaines ont besoin d'une mathématique appropriée à notre nature, à nos attitudes floues, à notre comportement nuancé, à nos dosages, à nos critères multiples. Si ce premier livre est suffisamment stimulant, de nombreux articles, concernant les aspects théoriques ou les applications, seront publiés par des lecteurs; des livres concurrents verront le jour. Tout ceci pour l'amélioration rapide de nos méthodes en vue d'aborder les sciences humaines."[9]

Beyond the fact that enthusiasm and generosity permeate this text, and the correctness of this prediction, it is also worth noticing that Kaufmann was considering that the applications of fuzzy set theory would be human-oriented sciences, while this is not so clear as of to-day. We have to remember that Kaufmann was writing this text at a time where information processing and artificial intelligence were still in infancy.

At the time when he got acquainted with fuzzy sets, Kaufmann was deeply interested in methods for helping creativeness[45], a topic on which he published a book later in 1979 [54]. So, it was one of the first areas where he considered

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[7]This list by no means claims to exhaustiveness!

[8]This more theoretical book, first written in Romanian in 1974 [75] would appear in English also in 1975 [76].

[9]"I would like to express a very sincere wish. I would like that my readers, taught and interested by my modest work, go farther, much farther and farther. Human-oriented sciences need a kind of mathematics that fits our nature, our fuzzy attitudes, our nuanced behavior, our balanced judgments, our multiple criteria. If this first book is sufficiently stimulating, many articles, dealing with theoretical or practical aspects, will be published by my readers; concurrent monographs will appear. All this, for the fast improvement of our methods for coping with human-oriented sciences."

applying fuzzy sets [1, 55, 53], for which he proposed a lattice and fuzzy relation-based approach, in the spirit of ideas and methods preiously advocated by Abraham Moles (1920-1992), an engineer by training, then a philosopher, working on the sociology and psychology of information and communication sciences [71, 72, 73] [74]. Kaufmann's approach to creativeness were also developed by his co-authors Michel Cools and Monique Peteau [17].

3   Elie Sanchez - Claude Ponsard - Robert Féron

The first three main followers of Arnold Kaufmann in France in the mid-1970's are Elie Sanchez, Claude Ponsard, and Robert Féron .

Elie Sanchez (1944-2014) [12, 96, 92] was the first in France, in 1974 in Marseilles, to defend a thesis on fuzzy set methods. His thesis is landmark piece of work on fuzzy relation equations, which contains important results on the solving of these equations [88]. This research was motivated by an attempt at a mathematical formalization of medical diagnosis. It had a very strong influence on the development of fuzzy set methods worldwide.

Claude Ponsard (1927-1990) [9, 26, 37], working in Dijon, started to propose in 1975 to apply fuzzy sets to various problems in economics [81, 82]. He then soon after led a small group of researchers on these questions, including Bernard Fustier [36] and Régis Deloche [18].

Robert Féron (b. 1921) [5] is a statistician working in econometrics in Lyon. He was the first to properly provide a theoretical basis for the study of fuzzy random sets and to advocate their interest [31, 32, 33, 34]. His writings, mostly in French, and mostly published in a journal having a limited circulation, would remain unfortunately largely ignored in the English American world. Let us also point out on the same kind of topic and published in the same place a paper by Robert Fortet and Mehri Kambouzia [35].

We should also mention the two pioneering papers by Jean-Pierre Aubin (b. 1939) on game theory with fuzzy cores (the set of multistrategies that are not rejected by any coalition) published at that time [3, 4].

4   1974-1976: The pivotal years

From 1974-1976 on, the number of young French researchers interested in fuzzy sets started to increase (even if the topic was not very popular and remained highly controversial especially in France (many people were considering that it was not a "serious" topic to work on - partly because of the name -, and whose connection / difference with probability was unclear). Let us provide a list of persons in France who began to use fuzzy sets in their works at that time:

- Bernadette Bouchon, a member of the team headed by Claude-François Picard (1926-1979) [79] started working on fuzzy questionaries [10],

- Jacques Brémont (1938-2014) defended his thesis on the use of fuzzy sets on speech recognition in 1975 [13] under the supervision of Michel Lamotte

[14, 64]. Altogether with Gérard Hirsch (b. 1938) [7] they later on formed an active research group on fuzzy set methods in Nancy for about two decades.

- The first French works on fuzzy systems were initiated by Pierre Vidal [63] in Lille in 1974, together with Noël Malvache (1943-2007) who then continued to work on fuzzy rule-based controllers with Didier Willaeys in Valenciennes [95].

- the year 1975 sees the publication of the first fuzzy set papers by two French pure mathematicians, Daniel Ponasse [80] and S. Ribeyre [85]. The former, once back in France, launched a seminar on "Fuzzy Mathematics" in Lyon, which would become very productive in the late 1970's and the 1980's. This group included Achille Achache (b. 1934), Nicole Blanchard, Odile Botta, Josette and Jean-Louis Coulon, Marianne Delorme, and Christiane Dujet. Let us also mention Michel Eytan [30] on this mathematical side.

Although the following people have been more briefly involved with fuzzy sets, one may still mention:

- in automation of production processes, the thesis of Moncef Ben Salem (1953-2015) [8] where a fuzzy multicriteria automatic decision-making procedure is proposed for determining the sequencing of operations accomplished by a machine tool. Later, Ben Salem became minister of Higher Education and Scientific Research (2011-2014), after the Tunisian revolution. Lucas Pun [84] was one of the very first in France to foresee the potential interest of fuzzy sets in the modeling of production processes.

- Jean-Marc Adamo (b. 1943) [2] started working in the second half of the 1970's for some years on dynamical systems and then on fuzzy programming languages.

- Jean-Philippe Massonie [65] in Besançon was the first in France to foresee the potential interest of fuzzy sets in geographical modeling, thus initiating a line of research that still exists in France.

What is more unexpected is that fuzzy sets were also a source of inspiration in the French avant-garde literature. The French novelist Claude Ollier [77], a writer close to the "Nouveau Roman" movement, seems to have met Arnold Kaufmann in September 1970, at a meeting about creativeness in art and science [1], where Kaufmann already spoke about fuzzy sets. In this French novel with an English title "Fuzzy sets", the author plays with the roles of the protagonists and the reader in the story, as well as with the display of the text on the pages. The novel was reprinted with a slightly less exotic page display two decades later [78]. Let us mention a more classical writer, Jacques Laurent, who published a novel also with the title "Les sous-ensembles flous" (this time in French) a bit later in 1981 [62]. In this book, the fuzzy set idea applies at several levels to the links between the characters and the forces that drive them.

Lastly, it is in 1976 that the authors of this note produced their first (hand-written !) research report of [24] (now indexed by Google Books). This was mainly a survey and a status report. Our first published contributions only appeared one year later [83, 25, ?]

5 Conclusion

This note is an attempt at offering a short overview of the first years of research in France regarding fuzzy sets and their applications. Only references from mid-seventies and before have been reported, without mentioning further developments of the works of the authors cited. In fact, some authors have encountered fuzzy sets very briefly in their research in this time period, while others have continued to contribute to fuzzy sets for several decades.

As shown by this brief overview, fuzzy set research in France (see [11, ?] for general overviews), starts in the years 1973-1976, and immediately deals with very different issues. They are led by researchers relatively isolated from one another, who often face suspicion, negative critique, and sometimes disparagement and bashing from their academic colleagues. Nevertheless, those times were more open-minded than the present period that seems to be under the tyranny and normalization of citation rates and impact factors!

It is also worth noticing that the first works rely mainly on fuzzy set operations and on fuzzy relations, and that many important notions have no role in these works, even if they already exist as the extension principle [98], or the notions of fuzzy measures and fuzzy integrals in the sense of Sugeno [94][10]. It is a matter of facts that some crucial developments would only appear a bit later, such as possibility theory [99], or the linkage between fuzzy set connectives and triangular norms, originally introduced in the study of probabilistic metric spaces, whose father was precisely Karl Menger, the man who first used the phrase "ensemble flou"!

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[10]Interestingly enough, Michio Sugeno spent the academic year 1976-1977 in Toulouse, at the LAAS laboratory, in the research group of José Aguilar Martin [91] and Gérald Banon [6]. The authors of this note were lucky enough to meet Sugeno and learn about his research by the end of his stay in Toulouse.

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