Pupil Center as a Function of Pupil Diameter

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Abstract

We investigate the gaze estimation error induced by pupil size changes using simulated data. We investigate the influence of pupil diameter changes on estimated gaze point error obtained by two gaze estimation models. Simulation data show that at wider viewing angles and at small eye-camera distances, error increases with increasing pupil sizes. The maximum error recorded for refracted pupil images is 2.4° of visual angle and 1.5° for non-refracted pupil projections.

Keywords: image processing, eye tracking, gaze estimation, pupil dynamics, human computer interaction

Concepts: •**Computing methodologies** → **Image manipulation; Image processing;** *Model verification and validation;* •**Human-centered computing** → *Human computer interaction (HCI);*

1 Introduction

The systematic error in video-based eye trackers may vary from a fraction of degree to several degrees of visual angle [Zhang and Hornof 2010]. A typical video-based eye tracker loops through image acquisition, feature extraction, and gaze estimation stages to estimate gaze direction. The artifacts produced at one stage are consumed and transformed by the following step. The transformed artifacts are then used by the next stage and so on. The inaccuracy induced in the artifacts at any stage translates among the stages and ultimately ends up in erroneous gaze direction estimation i.e. the systematic error. The factors such as camera perspective distortion, refraction, eye-camera distance, viewing angle, and image artifacts seem to influence the shape, size and position of the features during feature extraction stage.

The pupil center is one of the principle features used in gaze estimation methods. Most of the gaze estimation methods use pupil center as an input, as opposed to appearance based methods [Hansen and Ji 2010]. In model-based approaches gaze direction vector passes through corneal center and pupil center. Inaccurate pupil center approximation results into an error contribution to the final gaze estimation inaccuracy. The gaze direction estimation is highly influenced by the pupil image center approximation.

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In controlled lighting conditions, changes in pupil size are negligible but in variable lighting conditions, for example outdoor or luminance of monitor screen, pupil reacts to variations in light, and change its size to regulate the amount of light entering into the eye. How does the changes in pupil size, caused by variations in illumination conditions, influence the approximation of pupil image center is the main point of this study.

It is shown in [Villanueva and Cabeza 2008] that there exist an offset, caused by perspective effect and refraction, between pupil image center and projected pupil center in the eye. This offset induces errors in pupil center approximation. The shortcoming in their study is that they have not studied the affects of variation in pupil diameter, eye-camera distance, and viewing angle on the offset.

In this paper, through a systematic approach based on simulation, we investigate the amount of offset in pupil image center caused by changes in pupil diameter for different viewing angles and eye-camera distances. We also investigate the effect of changes in pupil diameter on estimated gaze point obtained from two gaze estimation methods.

Our simulation results show that at smaller eye-camera distances the offset increases with increasing pupil diameter.

2 Related Work

The perspective effect induces an offset between projected pupil center and the center of the pupil image [Villanueva and Cabeza 2008]. This offset is negligible for larger eye-camera distances, for example remote eye trackers. But for head mounted setups where eye-camera distance is small, this offset can induce significant gaze estimation error.

Refraction is another source of error contribution to the gaze-estimation functions because refractive properties of the cornea modify the pupil contour shape, size and position in the image with respect to its projection [Villanueva and Cabeza 2008]. They describe three different methods, used in alternative gaze estimation techniques, for pupil center approximation: 1). The Refraction-Ignored Method: The corneal refraction is not considered in this method, and the pupil image is a simple projection of the actual pupil in 3D. At larger camera-eye distances, affine projection allows to assume linear 3D-2D relationships which means that under such conditions approximated methods assume that the center of the pupil image contour coincides with the projection of actual pupil center. 2). The Approximated Method: The center of the refracted pupil contour is considered as the projection of actual pupil center. Guestrin and Eizenman [2006] and Shih et al [2000] assume pupil image contour center approximation as actual pupil center. 3). The Backprojected Method: In this method refracted pupil contour points in the image are back projected in 3D space so that they intersect the corneal sphere, and after refraction they make the required pupil contour on a plane inside cornea. Hennessey et al [2006], Beymer et al [2003], Ohno et al [2002] and Villanueva and Cabeza [2007] use this approach to estimate pupil center.

3 Method

The eye models used in different gaze-estimation methods assume certain simplifications to reduce physiological and eyeball kinematic complexities. Despite variations in the models there exist some agreed upon fundamental aspects of the eyeball geometry [Villanueva and Cabeza 2008]. Figure 1 depicts the eye model used in our study. The center of the eye ball is denoted by 'e_c'. 'c_c'is the center of the corneal sphere with corneal radius 'c_r'. The pupil is considered as a circular disk placed inside the cornea with its center 'p_c'and diameter 'p_d'. Pupil center and corneal center are connected perpendicularly at a distance 'd'from the corneal center. Suppose 'rp_c'is the projection of actual pupil center in the eye (regarded as 'reference pupil center ') and 'ap_c'is the approximated pupil center of the projected pupil contour in the image then the offset, denoted by 'ϵ', between projected pupil center and approximated pupil center is given by '$\epsilon = rp_c - ap_c$'.

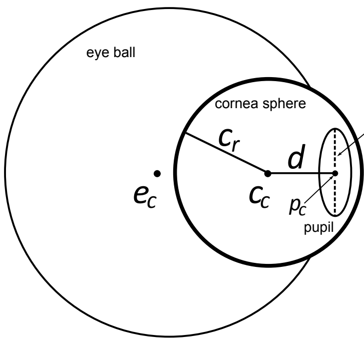

Figure 1: *Eye Model Illustration: 'e_c': Center of the eye, 'c_c': Center of corneal sphere, 'c_r': Corneal radius, 'p_c': Center of the pupil, 'p_d': Pupil diameter, 'd': perpendicular distance between cornea center and pupil center*

3.1 Simulation Model

We have used Böhme et al [2008]'s framework for the simulation because it assumes same simplifications as described in section 3. The figure 2(a) depicts the parametrized simulation model used in this study while the table 1 shows all possible parameter initialization values. The target plane is placed at eye-target plane distance, $d_{ep} = 65$cm which stays fixed for all simulation scenarios. An array of equidistant targets (horizontal array of black circles on the target plane), passing through the center of the target plane, 't_c', is placed horizontally on the plane. The number of interpolated target points, n, can be increased to get more intermediate results. In order to traverse all the targets, from one end to the other, optical axis of the eye subtends and angle of $56°$s. Camera has a fixed orientation and its optical axis always passes through the center of the eye, e_c and the center of the target plane, t_c. The camera-eye distance, d_{ec} can be changed for different simulation scenarios. The 'r'parameter, see table 1, is a binary indicator of whether refraction is modelled. The figure 2(b) depicts the shape of pupil inside corneal sphere. The diameter of the pupil, 'p_d', can be varied to generate pupils of different diameters — the table 1 shows five randomly chosen values for this particular study.

3.2 Workflow of a Simulated Scenario

A typical simulation scenario starts by initializing the simulation parameters. An example parameter initialization can be: $d_{ep} = 65$cm, $d_{ec} = 4.0$cm, $p_d = \{3.1, 4.1, 5.1, 6.0, 6.9\}$mm, $r = 0$, $n = 51$.

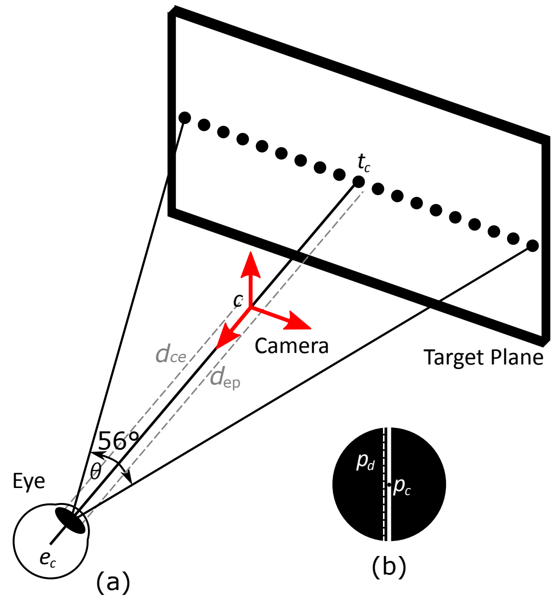

Figure 2: *Simulation Model Illustration: 'd_{ep}': Eye-target plane distance, 'd_{ec}': Eye-camera distance, 'e_c': Center of the eye, 't_c': Center of the target plane, 'θ': Angle subtended at eye center to traverse all the targets, 'p_c': Center of the pupil, 'p_d': Pupil diameter. Horizontal array of black circles on the target plane depicts the eye targets*

Once all the parameters are initialized, the eye is oriented towards all target points one by one. While the eye is looking at a particular target on the target plane, pupil image center offset ϵ is calculated for five randomly chosen pupil diameters — that is, $p_d = \{3.1, 4.1, 5.1, 6.0, 6.9\}$mm — by conforming to the following steps:

1. The actual pupil center in the eye, a 3D point, is projected to the image plane. The position of the projected pupil center 'rp_c'is calculated in the image plane and is regarded as the 'reference pupil center '.
2. The refracted or non-refracted pupil contour is projected to the image plane.
3. An ellipse is fitted to the projected contour points and ellipse center 'ap_c'(approximated pupil center) is calculated.
4. The distance between the reference pupil center and the ellipse center '$\epsilon = rp_c - ap_c$', is calculated.

Table 1: *Simulation Initialization Parameters*

Description	Parameter	Value	Unit
eye target plane distance	d_{ep}	65	centimeter
eye camera distance	d_{ec}	{3.0, 3.1, 3.2, ..., 65.0}	centimeter
pupil diameter	p_d	{3.1, 4.1, 5.1, 6.0, 6.9}	millimeter
refraction	r	{0, 1}	binary
number of target points	n	{11, 21, 31, ...}	

4 Results

In figure 3 pupil image center offset is plotted against viewing angle for the five pupil diameters. The broken lines represent the offsets for refracted pupil images while solid lines are the offsets for non-refracted pupil projections. This data is generated by a complete execution of a simulation scenario for which eye-camera distance $d_{ec} = 4.0$cm, number of target points $n = 51$. It can be clearly seen that the pupil image center offset increases with increasing pupil diameter and viewing angle. It can also be observed that for smaller pupil sizes, the center approximations for refracted pupil image are comparatively accurate — in terms of less offset — than the non-refracted pupil projections, and it is opposite for the larger pupil diameters. According to [Wyatt 2010] 0.1mm offset in pupil center corresponds to 1° in terms of visual angle. Therefore, results in figure 3 show that for refracted pupil contours the maximum pupil image center offset is 0.24mm which means that it can cause an error of 2.4° in terms of visual angle. In case of non-refracted pupil contours the maximum error caused by pupil image center offset could be approximately 1.5°.

The effect of longer eye-camera distance on pupil image center offset is investigated by running simulation scenarios for different eye-camera distances while keeping rest of the simulation parameters constant. It is observed that pupil image center offset decreases with increasing eye-camera distance. It is also observed that the offset in pupil image center becomes negligible at longer eye-camera distances — for example at eye-camera distance $d_{ec} = 65$cm (a typical eye-camera distance for remote eye trackers). Figure 4 depicts the simulation results when distance between eye and camera is 65cm while rest of the simulation parameters are identical to the parameters for figure 3. It can be seen from figure 4 that pupil image center offset is negligible for both refracted and non-refracted pupil contours at eye-camera distance $d_{ec} = 65$cm.

Figure 5 shows the density estimate constructed for the offset data for same simulation scenario. It complements the trends seen in figure 3 and 4. Each circle in the plot depicts the pupil image center offset (in millimeters) calculated for a target point in the target plane. The broken lines are the refracted pupil contours while the solid lines are non-refracted pupil contours. The peaks in the curves show the density of an offset in that particular region. It can be seen that with increasing pupil size, pupil image center offset starts increasing and spreading over a certain range for each pupil size. The short offset spreads for smaller pupil sizes means that approximated pupil center stays consistent even at larger viewing angles and smaller eye-camera distances. It can also be observed that there is less offset in case of smaller refracted pupil images compared to the same non-refracted images.

4.1 Validation

The influence of the pupil center offset is validated on homography and polynomial gaze estimation methods. Both the methods were tested against the same parameter settings which were used to generate the simulation scenario in section 3.2. Mean error and maximum error were calculated for both the gaze estimation methods. Each gaze estimation method was tested for refractive and non-refractive projections of the pupil contour points in the eye. The error caused by pupil image center offset was calculated for five pupil diameters, $p_d = \{3.1, 4.1, 5.1, 6.0, 6.9\}$mm, respectively. The target plane was placed at a distance of 65cm from eye. A 16x16 grid of targets was placed on the target plane. The target plane was 0.7meters wide and 0.7meters high — that is 70cm x 70cm. The eye-camera distance d_{ec} was set to 4.0cm.

The results show that the accuracy of the methods decreases with increasing pupil diameter. The respective maximum errors — for polynomial method with non-refractive projection of the pupil contour points and with increasing pupil diameter — are: 0.94°, 1.09°, 1.41°, 1.64° and 1.77°. Similarly, the respective maximum errors — for homography method with non-refractive projection of the pupil contour points and with increasing pupil diameter — are:

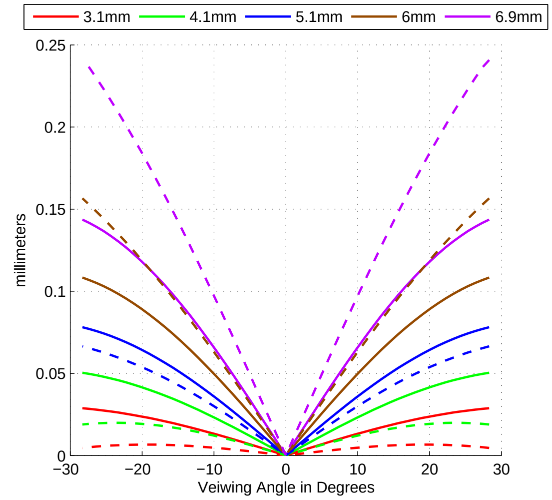

Figure 3: *Pupil center offset from reference pupil center with varying pupil diameters while fixating at same target point. eye-camera distance $d_{ec} = 4$cm. Curve colors: pupil diameters. Broken lines: refracted pupil images, solid lines: non-refracted pupil images.*

3.24°, 3.5°, 4.09°, 4.56° and 4.86°.

Figure 6 sums up the results for the tested gaze estimation methods. Both the methods were tested for refractive and non-refractive projections. All the methods show increasing trend in error with increasing pupil diameters. There is only one exception in all the results, polynomial method with refraction, where error decreases to 1.84° from previous value of 2.43°, while pupil diameter increases from 5.1mm to 6.0mm.

5 Conclusion and Discussion

Our results confirm that there exist an offset between approximated center of the pupil image and the projected center of the actual pupil in the eye. We also found that at wider viewing angles and small eye-camera distances, this offset increases with increasing pupil sizes. Consequently it means that all those gaze estimation methods which consider approximated center of the pupil image as actual pupil center; are likely to experience larger errors with increased pupil sizes. The validation tests on two different gaze estimation methods show an increasing trend in gaze estimation error with increasing pupil diameter.

The results of our investigation also confirm the outcome of the study conducted by [Villanueva and Cabeza 2008] but in their study the variations in viewing angle, eye-camera distance and pupil diameter are not considered. Our results show that variations in these parameters influence the amount of offset between pupil image center and projected center of the actual pupil in the eye.

It is also observed (figure 4) that at longer eye-camera distances the offset in pupil image center is negligible which means that remote eye tracker setups — where camera is placed far away from eye compared to the head mounted eye tracker setups — are least prone to the error caused by the offset in pupil image center.

In head mounted eye tracker setups eye-camera is placed very close to the eye which adds to the perspective distortion and as a result pupil shapes become more elliptical at wider viewing angles. According to our results (figure 3), this setup is more prone to the error

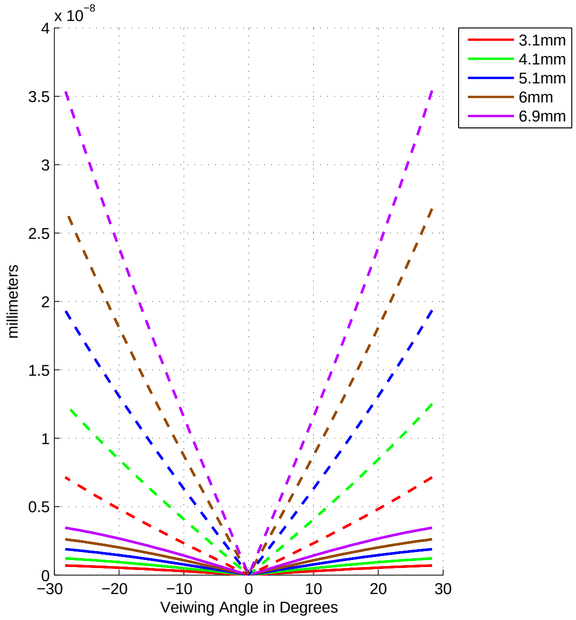

Figure 4: *Pupil center offset from reference pupil center with varying pupil diameters while fixating at same target point. eye-camera distance* $d_{ec} = 65cm$*. Curve colors: pupil diameters. Broken lines: refracted pupil images, solid lines: non-refracted pupil images.*

caused by the offset in pupil image center.

These results are based on simulated data. In future, we intend to confirm this phenomenon with real experimentation. We will also try to calculate the offset between reference pupil center and estimated pupil center systematically.

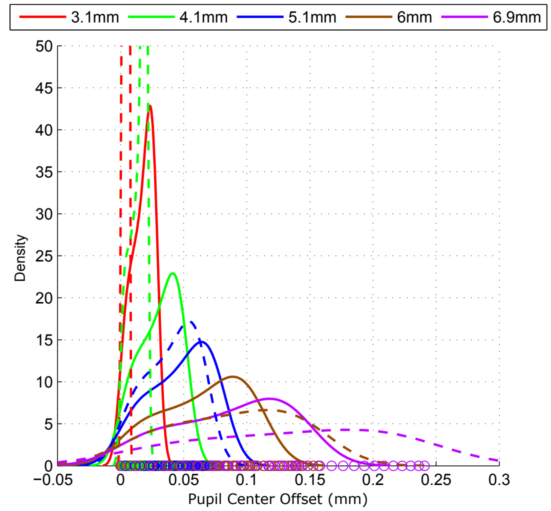

Figure 5: *Kernel Density Estimation plot for pupil center offset data. Each circle in the plot depicts the pupil image center offset (in millimeters) calculated for a target point in the target plane. The peaks in the curves show the density of an offset in that particular region. Narrower the peak more consistent is the offset.*

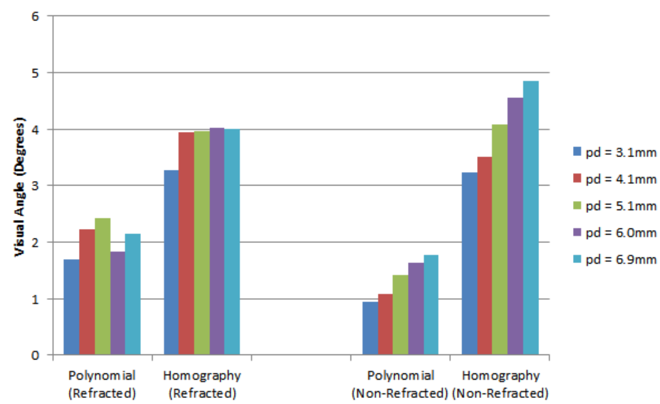

Figure 6: *Summary of the results for the tested gaze estimation methods. Both the methods were tested for refractive and non-refractive projections. Results are shown for five pupil diameters*

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