On Cumulant Techniques in Speech Processing

Vladimir Zaiats and Jordi Solé-Casals

Digital Technologies Group, Universitat de Vic, c/. Sagrada Família, 7, 08500 Vic (Barcelona), Spain,
{vladimir.zaiats,jordi.sole}@uvic.cat

Abstract. This paper analyzes applications of cumulant analysis in speech processing. A special focus is made on different second-order statistics. A dominant role is played by an integral representation for cumulants by means of integrals involving cyclic products of kernels.

Keywords: Cumulants, higher-order statistics, correlogram, speech enhancement

1 Introduction

Different methods in speech recognition use linear and non-linear procedures derived from the speech signal by matching the autocorrelation or the power spectrum, [4, 6, 5]. Many of these methods perform well for clean speech, while their performance decreases strongly if noise conditions mismatch for training and testing.

We obtain a representation for cumulants of second-order statistics containing a special type of integrals that involve cyclic products of kernels. Our techniques are based on [1–3, 7].

2 Integrals Involving Cyclic Products of Kernels

For $m \in \mathbb{N}$, define $\mathbb{N}_m := \{1, \dots, m\}$. Assume that $(\mathbb{V}, \mathcal{F}_{\mathbb{V}})$ is a measurable space and $\mu_1, \dots, \mu_m$ are σ-finite (real- or complex-valued) measures on $(\mathbb{V}, \mathcal{F}_{\mathbb{V}})$. For $m \in \mathbb{N}$, $m \geq 2$, consider the following integral:

$$\overset{\curvearrowleft}{I}\,(K_1, \dots, K_m;\ \boldsymbol{\varphi}) \tag{1}$$
$$:= \int \cdots \int_{\mathbb{V}^m} \left[\prod_{p=1}^{m} K_p(v_p, v_{p+1}) \right] \boldsymbol{\varphi}(v_1, \dots, v_m) \mu_1(dv_1) \dots \mu_m(dv_m)$$

where $v_{m+1} := v_1$. Integral (1) will be called an integral involving a cyclic product of kernels (IICPK).

We will denote:

$$\overset{\curvearrowleft}{\prod_{p \in \mathbb{N}_m}} K_p(v_p, v_{p+1}) := \prod_{p=1}^{m} K_p(v_p, v_{p+1})$$

with $v_{m+1} := v_1$. This function will be called a cyclic product of kernels $K_1, \dots, K_m$.

3 Cumulants of General Bilinear Forms of Gaussian Random Vectors

Suppose that $m \in \mathbb{N}$; $n_{j,1}, n_{j,2} \in \mathbb{N}$, $j \in \mathbb{N}_m$, and write

$$\boldsymbol{X}_{j,1} := (X_{j,1}(k),\ k \in \mathbb{N}_{n_{j,1}}), \qquad \boldsymbol{X}_{j,2} := (X_{j,2}(k),\ k \in \mathbb{N}_{n_{j,2}}), \quad j \in \mathbb{N}_m.$$

Assume that $\boldsymbol{X}_{j,1}$ and $\boldsymbol{X}_{j,2}$, $j \in \mathbb{N}_m$, are real-valued zero-mean random vectors and consider the following bilinear forms:

$$U_j := \sum_{k,l=1}^{n_{j,1},n_{j,2}} a_j(k,l) X_{j,1}(k) X_{j,2}(l), \quad j \in \mathbb{N}_m,$$

where

$$\sum_{k,l=1}^{n_{j,1},n_{j,2}} := \sum_{k=1}^{n_{j,1}} \sum_{l=1}^{n_{j,2}}.$$

If we put

$$(a_j(k,l)) := \begin{pmatrix} a_j(1,1) & \ldots & a_j(1,n_{j,2}) \\ \vdots & \ldots & \vdots \\ a_j(n_{j,1},1) & \ldots & a_j(n_{j,1},n_{j,2}) \end{pmatrix}, \quad j \in \mathbb{N}_m,$$

then for any $j \in \mathbb{N}_m$

$$\begin{aligned} U_j &= \boldsymbol{X}_{j,1} \, (a_j(k,l)) \, \boldsymbol{X}_{j,1}^\top \\ &= (X_{j,1}(1), \ldots, X_{j,1}(n_{j,1})) \begin{pmatrix} a_j(1,1) & \ldots & a_j(1,n_{j,2}) \\ \vdots & \ldots & \vdots \\ a_j(n_{j,1},1) & \ldots & a_j(n_{j,1},n_{j,2}) \end{pmatrix} \begin{pmatrix} X_{j,2}(1) \\ \vdots \\ X_{j,2}(n_{j,2}) \end{pmatrix}. \end{aligned}$$

Consider the joint simple cumulant cum $(U_1, \ldots, U_m)$ of the random variables $U_1, \ldots, U_m$ assuming that this cumulant exists. By general properties of the cumulants, we obtain

$$\begin{aligned} &\operatorname{cum}(U_1, \ldots, U_m) \\ &= \sum_{k_{1,1},k_{1,2}=1}^{n_{1,1},n_{1,2}} \cdots \sum_{k_{m,1},k_{m,2}=1}^{n_{m,1},n_{m,2}} \left[\left(\prod_{j=1}^{m} a_j(k_{j,1},k_{j,2}) \right) \right. \\ &\qquad\qquad \left. \times \operatorname{cum}\left(X_{j,1}(k_{j,1}) X_{j,2}(k_{j,2}), j \in \mathbb{N}_m\right) \right]. \end{aligned}$$

Since any general bilinear form can be represented as a sum of diagonal bilinear forms, the following result holds.

Theorem 1. *Let $m \in \mathbb{N}$; $n_{j,1} = n_{j,2} = n_j \in \mathbb{N}$, $j \in \mathbb{N}_m$. Assume that $(\boldsymbol{X}_{j,1}, \boldsymbol{X}_{j,2}, j \in \mathbb{N}_m)$ is a jointly Gaussian family of zero-mean random variables and suppose that for any $j, \tilde{j} \in \mathbb{N}_m$ and any $\alpha, \tilde{\alpha} \in \{1,2\}$ there exists a complex-valued measure $M_{j,\tilde{j}}^{\alpha,\tilde{\alpha}}$ such that*

$$\mathsf{E} X_{j,\alpha}(k) X_{\tilde{j},\tilde{\alpha}}(\tilde{k}) = \int_{\mathbb{D}} e^{i(k-\tilde{k})\lambda} M_{j,\tilde{j}}^{\alpha,\tilde{\alpha}}(d\lambda).$$

Then

$$\begin{aligned}
&\operatorname{cum}(U_1, \ldots, U_m) \\
&= \sum_{\boldsymbol{l} \in \mathcal{L}_m(n_j, j \in \mathbb{N}_m)} \sum_{(\boldsymbol{\jmath},\boldsymbol{\alpha}) \in \{P,2\}_{m-1}} \int \cdots \int_{\mathbb{D}^m} \left[\overset{\frown}{\prod_{p \in \mathbb{N}_m}} K_p^{(\boldsymbol{\jmath},\boldsymbol{\alpha},\boldsymbol{l})}(v_p - v_{p+1}) \right] \\
&\qquad \times \mu_1^{(\boldsymbol{\jmath},\boldsymbol{\alpha},\boldsymbol{l})}(dv_1) \ldots \mu_m^{(\boldsymbol{\jmath},\boldsymbol{\alpha},\boldsymbol{l})}(dv_m),
\end{aligned}$$

that is $\operatorname{cum}(U_1, \ldots, U_m)$ *is represented as a finite sum of integrals involving cyclic products of kernels. Here, $\boldsymbol{\jmath} := (j_1, j_2, \ldots, j_m)$, $\boldsymbol{\alpha} := (\alpha_1, \alpha_2, \ldots, \alpha_m)$, $j_{m+1} = j_1 = 1$, $\alpha_{m+1} = \alpha_1 = 2$, and the sum $\sum_{(\boldsymbol{\jmath},\boldsymbol{\alpha})}$ is extended to all*

$$((j_2, \ldots, j_m), (\alpha_2, \ldots, \alpha_m)) \in \operatorname{Perm}\{2, \ldots, m\} \times \{1,2\}^{m-1}. \tag{2}$$

The notation $(\boldsymbol{\jmath},\boldsymbol{\alpha}) \in \{P,2\}_{m-1}$ for fixed $j_1 = 1$ and $\alpha_1 = 2$ is equivalent to (2).

Here, we put $\mathbb{Z}_{|n_j-1|} := \{-(n_j-1), \ldots, -1, 0, 1, \ldots, n_j - 1\}$ for $j \in \mathbb{N}_m$ and $\mathcal{L}_m(n_j, j \in \mathbb{N}_m) := \mathbb{Z}_{|n_1-1|} \times \ldots \times \mathbb{Z}_{|n_m-1|}$.

4 Applications

We apply the above obtained integral representations to some problems in speech recognition. Let us consider a setting where sample correlograms and sample cross-correlograms of stationary time series appear.

Let $\boldsymbol{Y}(t) := (Y_1(t), Y_2(t))$, $t \in \mathbb{Z}$, be a weak sense stationary zero-mean bidimensional vector-valued stochastic process with real-valued components whose matrix-valued autocovariance function is as follows:

$$\mathbf{C}_{\boldsymbol{Y}}(t) := \begin{pmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{pmatrix}, \quad t \in \mathbb{Z},$$

and let

$$\mathbf{F}_{\boldsymbol{Y}}(\lambda) := \begin{pmatrix} F_{11}(\lambda) & F_{12}(\lambda) \\ F_{21}(\lambda) & F_{22}(\lambda) \end{pmatrix}, \quad \lambda \in [-\pi, \pi],$$

be the matrix-valued spectral function of the vector-valued process $\boldsymbol{Y}(t), t \in \mathbb{Z}$. Let $\gamma, \delta \in \{1,2\}$. Consider the following random variables:

$$\hat{C}_{\gamma\delta}(\tau; N) := \sum_{k=1}^{N} b_{\gamma\delta}(k;\ \tau, N) Y_\gamma(k+\tau) Y_\delta(k), \quad \tau \in \mathbb{Z}, \quad N \in \mathbb{N},$$

where $b_{\gamma\delta}(k;\ \tau, N)$, $k \in \mathbb{N}_N$, $\tau \in \mathbb{Z}$, $N \in \mathbb{N}$, are non random real-valued weights. It is often assumed that

$$\sum_{k=1}^{N} b_{\gamma\delta}(k;\ \tau, N) = 1, \quad \tau \in \mathbb{Z}, \quad N \in \mathbb{N}. \tag{3}$$

For example, let $N \in \mathbb{N}$ be given and let

$$b_{\gamma\delta}(k;\ \tau, N) = \frac{1}{N}, \quad k \in \mathbb{N}_N, \quad \tau \in \mathbb{Z}.$$

Then (3) holds and

$$\hat{C}_{\gamma\delta}(\tau; N) = \frac{1}{N} \sum_{k=1}^{N} Y_\gamma(k+\tau) Y_\delta(k), \quad \tau \in \mathbb{Z}, \quad N \in \mathbb{N}. \tag{4}$$

The following sample correlograms are also often used in spectral analysis and speech recognition as estimates of $C_{\gamma\delta}(\cdot)$, $\gamma, \delta \in \{1,2\}$:

$$\tilde{C}_{\gamma\delta}(\tau; N) = \begin{cases} \dfrac{1}{N} \displaystyle\sum_{k=1}^{N-|\tau|} Y_\gamma(k+|\tau|) Y_\delta(k), & \text{for } |\tau| < N; \\ 0, & \text{for } |\tau| \geq N, \end{cases}$$

$$\tilde{\tilde{C}}_{\gamma\delta}(\tau; N) = \begin{cases} \dfrac{1}{N-|\tau|} \displaystyle\sum_{k=1}^{N-|\tau|} Y_\gamma(k+|\tau|) Y_\delta(k), & \text{for } |\tau| < N; \\ 0, & \text{for } |\tau| \geq N. \end{cases}$$

Let $\gamma, \delta \in \{1,2\}$, $N \in \mathbb{N}$, $m \in \mathbb{N}$, and $\tau_j \in \mathbb{Z}$, $j \in \mathbb{N}_m$. Put

$$\begin{aligned}
&\mathrm{cum}_{\gamma\delta}^{(N)}(\tau_1, \ldots, \tau_m) := \mathrm{cum}(\hat{C}_{\gamma\delta}(\tau_j; N),\ j \in \mathbb{N}_m); \\
&n_j = N, \quad j \in \mathbb{N}_m; \qquad a_j(k) = b_{\gamma\delta}(k;\ \tau_j, N), \quad k \in \mathbb{N}_N,\ j \in \mathbb{N}_m; \\
&X_{j,1}(k) = Y_\gamma(k+\tau_j), \quad k \in \mathbb{N}_N,\ j \in \mathbb{N}_m; \\
&X_{j,2}(k) = Y_\delta(k), \quad k \in \mathbb{N}_N,\ j \in \mathbb{N}_m.
\end{aligned}$$

Under these conditions the results obtained in Section 3 can be applied to the cumulants. These results imply that the Gaussian component of the cumulant $\mathrm{cum}_{\gamma\delta}^{(N)}(\tau_1, \ldots, \tau_m)$ is represented as a finite sum of integrals involving cyclic products of kernels.

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