The Performance Accident in wireless multi-hop networks

Philippe Jacquet, Prateek Mittal

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

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The Performance Accident in wireless multi-hop networks

Philippe Jacquet[*†] , Prateek Mittal [‡†]

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Abstract: We study the maximum throughput and the optimum transmit probability of wireless networks under singlehop and multihop configurations. We show that as the number of nodes in the network increases, there is a point where the optimum probability of transmission to achieve the maximum throughput undergoes a dramatic shift. In this report, we present our analysis for this "network accident".

Key-words: multi-hop adhoc network, analytic model, maximum throughput, optimal transmission probability, network accident

* Email:Philippe.Jacquet@inria.fr
† Project HIPERCOM, INRIA Rocquencourt, BP 105, 78153 Le Chesnay Cedex, France
‡ Email:Prateek.Mittal@inria.fr

Unité de recherche INRIA Rocquencourt
Domaine de Voluceau, Rocquencourt, BP 105, 78153 Le Chesnay Cedex (France)
Téléphone : +33 1 39 63 55 11 — Télécopie : +33 1 39 63 53 30

L'Accident dans les performances des ré seaux sans fil à sauts multiples

Résumé : Dans ce rapport, nous étudions le débit maximum et la probabilité de transmission optimale dans des réseaux sans fil à un saut et à multi sauts. Nous démontrons que lorsque le nombre de noeuds augmente dans le réseau, il existe une limite pour laquelle la probabilité de transmission optimale correspondant au débit maximum d'émission change de valeur subitement. Dans ce rapport nous présentons notre analyse de cet "accident réseau".

Mots-clés : réseau ad hoc multi-sauts, modèle analytique, débit maximum, probabilité optimale de transmission, accident de réseau

Contents

1 Introduction

Characterizing the throughput of a wireless network is a challenging task, especially so, for the multi-hop wireless ad hoc networks. In this report, we focus on studying the optimum channel access probability to achieve the maximum throughput. Many routing protocols in adhoc networks do not take into consideration, the effect of medium access over transport performance. [2] was the first paper to give the throughput equations for packet radio network and showed that the critical parameter for network optimization is p, the probability of transmission in any given slot.

Global throughput of adhoc networks is inherently limited. Gupta and Kumar[3] placed an upper bound on the asymptotic throughput of multi-hop ad hoc networks to be of the order of $\sqrt{\frac{N}{logN}}$, where N is the number of nodes in the network. The access methods are also not optimized for multi-hop architecture leading to the problem of exposed nodes and hidden collisions, that greatly affect performance and scalability. The IEEE 802.11 is not at all optimized for the multi-hop scenario. Recent results [4] of an experimental study of scaling laws in adhoc networks employing IEEE 802.11 technology show that the per node throughput declines like $\frac{c}{N^{1.68}}$ which is considerably worse than what is attainable in theory.

Recently the authors of [1] have given an exact representation of the Singal to Noise and Interference ratio (SINR) at the receiver, taking into account all interferers. They show how the the transport capacity is maximized by selecting the probability of channel access appropriately. In this report, we study how the behavior of the throughput changes wrt to p.

The Gupta and Kumar throughput characterization is an asymptotic result, and it is of interest to study the behavior of the throughput of a wireless network, for varying number of nodes. We use an analytic model for our experiments. In this model, every node that has a packet to transmit, transmits the packet with a probability p independent of other nodes. The detailed system hypothesis for this model are discussed in section 2. We vary the transmission probability p, and study its effect on the network throughput.

Our main result is that as the number of nodes in the network increases, there is a value of N, for which the optimal p, to achieve the maximum throughput, undergoes a dramatic shift. In this report, we focus on investigating this dramatic shift, which we call the 'network accident'. In section 3, we give a description of this network accident and predict its occurance.

To compute the throughput of the network, we use a simulator, which in turn uses some results of the analytic model. This computation is done for two configurations:

- Single-hop configuration: In this scenario a node strives to send a packet directly to its destination.
- Multi-hop configuration: Here, a node sends a packet to its destination via a multi hop route.

The simulation technique involves the probability distribution of received energy at each node. Now, there are 2^N atoms in the distribution of the received energy at a node, which is a problem when the number of nodes increases. To circumvent this problem, for large values of N, some nodes are merged. The results of the analytic model used by the simulator and the the precise simulation technique are discussed in section 4.

We present the throughput results in setion 5. We vary the parameters p and N, and show the nature of the throughput for single-hop and multi-hop configurations. We show our main result, the sudden shift in optimum transmission probability.

We conclude with an analysis of the "network accident" in section 6, in light of these results.

2 System Hypothesis

The following are the major assumptions of our model.

2.1 Protocol Hypothesis

- The time is slotted.
- Every node transmits a packet on any given slot with probability p, independently of the other nodes.

2.2 Physical Model

- All transmitting nodes(emitters) have the same transmission power Q.
- The reception signal at distance r of emitter is $\frac{Q}{r^\alpha}$. We have assumed $\alpha = 2.5$.
- A packet is correctly received iff its SNR (Signal to Noise ratio) is greater than K, noise is the sum of signal powers received from all the transmitters on the same slot. We have assumed K=10.

2.3 Traffic Model

- The traffic model is uniform, every node sends equal traffic to any other destination in the network.
- We assume that every node has a transmit buffer filled with an infinite queue of packets.(Since we want to compute the ultimate throughput)

2.4 Network Map

- The Network map model is a square with N nodes randmoly and uniformly dispatched.

3 Network accident

We know that the singlehop throughput $\lambda_S(p, N) \geq Np(1-p)^{N-1}$, therefore an optimal value of $p \approx \frac{1}{N}$, would give $\lambda_S(p, N) \geq \frac{1}{e}$. Moreover $\lambda_S(p, N)$ remains bounded when $N- > \infty$ and tends to zero when p is fixed an does not depend on N.
Also, the multi-hop throughput $\lambda_M(p, N) \geq \lambda_S(p, N)$ since the single-hop configuration is just a special case of multi-hop configuration. When N is large, we expect to get the maximum throughput of the order for p of the order $\frac{1}{logN}$, which yields $\lambda_M(p, N)$ of the order of $\sqrt{\frac{N}{logN}}$. **Therefore there will be a value of N, where the optimal value of p will dramatically shift from 1/N to the order of 1/log N.** When p is fixed (p « 1), one should observe a slow increase of $\lambda_M(p, N)$ and then a strong decrease, when N increases and then a slow increase in $\sqrt{N}$.

Figure 1 depicts the plot of the asymptotic multihop throughput with our slotted model. It increases in $A * \sqrt{\frac{N}{logN}}$. The network accident should occur when the multihop capacity reaches the single hop capacity of around 0.40.

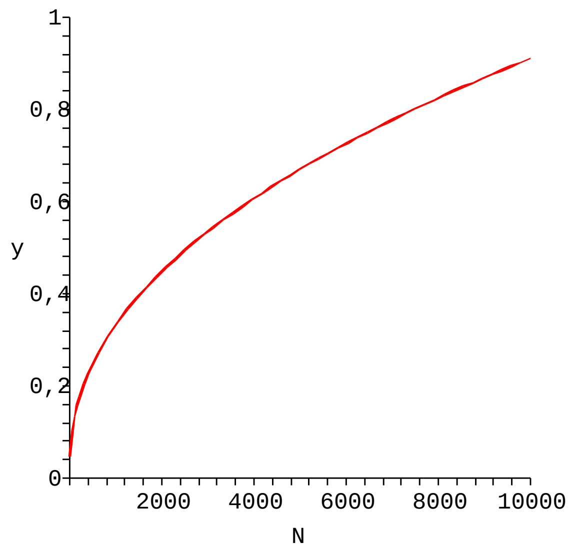

Figure 1: Maximum Asymptotic Throughput vs Number of Nodes

4 Throughput Computation

The goal of a simulation run with N nodes, can be limited to the measurement of the **Average Reception Matrix**. The average reception matrix is the matrix whose coefficient c_{ij} is equal to the statistic of successful reception by node j of a packet transmitted by node i.
The way the nodes select their destination and route their packets does not interfere with the internal functional of the simulation. Every node transmits a packet with a probability p, whatever its destination, since we consider that the packet waiting queues are full.

4.1 Analytic results for single-hop throughput

The average number of retransmissions (including the first transmission) of a packet from node i to node j is $\frac{1}{c_{ij}}$. Let us call the matrix $\frac{1}{c_{ij}}$, the transmission cost matrix D, where $D_{ij} = \frac{1}{c_{ij}}$ (we set $D_{ii} = 0$ for all i). Therefore the average number of retransmission of a packet transmitted by node i to node j is D_{ij}.
During T slots, there are in average pNT packet transmission attempts in the network. If $\lambda_s(p, N)$ is the throughput of the system, the number of successfully delivered packets is $\lambda_s(p, N)T$. Since we assume that the packets delivered in the network have source and destinations uniformly distributed on the nodes and that there are $(N-1)N$ pair of source and destinations (i,j), the number of packets delivered from source i to destination j during T slots, is $\frac{\lambda_s(p,N)T}{(N-1)N}$.
Each transmission from i to j costs in average D_{ij} retransmissions, therefore $\sum_{i,j} \frac{\lambda_s(p,N)TD_{ij}}{(N-1)N}$ is equal to the total number of transmission attempts in the network during T slots and should be equal to pNT. Therefore

$$\lambda_s = \frac{pN^2(N-1)}{\sum_{(i,j)} D_{ij}}$$

4.2 Analytic results for multi-hop throughput

Each node i computes its best route to any destination j. To this end it performs a Dijkstra algorithm on the matrix D. The best route is the route $(i_1, i_2, ..., i_k)$ such that $i_1 = i$ and $i_k = j$ and sum $\sum D_{i_l i_{l+1}}$ is minimal. Let H_{ij} be the minimal multihop value.

We use the (min,+) algorithm to compute the min hop matrix H. We set $H = D$ and we iterate $H \leftarrow H * D$ until H is stationary, with $(H * D)_{ij} = min_l(H_{il} + D_{lj})$. there could be only a maximum of log N iterations. Taking simlar reasoning as in computation of singlehop throughput, we get the maximum throughput of the multihop system

$$\lambda_M(p, N) = \frac{pN^2(N-1)}{\sum_{(i,j)} H_{ij}}$$

4.3 Simulation Technique

The average reception matrix C is computed via simulations. Now, there can be two approaches for calculation of matrix C. The first approach uses the simulation of packet transmission while the second approach involves computing the probability distribution of received energy at each node. While we have implemented both the approaches, the problem with the simulation of packet transmission is that we are not able to scale our computations to an acceptable level. The results presented herein use the second approach of computing the probability distribution of received energy at each node.

The distribution of the energy received at node i, W_i, has 2^{N-1} atoms, which is a problem for large values of N. We split the interval of logarithm of noise into even segments and merge all noise levels on each segment. This enables us to scale up our computations to thousands of nodes. The merging on nodes for large values of N enables us to reduce the complexity of the problem from exponential time to polynomial time.

The energy distribution at node i (W_i) is computed as follows:

- For each node j (distinct from i), the energy received is $\frac{1}{{r_{ij}}^{\alpha}}$ with probability p, and 0 with probability 1-p.
- The probability of correct reception of a packet sent by node j at node i is $\frac{P(W_i < \frac{r_{ij}^{-\alpha}}{K})}{(1-p)}$.

The factor of $\frac{1}{1-p}$ is to remove the contribution of node j in W_i. The value of K for our model is 10.

5 Results

We vary the transmission probability and the number of nodes in the network, in an attempt to understand the nature of the throughput wrt to these parameters.

5.1 Throughput variation wrt number of nodes

Keeping the transmission probability constant, we scale up the number of nodes. Figure 2 shows how the singlehop throughput varies for two cases, p=0.005 and p=0.01. Figure 3 depicts the Multihop throughput for p=0.005 and p=0.01.

5.1.1 Singlehop Configuration

We observe that the maximum singlehop throughput for fixed p is at $N \approx \frac{1}{p}$. Also, as $N -> \infty$, $\lambda_S(p, N)$ tends to 0 when p is fixed and does not depend upon N.

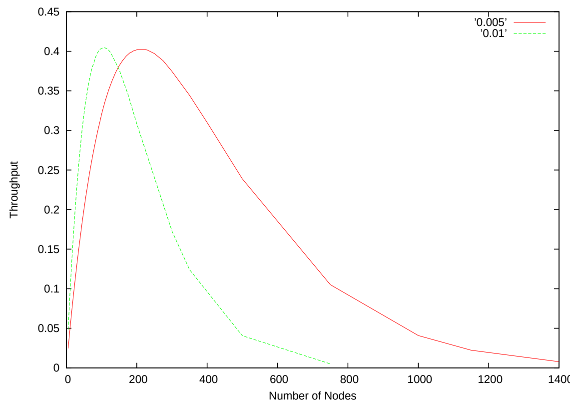

Figure 2: Throughput vs N for p=0.005, 0.01

5.1.2 Multihop Configuration

We can observe that there is an initial increase in throughput, followed by a sharp drop and then a slow increase in order of $\sqrt{N}$. The local maxima at $N \approx \frac{1}{p}$ occurs due to the singlehop nature of transmissions (Singlehop configuration is just a special case of Multihop configuration), which accounts for the initial increase and the subsequent drop in throughput. As we keep on increasing the numder of nodes in the network, the throughput begins to slowly increase, showing is its asymptotic character.

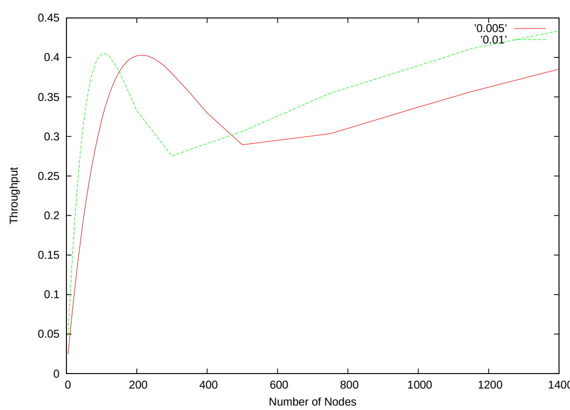

Figure 3: Throughput vs N for p=0.005, 0.01

5.2 Throughput variation wrt transmission probability

In this scenario, we keep the number of nodes in the network fixed and vary the transmission probability. Figures 4 shows the singlehop throughput variation wrt p for 100, 200 and 300 nodes. Figure 5 shows the same for 500, 750 and 1000 nodes. The multihop throughput variation wrt p is depicted in figures 6,7,8.

5.2.1 Single-hop throughput

From figures 4 and 5 ,we observe that the maximum singlehop throughput is near 0.4, at $p \approx \frac{1}{N}$. Also $\lambda_S(p, N)$ remains bounded when $N - > \infty$.

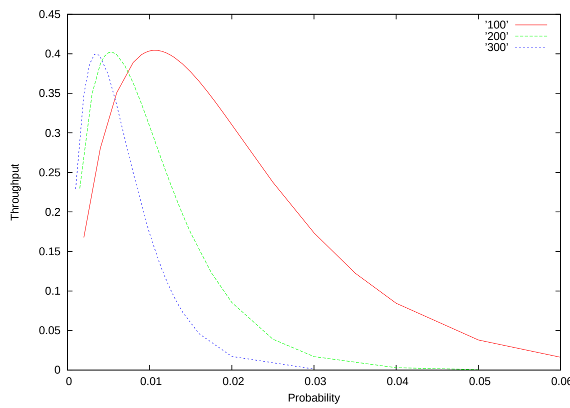

Figure 4: Throughput vs Probability for 100, 200 and 300 nodes

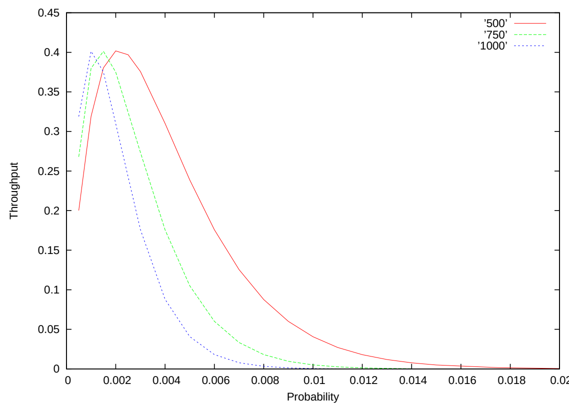

Figure 5: Throughput vs Probability for 500, 750 and 1000 nodes

5.2.2 Multi-hop Throughput

In figure 6, local maxima at $p \approx \frac{1}{N}$ is due to the singlehop nature of the network. It is very interesting to see the development of a second local maxima as we scale up the nodes in figure 7. Importantly, the scaling up of nodes also increases the value of this maxima, as it can be seen in figure 8. We note that the second local maxima occurs at $p \approx \frac{c}{logN}$, where $c \approx 0.07$ It is difficult to estimate the precise value of c due to the curvature at the second maxima being of a very flat nature.

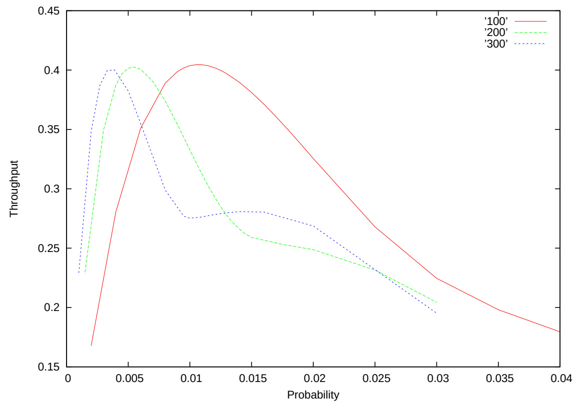

Figure 6: Throughput vs Probability for 100, 200 and 300 nodes

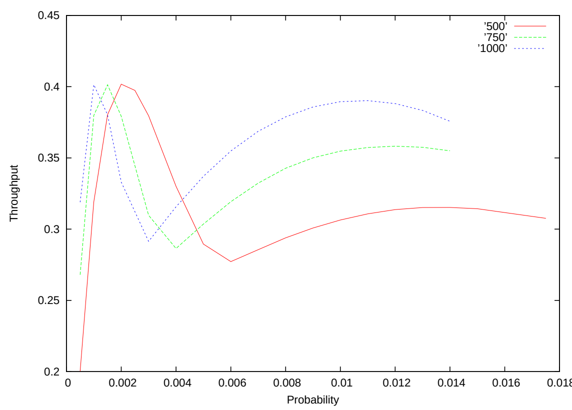

Figure 7: Throughput vs Probability for 500, 750 and 1000 nodes

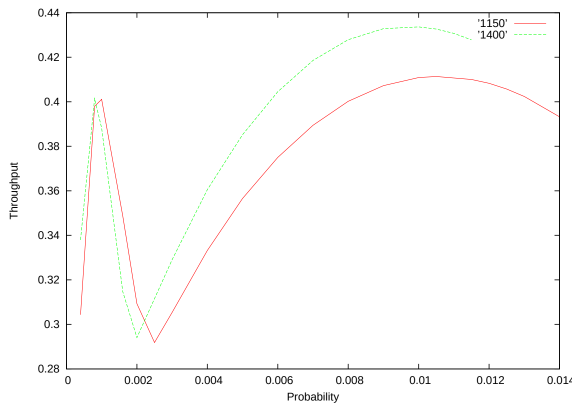

Figure 8: Throughput vs Probability for 1150 and 1400 nodes

5.3 Network Accident

Variation between the optimal value of p and the number of nodes is shown in figure The "network accident" is easily observed. At the point when the singlehop throughput is outperformed by the multihop throughput, the optimal transmission probability will shift from $\frac{1}{N}$ to the order of $\frac{1}{logN}$.

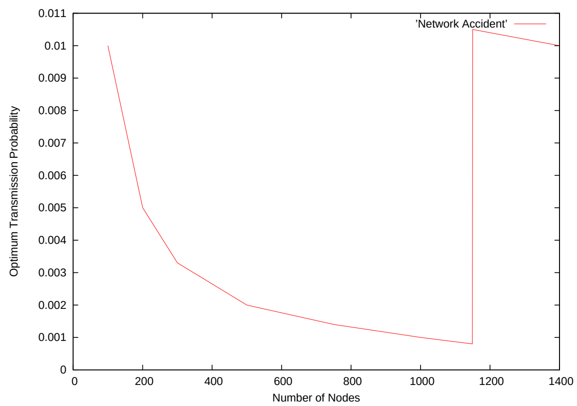

Figure 9: Optimal Probability of Transmission(p) vs Number of Nodes (N)

6 Conclusion

We show that in a dense network, as the number of nodes scales up, the multihop throughput begins to increase and there is a value of N, after which the multihop throughput outperforms the singlehop throughput. The singlehop throughput is shown to be optimal at $p \approx \frac{1}{N}$ and remains bounded for large values of N. We show that the multihop throughput variation wrt p has two local maximas. The first maxima at $p \approx \frac{1}{N}$ is due to the singlehop nature of the network, and the throughput at this maxima remains constant at approximately 0.4. The second maxima occurs at $p \approx \frac{c}{logN}$, and the throughput value keeps on increasing as the number of nodes increases. We thus show that there is a value of N, where the multihop throughput outperforms the singlehop throughput, and the optimal value of p dramatically shifts from $\frac{1}{N}$ to the order of $\frac{1}{logN}$.

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Unité de recherche INRIA Rocquencourt
Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex (France)

Unité de recherche INRIA Futurs : Parc Club Orsay Université - ZAC des Vignes
4, rue Jacques Monod - 91893 ORSAY Cedex (France)
Unité de recherche INRIA Lorraine : LORIA, Technopôle de Nancy-Brabois - Campus scientifique
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Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
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