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Cost optimisation in water supply system

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Thème 4 — Simulation et optimisation
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Abstract: This paper presents a water supply problem arising from the regions where power supply is not regulated and the water supplier has to manage the demand at minimum possible cost. The consumption is constrained to organize rationing, and thus known day after day. Furthermore, the same consumption profile is maintained during the summer season, and this explains the importance of the reservoir content at the beginning of each day. The criterion to be minimized is the cost of energy that evolves during the day.

Key-words: Water Supply, Optimisation, Cyclic system, and Reservoir capacity

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Optimisation des coûts dans un problème dapprovisionnement en eau

Résumé : Cette communication présente un problème dapprovisionnement en eau dans un environnement où le coût de lénergie varie au cours de la journée et où leau doit être rationnée. La régulation se fait en contraignant le profil de consommation durant la saison sèche : cela explique limportance du niveau de leau dans le réservoir au début de la période (la journée). Lobjectif est de minimiser le coût de lénergie consommée.

Mots-clés : Approvisionnement en eau, Optimisation, Réservoir, Système cyclique

1 Introduction

Water supply is a prominent problem in regions where water resources are scarce and energy is limited. In this kind of environment 24 hours pumping to circulate water in residential colony or city is not feasible. Since, the electric supply is not regulated, the supplier also depends upon the private power supply provider or self generated power by electric generator that run on diesel, and thus the electric cost varies during a day. To overcome the problem, the water supplier monitors the water consumptions and tries to refill the tank during the day when energy is cheap. Indeed, the choice of these periods is constrained by the capacity of the tank and its initial level.

In this paper, we consider a problem where the initial level of water content in the tank is known and the consumption is planned for the next day. The goal is to achieve the given consumption at the lowest cost and to return the same content of the tank at the end of the day. Assuming that the same consumption profile is applied during a large sequence of days, another problem to be solved is the selection of the best initial content of the reservoir, that is the initial level that leads to the lowest cost.

Researches have been conducted to find ways to face drought. Some of these researches concern tank/reservoir design, which is not the objective of this paper. Techniques for controlling water supply of reservoirs during drought, in accordance with consumption, did not attract many researchers.

One method is based on mathematical programming. As far as we know, [1] was the first who used linear programming in this field. [5] used mathematical programming approach to compute volumes that trigger rationing phases at different levels. [4] developed a multi-objective linear programming approach to manage water supply system during drought. Simulation is more widely represented in the literature. [2] has used simulation, associated with risk analysis. Other authors proposed approaches based on management rules, for instance, [3] introduced expert systems to apply a set of rules for managing water supply systems.

The problem at hand is different from the classical water supply problem in the sense that consumption is fixed during a given period (a day) and that the goal is to meet the consumption at the lowest price and return to the same tank level at the end of the period. Constraints to apply are the delivery rate of the pump and the capacity of the reservoir. Note that fixing consumption is a way to organize rationing.

The rest of the paper is organized as follows. Section 2 devoted to problem formulation. In section 3, we present the properties of an optimal solution. An optimal algorithm based on the properties is proposed in section 4. Section 5 illustrates the algorithm by a numerical example. In section 6, we show how the initial reservoir content influences the cost of water supply during a period and finally section 7 is the conclusion.

2 Problem formulation

A water tank is used to supply water in the city. A high power electric pump is used to replenish the tank from external reservoir (say underground water). The water consumption of the city is assumed to be periodic during a horizon T (say a day). The following notations are used in the formulation:
$g(t)$ gives the instantaneous water consumption at time $t \in [0, T]$.
q is the pumping rate i.e. the quantity pump is able to provide per unit time.
$c(t)$ gives the operating cost of pump when pump is operating at time $t \in [0, T]$. It is the cost of power supply. This cost is piecewise continuous.
s_0 represents the water level at time 0.
m is the capacity of the tank, and
$s(t)$ denotes the water level in the tank at time $t \in [0, T]$.

Furthermore, we denote by $\delta : t \rightarrow \{0, 1\}$ the function defined as follows:

$$\delta(t) \quad = \quad \left\{ \begin{array}{l} 1 \text{ if the pump is operating in time } t \\ 0 \text{ otherwise} \end{array} \right\}$$

In the rest of the paper we assume that if $\delta(t) = 1$ (respectively 0), there exists a closed interval $E_t = [\alpha_t, \beta_t]$ such that $\alpha_t < t < \beta_t$ and $|E_t| < \epsilon$ where ϵ is a very small number, and that $\delta(h) = 1$ (respectively 0) for any $h \in E_t$. We refer to this property as property 0. In this model we assume that the wearing and tearing cost due to start-up and shutdown of the pump is negligible with respect to power cost.

Let D be the set of functions δ that satisfy:

$$q. \int_{t=0}^{T} \delta(t)dt = \int_{t=0}^{T} g(t)dt$$

D characterizes the exact replenishment that is needed to fulfil the demand during period $[0, T]$. These replenishments are the only one of interest due to the periodicity of the consumption.

The problem at hand consist of selecting $\delta^* \in D$ that satisfy:

$$\int_{t=0}^{T} c(t)\delta^*(t)dt = Min_{\delta \in D} \int_{t=0}^{T} c(t)\delta(t)dt \tag{1}$$

under the following constraints:

$$s(h) = s_0 + q \int_{t=0}^{h} \delta^*(t)dt - \int_{t=0}^{h} g(t)dt \in [0, m], \forall \;\; h \in [0, T] \tag{2}$$

The objective, expression (1), is to minimize the replenishment cost during a given horizon. Constraint (2) guarantees that the tank level lies between 0 and m. A solution $\delta^* \in D$ that verifies (2) is said to be feasible and it is optimal if, in addition, it minimizes (1). Note that the set of feasible solutions may be empty even if $D \neq \emptyset$.

This is a non-linear problem and an analytical exact solution of this problem is very difficult. To make this problem tractable we discretize the problem by dividing the horizon into N intervals. Now, the discrete version of this problem can be formulated as a binary integer program. The formulation of the binary problem is presented in appendix. Again, it is well known that the binary integer programming method is not an effective method if the size of the problem is very large. Here the size (see appendix) of the binary problem depends upon N. In this work we show that the algorithm presented for this problem is far much better than the binary integer programming methods or dynamic programming.

In the following section, we introduce some properties of an optimal solution for the continuous problem. Later these properties will help us to develop an efficient heuristic to solve this problem. Though, heuristic algorithm comes up with the optimal solution of the discritized problem we call it heuristic because the problem we solved is discrete and the optimal solution of the problem may not be the optimal for the continuous problem. Of course the difference between the two solution can be reduced by selecting N sufficiently large.

3 Properties of an optimal solution

The first property shows that, if the optimal inventory level is not extreme i.e. neither maximal nor minimal during $[0, T]$, then the replenishments are made only during the periods when the power supply is the cheapest. The mathematical proof is presented as result 1.

Indeed, we assume that if the pump is operating (resp. not operating) at time t, it also operates (resp. does not operate) in a closed interval containing t, as mentioned in the previous section.

Result 1

Let δ^* be an optimal solution and $s^*(h), h \in [0, T]$ denotes the water level in the tank during the period $[0, T]$ and $U \subset [0, T]$ the union of the periods in which replenishment takes place, i.e. the periods in which $\delta^*(h) = 1$ if $h \in U$. Assume that $0 < s^*(h) < m$, $\forall h \in [0, T]$. Then, $\forall\ t \in U$ and $\tau \in [0, T] \setminus U$, $c(t) \leq c(\tau)$.

Proof

We conduct a demonstration *ad absurdum*. Assume that there exists $t \in U$ and $\tau \in [0, T] \setminus U$ such that $c(t) > c(\tau)$. Then, according to hypothesis (0) and the hypothesis of result 1, there exist two closed intervals E_t containing t and E_τ containing τ such that:

$$c(x) > c(y) \text{ for } x \in E_t \text{ and } y \in E_\tau \tag{3}$$

$$|E_t| = |E_\tau|, \text{i.e. the intervals } E_t \text{and } E_\tau \text{are equal.} \tag{4}$$

$$Min_{h\in[0,T]}[s^*(h), (m - s^*(h))]/q \geq |E_t| = |E_\tau| \tag{5}$$

We build δ^{**} as follows:

$$\delta^{**}(h) = \left\{ \begin{array}{lll} \delta^* & \text{if} & h \in [0,T] \setminus (E_t \cup E_\tau) \\ 1-\delta^*(h) & \text{if} & h \in E_t \cup E_\tau \end{array} \right\} \tag{6}$$

δ^{**} consists of non replenishment in E_t and replenishment in E_τ. We denote by s^{**} the function that gives the content of the tank corresponding to δ^{**}. Now:
- According to (4), $s^{**}(T) = s_0$.
- And, according to (5), $0 \leq s^{**}(h) \leq m, \quad \forall\, h \in [0,T]$.
Thus $\delta^{**} \in D$ and δ^{**} is feasible. Furthermore, according to (3) and (6):

$$\int_{h=0}^{T} c(h)\delta^*(h)dh = \int_{h=0}^{T} c(h)\delta^{**}(h)dh - \int_{h\in E_\tau} c(h)dh + \int_{h\in E_t} c(h)dh > \int_{h=0}^{T} c(h)\delta^{**}(h)dh$$

The above relation completes the proof, as it contradicts the hypothesis.

Result 1 is summarized in figure 1. The cost at any point in time of $l_1 \cup l_2$ is less than or equal to the cost at any point of $[0,T] \setminus \{l_1 \cup l_2\}$.

The following corollary is derived from result 1.

Result 2
Let $U \subset [0,T]$ denotes a set of the union of the sets of periods such that:

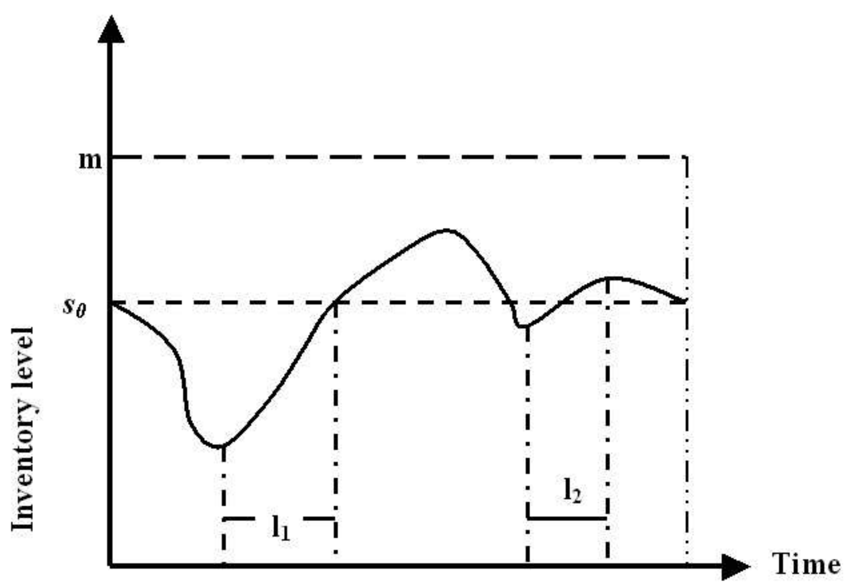

Figure 1: Never extremal inventory level (l_1, l_2 represents the replenishments periods)

- $\forall\ t \in U$ and $\tau \in [0,T] \setminus U, c(t) \leq c(\tau)$
- $q.|U| = \int_{t=0}^{T} g(t)dt$

And, we consider the solution δ defined as follows:

$$\delta(h) = \left\{ \begin{array}{lll} 1 & \text{if} & h \in U \\ 0 & \text{if} & h \in [0,T] \setminus U \end{array} \right\}$$

Then, if the above solution represented by δ is feasible and the water level is neither maximum (m) nor minimum (0) then the feasible δ is an optimal solution. The proof is straightforward and hence omitted. Please note that the optimal control may not be unique.

For an optimal solution, say δ^*, we denote by $t_1, t_2, ..., t_n$ where $0 = t_0 < t_1 < t_2 < ... < t_n < t_{n+1} = T$ the instants where the water level ($s^*(h), h \in [0,T]$) is maximal (i.e. equal to m). We define by $\theta_i^1, \theta_i^2, ..., \theta_i^{r_i}$ the instants, where water level is minimum (i.e. equal to 0), between the two instants t_i and t_{i+1}, $i \in \{0,1,...,n\}$. r_i denotes number of instance where the tank is empty between t_i and t_{i+1}. Indeed, we may have $r_i = 0$, $i \in \{0,1,2,...,n\}$ for some interval. We set $t_i = \theta_i^0$ and $t_{i+1} = \theta_i^{r_i+1}$.

Let U_i^k be the union of the periods included in $[\theta_i^k, \theta_i^{k+1}]$ such that

$$\delta^*(h) = \left\{ \begin{array}{lll} 1 & \text{if} & h \in U_i^k \\ 0 & \text{if} & h \in [\theta_i^k, \theta_i^{k+1}] \setminus U_i^k \end{array} \right\} k = 0,1,...,r_i.$$

Now, taking into account property 0, the following results, result 3 and result 4, holds.

Result 3 shows that for an optimal solution if there are, at least, three consecutive instants (say 1,2,3,4,..) where water level is minimum and there is no point between any two instants where the water level is maximal then the cost at point where pumping is not done, between the any two instants (say 1,2), is always costlier than the pumping cost during the periods where pumping is done between any two instants onwards (i.e. 2,3,4,..).

The result is also valid in the case where there are two consecutive instants with minimal water level and one next instant with maximal water level.

Result 3

If δ^* represents an optimal solution then for any $t \in \cup_{k=j}^{r_i} U_i^k$ and $\tau \in [\theta_i^j, \theta_i^{j+1}] \setminus U_i^j$, $t, \tau \neq \theta_i^j, \theta_i^{j+1}$ for $i = 0,1,...,n$ and $j = 0,1,...,r_i$, inequality $c(t) \leq c(\tau)$ holds.

Proof

We conduct a demonstration ***ad absurdum***. Assume that there exists $t \in \cup_{k=j}^{r_i} U_i^k$ and $\tau \in [\theta_i^j, \theta_i^{j+1}] \setminus U_i^j$, $t, \tau \neq \theta_i^j, \theta_i^{j+1}$ such that $c(t) > c(\tau)$. Then, according to property (0) and the hypothesis of result 3, there exists two closed intervals E_t containing t and E_τ containing τ that satisfy (3), (4) and

$$\theta_i^j, \theta_i^{j+1} \notin E_t \cup E_\tau \tag{7}$$

$$Min_{h \in Z(t,\tau)}[s^*(h), (m - s^*(h))]/q \geq |E_t| = |E_\tau|, \tag{8}$$

where $Z(t,\tau) = [t,\tau] \cup E_t \cup E_\tau$.

Now derive δ^{**} from δ^* as shown in (6). If $t \in U_i^j$, the trajectory of $s^{**}(h)$, water level correspond to δ^{**} :
- decreases between the lower bound of E_t and the upper bound of E_τ, but remains greater than or equal to 0 due to (7) and (8) if $t < \tau$,
- increases between the lower bound of E_τ and the upper bound of E_t, but remains less than or equal to m due to (7) and (8), if $t > \tau$.
In both the cases, the set $\{\theta_i^j\}_{j=0,1,\dots,r_i+1}$ remains unchanged, δ^{**} is feasible and

$$\int_{h=0}^{T} c(h)\delta^*(h)dh = \int_{h=0}^{T} c(h)\delta^{**}(h)dh - \int_{h\in E_\tau} c(h)dh + \int_{h\in E_t} c(h)dh > \int_{h=0}^{T} c(h)\delta^{**}(h)dh \tag{9}$$

Relation (9) shows that δ^* is not optimal.

If $j+1 \le r_i$ and $t \in \cup_{k=j+1}^{r_i} U_i^k$, then the trajectory of $s^{**}(h)$ increases between the lower bound of E_τ and the upper bound of E_t, but remains less than or equal to m due to (7) and (8). Furthermore, if $t \in U^{k_i^*}$, $k^* \in \{j+1,...,r_i\}$, then $s^{**}(h) > 0$ for $h \in \{\theta_i^j,...,\theta_i^{k^*}\}$. As in the case $t \in U_i^j$, we see that (9) holds and thus δ^* is not optimal. Note that this part of proof vanishes if $r_i = 0$. This completes the proof.

Result 3 is important since it shows that if a solution is optimal and if the water level corresponding to this solution is maximal in some points in time then:
- If the water level is never equal to zero between two points, t_i and t_{i+1}, defined as follows:
(i) $t_i = 0$ or the water level is maximal at point t_i,
(ii) the water level is maximum at point t_{i+1} or $t_{i+1} = T$,
(iii) the inventory level is never maximal in (t_i, t_{i+1})
then the cost at a point in $[t_i, t_{i+1}]$ where pump is idle is always greater than the cost at a point in $[t_i, t_{i+1}]$ where replenishment occurs. This result is pictorially illustrated in figure 2.

- If the water level is maximal at some points of time $t_1 < t_2 < ... < t_n$ and minimal, i.e. equal to zero, at some other points in time, say $\theta_i^1 < \theta_i^2 < ... < \theta_i^{r_i}$ on intervals $[t_i, t_{i+1}], i = 0,1,...,n$, then if there is a replenishment at $t \in [\theta_i^j, t_{i+1}]$ and no replenishments at time $\tau \in [\theta_i^j, \theta_i^{j+1}]$, we have $c(t) < c(\tau)$.

The general case is illustrated in figure 3. For instance, if there is replenishments at time $t \in [\theta_1^1, t_2]$ and no replenishment at time $\tau \in [\theta_1^1, \theta_1^2]$, then $c(t) < c(\tau)$. Similarly, if there is replenishment at time $t \in [\theta_2^1, T]$ and no replenishment at time $\tau \in [\theta_2^1, \theta_2^2]$, then $c(t) < c(\tau)$. Also, if there is replenishment at time $t \in [\theta_2^2, T]$ and no replenishment at time $\tau \in [\theta_2^2, \theta_2^3]$ then $c(t) < c(\tau)$.

Following result shows, for an optimal solution the pumping cost for any interval where the pump is not operating between the two instants where the water level is minimal (i.e. $\theta_i^k, \theta_i^{k+1}$) is always costlier than (or as costly as) the pumping cost at any instant between θ_i^{k+1}, and θ_i^{k+2} or θ_i^{k+1}, and t_{i+1} where the pump is operating.

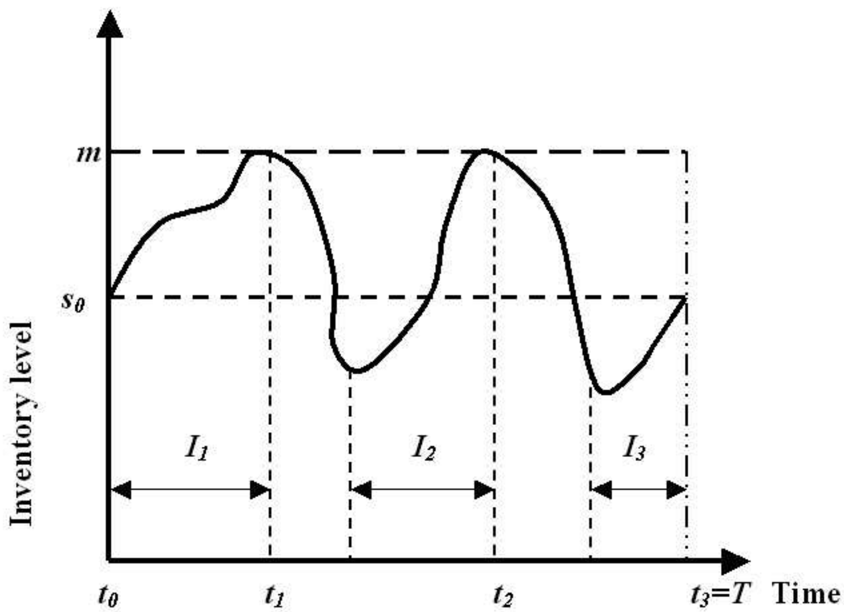

Figure 2: Case with no minima (I_1, I_2, I_3 represents the replenishment period)

Result 4
Consider a feasible solution δ^* defined as follows:

$$\delta^*(h) = \left\{ \begin{array}{lll} 1 & \text{if} & h \in U \\ 0 & \text{if} & h \in [0,T] \setminus U \end{array} \right\}$$

where U is the union of subsets of $[0,T]$, and that $q.|U| = \int_{t=0}^{T} g(t)dt$. As solution δ^* is feasible, the water level may be zero or maximum at some point of time. We use the same notations as in result 3 and introduce $\overline{U} = [0,T] \setminus U$. Then, for any $t \in U \cap [\theta_i^j, t_{i+1}]$ and any $\tau \in \overline{U} \cap [\theta_i^j, \theta_i^{j+1}]$, $i \in \{0,1,2,...,n\}$ and $j \in \{0,1,...,r_i\}$ if we have $c(t) < c(\tau)$ then the solution δ^* is optimal.

Proof
Derive δ^{**} from δ^* as shown in (6) for any pair $\{t,\tau\}$, where $t \in U \cap [\theta_i^j, t_{i+1}]$ and $\tau \in \overline{U} \cap [\theta_i^j, \theta_i^{j+1}]$. Since $q.|U| = \int_{t=0}^{T} g(t)dt$, the solution is feasible and cost increases. This completes the proof.

The algorithm presented in the next section is derived from the above results. It converges toward a solution that satisfies result 2 or/and result 4.

4 An optimal algorithm

The idea behind this algorithm is to start with continuous replenishment and then modify the solution step by step to converge towards the solution that satisfies result 2 or/and result 4.

Algorithm

1. Initialise the solution as follows:

$$\delta_1(t) = 1 \quad \forall \quad t \in [0,T] \text{ and set } U_1 = [0,T]$$

   δ_1 is an initial solution and we denote by $s_1(t)$ the corresponding water level. δ_1 may be infeasible.

2. Set $k = 1$.

3. Set $\varepsilon_k = q.max_{t \in U_k}(|E_t \cap U_k|)$ and $\eta_k = q.min_{t \in U_k}(|E_t \cap U_k|)$.
   E_t has been defined in the problem formulation. ε_k and η_k denotes respectively the maximal and the minimal decrement of the water level.

4. Define t_1 as follows:

   (a) t_1 is the smallest $t \in [0,T)$ such that $s_k(t) > m + \varepsilon_k$. If t_1 exists, go to 5.

   (b) If t_1 doesn't exist then define $t_1 = T$ if $s_k(T) > s_0 + \varepsilon_k$. Go to 5 if t_1 exists.

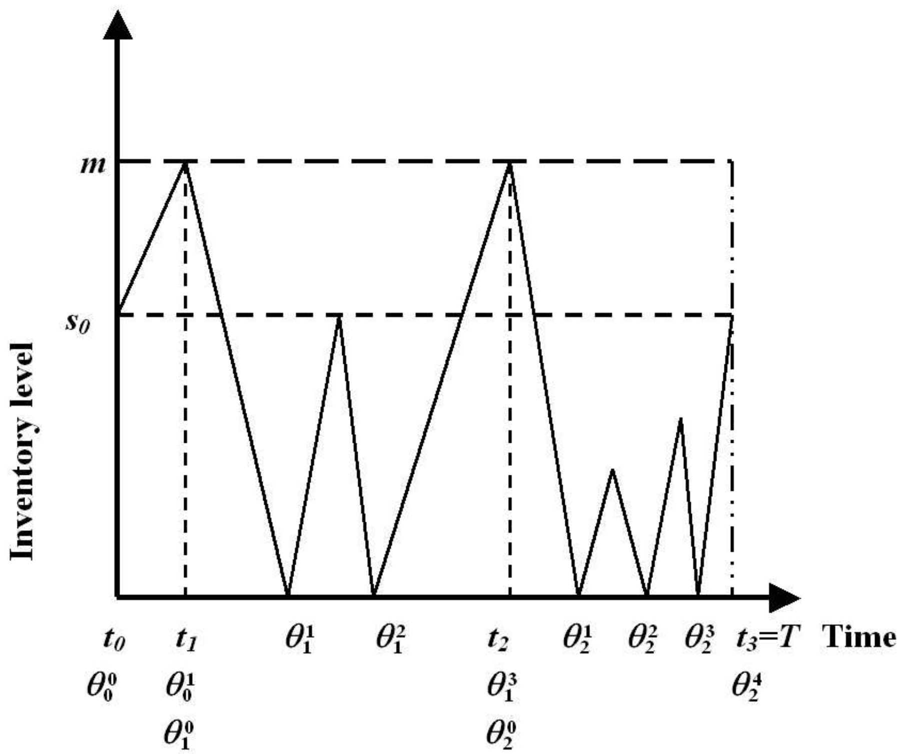

Figure 3: The general case

(c) If $s_k(t) > -\eta_k$ for any $t \in [0, T)$ and $s_k(T) > s_0 - \eta_k$ the optimal solution is obtained. Stop.

(d) Otherwise there is no solution to the problem. Stop.

5. Define θ_1 as follows:
θ_1 is the greatest $t \in [0, t_1)$ such that $s_k(t) < \eta_k$ or if such t doesn't exist then set $\theta_1 = 0$.

Steps 4 and 5 are derived from the results 2 and 4. The use of ε_k and η_k is required since the reduction of replenishment is made in closed intervals.

6. Define $Z = \{t \in (\theta_1 + \eta_k/q,\ t_1] \text{ s.t. } \delta_k(t) = 1\}$ and $\tau \in Z$ such that $c(\tau) = Max_{t \in Z} c(t)$.

7. Let E_τ denotes the closed interval corresponding to τ.
Now, set$F_\tau = E_\tau \cap Z$. If $|F_\tau| > \varepsilon_k/q$ then replace F_τ by a closed interval included in F_τ
For simplicity we call this new interval F_τ.

8. Define δ_{k+1} as follows:
$$\delta_{k+1}(t) = \left\{ \begin{array}{lll} 0 & \text{if} & t \in F_\tau \\ \delta_k(t) & \text{if} & t \notin F_\tau \end{array} \right\}$$

9. Set $k = k + 1$.

10. Set $U_k = \{t \mid \delta_k(t) = 1\}$

11. Go to 3.

According to result 2 and result 4, the solution obtained by the above algorithm, if any, can be as closed to the optimal as we want. Note that $U_1 \supset U_2 \supset\supset ... \supset U_i \supset ...$ As a consequence series ε_k and η_k are decreasing.

Explanation

In step 1 we initialise the algorithm by pumping the water in all periods $[0, T]$. Indeed this solution may not be feasible. Step 3 defines the minimum and the maximum decrement in the water level if the pump is off for a very small interval for the current solution. Step 4.a defines the instant t_1 at which the water overflows i.e. the water volume is greater than the tank's capacity. If water doesn't overflow then we check whether the water level at the last interval is same as the starting level. If greater than the starting level, we define t_1 as T. If t_1 doesn't exist this only means that either the optimal solution is obtained or any feasible solution doesn't exist (4.c and 4.d). Step 5 defines the first instant when the water level is minimum i.e. lower than the unit-interval-pumping volume that is the total quantity of water pumped during an elementary interval. It is obvious that we cannot stop pumping before this interval otherwise the tank will become empty. We select an instant after this point and before the point where tank overflows i.e. (t_1) and such that the cost is very high (step 6). In step 8 we shut down the pump during this interval and in step 9 and 10 we define the new solution by incorporating the new pump schedule. We repeat the process until we reach the optimal solution, if exists (step 4).

Complexity

From step 8 of the algorithm we can see that at each iteration the pump is stopped for one elementary interval. Since we have divided the total horizon into N periods, we have total of N elementary intervals. Let $\psi = T/(N)$ denotes the length of the one elementary interval and R denotes the number of intervals necessary for pumping operation to satisfy the demand i.e. $R = \lceil \int_{t=0}^{T} g(t)dt/(q.\psi) \rceil$, where $\lceil a \rceil$ represents the smallest integer number greater or equal to a; and thus the maximum number of iterations required by the algorithm are $(N - R)$. We can see that only step 3 to step 10 involves in iteration. The complexity of each step (3 to 10) is on the order of $O(N)$ as all the operations performed by these steps required to either compute the water level ($s(t)$) or cost ($\delta(t)$) and we have maximum of N intervals to compute. Therefore the complexity of the full algorithm is $O(8 \times (N) \times (N - R)) \approx O(8(N)^2)$.

5 Numerical illustration

In this section we present a numerical example to present the approach. We consider this example for the horizon $T = 10$. The pumping rate is 600 volume units per time unit. The capacity of water tank is 1000 volume units and it the initial water level is 800 volume units. The demand and the cost functions are as follows:

$$\text{Demand} \quad \phi(t) = 300[cos(\pi.t/2.5) + 1] \text{ for } \quad t \in [0, T]$$

$$\text{Cost} \quad c(t) = 240[cos(\pi t/2.5) + 1] \text{for} \quad t \in [0, T]$$

From the above functions we can see that cost and demand evolve similarly. Both are maximum around 0, 5 and 10 and minimum around 2.5 and 7.5. We first divide period $[0, T]$ into N elementary periods and assumed that the pump is operating or not within these periods. In this example we divided the horizon into 100 elementary periods. On applying the algorithm we obtained operating cost of the pump corresponding to the given consumption profile, which is 684. Table 1 presents the state of the pump during 100 elementary periods. 1 represents the pumping operation and zero represent the idleness of the pump. The $k - th$ position corresponds to $[0.1.(k - 1), 0.1.k]$ interval.

Table 1: State of the pump

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 1 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 0 1 1 1 0 0 1 1 1 1 1 0 0

Table 2: Optimal sequence of reservoir contents

	1	2	3	4	5	6	7
0.	800.00 (0)	740.08 (0)	680.63 (0)	622.12 (0)	564.99 (0)	509.68 (0)	456.58 (0)
1.	406.05 (0)	358.43 (0)	313.99 (0)	272.95 (0)	235.50 (0)	201.74 (0)	171.74 (0)
2.	145.50 (0)	122.95 (0)	103.99 (0)	**88.43 (1)**	136.05 (1)	186.58 (1)	239.68 (1)
3.	294.99 (1)	352.12 (1)	410.63 (1)	470.08 (1)	530.00 (1)	589.92 (1)	649.37 (1)
4.	707.88 (1)	765.01 (1)	820.32 (1)	873.42 (1)	923.95 (1)	971.57 (0)	956.01 (1)
5.	997.05(0)	974.50 (0)	948.26 (1)	978.26 (0)	944.50 (0)	907.05 (1)	926.01 (1)
6.	941.57 (0)	893.95 (0)	843.42 (0)	790.32 (0)	735.01 (0)	677.88 (0)	619.37 (0)
7.	559.92 (0)	500.00 (0)	440.08 (0)	380.63 (0)	322.12 (0)	264.99 (0)	209.68 (0)
8.	156.58 (0)	106.05 (0)	58.43 (1)	73.99 (0)	**32.95 (1)**	55.50 (1)	81.74 (0)
9.	51.74 (1)	85.50 (0)	62.95 (0)	43.99 (1)	88.43 (1)	136.05 (1)	186.58 (1)
10.	239.68 (1)	294.99 (1)	352.12 (1)	410.63 (1)	470.08 (1)	530.00 (1)	589.92 (1)
11.	649.37 (1)	707.88 (1)	765.01 (1)	820.32 (1)	873.42 (1)	923.95 (1)	971.57 (0)
12.	956.01 (0)	937.05 (1)	974.50 (0)	948.26 (0)	918.26 (1)	944.50 (1)	967.05 (1)
13.	986.01(0)	941.57 (0)	893.95 (1)	903.42 (1)	910.32 (1)	915.01 (1)	917.88 (1)
14.	919.37 (0)	859.92 (0)	800.00				

6 Initial reservoir contents

The data used in this example are the same as the one used in section 5. Only capacity (m) and initial inventory (s_0) are different. They evolve as showed in Table 3. This table provides the optimal cost for each pair (m, s_0).

Table 3: Cost as a function of the pair (m, s_0)

Capacity ⇒ / Initial inventory⇓	500	600	700	800	900	1000	1100	1200
100	1295.37	1154.44	1043.05	966.26	920.89	892.67	885.79	885.79
200	<u>1284.71</u>	1133.72	1009.14	913.12	856.98	828.76	821.89	821.89
300	1285.28	<u>1126.67</u>	<u>993.00</u>	883.63	808.93	776.88	770.01	770.01
400	1297.03	1133.72	993.81	<u>874.38</u>	<u>786.50</u>	740.05	733.18	733.18
500		1154.76	1011.00	883.95	786.98	<u>727.19</u>	<u>714.81</u>	714.81
600			1045.43	913.67	810.46	740.77	715.15	<u>713.44</u>
700				966.24	859.15	778.28	733.72	715.15
800					922.90	830.61	770.43	733.72
900						894.35	822.20	770.43
1000							885.66	822.20
1100								885.66

As the reader can see, the optimal cost decreases as the capacity of the reservoir increases, but more and more slowly. The costs corresponding to the best initial content are underlined. Furthermore, the optimal initial content is equal to half the capacity of the reservoir: this is due to the shapes of cost and consumption

7 Conclusion

In this paper, we presented the case of water management problem when consumption profile is known during a given horizon (a day). The goal is to reach the initial water level in the tank at the end of the horizon. It will, at least, assure that for the same water consumption profile we don't need to run the algorithm again. Our assumption of minimum water level (zero) can be easily changed to any positive value if the safety stock is also the requirement. The problem is not new and can easily be formulated as binary integer programming or dynamic programming, but the problem structure revealed certain properties which allow us to solve this problem easily without using DP and IP. The algorithm is helpful in the case when the horizon is long and is divided into a huge number of intervals. We proved that the algorithm come up with an optimal solution under the limitation of descritization. Furthermore, since the algorithm is fast, it can be further used to find optimal initial water level of tank corresponding to given consumption profile. Indeed, the same approach also applies when the final reservoir content is different from the initial one. Further research will focus on the definition of the consumption in case of drought. The pivotal question, in this case, is the rationing policy associated with the cost.

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Appendix

The discrete version of this problem can easily be formulated as binary integer programming problem as follows:

$$\text{Minimize} \quad \sum_{k=1}^{N} c(k)\delta_k$$

Subjected to:

$$\sum_{k=1}^{N} \delta_k = R$$

$$s(k) = s(k-1) + q.\delta_k - g(k) \ \forall \ k = 1, 2, ..., N$$

$$0 \geq s(k) \geq m$$

$$\delta_k = \{0, 1\}$$

where R is the smallest integer greater than equal to $\int_{t=0}^{T} g(t)dt/(q.\psi)$.

The above formulation shows that this problem has $2N+1$ constraints with N binary variables. Small version of this problem can easily be solved using Branch and Bound algorithm. Optimum solution of discrete version is generally different from the optimal solution of corresponding continuous problem and the only way to achieve the solution close to continuous problem is to make the intervals as small as possible i.e. to increase the N. The worst-case complexity of the branch and bound algorithm is 2^N.

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